Bubble Nucleation Rates from the Functional Renormalisation Group

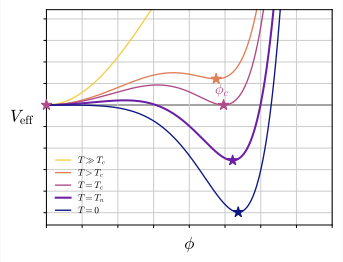
Manuel Reichert

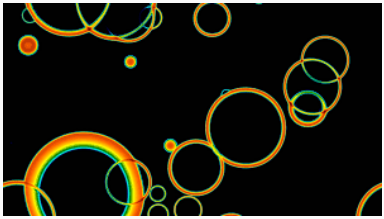
Ernest Rutherford Fellow at University of Sussex, Brighton, UK

VAST 2026: From Vacuum Stability to Gravitational Waves
Madrid, 21 October 2026

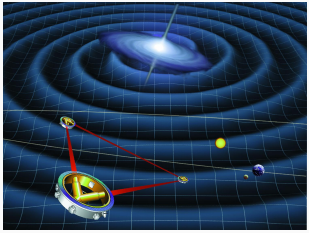
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Motivation

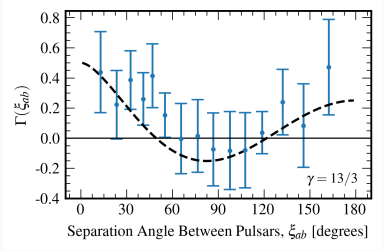




[Weir '17]

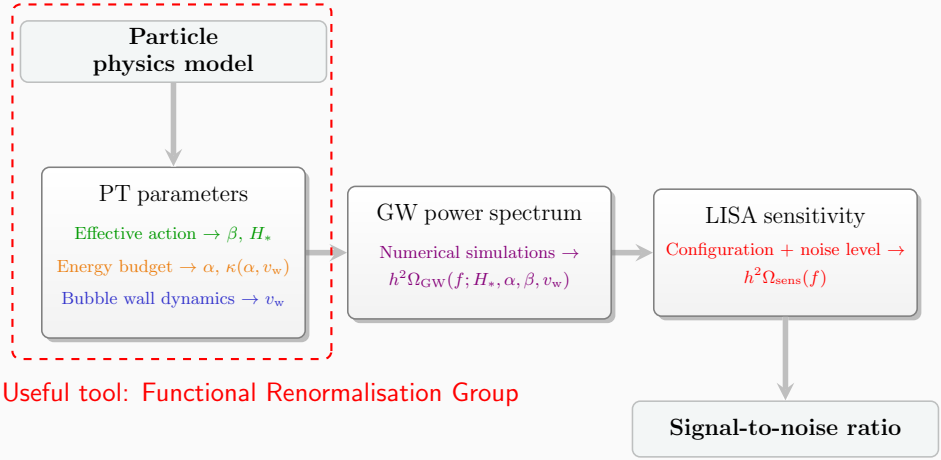


[LISA, Image Wikipedia]



[NANOGrav '23]

From Phase Transitions to Gravitational Waves



Useful tool: Functional Renormalisation Group

[Caprini et al '19]

Bubble nucleation

- Bubble nucleation rate [Coleman '77; Linde '81, '83, Affleck '81]

$$\frac{\Gamma}{\mathcal{V}} \propto \left| \frac{\det' S_3''[\phi_B]}{\det S_3''[\phi_F]} \right|^{-1/2} e^{-S_3[\phi_B]/T}$$

  where $S_3[\phi_B]$ is the $d = 3$ Euclidean action evaluated on the bounce solution

- Which action should be used? Tree-level? Effective action?
- Consistent treatment through EFT approach [Gould, Hirvonen '21]

$$\frac{\Gamma}{\mathcal{V}} \propto \underbrace{\left| \frac{\det' S_{3,\text{eff}}''[\phi_B]}{\det S_{3,\text{eff}}''[\phi_F]} \right|^{-1/2}}_{\text{modes } E<\Lambda_{\text{eff}}} \underbrace{e^{-S_{3,\text{eff}}[\phi_B]/T}}_{\text{modes } E>\Lambda_{\text{eff}}}$$

