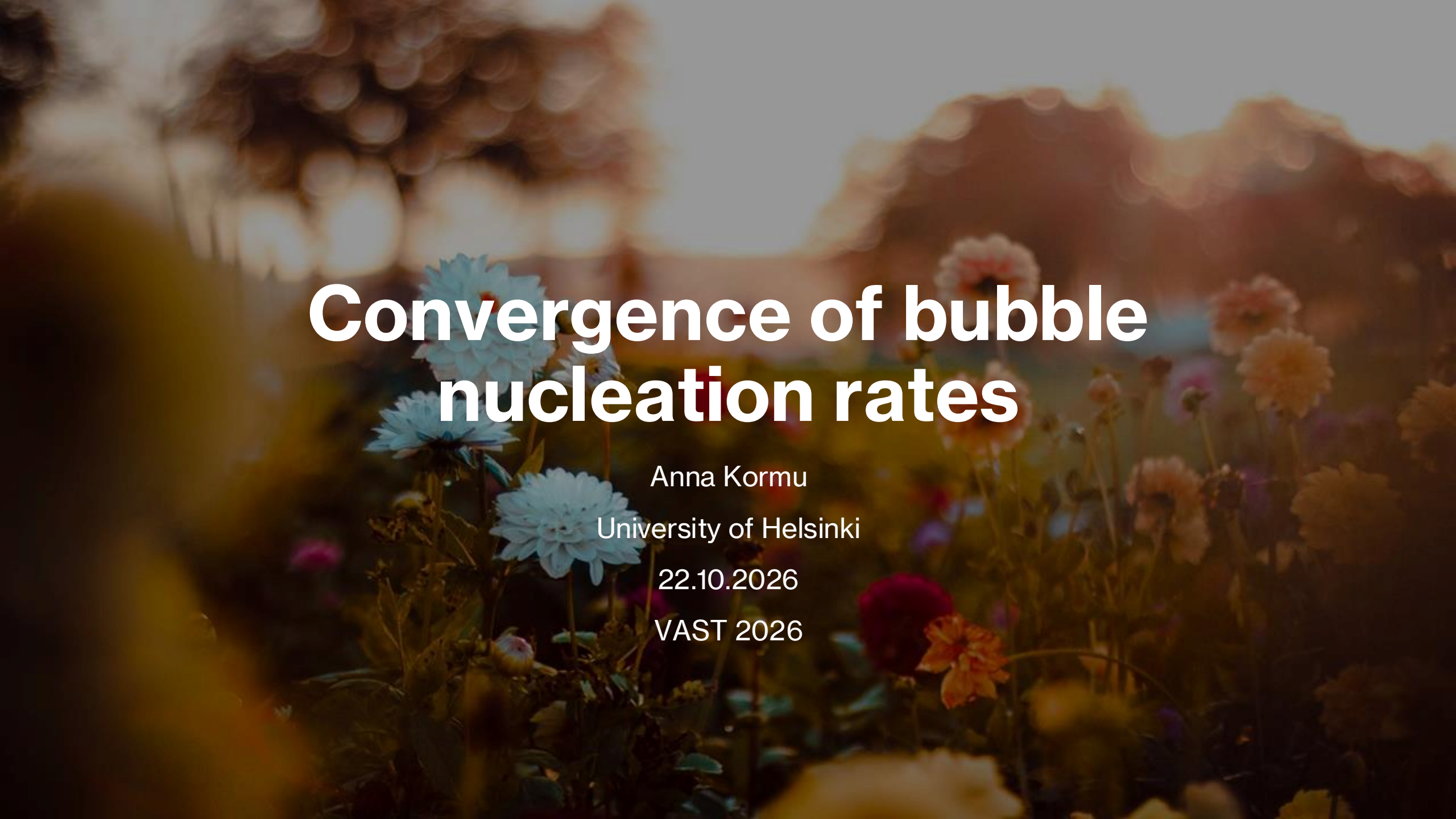


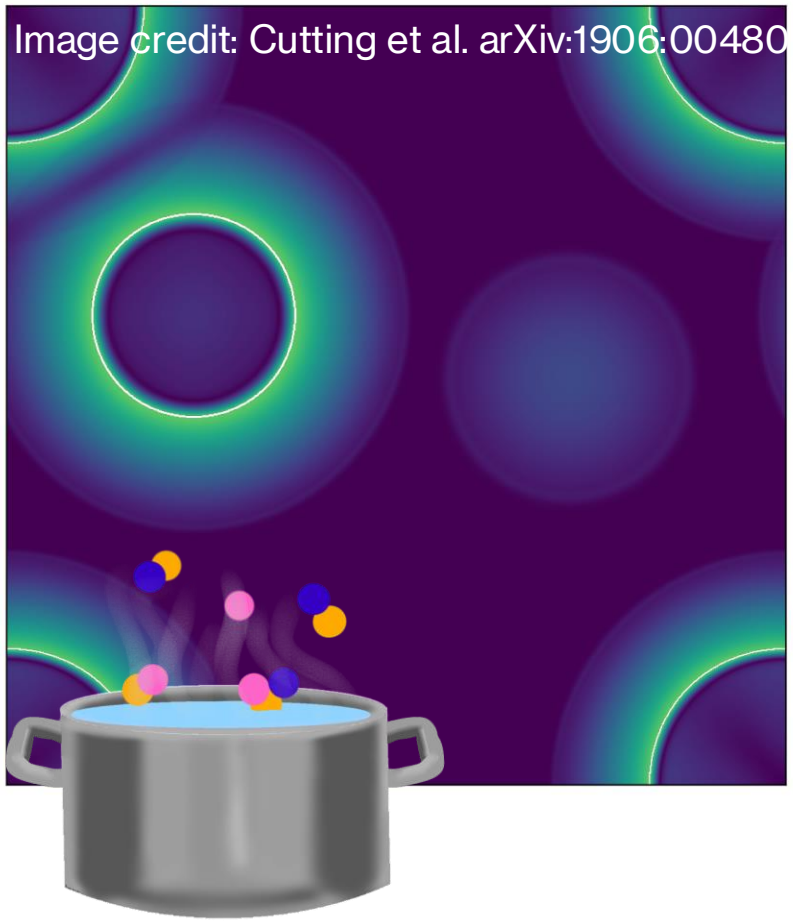
Convergence of bubble nucleation rates

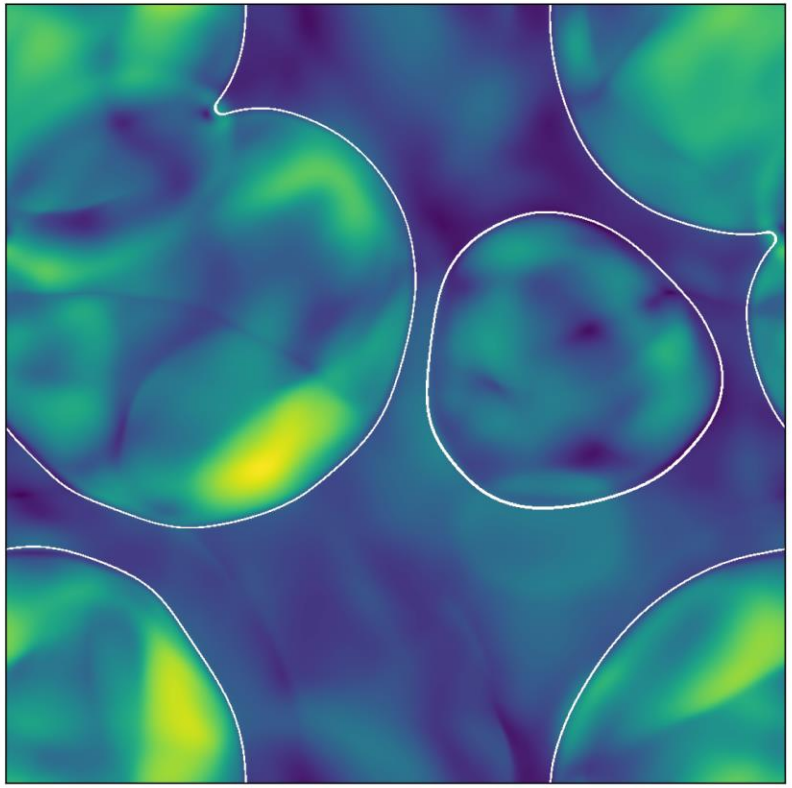
Anna Kormu

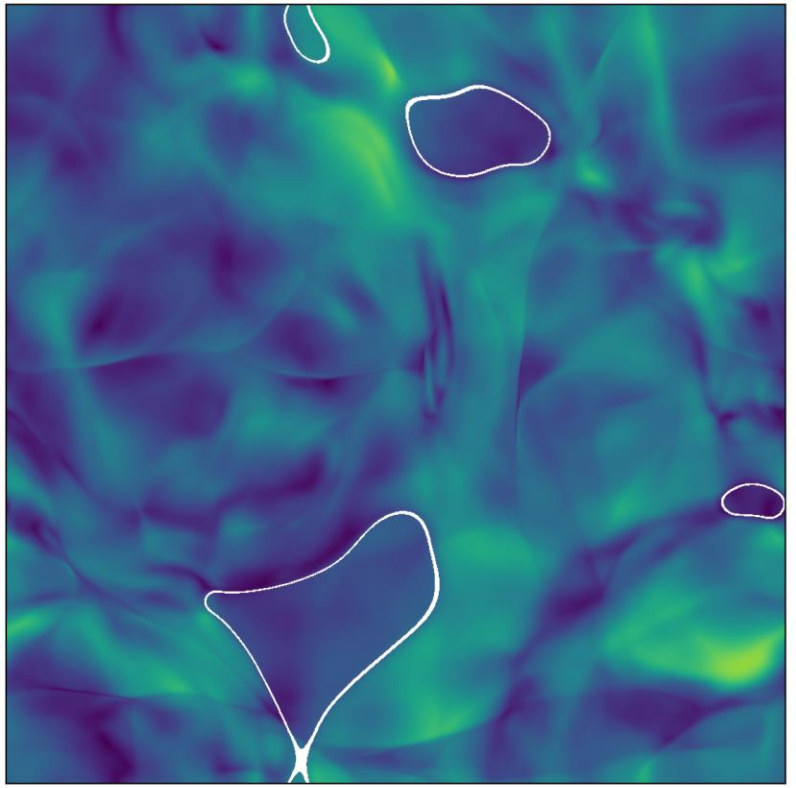
University of Helsinki

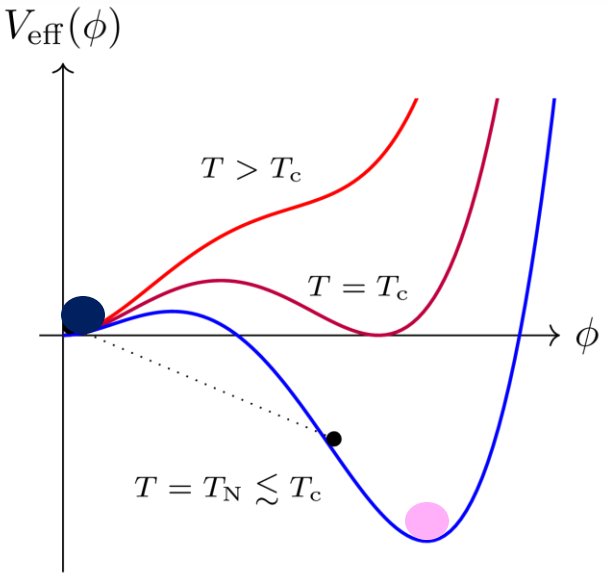
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VAST 2026









First-order phase transitions

Fate of the False Vacuum

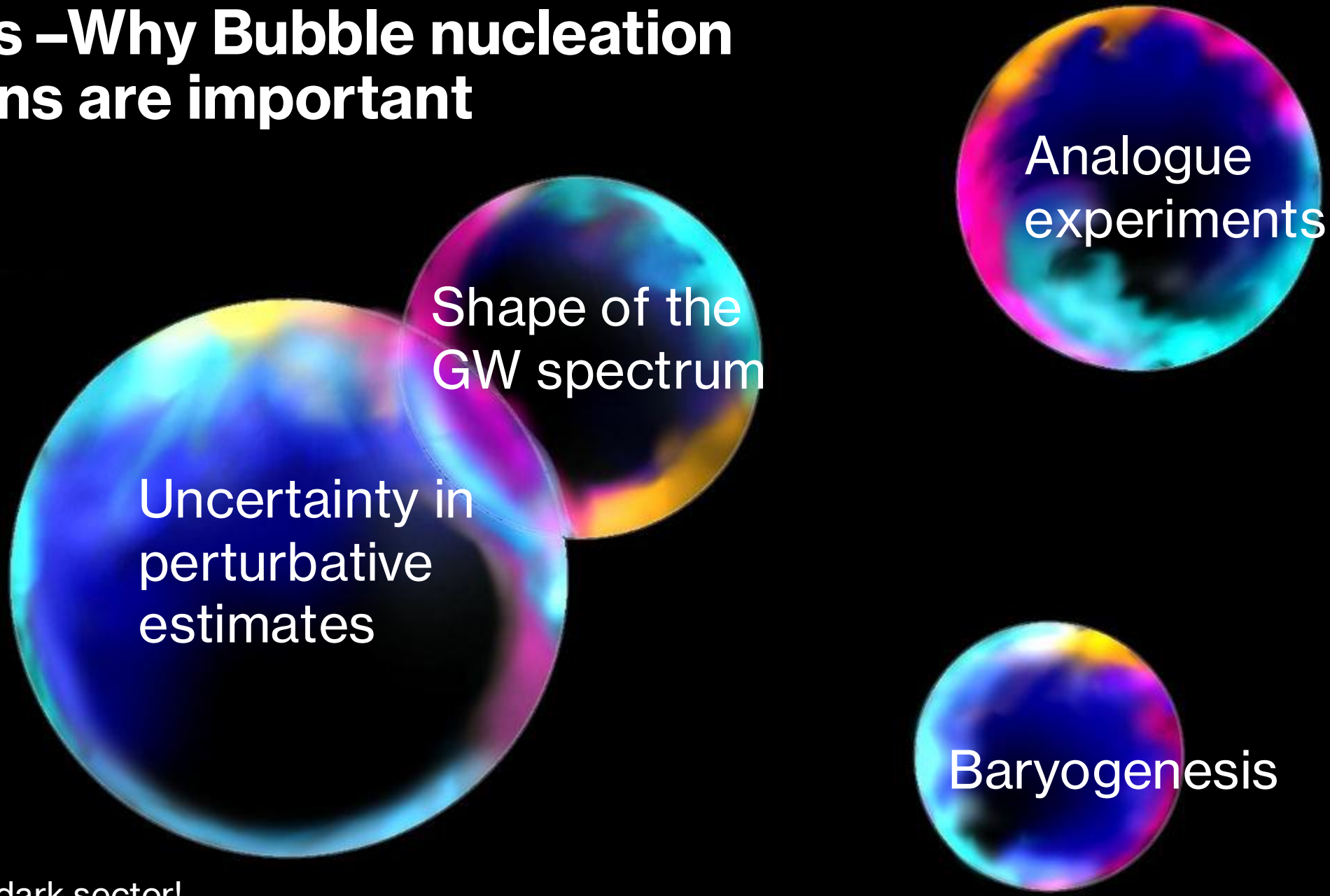
- Classical field theory (a complete one-loop calculation) by Langer Annals Phys. 41, 108 (1967), Annals Phys. 54, 258 (1969)
- At zero temperature: relativistic field theory generalisation by Callan & Coleman Phys. Rev. D 16, 1762 (1977)
- Finite temperature approach in quantum field theory introduced later by Affleck & Linde Phys. Rev. Lett. 46, 388 (1981), Phys. Lett. B 100, 37 (1981)
- More recent studies, crucially showing that the nucleation rate factorises, Gould, Hirvonen arXiv:2108.04377, Hirvonen arXiv:2403.07987, Ekstedt arXiv:2201.07331

$$\Gamma = A_{\rm dyn} \times A_{\rm stat}$$

Usually not computed!

$$\Gamma = A_{\rm dyn} \times \sqrt{\left|\frac{\det(S''[\phi_0]/2\pi)}{\det'(S''[\phi_{\rm b}]/2\pi)}\right|} \left(\frac{\Delta S[\phi_{\rm b}]}{2\pi}\right)^{3/2} e^{-\Delta S[\phi_{\rm b}]}$$

Key points –Why Bubble nucleation calculations are important



And dark matter/dark sector!

