

# Higgs vacuum stability and a gravity-induced UV closure of the Standard Model


## Aaron Held

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École Normale Supérieure



Oct 21, 2026: VAST 2026: From Vacuum Stability to Gravitational Waves
Real Jardín Botánico, CSIC, Madrid



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**LUX** – **L**aboratory for the study of the **U**niverse and e**X**treme phenomena
Observatoire de Paris / CNRS


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# Motivation: Consistency of the Standard Model
and a first hint of Planck-scale physics

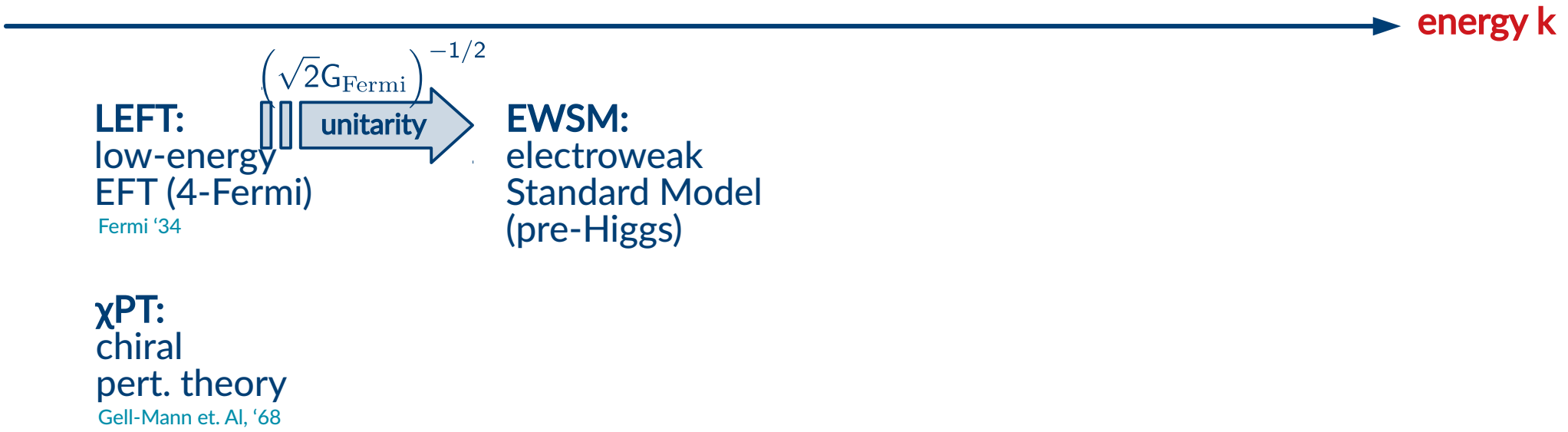
# The Standard Model (and gravity) …




**LEFT:**
low-energy
EFT (4-Fermi)
Fermi ‘34

**χPT:**
chiral
pert. theory
Gell-Mann et. Al, ‘68

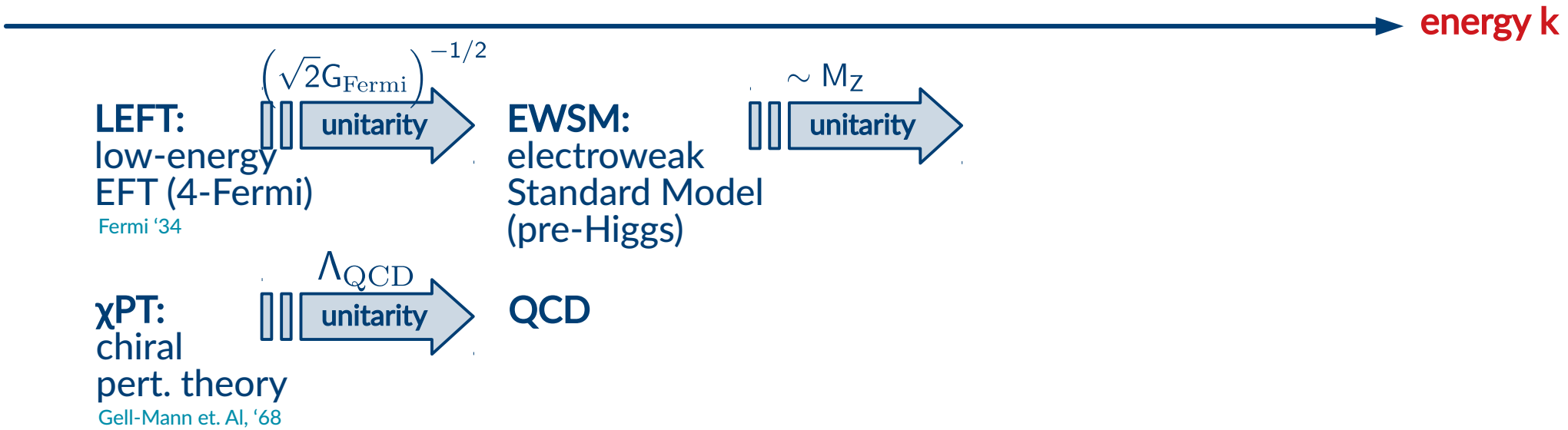
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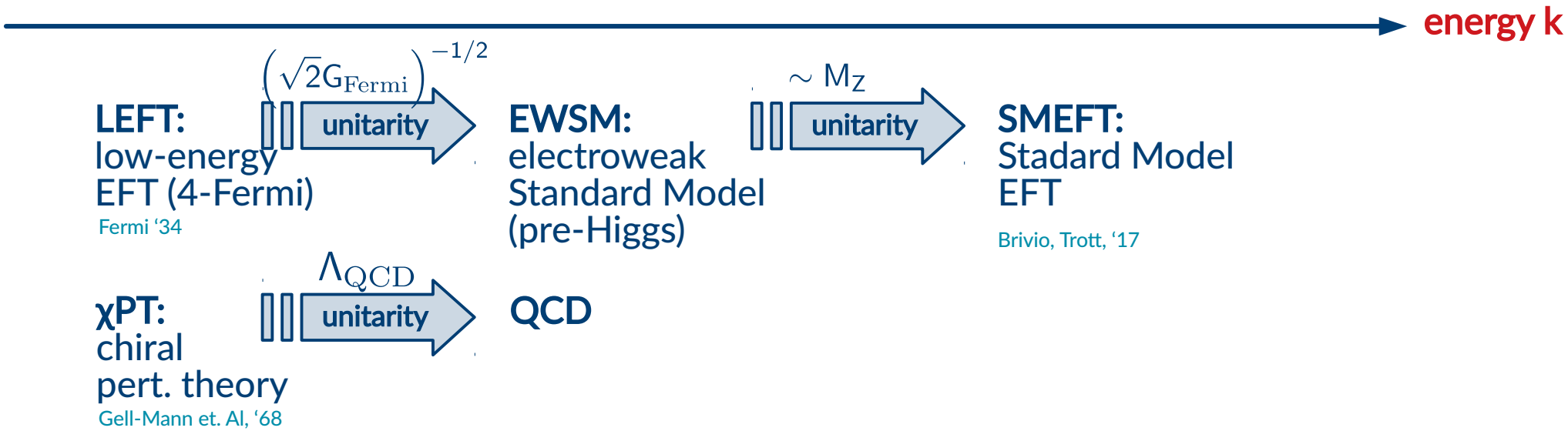
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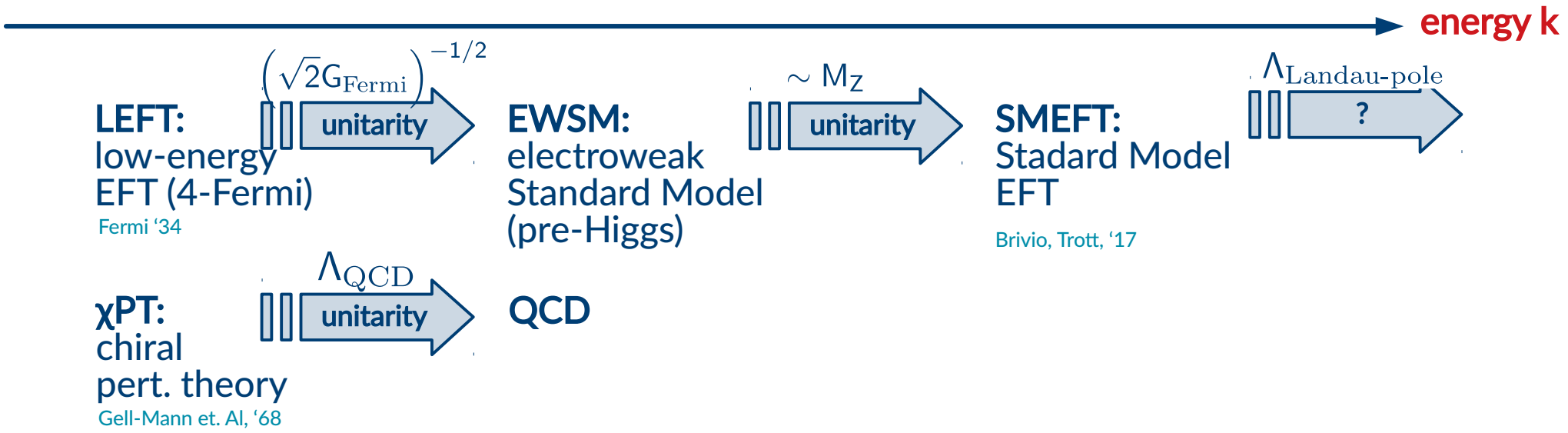
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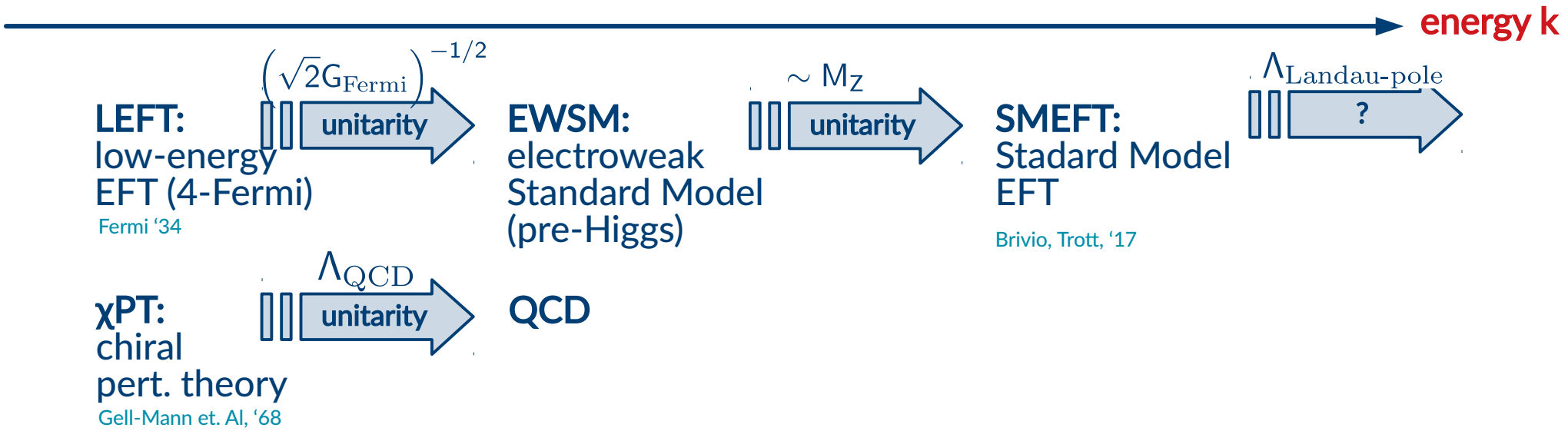
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# The Standard Model …

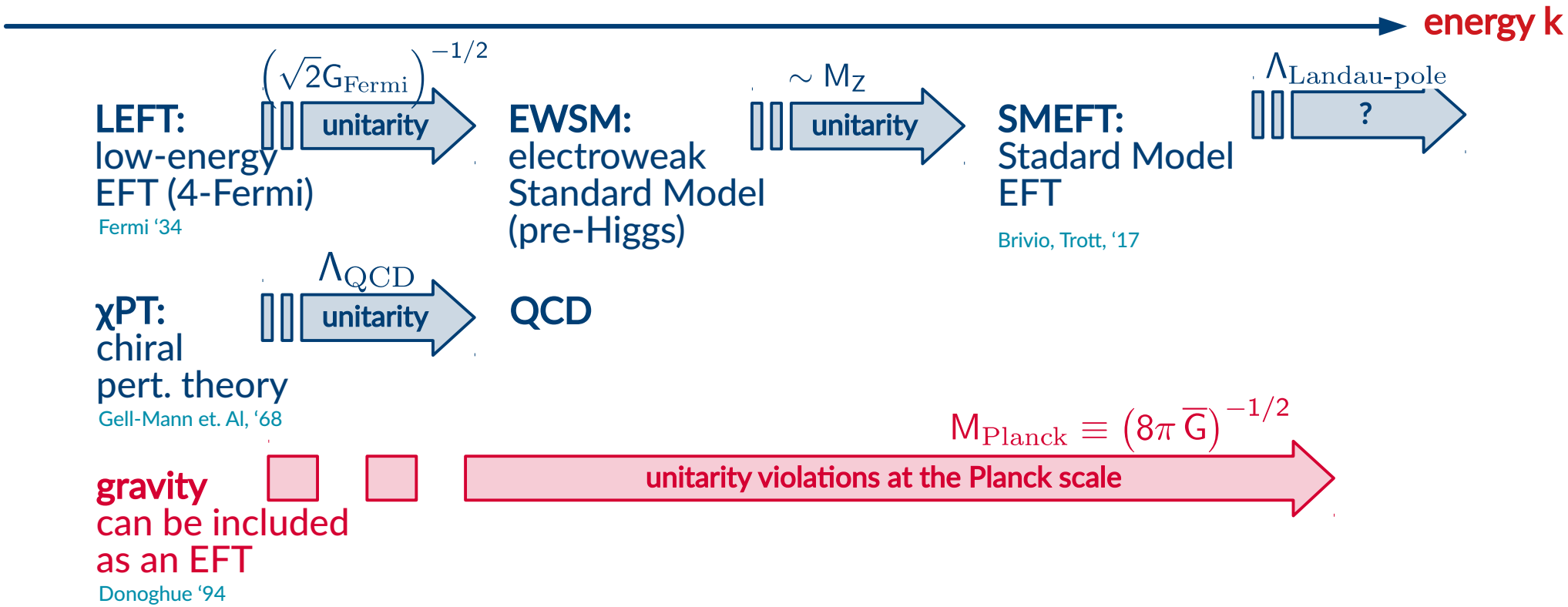


# The Standard Model ...




# ... can be extended up to the Planck scale.

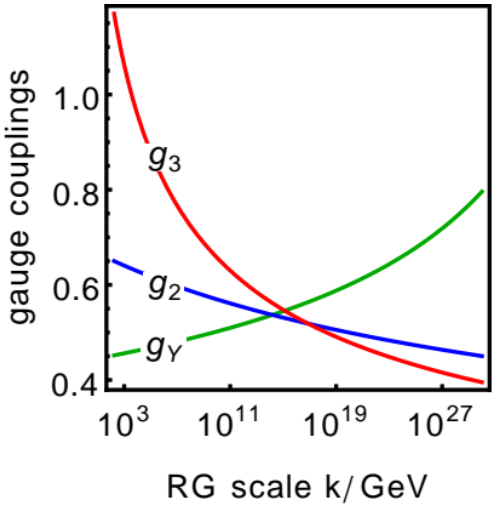
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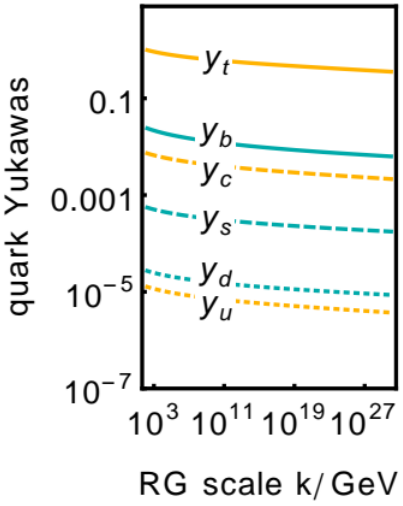

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# The Standard Model …

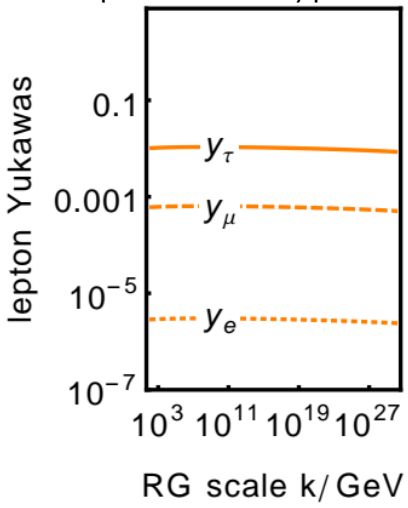
1-loop running of gauge couplings:
~ unification at $10^{15}$ GeV
consistent beyond $10^{19}$ GeV
U(1)-Landau pole at $10^{41}$ GeV