- Functional Renormalisation Group provides direct access to S_{eff} at the scale Λ_{eff}

Non-perturbative functional approach

- Path integral with suppressed IR modes

$$Z_k[J] \sim \int \mathcal{D}\varphi_{p^2 \geq k^2} \; e^{-S[\varphi] + \int_x J(x)\,\varphi(x)}$$

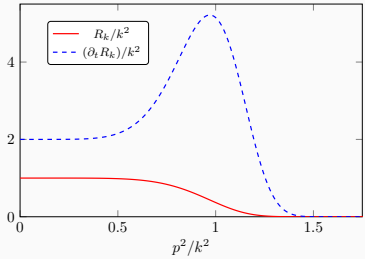
- Regulator function R_k to suppress IR modes

$$\int \mathcal{D}\varphi_{p^2 \geq k^2} = \int \mathcal{D}\varphi \, e^{-\frac{1}{2}\int_p \phi R_k \phi}$$

- Wilsonian integrating out of momentum shells

$$k\partial_k Z_k[J] = -\frac{1}{2}\int_p \frac{\delta^2 Z_k[J]}{\delta J(p)\delta J(-p)} k\partial_k R_k(p^2)$$

- Turns path integral into functional differential equation
  - For $k \to \Lambda$: no quantum effects / classical action
  - For $k \to 0$: full quantum theory



The Functional Renormalisation Group

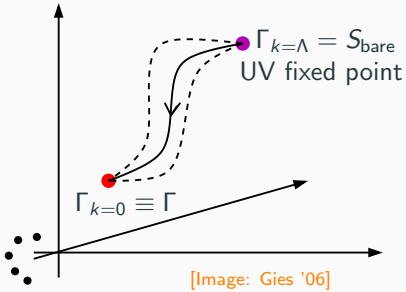
Legendre transform of $\log(Z_k[J])$ leads to a flow of scale-dependent effective action:

Non-perturbative renormalisation group equation [Wetterich '93]

$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2)} + R_k} k\partial_k R_k\right]$$

Flow provides connection between

- Classical action
- 1PI quantum effective action Γ
- Through Wilsonian integrating out of momentum modes



[Image: Gies '06]

Flow of the effective potential

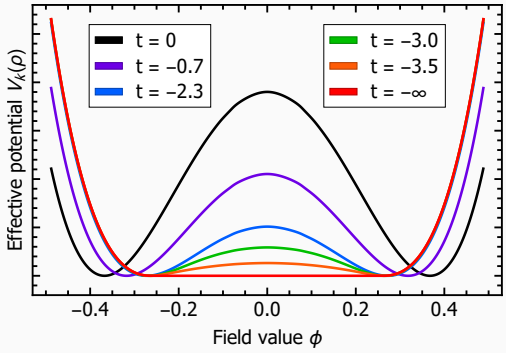
For a single scalar field at finite temperature

$$k\partial_k V_k(\phi) = \frac{1}{2} T \sum_{n=-\infty}^{\infty} \int \frac{\mathrm{d}^{d-1}\mathbf{q}}{(2\pi)^{d-1}} \frac{\partial_t R_k(\mathbf{q})}{\omega_n^2 + \mathbf{q}^2 + V_k''(\phi) + R_k(\mathbf{q})}$$

$$\overset{R_k = k^2}{=} \frac{k^2}{(2\pi)^2} \int_0^\infty \mathrm{d}q \Bigg(\underbrace{\frac{q^2}{\sqrt{q^2 + k^2 + V_k''(\phi)}}}_{\text{Vacuum}} + \underbrace{\frac{2q^2}{\sqrt{q^2 + k^2 + V_k''(\phi)}} \frac{1}{\exp\left(\frac{\sqrt{q^2 + k^2 + V_k''(\phi)}}{T}\right) - 1}}_{\text{Thermal}} \Bigg)$$