Real scalar theory

For equilibrium dynamics, see Gould, arXiv:2101.05528

- Toy model possessing key features of BSM models
  - Potential has a tree-level barrier
  - Strong phase transition
  - Perturbative expansion simpler (we understand the dynamics)
- Dimensional reduction (imaginary time, high temp)

$$\mathscr{L} = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi) - J_1\varphi - J_2\varphi^2$$

Interaction terms

$$V(\varphi) = \sigma\varphi + \frac{1}{2}m^2\varphi^2 + \frac{1}{3!}g\varphi^3 + \frac{1}{4!}\lambda\varphi^4,$$

Model parameters

Real scalar theory

For equilibrium dynamics, see Gould, arXiv:2101.05528

- At high temperatures system looks 3d
- Dimensional reduction 4d cont $\rightarrow$ 3d cont
- Integrate out heavy modes, match correlation functions

$$\mathcal{L}_{4d}$$

$$\downarrow$$

$$\mathcal{L}_{3d}$$

$$\downarrow$$

$$\mathcal{L}_{3\mathrm{d,lattice}}$$

$$H[\phi,\pi] = \int \mathrm{d}^3x \left(\frac{1}{2}\pi^2 + S_3[\phi]\right) ,$$

$$S_3[\phi] = \int \mathrm{d}^3x \left(\frac{1}{2}\partial_i\phi\partial_i\phi + V_3(\phi)\right) ,$$

$$V_3(\phi) = \Lambda_3 + \sigma_3\phi + \frac{1}{2}m_3^2\phi^2 + \frac{1}{3!}g_3\phi^3 + \frac{1}{4!}\lambda_3\phi^4 .$$

3d effective params

Real scalar theory

For equilibrium dynamics, see Gould, arXiv:2101.05528

- 3d continuum → 3d lattice
- Lattice action needs to be renormalised due to to thermal fluctuations
- Note that we perform a basis shift such that the cubic term disappears

$$\mathcal{L}_{4d}$$

$$\downarrow$$

$$\mathcal{L}_{3d}$$

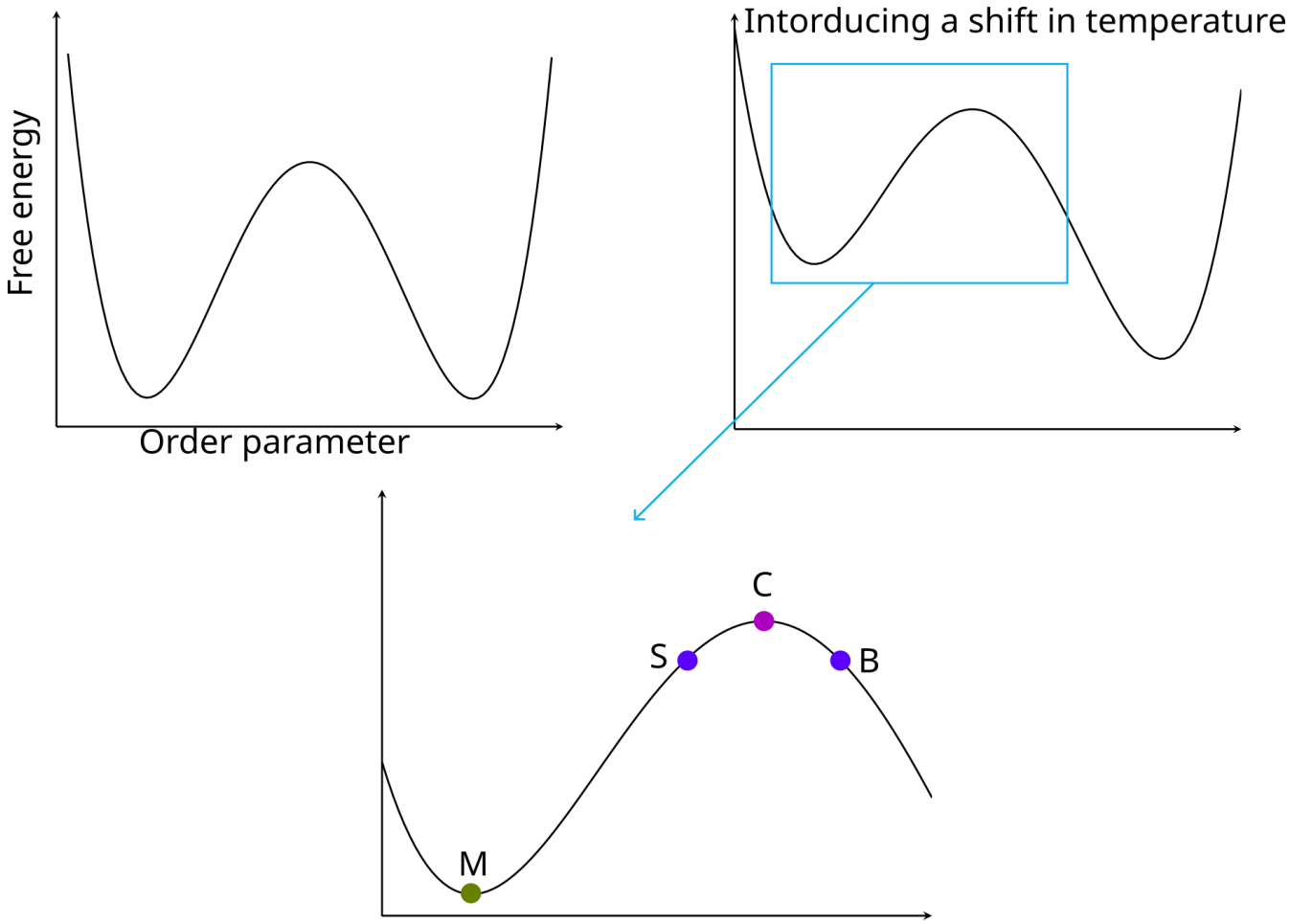
$$\downarrow$$

$$\mathcal{L}_{3\mathrm{d,lattice}}$$

Lattice spacing Lattice counterterms

$$S_{\mathrm{lat}} = \sum_x a^3 \Big[-\frac{1}{2} Z_\phi \phi_x (\nabla^2_{\mathrm{lat}} \phi)_x + \sigma_3 \phi_x + \frac{1}{2} Z_\phi Z_m (m_3^2 + \delta m_3^2) \phi_x^2 + \frac{1}{4!} Z_\phi^2 (\lambda_3 + \delta\lambda_3) \phi_x^4 \Big]$$

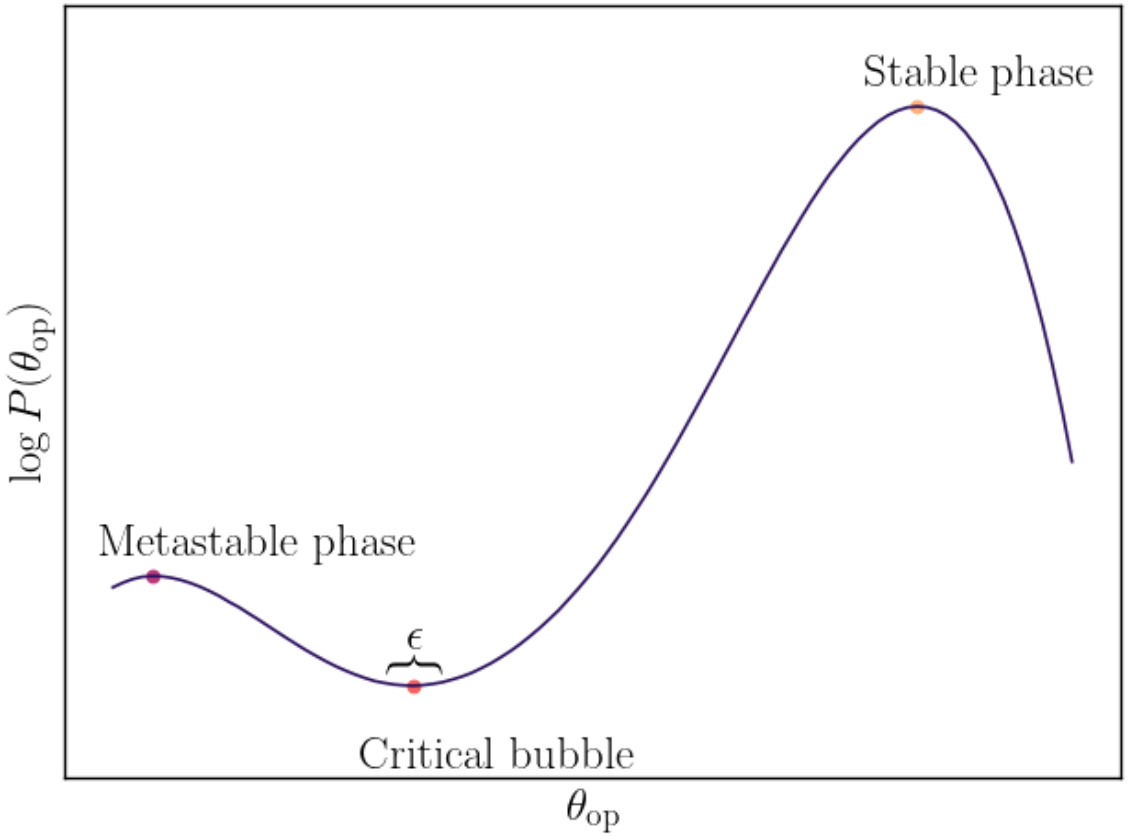
Statistical approach to bubble nucleation



- Moore & Rummukainen: statistical approach to calculating bubble nucleation on the lattice
- Method is similar to Langer's: Monte Carlo methods replace the saddle-point approximation, dynamics of the barrier crossing modeled with Lengevin dynamics

