
SURPRISES IN VACUUM DECAY:
AN UPDATE
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PLAN

I Vacuum decay : Euclidean approach

- Pseudo-bounces

II Surprises : From 0 to 2n+1 bounces

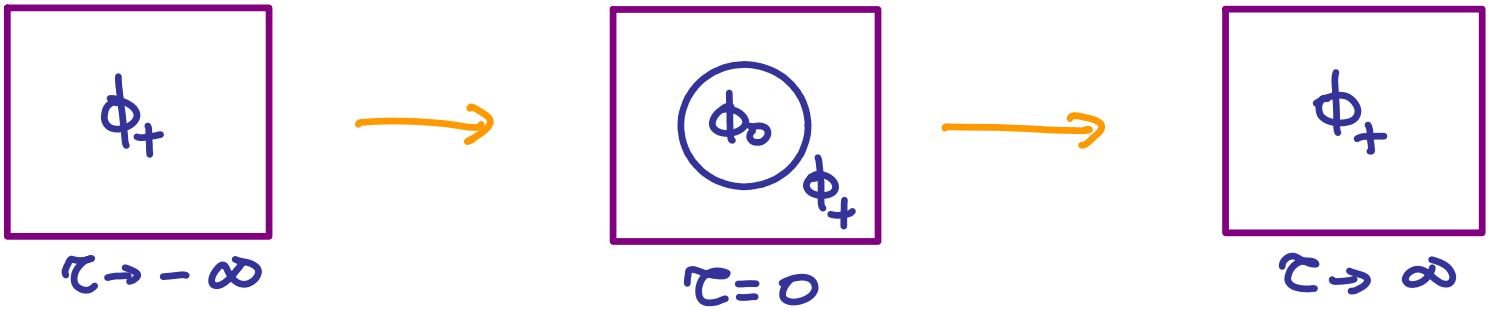
III Which solutions describe decay ? New!

IV Which potentials have more than one bounce ? New!

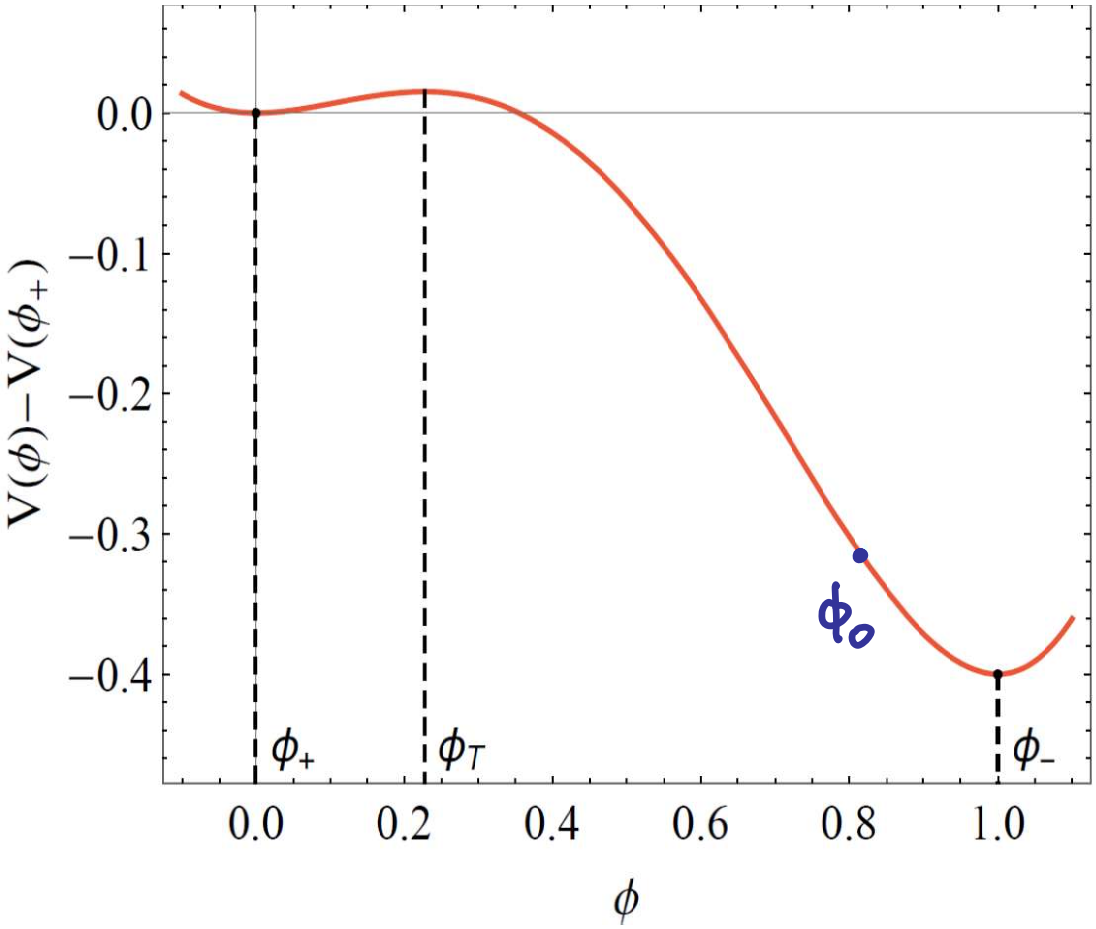
I Vacuum Decay



Euclidean approach



THE PROBLEM



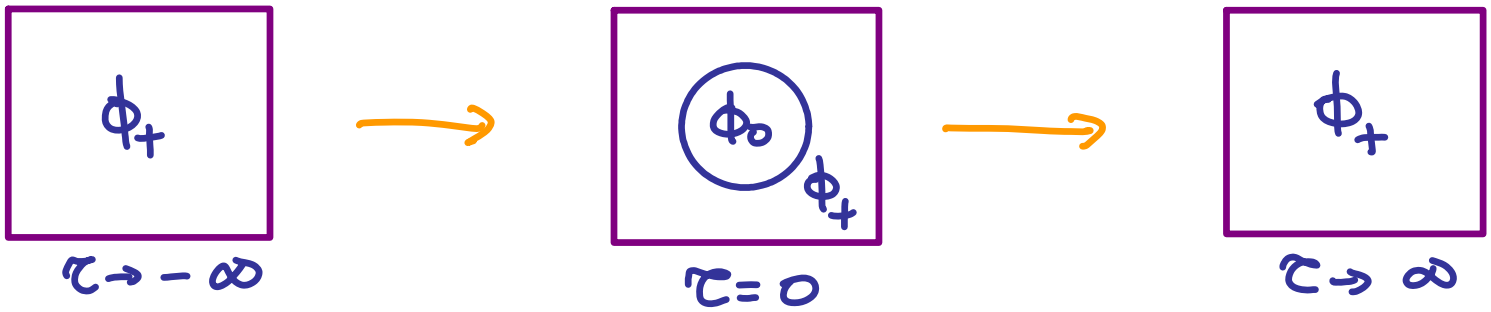


$$\Gamma/V = A e^{-S/\hbar}$$

EUCLIDEAN FORMALISM

Coleman '77

Euclidean bounce $\phi_B(\vec{x},\tau)$ extremizes S_E



EUCLIDEAN FORMALISM

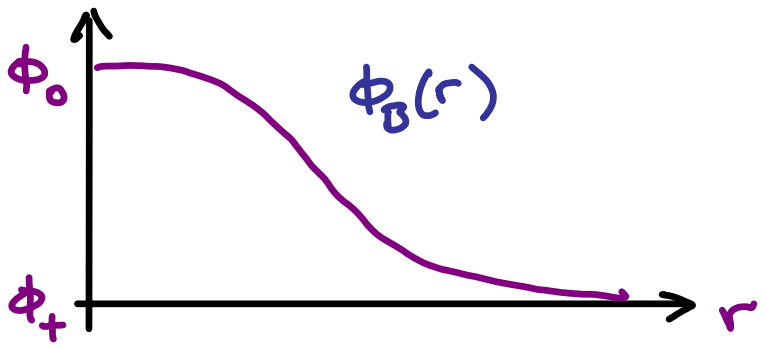
Coleman '77

Euclidean bounce $\phi_B(r)$, O(4)-sym, extremizes

$$S_E = 2\pi^2 \int_0^\infty dr\, r^3 \left[\frac{1}{2}\dot{\phi}^2 + V(\phi) \right]$$

$$\Rightarrow \quad \ddot{\phi} + \frac{3}{r}\dot{\phi} = \frac{\partial V}{\partial \phi}$$

with $\phi(0) = \phi_0 \quad \dot{\phi}(0) = 0 \quad \phi(\infty) = \phi_+$



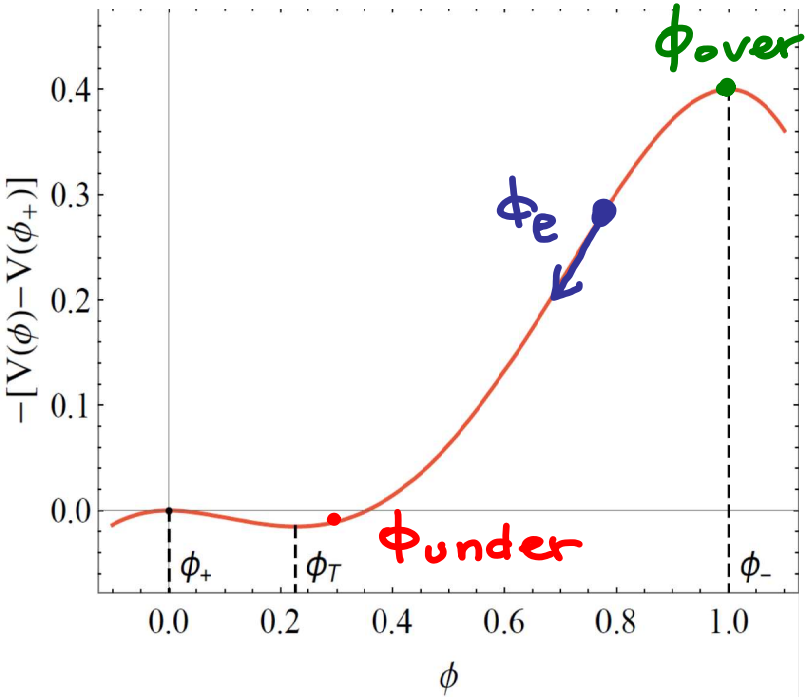
$$S = \Delta S_E \equiv S_E[\phi_B] - S_E[\phi_+]$$

