Introduction to inflationary and post-inflationary dynamics

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Fundación BBVA

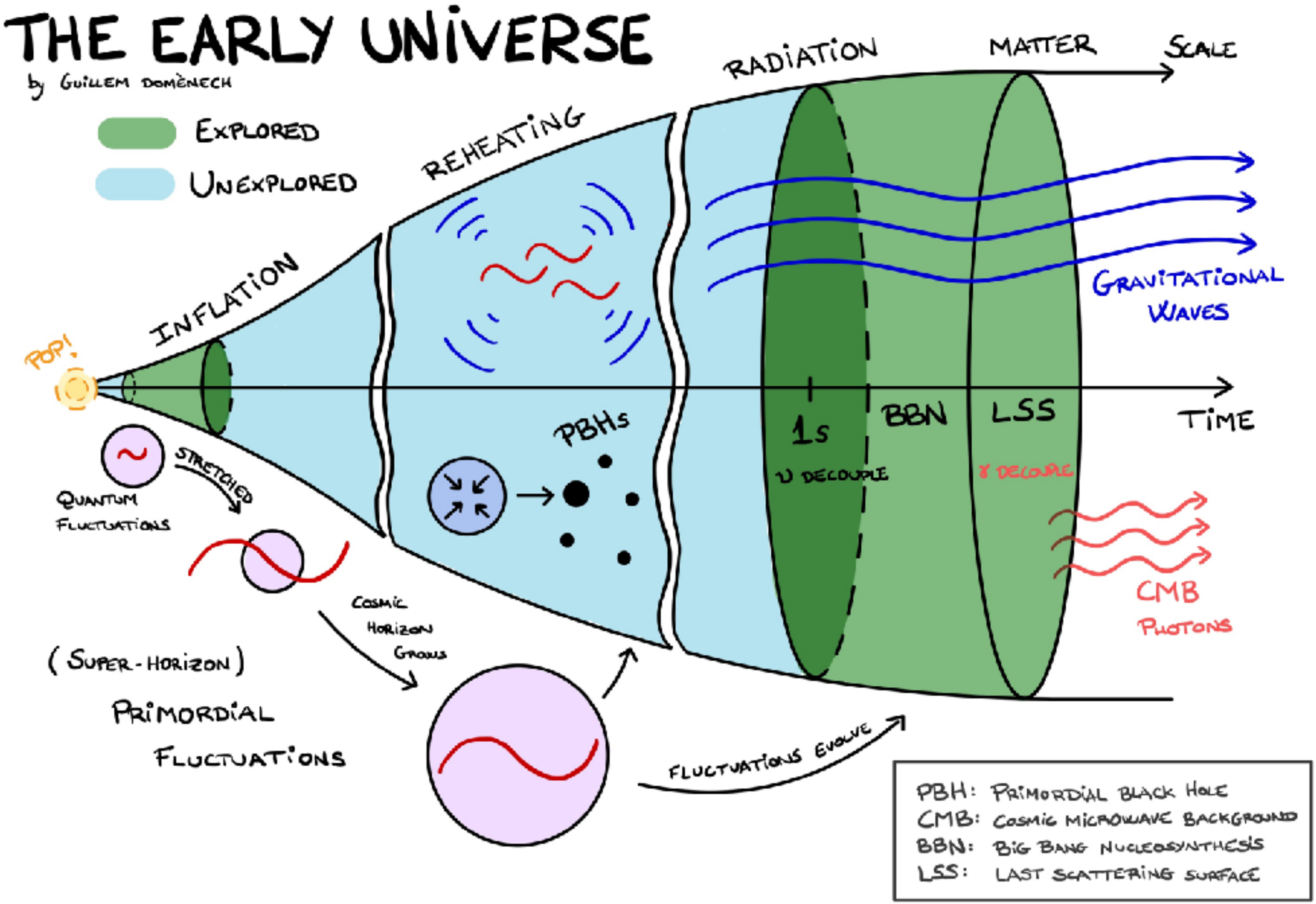
Outline

1. The inflationary paradigm

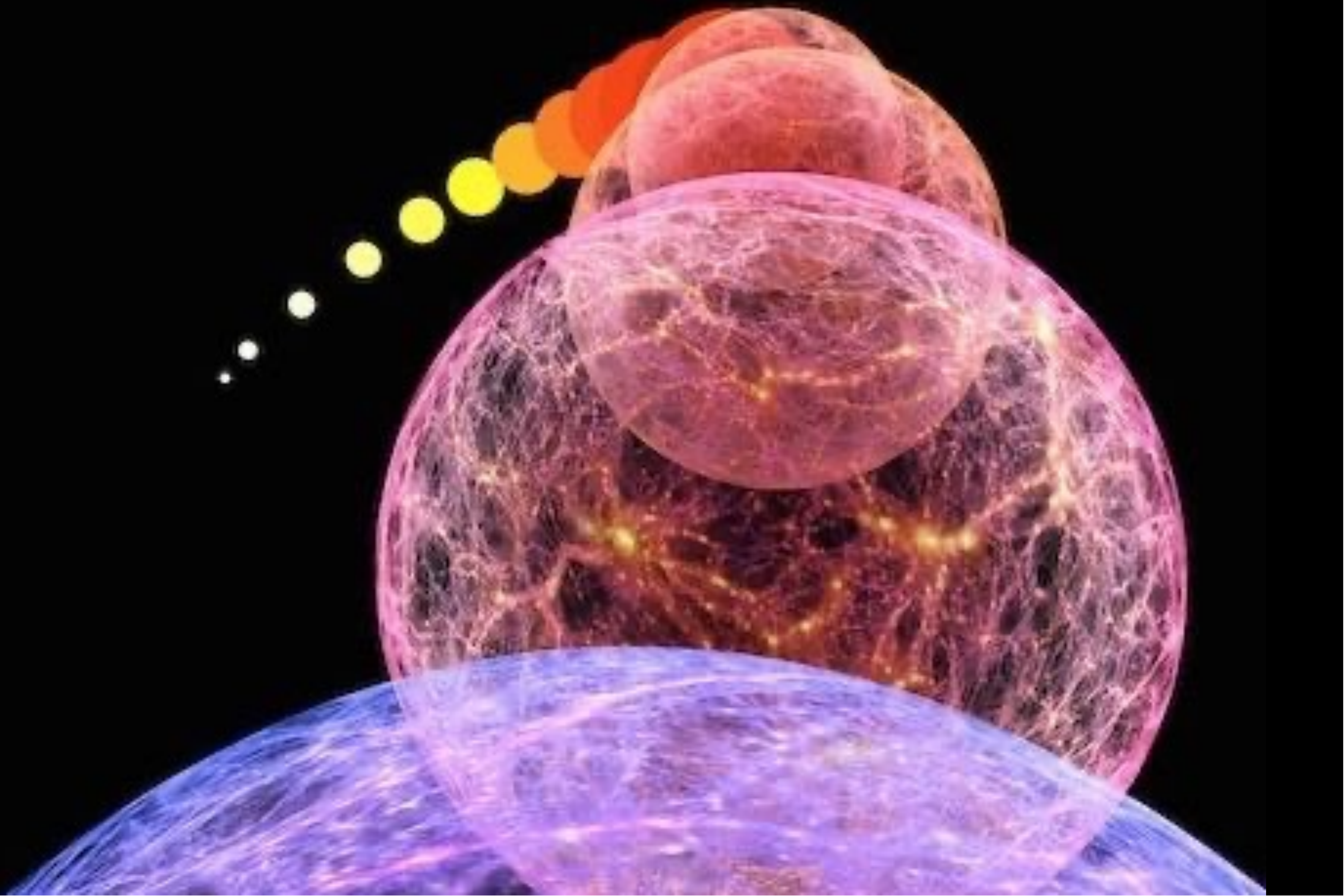
2. The onset of the hot Big Bang

3. A worked-out example: Preheating in λφ4

4. Program variables in CosmoLattice


THE EARLY UNIVERSE
by Guillem Domènech
Explored
Unexplored
Inflation
Reheating
Radiation
Matter
Scale
Time
Pop!
Quantum Fluctuations
Stretched
(Super-Horizon)
Primordial Fluctuations
Cosmic Horizon Grows
Fluctuations Evolve
PBHs
1s
ν decouple
BBN
LSS
γ decouple
Gravitational Waves
CMB Photons
PBH: Primordial Black Hole
CMB: Cosmic Microwave Background
BBN: Big Bang Nucleosynthesis
LSS: Last Scattering Surface

The inflationary paradigm



The basic idea

Inflation: accelerated expansion before RD with

a negative pressure fluid, **a nearly constant energy density**

$$\frac{\ddot{a}}{a} = -\frac{\rho}{6}(1+3w) > 0$$

$$\frac{\ddot{a}}{a} = (H^2 + \dot{H}) > 0$$

$$w < -\frac{1}{3}$$

and a CLOCK

$$H = \sqrt{\frac{\rho}{3}} \approx const.$$

Λ

Large vacuum energy

Time-translation invariance

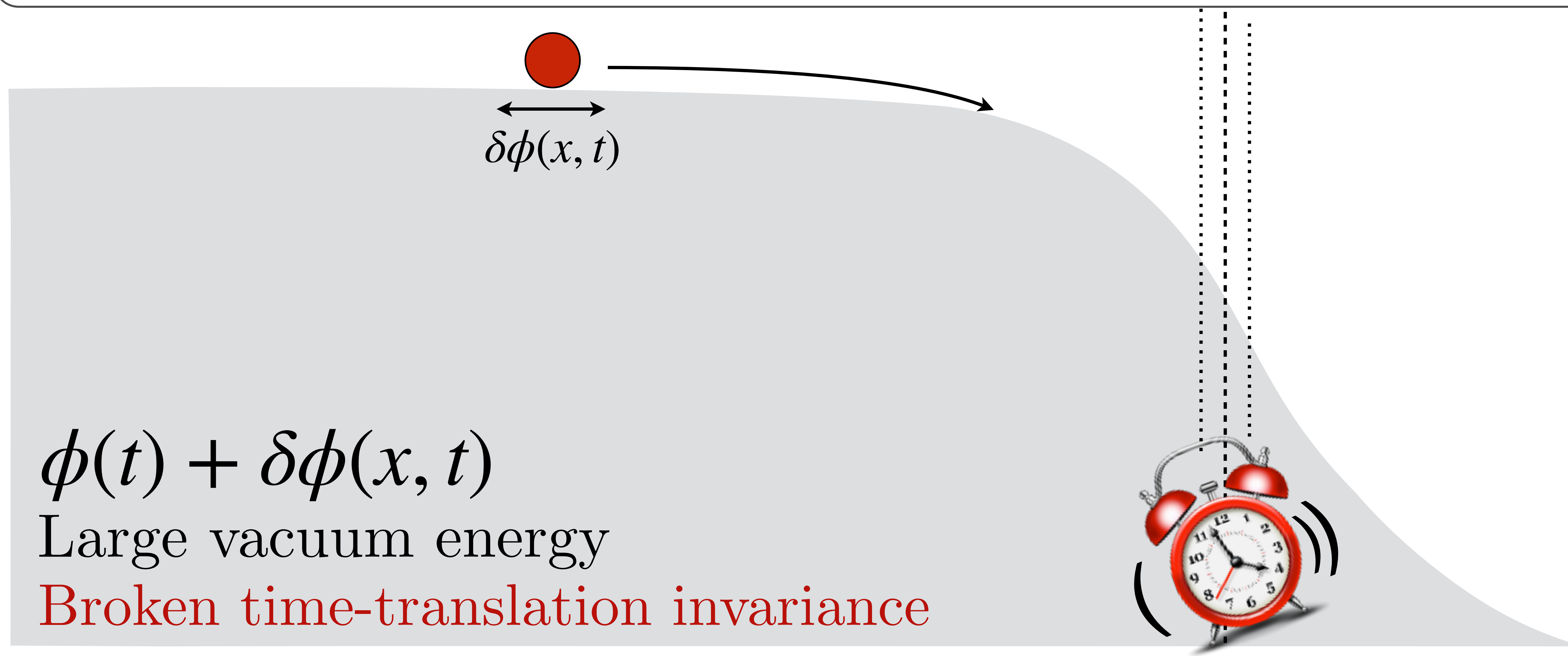
The slow-roll approximation

Inflation must…

…happen… $w_\phi \equiv \dfrac{p_\phi}{\rho_\phi} = \dfrac{\cancel{\frac{1}{2}\dot\phi^2} - V(\phi)}{\cancel{\frac{1}{2}\dot\phi^2} + V(\phi)} \simeq -1 \Rightarrow \dot\phi^2 \ll |V(\phi)|$ $\qquad \epsilon_V \equiv \dfrac{m_p^2}{2}\left(\dfrac{V'(\phi)}{V(\phi)}\right)^2 \ll 1$

…last enough… $\cancel{\ddot\phi} + 3H\dot\phi + V_{,\phi}(\phi) = 0 \Rightarrow |\ddot\phi| \ll 3H|\dot\phi|, V_{,\phi}(\phi)$ $\qquad \eta_V \equiv m_p^2\left(\dfrac{V''(\phi)}{V(\phi)}\right) \ll 1$