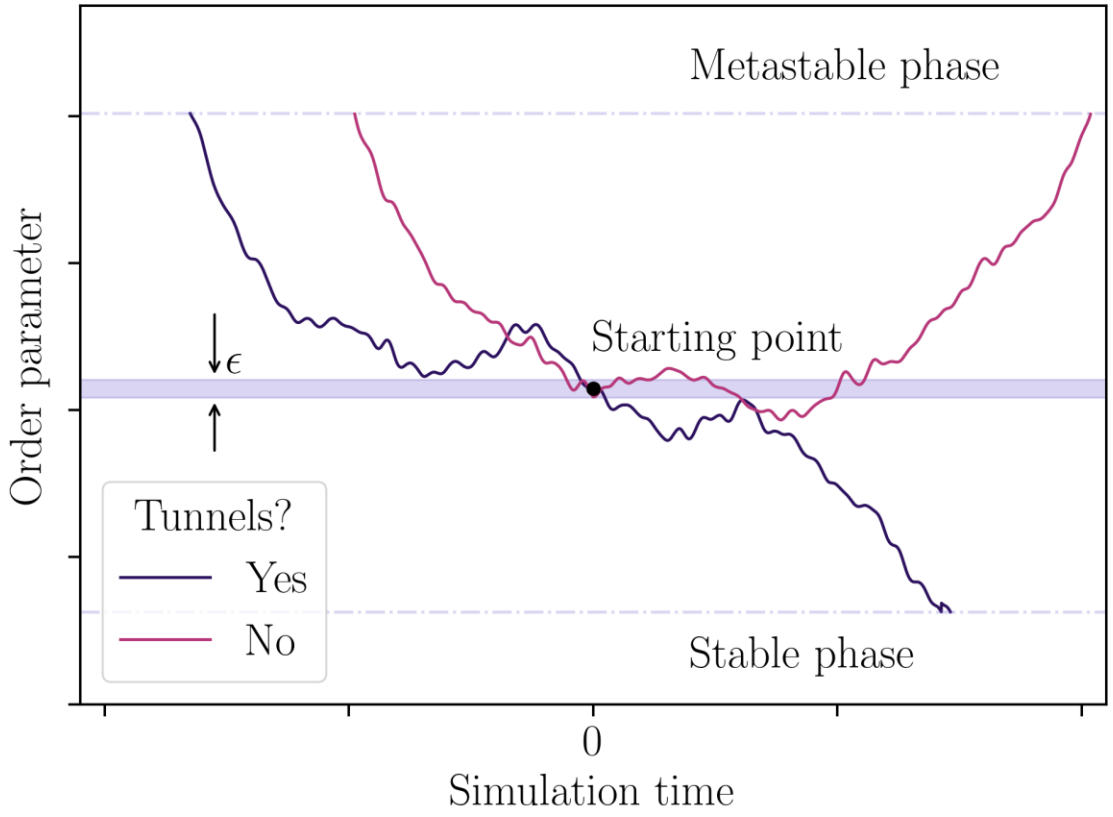
Calculating bubble nucleation rates non-perturbatively

1. Simulate probability distribution (multicanonical MC) P_c
2. Calculate the probability of critical bubbles relative to the metastable state
3. Draw separatrix configurations from a small area ϵ around the critical bubble peak



Calculating bubble nucleation rates non-perturbatively

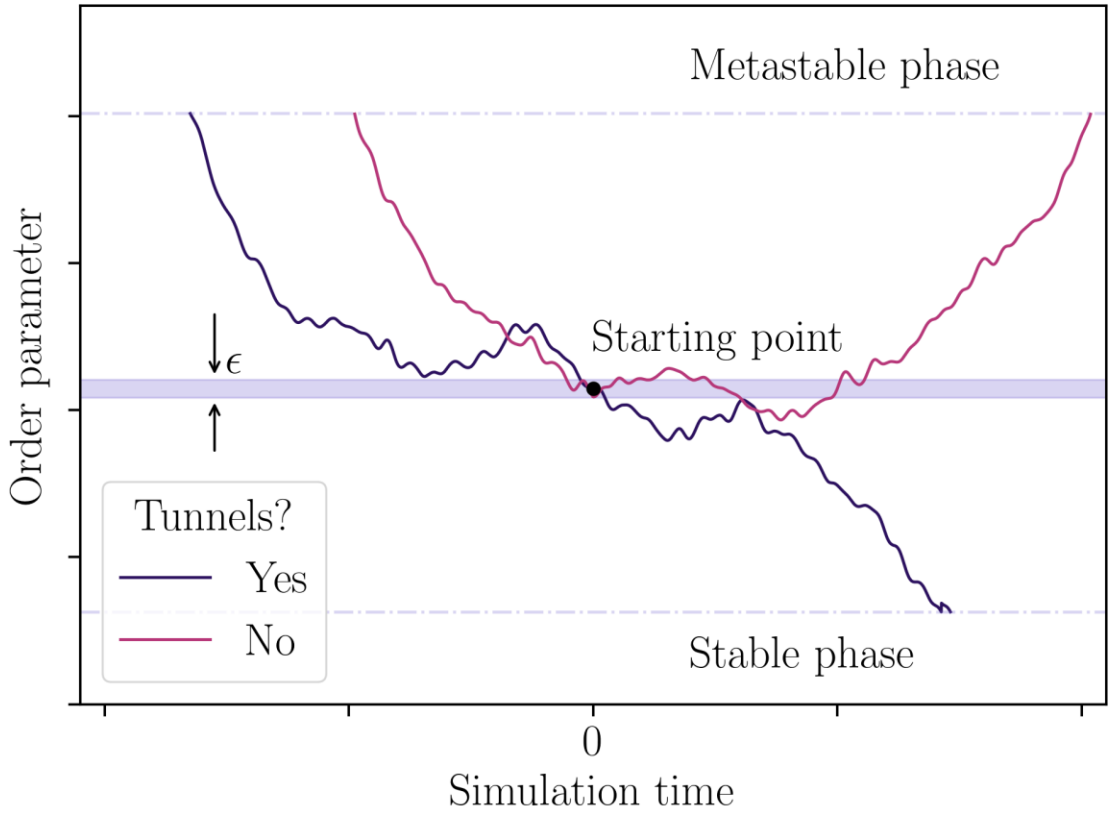
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4. Evolve trajectories backwards and forwards in time to calculate the tunnelling fraction



Calculating bubble nucleation rates non-perturbatively

4. Evolve trajectories backwards and forwards in time to calculate the tunnelling fraction

$$\mathbf{d} = \frac{\delta_{\text{tunnel}}}{N_{\text{crossings}}}$$



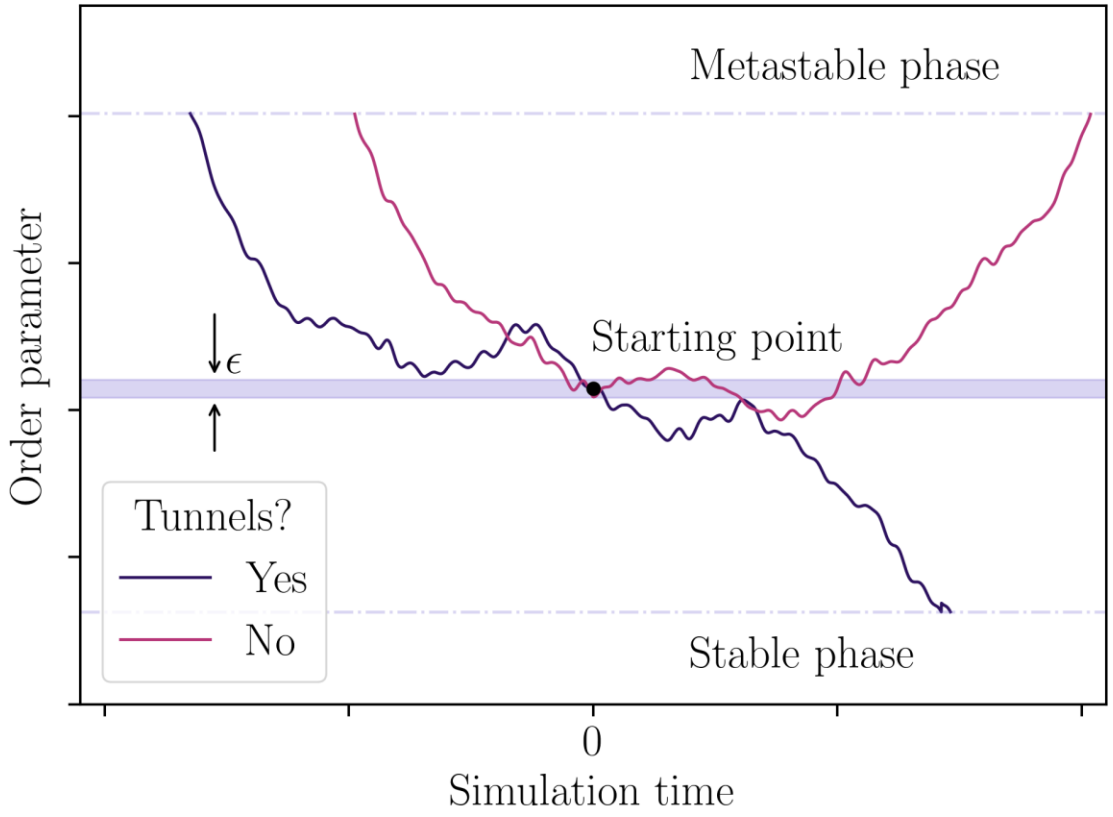
Calculating bubble nucleation rates non-perturbatively