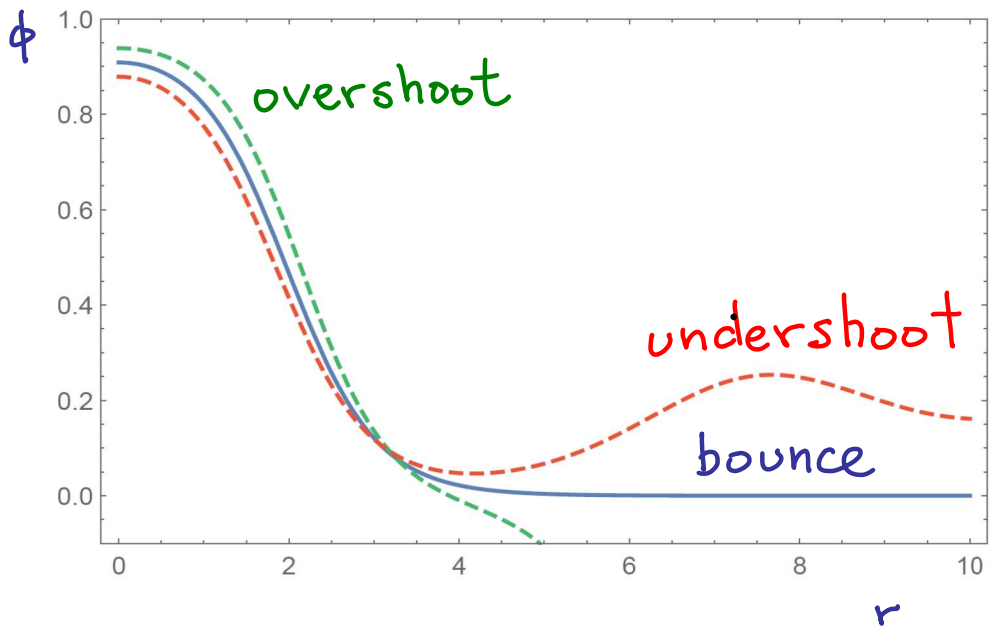
OVER/UNDERSHOOTING

$\phi \rightarrow$ particle's position $\qquad r \rightarrow$ time

$$\ddot{\phi} + \frac{3}{r}\dot{\phi} = +V'$$

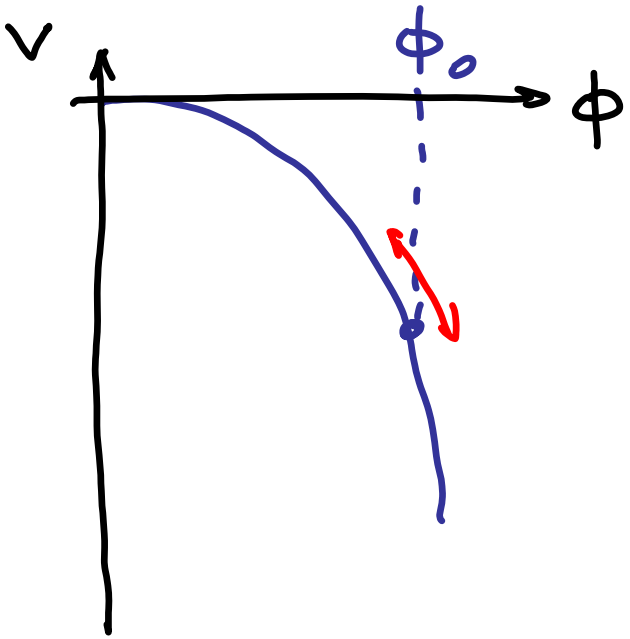
motion on $-V$ with time-dependent friction





Algorithm: If undershot, $\phi_e \uparrow$. If overshot, $\phi_e \downarrow$.

$$V(\phi) = -\frac{\lambda}{4!}\phi^4$$



$\Rightarrow$ ϕ_0 arbitrary

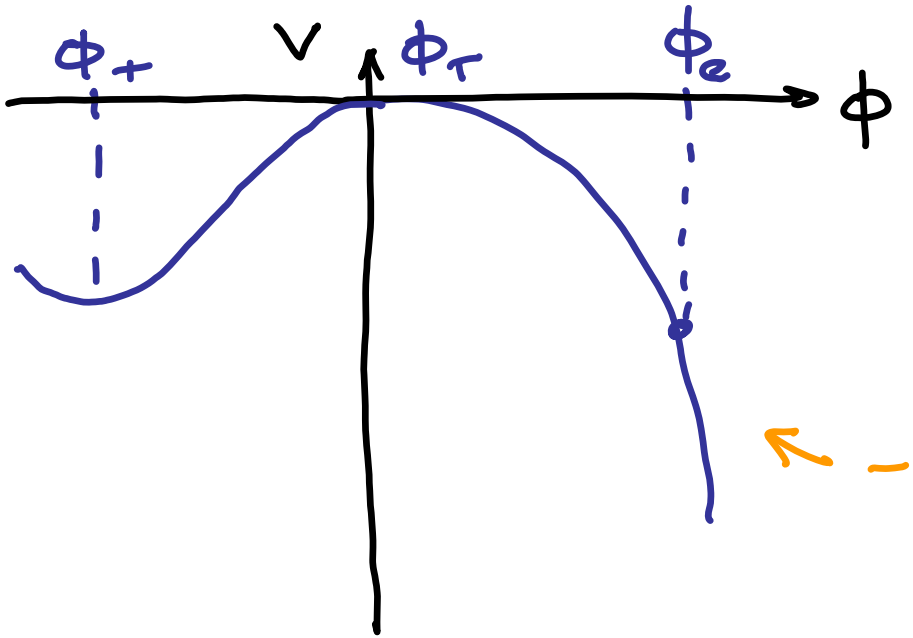
scale invariance of V

$$S[\phi_B] = \frac{16\pi^2}{\lambda}$$

$\Rightarrow$ ϕ_0 - independent

$\Rightarrow$ one-parameter (ϕ_0) family of bounces with same action

NO BOUNCE

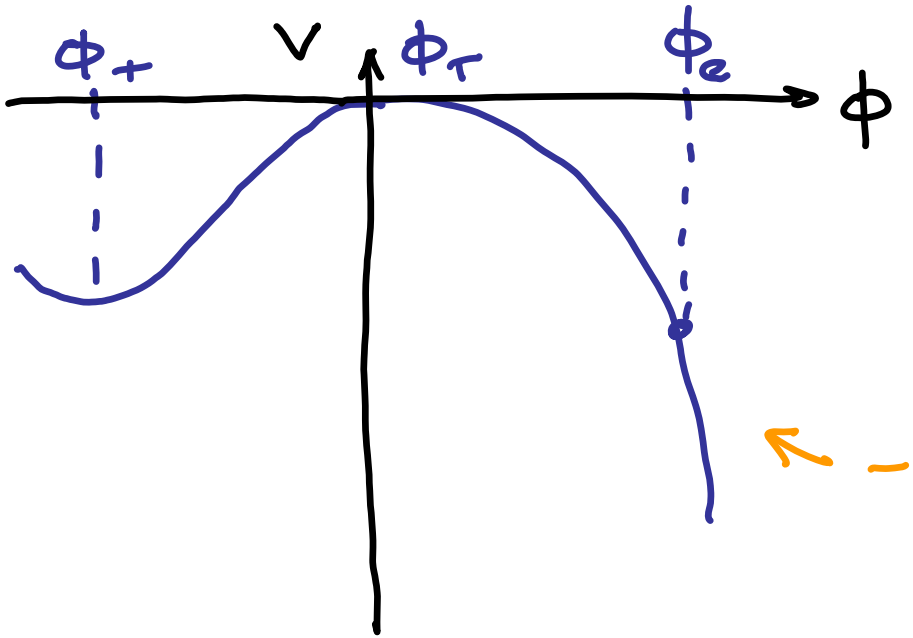


$\leftarrow -\frac{\lambda}{4!}\phi^4$ piece

For any $\phi_e \rightarrow \phi(\infty) = \phi_T$ not ϕ_+ (all undershots)

$\Rightarrow$ No bounce ! $\Rightarrow$ No decay ?

NO BOUNCE



$\nwarrow -\frac{\lambda}{4!}\phi^4$ piece

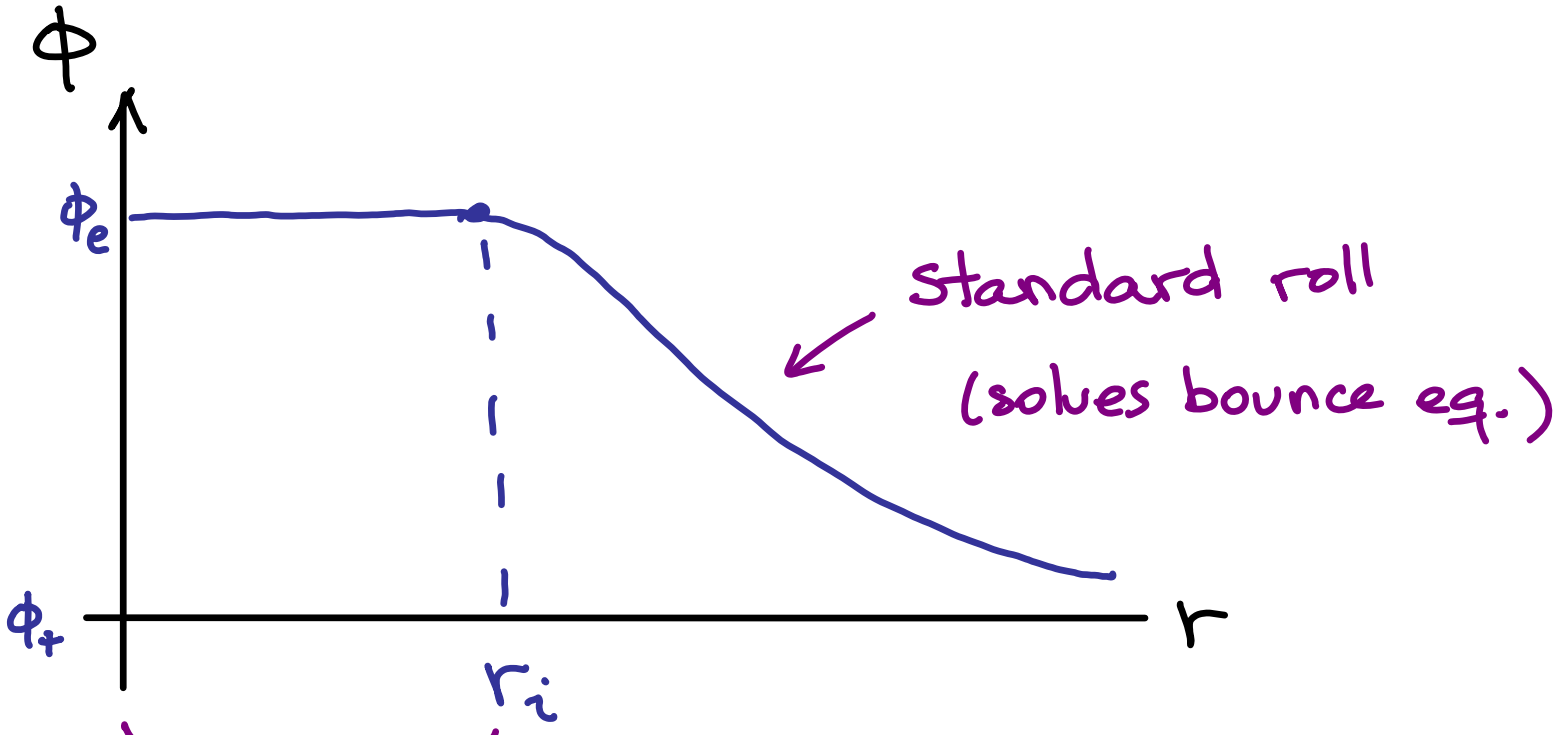
For any $\phi_e \rightarrow \phi(\infty) = \phi_T$ not ϕ_+ (all undershots)

$\Rightarrow$ No bounce !

Decay via pseudo-bounces

JRE '19

PSEUDO-BOUNCE PROFILE

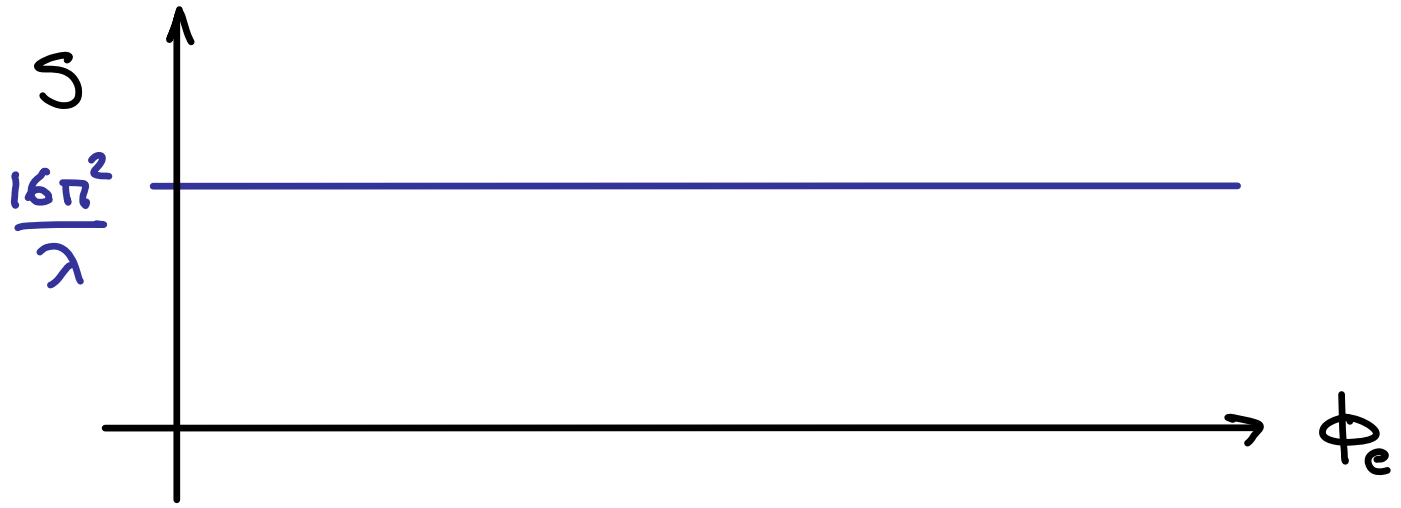


Waiting "time" to decrease friction & avoid undershot. Tuned to give $\phi(\infty) = \phi_+$

- Do for different ϕ_e : family of pseudo-bounces.

