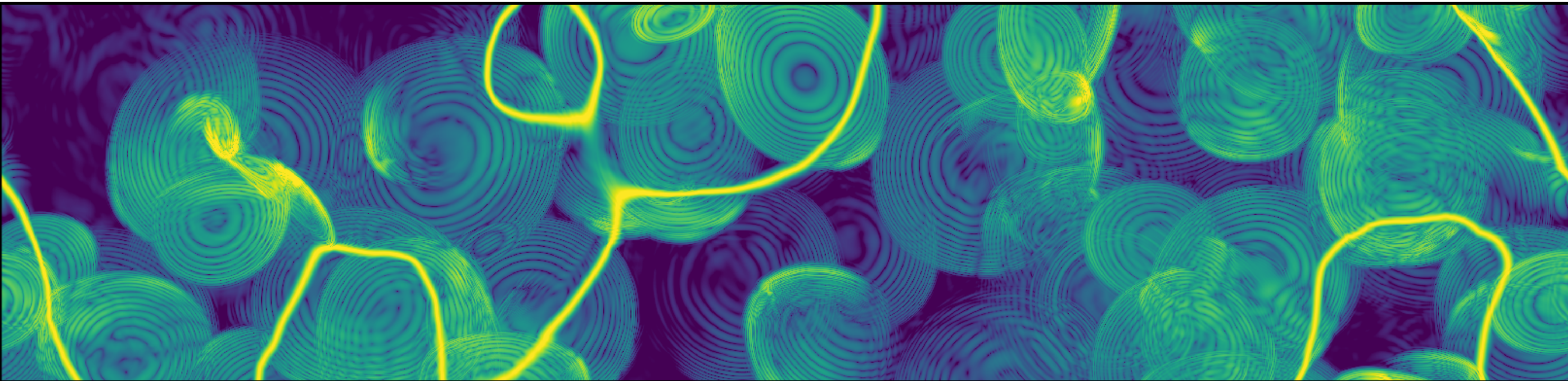


SIMULATING THE EARLY UNIVERSE:

A hands-on CosmoLattice workshop

INTRODUCTION



Francisco Torrentí

U. Carlos III, Madrid

SIMULATING THE EARLY UNIVERSE

A Hands-On **CosmoLattice** Workshop

Nonlinear dynamics
Gauge fields

Reheating
Gravitational waves

— Speakers —

N. Loayza · K. Marshall · J. Rubio · F. Torrenti

WELCOME!!!

7-8 May 2026
Madrid

REGISTER NOW!

Fundación
BBVA



UNIVERSIDAD
COMPLUTENSE
MADRID



CosmoLattice Com... +

Find channel

CHANNELS

Town Square

Add channels

EVENTS

CL Workshop Madrid '26

DIRECT MESSAGES

CL Workshop Madrid '26

Public channel created by Paco Torrentí on April 13, 2026.

This is the start of CL Workshop Madrid '26. Any team member can join and read this channel.

Favorite Add people Set header Notifications

April 13

System 5:53 PM
You joined the channel.
@Nicolás Loayza Romero and @Kenneth Marschall added to the channel by you.

System 6:02 PM
@Paco Torrentí updated the channel header to:
This the channel for "Simulating the early universe: A Hands-On CosmoLattice workshop", held on May 7–8 2026, at Complutense University, Madrid, Spain.

