Can the Universe be matter dominated after a supercooled phase transition ?

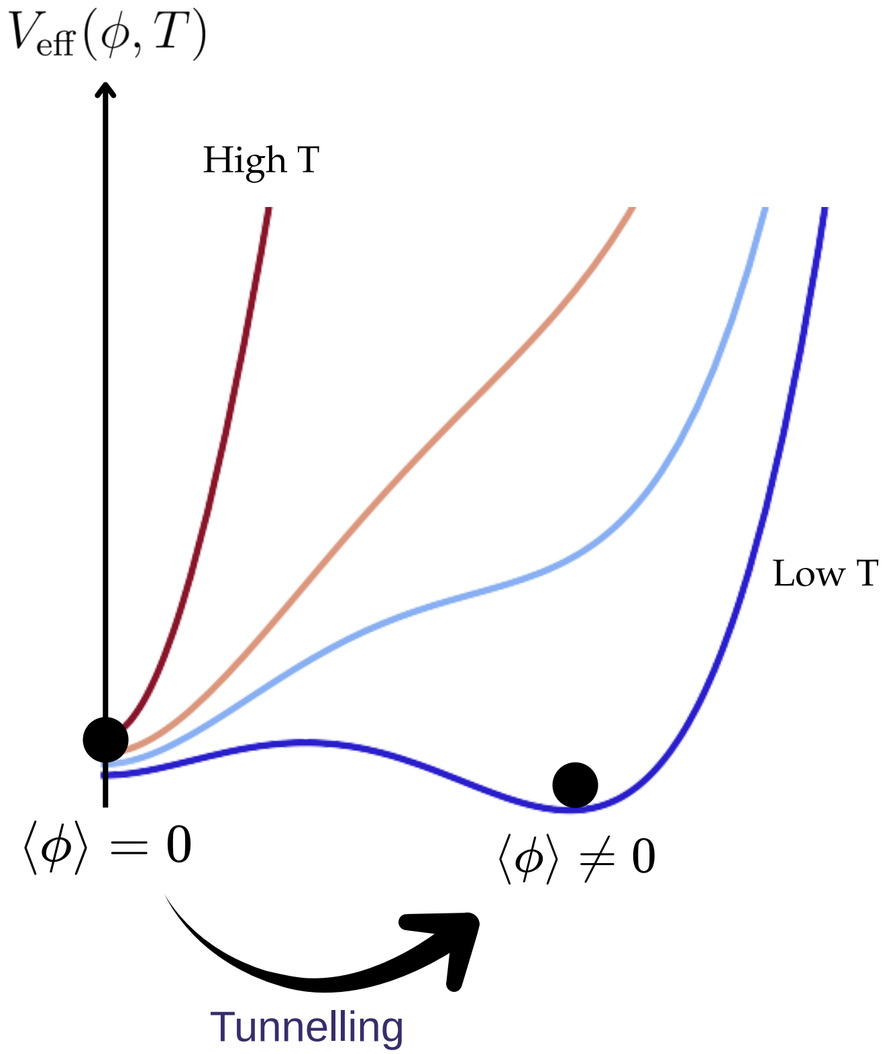
Speaker: Henda Mansour

@VAST 2026

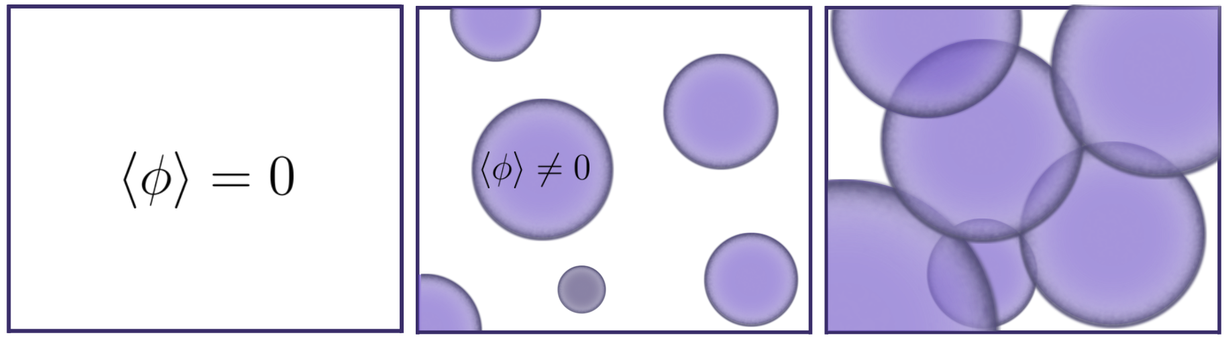
Based on: 2607.19469

with Yann Gouttenoire and Felix Kahlhoefer

First-Order Phase Transitions

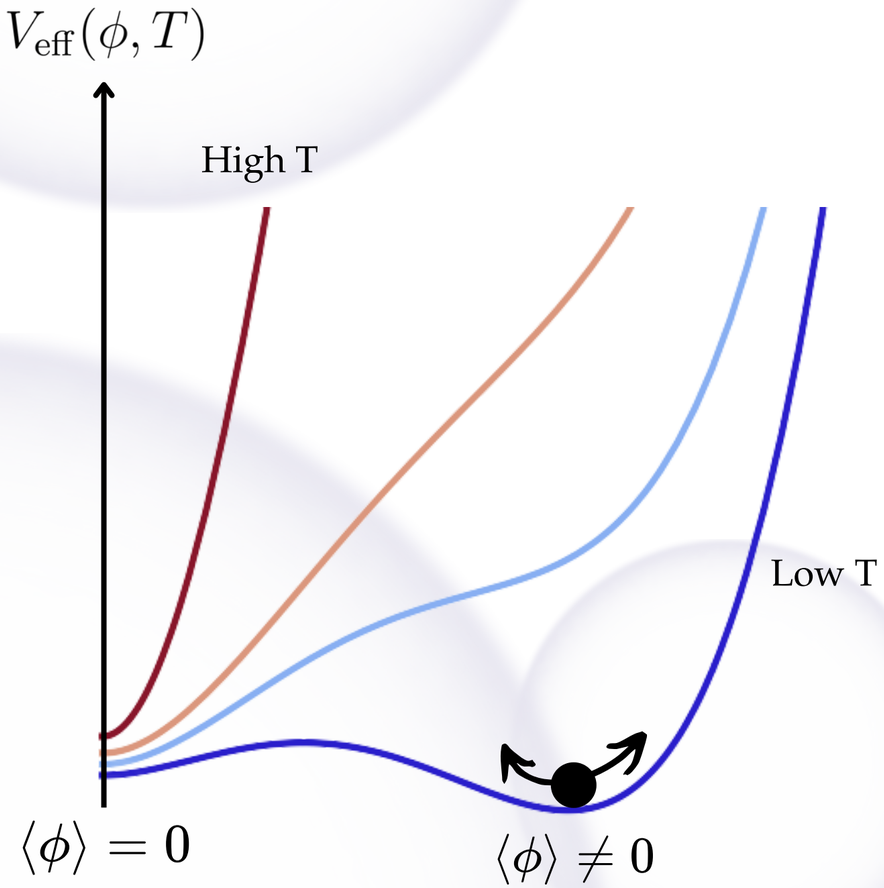


The transition proceeds through bubble nucleation:



- Vacuum energy of the scalar is converted into kinetic and gradient energy of the bubbles/bubble walls
- For strong supercooling the scalar field dominates the energy density of the universe $\Delta V > \rho_{\text{rad}}(T_{\text{PT}})$
- Promising source of stochastic gravitational wave backgrounds + other BSM physics

Delayed reheating: scalar-field domination



Assume that reheating is slow, $\Gamma_\phi/H_\star \ll 1$, and the energy density in the scalar field dominates:
After the transition, the field (on average) oscillates around the new minimum

→ **Early matter domination?** i.e. $w_\phi = 0$

(e.g: Ellis et al. (2019) and (2020) , Gonstal et al. (2025))

At the same time, the bubble walls are relativistic. A significant amount of the vacuum energy is converted into gradients

→ **radiation domination?** i.e $w_\phi = \frac{1}{3}$

Why does it matter ?