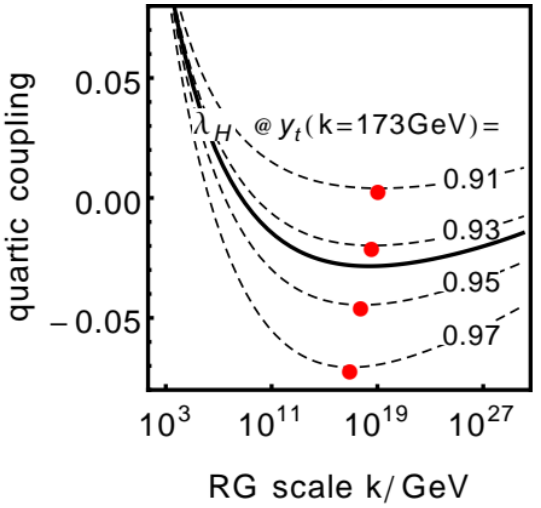

1-loop running of quark-Yukawas
Consistent >>$10^{19}$ GeV
peculiar hierarchy pattern




1-loop running of lepton-Yukawas:
consistent >>$10^{19}$ GeV;
peculiar hierarchy pattern



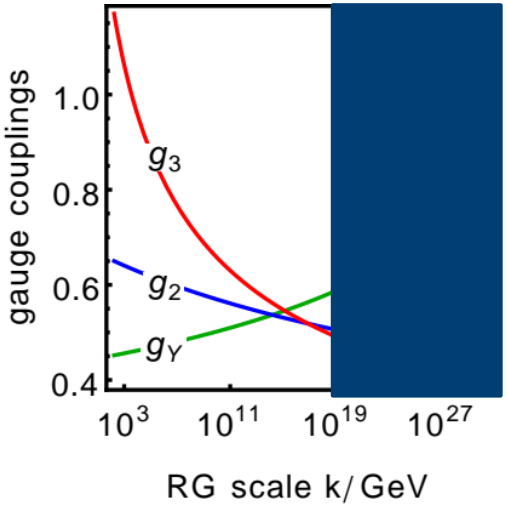

1-loop running of the Higgs-quartic for different values of the top-Yukawa coupling:
experimental values are consistent with exact **scale symmetry** at $10^{19}$ GeV



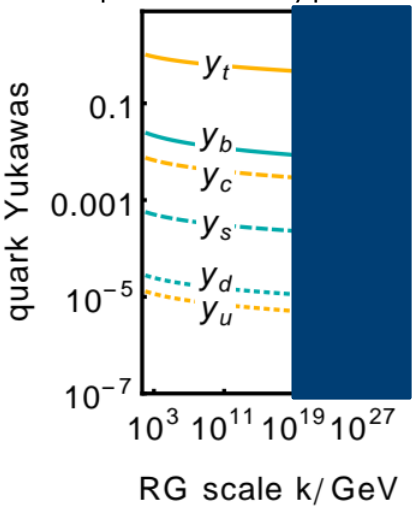

# … points to the Planck scale.

# The Standard Model …

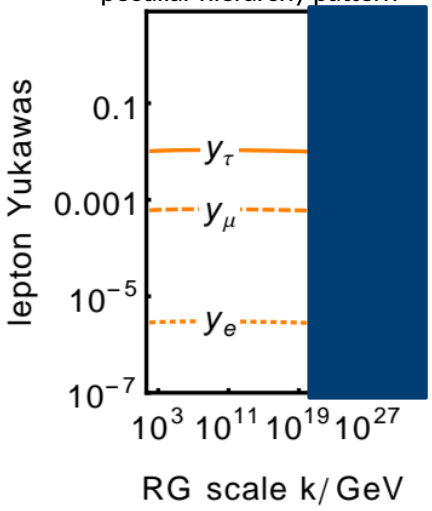
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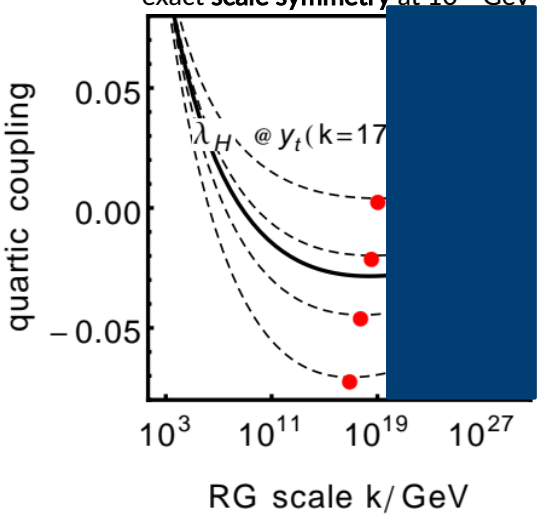

1-loop running of quark-Yukawas
Consistent >>$10^{19}$ GeV
peculiar hierarchy pattern




1-loop running of lepton-Yukawas:
consistent >>$10^{19}$ GeV;
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1-loop running of the Higgs-quartic for different values of the top-Yukawa coupling:
experimental values are consistent with exact **scale symmetry** at $10^{19}$ GeV




# … points to the Planck scale.

# Motivation: The EFT of gravity – The difficulties of quantising gravity

# General Relativity is only the leading term ...

$$\mathcal{L}^{(1)}_{\mathrm{EFT}} = \mathrm{M}^2_{\mathrm{Pl}}\,\mathrm{R}$$

$$\mathcal{L}^{(2)}_{\mathrm{EFT}} = \left[\alpha_0 \mathrm{R}_{\mathrm{ab}}\mathrm{R}^{\mathrm{ab}} - \beta_0\,\mathrm{R}^2\right]$$

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After reduction via

(i) index symmetries

(ii) geometric identities

(iii) 4D-specific identities (e.g. Gauss-Bonnet)

see Fulling CQG 9 (1992); Martin-Garcia, Yllanes, Portugal, CPC 179 (2008)

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$$\mathcal{L}^{(2)}_{\rm EFT} = \left[\alpha_0 \mathrm{R}_{\rm ab}\mathrm{R}^{\rm ab} - \beta_0\,\mathrm{R}^2\right]$$

$$\mathcal{L}^{(3)}_{\rm EFT} = \frac{1}{\mathrm{M}^2_{\rm Pl}}\Big[\alpha_1\,\mathrm{R}^{\rm ab}\,\Box\,\mathrm{R}_{\rm ab} - \beta_1\,\mathrm{R}\,\Box\,\mathrm{R} + \gamma_3\,\mathrm{C}_{\rm ab}{}^{\rm cd}\mathrm{C}_{\rm cd}{}^{\rm ef}\mathrm{C}_{\rm ef}{}^{\rm ab} + \delta_{3,1}\,\mathrm{C}_{\rm abcd}\mathrm{C}^{\rm abcd}\mathrm{R} + \delta_{3,2}\,\mathrm{C}_{\rm abcd}\mathrm{R}^{\rm ac}\mathrm{R}^{\rm bd} + \delta_{3,3}\,\mathrm{R}^{\rm b}_{\rm a}\,\mathrm{R}^{\rm c}_{\rm b}\,\mathrm{R}^{\rm a}_{\rm c} + \delta_{3,4}\,\mathrm{R}_{\rm ab}\mathrm{R}^{\rm ab}\mathrm{R} + \delta_{3,5}\,\mathrm{R}^3\Big]$$

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$$\mathcal{L}^{(4)}_{\rm EFT} = \frac{1}{\mathrm{M}^4_{\rm Pl}}\Big[\alpha_2\,\mathrm{R}^{\rm ab}\,\Box^2\,\mathrm{R}_{\rm ab} - \beta_2\,\mathrm{R}\,\Box^2\,\mathrm{R} + \gamma_{4,1}\,(\mathrm{C}_{\rm abcd}\,\mathrm{C}^{\rm abcd})^2 + \gamma_{4,2}(\mathrm{C}_{\rm abcd}\,{}^*\mathrm{C}^{\rm abcd})^2 + \dots\Big]$$

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order-by-order field redefinitions of the form

$$g_{ab} \to g_{ab} + c_1\, g_{ab}\, X + c_2\, X_{ab}$$

can remove any term containing Ricci variables

$$\mathcal{L}^{(3)}_{\rm EFT} = \frac{1}{M_{\rm Pl}^2}\Big[\alpha_1\, R^{ab}\,\Box\, R_{ab} - \beta_1\, R\,\Box\, R + \gamma_3\, C_{ab}{}^{cd} C_{cd}{}^{ef} C_{ef}{}^{ab}$$
$$+ \delta_{3,1}\, C_{abcd}C^{abcd}R + \delta_{3,2}\, C_{abcd}R^{ac}R^{bd} + \delta_{3,3}\, R_a^b\, R_b^c\, R_c^a + \delta_{3,4}\, R_{ab}R^{ab}R + \delta_{3,5}\, R^3\Big]$$

$$\mathcal{L}^{(4)}_{\rm EFT} = \frac{1}{M_{\rm Pl}^4}\Big[\alpha_2\, R^{ab}\,\Box^2\, R_{ab} - \beta_2\, R\,\Box^2\, R + \gamma_{4,1}\,(C_{abcd}\, C^{abcd})^2 + \gamma_{4,2}(C_{abcd}\, {}^*C^{abcd})^2 + \dots\Big]$$

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Goroff, Sagnotti, Nucl.Phys.B 266 (1986)
Bueno, Cano, PRD 94 (2016) 10
de Rham, Francfort, Zhang, PRD 102 (2020) 2

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Endlich, Gorbenko, Huang, Senatore, JHEP 09, 122 (2017)

# ... in the vacuum EFT of gravity.

# Perturbative quantisation of General Relativity ...

- What happens if we just treat $h_{\mu\nu}$ as a quantum-field theory around a background $\bar{g}_{\mu\nu}$?

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu}$$

$$g^{\mu\nu} = \bar{g}^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu\lambda} h_{\lambda}{}^{\nu} + \dots$$

$$\kappa = \sqrt{32\pi G}$$

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- The GR path-integral:

$$Z = \int [dh_{\mu\nu}] \exp\left\{ i \int d^4x \sqrt{-\bar{g}} \left[ \mathcal{L}_{GR} + \mathcal{L}_{matter} + \mathcal{L}_{gf} + \mathcal{L}_{gh} \right] \right\}$$

**coupling constant has negative mass dimension**

$$\mathcal{L}_{GR} = \mathcal{L}_{GR}^{0\text{-loop}} + \mathcal{L}_{GR}^{1\text{-loop}} + \mathcal{L}_{GR}^{2\text{-loop}} + \dots \qquad \mathcal{L}_{GR}^{0\text{-loop}} = \frac{2}{\kappa^2} R$$

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- Superficial degree of divergence:

$$D = 4L - 2I + 2V$$
$$\overset{L=I-V+1}{=} 2L + 2$$

L : # loops

I : # internal propagators $\sim 1/p^2$

V : # vertices $\sim p^2$

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# ... fails because of momentum-dependent interactions.

# Perturbative renormalisation of General Relativity ...

| Renormalized Lagrangian | New Divergences |
| --- | --- |
| $\mathcal{L}_{\text{GR}}^{\text{0-loop}} = \frac{2}{\kappa^2}\text{R}$ | $\mathcal{L}_{\text{div}}^{\text{1-loop}} = \frac{1}{8\pi^2\epsilon}\left\{\frac{1}{120}\bar{\text{R}}^2 + \frac{7}{20}\bar{\text{R}}_{\mu\nu}\bar{\text{R}}^{\mu\nu}\right\}$ |

# Perturbative renormalisation of General Relativity ...

## Renormalized Lagrangian

$$\mathcal{L}_{\text{GR}}^{\text{0-loop}} = \frac{2}{\kappa^2}\mathsf{R}$$

$$\mathcal{L}_{\text{GR}}^{\text{1-loop}} = \mathsf{c}_1^{(\mathsf{r})}\,\mathsf{R}^2 + \mathsf{c}_2^{(\mathsf{r})}\,\mathsf{R}_{\mu\nu}\mathsf{R}^{\mu\nu}$$

$$\mathsf{c}_1^{(\mathsf{r})} = \mathsf{c}_1 + \frac{1}{960\pi^2\epsilon} \qquad \mathsf{c}_2^{(\mathsf{r})} = \mathsf{c}_2 + \frac{7}{160\pi^2\epsilon}$$

## New Divergences

$$\mathcal{L}_{\text{div}}^{1-\text{loop}} = \frac{1}{8\pi^2\epsilon}\left\{\frac{1}{120}\bar{\mathsf{R}}^2 + \frac{7}{20}\bar{\mathsf{R}}_{\mu\nu}\bar{\mathsf{R}}^{\mu\nu}\right\}$$

$$\mathcal{L}_{\text{div}}^{2-\text{loop}} = \frac{209\kappa^2}{2880(16\pi^2)}\frac{1}{\epsilon}\bar{\mathsf{R}}^{\alpha\beta}{}_{\gamma\delta}\bar{\mathsf{R}}^{\gamma\delta}{}_{\eta\sigma}\bar{\mathsf{R}}^{\eta\sigma}{}_{\alpha\beta}$$

# Perturbative renormalisation of General Relativity ...

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$$\mathcal{L}_{\text{div}}^{\text{2-loop}} = \dots$$

# Perturbative renormalisation of General Relativity ...

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| $\mathcal{L}_{\text{GR}}^{\text{1-loop}} = c_1^{(r)}\,\mathrm{R}^2 + c_2^{(r)}\,\mathrm{R}_{\mu\nu}\mathrm{R}^{\mu\nu}$ <br> $c_1^{(r)} = c_1 + \frac{1}{960\pi^2\epsilon} \quad c_2^{(r)} = c_2 + \frac{7}{160\pi^2\epsilon}$ | $\mathcal{L}_{\text{div}}^{1-\text{loop}} = \frac{1}{8\pi^2\epsilon}\left\{\frac{1}{120}\bar{\mathrm{R}}^2 + \frac{7}{20}\bar{\mathrm{R}}_{\mu\nu}\bar{\mathrm{R}}^{\mu\nu}\right\}$ |
| $\mathcal{L}_{\text{GR}}^{\text{2-loop}} = d_1^{(r)}\,\mathrm{R}^{\alpha\beta}{}_{\gamma\delta}\mathrm{R}^{\gamma\delta}{}_{\eta\sigma}\mathrm{R}^{\eta\sigma}{}_{\alpha\beta}$ | $\mathcal{L}_{\text{div}}^{2-\text{loop}} = \frac{209\kappa^2}{2880(16\pi^2)}\frac{1}{\epsilon}\bar{\mathrm{R}}^{\alpha\beta}{}_{\gamma\delta}\bar{\mathrm{R}}^{\gamma\delta}{}_{\eta\sigma}\bar{\mathrm{R}}^{\eta\sigma}{}_{\alpha\beta}$ |
| | $\mathcal{L}_{\text{div}}^{2-\text{loop}} = \dots$ |

**... leads to an infinite tower of counter terms.**

# Perturbative quantisation of Quadratic Gravity ...

- The quadratic path-integral: $$Z = \int [dh_{\mu\nu}] \exp\left\{ i \int d^4x \sqrt{-\bar{g}} \left[ \mathcal{L}_{QG} + \mathcal{L}_{matter} + \mathcal{L}_{gf} + \mathcal{L}_{gh} \right] \right\}$$

**Effective coupling has vanishing mass dimension**

$$\mathcal{L}_{QG} = M_{Pl}^2 \left[ R + \frac{1}{12 m_0^2} R^2 + \frac{1}{4 m_2^2} C_{abcd} C^{abcd} \right]$$

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$$D = 4L - 4I + 4V \overset{L = I - V + 1}{=} 4$$

L : # loops
I : # propagators
V : # vertices $\sim p^4$

$$\sim \frac{1}{p^2} - \frac{1}{p^2 + m^2} \overset{p \to \infty}{\longrightarrow} \frac{1}{p^4}$$

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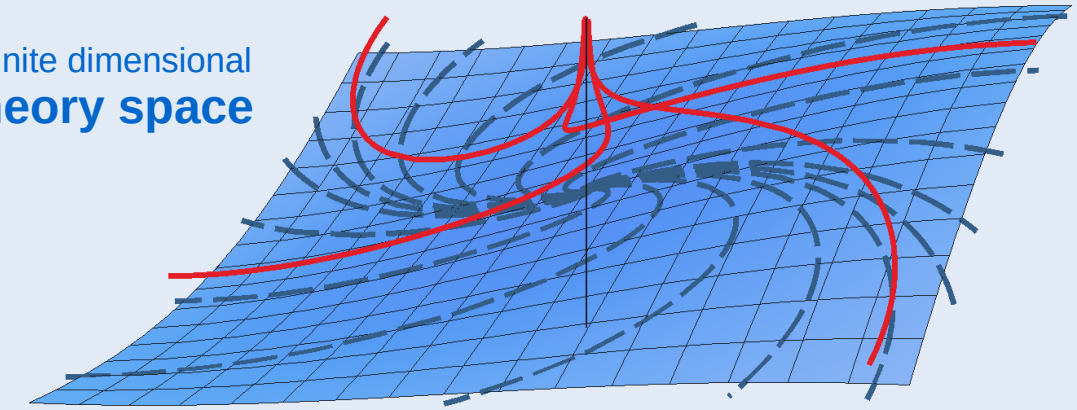
Stelle, PRD 16 (1977) 953-969