…and end… $\epsilon_V \approx 1$

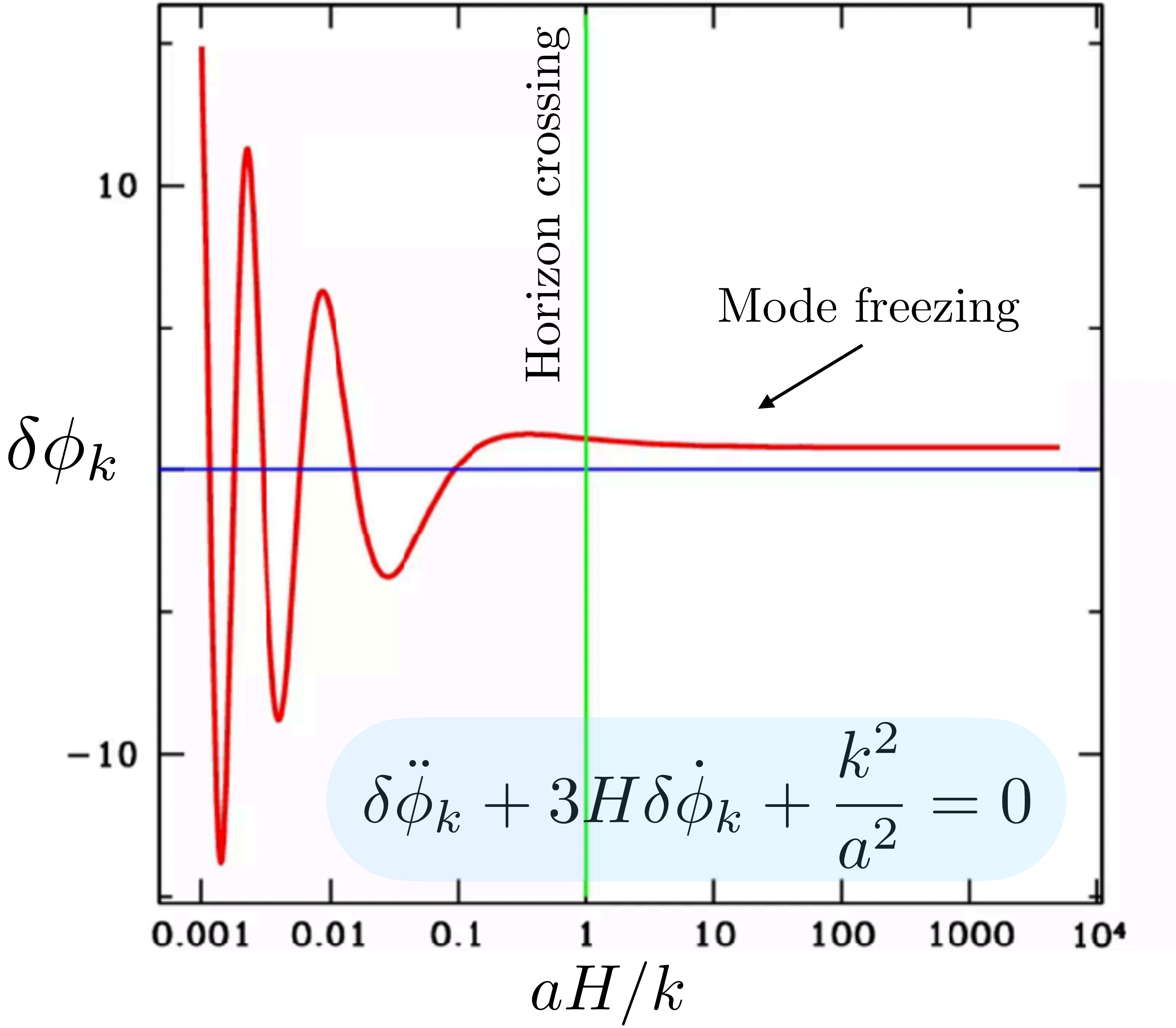


$\phi(t) + \delta\phi(x,t)$

Large vacuum energy

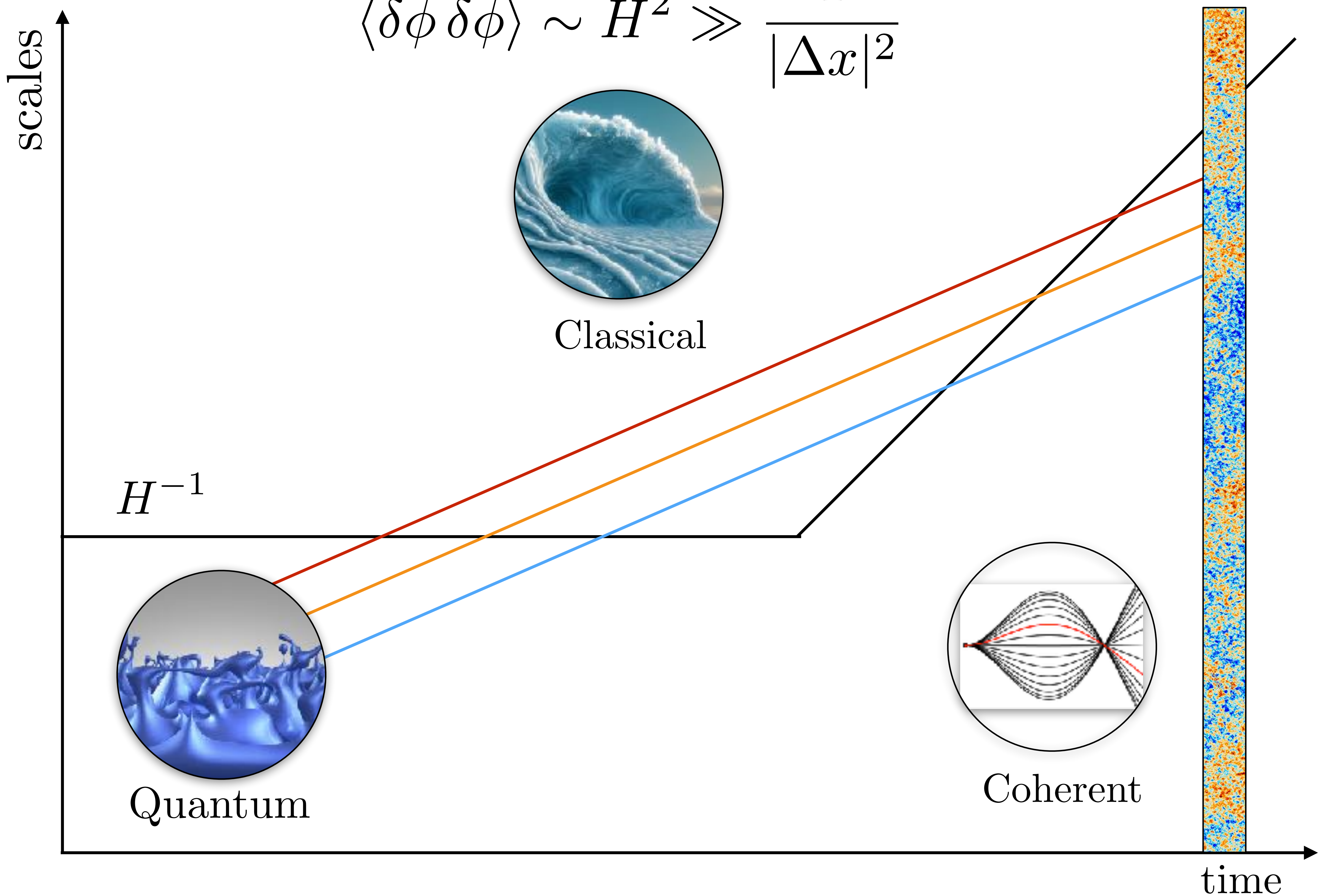
Broken time-translation invariance

From quantum to classical

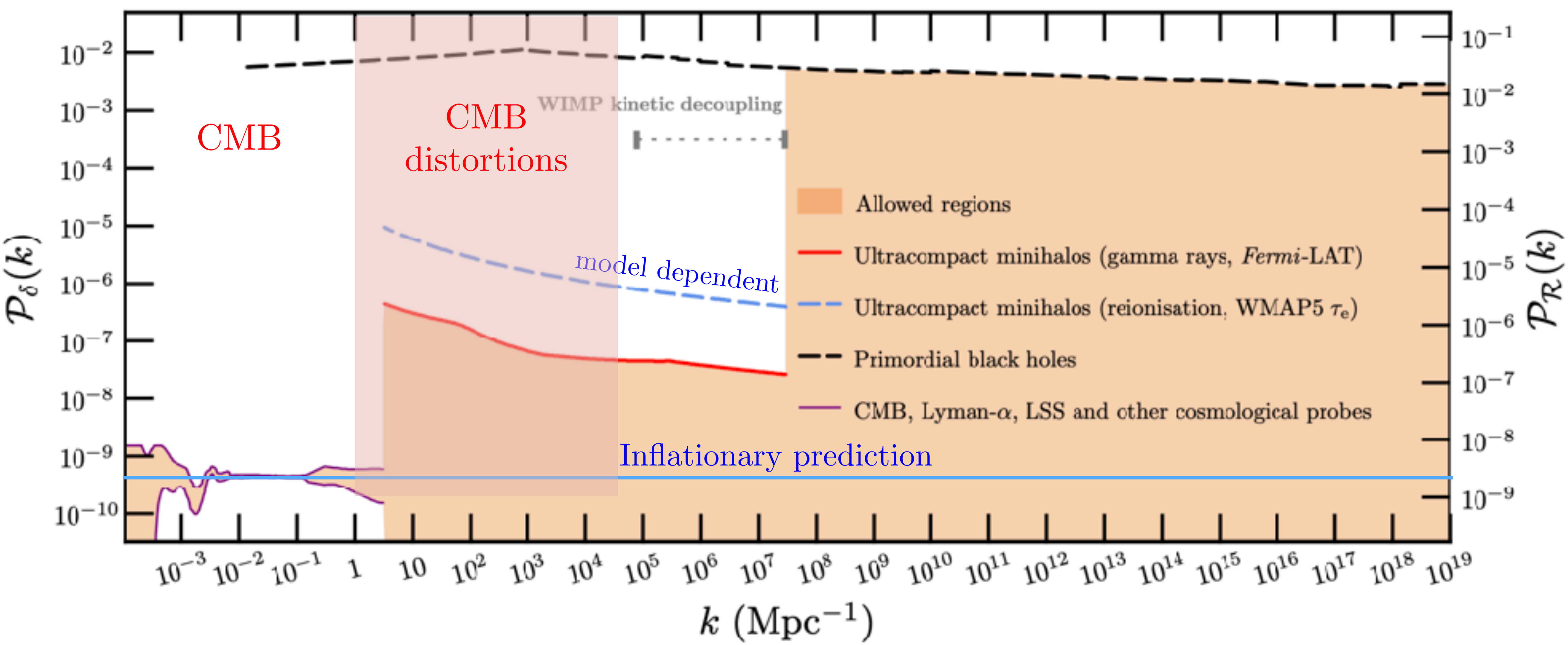


Out and in the horizon

$$\langle \delta\phi\, \delta\phi \rangle \sim H^2 \gg \frac{1}{|\Delta x|^2}$$

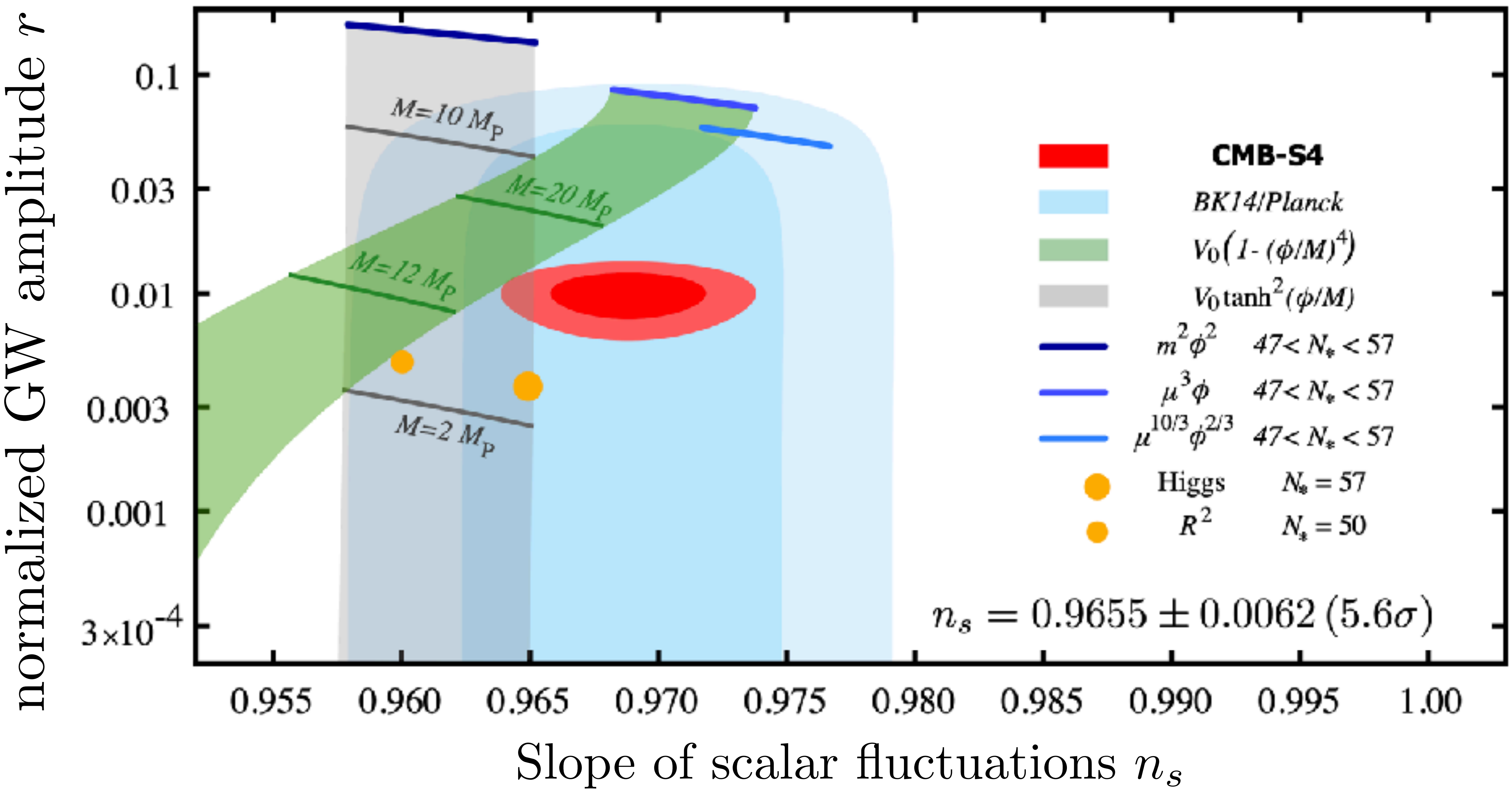


Primordial density spectrum: prediction



CMB is only sensitive to the first 10 e-folds of inflation

Inflationary observables

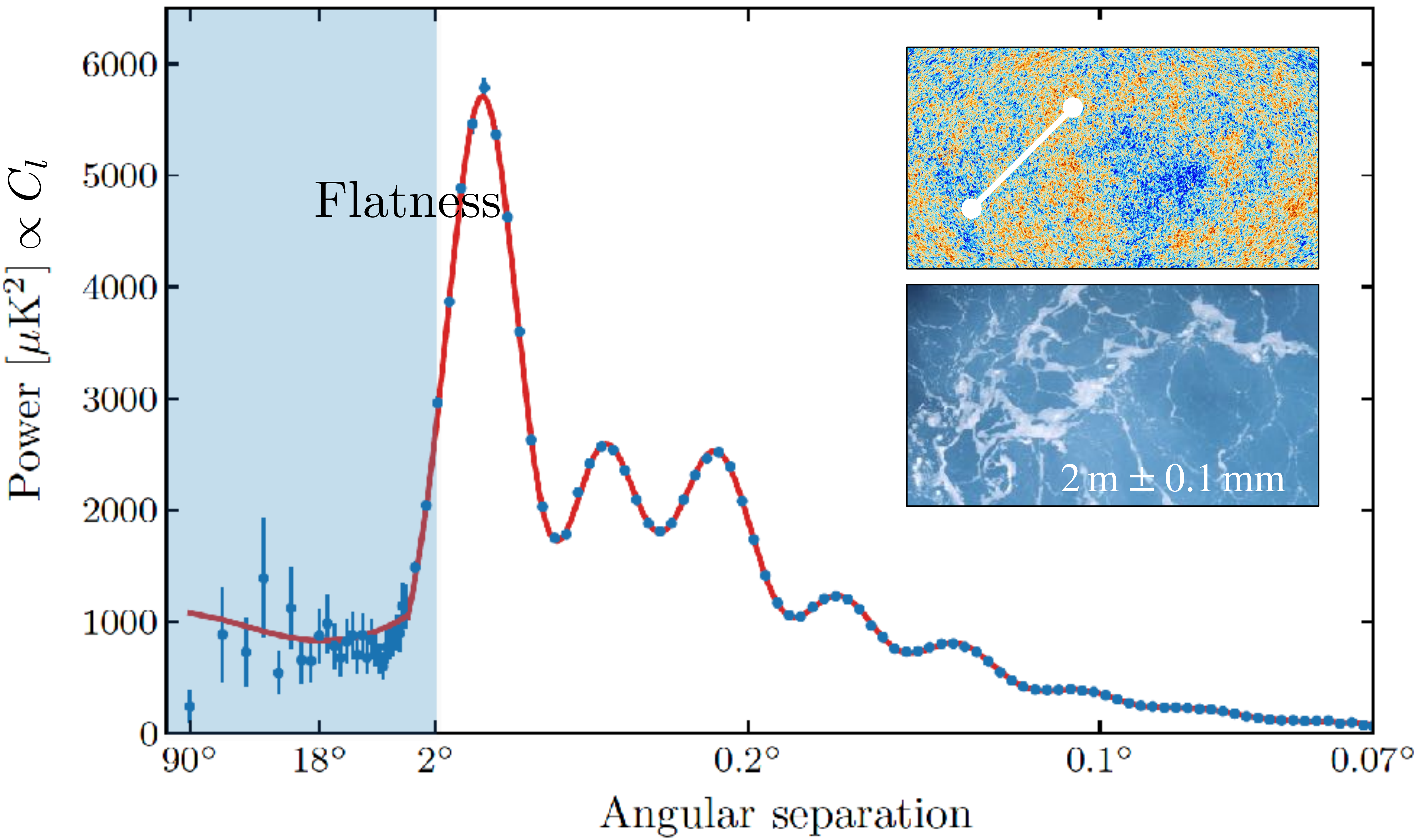


Uncertainty from $\Delta N_{\rm RH}$, ω_{RH}: relevant from precision cosmology

$$N_* \approx 67 - \ln\left(\frac{k_*}{a_0 H_0}\right) + \frac{1}{4}\ln\left(\frac{V_*^2}{M_{\rm pl}^4 \rho_{\rm end}}\right) + \frac{1-3w_{\rm int}}{12(1+w_{\rm int})}\ln\left(\frac{\rho_{\rm th}}{\rho_{\rm end}}\right) - \frac{1}{12}\ln(g_{\rm th})$$