Gravitational Waves:

Impacts the GW spectrum : dilution and IR spectral shape

General impact of MD : (Kazunori Nakayama et al. (2008), Boyle and Steinhardt (2008), Seto and Yokoyama (2003), D'Eramo and Schmitz (2019)....)
More specific to FOPTs: (Ellis et al. (2019) and (2020) , Gonstal et al. (2025))

Particle production :

Dilution of pre-existing abundances in the case of matter domination (MD)

(Co. et al. (2015), Cirelli et al. (2016), Bishara et al. (2024) and many others)
More specific to FOPT: Benso, Kahlhoefer, HM (2025) [2504.10593]

Production of primordial black holes :

Smaller overdensity threshold for collapse during MD

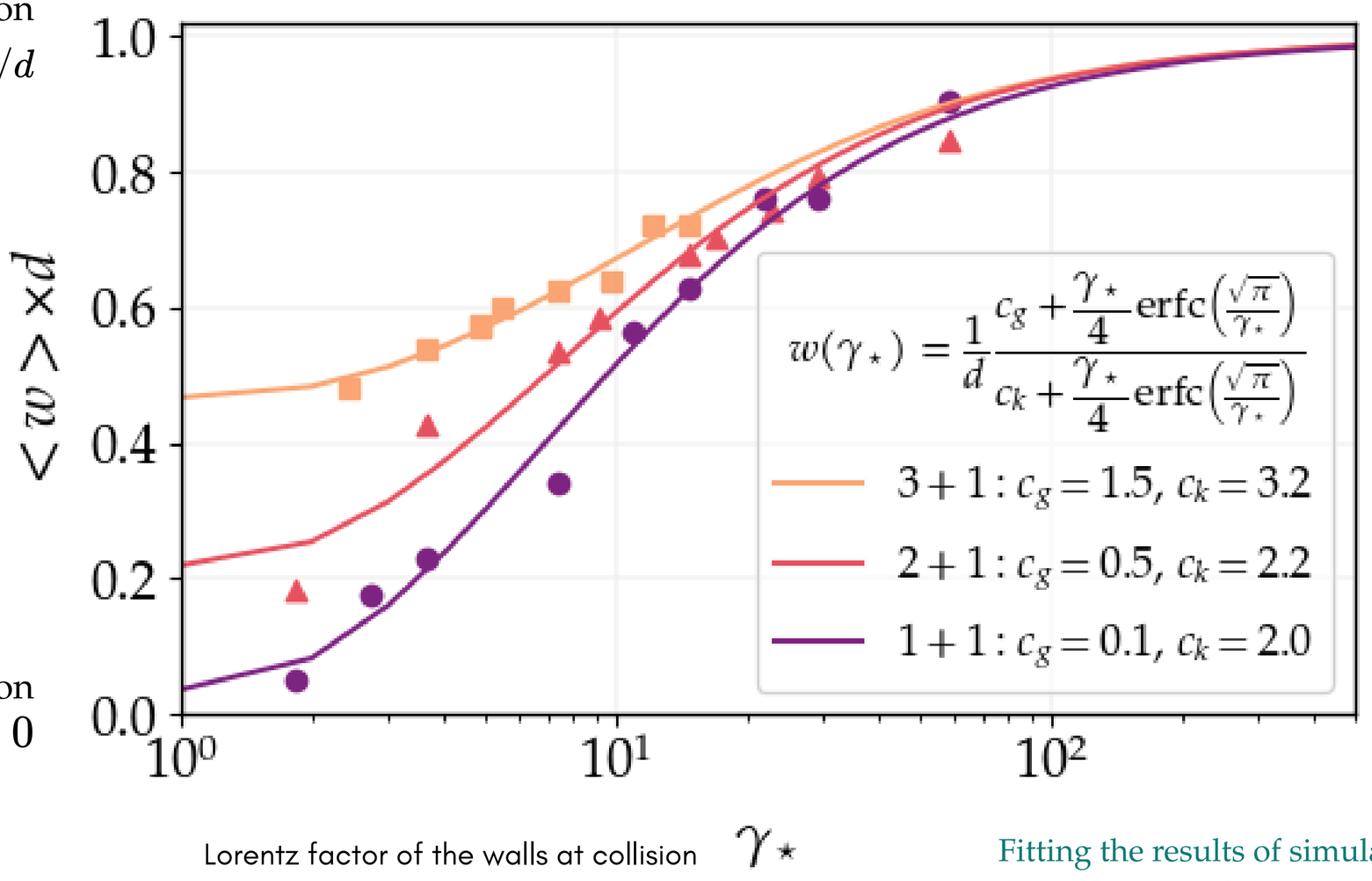
(PBH production during matter domination : Harada et al. [1609.01588],
PBH production during FOPTs: Y. Gouttenoire and T. Volansky, [2305.04942]
More recently: Ai et al. [2605.11332])

EoS at percolation: not matter-like

(Short answer)

radiation domination
$w = 1/d$

matter domination
$w = 0$

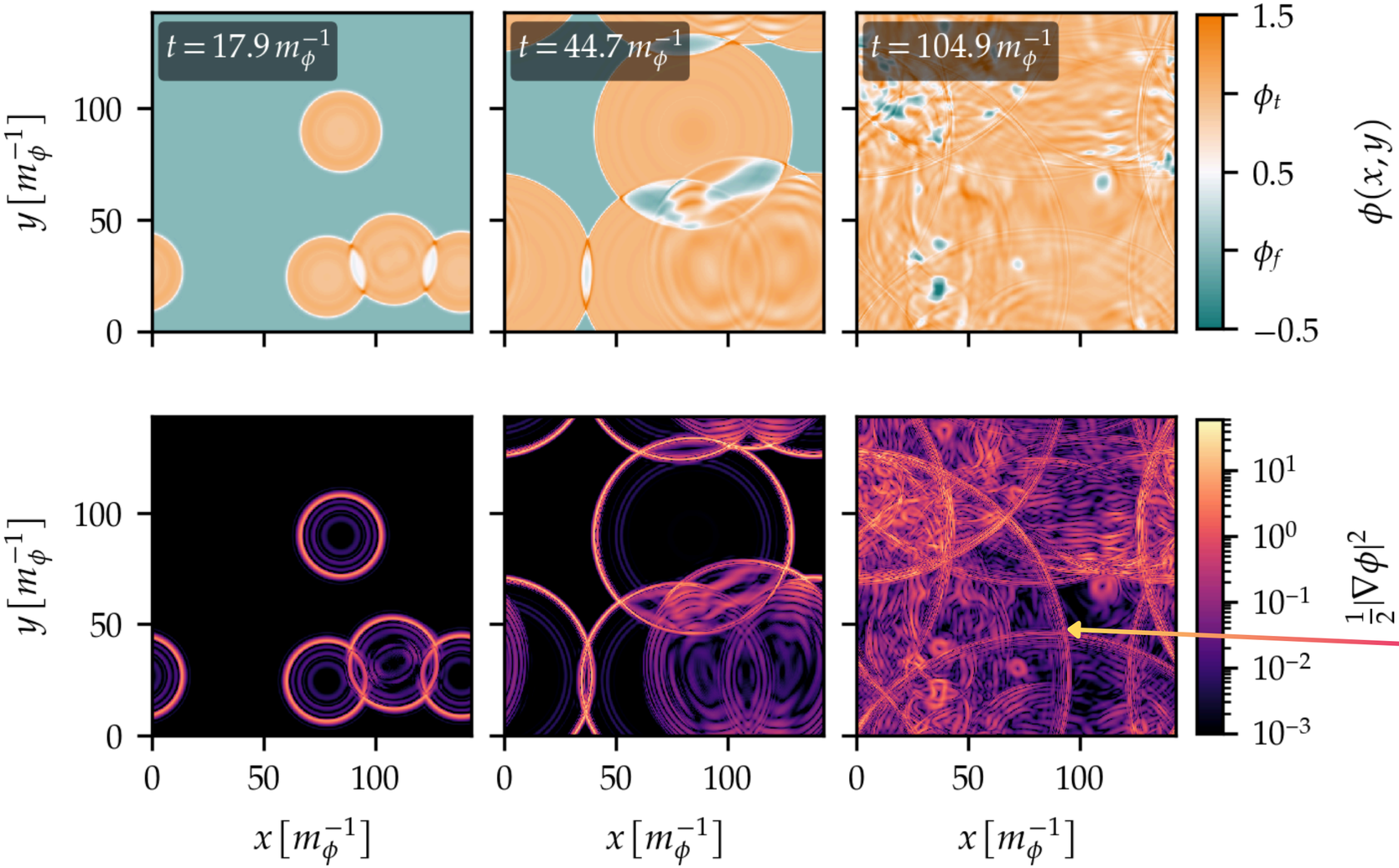


For $\gamma_\star \gg 10^2$, the scalar field behaves like radiation after percolation.

Fitting the results of simulations in 1+d dimensions
(d = 1, 2, 3 spatial dimensions, flat static background)

Why not matter-like?