- Partial differential equation
- Manifestly IR finite due to regulator function
- Includes Daisy, super-Daisy resummations and more
- Useful numerical tools: discontinuous Galerkin methods
  [Grossi, Wink '19; Ihssen, Pawlowski, Sattler, Wink '22; Sattler, Pawlowski '24]

Evolution of the Effective Potential



At $k = 0$ ($t = \ln \frac{k}{\Lambda} = -\infty$) the effective potential is convex and has non-analyticities

Fluctuation determinant with the FRG

Regularisation of the fluctuation determinant to only includes modes with $q^2 \lesssim k^2$

[Strumia, Tetradis '98; '99]

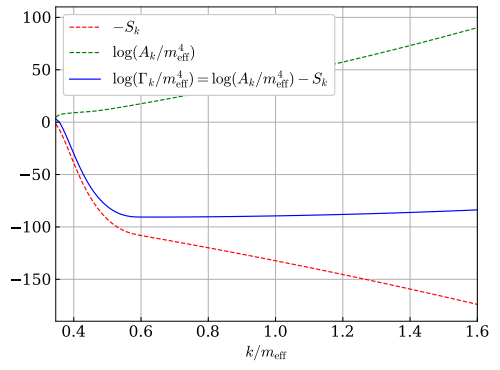
$$\left| \frac{\det{}' \left(-\nabla^2 + V_k''(\phi_\mathrm{b}(r))\right)}{\det\left(-\nabla^2 + V_k''(\phi_\mathrm{FV})\right)} \right|^{-1/2}$$

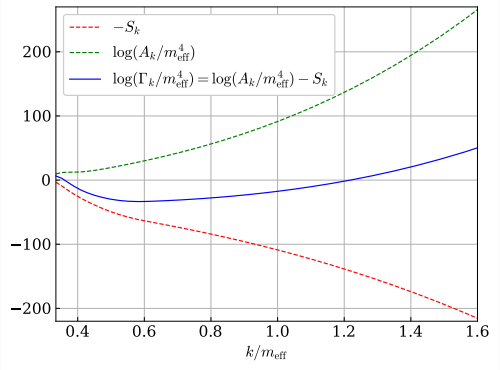
$$\longrightarrow \left| \frac{\det{}' \left(-\nabla^2 + V_k''(\phi_\mathrm{b}(r))\right)}{\det\left(-\nabla^2 + R_k(-\nabla^2) + V_k''(\phi_\mathrm{b}(r))\right)} \cdot \frac{\det\left(-\nabla^2 + R_k(-\nabla^2) + V_k''(\phi_\mathrm{FV})\right)}{\det\left(-\nabla^2 + V_k''(\phi_\mathrm{FV})\right)} \right|^{-1/2}$$

- Evaluated with Gelfand-Yaglom theorem [Gelfand '59; see also Ekstedt, Gould, Hirvonen '23]
- Mass-like regulator $R_k = k^2$ favourable for evaluation but requires additional UV regularisation [Braun, MR, et al '22]

Decay Rate

Toy model: Vacuum decay rate for single scalar field in $d = 3$ in LPA approximation



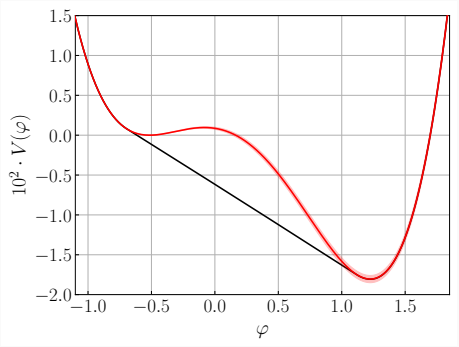


[Huber, Phipps, MR (in prep)]