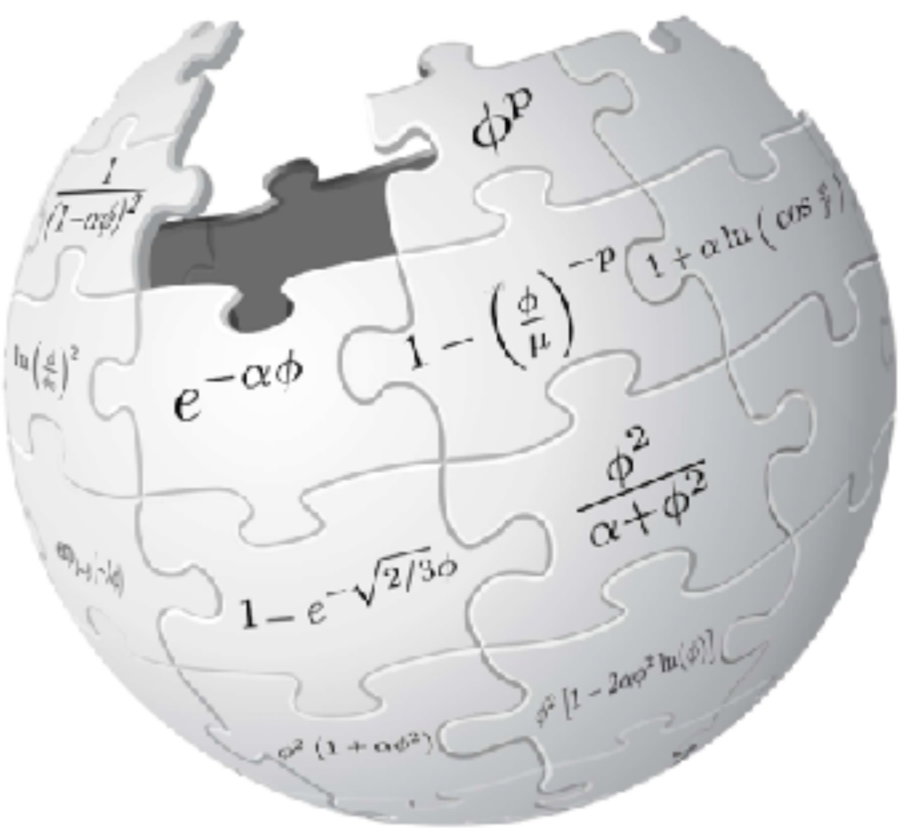
A bunch of (confirmed) predictions

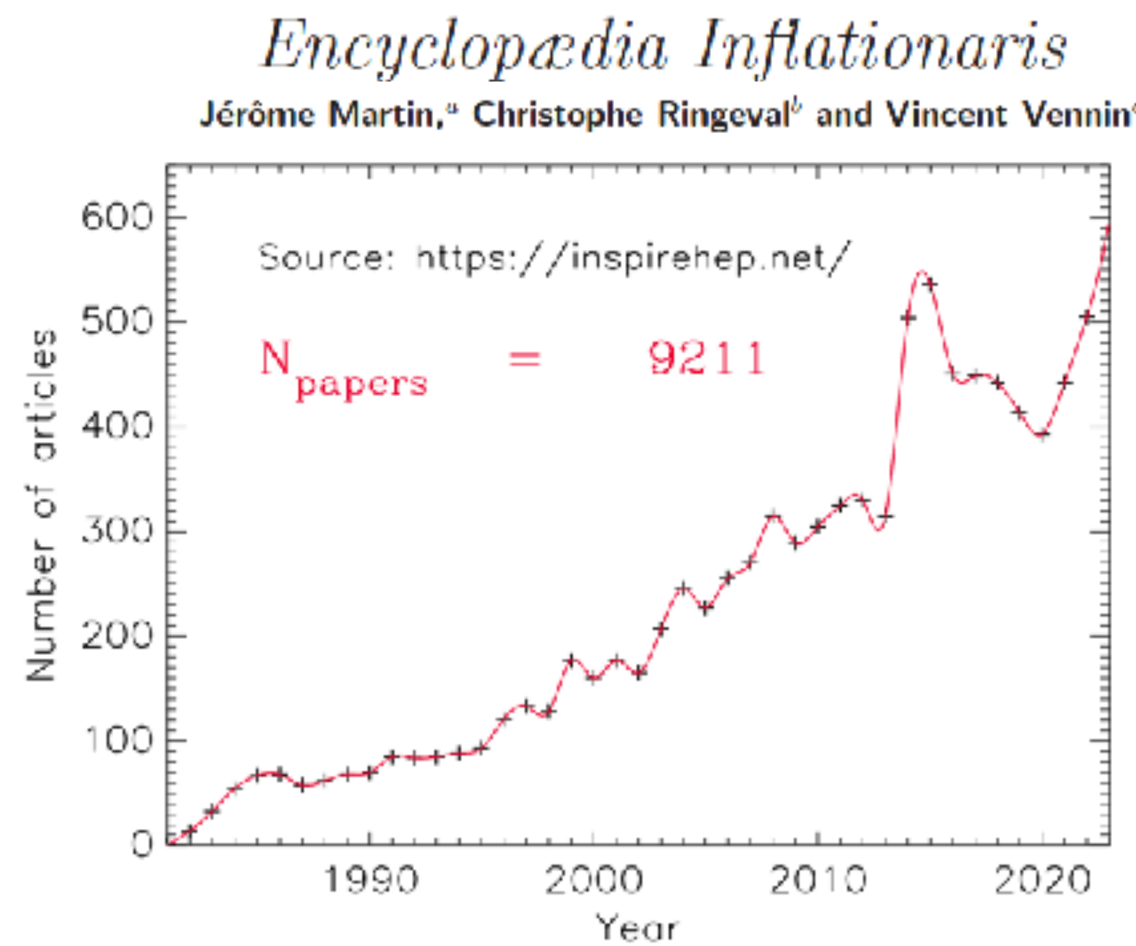
Flatness, homogeneity, scale invariance, Gaussianity, coherence



Many candidates...too many

- Often no guidance on $V(\phi)$ beyond need for inflation
- Weakly coupled or tuned to protect flatness from loop corrections



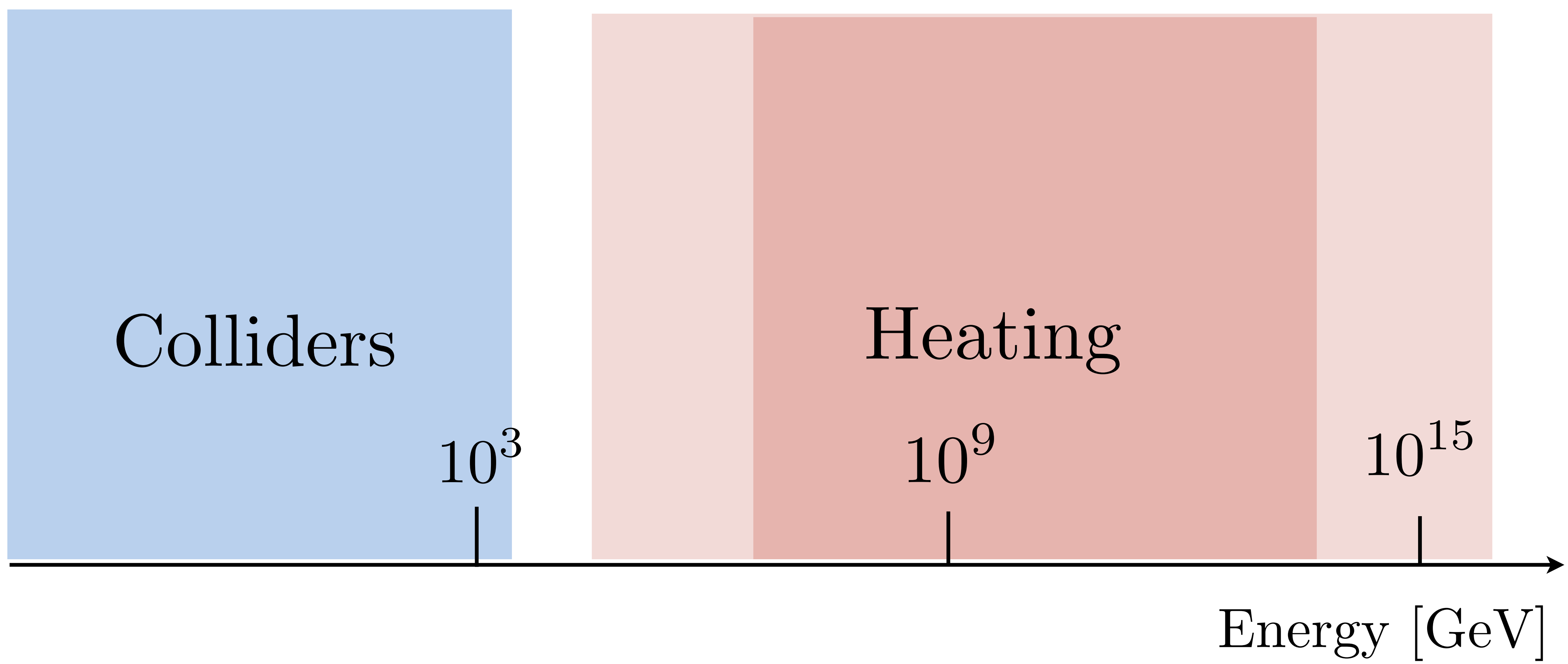


"Inflation consists of taking a few numbers we don't understand and replacing it with a function that we don't understand"

Lots of unknown $\mathscr{L} = \mathscr{L}(\phi, \chi_i, \psi_j, A_\mu, h_{\mu\nu}, \ldots)$

- Unknown energy scale
- Unknown fields
- Unknown masses
- Unknown couplings

$$E_{\rm inf} \equiv V^{1/4} = 10^{16}\,{\rm GeV}\left(\frac{r}{0.01}\right)^{1/4}$$



Details of the early universe evolution depends on the high-energy physics model.

