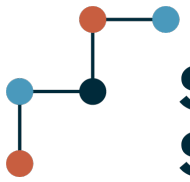
On the validity of finite-temperature EFTs

Fabio Bernardo (University of Geneva)

Based on

[**FB,** Klose, Schicho, Tenkanen '25- **2503.18904**]
[**FB,** Chala, Gil, Schicho '26- **2602.06962**]
[**FB,** Reinle, Schicho '26- **2605.15176**]
[**FB,** Kierkla, Leister, Schicho, Schwaller '26- **2610.12284**]



Swiss National Science Foundation

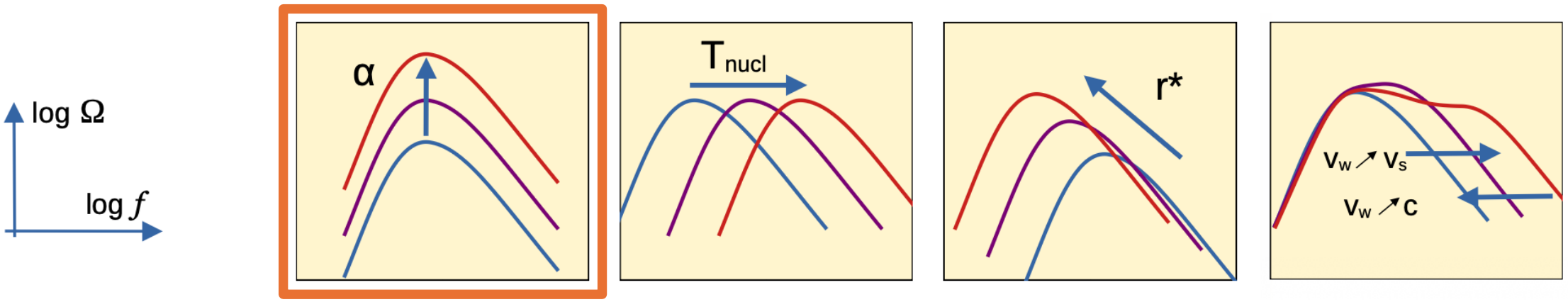
SNSF grant n. **PZ00P2-21599**

First order transitions: from micro-physics to detection

Phase transition parameters

$$\mathcal{L} \xrightarrow{\textbf{EFTs}} \boxed{\alpha,}\ \beta/H,\ T_n,\ v_w,\ \ldots \longrightarrow \textbf{GWs}$$

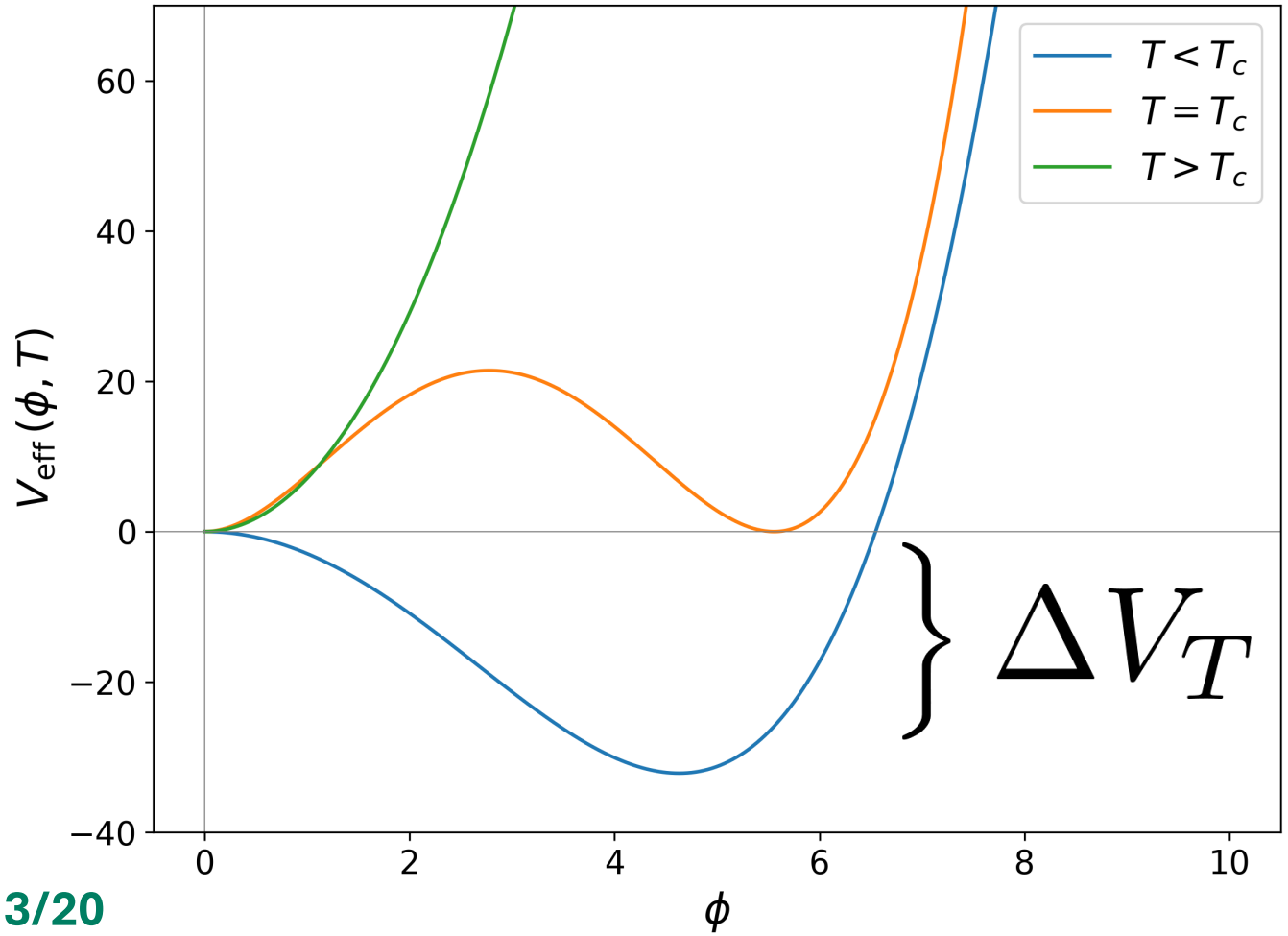
Transition strength



[credit to Rummukainen «Gravitational waves and phase transitions in the early Universe» '25]

Hierarchy in first order phase transitions (FOPT)

There is a **parametric separation** between the temperature and the zero temperature mass



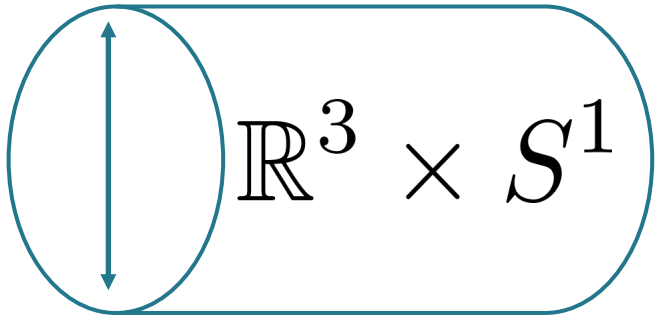
$$\frac{\Delta V_T}{V_{\text{LO}}} \sim \frac{g^2 T^2}{m^2} \sim 1$$

for weakly coupled theories

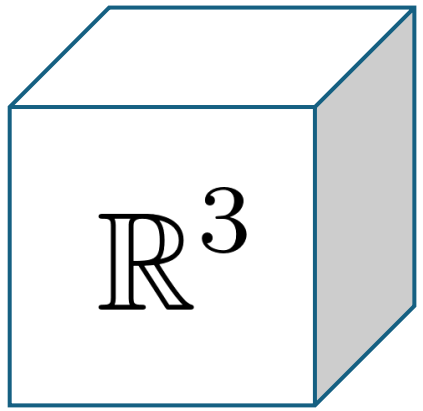
$$T \gg m$$

The 3d EFT approach

Through the imaginary time formalism, temperature can be seen as a **compactified dimension**:

$1/T \sim$  $\mathbb{R}^3 \times S^1$

DIMENSIONAL REDUCTION (DR)



$T \gg m$

The wrapped dimension is **very small**

Effective 3d couplings depend on T

$g(T)$

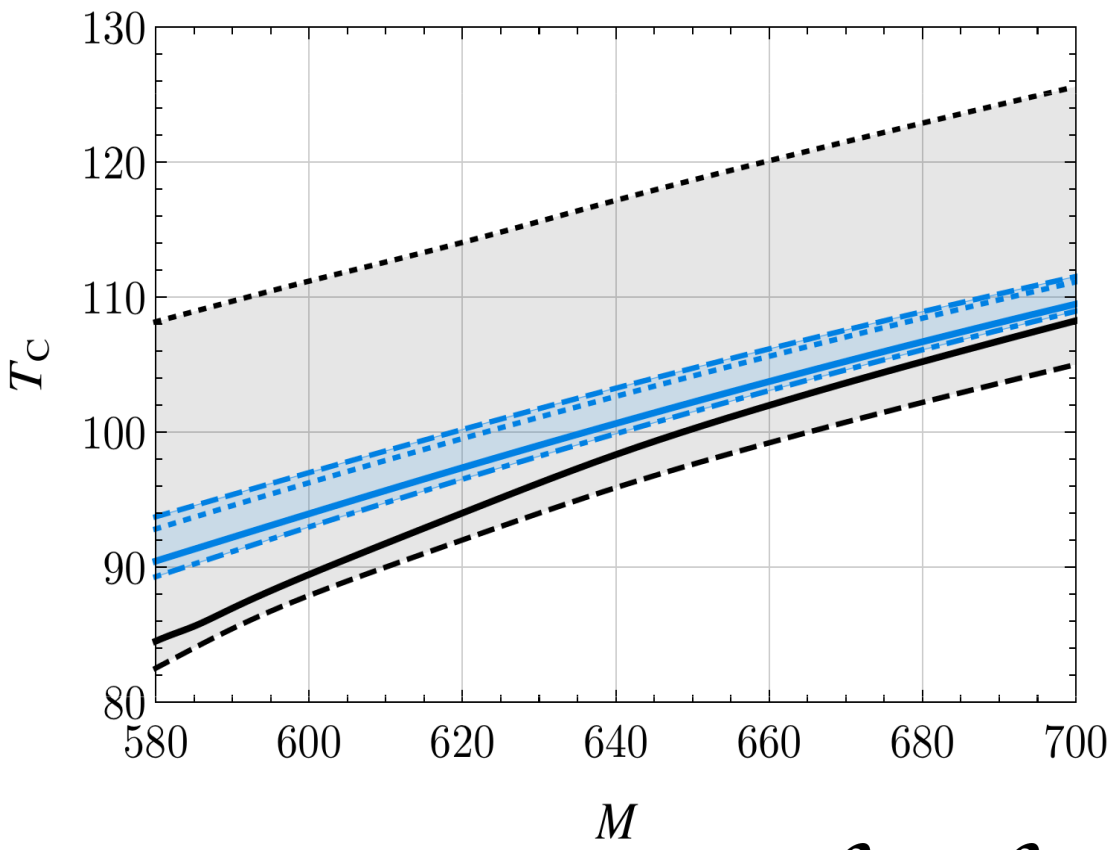
[Ginsparg '80]

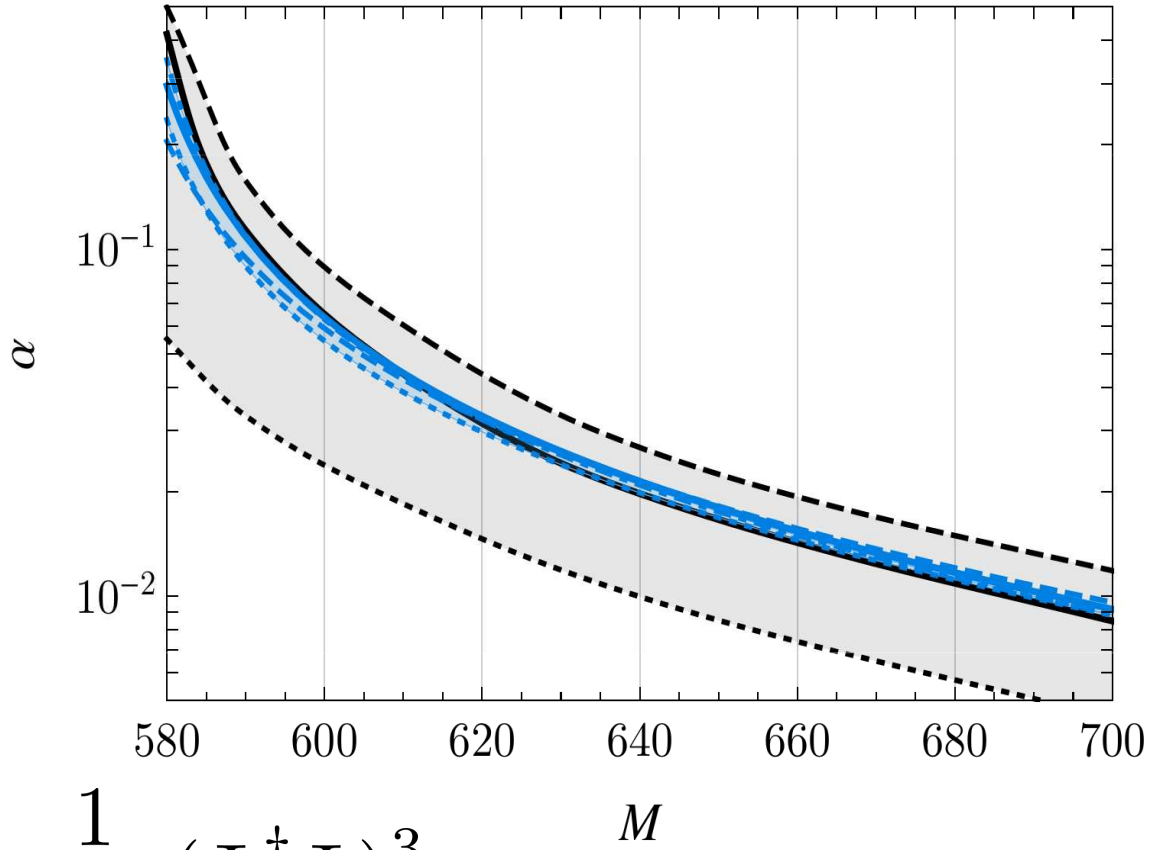
[Kajantie, Laine, Rummukainen, Shaposhnikov '95]

[Kajantie, Laine, Rummukainen, Schröder '03]

The 3d EFT approach is more precise

The uncertainties of PT parameters computed with **3d EFTs** are smaller than the **4d approach**





$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{M^2}(\Phi^\dagger\Phi)^3$$

[Croon, Gould, Schicho et al. '20]

The EFT is not always valid !

The EFT is valid as long as we have **convergence** in the **operator expansion**