5. Calculate the flux – the rate of change of the order parameter

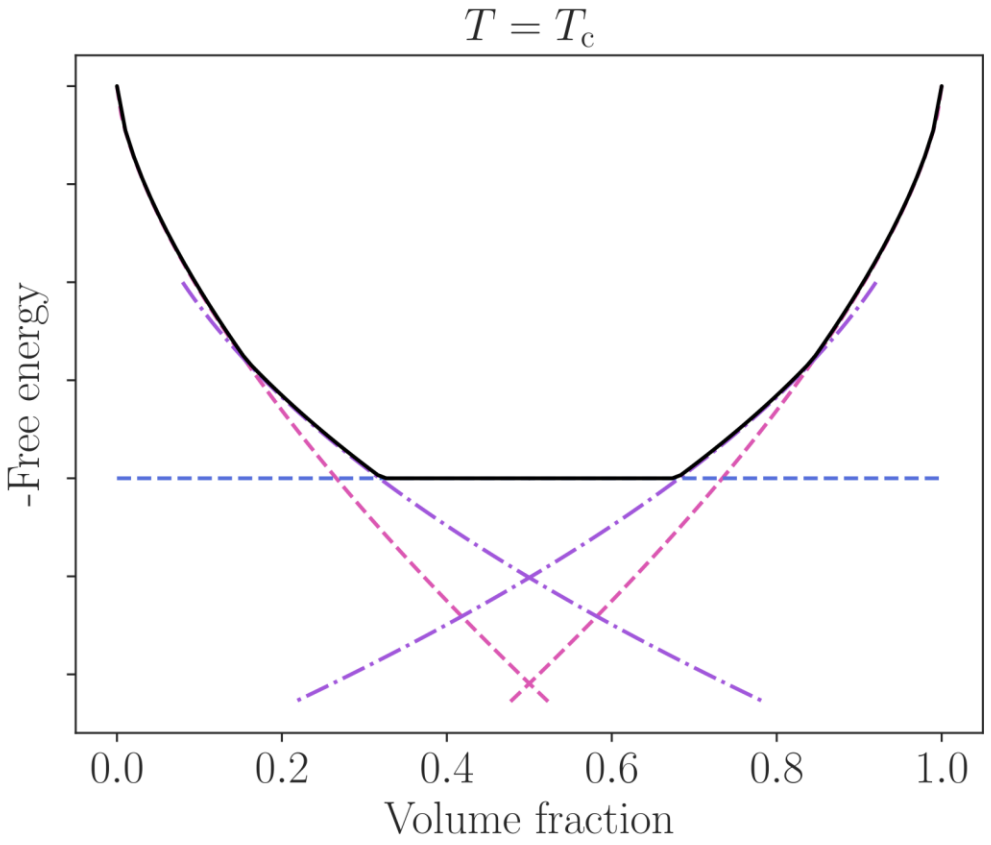
$$\langle \text{flux} \rangle = \left\langle \left| \frac{\Delta\theta}{\Delta t} \right|_{\theta_c} \right\rangle = \sqrt{\frac{8}{\pi\mathcal{V}}(\theta_c + A^2)}$$

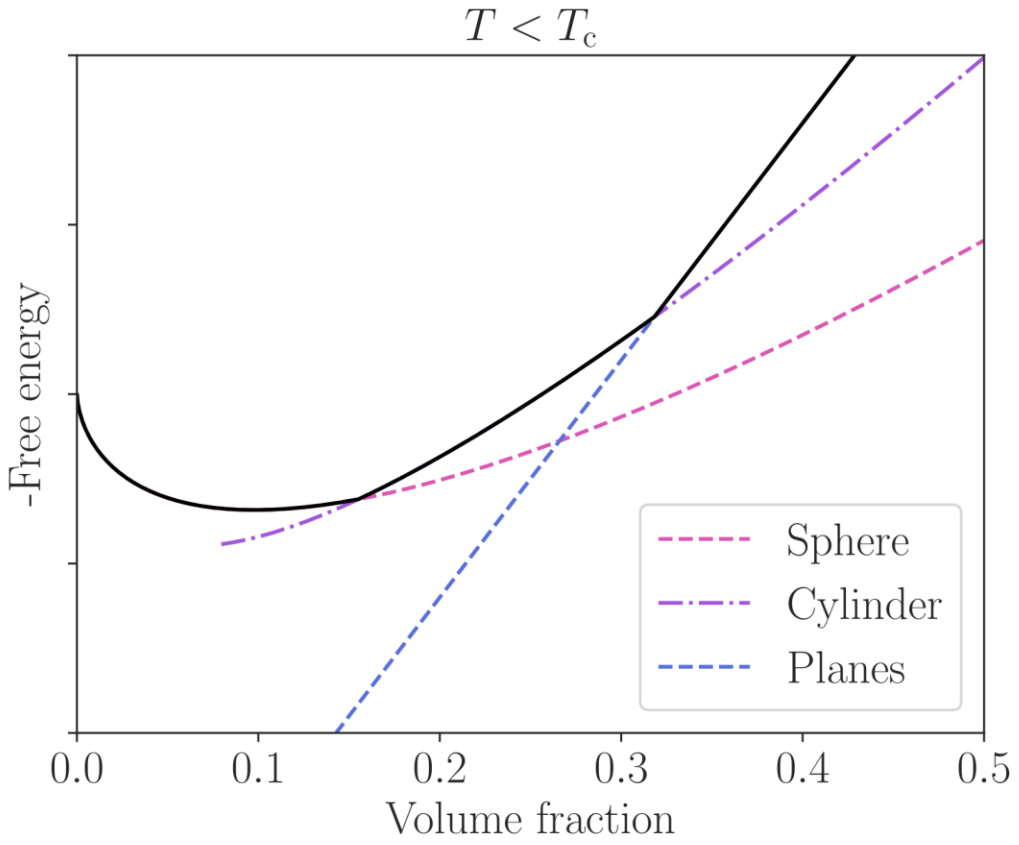
6. Compute the full rate

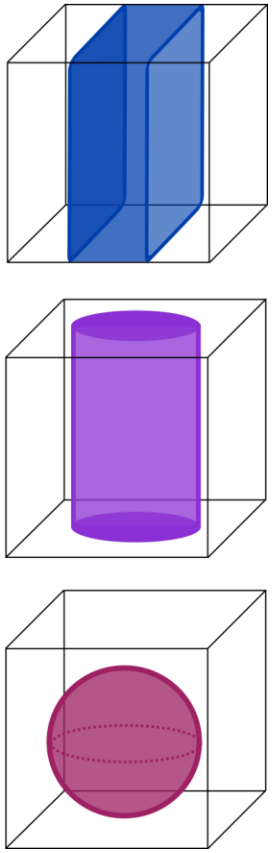
$$\Gamma\mathcal{V} \approx \frac{1}{2} P_c \langle \text{flux} \rangle \langle \mathbf{d} \rangle$$



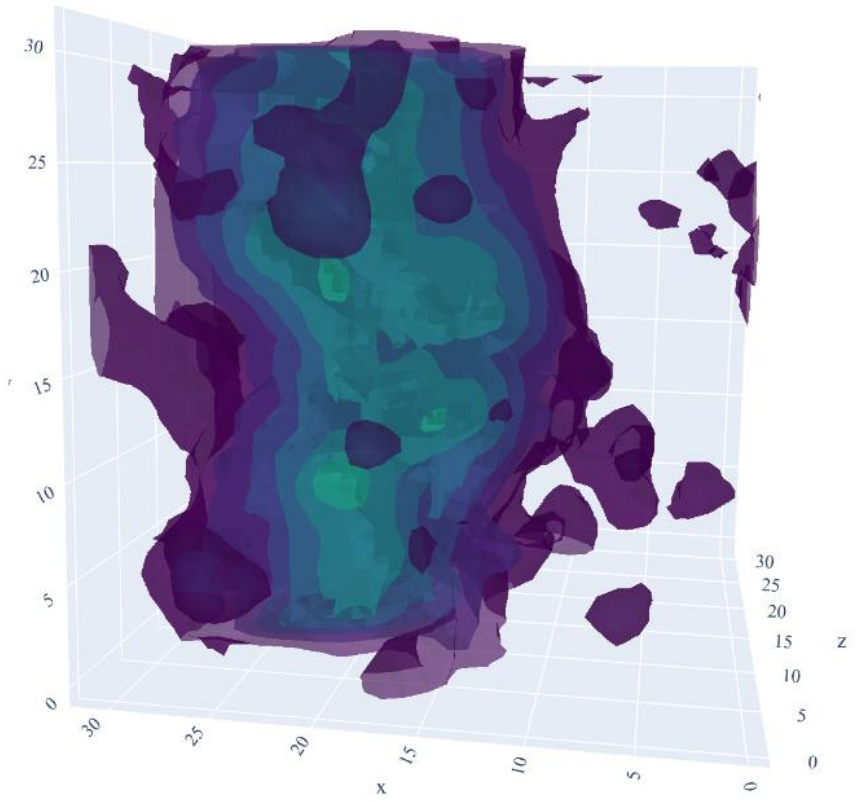
Finite volume battles vol 1: does the geometry allow for a bubble?