**... leads to a renormalisable theory with ghosts.**

While the GR is **perturbatively non-renormalisable**
it may be **non-perturbatively renormalisable** as an EFT

# Nonperturbative quantisation of gravity ...

infinite dimensional
**theory space**



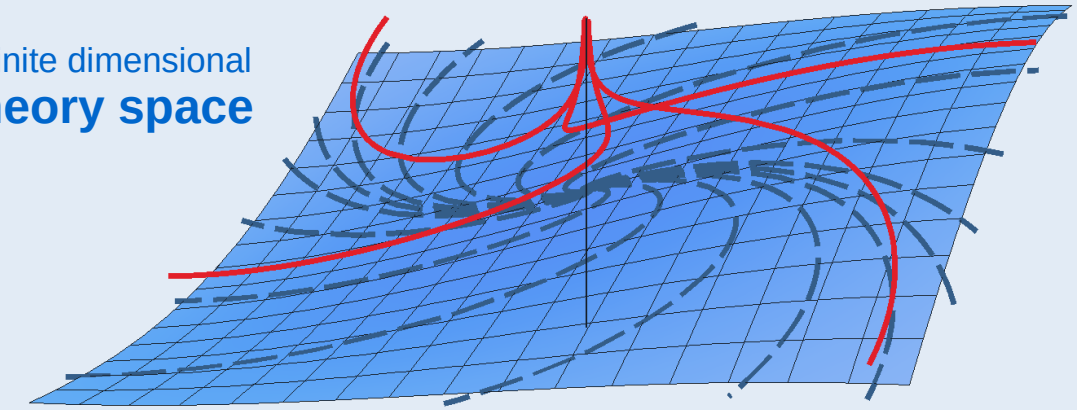
$$S = \frac{-1}{16\pi \bar{G}_N} \int d^4x \sqrt{g} \Big[ R - 2\bar{\Lambda} + \bar{a}R^2 + \bar{b} R_{\mu\nu} R^{\mu\nu} + \dots \text{ all terms allowed by symmetry} \Big]$$

infinite dimensional
**theory space**

- fixed point of RG flow
  $\beta_{C_i}|_* = 0 \quad \forall\, C_i$
- with a finite number of relevant (IR-repulsive) directions (**predictivity**)
  $$\theta_k = -\left[ \text{ev}_k \left( \frac{\partial \beta_{C_i}}{\partial C_j}\bigg|_* \right) \right] > 0$$
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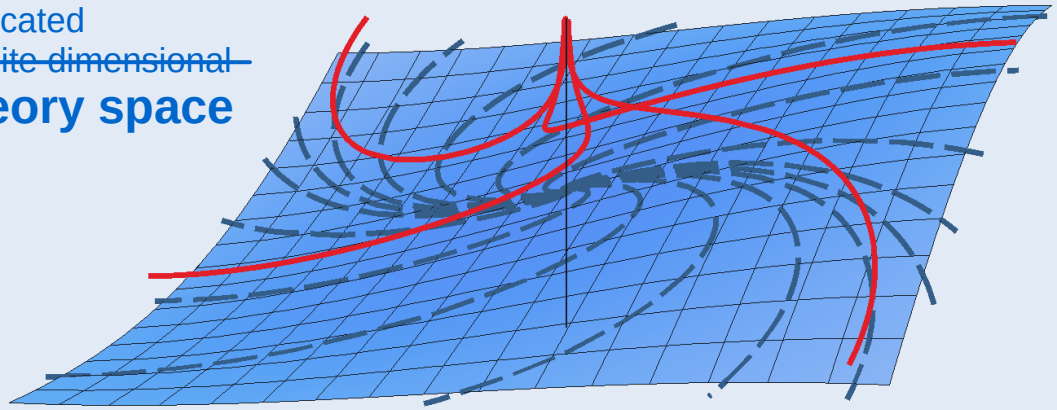
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## ... at an asymptotically safe fixed point.

# Nonperturbative quantisation of gravity ...

truncated
~~infinite dimensional~~
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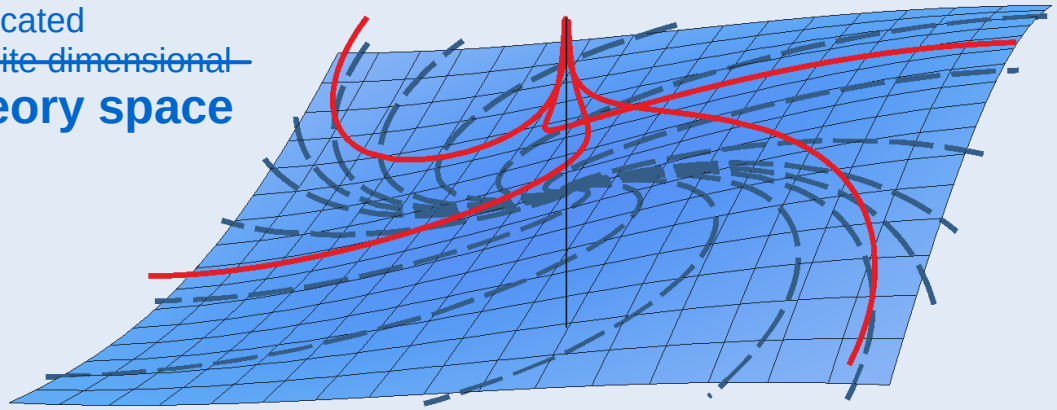
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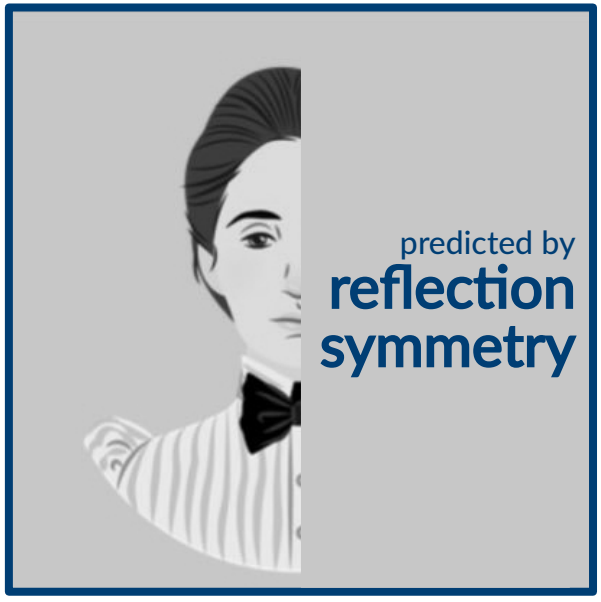
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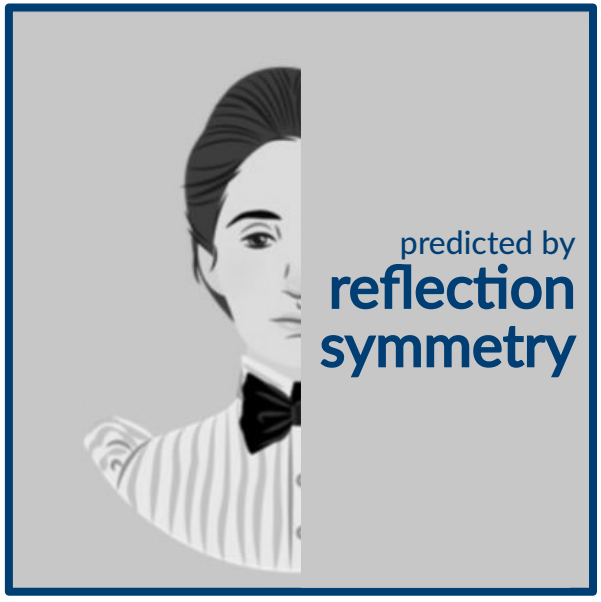
| fixed point | symmetry invariants | $\theta > 0$ | $\theta < 0$ |
|---|---|---|---|
| ☑ | $R$ | ☒ | |
| ☑ | $\Lambda + R$ Reuter '96 | ☒ | |
| ☑ | $R^2 + R_{\mu\nu}R^{\mu\nu}$ Benedetti, Machado, Saueressig '09 | ☒ | ☒ |
| ☑ | $C^{\rho\sigma}_{\mu\nu} C^{\kappa\lambda}_{\rho\sigma} C^{\mu\nu}_{\kappa\lambda}$ Gies, Knorr, Lippoldt, Saueressig '16 | | ☒ |
| ☑ | $R^3$ Reuter, Lauscher '02 | | ☒ |
| ⋮ | ⋮ | | ⋮ |
| ☑ | $R^{70}$, $R^{\mu\nu}{}^{34}$ Codello, Percacci, Rahmede '07, '08; Machado, Saueressig '07; K. Falls et.Al '13, '18 | | ☒ |

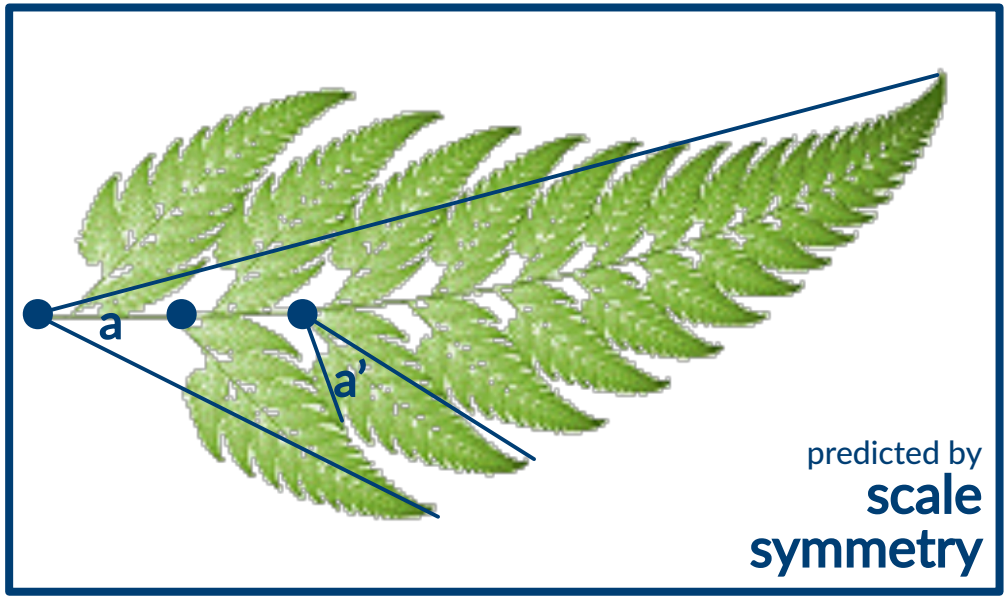
**... at an asymptotically safe fixed point.**

**Quantum scale symmetry:**
Increased predictive power from irrelevant directions

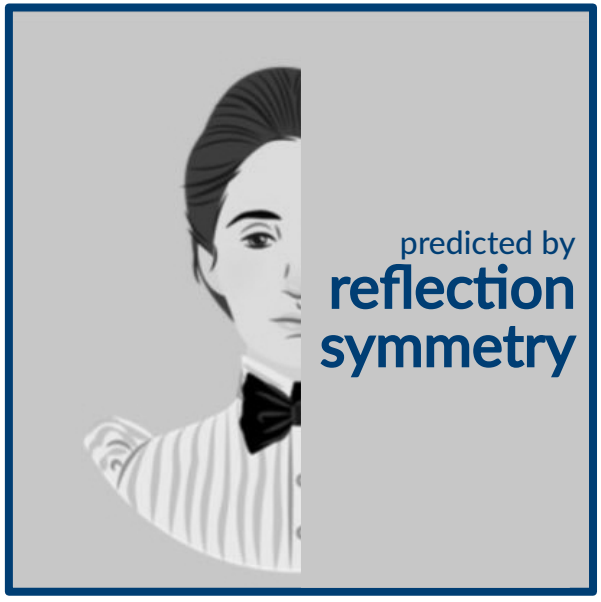


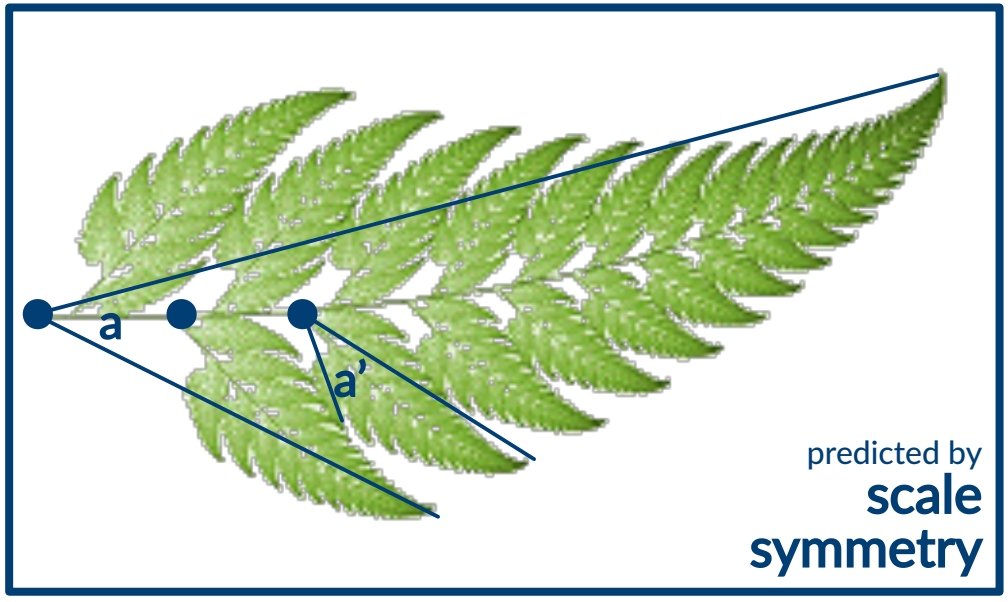

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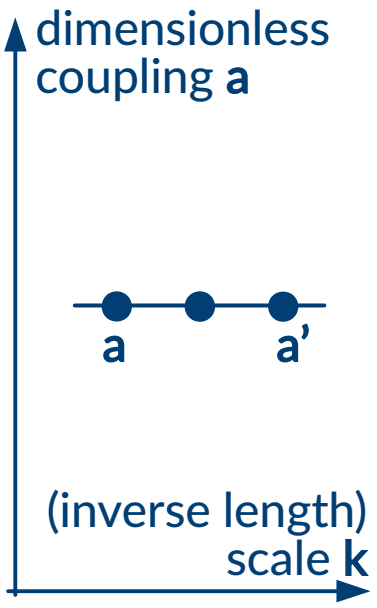



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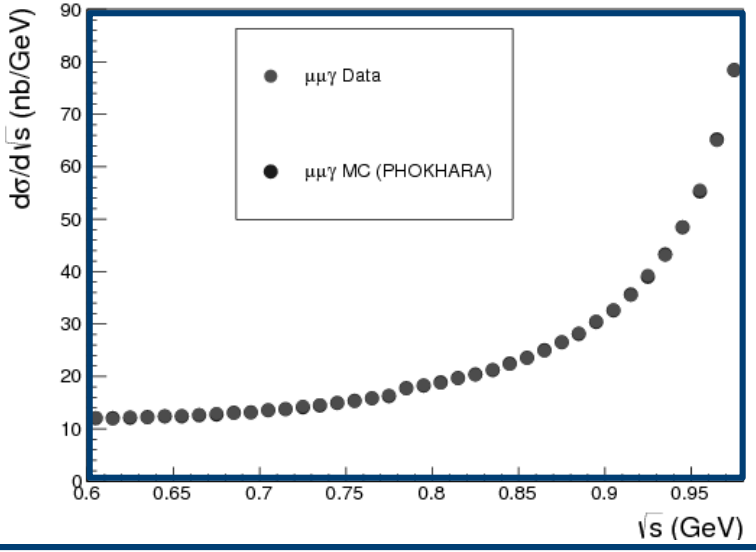