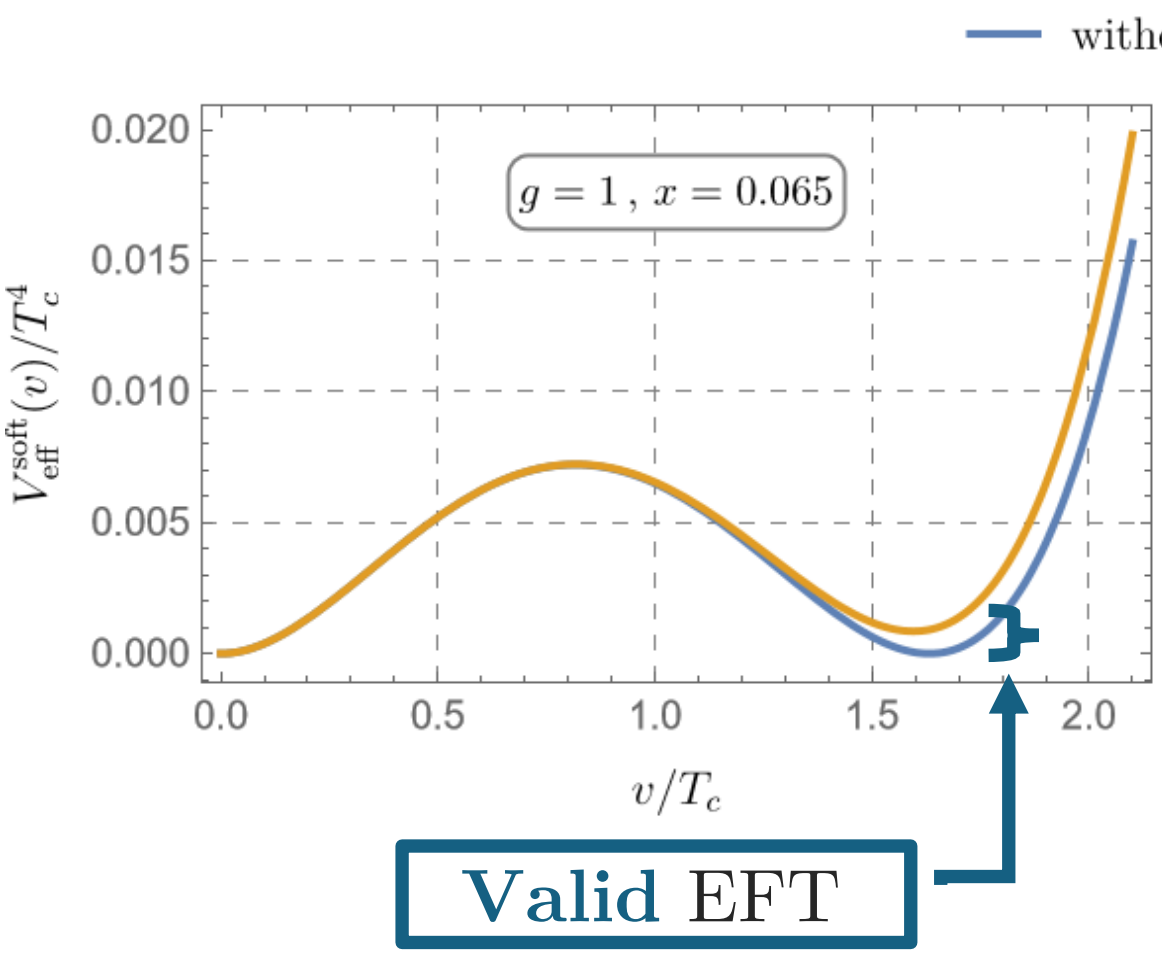
$$\mathcal{L}_{3d} \sim \widehat{\mathcal{L}}\left(\frac{g\,\phi}{4\pi T}, \frac{\partial}{4\pi T}, \frac{g^2}{(4\pi)^2}\right)$$

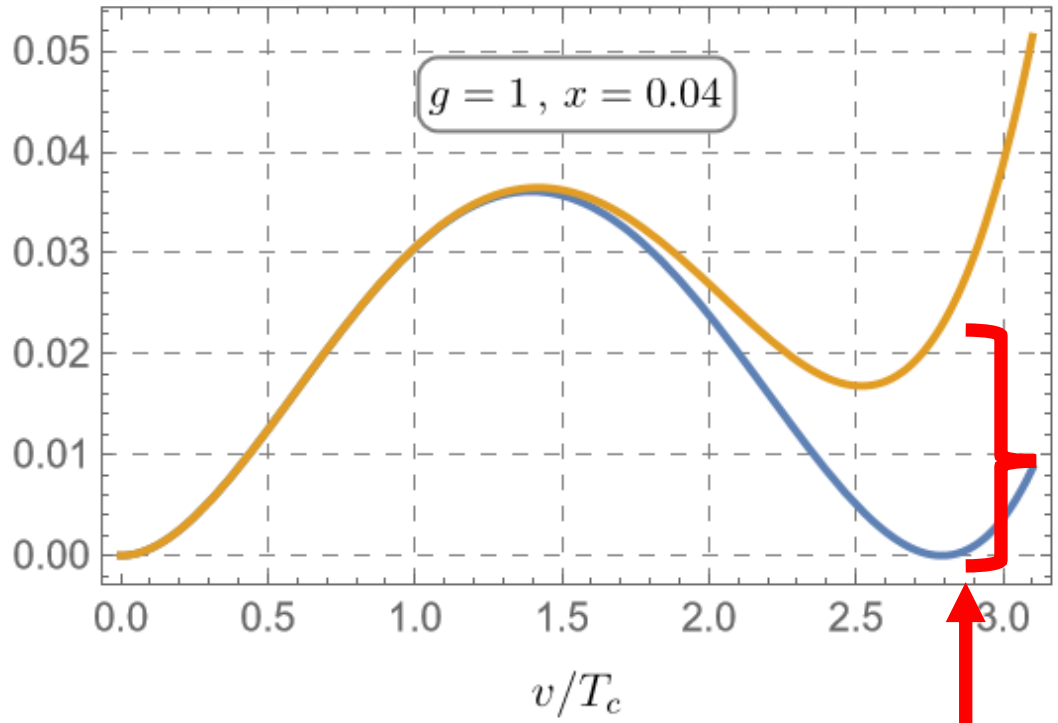
What is the size of **Higher Dimensional Operators (HDO)** ?

$$V_{\rm eff}(v) = \frac{1}{2}\bar{\mu}_3^2\, v^2 - \frac{1}{6\pi}\,\bar{g}_3^3\, v^3 + \frac{1}{4}\bar{\lambda}_3\, v^4 + \boxed{\frac{1}{8}\bar{c}_6\, v^6}$$

EFTs fail for sufficiently strong transitions …

… which are the ones that we may detect with future experiments





Valid EFT

Invalid EFT

PT **too weak** to be detected

PT **strong enough** to be detected

Q: Can 3d EFTs be used for detectable phase transitions?

Abelian Higgs Model

$$\mathcal{L}^{4d}_{\rm AH} = \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + (D_\mu\phi)(D_\mu\phi^\dagger) + \mu^2(\phi\phi^\dagger) + \lambda(\phi\phi^\dagger)^2$$

Step 1 : **DIMENSIONAL REDUCTION**

$$B_\mu = (B_0, B_i)$$

temporal scalar
&
3d gauge vector

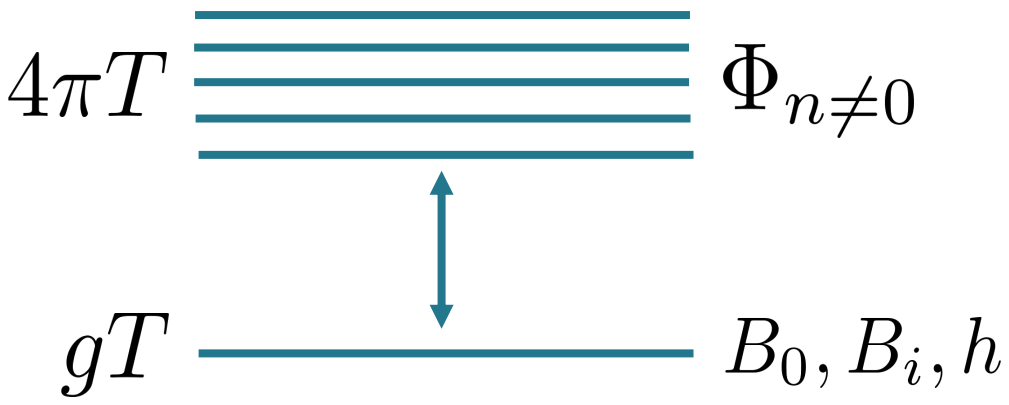
$$\mathcal{L}^{3d}_{\rm DR} = \mathcal{L}^{3d}_{\rm AH} + \frac{1}{2}(\partial_i B_0)^2 + \frac{1}{2}m_D^2 {B_0}^2 + \kappa_3(\phi_3^\dagger\phi_3){B_0}^2 + h_3 {B_0}^4$$