PSEUDO-BOUNCES AND DECAY VALLEYS

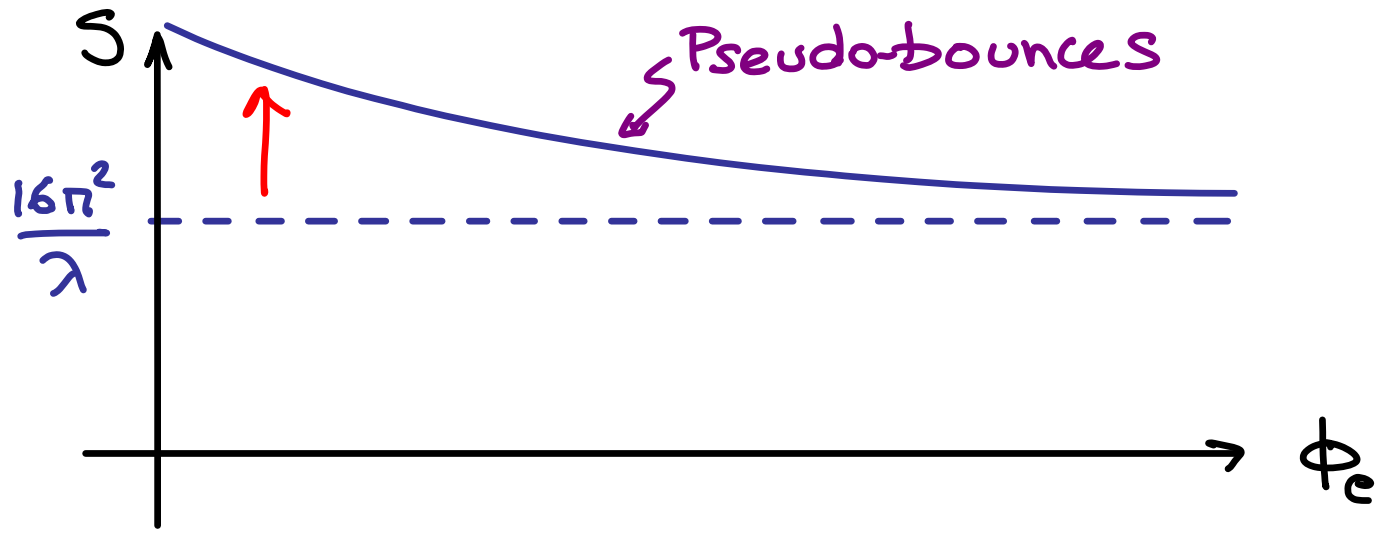
For $V = -\frac{\lambda}{4!}\phi^4$



Flat direction of $S[\phi]$ in config. space

PSEUDO-BOUNCES AND DECAY VALLEYS

For $V = -\frac{\lambda}{4!}\phi^4 + \frac{1}{2}m^2\phi^2$



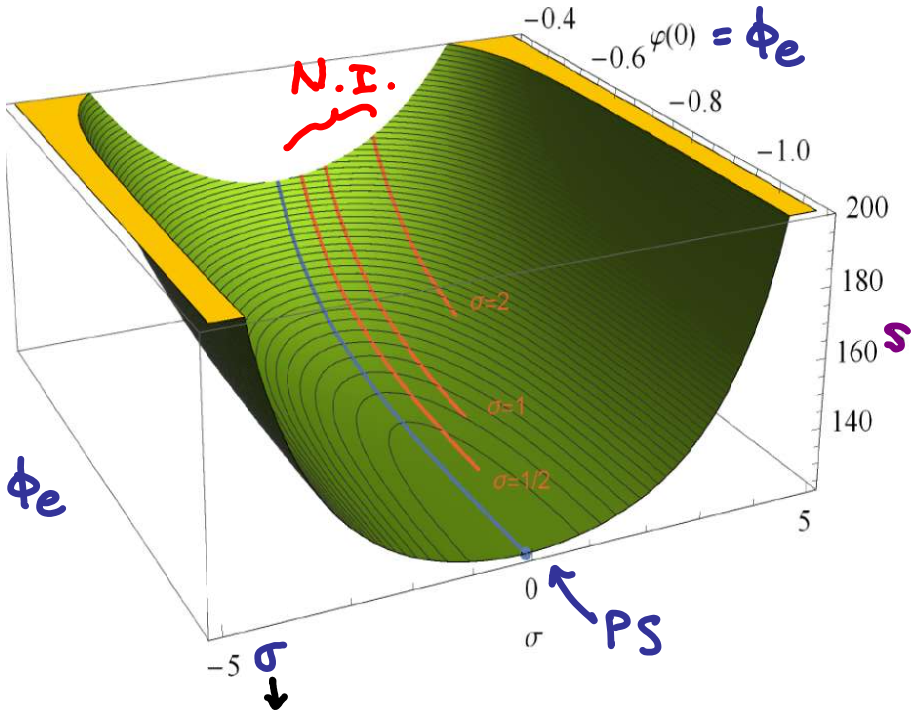
Flat direction of $S[\phi]$ in config. space

lifted to a sloping valley

→ Bounce is pushed to $\phi_e \to \infty$

PSEUDO-BOUNCES TRACK DECAY VALLEYS

Example:



parameter independent of ϕ_e

JRE, Huertas '21
vs
Mukharov et al. '20

Pseudo-bounces = O(4) Field configurations $\phi(r)$ that minimize S for fixed $\phi(0)$

PSEUDO-BOUNCE PROPERTIES

- Not solutions of bounce eq. $\ddot{\phi} + \frac{3}{r}\dot{\phi} = \frac{\partial V}{\partial \phi}$

  ➔ Not extremals of $S[\phi]$

  PS sit on a sloping valley ✓

$$\frac{dS}{d\phi_e} = \frac{\pi^2}{2} r_i^4 V'(\phi_e)$$

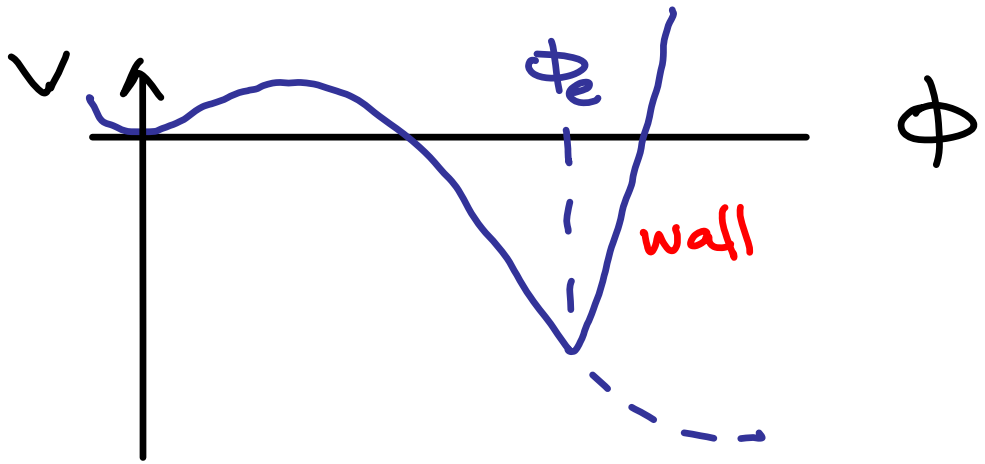
- Their $\chi = 0$ slice (nucleated bubble) has

$$E = 4\pi \int_0^\infty dr\, r^3 \left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] = 0 \quad ✓$$

Proper final state for decay

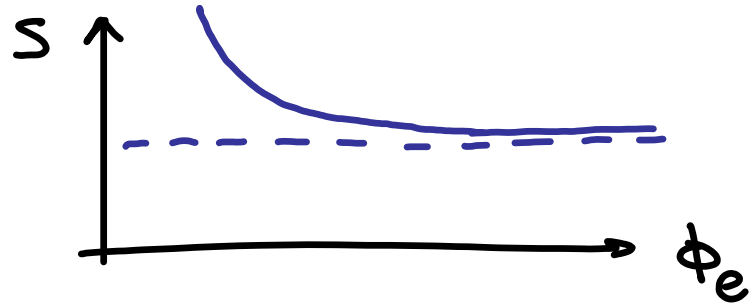
PSEUDO-BOUNCE PROPERTIES

- Origin of their good properties
  - They would be proper bounces for

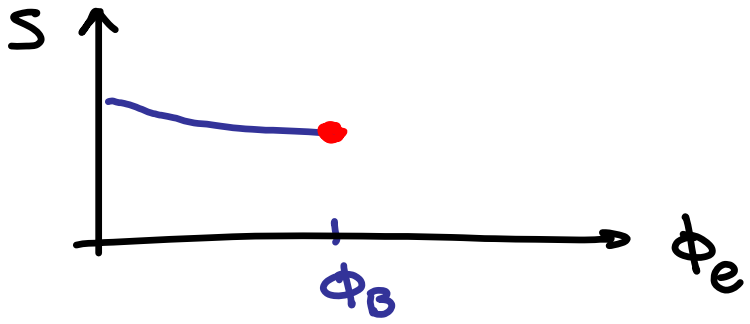


USES OF PSEUDO-BOUNCES

- cases w/o bounce



- With bounce, when flattish S valleys:



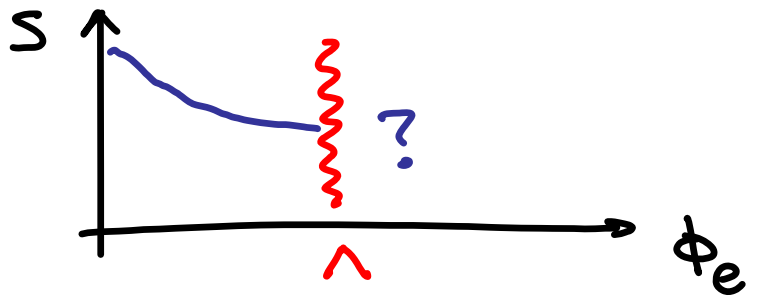
SM ?

