A consistent EFT expansion of the bubble nucleation rate

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in collaboration with

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Effective Field Theories in Dimensional Reduction

$$\mathcal{L}_{\rm QCD} = \sum_q \overline{\psi}_q (i\gamma^\mu D_\mu - m)\psi_q - \frac{1}{4} G^{\mu\nu\,a} G^a_{\mu\nu} \longrightarrow V_{\rm eff}(T)\,?$$



High-Temperature
Dimensional Reduction

$$\left[\mathcal{L} \implies \mathcal{L}_{\rm 3D\ EFT}\right]$$

$$\begin{aligned}\mathcal{L}_{\rm hot\ QCD} =& \frac{1}{4} G^a_{ij} G^a_{ij} + \frac{1}{2}(D_i G^a_0)^2 + \frac{1}{2} m_D(T)\,(G^a_0)^2 + \lambda(T)\,(G^a_0)^4 \\ &+ c_1(T)(D_i G^a_{ij})^2 + c_2(T) f^{abc} G^a_{ij} G^b_{jk} G^c_{kl} + c_3(T)(D^2 G^a_0)^2 + c_4(T)(G^a_0)^2 (G^a_{ij})^2 \\ &+ c_5(T)(G^a_0)^6 + \dots\end{aligned}$$

[Fabio's and Luis' talks]

Effective Field Theories in Dimensional Reduction

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_0 + \varepsilon\mathcal{L}_1 + \dots \qquad \left(\varepsilon = \frac{m}{\Lambda}\right)$$

Equivalence Theorem: Observables a recomputed through a path integral $\int \mathcal{D}\phi$



Naturally invariant under invertible field redefinitions (FRs)

—perturbative—

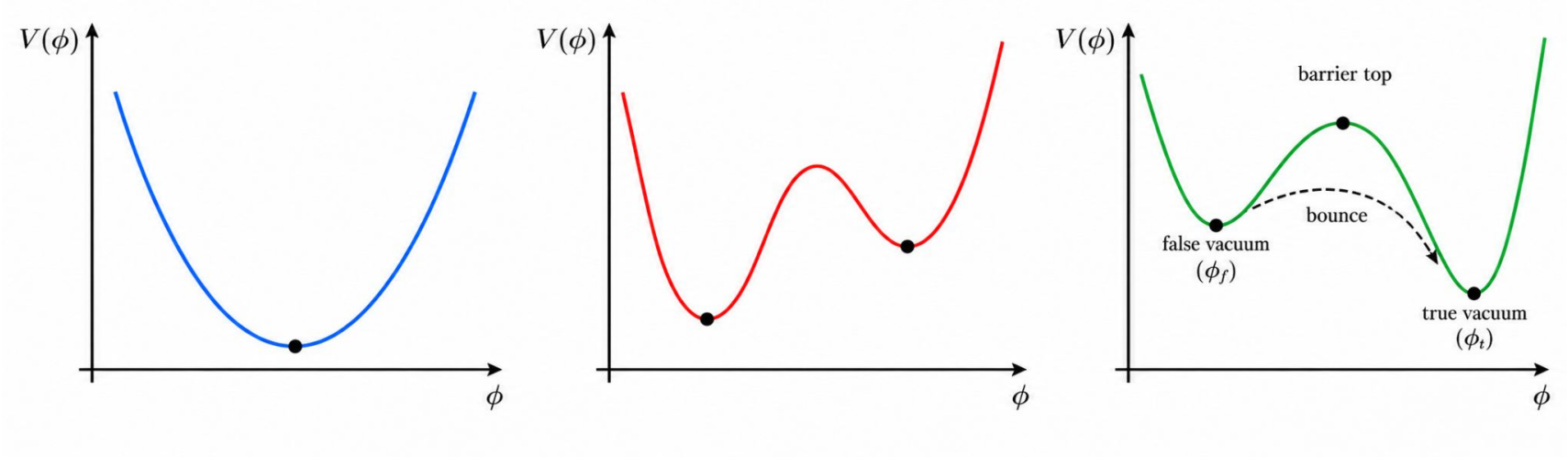
$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 + \varepsilon\left[2c\phi(\partial_\mu\phi)^2 - cm^2\phi^3\right]$$

$\mathcal{A}(\phi\phi \to \phi\phi) = 0$

$\phi \to \phi - \varepsilon c\phi^2$ $\Rightarrow$ $\mathcal{L}' = \frac{1}{2}(\partial_\mu\phi)^2 + \mathcal{O}(\varepsilon^2)$