# Quantum scale symmetry:
Increased predictive power from irrelevant directions


QED
QCD
scale symmetry is
broken by
quantum fluctuations
energy

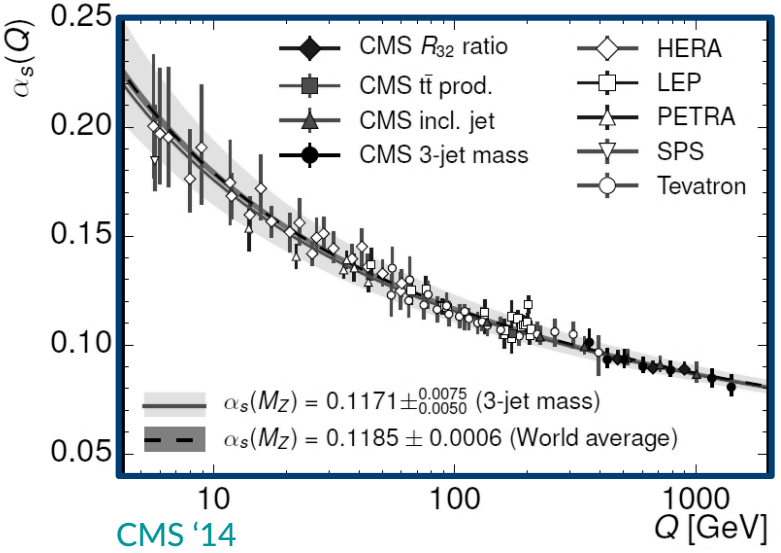
KLOE-2 collaboration ‘17




**QED**

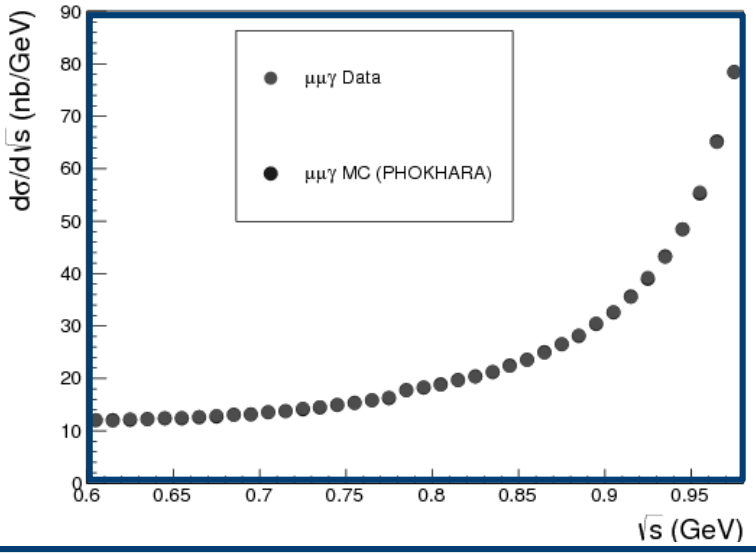
**QCD**

scale symmetry is broken by quantum fluctuations → energy



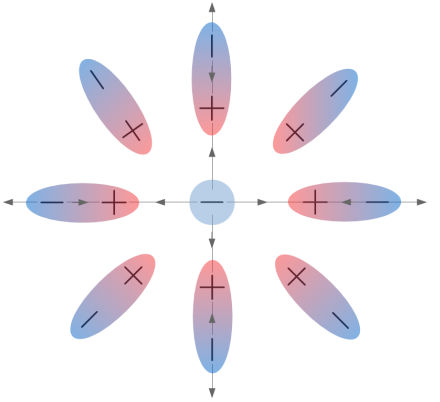

CMS ‘14

KLOE-2 collaboration ‘17




**QED**

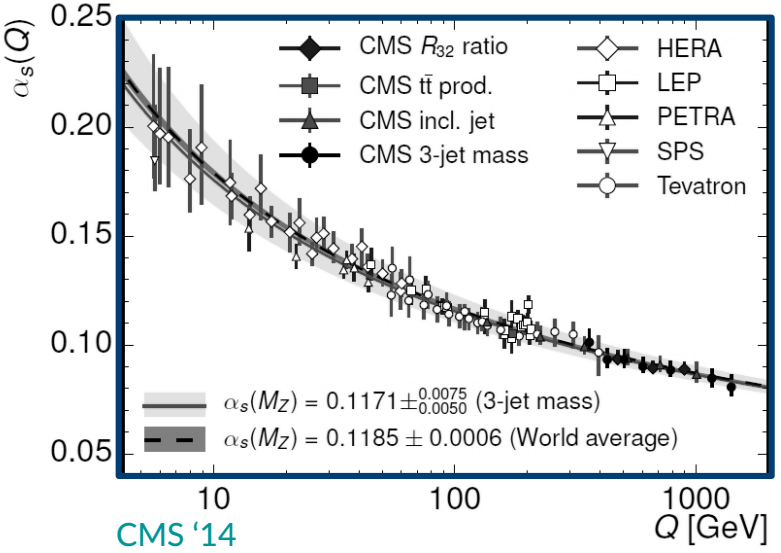
**screening**



scale symmetry is broken by energy

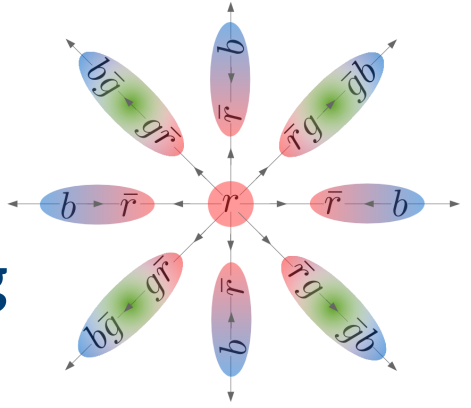
quantum fluctuations

**QCD**

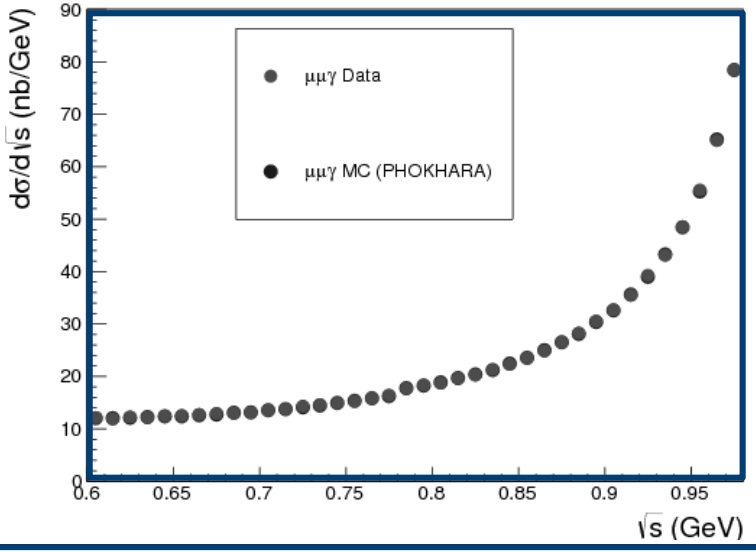



CMS ‘14

**antiscreening**



KLOE-2 collaboration ‘17



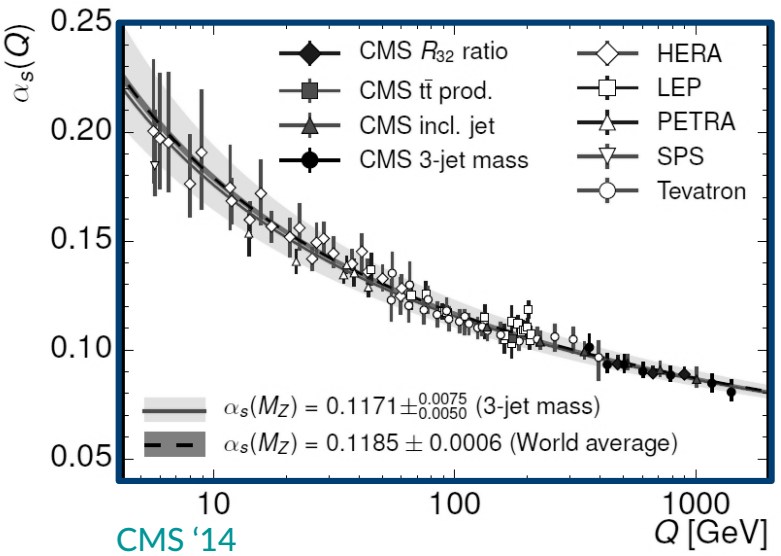

**screening**

$$\beta_{g_y} \equiv k\,\partial_k g_y > 0$$

**QED**

scale symmetry is broken by quantum fluctuations → energy

**QCD**

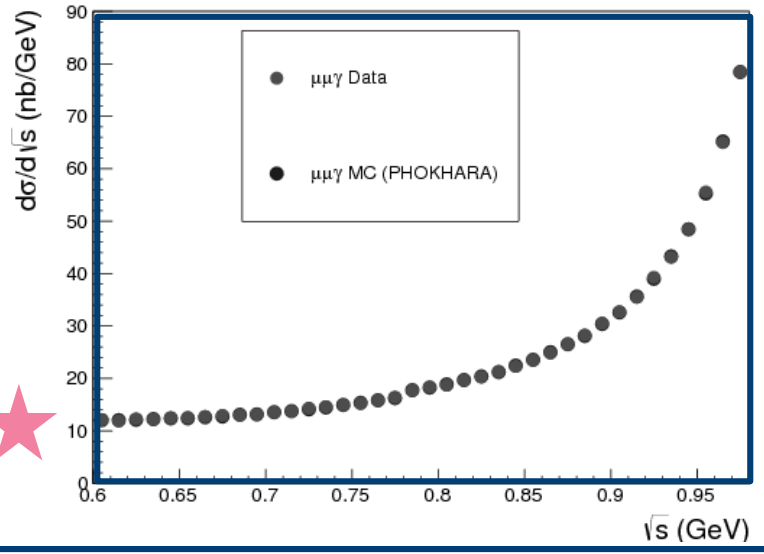



CMS ‘14

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**antiscreening**

KLOE-2 collaboration ‘17




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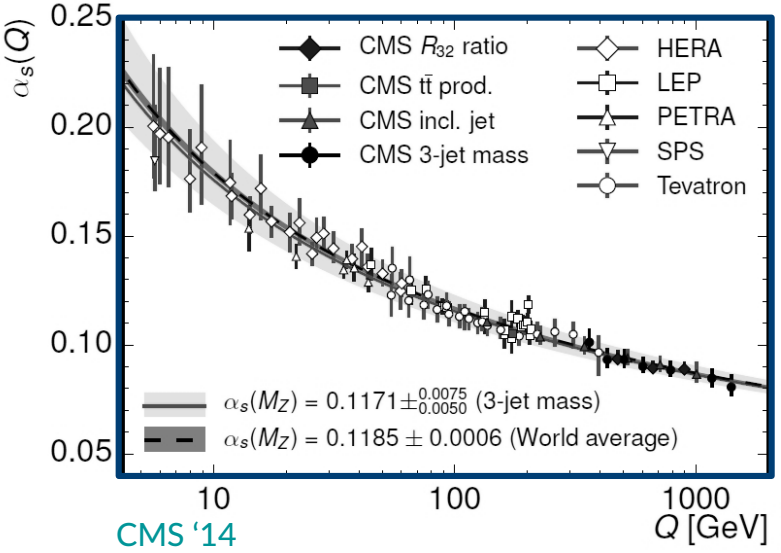
**screening**

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scale symmetry is **restored** at a ⟶ energy

**fixed point**, i.e., **β=0**

**QCD**

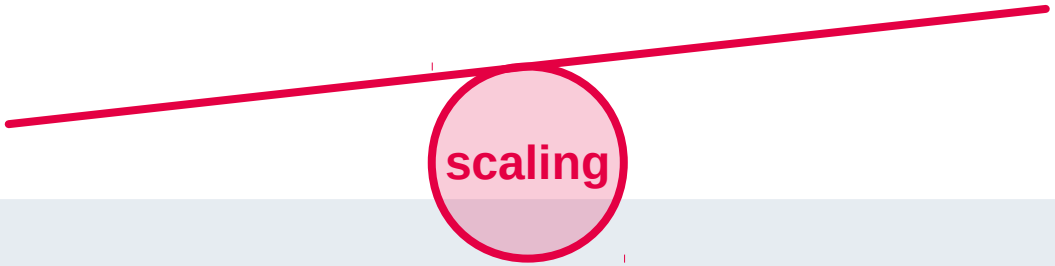



CMS ‘14

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**antiscreening**

# The basic mechanism for asymptotic safety ...

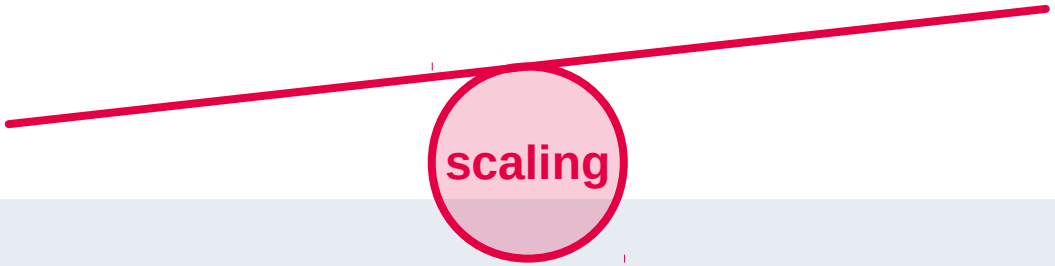



... balances out quantum fluctuations.

# The basic mechanism for asymptotic safety ...

- **competing loop orders**
  - gauge-Yukawa theories

1-loop vs 2-loop



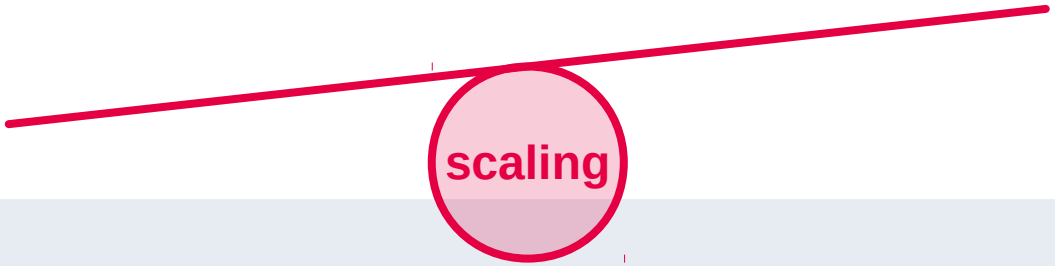

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- **competing loop orders**
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- **above/below the critical dimension**
  - gravity in 2+ε dimensions
  - gravity in 4 dimensions
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1-loop vs 2-loop

dimensional scaling vs quantum fluctuations




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- **competing degrees of freedom**
  - fermions vs bosons
  - matter vs gauge/gravity

1-loop vs 2-loop

dimensional scaling vs quantum fluctuations

Standard Model fluctuations vs gravitational fluctuations




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non-perturbative methods

1-loop vs 2-loop

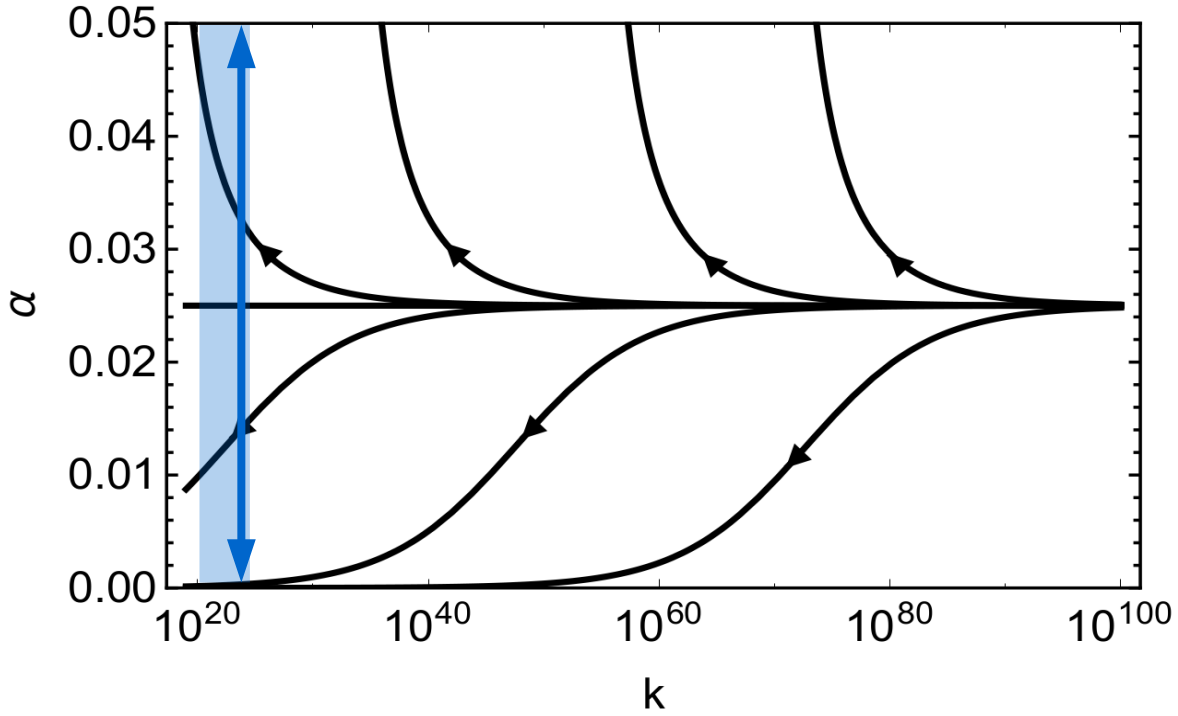
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... balances out quantum fluctuations.