Write to CL Workshop Madrid '26

B I S H | | | | | Aa ^ | | | >



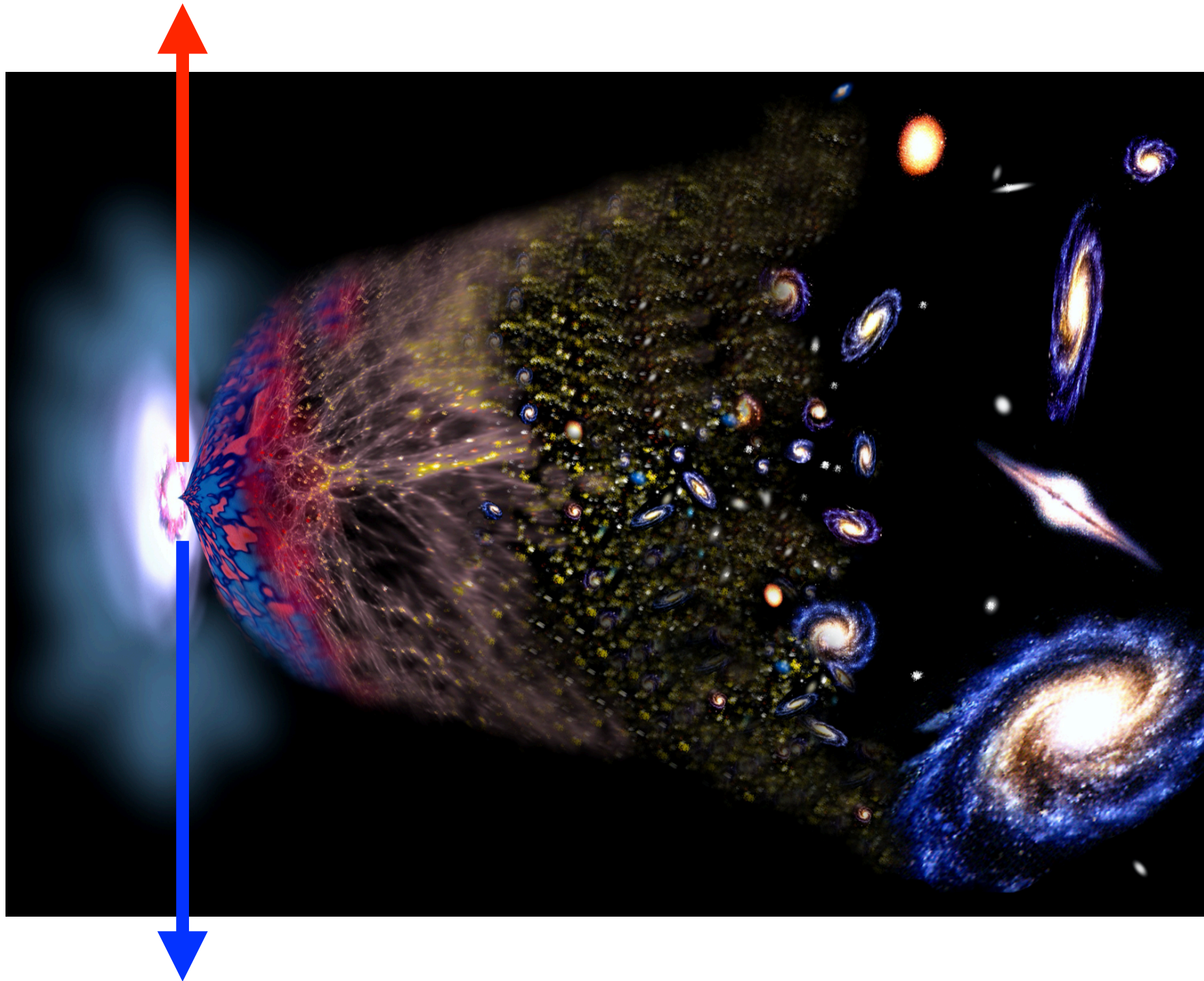
“CosmoLattice Community” Mattermost team:

https://mattermost.web.cern.ch/signup_user_complete/?id=n5mh3j99obrmxcrcubacpbbhya&md=link&sbr=su

Madrid Workshop channel:

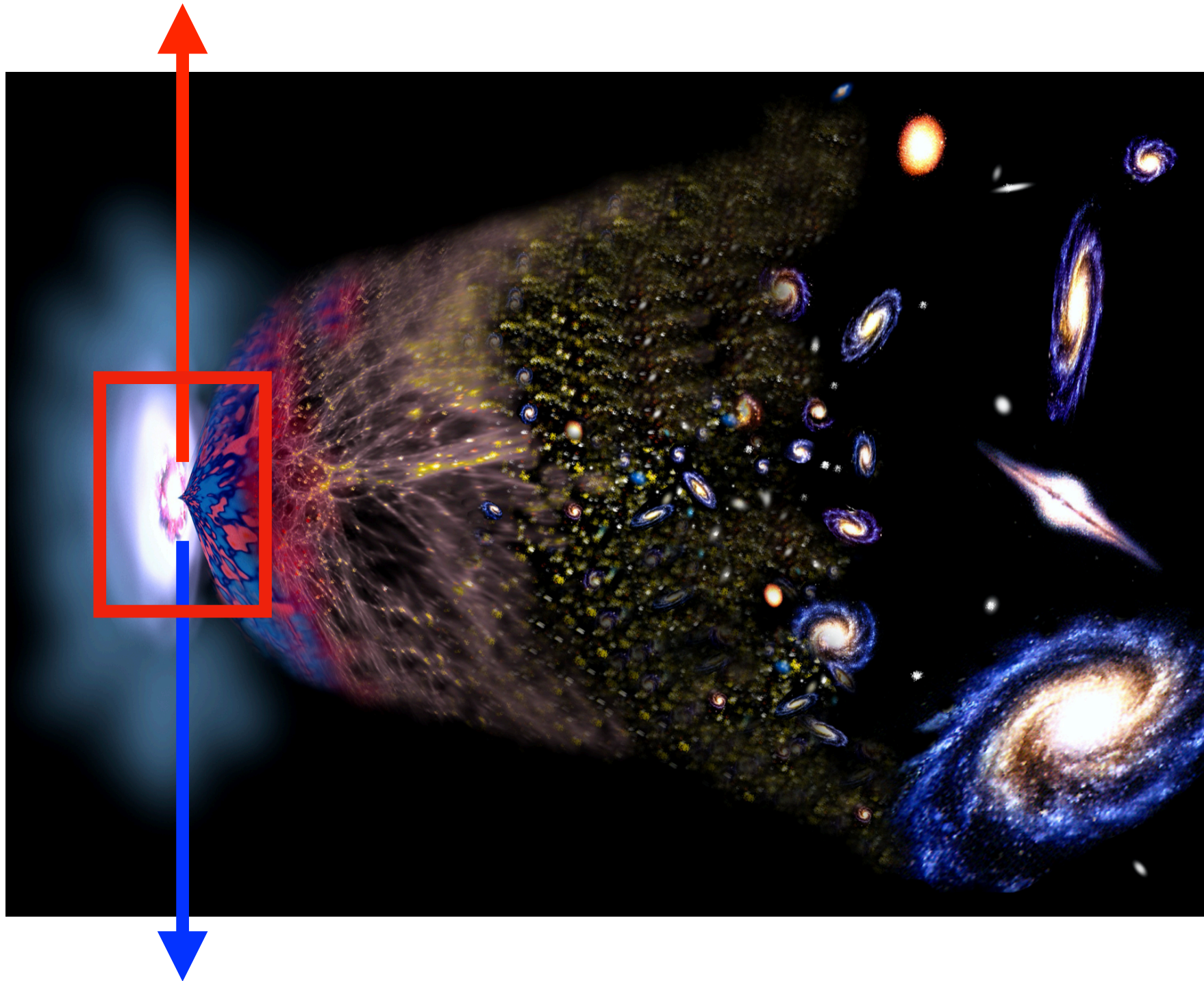
<https://mattermost.web.cern.ch/cosmolattice2/channels/cl-workshop-madrid-26>

How can we simulate the very first
instants of the Universe?