(Short answer)



Field

Gradient

Still concentrated in shells even after the new vacuum is established

EoS at percolation: analytic

Build some analytical understanding of the equation of state:

General EoS for an inhomogeneous scalar field:

$$\omega \equiv \frac{p_\phi}{\rho_{\rm tot}} = \frac{\rho_{\rm kin} + (d-2)\rho_{\rm grad}/d - \rho_{\rm pot}}{\rho_{\rm kin} + \rho_{\rm grad} + \rho_{\rm pot}} \quad \text{with} \quad d = \text{\# of spatial dimensions}$$

kinetic — gradient — potential

EoS at percolation: analytic

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[Turner 1983]

Assuming that after percolation the system virializes, and that the potential can be approximated as quadratic

Approximate around the minimum

kinetic

$$\langle \rho_{\rm kin} \rangle \simeq \langle \rho_{\rm grad} \rangle + \frac{n}{2} \langle \rho_{\rm pot} \rangle \quad \Rightarrow \quad V(\phi) \sim \phi^2 \quad \Rightarrow \quad \langle \rho_{\rm kin} \rangle \simeq \langle \rho_{\rm grad} \rangle + \langle \rho_{\rm pot} \rangle$$

We find a simplified expression:

$$\omega = d^{-1} \frac{\langle \rho_{\rm grad} \rangle}{\langle \rho_{\rm grad} \rangle + \langle \rho_{\rm pot} \rangle} \simeq d^{-1} \frac{\langle \rho_{\rm grad} \rangle}{\langle \rho_{\rm kin} \rangle}$$

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If wall gradients dominate → Radiation domination
If wall gradients negligible → Matter domination

This is why the EoS depends on the Lorentz factor $\gamma_\star$

EoS at percolation: lattice

Solve on the lattice: $\ddot{\phi} - a^{-2}\vec{\nabla}^2\phi + dH\dot{\phi} = -V_{,\phi}$ with $V(\phi) = \frac{m_\phi^2}{2}\phi^2 - \kappa\phi^3 + \alpha\phi^4$

Example: 3D simulation with random nucleation and static background

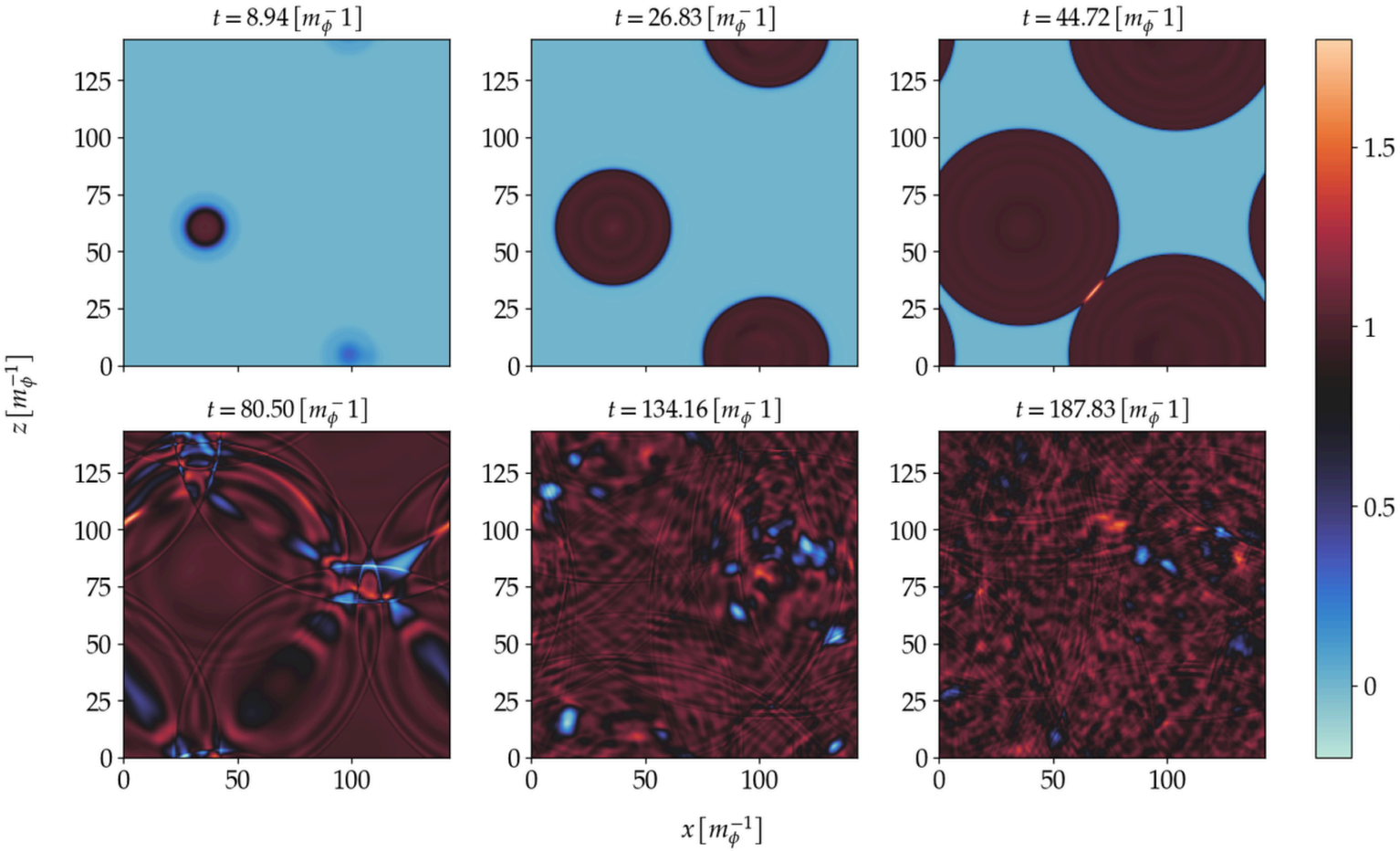
Initial configuration :
randomly nucleated critical bubbles at rest

The more the bubbles expand and accelerate the more Lorentz-contracted the walls become

$$\gamma_\star \propto \frac{R_\star}{R_n}$$

$R_\star$ Radius at collision

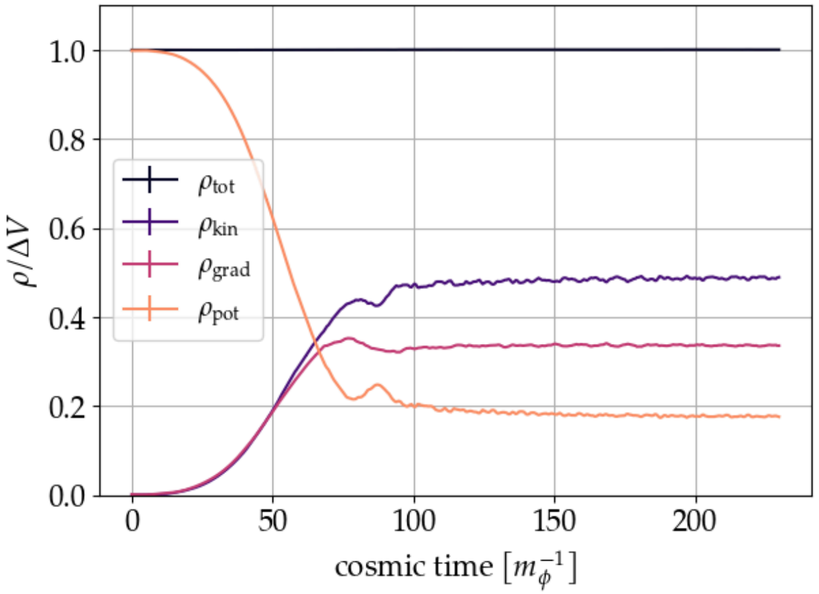
R_n Radius at nucleation

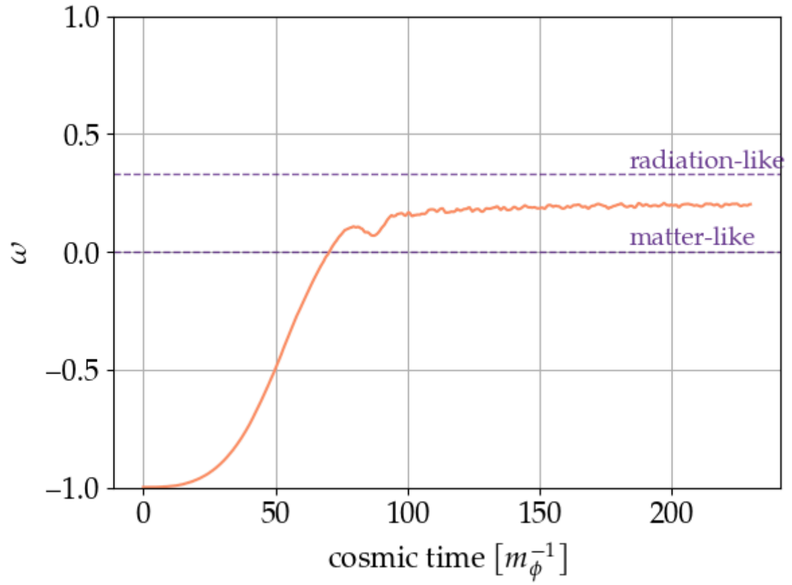


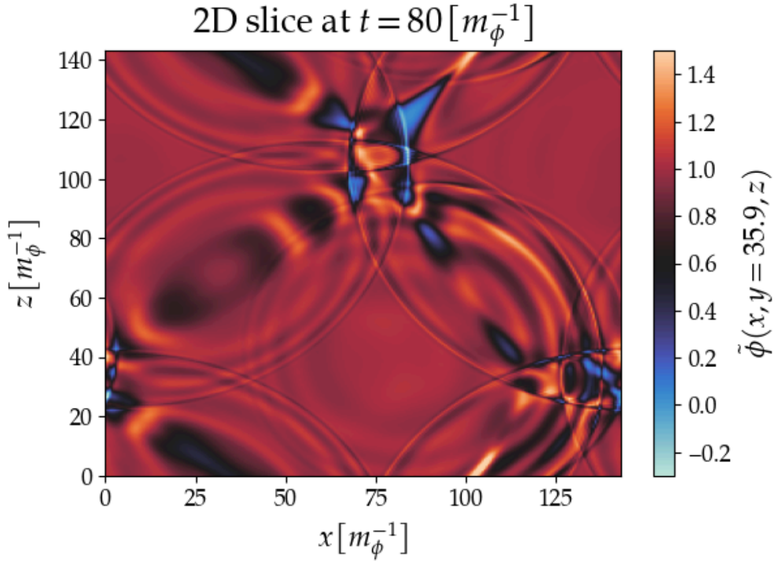
EoS at percolation: lattice

Solve on the lattice: $\ddot{\phi} - a^{-2}\vec{\nabla}^2\phi + dH\dot{\phi} = -V_{,\phi}$ with $V(\phi) = \frac{m_\phi^2}{2}\phi^2 - \kappa\phi^3 + \alpha\phi^4$

The EoS can be determined from the evolution of the energy densities: $\omega \equiv \frac{p_\phi}{\rho_{\rm tot}} = \frac{\rho_{\rm kin} + (d-2)\rho_{\rm grad}/d - \rho_{\rm pot}}{\rho_{\rm kin} + \rho_{\rm grad} + \rho_{\rm pot}}$







The lattice results show an equation of state value between matter and radiation domination.