Bubble nucleation: the nucleation rate

$$\Gamma = \frac{1}{\mathcal{T}\mathcal{V}} \mathrm{Prob}\left(\right)$$



Bubble nucleation: the nucleation rate

$$\Gamma_{\rm NLO} = A_{\rm dyn} \times \left[\frac{\widetilde{\det}(S_3''[\varphi_b])}{\det\left(S_3''[\varphi_{\rm FV}]\right)} \right]^{-1/2} e^{-S_3[\varphi_b]}$$

The $O(3)$ bounce solution satisfies:

$$S'[\varphi_b] = 0$$
$$\varphi_b'(0) = 0\,, \quad \varphi_b(\infty) = \varphi_{\rm FV}$$

How to find the bounce? ⟶ Shooting algorithm (`CosmoTransitions`, `FindBounce`...)

How to find the determinant? ⟶ Galfand-Yaglom (`BubbleDet`)

[Wainwright 1109.4189, Guada et al. 2002.00881, Esktedt et al. 2308.15652, ...]

Higher-order effective operator: challenges

One should keep in S_3 all **effective operators** coming from dimensional reduction

(up to a given power-counting order)

$$S_3[\phi] = \int_x \left[\frac{1}{2}(\partial_i \phi)^2 + V(\phi) + \text{higher derivative operators} \right]$$

Challenges

* $\ddot{\varphi}_b(r) + \frac{2}{r}\dot{\varphi}_b(r) - V'(\varphi_b(r)) + (\text{other derivative terms})(\varphi_b(r)) = 0$

  - non-trivial to solve
  - numerical instability
  - no public codes

* $\det\left[-\partial^2 + V''(\varphi_b) + (\text{other derivative terms})(\varphi_b(r))\right]$

  - out of validity of GY *
  - no public codes

*[Bernardo et al. 2610.12284]

Higher-order effective operator: challenges

How do we **consistently** include all kind of **higher-dimension effective operators** in the computation of Γ?

How can we be sure that Γ stays the **same** after exploiting **redundancies** in S (gauge fixing, field redefinitions, etc.)?

Strict perturbative expansion

A strict perturbative expansion

Start from:

$$S[\phi] = S_0[\phi] + \varepsilon S_1[\phi] + \dots$$

Goal:

$$\log \Gamma = \log \Gamma_0 + \varepsilon \log \Gamma_1 + \dots$$

Proposal:

1. Compute the perturbative correction to the bounce $\varphi_b(r) = \varphi_0(r) + \varepsilon \varphi_1(r) + \dots$
2. Compute the perturbative correction to $\log \Gamma = -S[\varphi_b] - \frac{1}{2} \log \det \left(S''[\varphi_b] \right)$
   - 2.a. $-S[\varphi_b] = -B_0 - \varepsilon B_1 + \dots$
   - 2.b. $-\frac{1}{2} \log \det \left(S''[\varphi_b] \right) = -F_0 - \varepsilon F_1 + \dots$