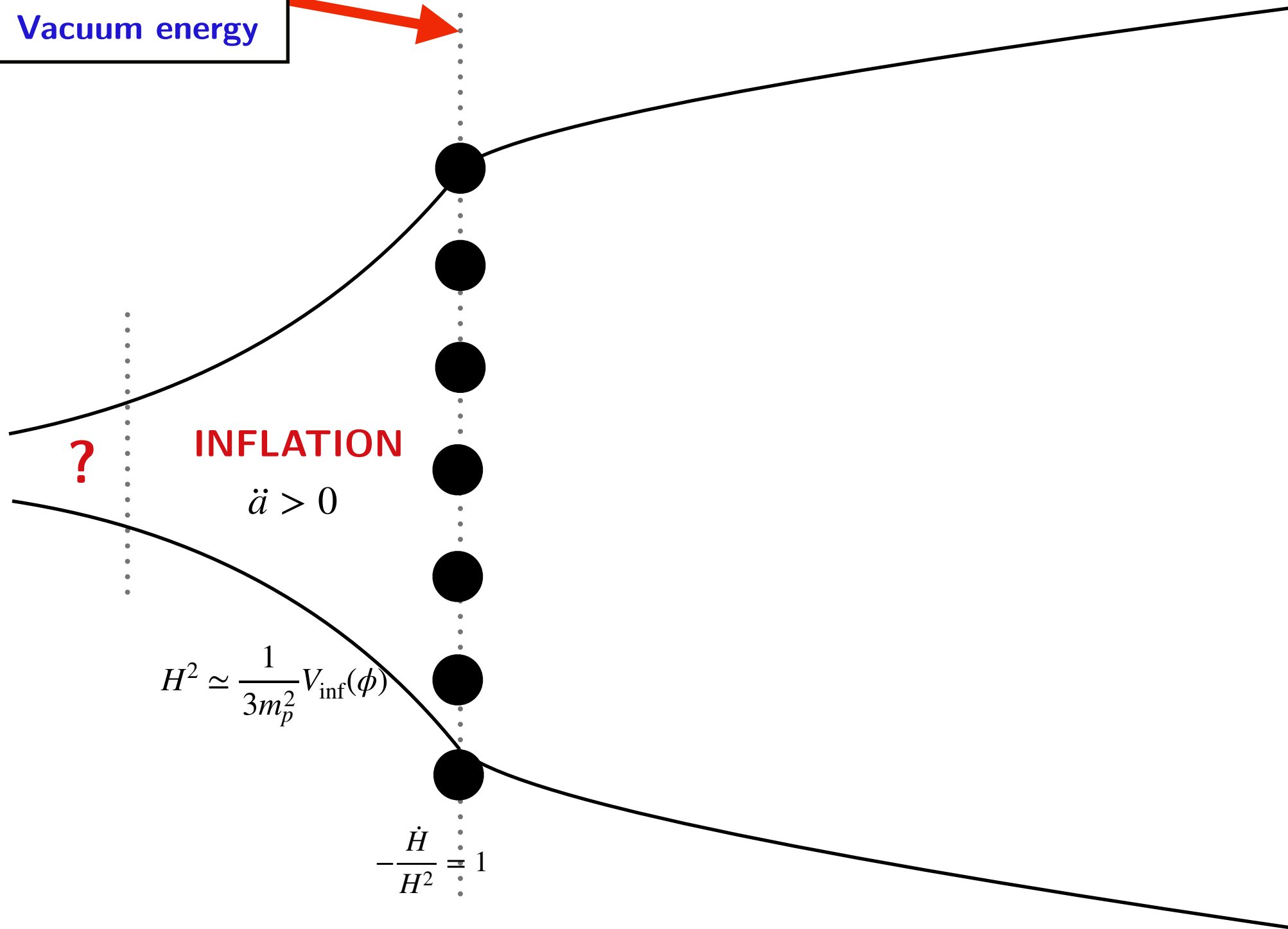
What can we learn from this epoch?

How can we simulate the very first
instants of the Universe?



What can we learn from this epoch?