JRE, A. Rajantie

- Bounce beyond EFT:

upper bound on S_{Bounce}

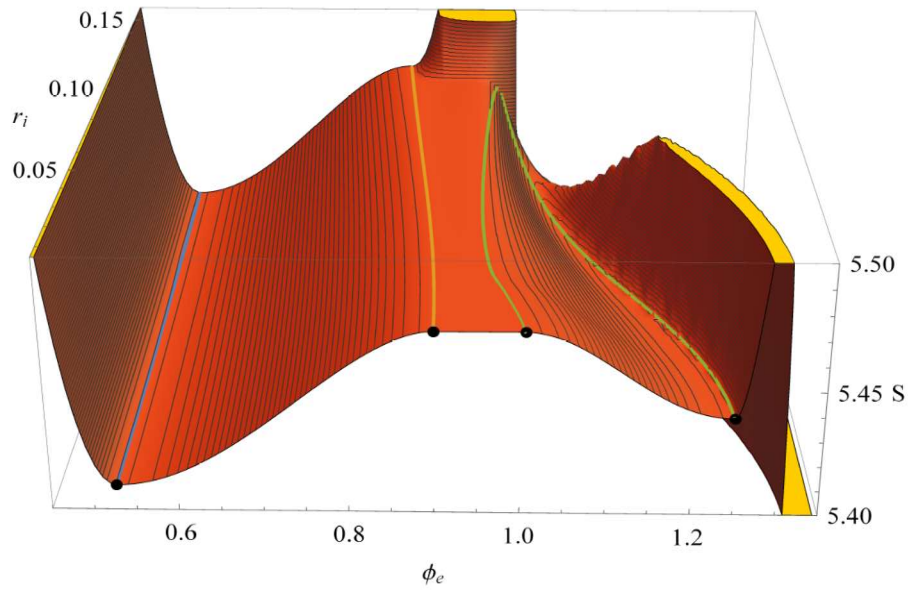
Mirbabayi, Villadoro '25



- Tool to study general decay structure

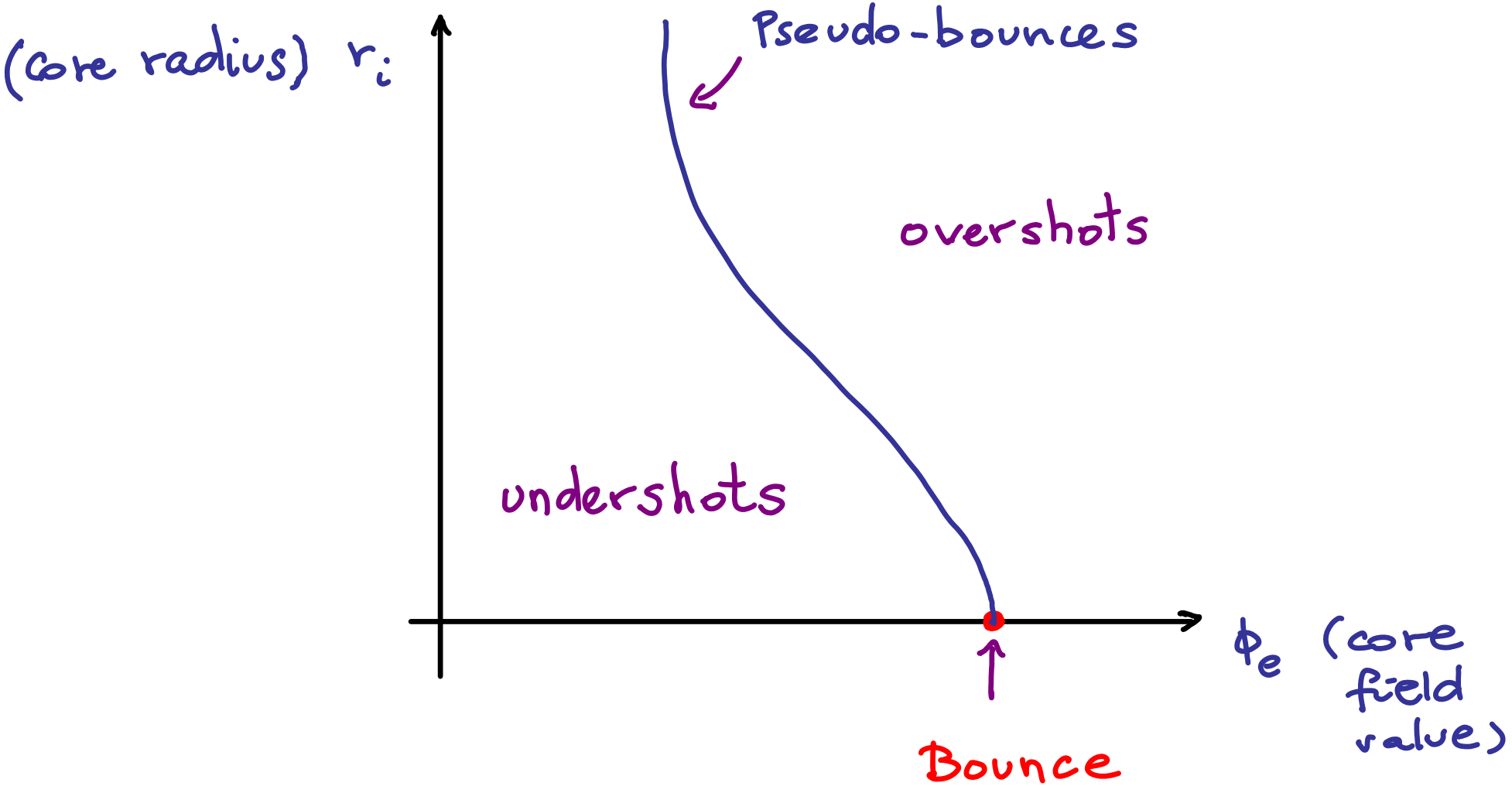
II Surprises

From 0 to 2n+1 bounces



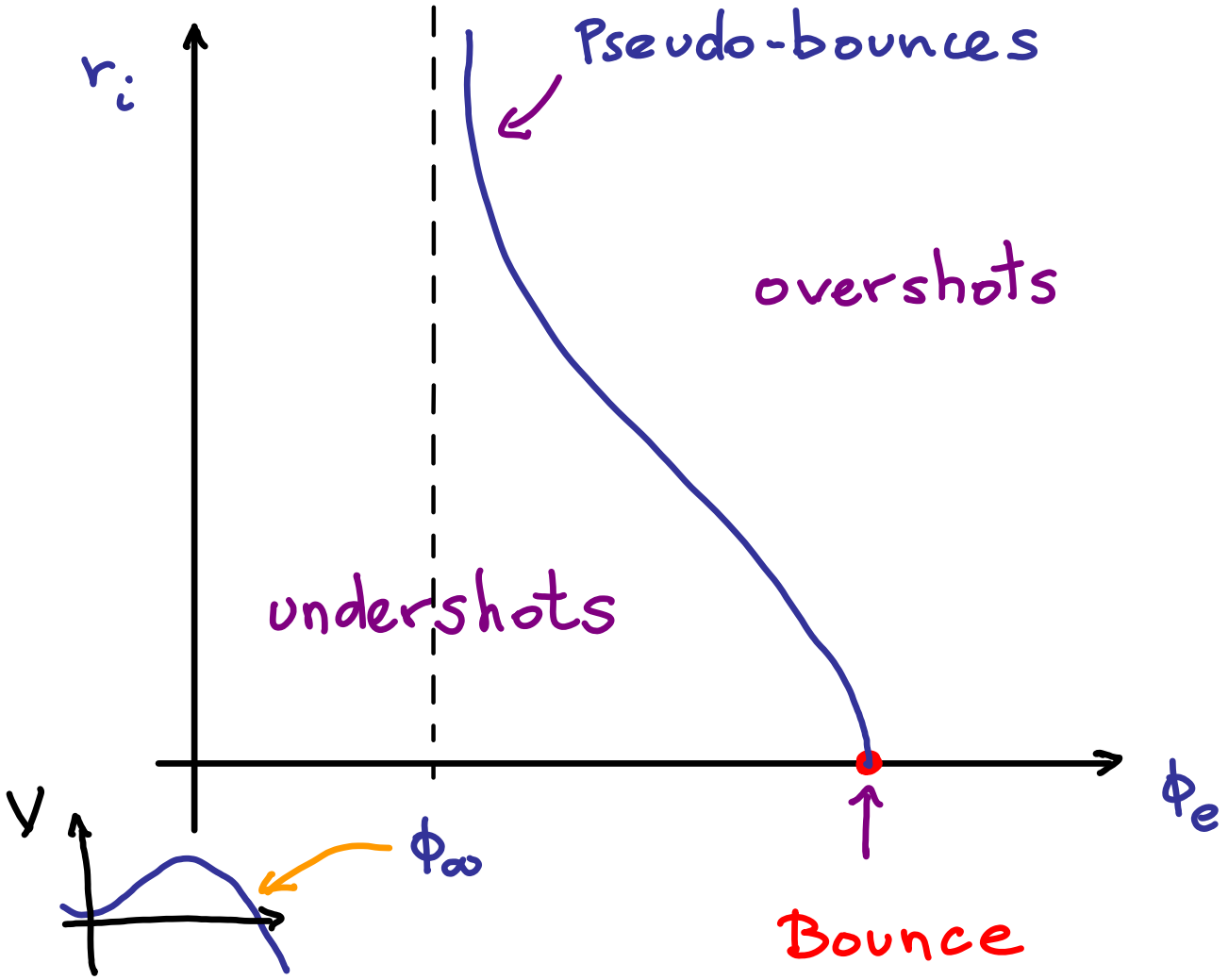
STANDARD CASE

Pseudo-bounces in (ϕ_e, r_i) plane :



STANDARD CASE

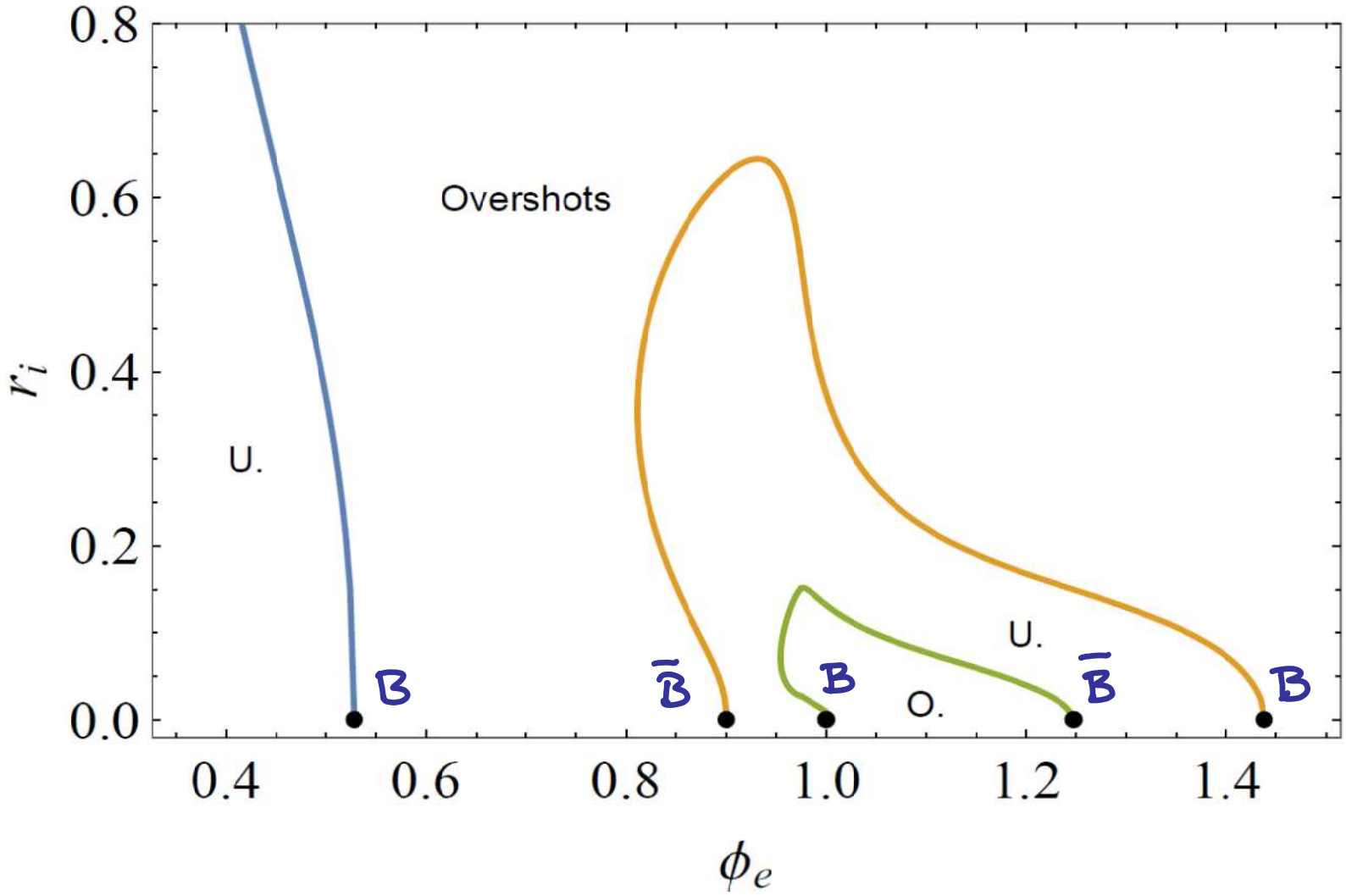
Pseudo-bounces in (ϕ_e, r_i) plane :