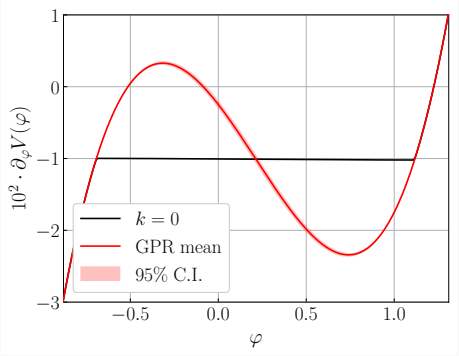
- Decay rate (prefactor + exponential) flattens out around an effective nucleation scale for some theory parameters [see also: Strumia, Tetradis '98; '99]
- Unsatisfactory from FRG perspective: need to use results with finite regulator

Potential Reconstruction

Potential



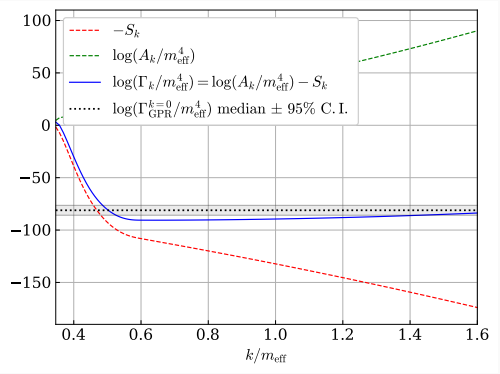
Derivative

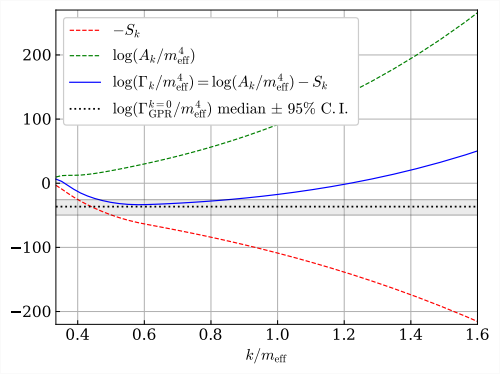


[Huber, Phipps, MR (in prep)]

- Idea: Convex potential still contains information about barrier through non-analyticities
- Reconstruct potential barrier through analyticity from outside of convexity region
- Tool: Gaussian Process Regression (GPR)

Reconstructed Decay Rate





[Huber, Phipps, MR (in prep)]

- Bounce solution from reconstructed effective potential at $k = 0$
- Fluctuation determinant to account for convexity generating modes
- Outlook: Quantitative comparison to lattice and EFT results

Application to the Electroweak Phase Transition

Electroweak Phase Transition

Running couplings and Higgs potential (all temperature and RG scale dependent)

$$V(\phi), y_t, g, g', g_s$$

FRG correctly determines order of the electroweak phase transition

- Crossover for SM Higgs mass
- First-order for small Higgs masses

In agreement with lattice [Kajantie, Laine, Rummukainen, Shaposhnikov '96; D'Onofrio, Rummukainen '15; ...]

TeV-scale new physics

Generic new physics at the TeV-scale via modified Higgs potential

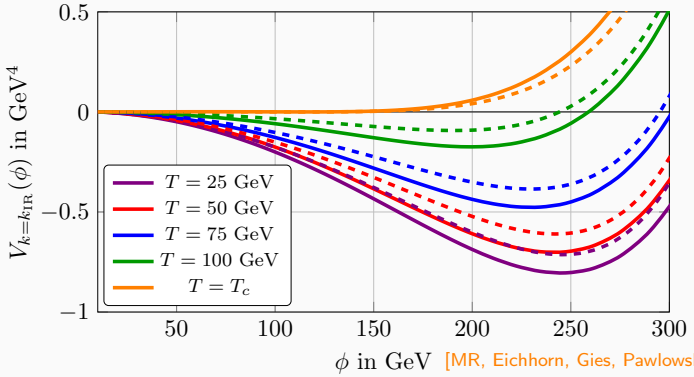
$$V_{k=M_{\mathrm{NP}}}(\phi) = \frac{\mu^2}{2}\phi^2 + \frac{\lambda_4}{4}\phi^4 + \Delta V(\phi)$$