The naive reheating picture

$$S_{\text{int}} \supset \int d^4x \sqrt{-g}\left(-\sigma\phi\chi^2 - h\phi\bar{\psi}\psi\right)$$

$$\Gamma_{\phi\to\chi\chi} = \frac{\sigma^2}{8\pi m}$$

$$\Gamma_{\phi\to\bar{\psi}\psi} = \frac{h^2 m}{8\pi}$$

$$\Rightarrow \quad \frac{d\left(a^3 n_{\bar{\phi}}\right)}{dt} = -\Gamma_{\text{tot}} a^3 n_{\bar{\phi}}$$

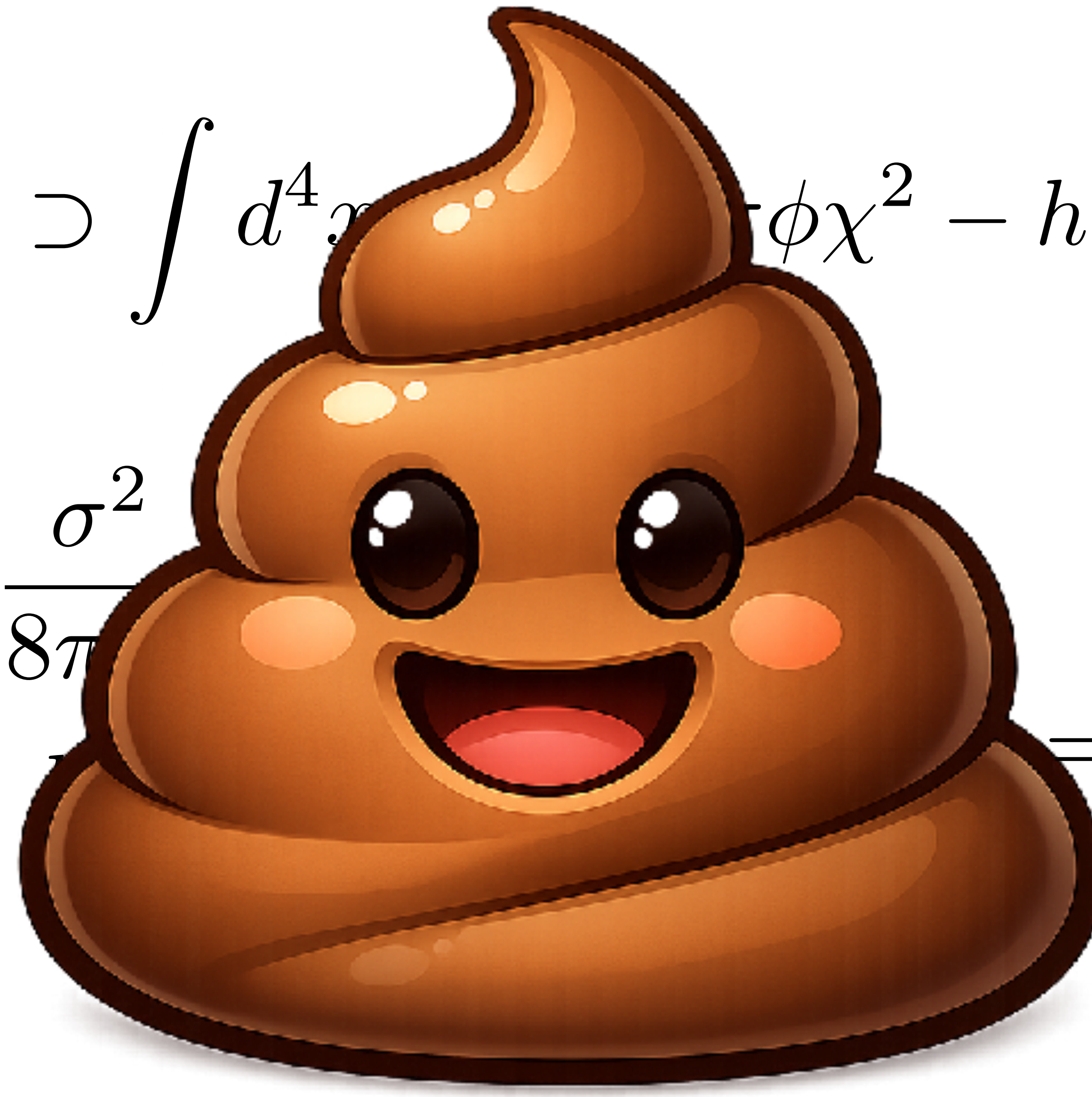
The naive reheating picture

$$S_{\mathrm{int}} \supset \int d^4x \;\ldots\; \phi\chi^2 - h\phi\bar{\psi}\psi)$$

$$\Gamma_{\phi\to\chi\chi} = \frac{\sigma^2}{8\pi \ldots}$$

$$\ldots = -\Gamma_{\mathrm{tot}}\, a^3 n_{\bar{\phi}}$$

$$\Gamma_{\phi\to\bar{\psi}\psi} = \ldots$$



Bose condensation effects can exponentially enhance the rate of energy transfer from the inflaton to the coupled bosonic fields

A short (and very nice) review

ELSEVIER

Nuclear Physics B

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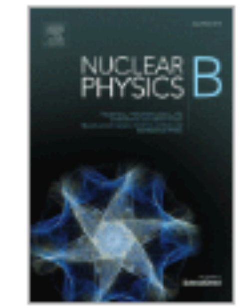
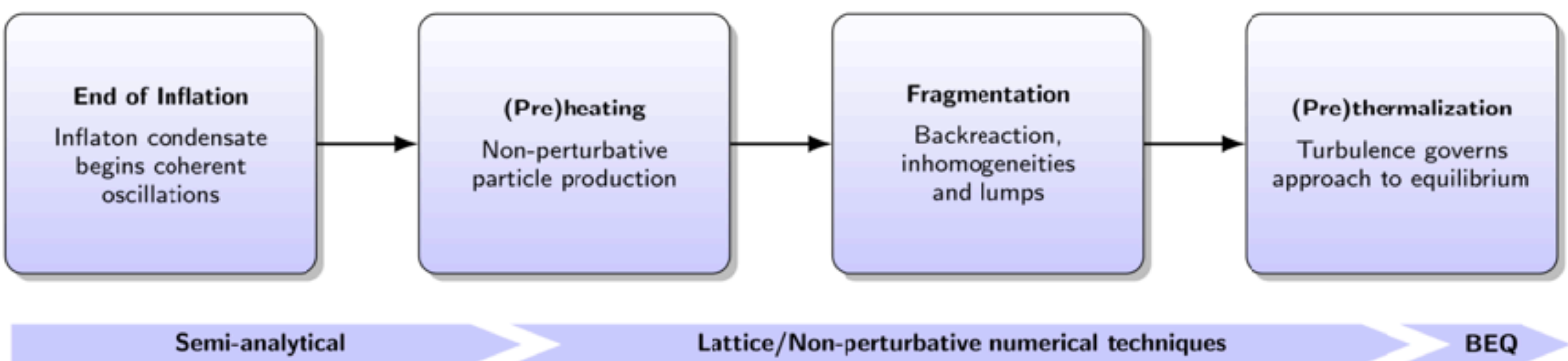


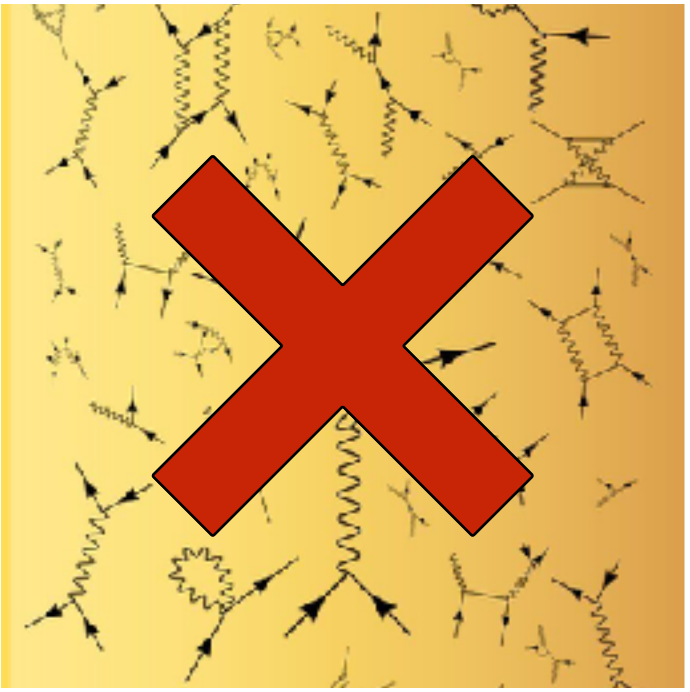
Special Issue on Clarifying misconceptions

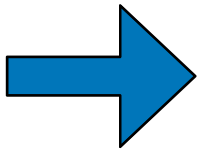
Two or three things particle physicists (mis)understand about (pre)heating

Basabendu Barman [a], Nicolás Bernal [b], Javier Rubio [c]



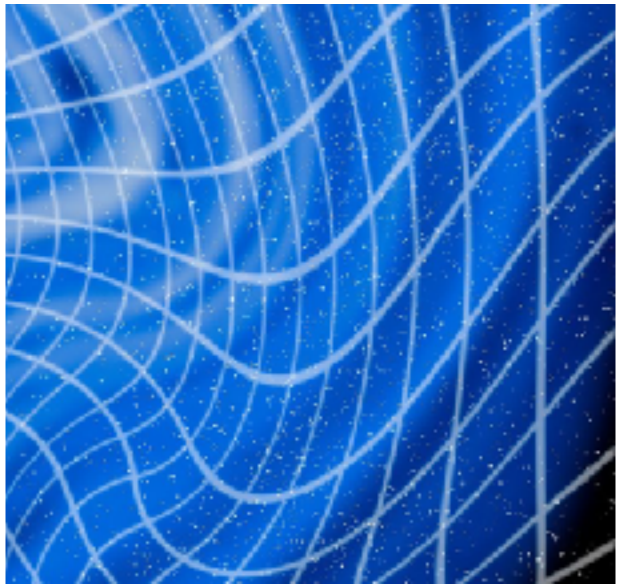
Very complicated dynamics



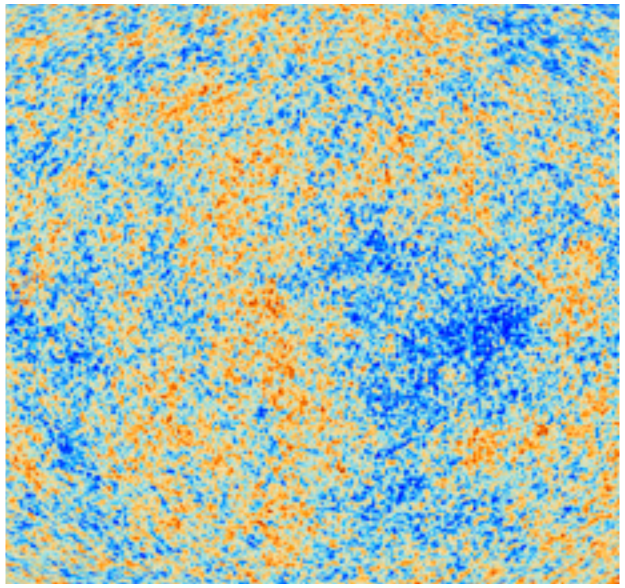


- Non-perturbative effects
- Non-linear interactions
- Backreaction/rescattering effects
- Far from equilibrium
- Expanding Universe
- Metric perturbations are relevant
- Consistency of several mechanisms