Not matter-like

(set by the Lorentz factor)

Systematic study of the simulation results for bubble collisions in 1, 2 and 3 spatial dimensions

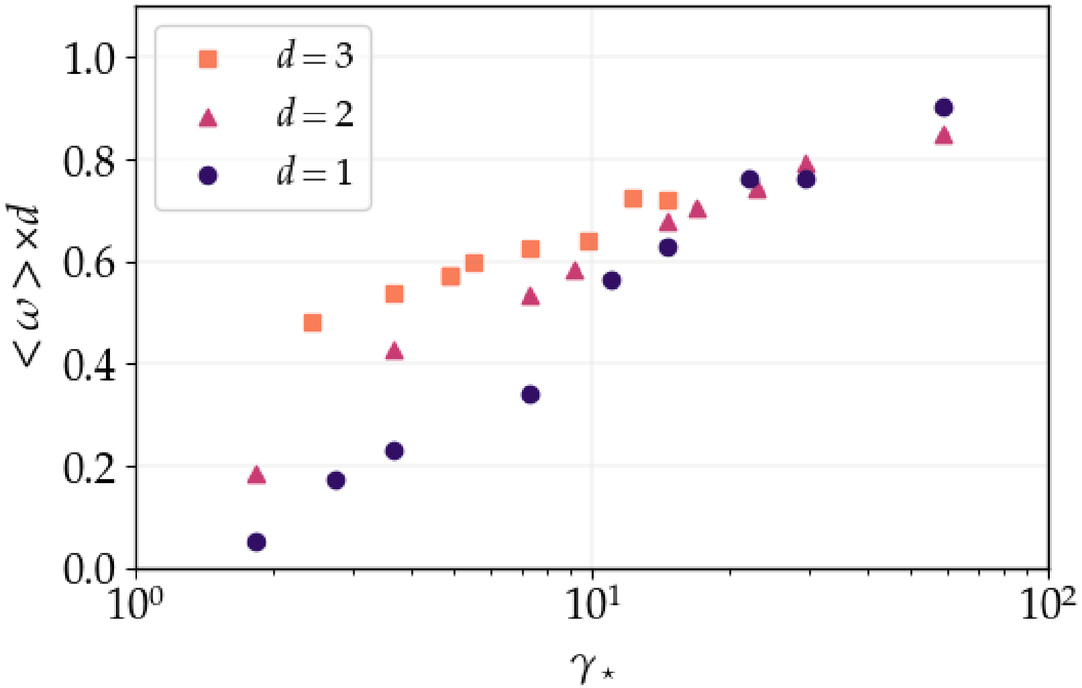
Varying the initial bubble separation to reach different bubble wall velocities:

$$\gamma_\star \propto \frac{R_\star}{R_n}$$

$R_\star$ — Radius at collision

R_n — Radius at nucleation

Summarized results for the EoS after percolation (1+1 , 1+2 and 1+3 simulations)



From the EoS to the power spectrum

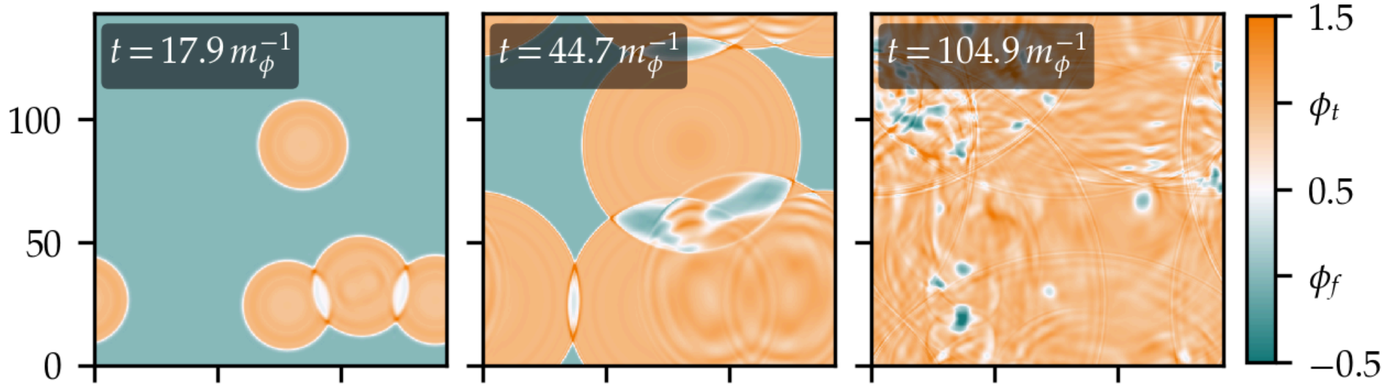
We introduce the dimensionless power spectrum

$$\Delta_\phi(k) = \frac{k^d}{(2\pi)^d} S_{d-1} P_\phi(k), \text{ with } S_{d-1} = 2\pi^{d/2}/\Gamma(d/2) \text{ and } \langle \phi_{\mathbf{k}'} \phi_{\mathbf{k}} \rangle = P_\phi(k)(2\pi)^3 \delta^{(3)}(\mathbf{k}+\mathbf{k}')$$

With some approximations the equation of state can be written in terms of the power spectrum :

dependent on the scale factor

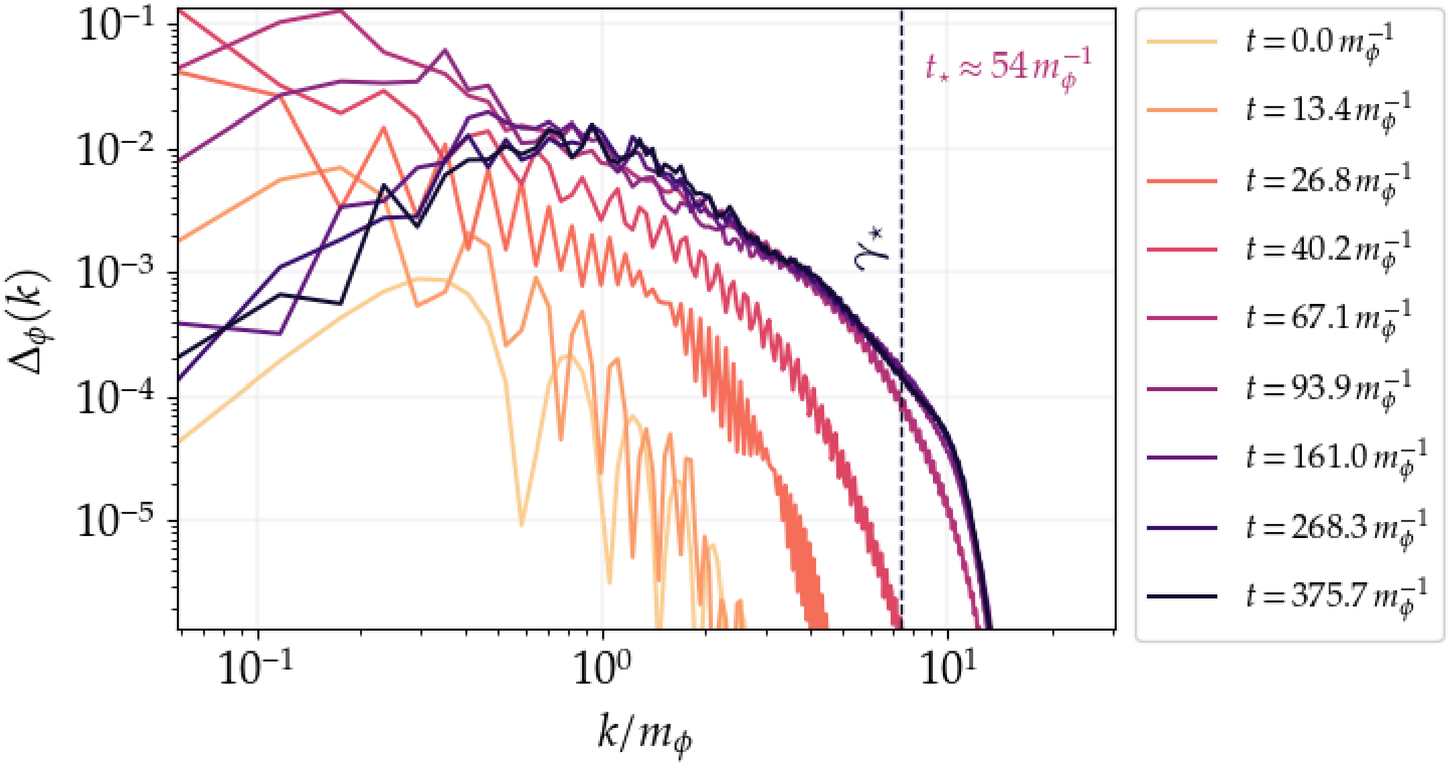
$$\omega = d^{-1} \frac{\int d\ln k (k/a)^2 \Delta_\phi(k,a)}{\int d\ln k \left(m_\phi^2 + (k/a)^2\right) \Delta_\phi(k,a)}$$