NON UNIQUENESS OF THE BOUNCE

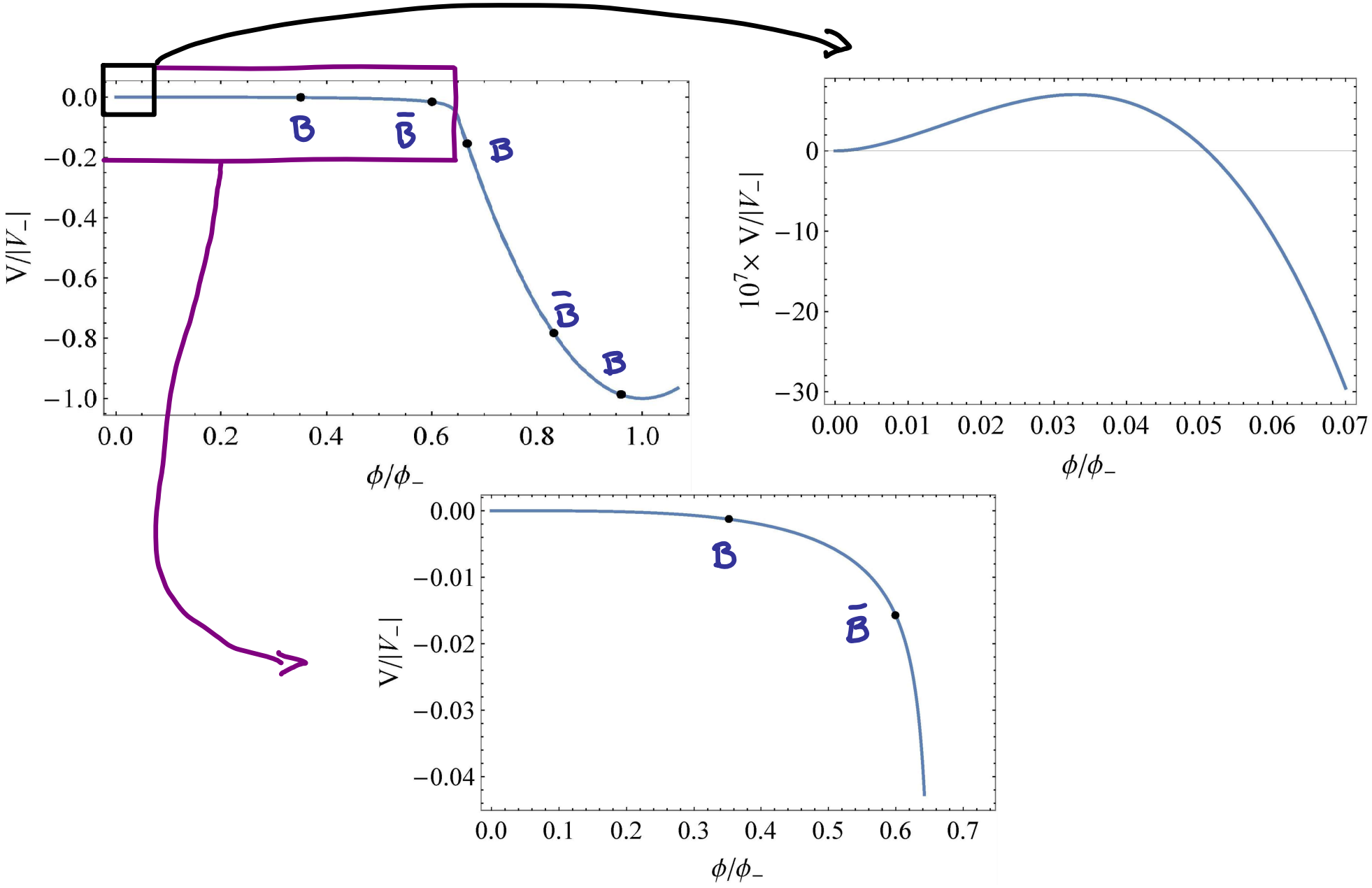
JRE, Konstandin '25

Some potentials have a richer decay structure :



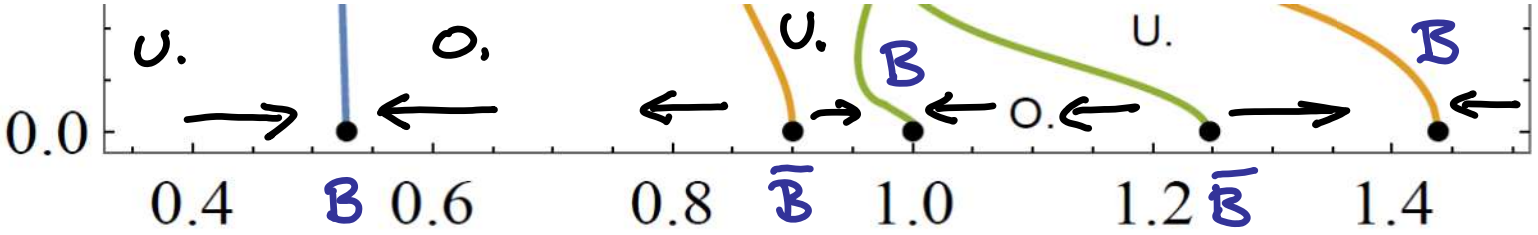
THE POTENTIAL

JRE, Konstandin '23 '25



ANTI BOUNCES

- missed in standard searches Mühlleitner et al '24



How are they possible ??

U: $\phi_e \to \phi_e + \Delta\phi$ ⇨ ϕ_e reached at higher r

⇩

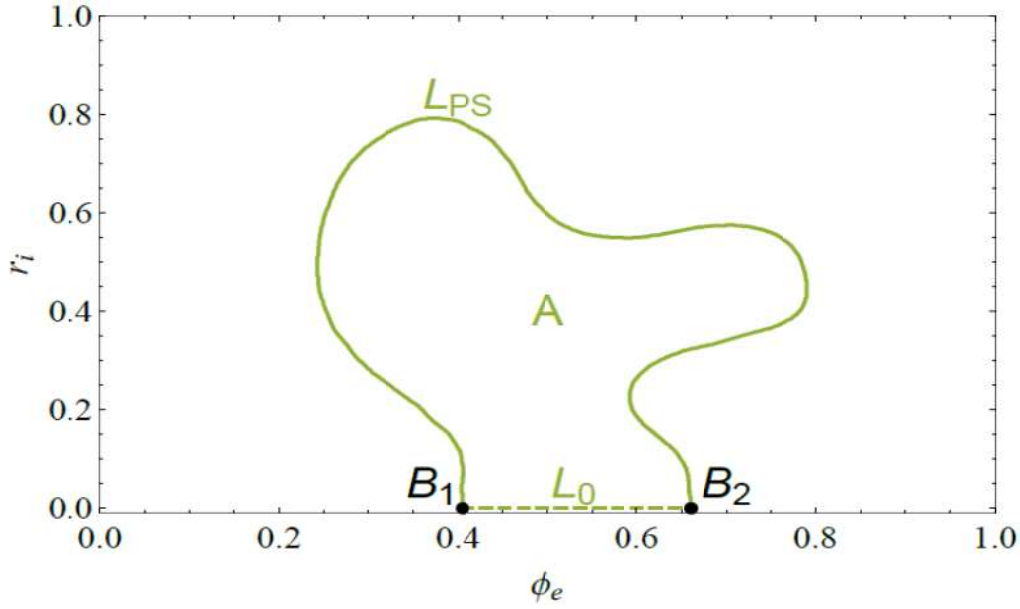
less friction $\sim \dot{\phi}/r$

But higher $\dot{\phi}$ can compensate r ⇨ Steep V required

Open question: How steep V should be? $(-V'')$

GENERAL STRUCTURE AND ACTION

- $2n+1$ bounces, with n $B\bar{B}$ pairs joined by pseudo-bounce lines



Using $\frac{dS}{d\phi_e} = \frac{\pi^2}{2} r_i^4 V'(\phi_e)$ and Stoke's theorem

$$S_{B_2} < S_{B_1}$$

GENERAL STRUCTURE AND ACTION

JRE, Konstandin '25

- For disconnected B_i, B_j no general proof $S_i \gtrsim S_j$

Typically S lower for higher ϕ_e

⇨ Lowest action for bounce closer to minimum

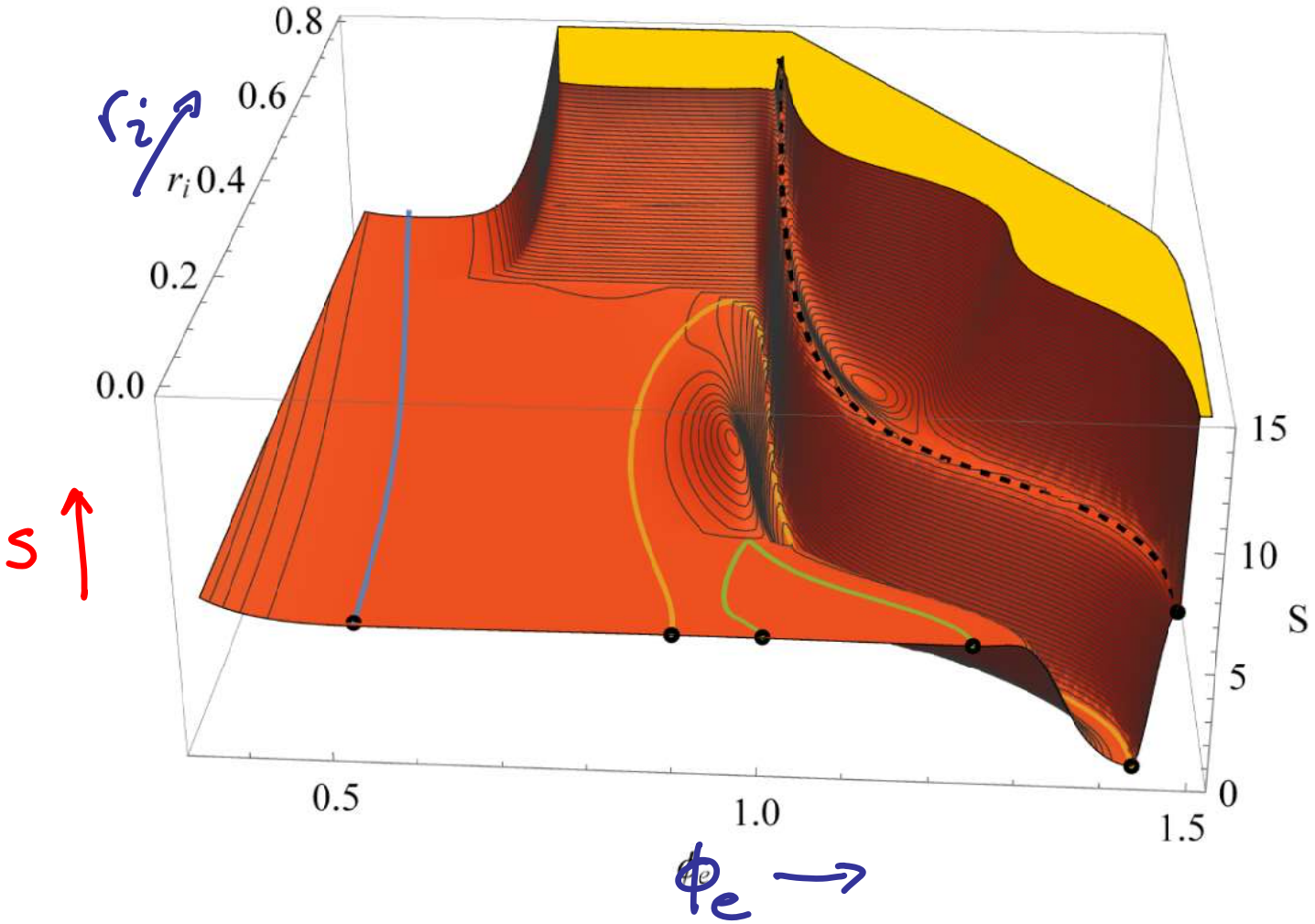
Exceptions ? (YES)