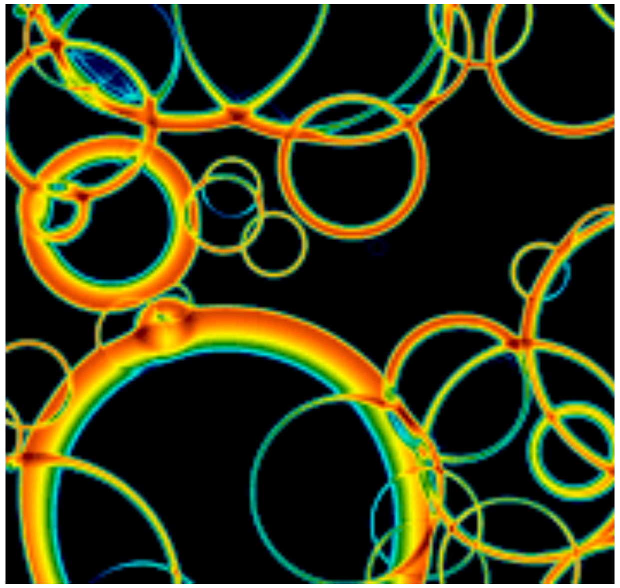
Grav. waves



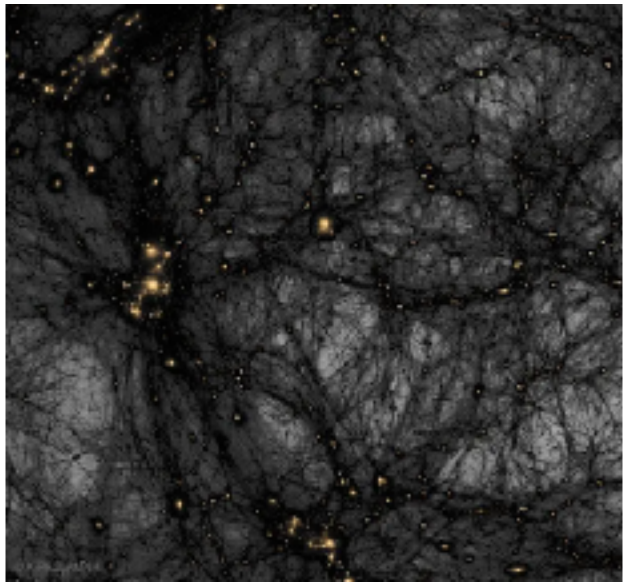
Cosmic history



Baryogenesis

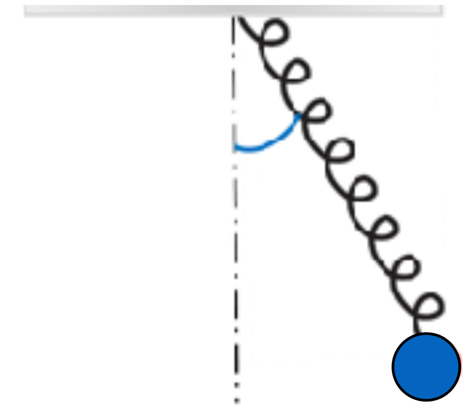


Dark matter



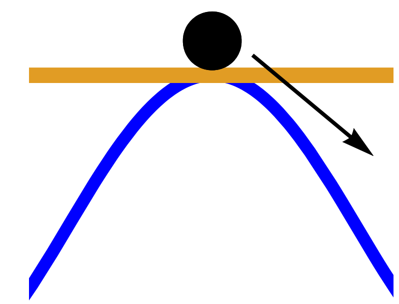
The hot Big Bang onset

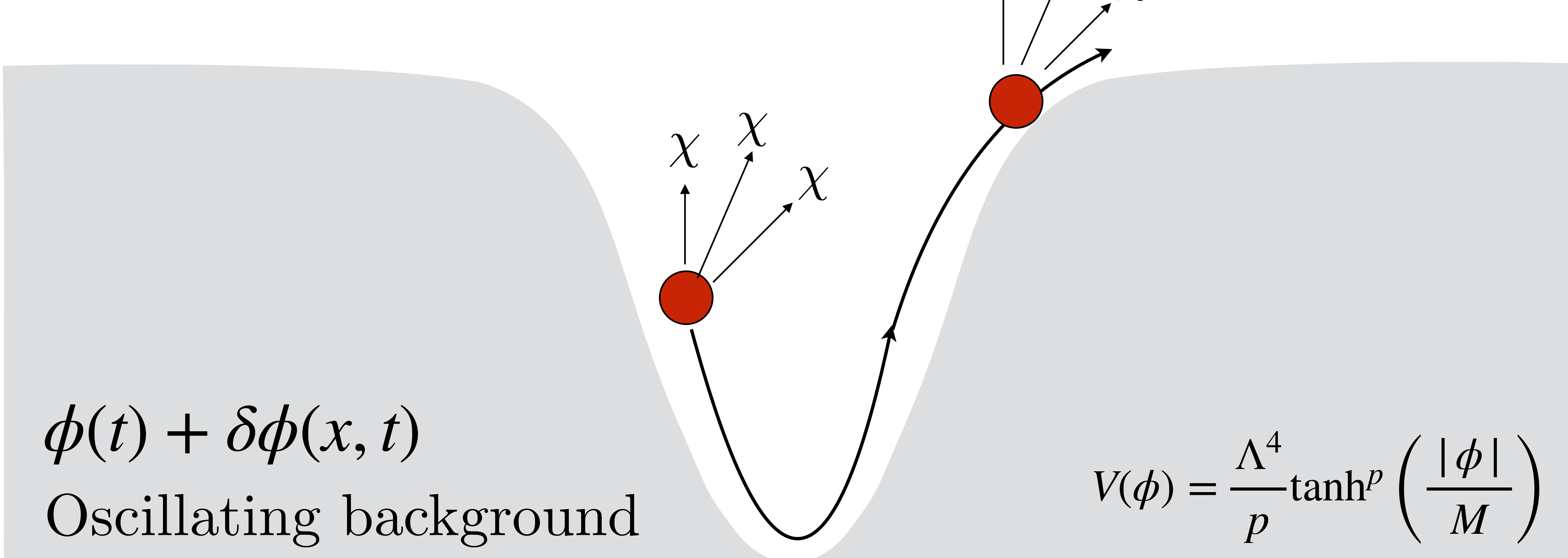
Particle production after inflation



$$\ddot{\chi}_k(\tau) + \left[k^2 + m_{\rm eff}^2(\tau)\right]\chi_k(\tau) = 0$$

$$m_{\rm eff}^2(\tau) = f\left[a(\tau), g^2\phi^2(\tau), \xi R(\tau), \ldots\right]$$





Classicalization

$$\hat{\chi}_k(t) = f_k(t)\hat{a}_k + f_k^*(t)\hat{a}^\dagger_{-k}$$

$$\Sigma_k(t) = \begin{pmatrix} \langle\{\hat{\phi}_k, \hat{\phi}_{-k}\}/2\rangle & \langle\{\hat{\phi}_k, \hat{\pi}_{-k}\}/2\rangle \\ \langle\{\hat{\pi}_k, \hat{\phi}_{-k}\}/2\rangle & \langle\{\hat{\pi}_k, \hat{\pi}_{-k}\}/2\rangle \end{pmatrix} = \begin{pmatrix} |f_k|^2 & \mathrm{Re}\Big(f_k \dot{f}_k^*\Big) \\ \mathrm{Re}\Big(f_k \dot{f}_k^*\Big) & |\dot{f}_k|^2 \end{pmatrix}$$

The quantum-to-classical transition can be characterized by comparing the symmetric and antisymmetric parts of the two-point functions. The genuinely quantum contribution is instead contained in the antisymmetric part

$$\langle\hat{\phi}_k\hat{\pi}_{-k}\rangle = \mathrm{Re}\Big(f_k\dot{f}_k^*\Big) + \frac{i}{2}$$

$$[\hat{\chi}_k, \hat{\pi}_{-k}] = i$$

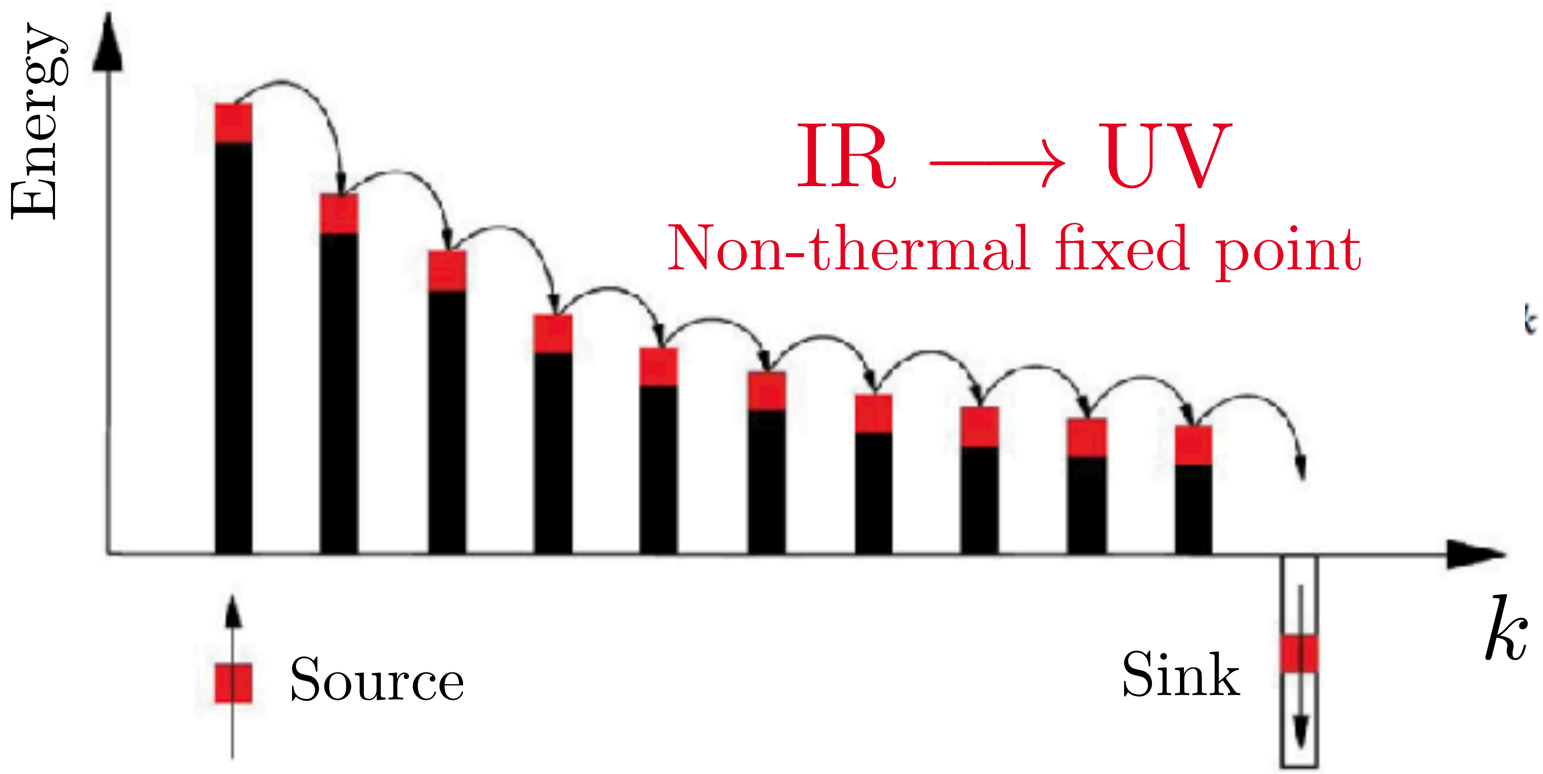
During preheating, resonant/tachyonic modes grow exponentially

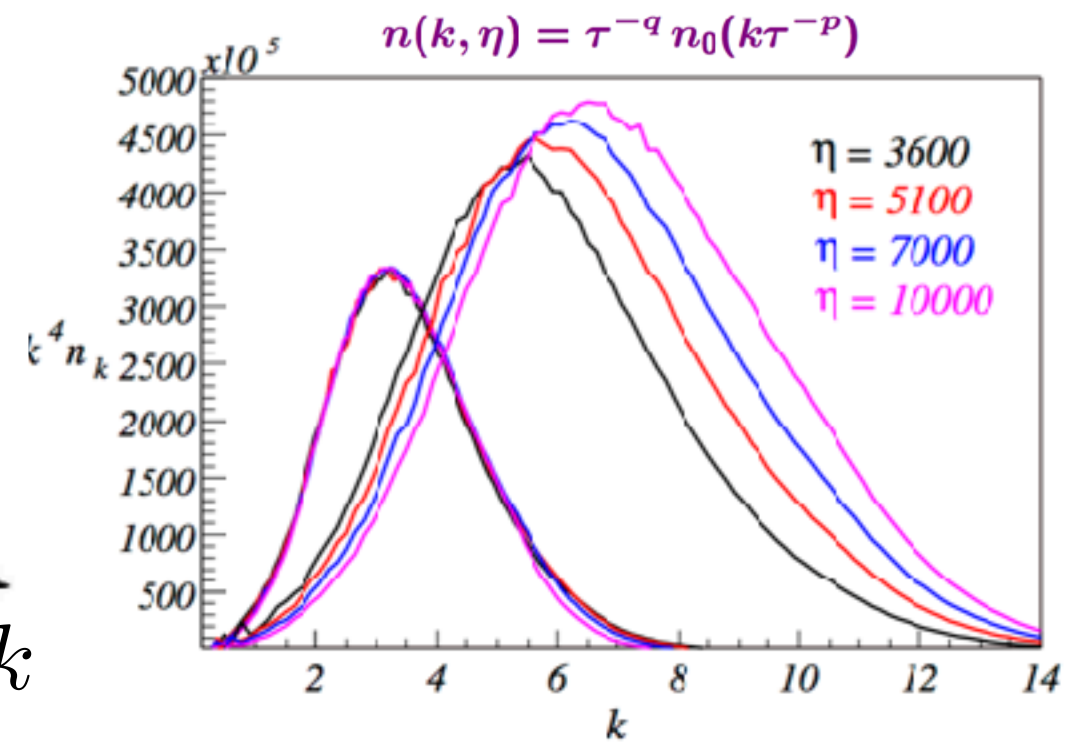
$$\frac{\big|[\hat{\chi}_k, \hat{\pi}_{-k}]\big|}{\big|\frac{1}{2}\langle\{\hat{\chi}_k, \hat{\pi}_{-k}\}\rangle\big|} = \frac{1}{\Big|\mathrm{Re}\Big(f_k\dot{f}_k^*\Big)\Big|} \sim e^{-2\mu_k t} \ll 1$$