Compute the power spectrum from the Fourier transform at different steps of the evolution

The power spectrum

Evolution of the power spectrum before and after collision/percolation (results of 3D simulation)



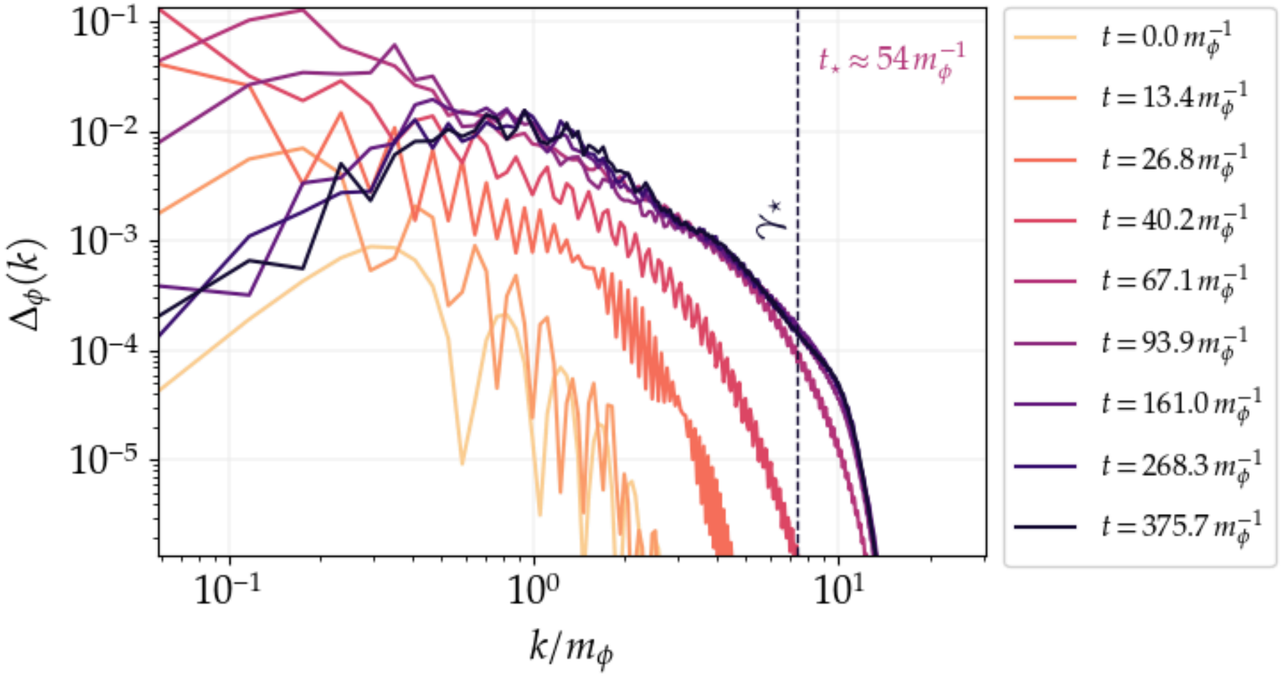
More and more high-k modes are occupied the more the bubbles expand

The UV-part of the power spectrum becomes roughly constant for $t > t_\star$, i.e. after the completion of the phase transition.

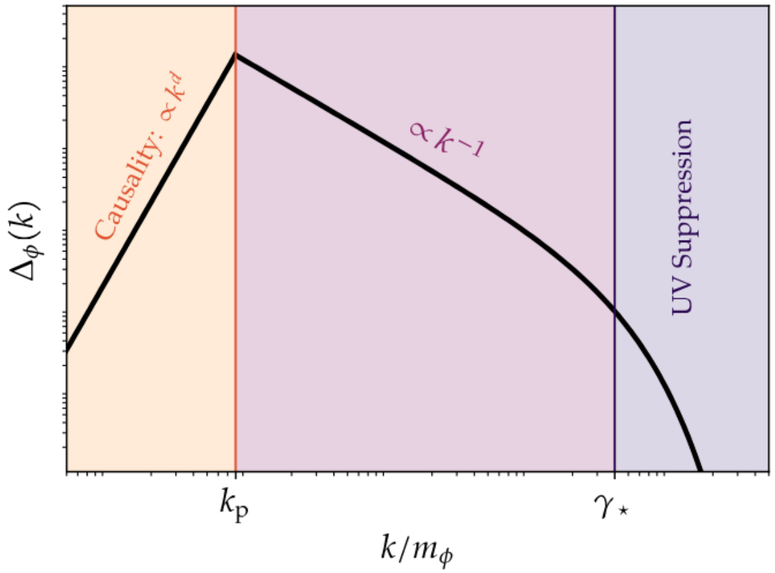
Our simulations suggest that the shape of the power spectrum for low-k is independent of $\gamma_\star$. The Lorentz factor sets the UV-cutoff of the power spectrum, and hence $w(\gamma_\star)$.

The power spectrum

Evolution of the power spectrum before and after collision/percolation (results of 3D simulation)



Main features of the late-time power spectrum:



Smooth template, with $k_\star \simeq \gamma_\star m_\phi$:

$$\Delta_\phi(k) = \Delta_p \left[\left(\frac{k}{k_p} \right)^{-3} + \left(\frac{k}{k_p} \right) \right]^{-1} \exp\left[-\pi k^2 / k_\star^2 \right]$$

The power spectrum can be approximated by a broken power law with a peak around $k_p \sim m_\phi$, and a cutoff corresponding to the boosted wall-width.

From the power spectrum to the EoS

Reminder:

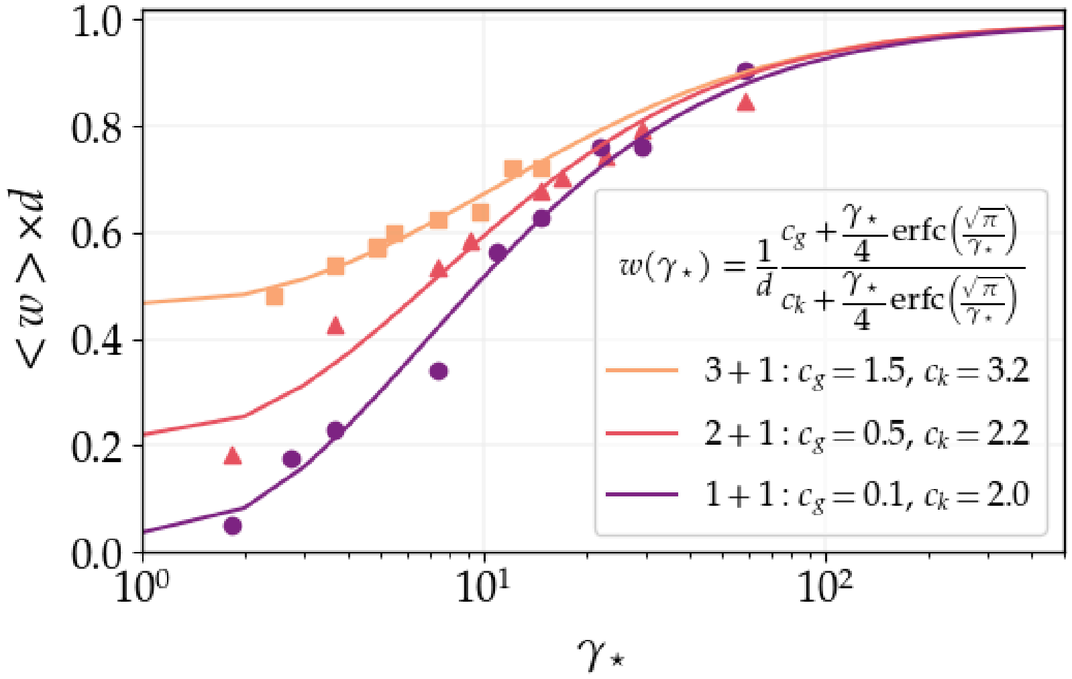
$$w \;=\; d^{-1}\frac{\int d\ln k (k/a)^2 \Delta_\phi(k,a)}{\int d\ln k \left(m_\phi^2 + (k/a)^2\right)\Delta_\phi(k,a)}$$