- Polynomial: $\Delta V(\phi) = \frac{\lambda_6}{M_{\mathrm{NP}}^2}\phi^6$ [Grojean, Servant, Wells '05; ...]
- Logarithmic: $\Delta V(\phi) = \lambda_{\ln}\phi^4 \ln\frac{\phi^2}{2M_{\mathrm{NP}}^2}$
- Exponential: $\Delta V(\phi) = \lambda_{\exp}\phi^4 \exp\left(-\frac{2M_{\mathrm{NP}}^2}{\phi^2}\right)$

Phenomenology

- Higgs self-couplings from effective potential at zero temperature
- Gravitational waves from effective potential at finite temperature
- Matter-Antimatter asymmetry (not considered here)

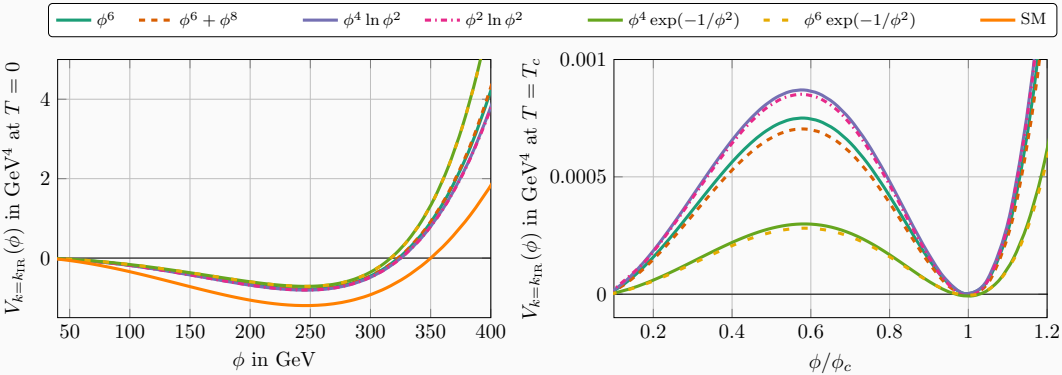
Evolution of the Higgs Potential



[MR, Eichhorn, Gies, Pawlowski, Plehn, Scherer '17]

Solid: $\phi^4 \ln \phi^2$ Dashed: $\phi^4 \exp(-1/\phi^2)$

Evolution of the Higgs Potential

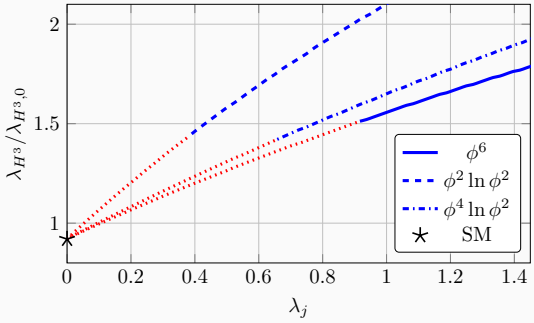


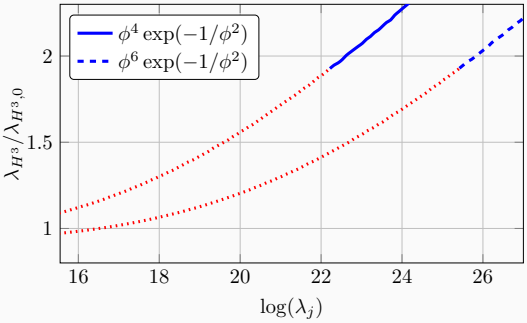
[MR, Eichhorn, Gies, Pawlowski, Plehn, Scherer '17]