Corrections to the bounce

1. Compute the perturbative correction to the bounce $\varphi_b(r) = \varphi_0(r) + \varepsilon\varphi_1(r) + \dots$

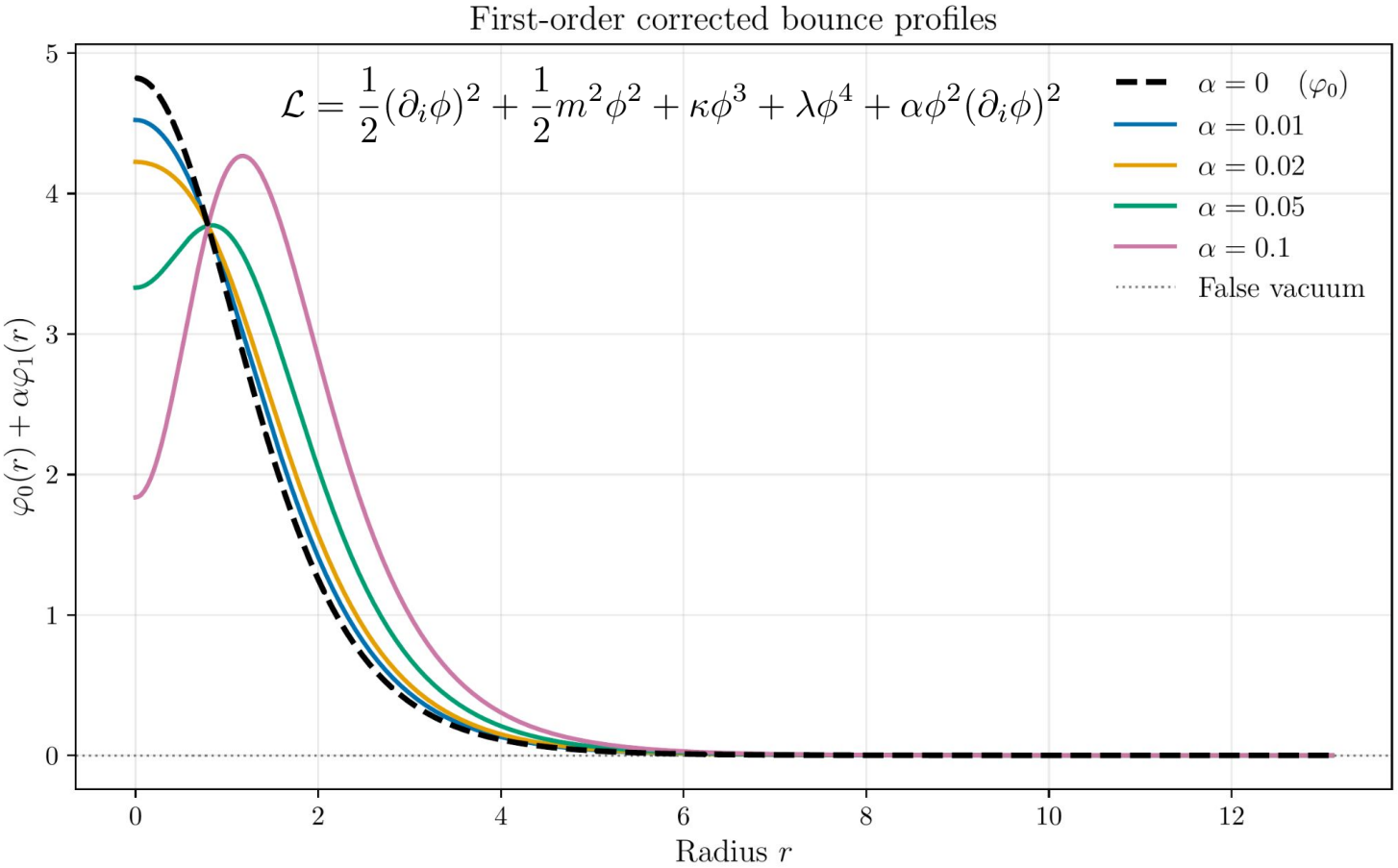
$$\begin{aligned} S'[\varphi_b] &= (S'_0 + \varepsilon S'_1)[\varphi_0 + \varepsilon\varphi_1] \\ &= S'_0[\varphi_0] + \varepsilon\big(S''_0[\varphi_0]\varphi_1 + S'_1[\varphi_0]\big) \end{aligned}$$

$$S'_0[\varphi_0] = 0 \implies \partial^2\varphi_0 - V'_0(\varphi_0) = 0 \qquad \text{(usual bounce eq.)}$$

$$S''_0[\varphi_0]\varphi_1 + S'_1[\varphi_0] = 0 \implies \big(-\partial^2 + V''_0(\varphi_0)\big)\varphi_1 + S'_1(\varphi_0) = 0 \qquad \text{(2nd order ODE)}$$

[Chala et al. 2406.02667]

Corrections to the bounce



Corrections to the nucleation rate

2. Compute the perturbative correction to $\log \Gamma = -S[\varphi_b] - \frac{1}{2} \log \det \left(S''[\varphi_b] \right)$

2.a. $-S[\varphi_b] = -B_0 - \varepsilon B_1 + \dots$

$$S[\varphi_b] = \underbrace{S_0[\varphi_0]}_{B_0} + \varepsilon \left(\underbrace{S_1[\varphi_0]}_{B_1} + \cancel{S_0'[\varphi_0]\varphi_1} \right) + \dots$$

→ Numerical integrals over φ_0 ✓

2.b. $-\frac{1}{2} \log \det \left(S''[\varphi_b] \right) = -F_0 - \varepsilon F_1 + \dots$