Assuming free-streaming, the smooth template for the power spectrum gives:

$$w(\gamma_\star, a) \;=\; \frac{1}{d}\frac{\dfrac{c_g}{a^2} + \dfrac{\gamma_\star}{4a}\,\mathrm{erfc}\left(\dfrac{\sqrt{\pi}a}{\gamma_\star}\right)}{c_k + \dfrac{\gamma_\star}{4a}\,\mathrm{erfc}\left(\dfrac{\sqrt{\pi}a}{\gamma_\star}\right)}$$

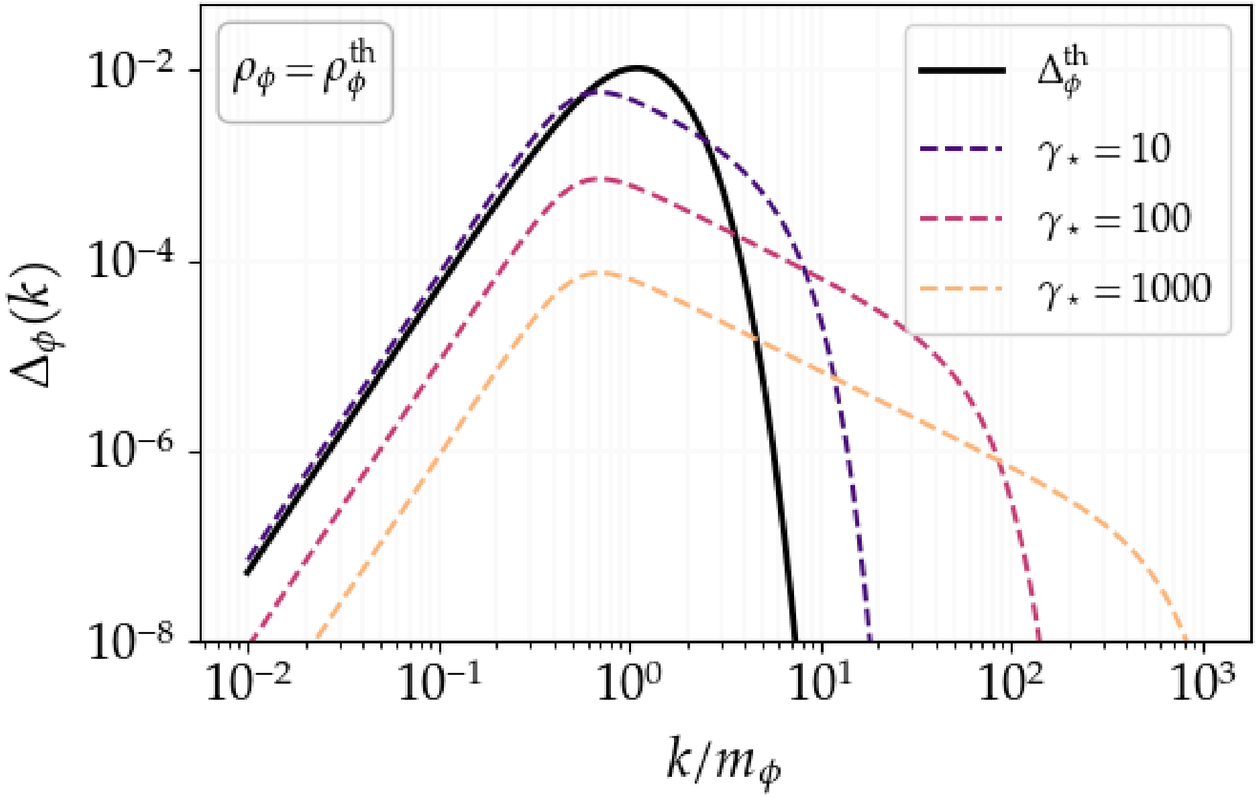
The parameters c_k and c_g are fixed by a fit to the simulation data.

Fitting the results of simulations in 1+1, 1+2 and 1+3 dimensions (flat static background)



The scalar post-percolation: far from thermal

Bubble collisions do not lead to thermalisation

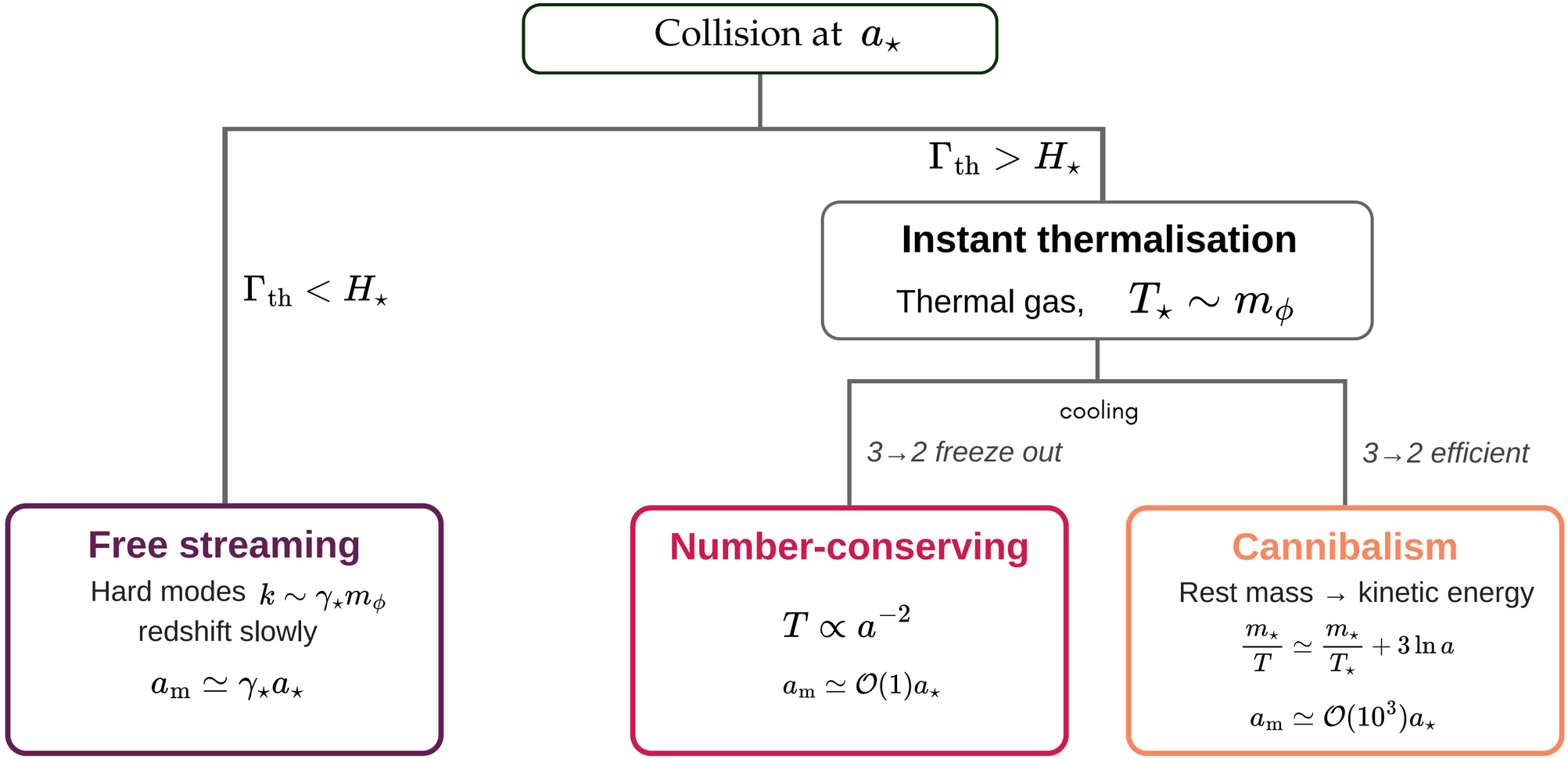


Post-collision power spectra for different values of the Lorentz factor against the thermal one as given by a Bose-Einstein distribution at equal energy density $\rho_\phi = \rho_\phi^{th}$