Differences in effective potential at $T = 0$ and $T = T_c$ despite fixed observables

($\phi_c = T_c$, $v = 246\,\mathrm{GeV}$, $m_H = 125\,\mathrm{GeV}$, $m_t = 173\,\mathrm{GeV}$)

Order of the Phase Transition



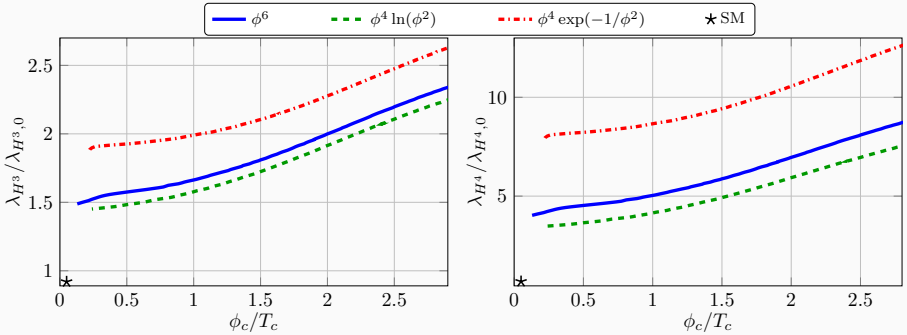


[MR, Eichhorn, Gies, Pawlowski, Plehn, Scherer '17]

Red: Crossover

Blue: first-order

Higgs-self coupling



[MR, Eichhorn, Gies, Pawlowski, Plehn, Scherer '17]

- Enhancement of the Higgs self-couplings necessary for first-order transition
- Strength of enhancement depends on UV physics
- Trilinear coupling measurable at the high-luminosity run of the LHC

Gravitational-wave parameters

Gravitational waves from sound waves in plasma after bubble collision

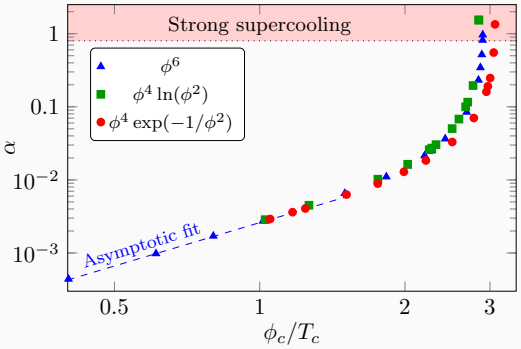
[Hindmarsh, Huber, Rummukainen, Weir '13; Giblin, Mertens '13; ...]

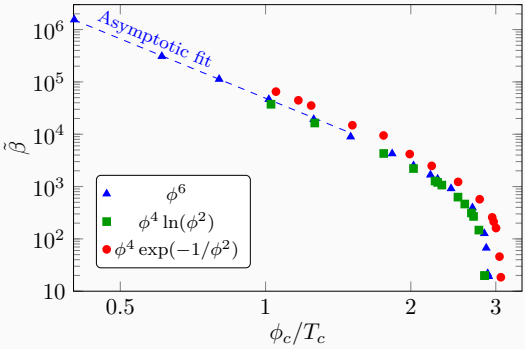
$$h^2\Omega_{\mathrm{GW}}(f) = h^2\Omega_{\mathrm{GW}}^{\mathrm{peak}} \left(\frac{f}{f_{\mathrm{peak}}}\right)^3 \left[\frac{4}{7} + \frac{3}{7}\left(\frac{f}{f_{\mathrm{peak}}}\right)^2\right]^{-\frac{7}{2}}$$

$$h^2\Omega_{\mathrm{GW}}^{\mathrm{peak}} \propto \frac{v_w}{\tilde{\beta}} \left(\frac{\kappa\,\alpha}{1+\alpha}\right)^2 \qquad f_{\mathrm{peak}} \propto \frac{T_n\,\tilde{\beta}}{v_w}$$