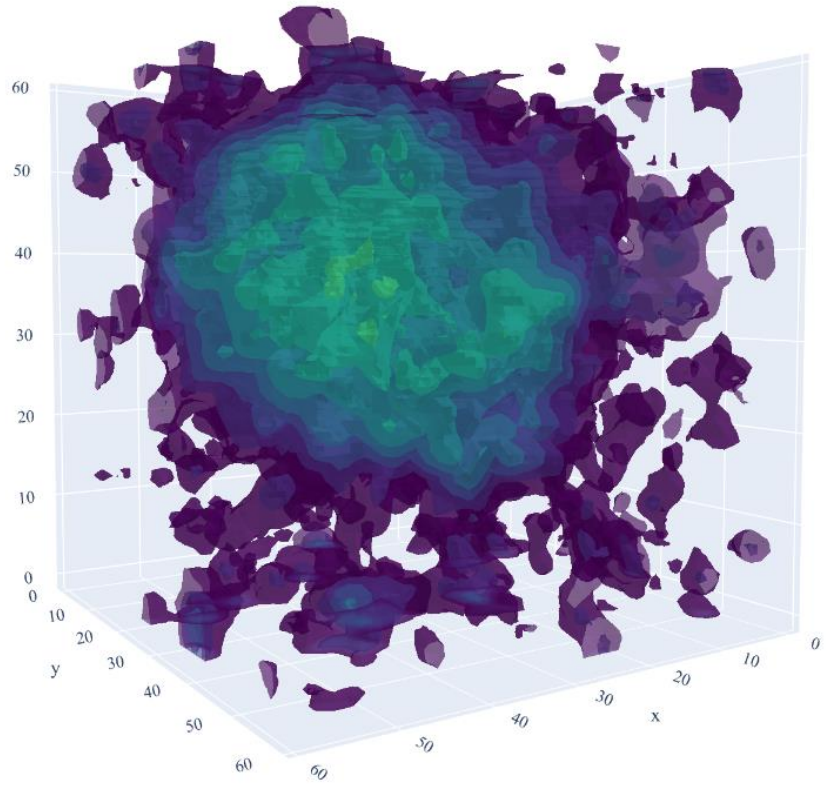




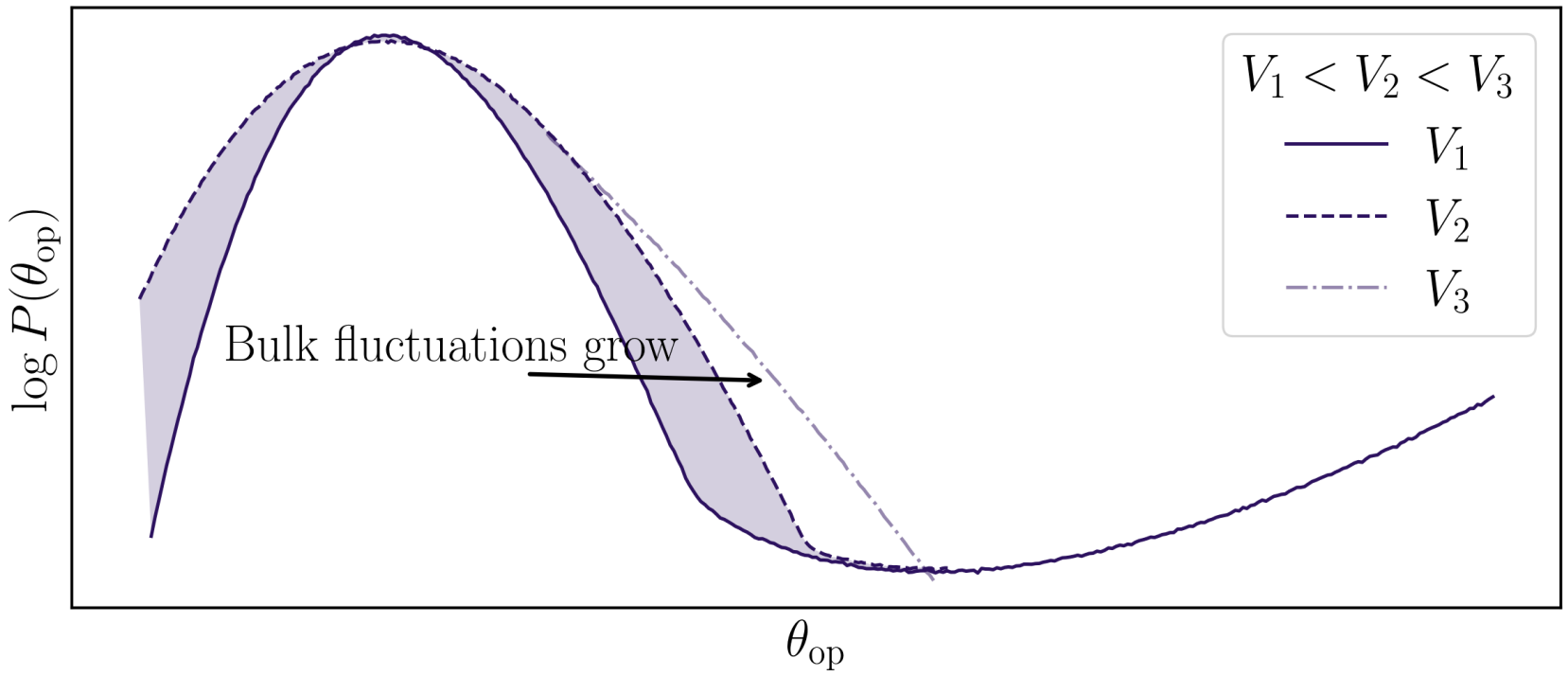


"Bubbles" in too small boxes

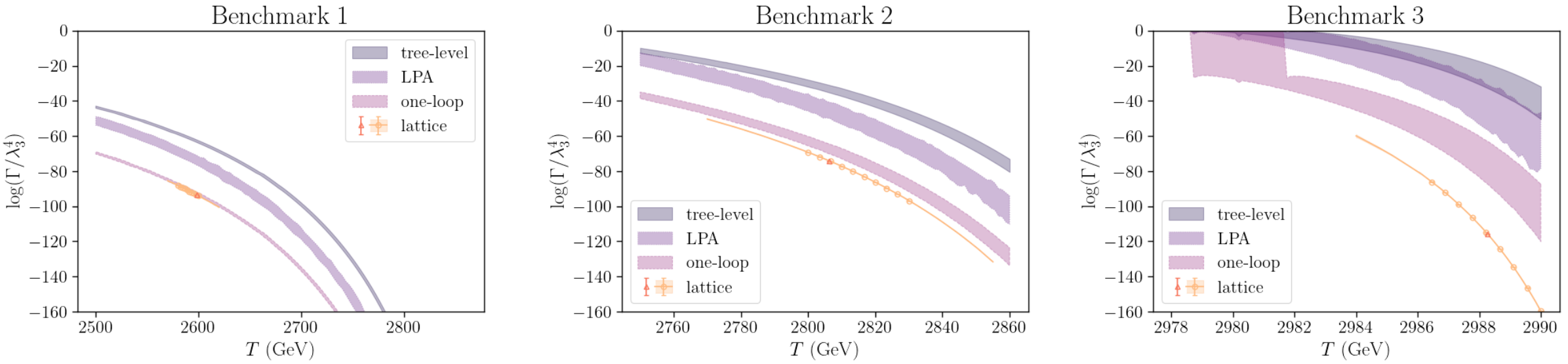




Finite volume battles vol 2: Fluctuations around the metastable phase



Lattice vs. Perturbation theory



Gould, AK, Weir (forthcoming)