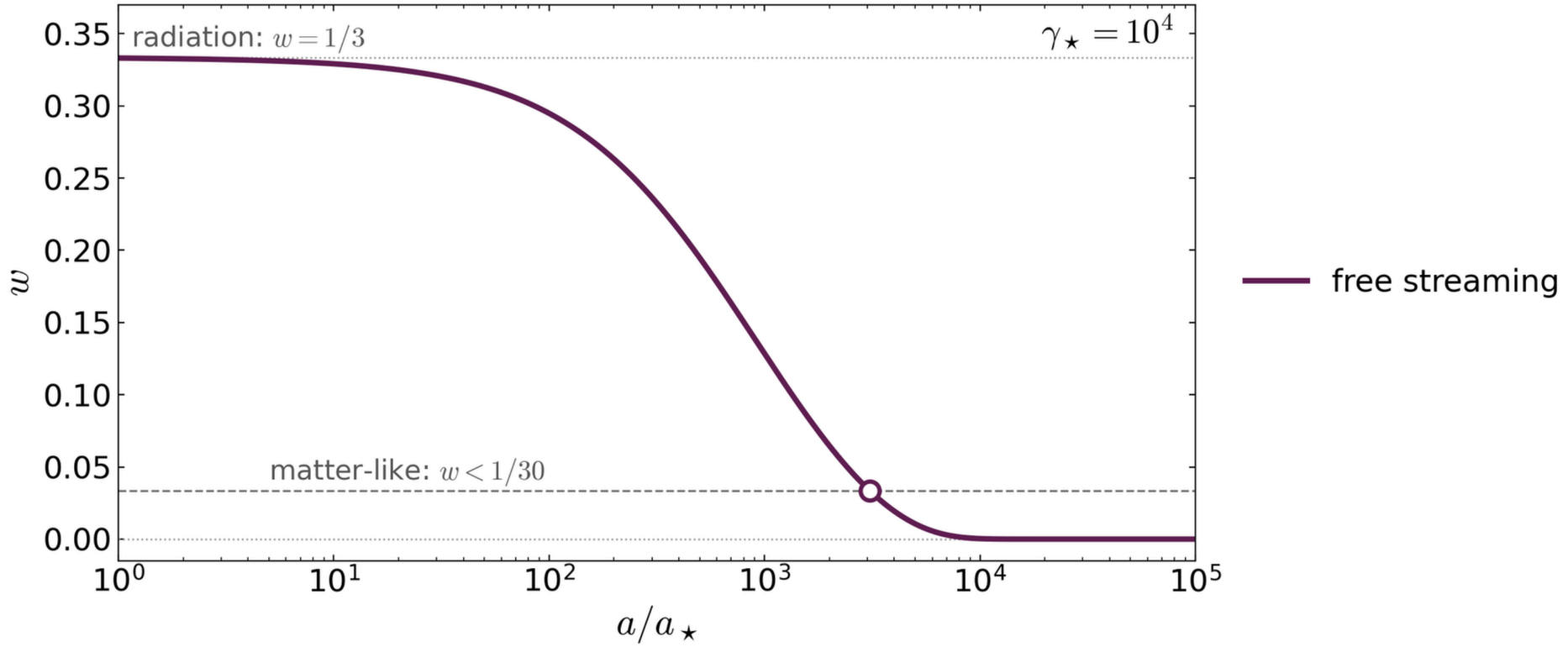
The more relativistic the walls, the further the configuration lies from a thermal distribution.

So when does matter domination start?

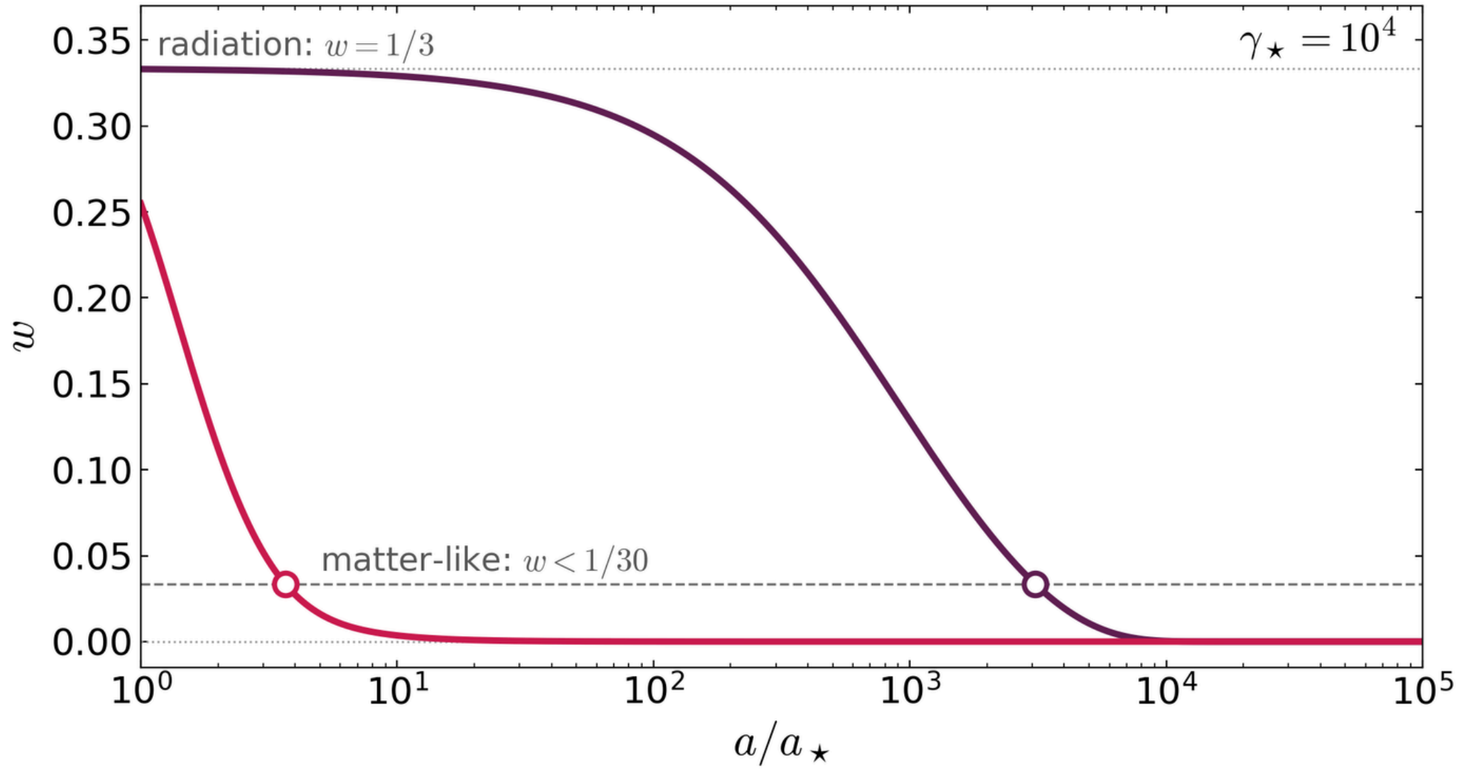
Late-time evolution: When does matter domination start?



Late-time evolution:
When does matter domination start?



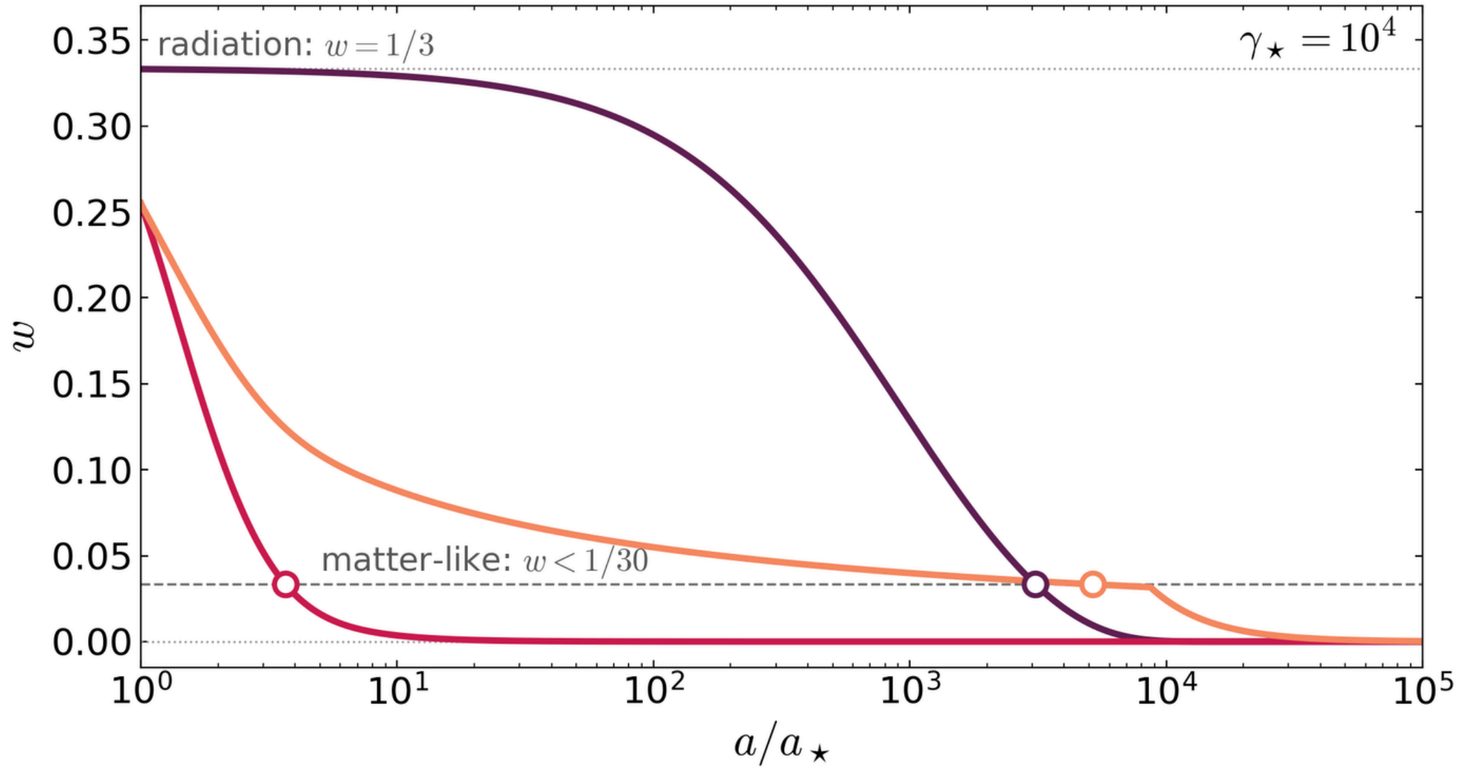
Late-time evolution:
When does matter domination start?