# Free EFT parameters ...




**Relevant directions:**
free parameters

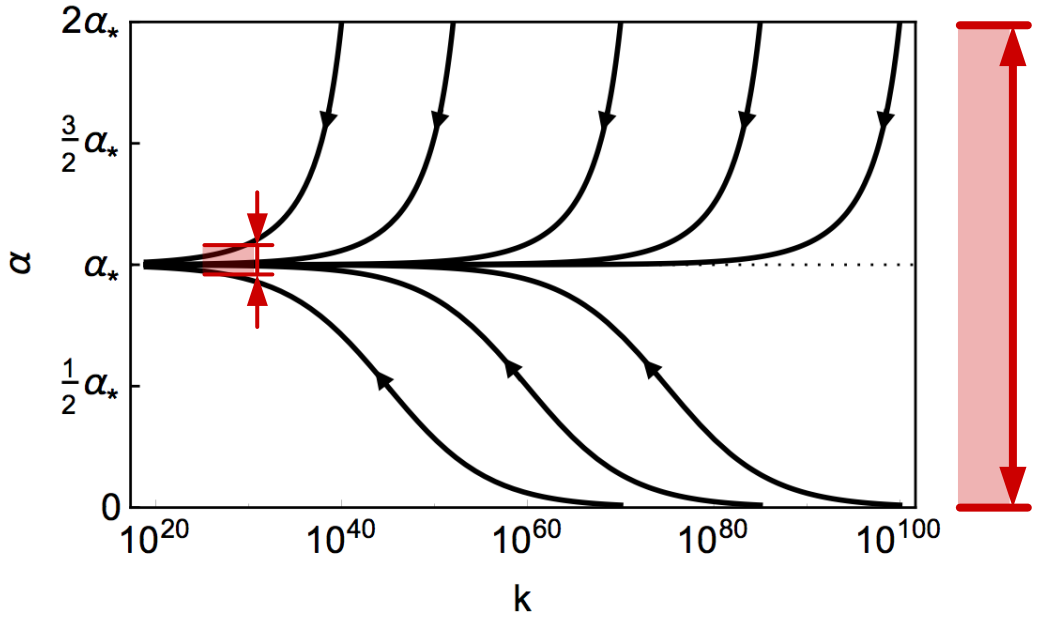
parameterize deviation
from scale invariance

## ... along relevant (UV attractive) directions.

# Predicted EFT parameters ...

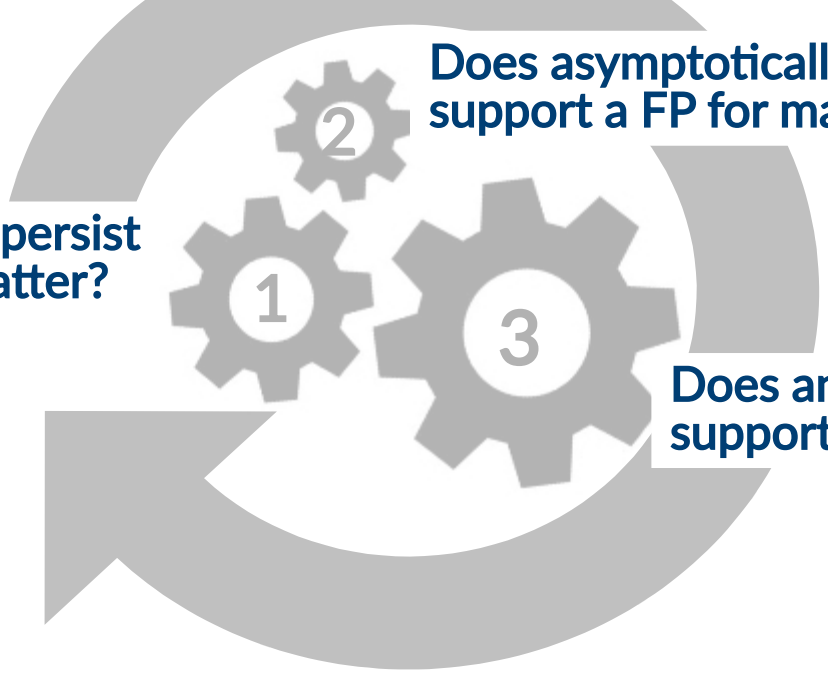
**Irrelevant directions:**
Predictions from asymptotic safety

**predictive power:**
consequence of a fixed point




**... along irrelevant (UV repulsive) directions.**

# Matter in asymptotically safe gravity




**Does asymptotically safe gravity support a FP for matter?**
Eichhorn, Held, Pawlowski ‘16,
Christiansen, Eichhorn ‘17, ...

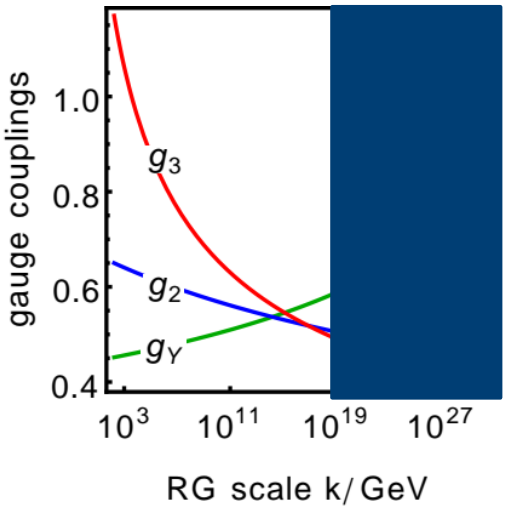
**Does the gravity FP persist upon inclusion of matter?**
Donà, Eichhorn, Percacci ‘13
Meibohm, Pawlowski, Reichert ‘16
Biemans, Platania, Saueressig ‘17
Christiansen et.Al ‘17

**Does an asymptotically safe FP support a viable phenomenology?**
Shaposhnikov, Wetterich ‘09,
Harst, Reuter ‘11,
Eichhorn, Held ‘17, ‘17 ‘18,
Eichhorn, Versteegen ‘17, ...

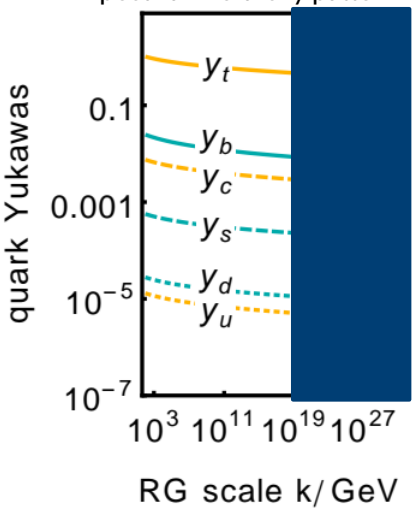
# Matter in asymptotically safe gravity

# Matter in asymptotically safe gravity: Parameterised UV closure of the Standard Model

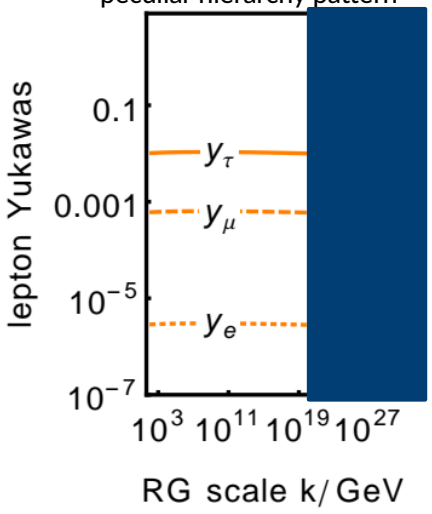
1-loop running of gauge couplings:
~ unification at $10^{15}$ GeV
consistent beyond $10^{19}$ GeV
U(1)-Landau pole at $10^{41}$ GeV



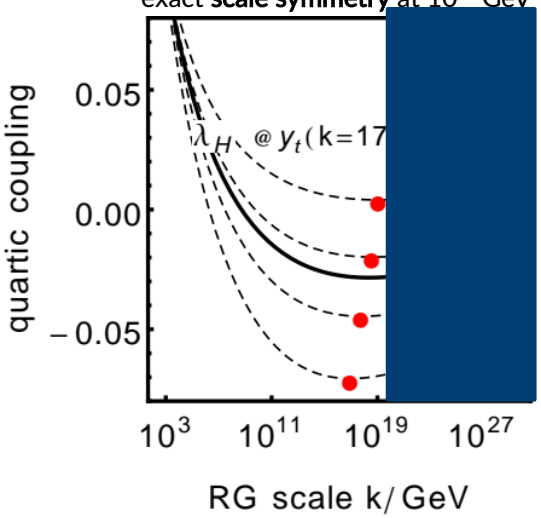

1-loop running of quark-Yukawas
Consistent >>$10^{19}$ GeV
peculiar hierarchy pattern




1-loop running of lepton-Yukawas:
consistent >>$10^{19}$ GeV;
peculiar hierarchy pattern

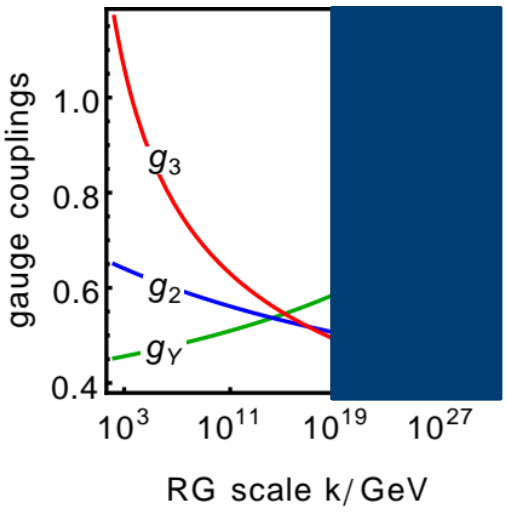



1-loop running of the Higgs-quartic for different values of the top-Yukawa coupling: experimental values are consistent with exact **scale symmetry** at $10^{19}$ GeV

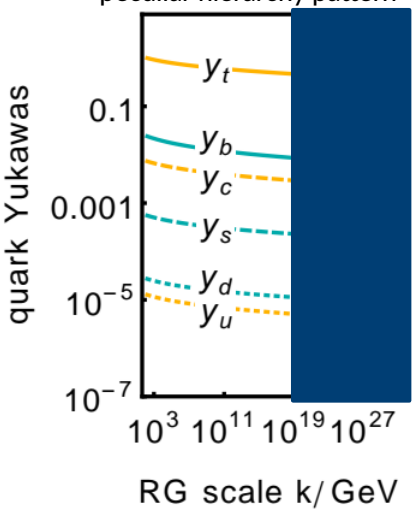



# Matter in asymptotically safe gravity: Parameterised UV closure of the Standard Model

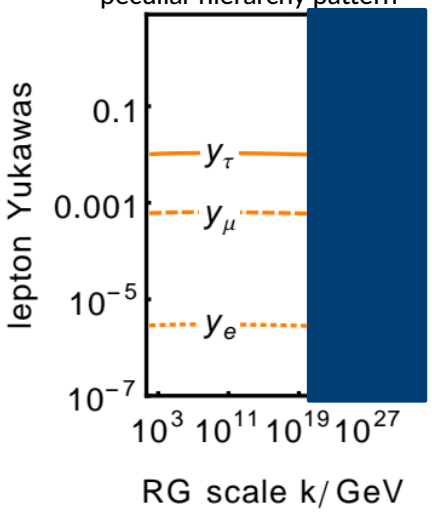
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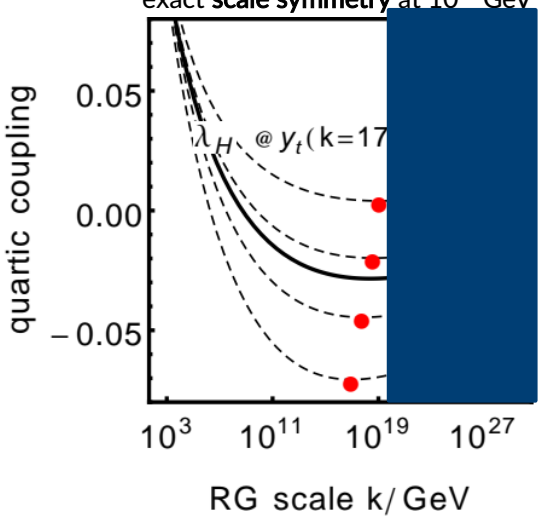

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# Matter in asymptotically safe gravity: Parameterised UV closure of the Standard Model

$$\beta_{g_i} = \frac{b_i\, g_i^3}{16\pi^2} \boxed{- f_g\, g_i}$$

$f_g \geqslant 0$ Daum, Harst, Reuter, ‘10
Folkerts, Litim, Pawlowski, ‘12
Eichhorn, Versteegen ‘17

$$\beta_{y_i} = \frac{9\, y_i^3}{16\pi^2} \boxed{- f_y\, y_i}$$

$f_y \geqslant 0$ Eichhorn, Held, Pawlowski, ‘16
Eichhorn, Held, ‘17
de Brito, Schiffer, Reichert ‘25

$$\beta_\lambda = \frac{3\,\lambda^2}{2\pi^2} \boxed{- f_\lambda\, \lambda}$$

$f_\lambda < 0$ Griguolo, Percacci ‘95
Percacci, Perini ‘03
Narain, Percacci ‘09

# Gauge couplings ...

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- **non-Abelian gauge couplings:**
  Asymptotic freedom
  is reinforced by gravity

- **Abelian gauge couplings:**
  Competition of gauge & gravity fluctuations
  balancing out at asymptotic safety
  causing an **increase of predictive power**

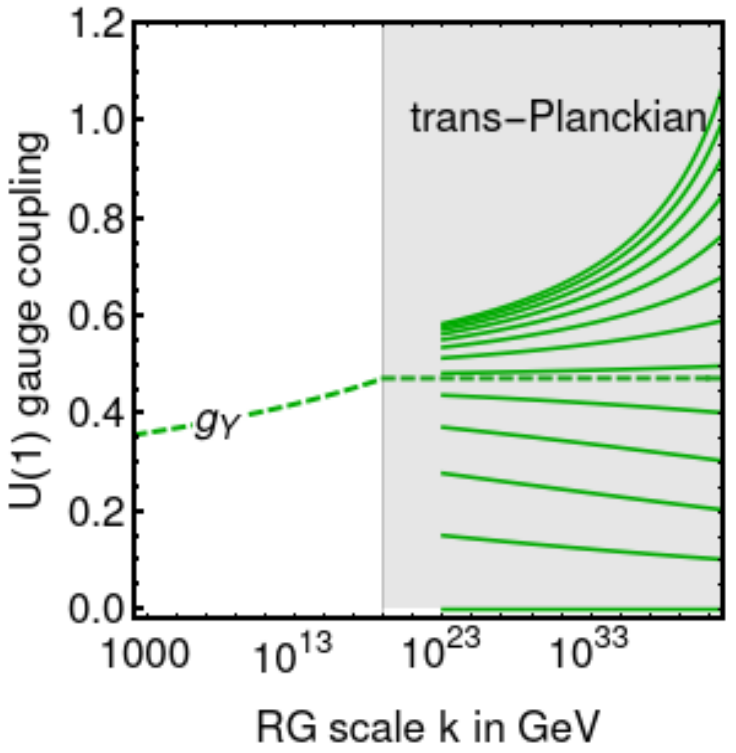
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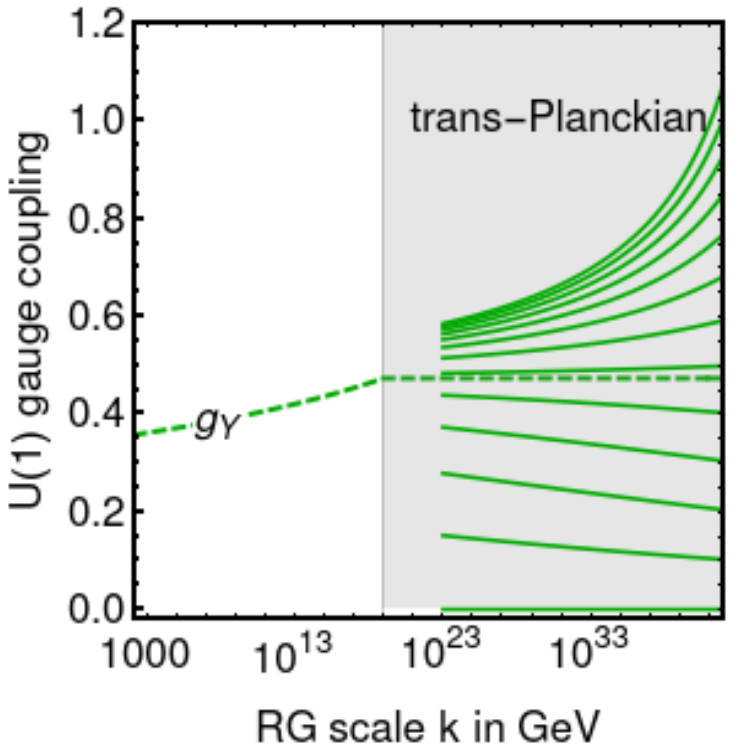
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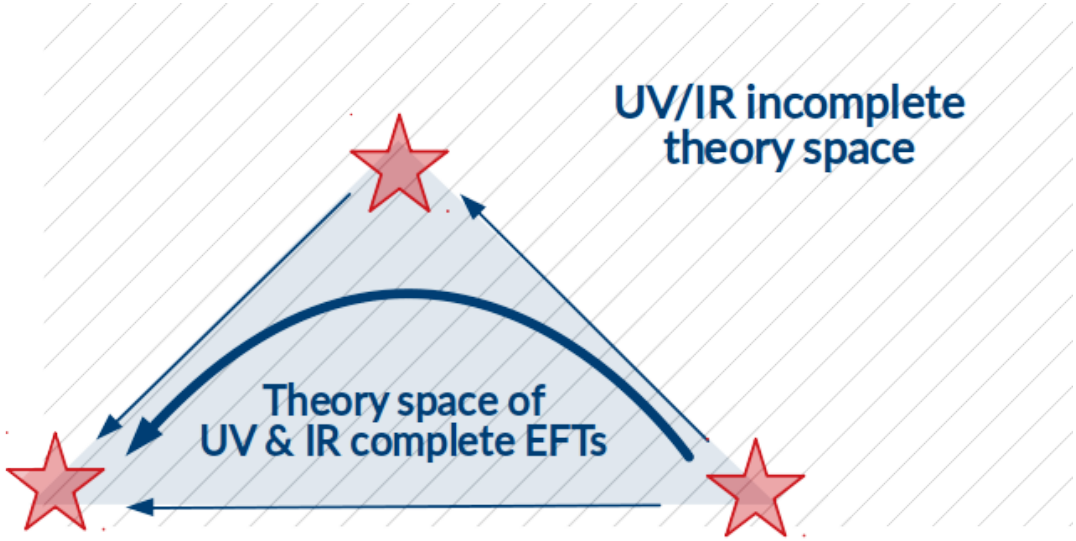