- Determine energy budget α, inverse duration $\tilde{\beta}$, and efficiency factor κ from V_{eff}
- v_w input parameter

α and β as a function of ϕ_c/T_c

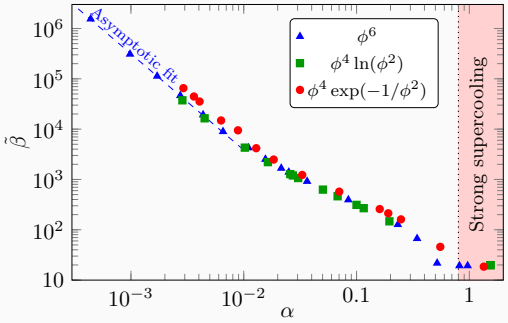


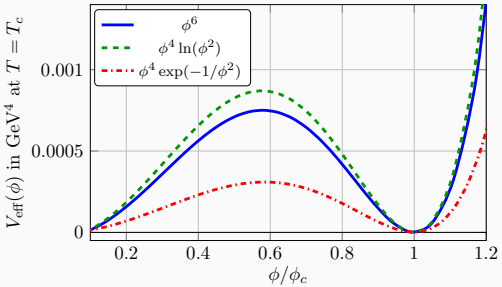


[Eichhorn, Lumma, Pawlowski, MR, Yamada '20]

All modifications lead to almost universal GWs depending only on ϕ_c/T_c

Universal gravitational-wave signal



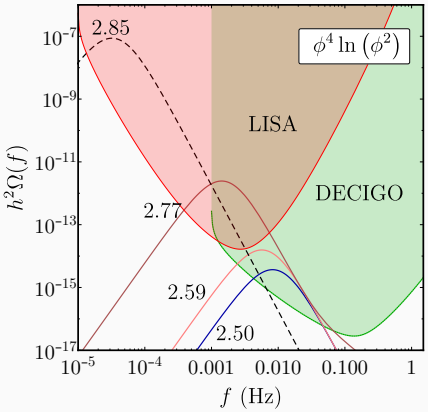


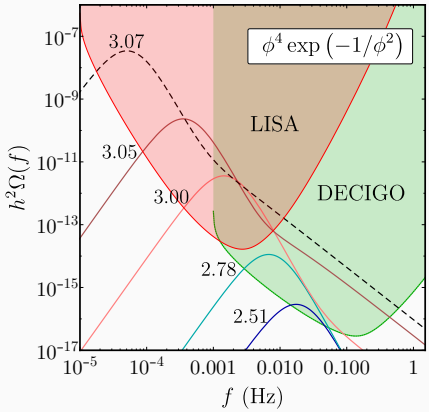
[Eichhorn, Lumma, Pawlowski, MR, Yamada '20]

All modifications lead to almost universal GWs depending only on ϕ_c/T_c

despite significant differences in the effective potential at $T = T_c$ and $T = 0$

Gravitational-wave spectrum





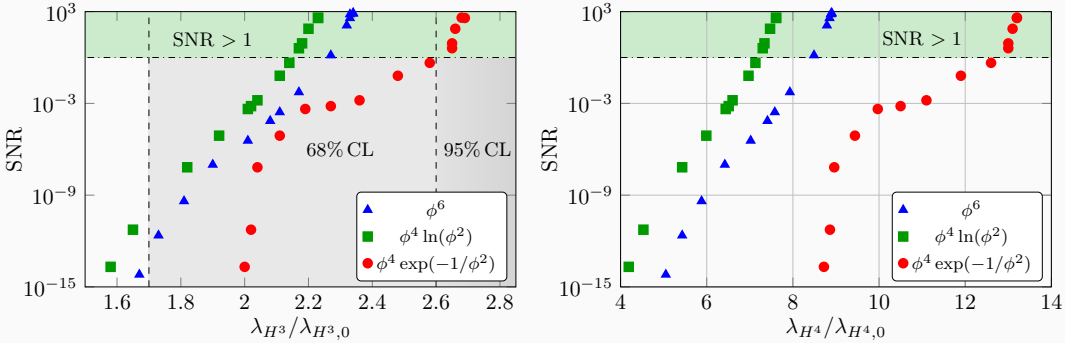
[Eichhorn, Lumma, Pawlowski, MR, Yamada '20]