Open question

Can antibounces dominate decay? ... (NO)

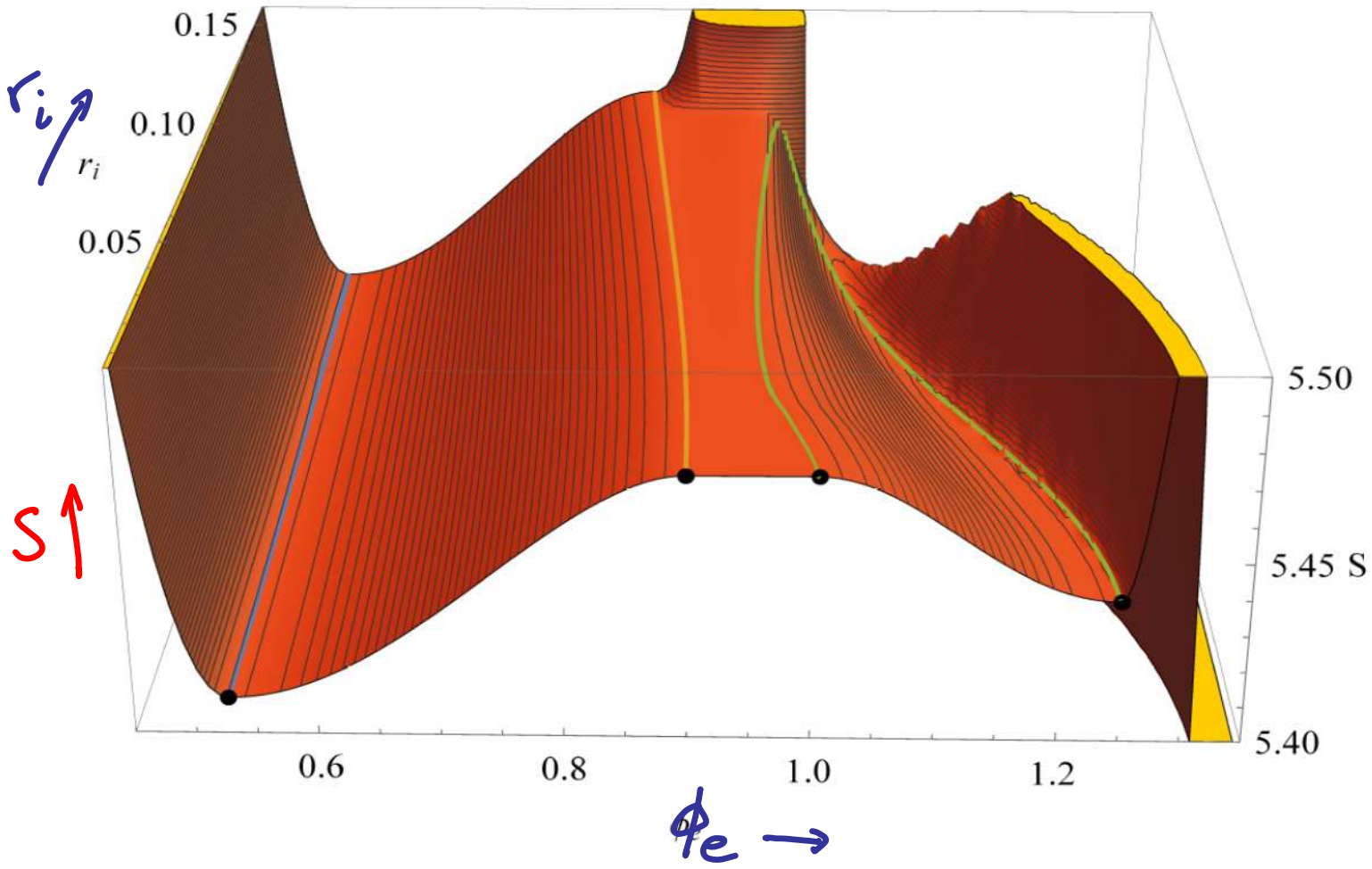
DECAY LANDSCAPE

JRE, Konstandin '25

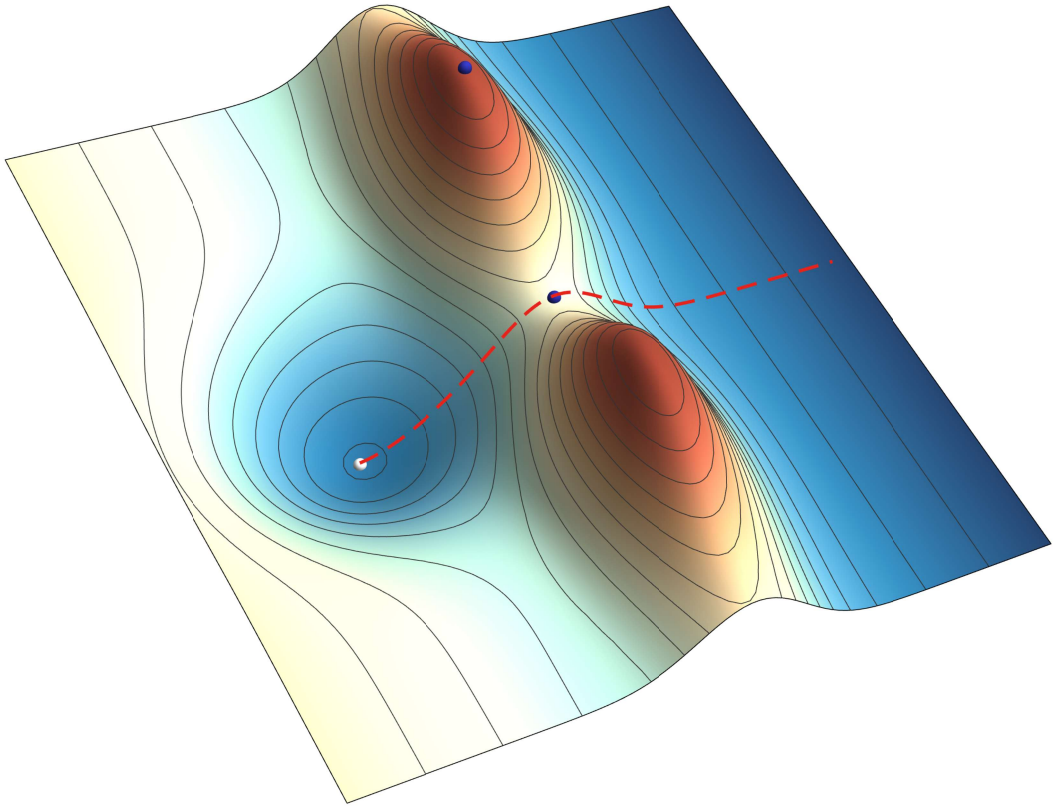


DECAY LANDSCAPE

JRE, Konstandin '2506



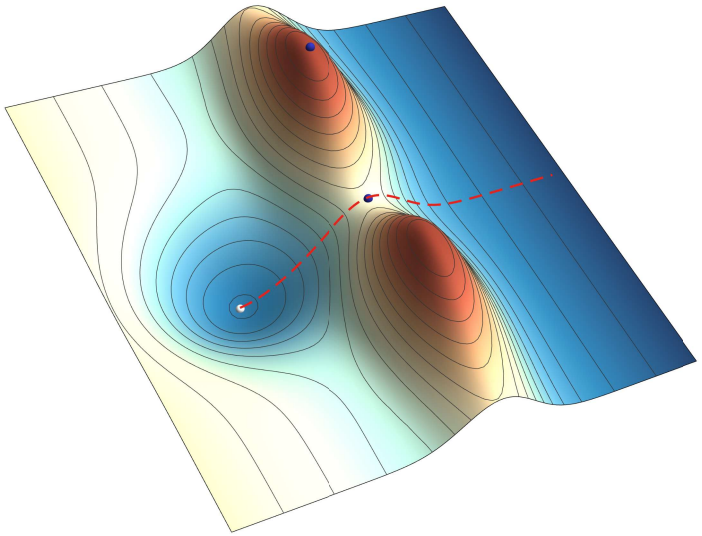
III Which solutions describe decay ?



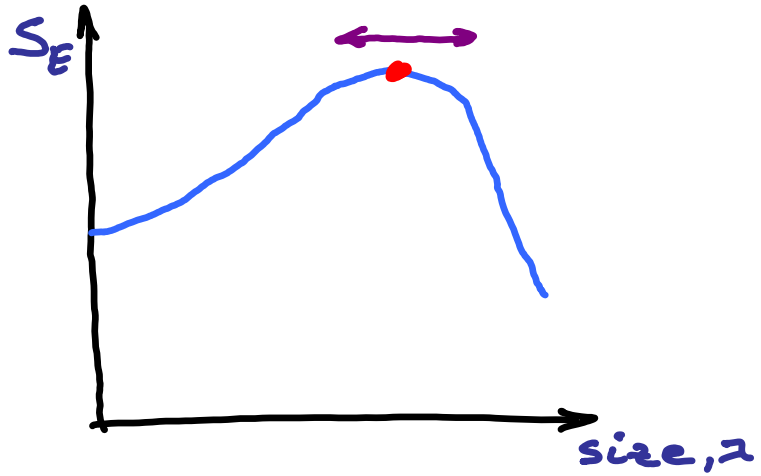
JUST ONE NEGATIVE MODE Coleman '88

Only saddles with one negative mode describe decay

Decay = crossing the mountain pass



Every saddle has at least one negative mode



$\phi(r/\lambda)$: $S_E = \lambda^2 S_T + \lambda^4 S_V$

COUNTING NEGATIVE MODES

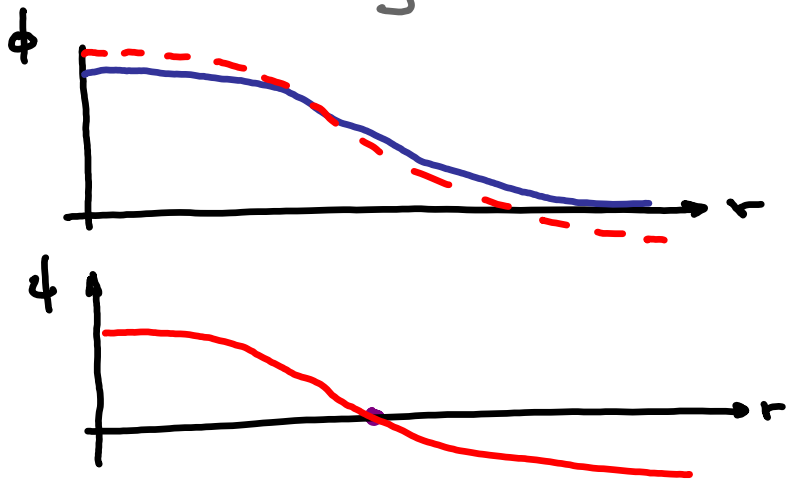
Shoot with slightly different ϕ_e

$$\psi \equiv \partial\phi/\partial\phi_e$$

solves $\ddot{\psi} + \frac{3}{r}\dot{\psi} = V''(\phi_B)\psi$ $\qquad \psi(0)=1\,,\ \dot{\psi}(0)=0$

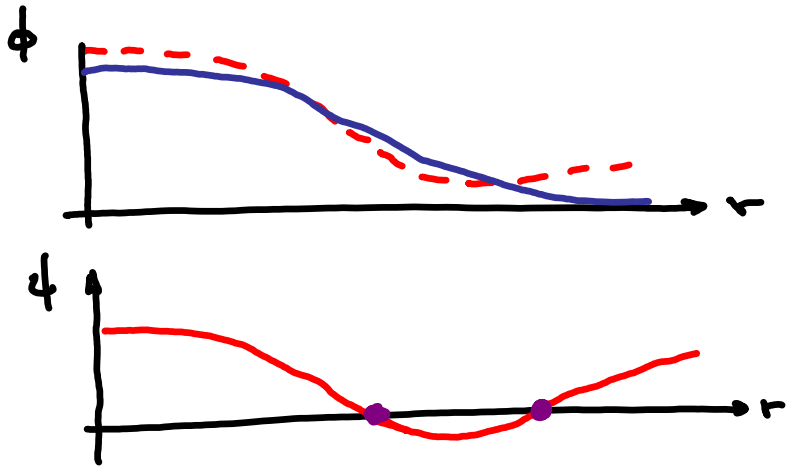
$N_- \equiv$ # negative modes of ϕ_B = # zeros of ψ