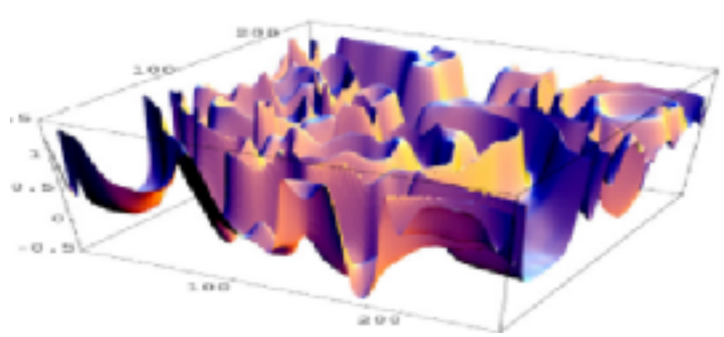
Amplified quantum modes $\rightarrow$ classical stochastic variables

A long way to thermalization

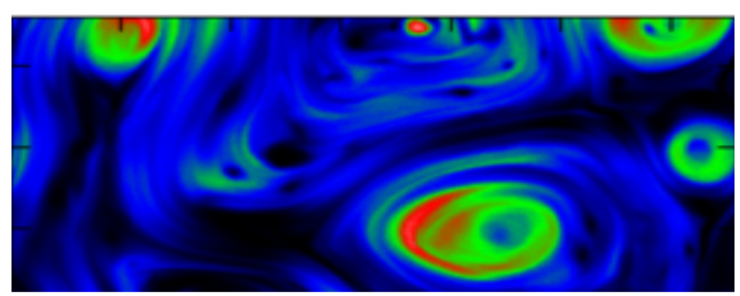
In many models zero mode does not decay completely







Non-linear fragmentation



Turbulent cascading

Eventual thermalization

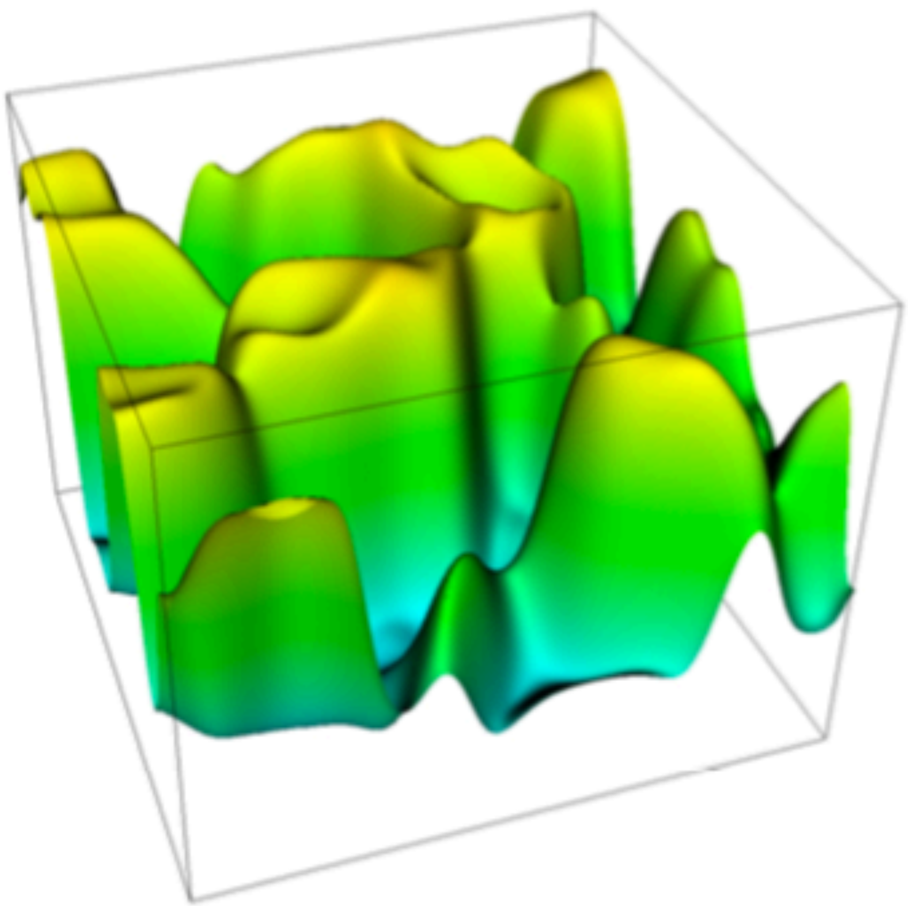
Classical Field Theory

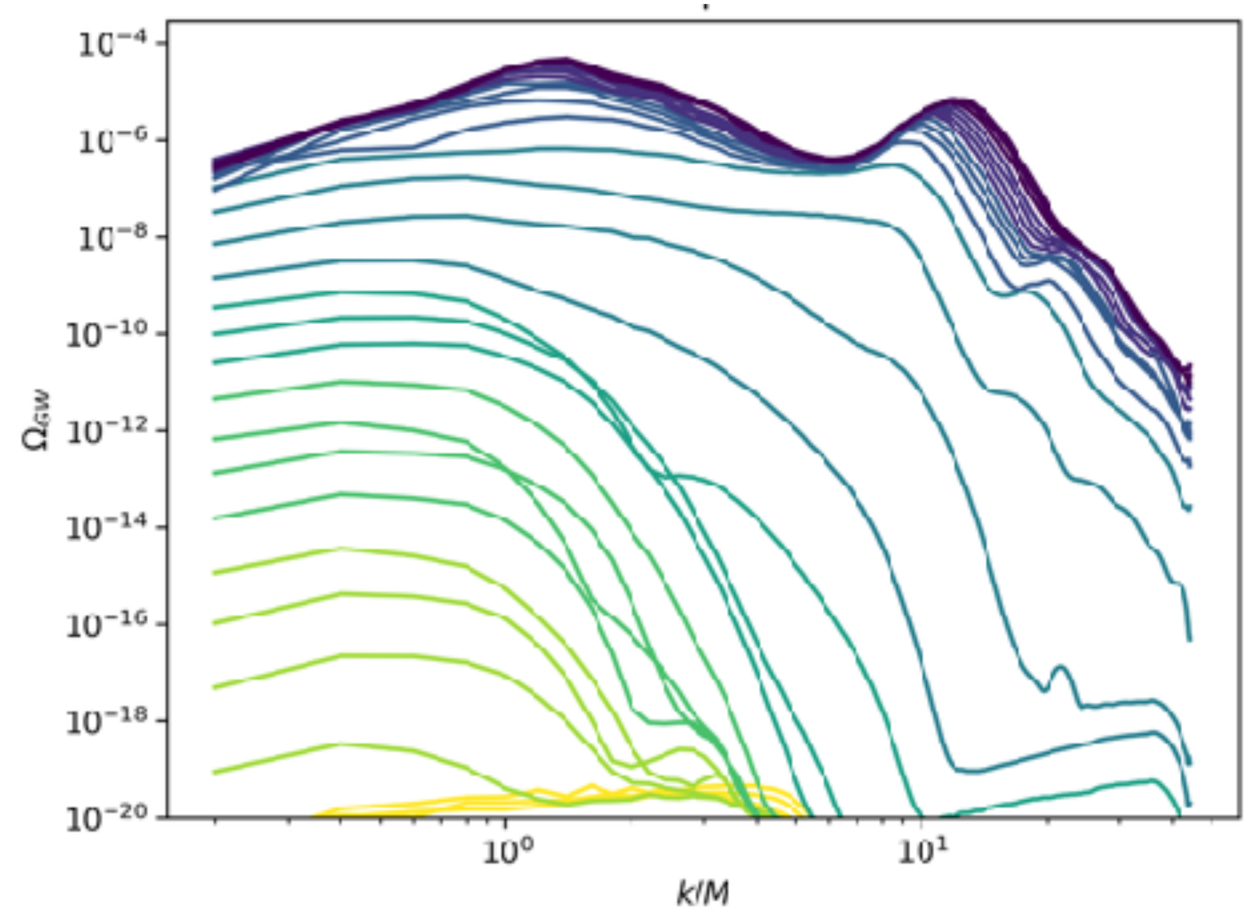
Kinetic theory

Reheating temperature = Onset of radiation domination

Gravitational waves

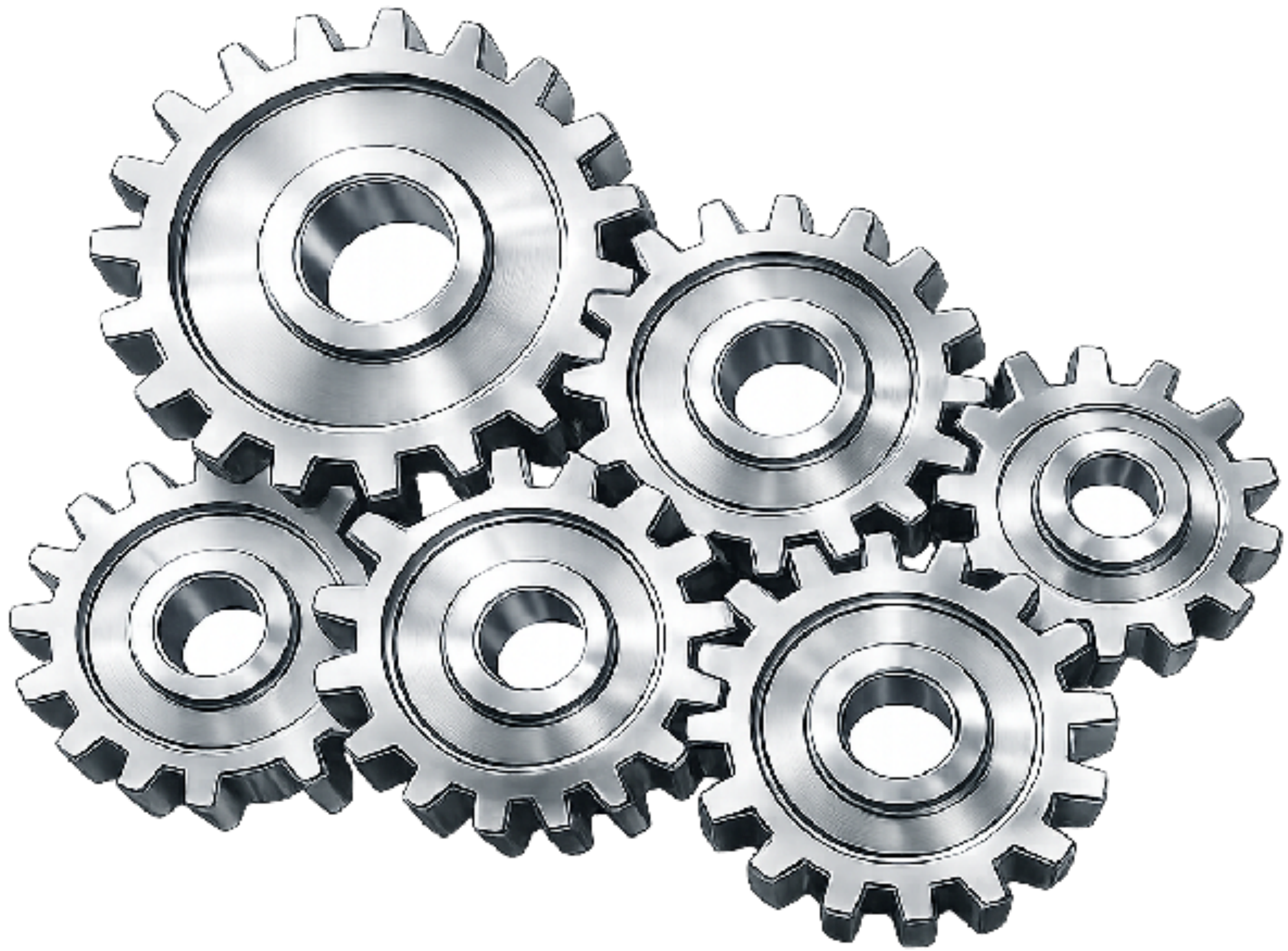
$$(h_{ij}^{TT})'' + 2H(h_{ij}^{TT})' - \frac{\nabla^2 h_{ij}^{TT}}{a^2} \simeq \frac{2a^2}{M_P^2}\Pi_{ij}^{TT}$$