**... can exhibit an upper bound and predictive power.**

# Gauge couplings ...

$$\beta_{g_i} = \frac{b_i\, g_i^3}{16\pi^2} - f_g\, g_i$$

$f_g \geqslant 0$ Daum, Harst, Reuter, '10
Folkerts, Litim, Pawlowski, '12
Eichhorn, Versteegen '17




- **non-Abelian gauge couplings:**
  Asymptotic freedom
  is reinforced by gravity

- **Abelian gauge couplings:**
  Competition of gauge & gravity fluctuations
  balancing out at asymptotic safety
  causing an **increase of predictive power**

**... can exhibit an upper bound and predictive power.**

# Yukawa couplings (in one generation) ...

- **up-type and down-type** quarks/Yukawas are **charged differently** under the U(1)

$$\beta_{y_{t\,(b)}} = \frac{y_{t\,(b)}}{16\,\pi^2}\left[\frac{3y_{b\,(t)}^2}{2} + \frac{9y_{t\,(b)}^2}{2} - \frac{9g_2^2}{4} - 8g_3^2\right] - \frac{3y_{t(b)}}{16\,\pi^2}\left(Y_Q^2 + Y_{t(b)}^2\right) g_Y^2 - f_y\, y_{t(b)}$$

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Eichhorn, Held, PRL 121 (2018)

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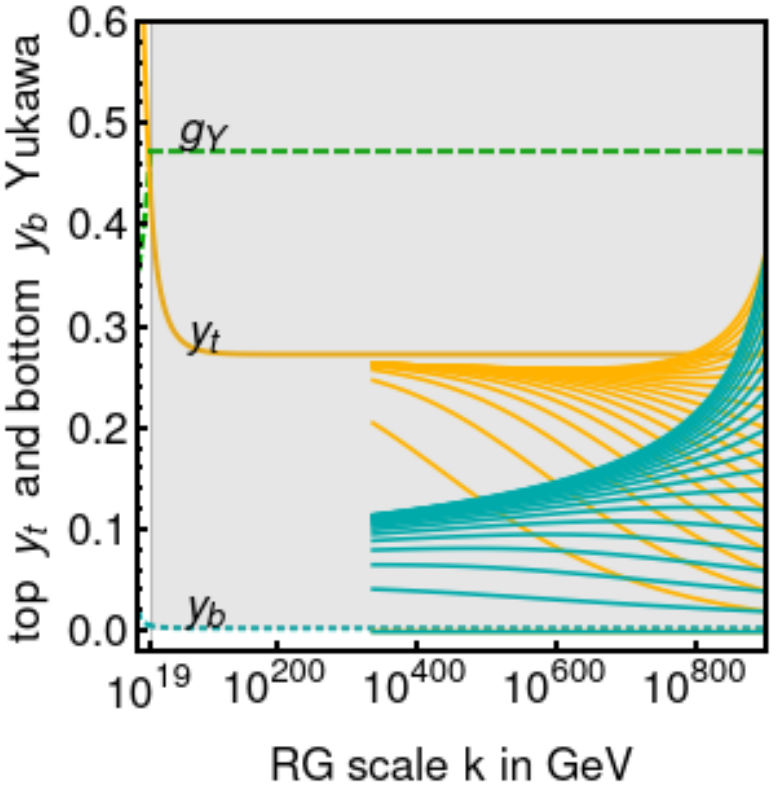
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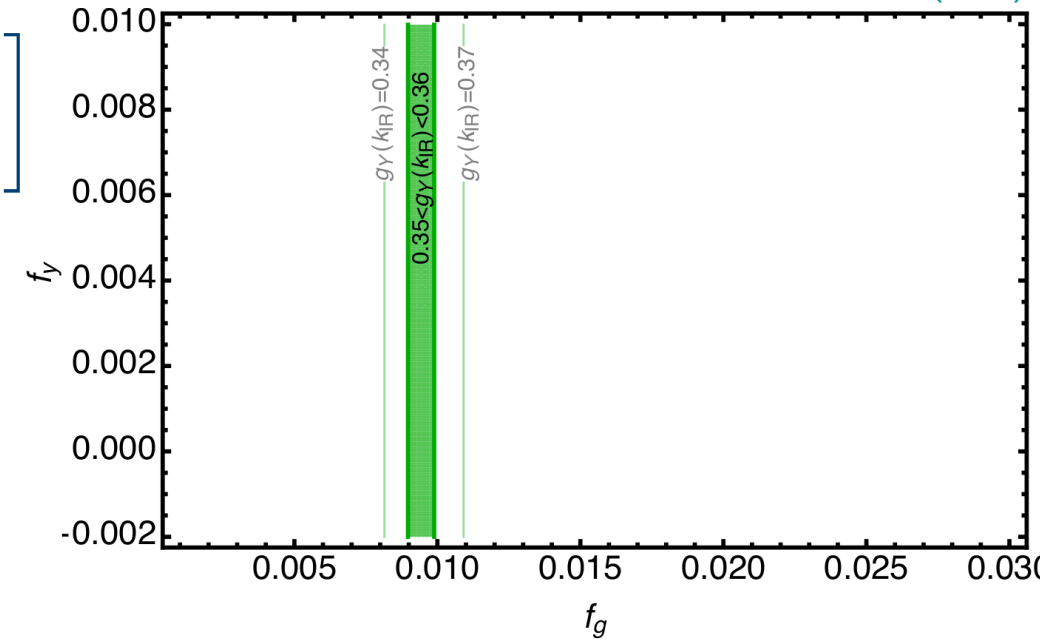
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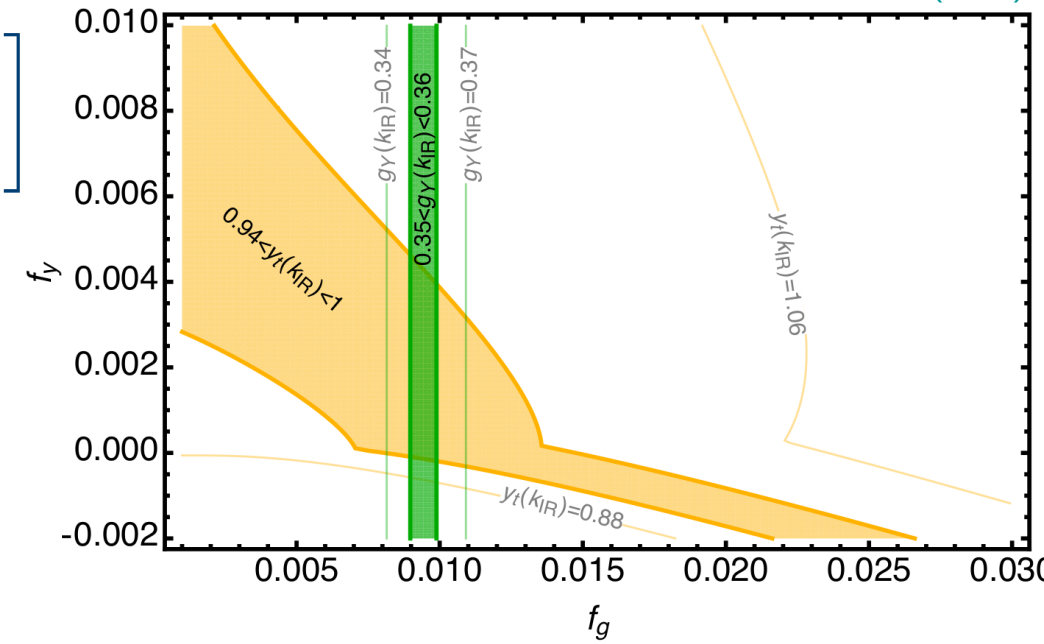
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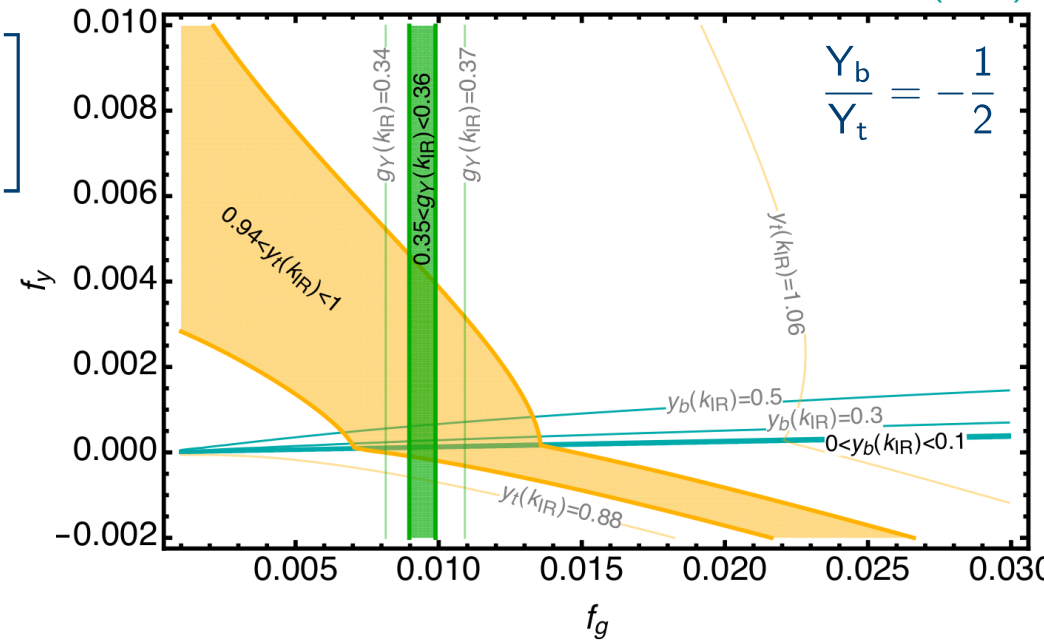
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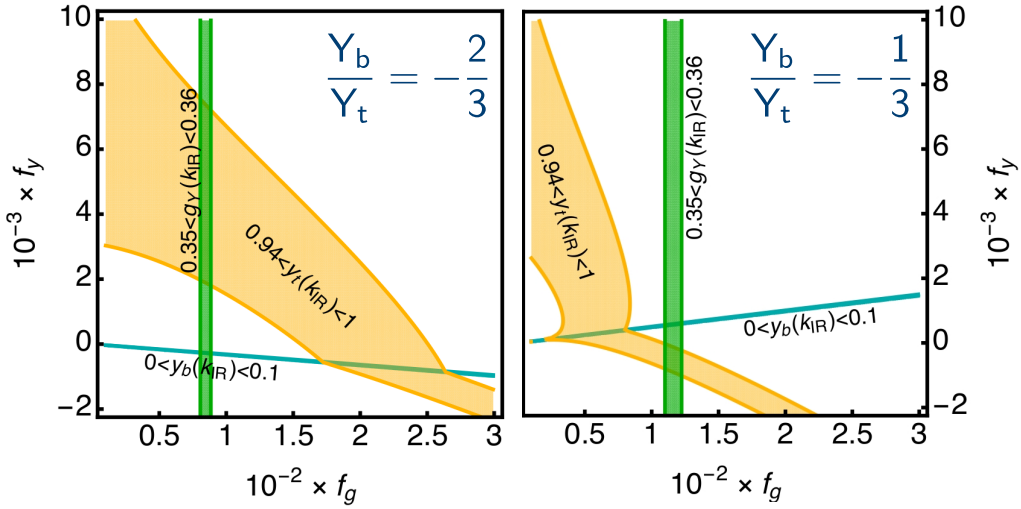
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$$V_{CKM}^{IR} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad V_{CKM}^{UV} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