+ Higher Dimensional operators

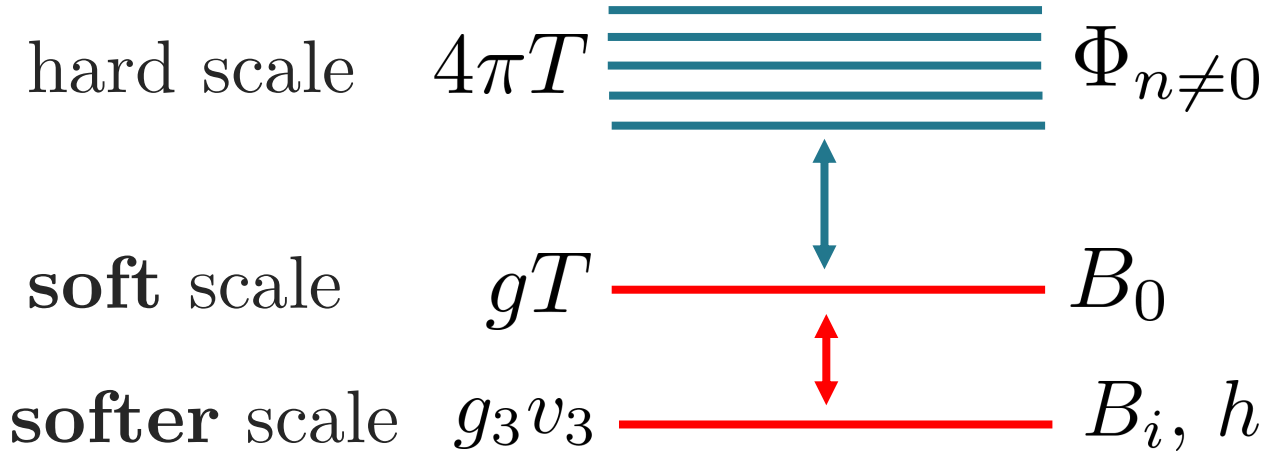
We compare two EFT approaches: SOFT and SOFTER

Masses (at T_c) $\quad m^2_{B_0} = m^2_D \sim g^2\,T^2 \quad m^2_{B_i} \sim g^2_3 v^2_3 \quad m^2_h \sim \lambda_3 v^2_3$

SOFT (DR)



SOFTER (DR+B_0)



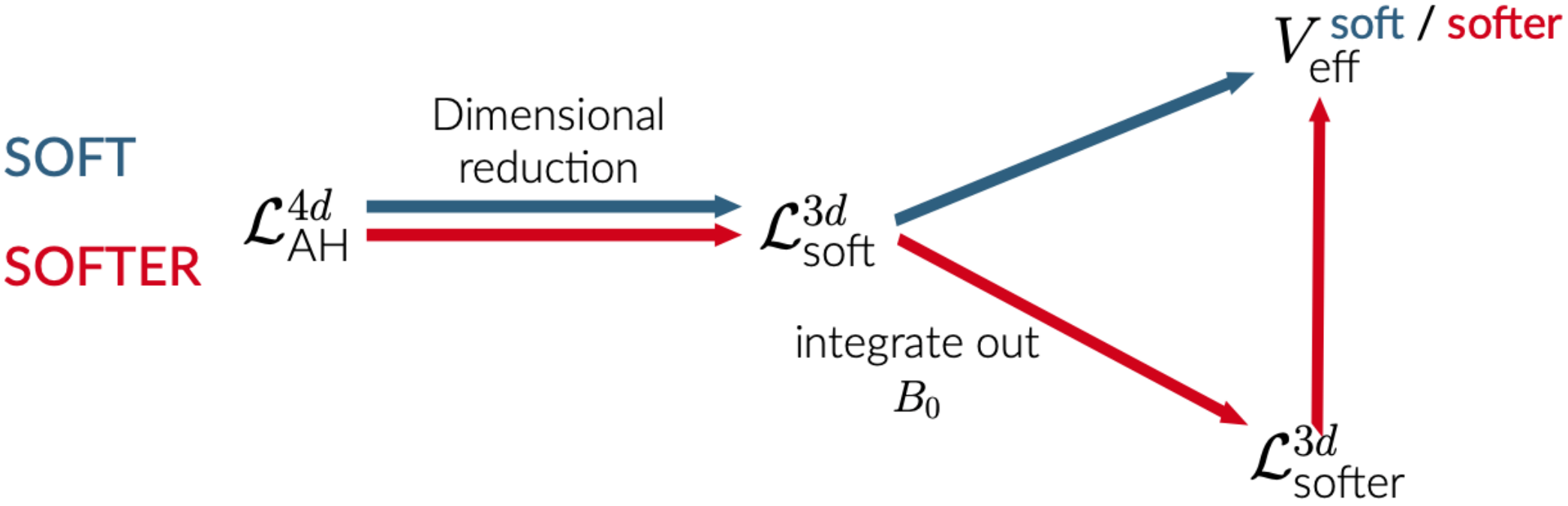
we can integrate out B_0

if $\quad v_3 \ll T^{1/2}$

$m_{B_i}\,,\, m_h \ll m_{B_0}$

Pros and Cons of SOFTER approach

Integrating out B_0 **improves the precision**
(if softer EFT is valid)...



... but **lowers the cut-off** of the EFT

Dimension 6 operator basis

dimension-six operator basis	
$F_{ij}F_{ij}B_0^2$	$(\partial_i F_{ij})^2$
$F_{ij}F_{ij}\phi^\dagger\phi$	$(\partial^2 B_0)^2$
$(D_i\phi^\dagger D_i\phi)(\phi^\dagger\phi)$	$B_0^3\partial^2 B_0$
$(D_i\phi^\dagger D_i\phi)B_0^2$	$(D^2\phi^\dagger)(D^2\phi)$
B_0^6	$(\phi^\dagger\phi)(\phi^\dagger D^2\phi + h.c.)$
$B_0^4(\phi^\dagger\phi)$	$(\partial_i F_{ij})i\phi^\dagger(D_j\phi)$
$B_0^2(\phi^\dagger\phi)^2$	$(\phi^\dagger\phi)B_0\partial^2 B_0$
$(\phi^\dagger\phi)^3$	$(\phi^\dagger D^2\phi)B_0^2 + h.c.$