Non-linear fragmentation dominates the emission

Classical in nature

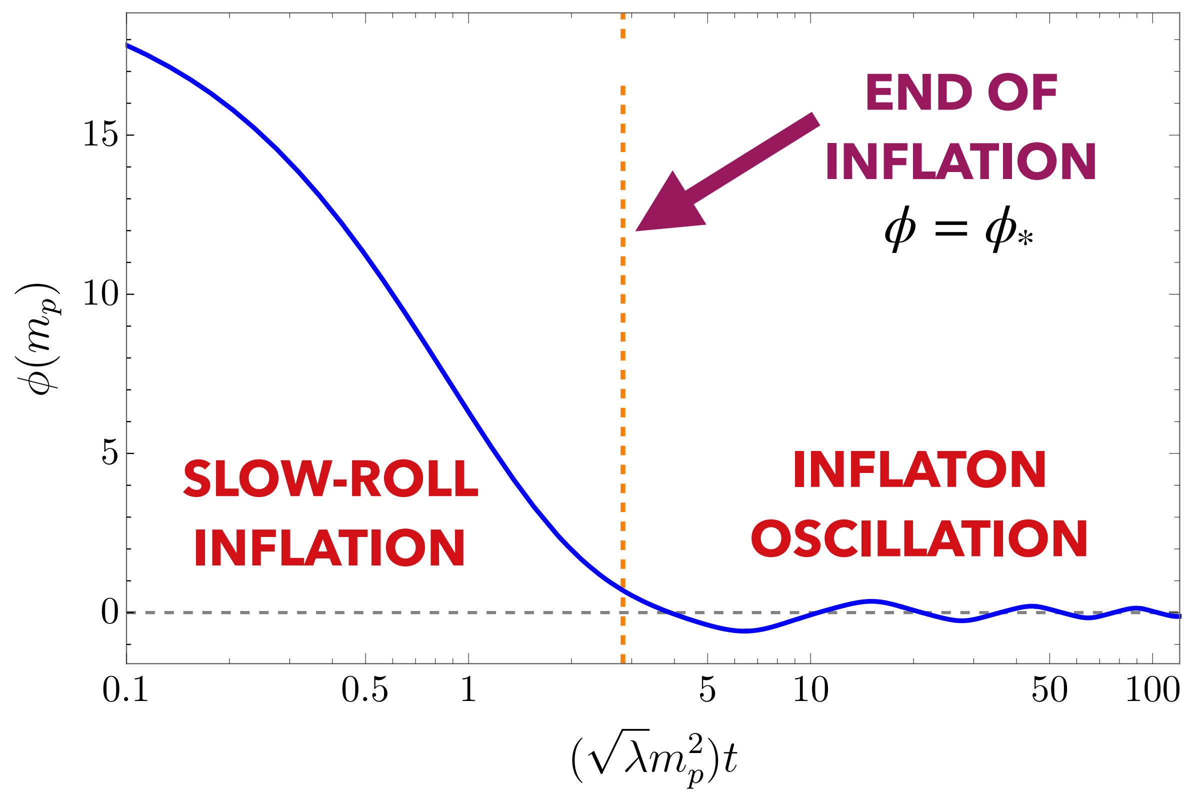


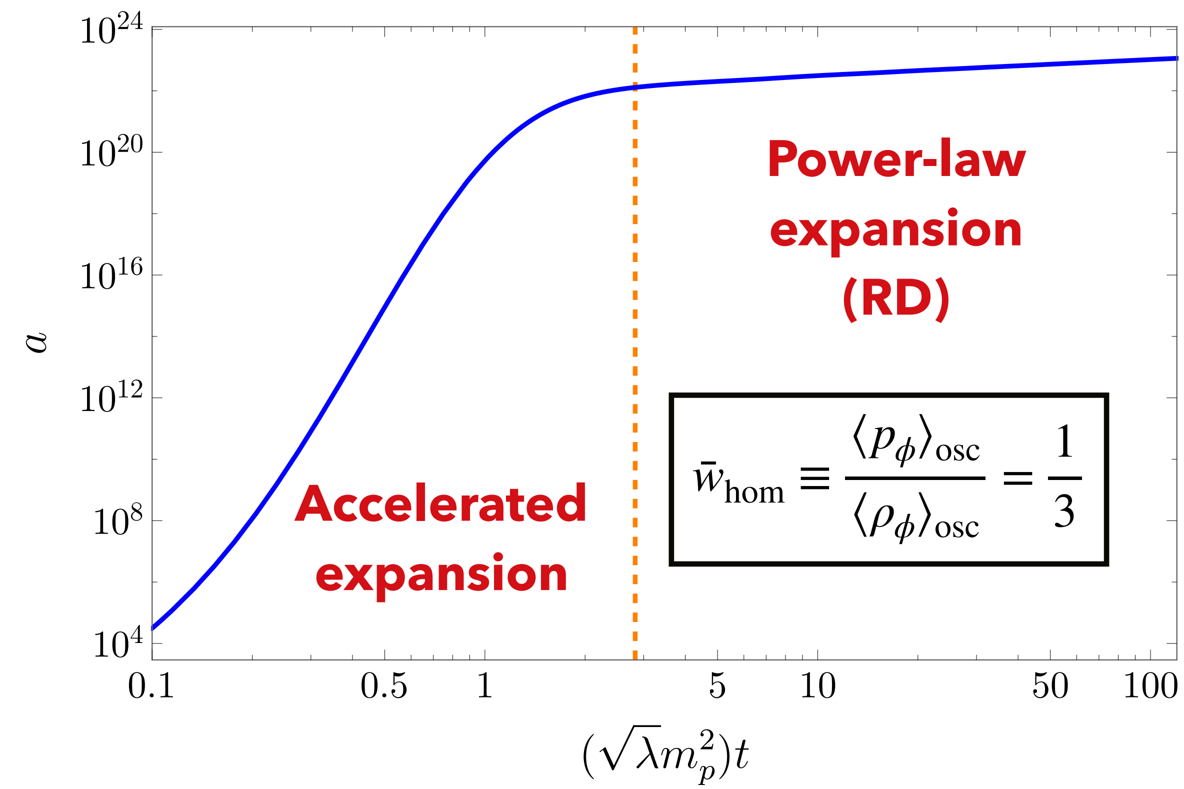
$$V(\phi, X) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2X^2$$

ϕ **inflaton**

X **daughter field**

Homogeneous approximation (with $g^2 = 0$)





Equation of state:

$$\bar{w}_{\rm hom} \equiv \frac{\langle p_\phi \rangle_{\rm osc}}{\langle \rho_\phi \rangle_{\rm osc}} = \frac{1}{3}$$

Amplitude:

$$A_\phi(t) \equiv \phi_* \left(\frac{t}{t_*}\right)^{-1/2} \propto a^{-1}$$

Frequency: $\omega_* \equiv \sqrt{\lambda}\phi_*$

$$\Omega_{\rm osc} \equiv \omega_* \left(\frac{t}{t_*}\right)^{-\frac{1}{2}} \propto a^{-1}$$

- **"Natural" variables** for quartic potential: Conformal rescaling of fields and time

$$\varphi \equiv \frac{1}{\phi_*} a\phi \qquad \chi \equiv \frac{1}{\phi_*} aX \qquad u \equiv \omega_* \int_{t_*}^{t} a(t')^{-1} dt' \qquad \vec{y} \equiv \omega_* \vec{x}$$

- **Simple solution** $\quad \bar{\varphi}'' + \bar{\varphi}^3 \simeq 0 \quad \longrightarrow \quad \bar{\varphi} \simeq \mathrm{cn}(u, 1/2) \approx \cos(0.85u)$

Complete field equations

$$V(\phi, X) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2 X^2$$

Field equations:

$$\ddot{\phi} - \frac{1}{a^2}\nabla^2\phi + 3H\dot{\phi} + \lambda\phi^3 + g^2X^2\phi = 0$$

$$\ddot{X} - \frac{1}{a^2}\nabla^2 X + 3H\dot{X} + g^2X^2\phi = 0$$

$$\frac{\ddot{a}}{a} = \frac{1}{3m_p^2}(-\dot{\phi}^2 - \dot{X}^2 + V(\phi, X))$$

Initial conditions:

$$\phi(t_*, \vec{x}) = \phi_* + \delta\phi(t_*, \vec{x})$$

$$X(t_*, \vec{x}) = \delta X(t_*, \vec{x})$$

$$a(t_*) = a_* = 1$$

(Classical) Initial Fluctuations

$$\left\langle \delta\phi^2 \right\rangle = \int d\log k \; \Delta_{\delta\phi}(k) \qquad \Delta_{\delta\phi}(k) \equiv \frac{k^3}{2\pi^2}\mathscr{P}_{\delta\phi}(k)$$

$$\mathscr{P}_{\delta\phi}(k) \equiv \frac{1}{2a^2\omega_{k,\phi}} \qquad \omega_{k,\phi} \equiv \sqrt{k^2 + a^2 m_\phi^2} \qquad m_\phi^2 \equiv \left.\frac{\partial^2 V}{\partial\phi^2}\right|_{\phi=\bar{\phi}_*}$$

Linear dynamics (in conformal variables)

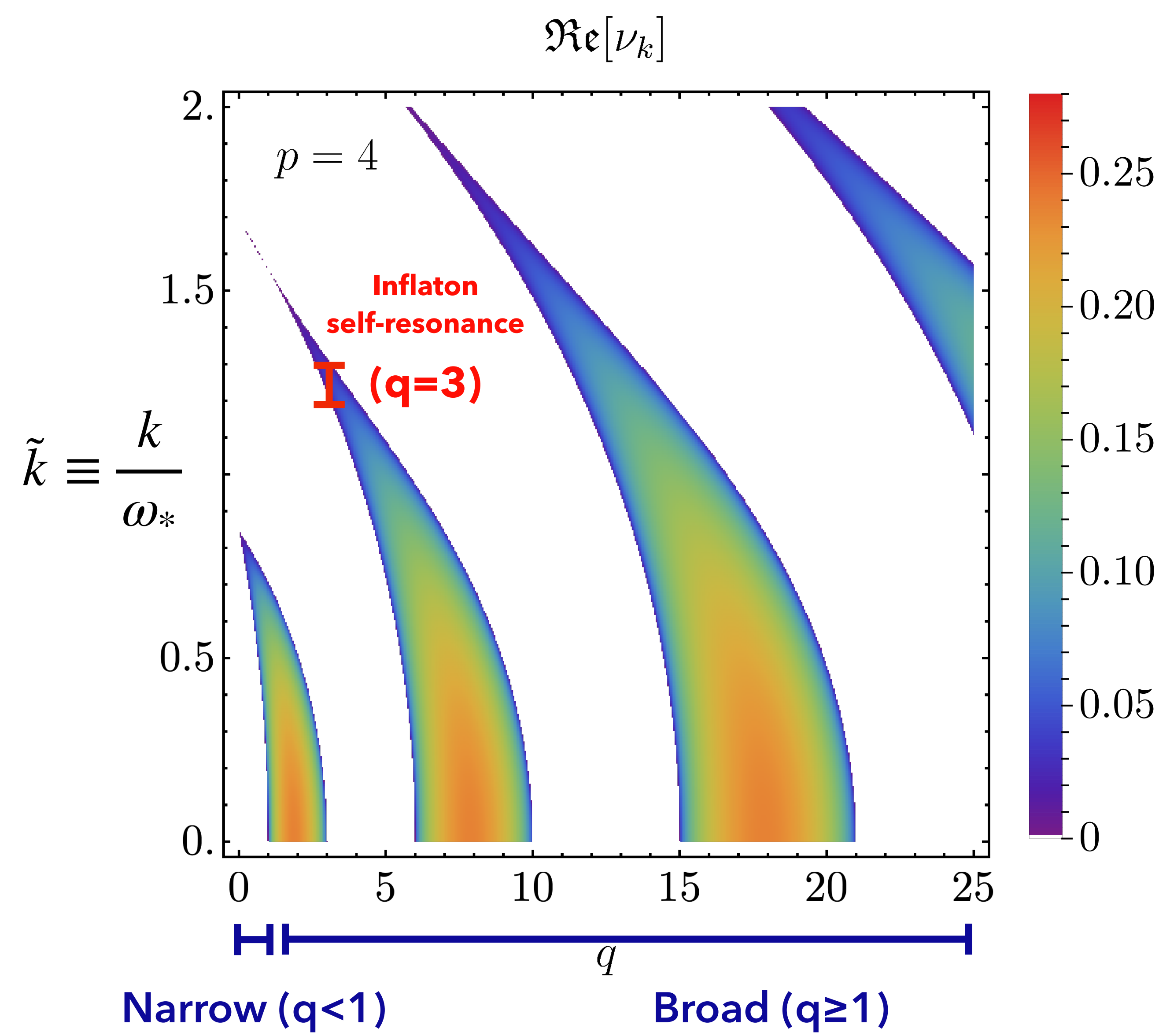
$$\varphi'' - \nabla_{\vec{y}}^2\varphi + \left(|\varphi|^2 + q\chi^2 - \underbrace{\cancel{\frac{a''}{a}}}_{\sim u^{-2}}\right)\varphi = 0$$

$$\chi'' - \nabla_{\vec{y}}^2\chi + \left(q\varphi^2 - \underbrace{\cancel{\frac{a''}{a}}}_{\sim u^{-2}}\right)\chi = 0$$