free streaming

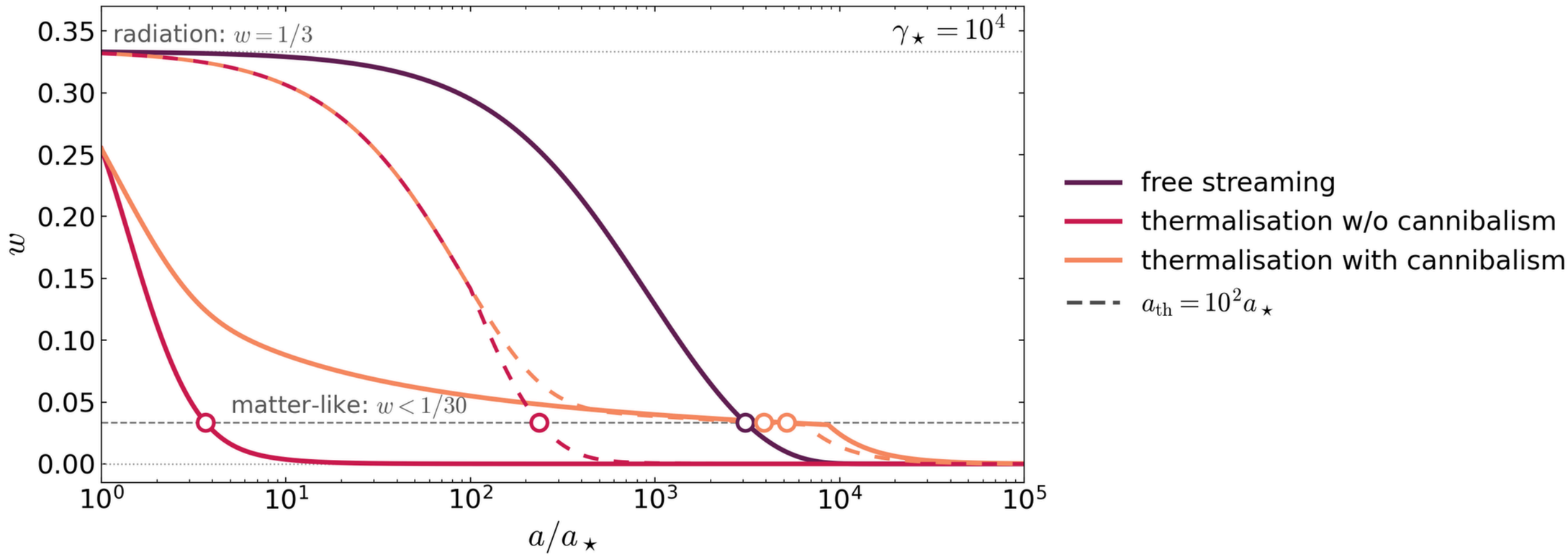
thermalisation w/o cannibalism

Late-time evolution:
When does matter domination start?



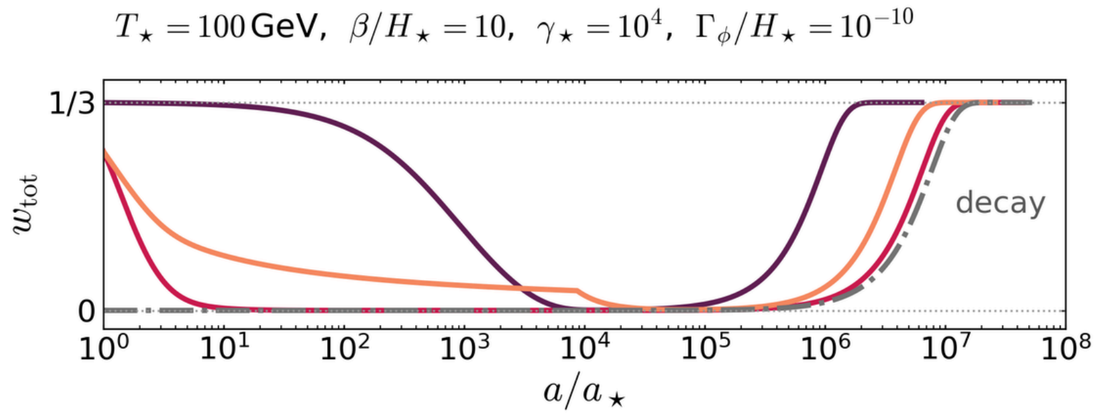
free streaming
thermalisation w/o cannibalism
thermalisation with cannibalism

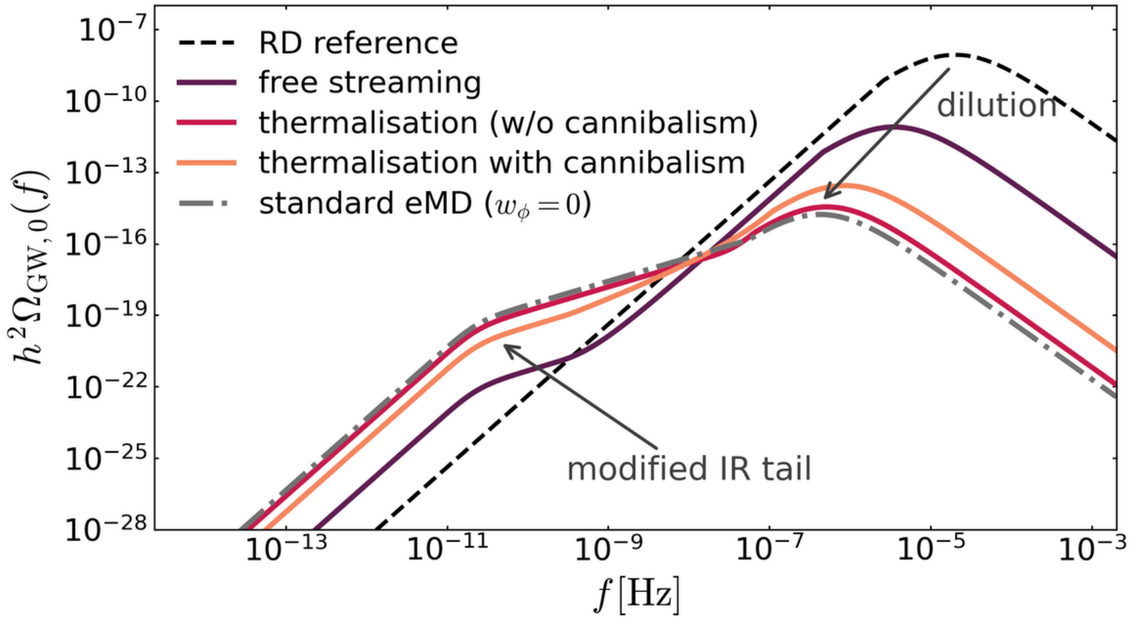
Late-time evolution: When does matter domination start?



How fast the scalar becomes matter-like depends on its self-interactions.

Impact on the gravitational waves spectrum





$w_{\rm tot}$ accounts for the decay of the scalar into a radiation bath:

$$\frac{\mathrm{d}\rho_\phi}{\mathrm{d}a} = -\frac{3[1+w(a,\gamma_\star)]}{a}\rho_\phi - \frac{\Gamma_\phi}{aH}\rho_\phi$$
$$\frac{\mathrm{d}\rho_r}{\mathrm{d}a} = -\frac{4}{a}\rho_r + \frac{\Gamma_\phi}{aH}\rho_\phi$$

Modes superhorizon at production are sensitive to the equation of state when they enter the horizon
→ modified IR tail $\propto f^{(1+15w)/(1+3w)}$

(Hook et al. 2020, arXiv:2010.03568)

An EoS $w < 1/3$ leads to the dilution of the overall amplitude.

→ The GW spectrum is sensitive to the thermalisation history

Conclusions

1. **EoS at percolation**
   - Immediately after percolation, the scalar configuration is highly inhomogeneous and dominated by relativistic modes.
   - For $\gamma_\star \gg 10^2$, its equation of state is radiation-like, not matter-like.

2. **Power spectrum**
   - The configuration is non-thermal and dominated by hard modes, with a UV cut-off at $k_\star \simeq \gamma_\star m_\phi$

3. **Late-time evolution**
   - How fast these modes redshift depends on the self-interactions of the field.
   - Early matter domination can be significantly delayed, especially for free-streaming modes: it starts at $a \sim \gamma_\star a_\star$, where $\gamma_\star$ can be up to $\mathcal{O}(10^{10})$ in runaway scenarios.

4. **Gravitational waves**
   - The GW spectrum is sensitive to the thermalisation/reheating history, through the dilution of the amplitude and a modified IR tail.

H. Mansour, Y. Gouttenoire, F. Kahlhoefer, arXiv:2607.19469

Analytical modelling of the power spectrum of a single expanding bubble