$\left[\log \det(A) = \operatorname{Tr} \log(A) \right]$

$\left[G \equiv (S_0''[\varphi_0])^{-1} \right]$

$$\operatorname{Tr} \log(S''[\varphi_b]) = \operatorname{Tr} \log \left[S_0''[\varphi_0] + \varepsilon \left(S_1''[\varphi_0] + S_0'''[\varphi_0]\varphi_1 \right) \right]$$

$$= \operatorname{Tr} \log \left(S_0''[\varphi_0] \right) + \operatorname{Tr} \log \left[1 + \varepsilon G \left(S_1''[\varphi_0] + S_0'''[\varphi_0]\varphi_1 \right) \right]$$

Corrections to the nucleation rate

2.b. $-\frac{1}{2}\log\det\left(S''[\varphi_b]\right) = -F_0 - \varepsilon F_1 + \dots$ $\quad\left[\log\det(A) = \mathrm{Tr}\log(A)\right]$

$\left[G \equiv \left(S_0''[\varphi_0]\right)^{-1}\right]$

$$\begin{aligned}
\mathrm{Tr}\log(S''[\varphi_b]) &= \mathrm{Tr}\log\left[S_0''[\varphi_0] + \varepsilon\left(S_1''[\varphi_0] + S_0'''[\varphi_0]\varphi_1\right)\right] \\
&= \mathrm{Tr}\log\left(S_0''[\varphi_0]\right) + \mathrm{Tr}\log\left[1 + \varepsilon G\left(S_1''[\varphi_0] + S_0'''[\varphi_0]\varphi_1\right)\right] \\
&= \underbrace{\mathrm{Tr}\log\left(S_0''[\varphi_0]\right)}_{F_0} + \varepsilon\,\underbrace{\mathrm{Tr}\left[G\left(S_1''[\varphi_0] + S_0'''[\varphi_0]\varphi_1\right)\right]}_{F_1} + \dots
\end{aligned}$$

$F_0 \longrightarrow \mathrm{Tr}\log(S_0''[\varphi_0]) = \log\det(S_0''[\varphi_0])$ GY method (`BubbleDet`) 

$F_1 \longrightarrow$ 

The functional trace

The functional trace is

$$\mathrm{Tr}(GM) = \int_x \int_y G(x,y) M(x,y) \delta(x-y)$$

The propagator $G(x,y)$ satisfies

$$G(x,y) \equiv (S_0''[\varphi_0])^{-1} \implies [-\partial_x^2 + V_0''(\varphi_0(x))] G(x,y) = \delta(x-y)$$

The trace is in general divergent. We use dimensional regularization and partial-wave decomposition

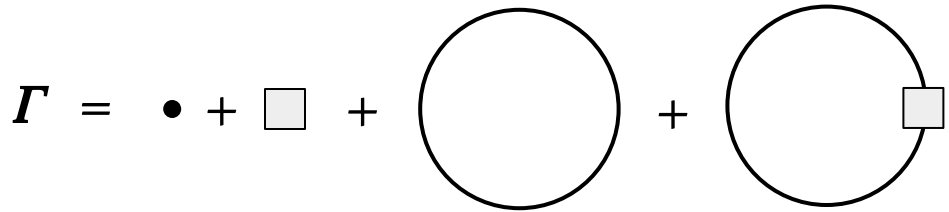
$$G(\vec{x},\vec{x}) = \sum_{\ell=0}^{\infty} (2\ell+1) G_\ell(r)$$

to transform the divergence in the integral into a divergence in the series:

$$\mathrm{Tr}(GM) = \sum_{\ell=0}^{\infty} \underbrace{\int_0^\infty \mathrm{d}r\, r^2 f_\ell(r)}_{\text{convergent}}$$

[Based on Ekstedt 2201.07331, Ekstedt et al. 2308.15652]