B: start higher: overshot



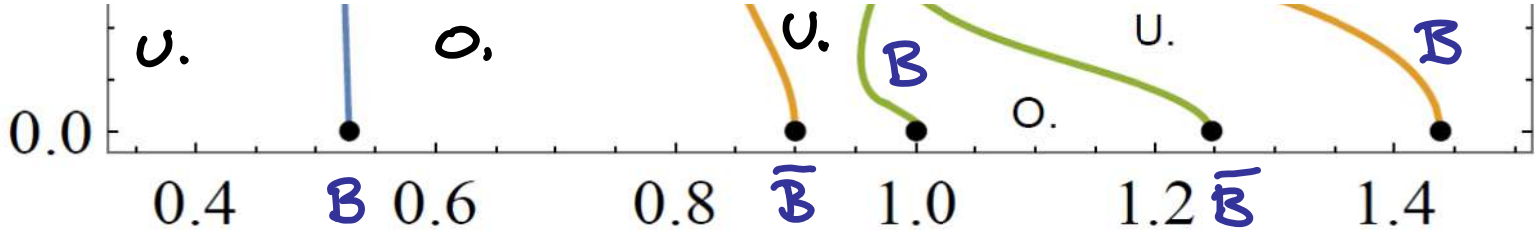
B : N_- odd

$\bar{B}$: start higher : undershot



$\bar{B}$: N_- even

COUNTING NEGATIVE MODES



$N_- =$ 1 2 3 2 1

★ Antibounces never relevant for decay

★ Neighboring solutions differ by $\Delta N_- = \pm 1$ typically

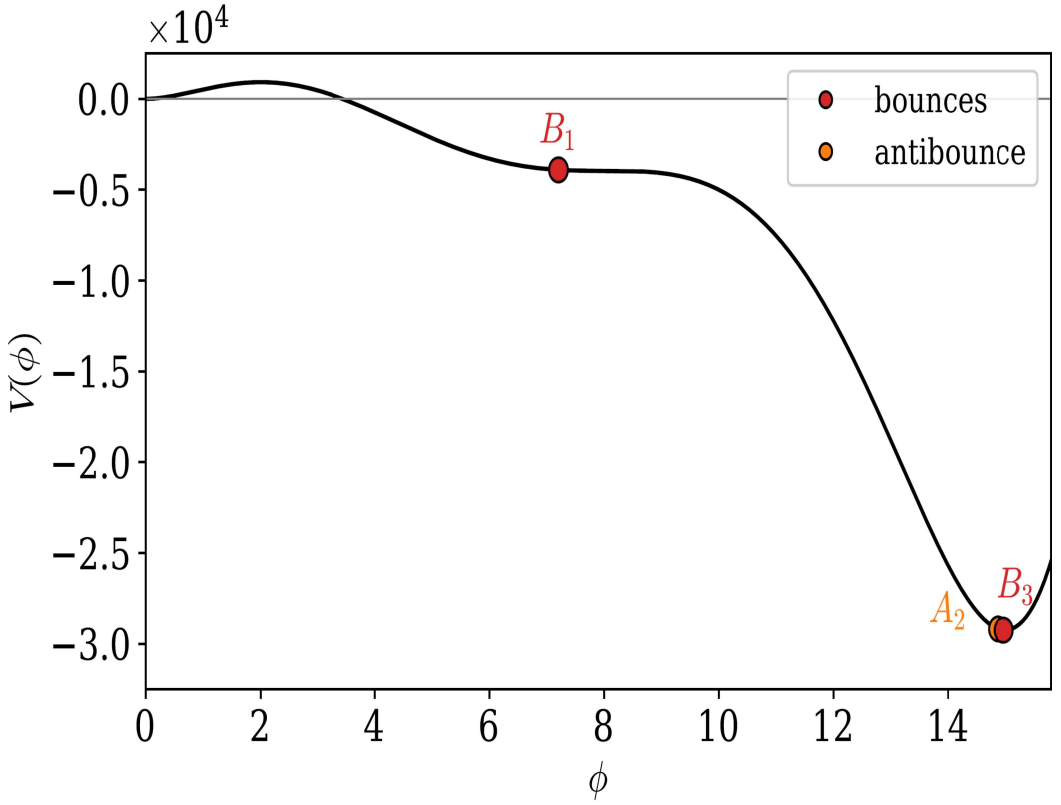
★ Lowest ϕ_e solution always $N_- = 1$. ✓

★ Which relevant ($N_- = 1$) bounce dominates is model dependent

Typically S lowest for highest ϕ_e, but not always

EXAMPLE

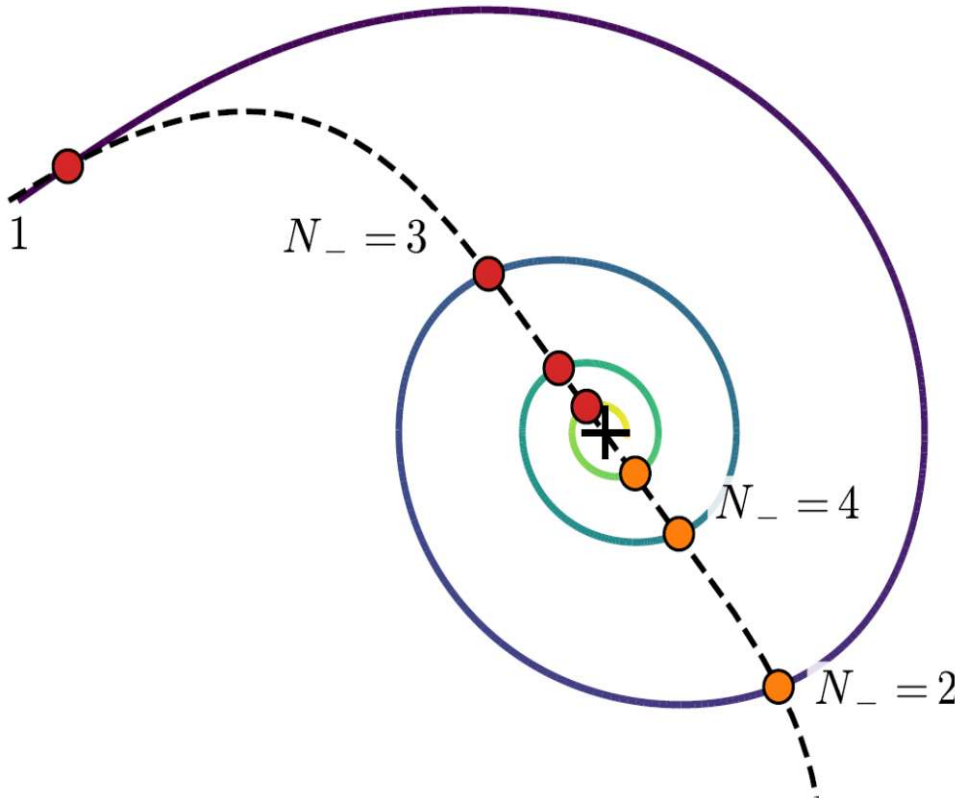
$V' = \phi(\phi-2)(\phi-8)^2(\phi-15)$



	N_
B_1	1
A_2	2
B_3	1

$S_{B_1} < S_{B_3}$

IV WHICH POTENTIALS HAVE MORE THAN ONE BOUNCE ?



WHEN DO EXTRA NEGATIVE MODES APPEAR?

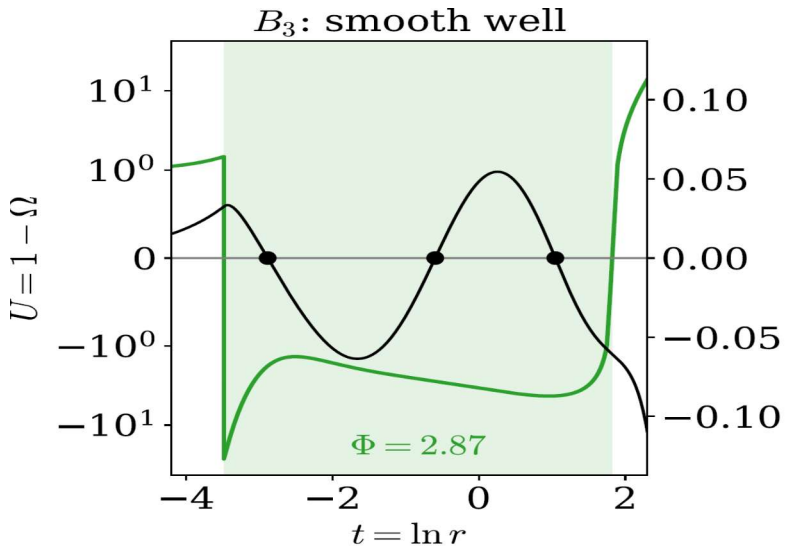
$2n+1$: Additional bounce ➔ Antibounce ➔ $N_- \geq 2$

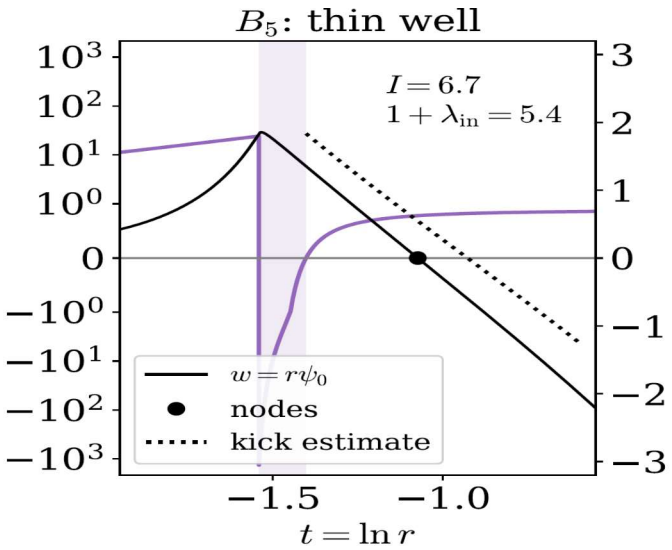
N_-: nodes of $\psi \equiv \partial\phi/\partial\phi_c$ or $\omega \equiv r\psi$

For $t \equiv \ln r$

$$\frac{d^2\omega}{dt^2} = \underbrace{[1 - \Omega(r)]}_{U(r)}\,\omega \qquad \Omega(r) = -r^2 V''(\phi_B(r))$$