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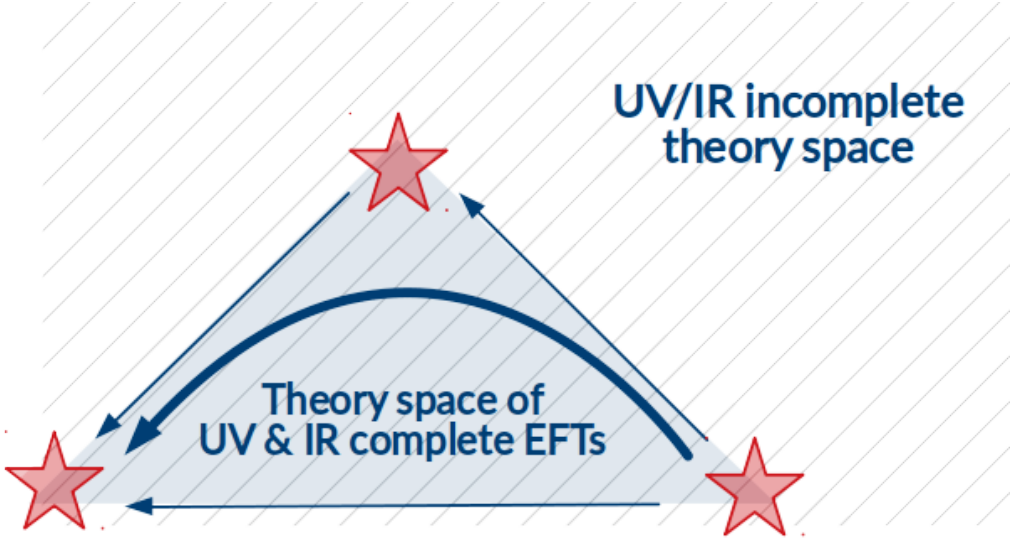
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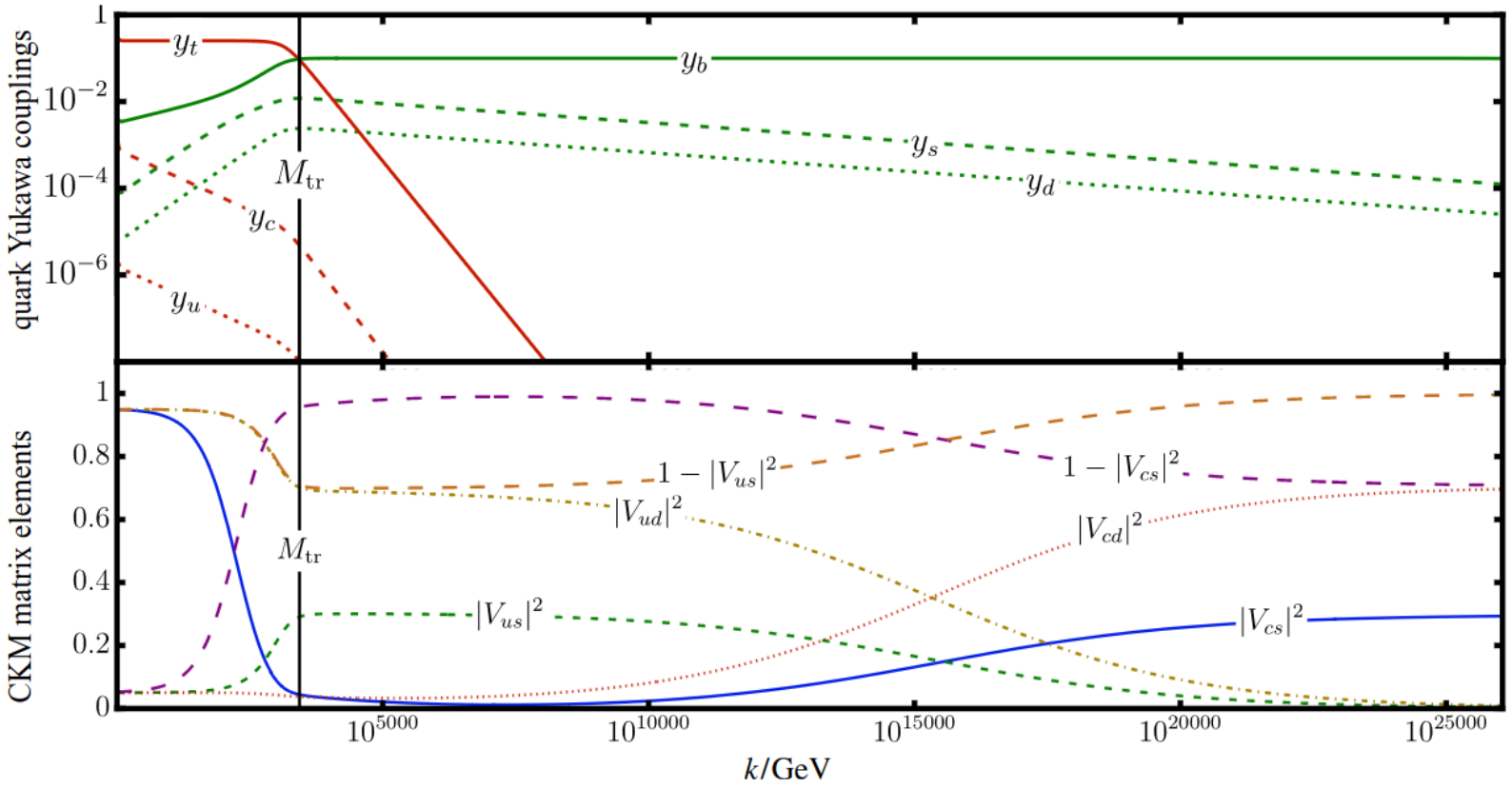
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**... can lead to a cascade of fixed points.**

# Yukawa couplings (in multiple generations) ...




This cascade explains the observed CKM ordering, e.g.,

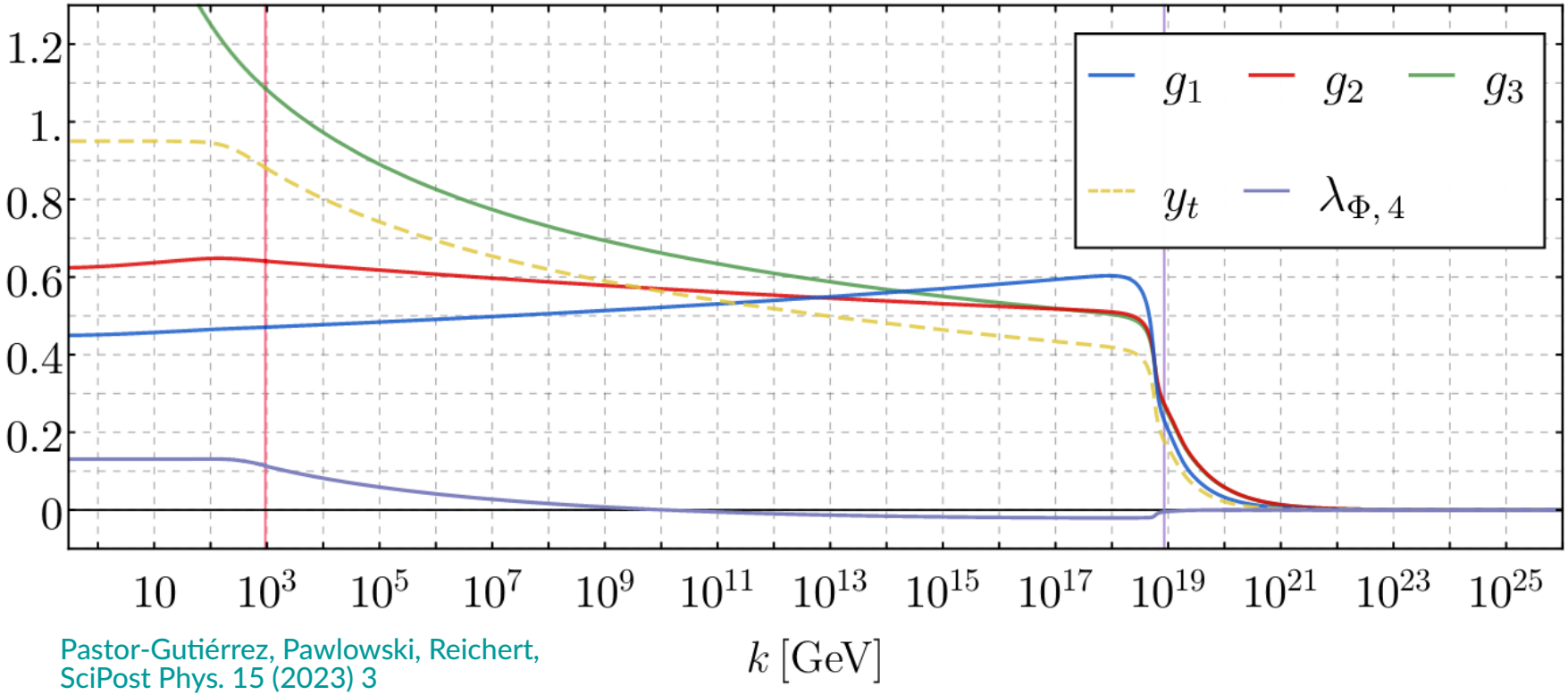
$|V_{cb}|^2 > |V_{ub}|^2$

Eichhorn, Gyftopoulos, Held, 2507.18304

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... can lead to a cascade of fixed points.

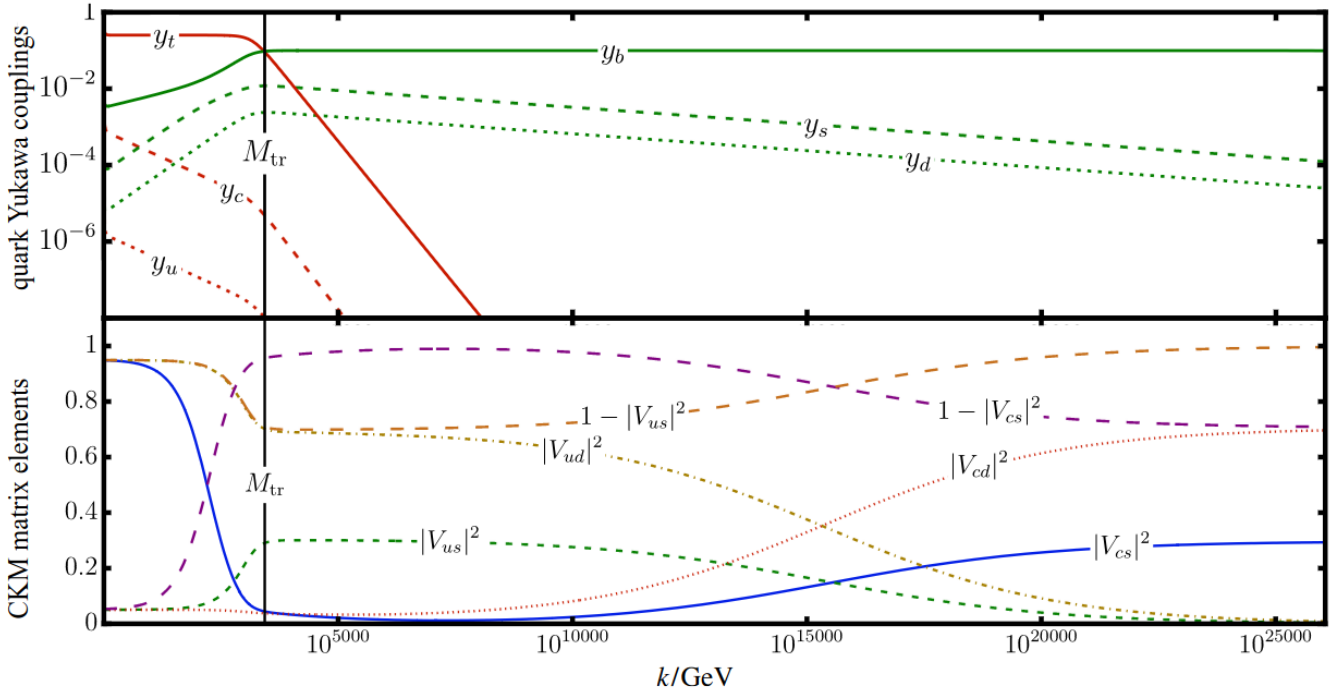
# The full Standard Model …




Pastor-Gutiérrez, Pawlowski, Reichert,
SciPost Phys. 15 (2023) 3

… can emerge from **gravity-induced asymptotic freedom.**

# The observed Standard-Model parameters …




- **mass-differences** dynamically tied to **charge differences**
- **CKM structure** matches expectations from a **cascade of fixed points**
- this agreement is a **quantitative hint**

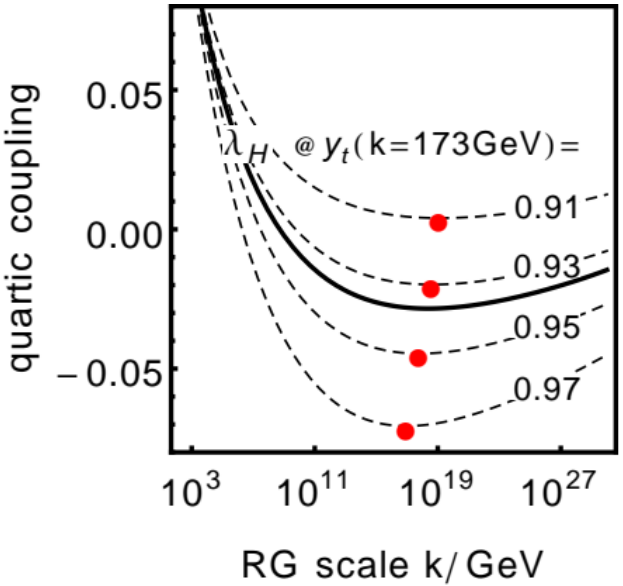
… point to the **predictive edge** of a **“UV-conformal window”.**

# **Scalar potentials**
and non-minimal coupling to curvature

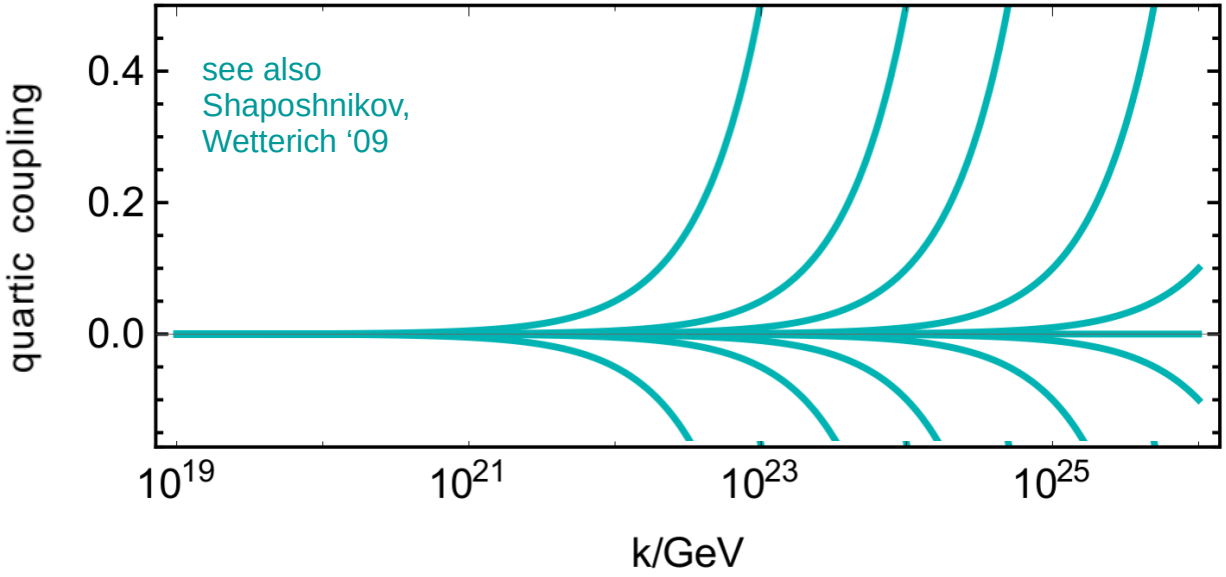
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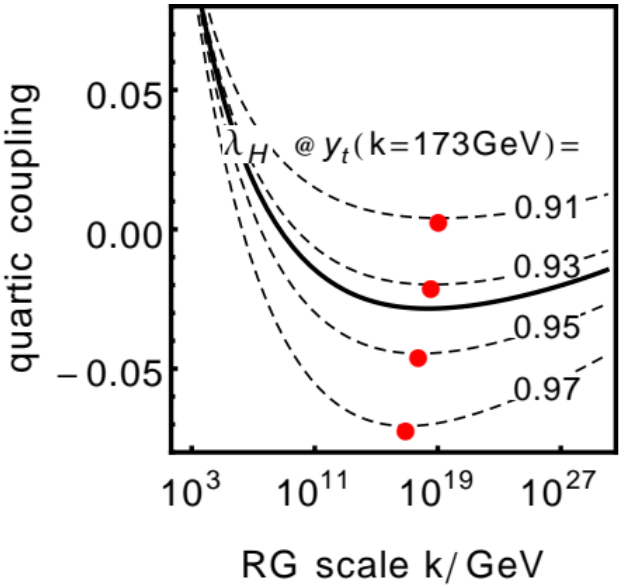

$\mathrm{f}_\lambda < 0$ Griguolo, Percacci ‘95
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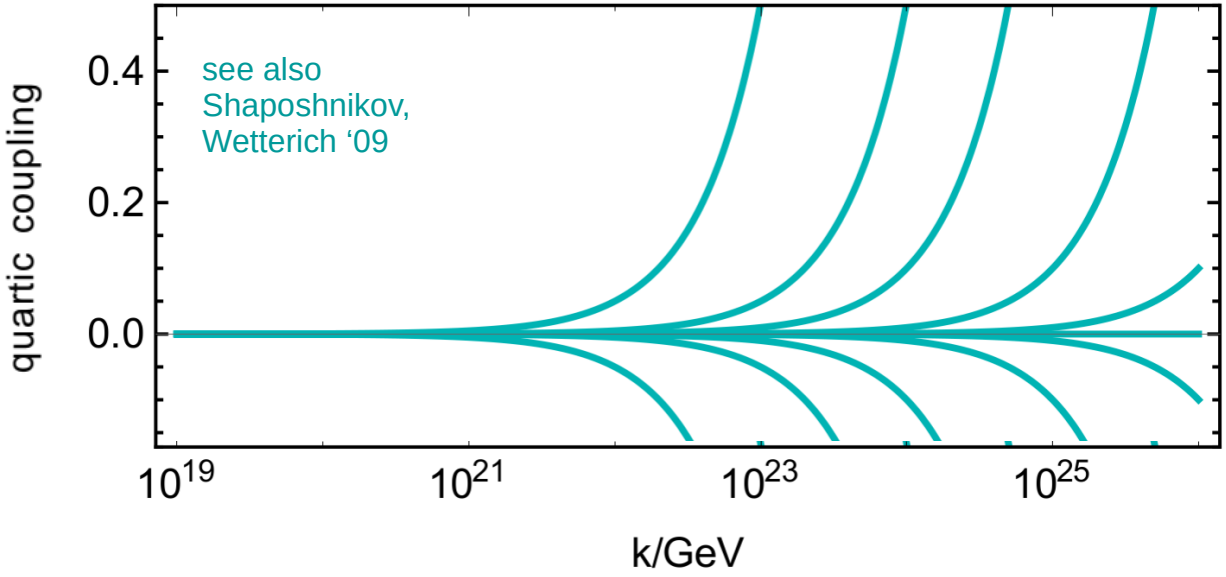
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**… is predicted [apart from the relevant mass term setting the EW scale].**