$$c_6(\phi^\dagger\phi)^3$$

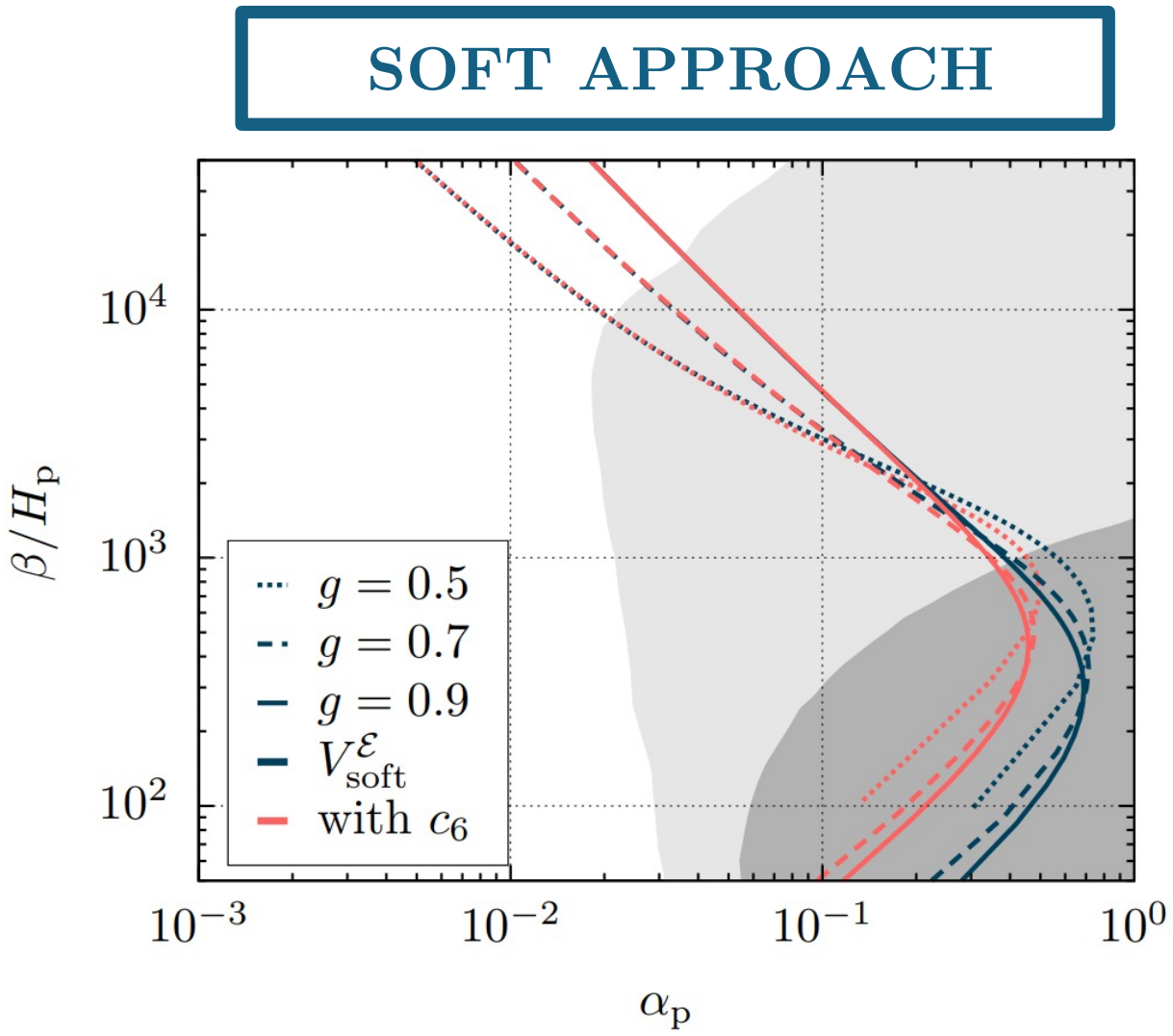


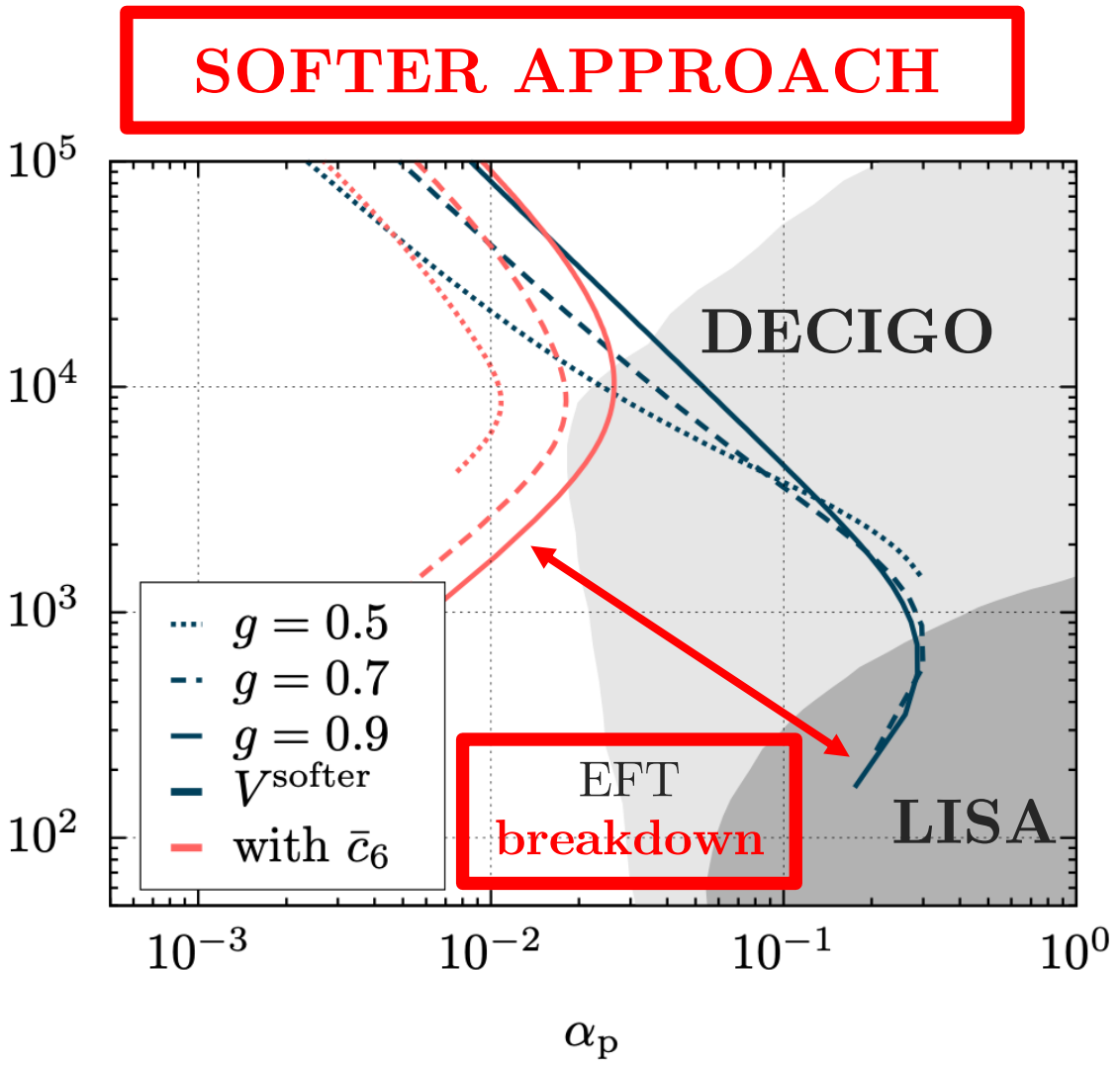
Generated by **DR** and by **integrating** B_0

$$\underbrace{\bar{c}_6}_{\text{softer}} = \underbrace{c_6}_{\text{from DR}} + \underbrace{\delta c_6}_{\text{from } B_0} \gg \underbrace{c_6}_{\text{soft}}$$

Integrating B_0 produces larger couplings, therefore **SOFTER EFT breaks down earlier**