Oscillations (➔ nodes!) require $\Omega(r) > 1$ ➔ large $-V''$





SELF-SIMILAR REGION OF V

For $V(\phi) \sim -(\phi-\phi_s)^p$

- power laws ($p>2$)
- exponentials ($p=\infty$)
- poles ($p<0$)

$$\mathcal{K} \equiv \frac{V'V'''}{(V'')^2}$$

↑ constant

Bounce Scaling solution

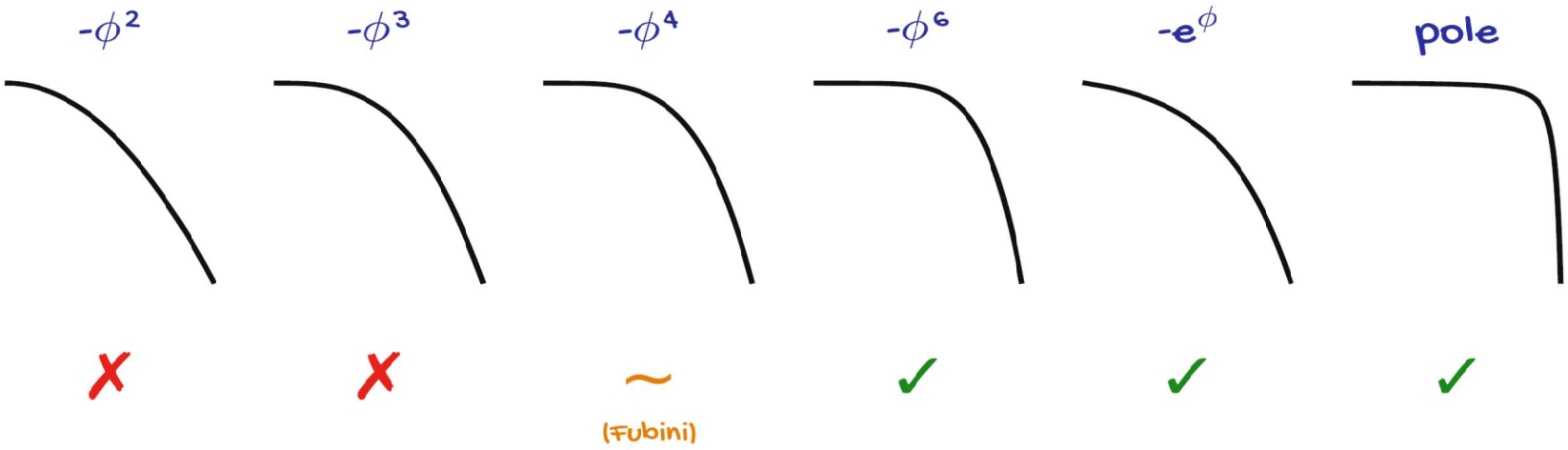
$$\phi - \phi_s \sim r^{-2/(p-2)} \qquad \Omega = \frac{4(p-1)(p-3)}{(p-2)^2}$$

Regular solutions with extra negative modes if

- attracted to scaling solution $\quad \frac{2}{p-2} < 1$
- oscillate around it $\quad \Omega > 1$

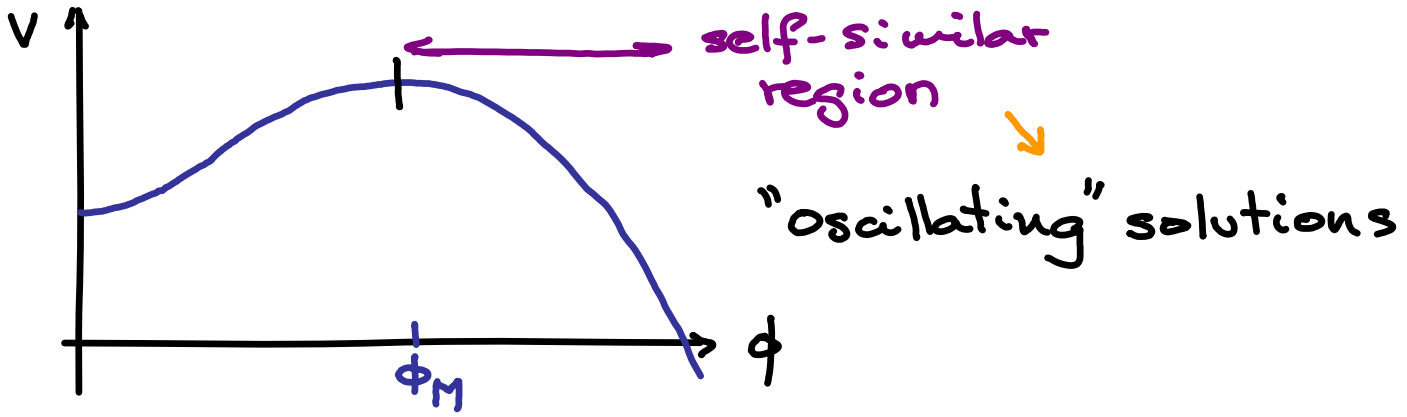
SELF-SIMILAR REGION OF V

Extra bounces if $\kappa = \frac{V'V'''}{(V'')^2} > 2/3$ ($p > 4$ or pole)

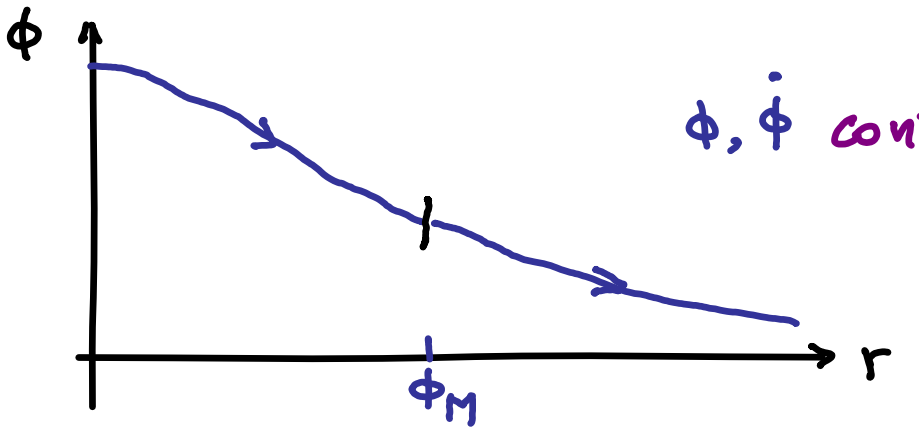


⚠ $\kappa > \frac{2}{3}$ not sufficient nor necessary.

MATCHING SOLUTIONS



Match at ϕ_M with solution from false vacuum:

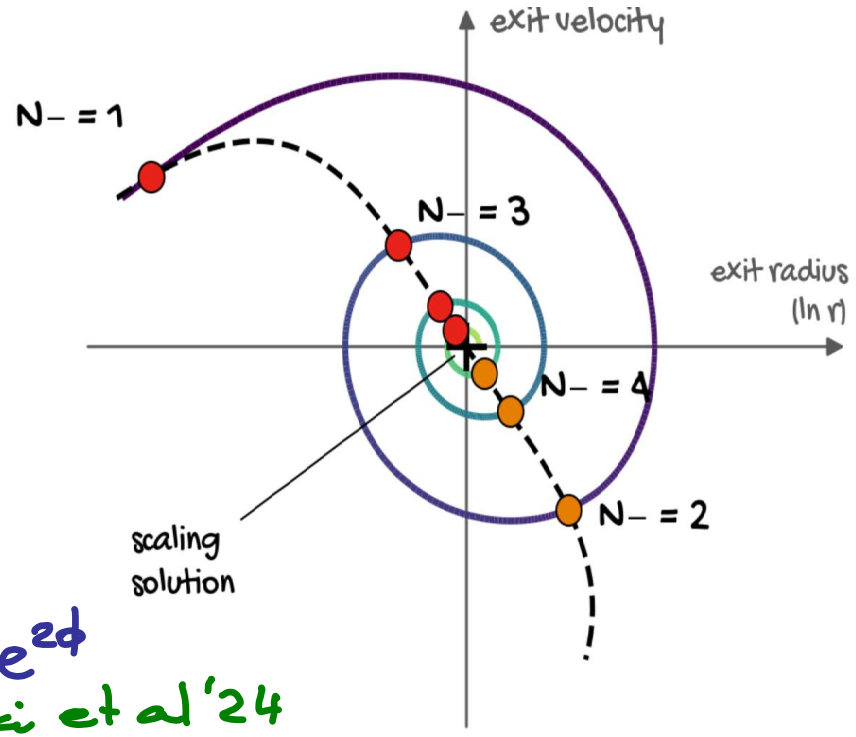


MATCHING SOLUTIONS

"oscillating" solutions ➔ Spiral in $(r, \dot{\phi})$ plane at ϕ_M around scaling solution

--- Solution to false vacuum

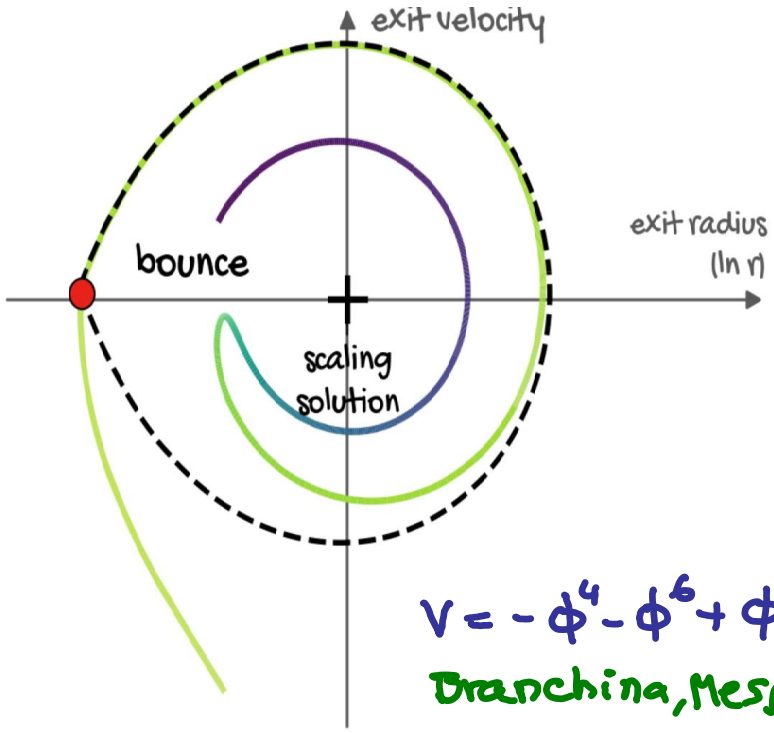
• • bounces



$V = -e^{2\phi}$

Sasaki et al '24

Many bounces



$V = -\phi^4 - \phi^6 + \phi^8$

Branchina, Messina' 13

only one bounce

PHYSICALLY MOTIVATED V(φ)'s

	$V'V'''/(V'')^2$	
Negative Exponentials (ekpyrotic, bubbles of nothing, ...)	1 ✓	needs gravity
SM Higgs	~ 2/3 ~	marginal
Higgs $-\phi^6/M_p^2$	~ 0.75 ✓	but single bounce
Thermal transitions (d=3)*	need > 4/5 ✗	not reached in general

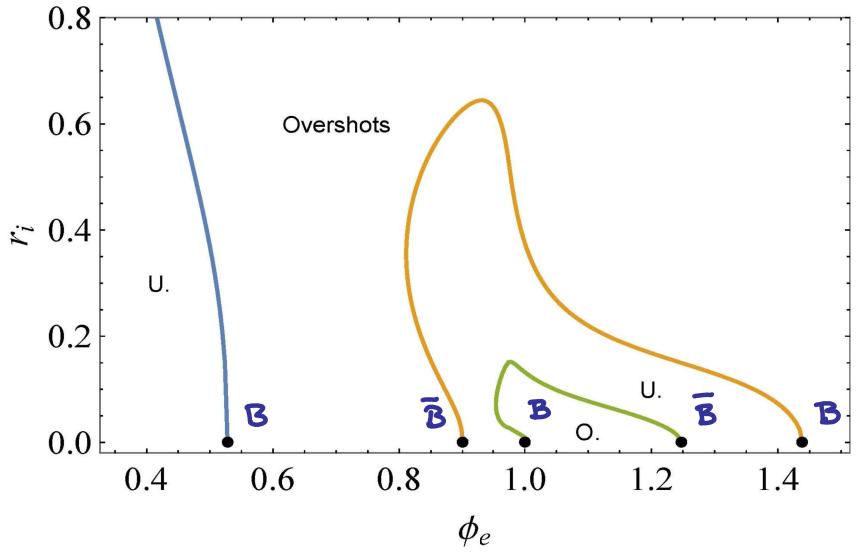
* $K > \frac{4}{d-2}$

CONCLUSIONS

- Pseudo-bounces (track action valleys) can be relevant even with bounce
- Rich vacuum decay structure is possible
- Most general case

  2n+1 bounces

  n $B\bar{B}$ pairs



CONCLUSIONS

- Only solutions with one negative mode are relevant for decay
  - Antibounces are never relevant
  - But they signal further bounces
  - Which bounce dominates is model dependent

- Extra bounces can appear from large $-V''(\phi)$
  - kicks
  - self-similar regions with $\frac{V'V'''}{(V'')^2} > \frac{2}{3}$