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| | stabilise | de-stabilise |
|---|---|---|
| $\mathrm{f}_\lambda$ | – Yukawa-contributions | + gauge-contributions |

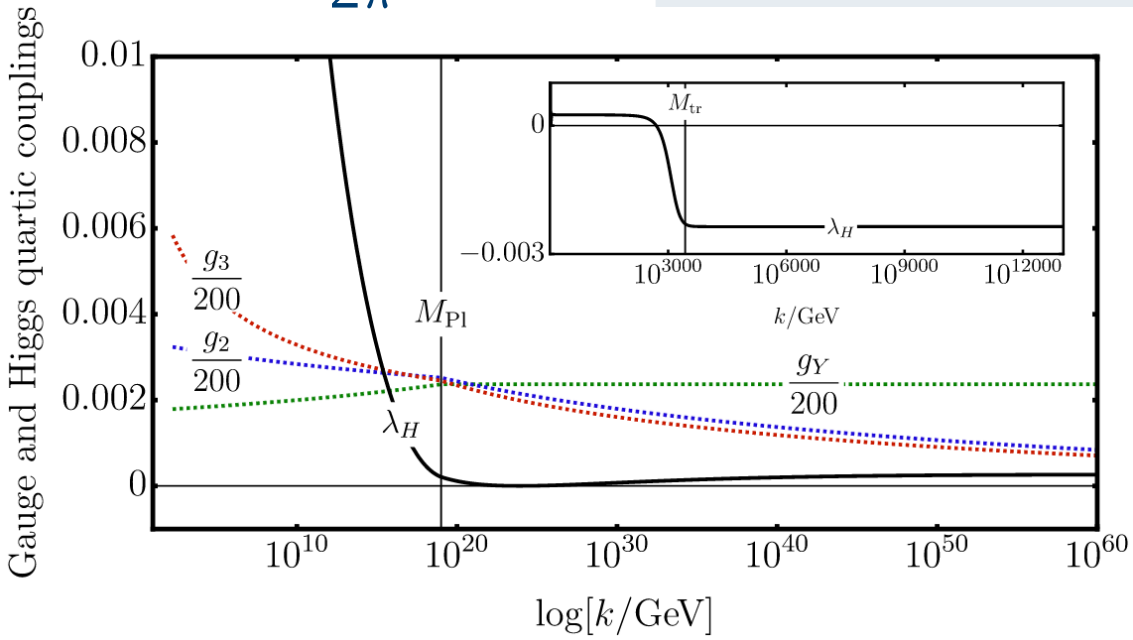
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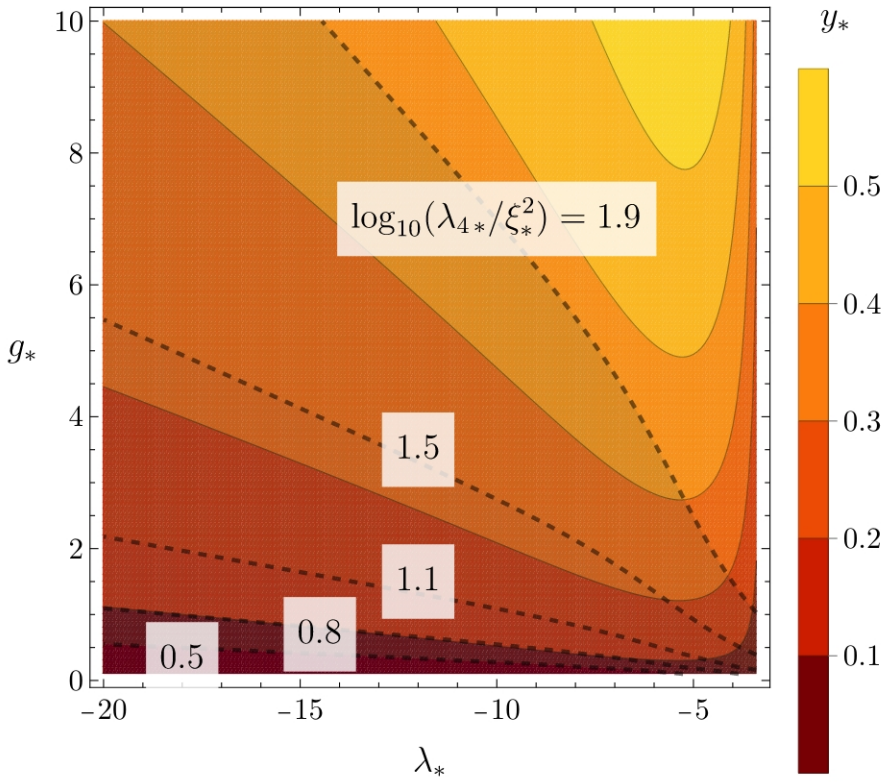
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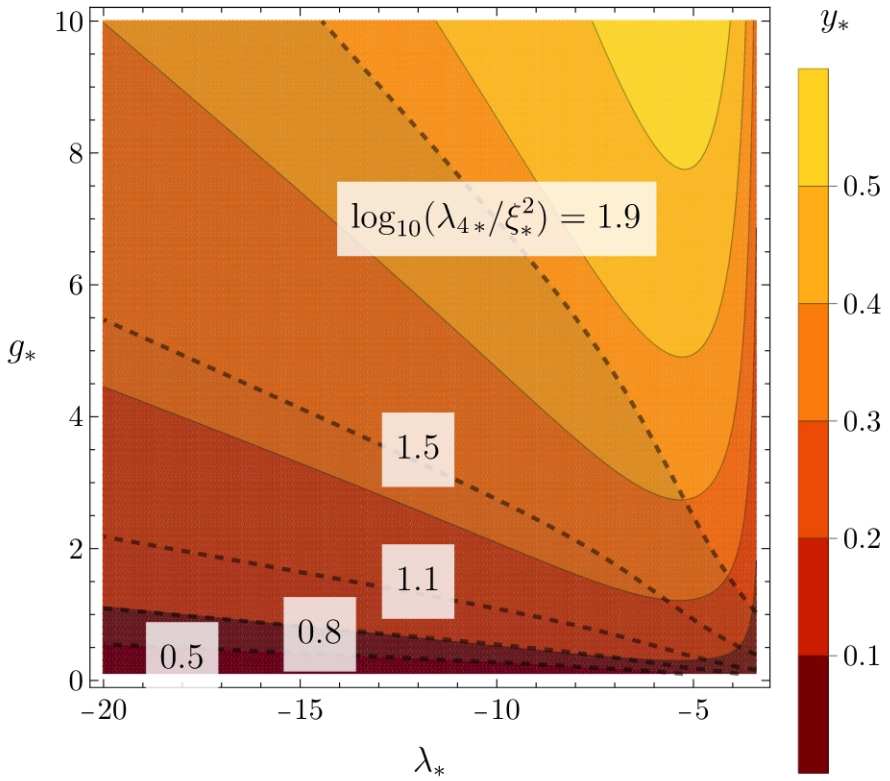


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… is a source of tension.

# How to solve nonlinear problems?
Well-posed time evolution in gravitational EFTs



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**Robustness of inflation**

**Primordial gravitational waves**

**Primordial black-hole formation**

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Goroff, Sagnotti, Nucl.Phys.B 266 (1986)
Bueno, Cano, PRD 94 (2016) 10
de Rham, Francfort, Zhang, PRD 102 (2020) 2

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Endlich, Gorbenko, Huang, Senatore, JHEP 09, 122 (2017)

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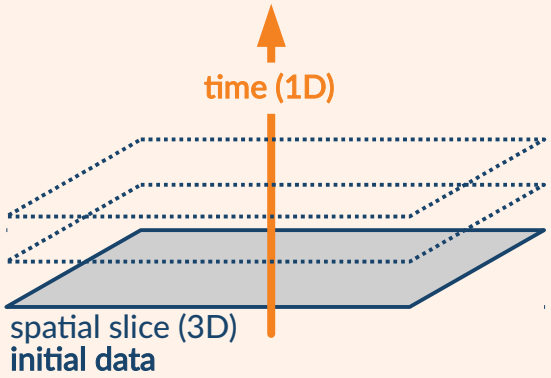
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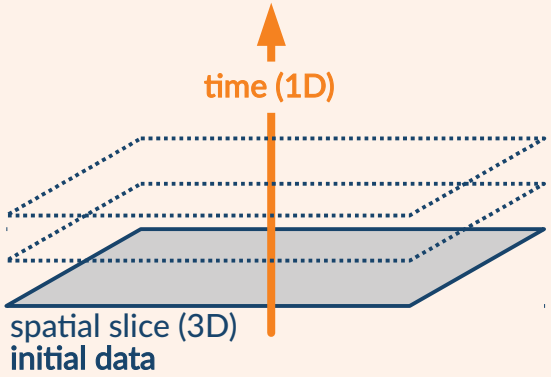
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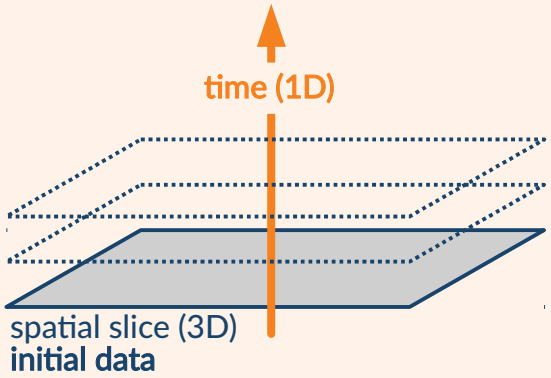
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Formal proof of existence and uniqueness
Yvonne Choquet-Bruhat '52



(3+1) numerical evolution
Pretorius '05; BSSN '87-'99; Sarbach et.Al '02-'04

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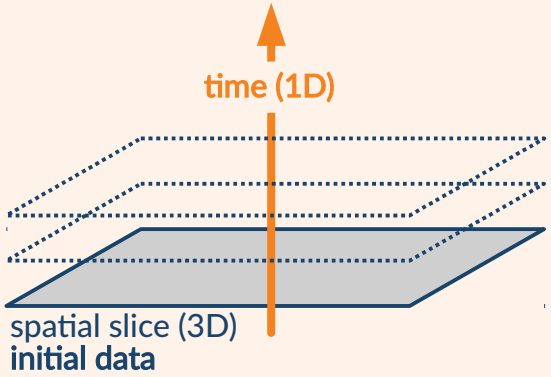
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Pretorius '05; BSSN '87-'99; Sarbach et.Al '02-'04

## ... and for Quadratic Gravity

Noakes, JMP 24, 1846 (1983)
**Held**, Lim, PRD 104 (2021) 8

**Held**, Lim, PRD 108 (2023) 10; 2503.13428
Cayuso '23; East, Siemonsen '23

# A well-posed initial value problem (IVP) ...




"An initial value problem is well-posed if a solution"

- **exists**
- is **unique**
- and depends **continuous**ly on the initial data

## ... for General Relativity

Formal proof of existence and uniqueness
Yvonne Choquet-Bruhat '52

(3+1) numerical evolution
Pretorius '05; BSSN '87-'99; Sarbach et.Al '02-'04

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## ... and for the EFT (at any order)

Figueras, **Held**, Kovacs, 2407.08775

# Leray weights: A simple example ...

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**... that can be diagonalised.**

# Leray weights: A simple example ...

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$$\Box u \overset{2\,|\,0}{=} v$$

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# General Relativity (in harmonic gauge) ...

- Gauge potential: $F^a \equiv -g^{cd}\Gamma^a_{cd}$
- Ricci curvature: $R_{ab} \sim \Box g_{ab} + g_{c(a}\nabla_{b)}F^c + \mathcal{O}(g, \partial g) = 0$
- In **harmonic gauge**, i.e., $F^a = 0$
  the vacuum Einstein equations, i.e., $R_{ab} = 0$
  are of **wave-like form**.

  For constraint propagation see
  Choquet-Bruhat ‘52

**... is already in wave-like form.**

# Quadratic Gravity ...

**Held**, Lim, PRD 104 (2021) 8

- recall $\mathcal{L} = M_{Pl}^2 \left[ R + \frac{1}{12 m_0^2} R^2 + \frac{1}{4 m_2^2} C_{abcd} C^{abcd} \right]$

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Stelle, PRD 16 (1977) 953-969
Noakes, JMP 24, 1846 (1983)
**Held**, Lim, PRD 104 (2021) 8

$2^{nd}$-order variables

$$\Box g_{ab} \sim R_{ab} \equiv S_{ab} + \frac{1}{4} g_{ab} R$$ **massless spin-2** (graviton)

$$\Box R = m_0^2 R$$ **massive spin-0** (scalar)

$$\Box S_{ab} = -\frac{1}{3}\left(\frac{m_2^2}{m_0^2} - 1\right)(\nabla_a \nabla_b R) - 2 S^{cd} C_{acbd} + \mathcal{O}_{\text{lower order}}$$ **massive spin-2** (ghost)

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## ... admits wave-like $2^{nd}$ order field equations.

# Cubic Gravity (after suitable field redefinitions) ...

Figueras, **Held**, Kovacs, 2407.08775

- recall $\mathcal{L}^{(3)}_{\rm EFT} = \frac{1}{M^2_{\rm Pl}}\left[\alpha_1\, R^{ab}\,\Box\, R_{ab} - \beta_1\, R\,\Box\, R + \gamma_3\, C_{ab}{}^{cd} C_{cd}{}^{ef} C_{ef}{}^{ab}\right]$

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- Inductively, this extends to $\mathcal{L}^{(n)}_{\text{reg}} = \sum_{k=0}^{n} \left[ \alpha_k \, R^{ab} \, \Box^k \, R_{ab} - \beta_k \, R \, \Box^k \, R \right]$ with $\alpha_n = 2\beta_n$

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**Not altered if supplemented with an action that only adds to the omitted lower-order terms.**

see Figueras, **Held**, Kovacs, 2407.08775 for a complete proof

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# Well-posed initial value formulation ...

$$\mathcal{L}^{(1)}_{\rm EFT} = M_{\rm Pl}^2\, R$$

✓ Choquet-Bruhat '52

$$\mathcal{L}^{(2)}_{\rm EFT} = \left[\alpha_0 R_{ab}R^{ab} - \beta_0\, R^2\right]$$

order-by-order field redefinitions of the form

$$g_{ab} \to g_{ab} + c_1\, g_{ab}\, X + c_2\, X_{ab}$$

can remove any term containing Ricci variables

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Noakes, JMP 24, 1846 (1983)
Held, Lim '21, '23, '25

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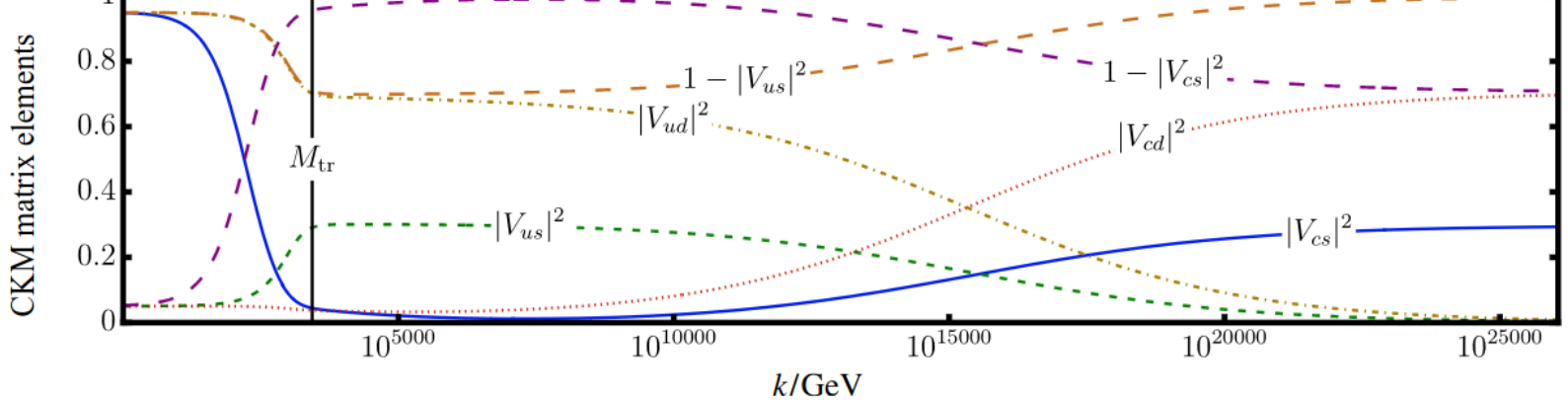
Figueras, Held, Kovacs 2407.08775 ✓

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# ... of general effective field theories of gravity.




**Asymptotic safety provides gravity-induced SM UV completions.**

**Enhanced predictivity at the edge of the UV-complete window.**

**Cosmological predictions require nonlinear evolution.**