Phase transition through EFTs with(out) HDO

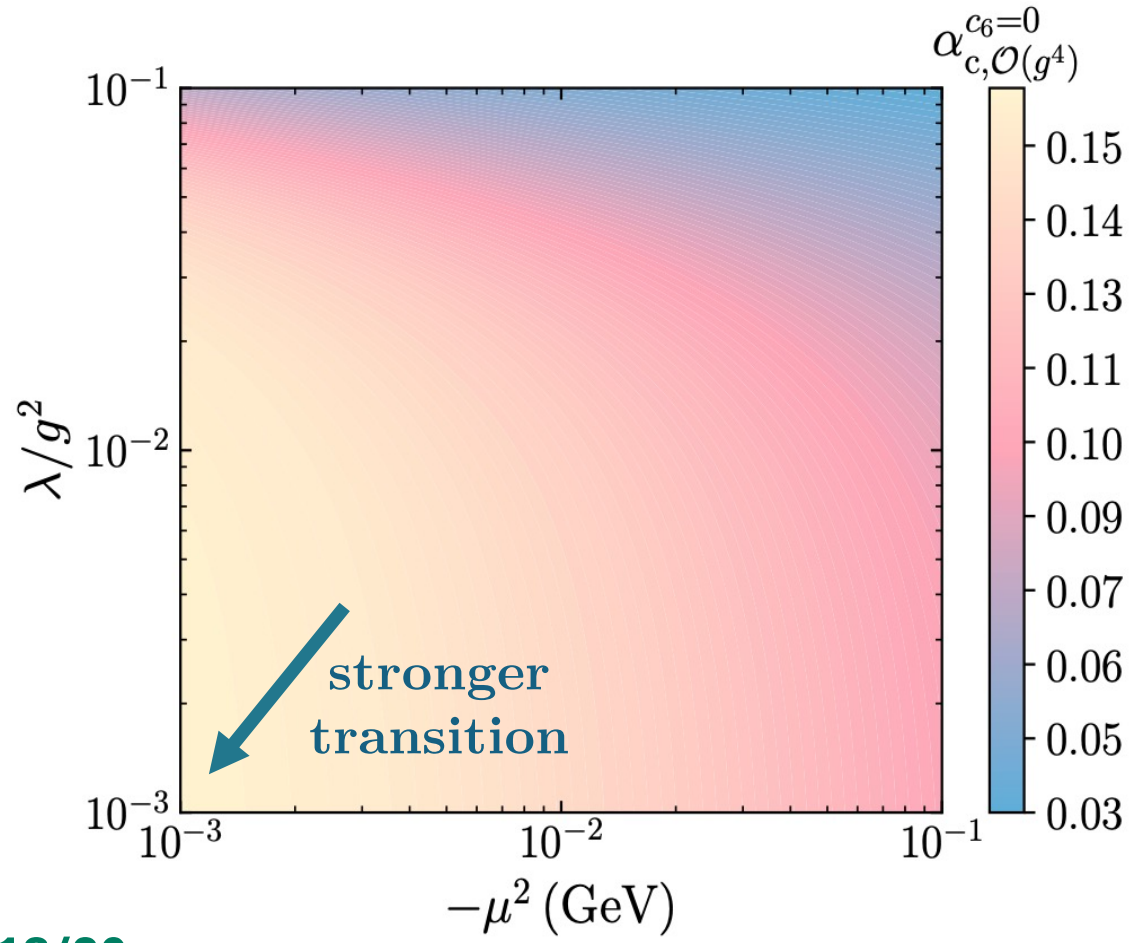


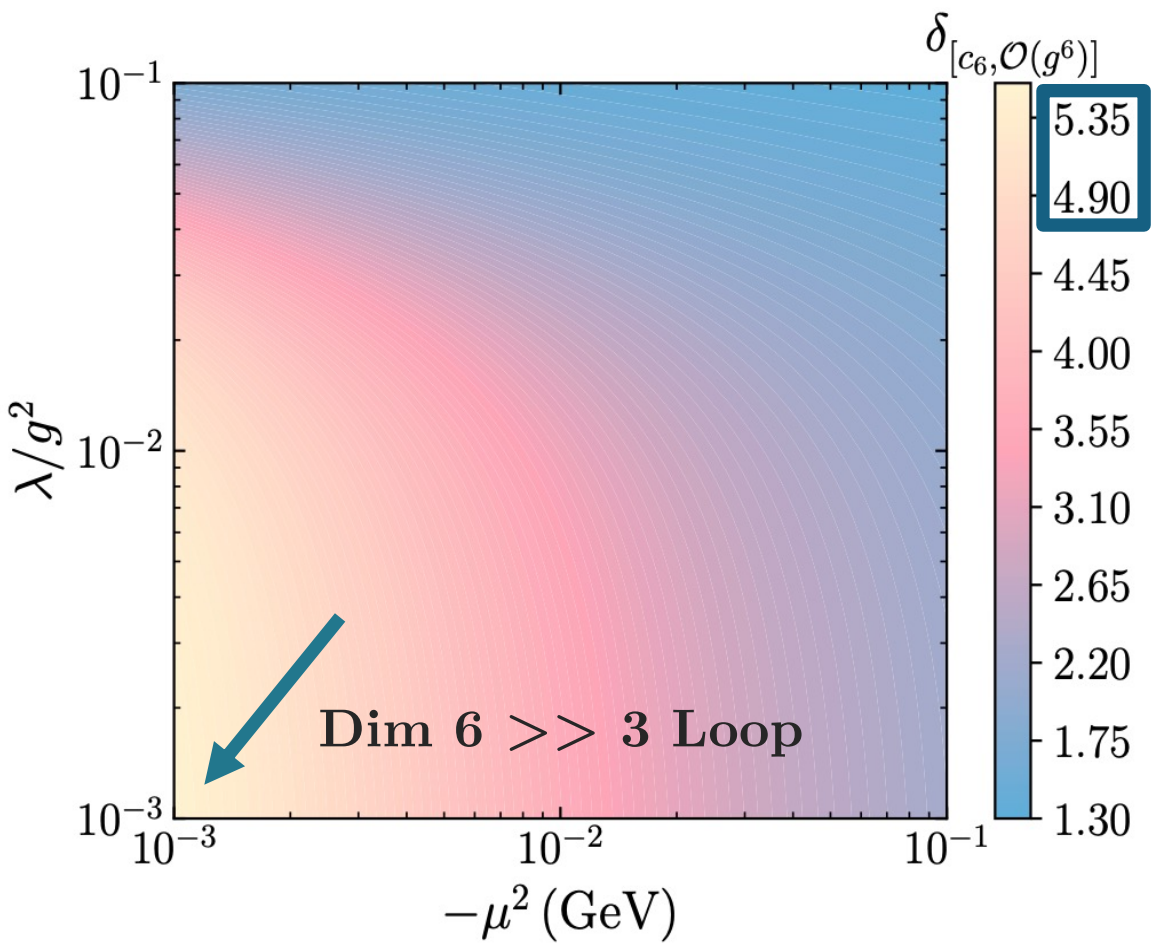


[**FB,** Klose, Schicho, Tenkanen '25- **2503.18904**]

Dim-6 operators beat 3-loop corrections

$$\delta_{[c_6,\mathcal{O}(g^6)]} \equiv \frac{\text{Corrections to } \alpha \text{ due to } \phi^6}{\text{Corrections to } \alpha \text{ due to 3 loop}}$$





[**FB,** Chala, Gil, Schicho '26- **2602.06962**]

HDO matching is now available in Dralgo v1.5 & v1.6

Higher dimensional operators matching and field redefinitions can be extremely tedious, but automated tools are now available



```
In[56]:= sol=Dimension6Matching[TensorList,NList,WC,d][[1]];
    128  sol//Factor//TableForm
```

Out[57]//TableForm=

$$\alpha\left[F^2\,\phi^2\right] \to \frac{1}{6}\,g1^2\,T\left(7\,g1^2-4\,\lambda\right)\,Zb[3,\,0]$$

$$\alpha\left[B0^2\,F^2\right] \to -\frac{1}{6}\,(-5+d)\,g1^4\,Zb[3,\,0]$$

$$\alpha\left[D^2\,\phi^4,\,1\right] \to -\frac{1}{3}\,T\left(-15\,g1^4+2\,d\,g1^4-80\,g1^2\,\lambda+8\,\lambda^2\right)\,Zb[3,\,0]$$

$$\alpha\left[D^2\,\phi^4,\,2\right] \to -\frac{1}{6}\,T\left(5\,g1^4+2\,d\,g1^4-20\,g1^2\,\lambda-12\,g1^2\,xi\,\lambda+16\,\lambda^2\right)\,Zb[3,\,0]$$

$$\alpha\left[B0^2\,D^2\,\phi^2,\,2\right] \to \frac{1}{6}\,g1^2\,T\left(-18\,g1^2+3\,d\,g1^2+6\,g1^2\,xi+16\,\lambda-4\,d\,\lambda\right)\,Zb[3,\,0]$$

$$\alpha\left[B0^2\,D^2\,\phi^2,\,3\right] \to \frac{1}{3}\,(-4+d)\,g1^2\,T\left(7\,g1^2-4\,\lambda\right)\,Zb[3,\,0]$$

$$\alpha\left[B0^2\,D^2\,\phi^2,\,1\right] \to -\frac{1}{3}\,g1^2\,T\left(-2\,g1^2+d\,g1^2-24\,\lambda+4\,d\,\lambda\right)\,Zb[3,\,0]$$

$$\alpha\left[B0^4\,D^2\right] \to -\frac{1}{9}\,(-5+d)\,(-4+d)\,g1^4\,T\,Zb[3,\,0]$$

$$\alpha\left[\phi^6\right] \to \frac{4}{3}\,T^2\left(d\,g1^6-3\,g1^2\,xi\,\lambda^2+28\,\lambda^3\right)\,Zb[3,\,0]$$