The nucleation rate as a diagrammatic expansion



$$= B_0 = S_0[\varphi_0(x)] = \widetilde{S}_0[\widetilde{\varphi}_0(p)]$$

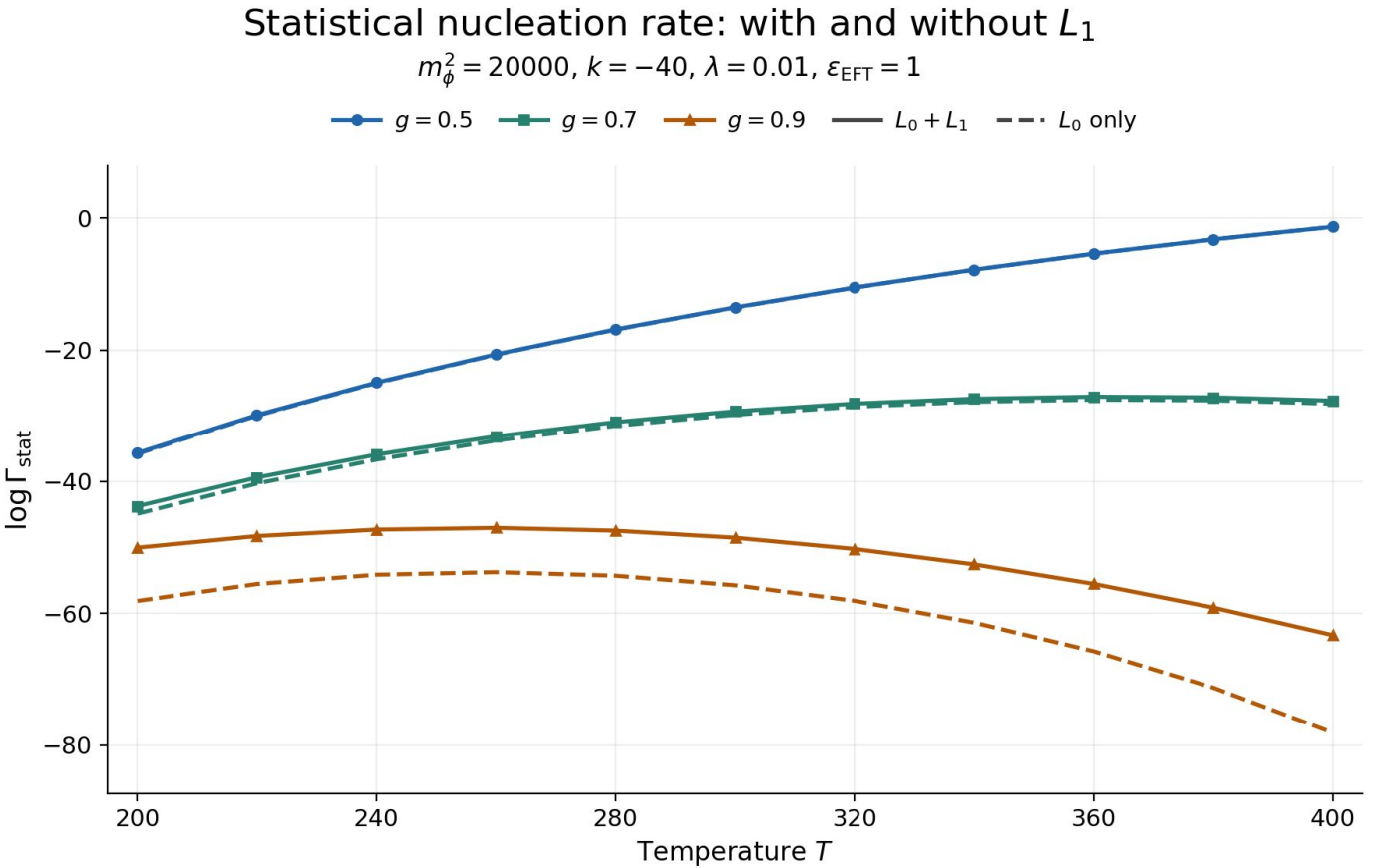
$$= B_1 = S_1[\varphi_0(x)] = \widetilde{S}_1[\widetilde{\varphi}_0(p)]$$

$$= F_1 = \log\det\Big(S_0''[\varphi_0(x)]\Big) = \int_p \log(p^2 + \widetilde{m}^2(p))$$

$$= F_2 = \mathrm{Tr}(GM) = \int_p \frac{1}{p^2 + \widetilde{m}^2(p)}\widetilde{M}(p)$$