Only very strong first-order phase transitions ($\phi_c/T_c \gtrsim 2.7$ model dependent)

from strong Higgs-potential modifications are detectable with LISA

[see also: Gould, Kozaczuk, Niemi, Ramsey-Musolf, Tenkanen, Weir '19]

Detectability at LISA vs LHC



[Eichhorn, Lumma, Pawlowski, MR, Yamada '20]

Combined LISA and LHC measurements can distinguish new physics modifications

Summary

Methodology:

- FRG is a useful tool for the computation of effective potentials and nucleation rates
- Implements resummations and mode splitting between fluctuation determinant and bounce action
- Analytic reconstruction allows for bubble nucleation at vanishing RG scale

Application to electroweak phase transition:

- Generic polynomial and non-polynomial TeV-scale new physics via Higgs potential
- Strong enhancement of Higgs self-couplings
- Almost universal gravitational-wave signal

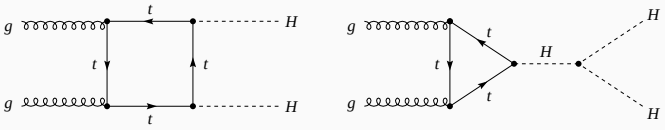
Outlook

- Application of new reconstruction to realistic models and quantitative comparison

Thank you for your attention!

Measurement of the Higgs self-coupling

Higgs pair production in gluon fusion



[Baur, Plehn, Rainwater '03]

Destructive interference in certain kinematic regimes at tree level

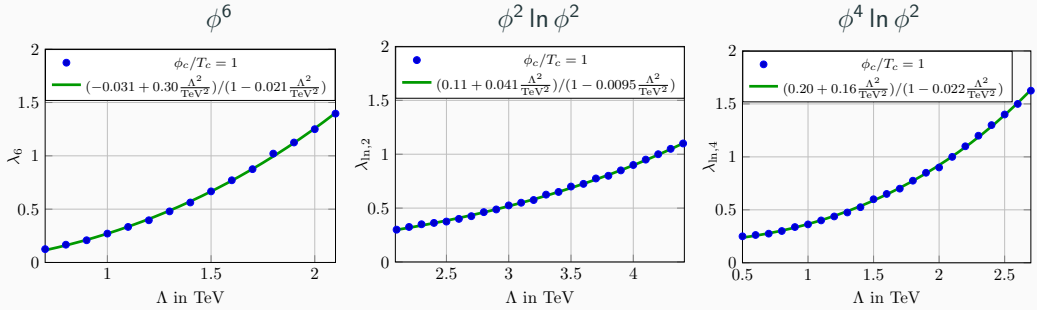
$$\mathcal{A} \propto \frac{\alpha_s}{12\pi v}\left(\frac{\lambda_{H^3}}{s-m_H^2}-\frac{1}{v}\right) \overset{\lambda_{H^3}=\lambda_{H^3,0}}{\longrightarrow} 0$$

High-luminosity LHC run: measurement with the precision

$\lambda_{H^3}/\lambda_{H^3,0} = 0.4 \ldots 1.7$ at 68% CL and for 3 ab^{-1}

[Kling, Plehn, Schichtel '16]

Scale of new physics



[MR, Eichhorn, Gies, Pawlowski, Plehn, Scherer '17]

$\Lambda_6^{\text{crit}} = 7.0$ TeV $\qquad \Lambda_{\ln,2}^{\text{crit}} = 10$ TeV $\qquad \Lambda_{\ln,4}^{\text{crit}} = 6.8$ TeV

Scale where the system becomes strongly coupled