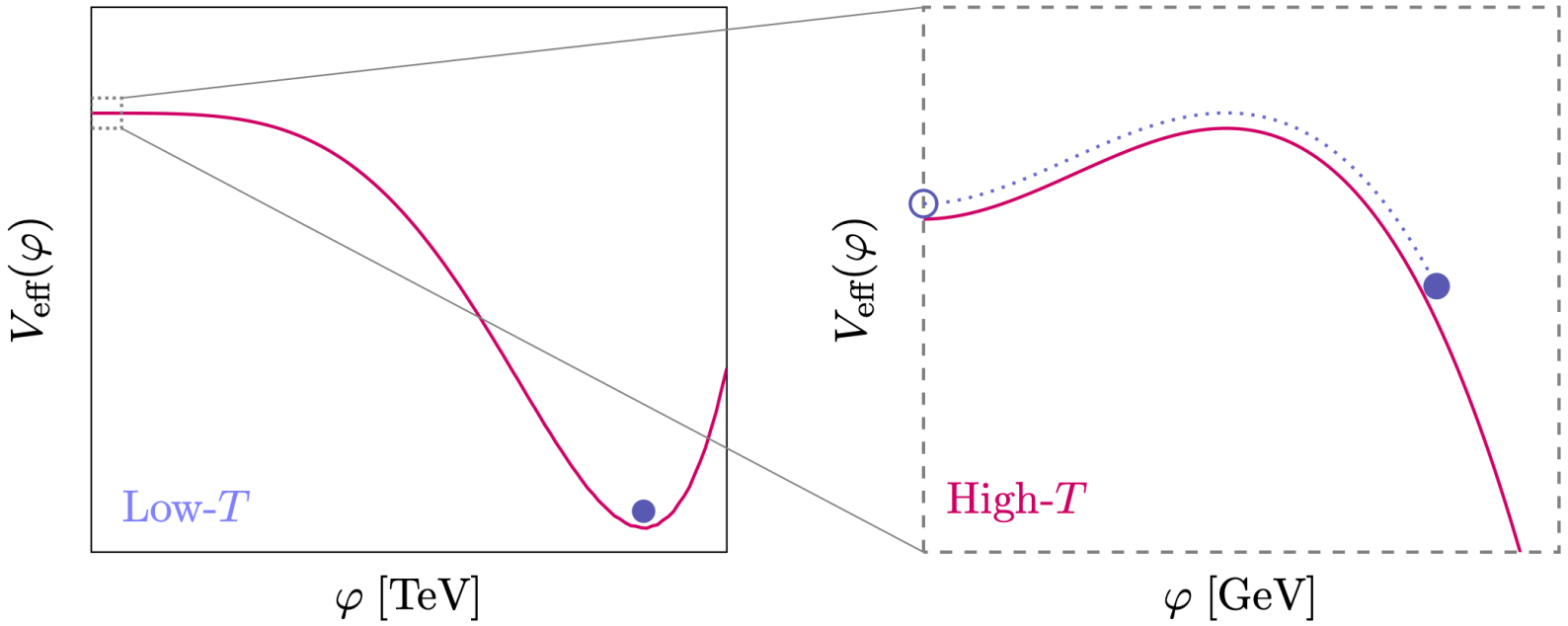
[**FB,** Reinle, Schicho '26- **2605.15176**]

Supercooled phase transitions

Classically scale-invariant Abelian Higgs Model

$$\mathcal{L}^{4d}_{\rm AH} = \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + (D_\mu\phi)(D_\mu\phi^\dagger) + \cancel{\mu^2(\phi\phi^\dagger)} + \lambda(\phi\phi^\dagger)^2$$

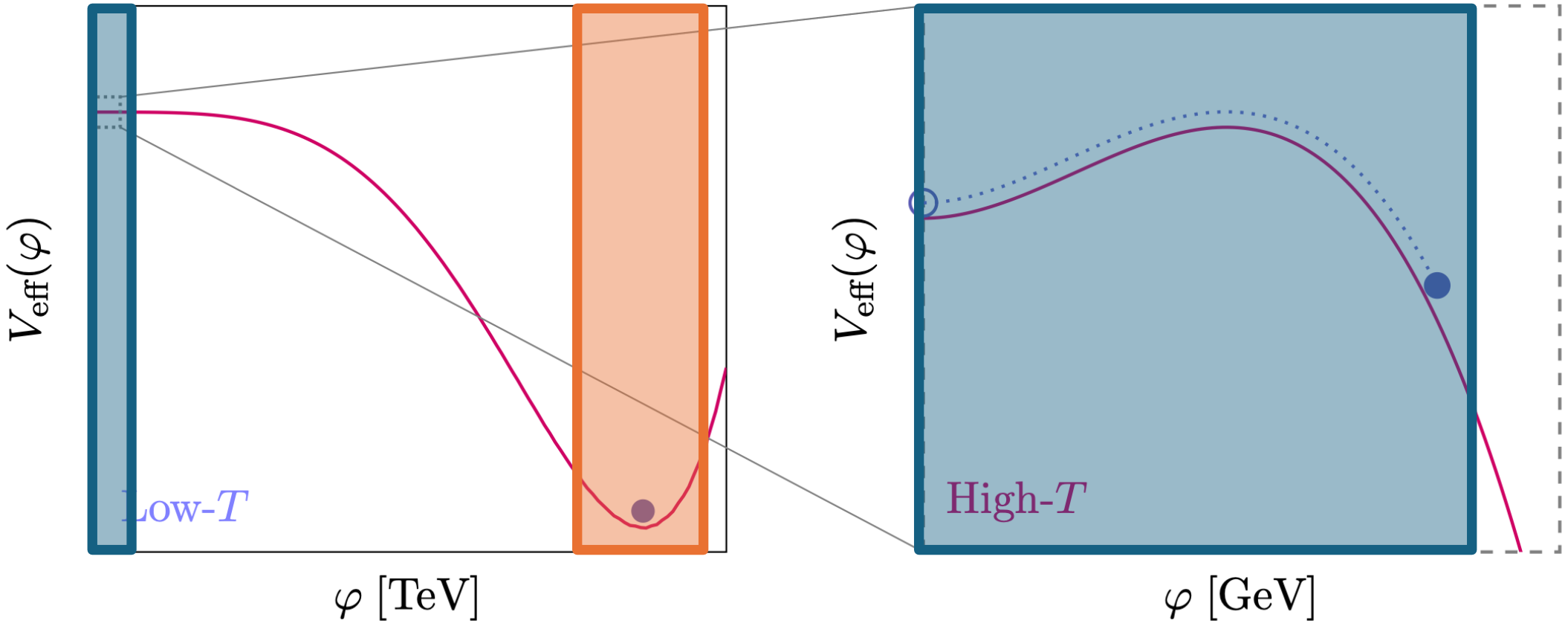
In this scenario, the universe remains trapped in the false vacuum longer, **nucleation** occurs at very low temperature



[Kierkla, Lewicki, Schicho et al. '26]