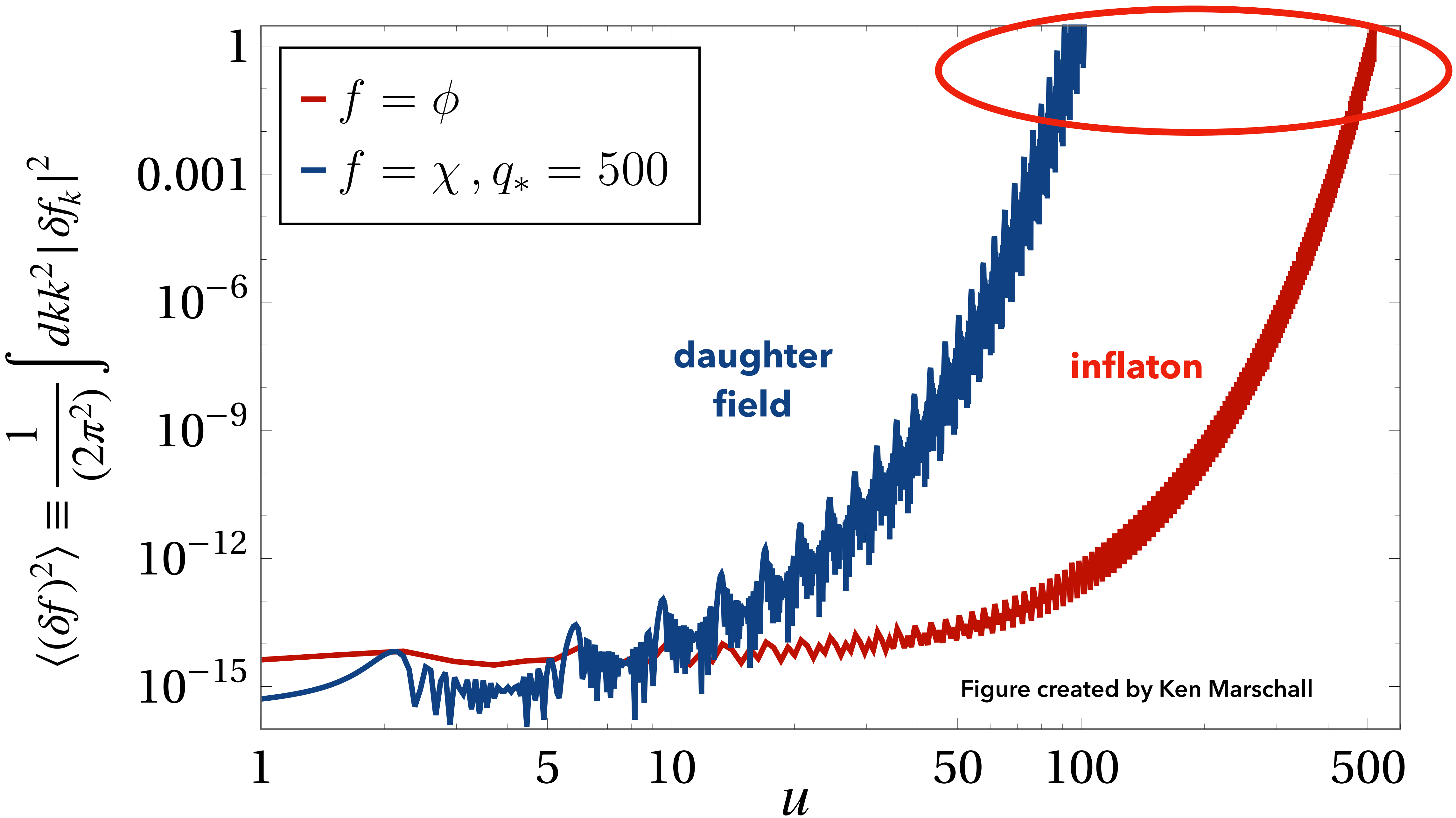
$$\delta\varphi_k'' + (\tilde{k}^2 + 3\bar{\varphi}^2)\delta\varphi_k \simeq 0$$

$$\delta\chi_k'' + (\tilde{k}^2 + q\bar{\varphi}^2)\delta\chi_k \simeq 0$$

$$\boxed{\begin{array}{l} \text{RESONANCE PARAMETER:} \quad q \equiv \dfrac{g^2}{\lambda} \\ |\varphi_{\mathbf{k}}|^2 \sim e^{2\mu_\kappa u} \\ |\chi_{\mathbf{k}}|^2 \sim e^{2\nu_\kappa(q) u} \end{array}}$$



Non-linearities & Backreaction



Fluctuations of daughter field grow faster than those of inflaton

Non-linearities become relevant

Lattice simulations needed!

Program variables

- In the lattice we operate in a set of dimensionless spacetime and field variables called **program variables (chosen in a case by case basis)**:

Choose:

$$\{\alpha, f_*, \omega_*\} \longrightarrow \boxed{d\tilde{\eta} \equiv a^{-\alpha}\omega_* dt \quad d\tilde{x}^i \equiv \omega_* dx^i} \qquad \boxed{\tilde{\phi} = \frac{\phi}{f_*}}$$

Number

Mass scales

Space and time

Scalar fields

$\dot{f} \equiv df/dt$ denotes derivatives with respect to cosmic time.

$f' \equiv df/d\tilde{\eta}$ denotes derivatives with respect to program time.

- We also build other (dimensionless) quantities from program variables for convenience. These are tagged with the diacritic "~".

Example: Program potential

$$\boxed{\tilde{V}(\tilde{\phi}) = \frac{1}{f_*^2\omega_*^2} V(f_*\tilde{\phi})}$$

Program fields

Physical fields

Program variables

- Program variables are chosen **in a case by case basis**.

- Optimal choice for **monomial inflationary potentials** is:

$$\boxed{V(\phi) = \frac{1}{p}\lambda\mu^{4-p}|\,\phi\,|^p} \longrightarrow \begin{cases} \alpha = 3\left(\dfrac{p-2}{p+2}\right) \\ f_* = \phi_* \\ \omega_* = \sqrt{\lambda}\mu^{\frac{4-p}{2}}\phi_*^{\frac{p-2}{2}} \end{cases}$$

(Cosmic time for p=2, conformal time for p=4)

Amplitude at the end of inflation

Oscillation frequency at the end of inflation

in these variables, the **oscillation frequency is approximately constant**, and **time and space scales are of order one.**

Example model

- **Inflaton with quartic potential, coupled to a "daughter" field through a quadratic-quadratic interaction term**

$$S = \int d^4x a(t)^3 \mathscr{L}_m$$

$$\boxed{-\mathscr{L}_m = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \frac{1}{2}\partial^\mu\chi\partial_\mu\chi + V(\phi,\chi)}$$

$$\boxed{V(\phi,\chi) \equiv \sum_{m=0}^{N_p-1} V^{(m)}(\phi,\chi) = \frac{\lambda}{4}\phi^4 + \frac{1}{2}g^2\phi^2\chi^2}$$

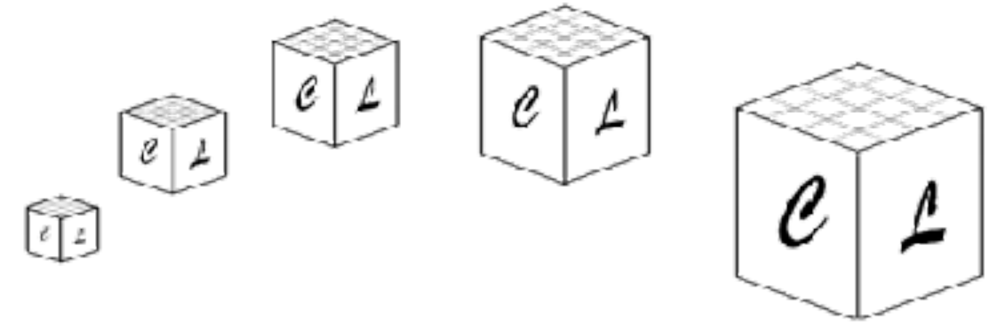
- **Number scalar singlets:** $N_s = 2$
- **Number potential terms:** $N_t = 2$

- We fix the **program variables** to:

$$\boxed{\begin{aligned} f_* &= \phi_* \\ \omega_* &= \sqrt{\lambda}\phi_* \\ \alpha &= 1 \end{aligned}} \longrightarrow \boxed{\tilde{\phi} = \frac{\phi}{\phi_*} \qquad d\tilde{\eta} \equiv \sqrt{\lambda}\phi_* a^{-1} dt \qquad d\tilde{x}^i \equiv \sqrt{\lambda}\phi_* dx^i}$$

[Note: unlike the "natural variables" introduced before, the program inflaton is not rescaled by the scale factor]

Example model



- **Program potential:**

$$\boxed{\tilde{V}(\tilde{\phi},\tilde{\chi}) \equiv \frac{1}{f_*^2\omega_*^2}V(f_*\tilde{\phi},f_*\tilde{\chi}) = \overset{\tilde{V}^{(0)}}{\overset{\downarrow}{\frac{1}{4}|\tilde{\phi}|^4}} + \overset{\tilde{V}^{(1)}}{\overset{\downarrow}{\frac{1}{2}q\tilde{\phi}^2\tilde{\chi}^2}}}$$

$$\boxed{q \equiv \frac{g^2}{\lambda}}$$

Resonance parameter

- Analogously, we define the **program energy density** and **program pressure density:**

$$\boxed{\tilde{\rho} \equiv \frac{\rho}{f_*^2\omega_*^2} = \widetilde{K} + \widetilde{G} + \widetilde{V}} \qquad \boxed{\tilde{p} \equiv \frac{p}{f_*^2\omega_*^2} = \widetilde{K} - \frac{1}{3}\widetilde{G} - \widetilde{V}}$$

Kinetic contribution:

$$\widetilde{K} \equiv \sum_{n=0}^{N_s-1}\widetilde{K}^{(n)} \qquad \widetilde{K}^{(n)} \equiv \frac{1}{2a^{2\alpha}}(\tilde{\phi}_n^{\,\prime})^2 \longrightarrow \widetilde{E}_K \equiv \langle \tilde{K} \rangle$$

Gradient contribution:

$$\widetilde{G} \equiv \sum_{n=0}^{N_s-1}\widetilde{G}^{(n)} \qquad \widetilde{G}^{(n)} \equiv \frac{1}{2a^2}\sum_i(\widetilde{\nabla}_i\tilde{\phi}_n)^2 \longrightarrow \widetilde{E}_G \equiv \langle \tilde{G} \rangle$$

Volume averages

Potential contribution:

$$\widetilde{V} \equiv \sum_{n=0}^{N_t-1}\widetilde{V}^{(t)} \qquad \widetilde{V}^{(t)} \equiv \frac{1}{f_*^2\omega_*^2}V^{(t)}(\tilde{\phi}_n) \longrightarrow \widetilde{E}_V \equiv \langle \tilde{V} \rangle$$

Example model

- The code solves the following equations:

$$\tilde{\phi}'' - a^{-2(1-\alpha)}\tilde{\nabla}^2\tilde{\phi} + (3-\alpha)\frac{a'}{a}\tilde{\phi}'_a = -a^{2\alpha}\widetilde{V}_{,\tilde{\phi}} = -a^{2\alpha}\left(\tilde{\phi}^3 + \frac{g^2}{\lambda}\tilde{\phi}\tilde{\chi}^2\right)$$

FIELD 0:
2*N³ equations

$$\chi'' - a^{-2(1-\alpha)}\tilde{\nabla}^2\tilde{\chi} + (3-\alpha)\frac{a'}{a}\tilde{\chi}'_a = -a^{2\alpha}\widetilde{V}_{,\tilde{\chi}} = -a^{2\alpha}\frac{g^2}{\lambda}\tilde{\phi}^2\tilde{\chi}$$

FIELD 1:
2*N³ equations

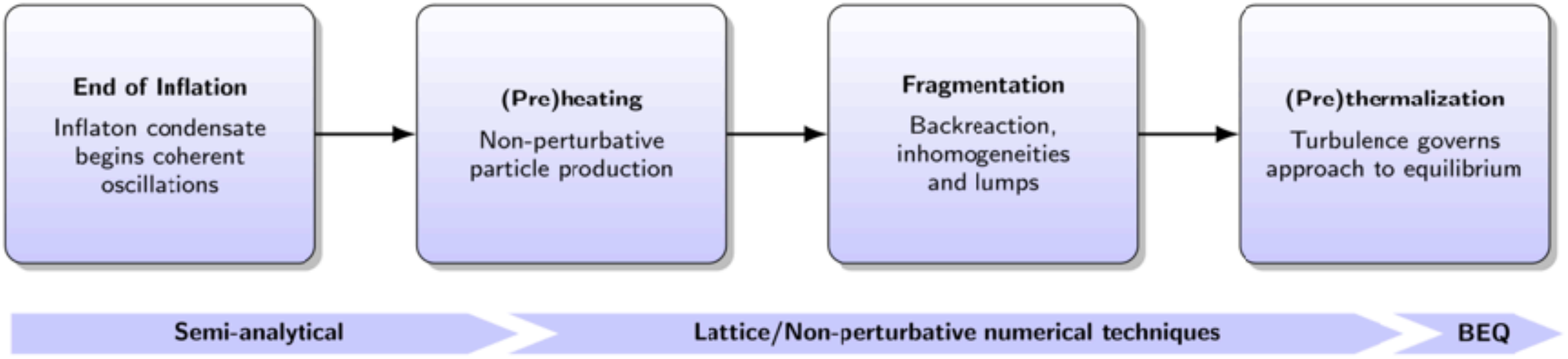
SCALE FACTOR:
1 equation

$$\frac{a''}{a} = \frac{a^{2\alpha}}{3}\left(\frac{f_*}{m_p}\right)^2\left[(\alpha-2)\widetilde{E}_K + \alpha\widetilde{E}_G + (\alpha+1)\widetilde{E}_V\right]$$

- The accuracy of our solution is monitored through the 1st Friedmann eq.:

$$a'^2 = \frac{a^{2\alpha+2}}{3}\left(\frac{f_*}{m_p}\right)^2\left[\widetilde{E}_K + \widetilde{E}_G + \widetilde{E}_V\right] \longrightarrow \Delta_E \equiv \frac{|\mathrm{LHS} - \mathrm{RHS}|}{|\mathrm{LHS} + \mathrm{RHS}|} \lesssim 1$$

Thank you!


End of Inflation
Inflaton condensate begins coherent oscillations
(Pre)heating
Non-perturbative particle production
Fragmentation
Backreaction, inhomogeneities and lumps
(Pre)thermalization
Turbulence governs approach to equilibrium
Semi-analytical
Lattice/Non-perturbative numerical techniques
BEQ