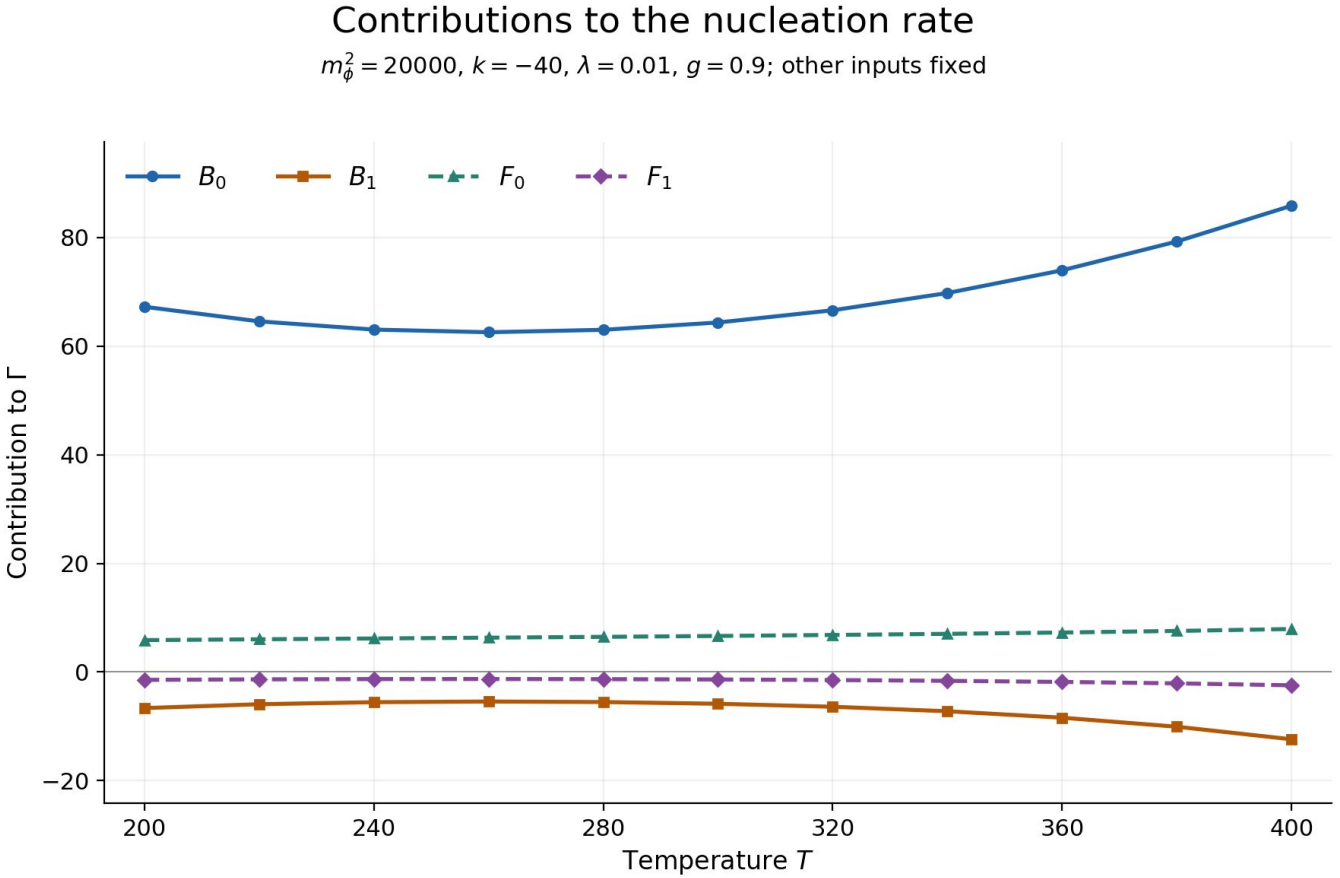
An example

$$\mathscr{L}_4 = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \kappa\phi^3 - \lambda\phi^4 + \overline{\psi}i\partial\!\!\!/\psi - g\phi\overline{\psi}\psi$$



An example

$$\mathscr{L}_4 = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \kappa\phi^3 - \lambda\phi^4 + \overline{\psi}i\partial\!\!\!/\psi - g\phi\overline{\psi}\psi$$



Take-home messages

- **Higher-order effective operators** are **unavoidable** in **thermal field theory**. This is a direct consequence of Dimensional Reduction.

- We derived a **systematic method** for treating **general operators** in an EFT. Effective operators are inserted as **corrections** in a **strict perturbative expansion** of Γ. No modifications to shooting algorithms or the Galfand-Yaglom method are needed.

- The value of Γ is **inherently invariant under field redefinitions**. The expansion allows for the evaluation of **different contributions individually**, providing **criteria** for the **consistent truncation** of the EFT expansion.