The supercooled 'hybrid' approach

Effective potential at **nucleation** in the presence of **supercooling**



T = 0 QFT for the real vacuum

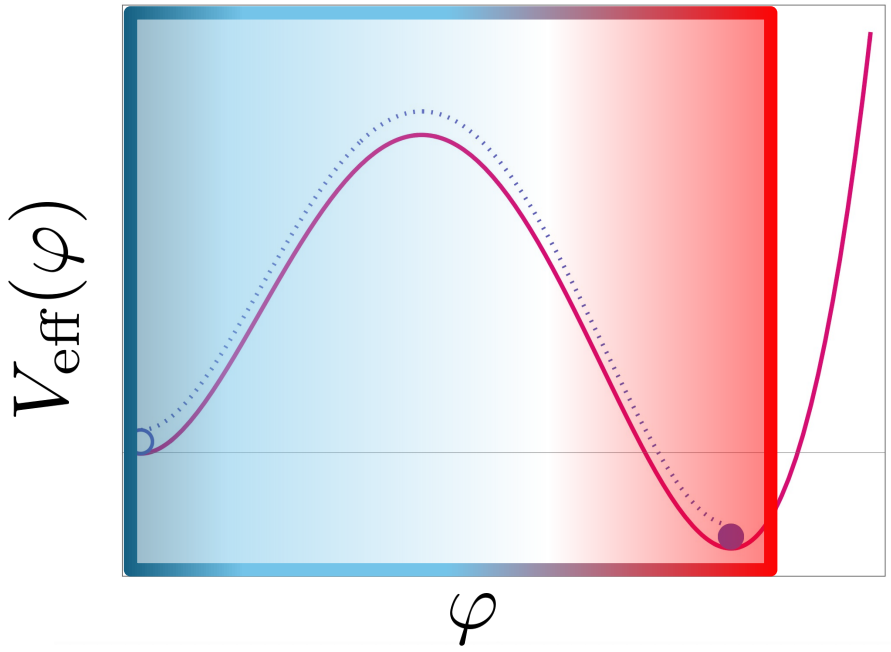
DR for the barrier

With vs without supercooling

Without supercooling

we had to use DR for the

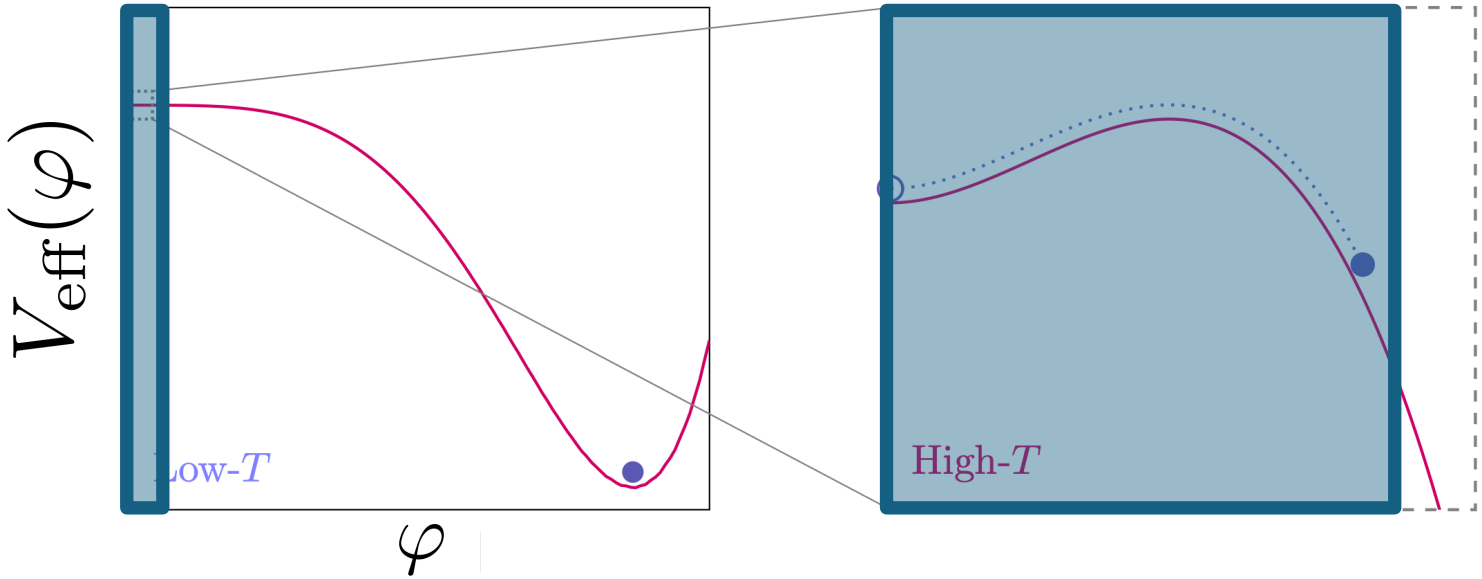
two vacua



With supercooling

we can use DR only

close the **barrier**



Q: Is DR really valid close to the barrier?

$$\mathcal{L}_{3d} \sim \widehat{\mathcal{L}}\left(\frac{g\,\phi}{4\pi\,T}, \frac{\partial}{4\pi T}, \frac{g^2}{(4\pi)^2}\right)$$

GOOD FOR EFT

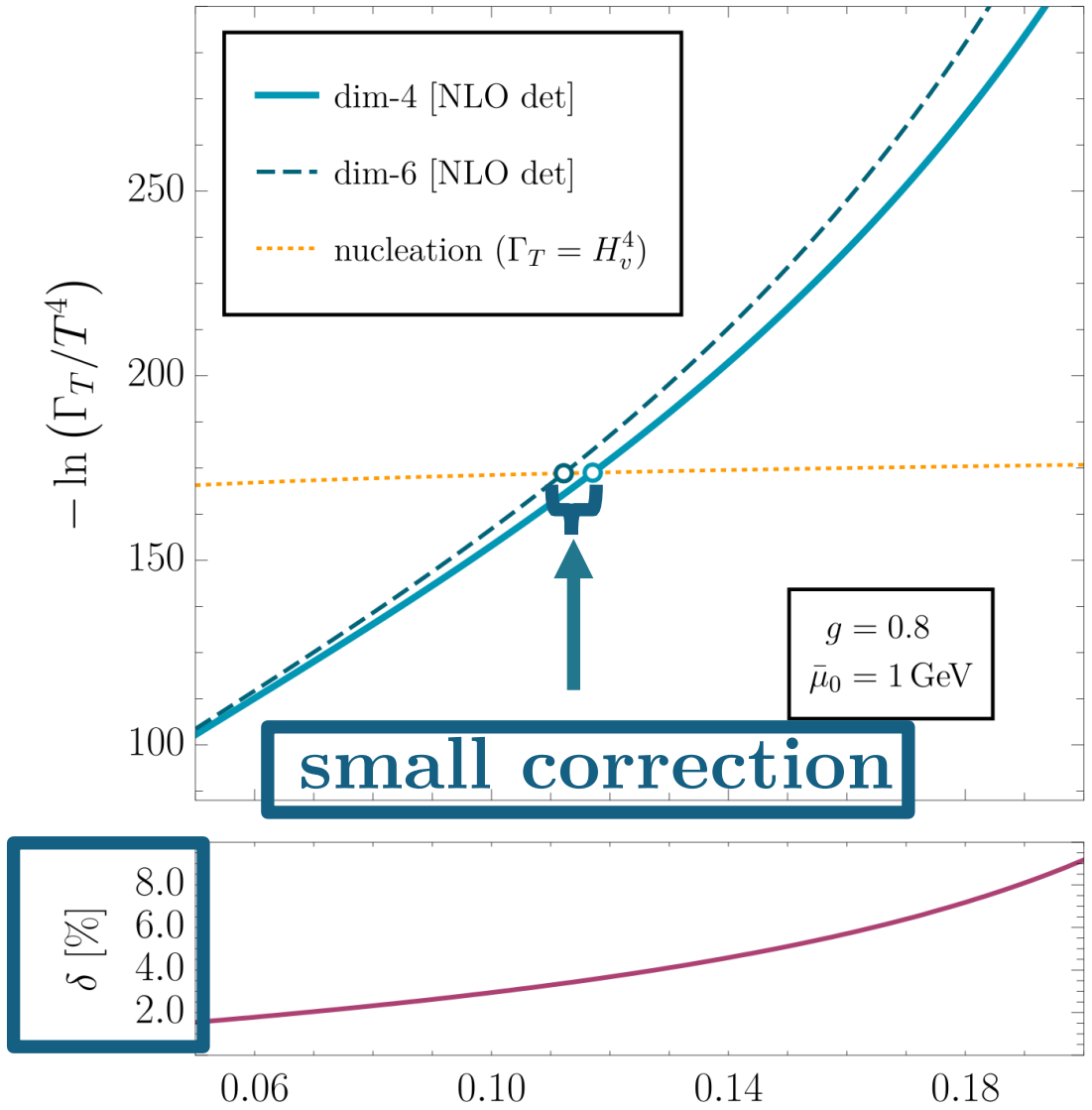
Close to the barrier, the **value of the field is small**, we may expect that operators with many ϕ can be negligible

BAD FOR EFT

Nucleation temperature is very low, hence also the cut-off of the EFT is very small

A: Dimensional reduction is valid !

Higher dimensional operators give corrections of the order of **a few percent**, we can therefore rely on **DR**



[**FB,** Kierkla, Leister, Schicho, Schwaller '26- **2610.12284**]

Conclusions

1. **3d EFTs** break down for sufficiently strong transitions manifesting in **non-negligible** higher dimensional operators

2. **Higher dimensional operators** are **more important** than higher loop matching for strong transitions

3. Higher dimensional operators **matching** is now **available** in DRalgo v1.5 & v1.6

4. **Dimensional reduction** is robust in the hybrid approach of supercooled phase transitions