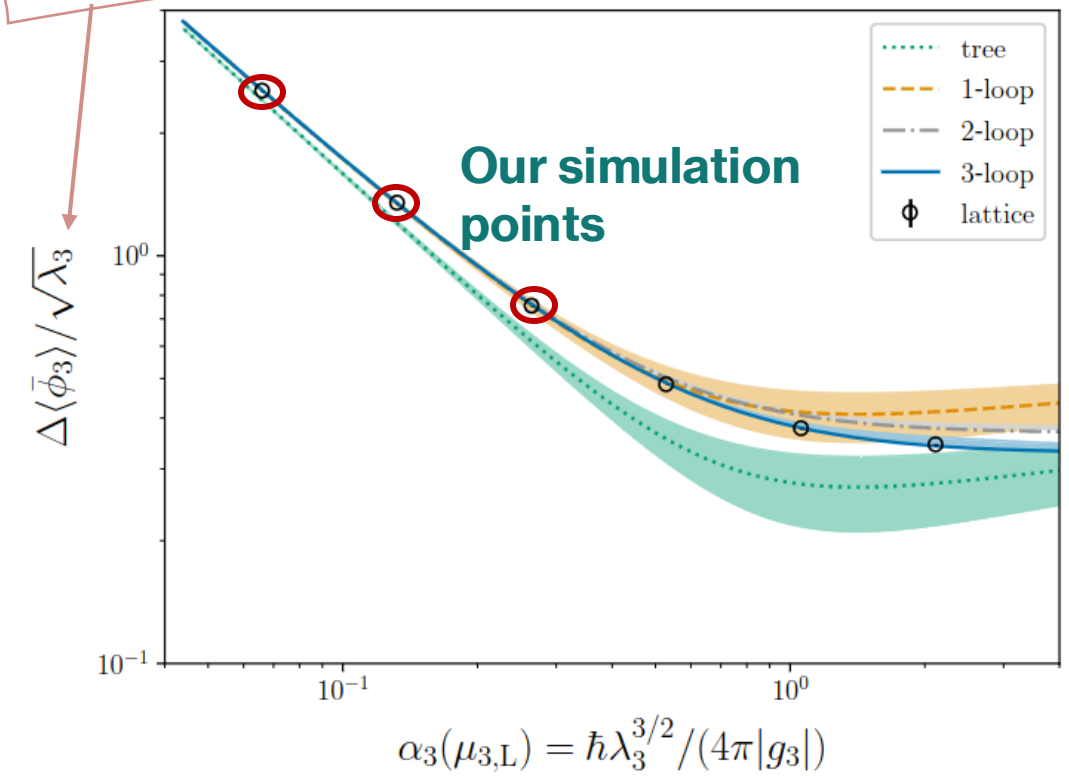
- **Tree-level**: functional determinant and dynamical prefactor approximated as T^4
- **LPA**: Local potential approximation
- **One-loop**: full one-loop calculation from BubbleDet, including the dynamical prefactor!
- **Lattice**: one simulated parameter point, reweighted to other temperatures

Do we fully understand bubble nucleation?

- Latent heat on the lattice vs. one-loop agree well at one-loop across all three benchmarks
- The disagreement on nucleation rates between one-loop and lattice is:
  - BM1: 1%
  - BM2: 9%
  - BM3: 33%
- Do we understand the discrepancy? Semiclassical breaks down? Something else?

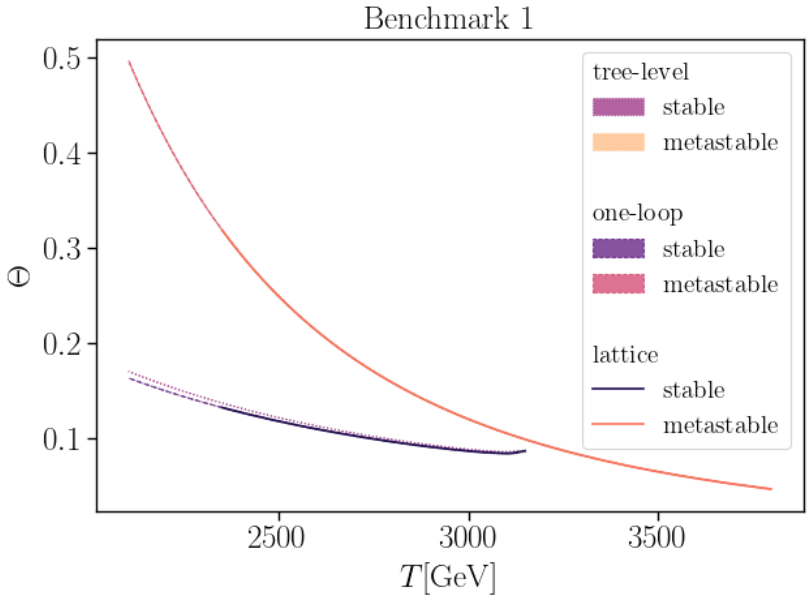
Proportional to latent heat

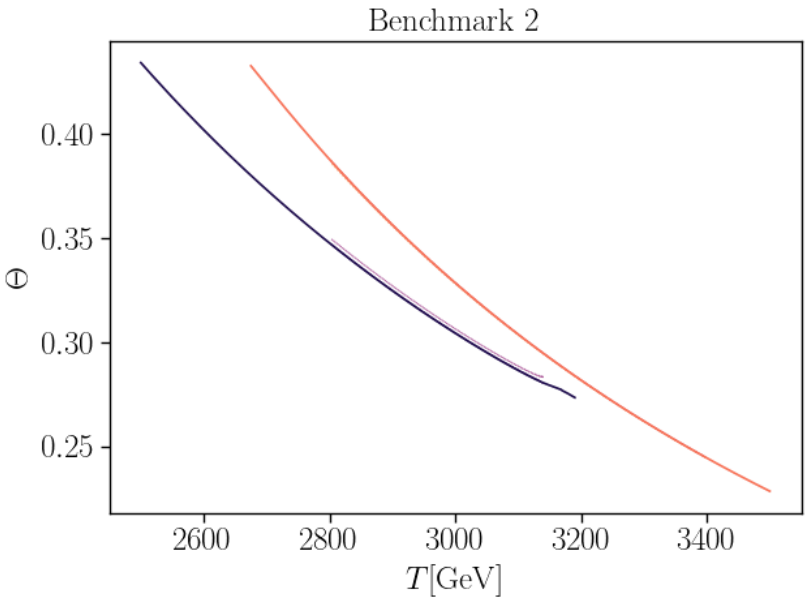
Gould, arXiv:2101.05528

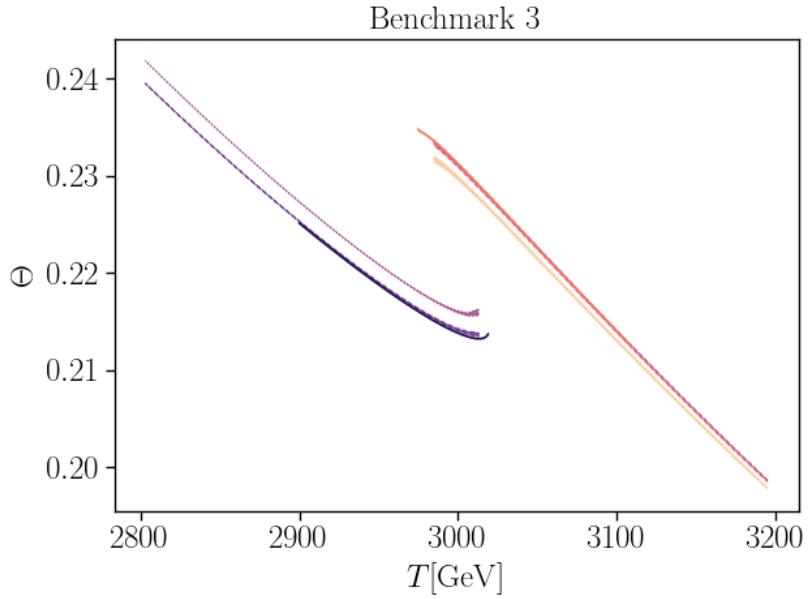


Loop expansion parameter

Gould, AK, Weir (forthcoming)





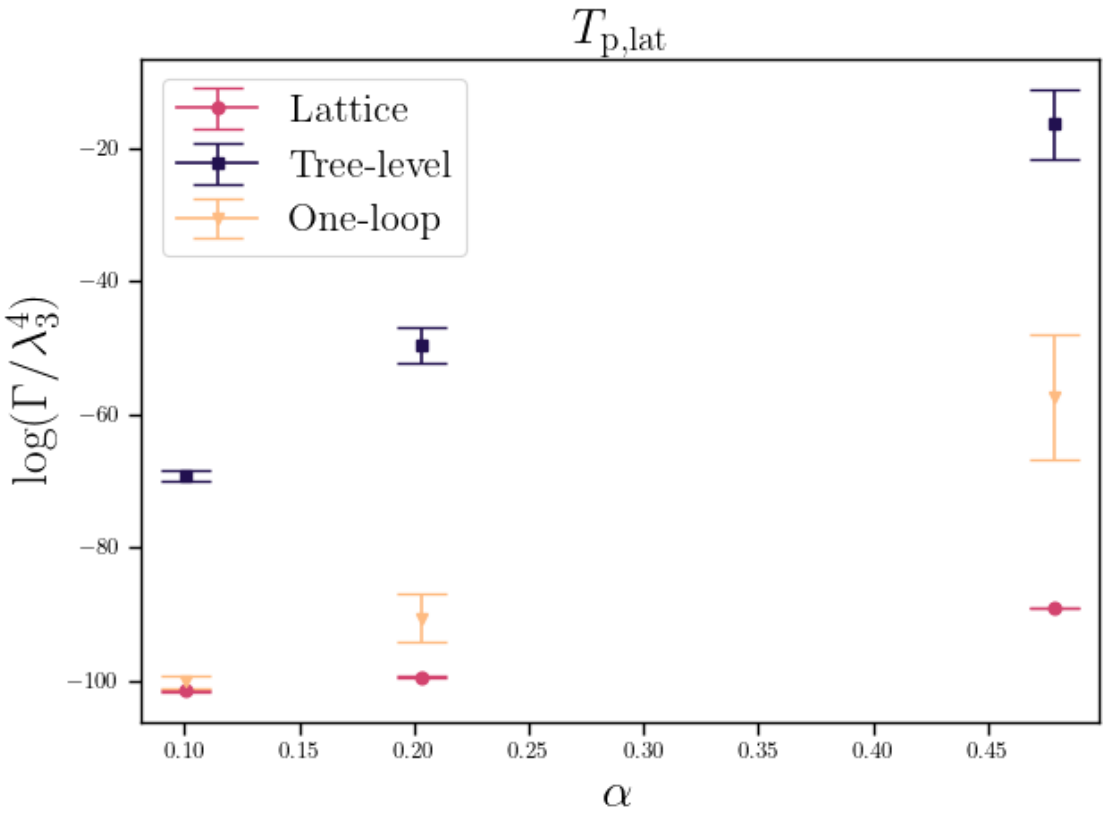


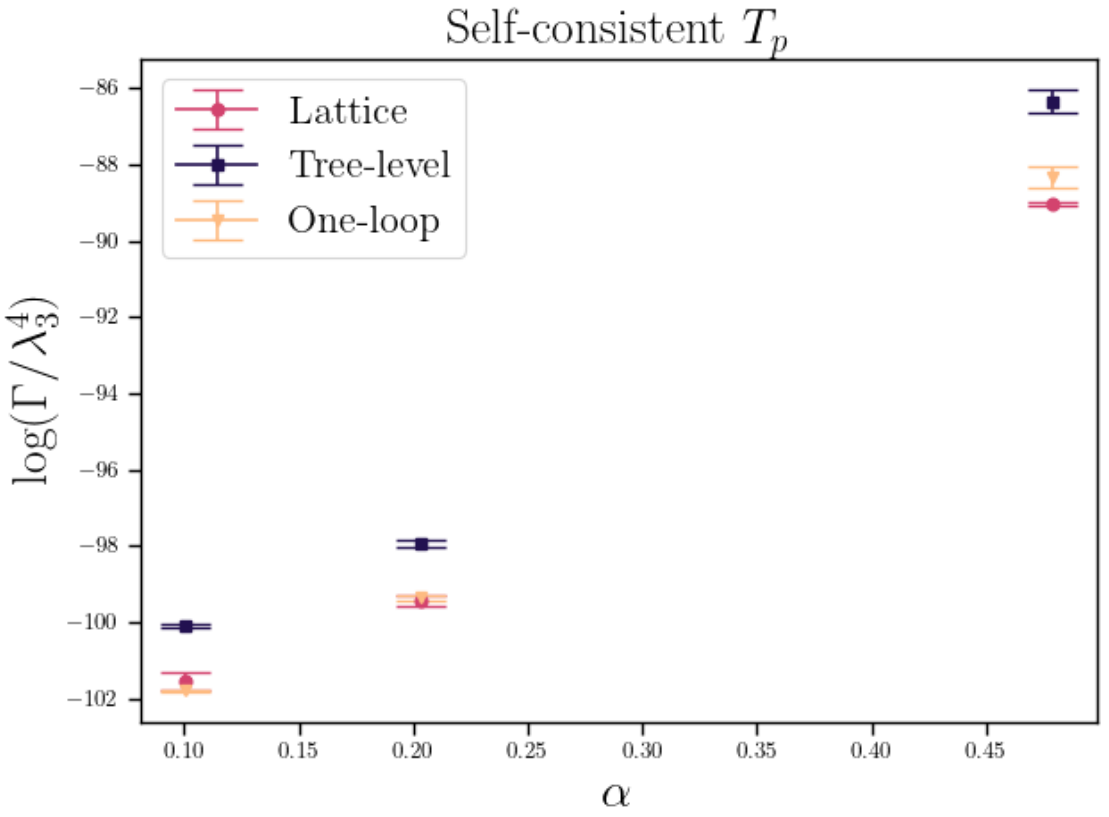
Thermodynamics

- Based on our lattice data we can determine the interaction measure (sometimes also referred to as the trace anomaly)

$$\Theta = T\frac{d}{dT}\left(\frac{p(T)}{T^4}\right)$$

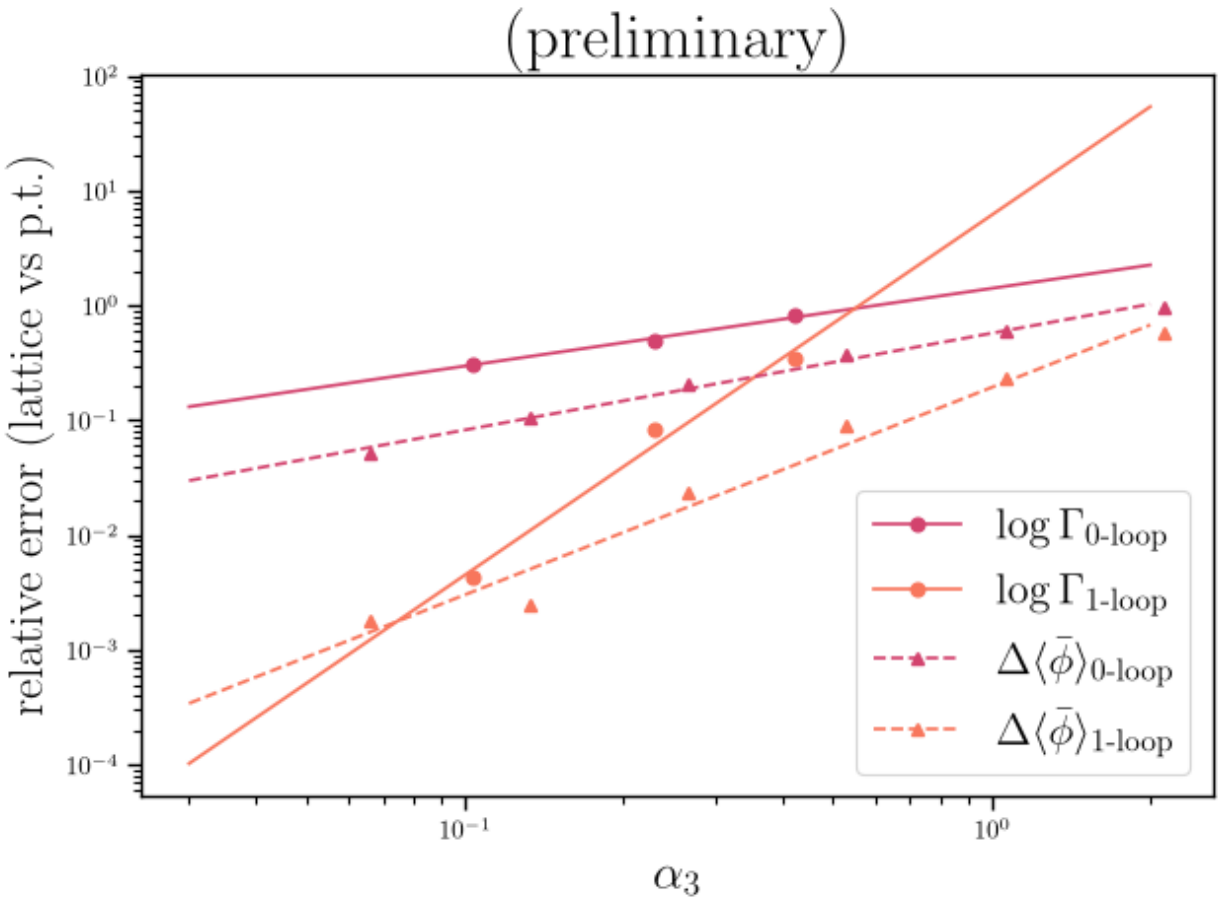
Nucleation rate evaluated at the percolation temperature





Gould, AK, Weir (forthcoming)

On convergence



Gould, AK, Weir (forthcoming)

- Linear condensate data from Gould, arXiv:2101.05528
- Erros larger in the nucleation rate, but perturbation theory converges (broadly) towards the lattice results as expected

Conclusions

- Allows us to calibrate the uncertainty in PT parameters when obtained from perturbative results
- The thermodynamics of the system are well captured by perturbation theory
- Perturbation theory struggles to capture the dynamics of the bubble nucleation at weaker transitions

One-bubble takeaway

Perturbative calculations do converge towards the lattice result as expected