INITIAL
CONDITIONS:
Vacuum energy



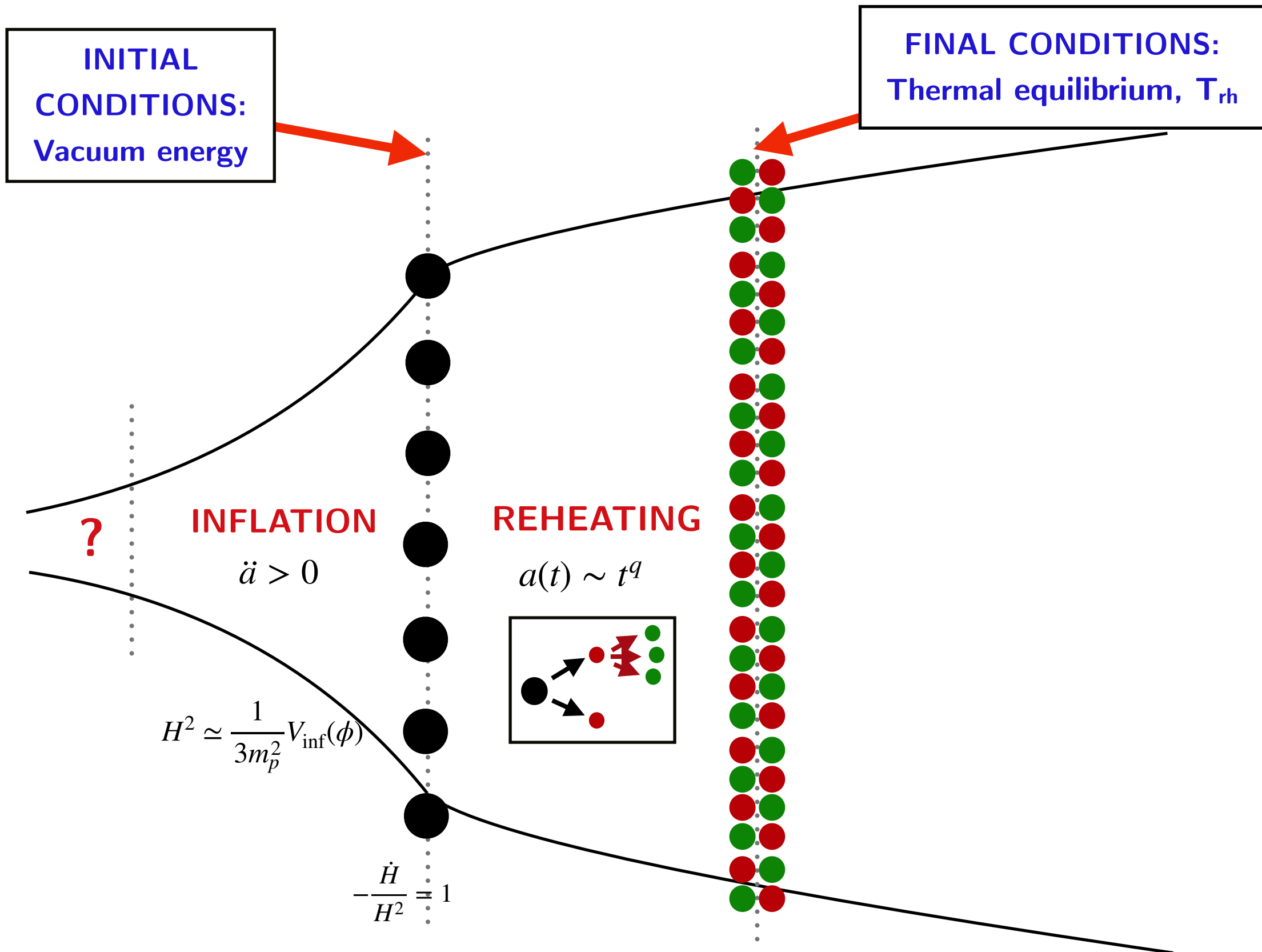
?

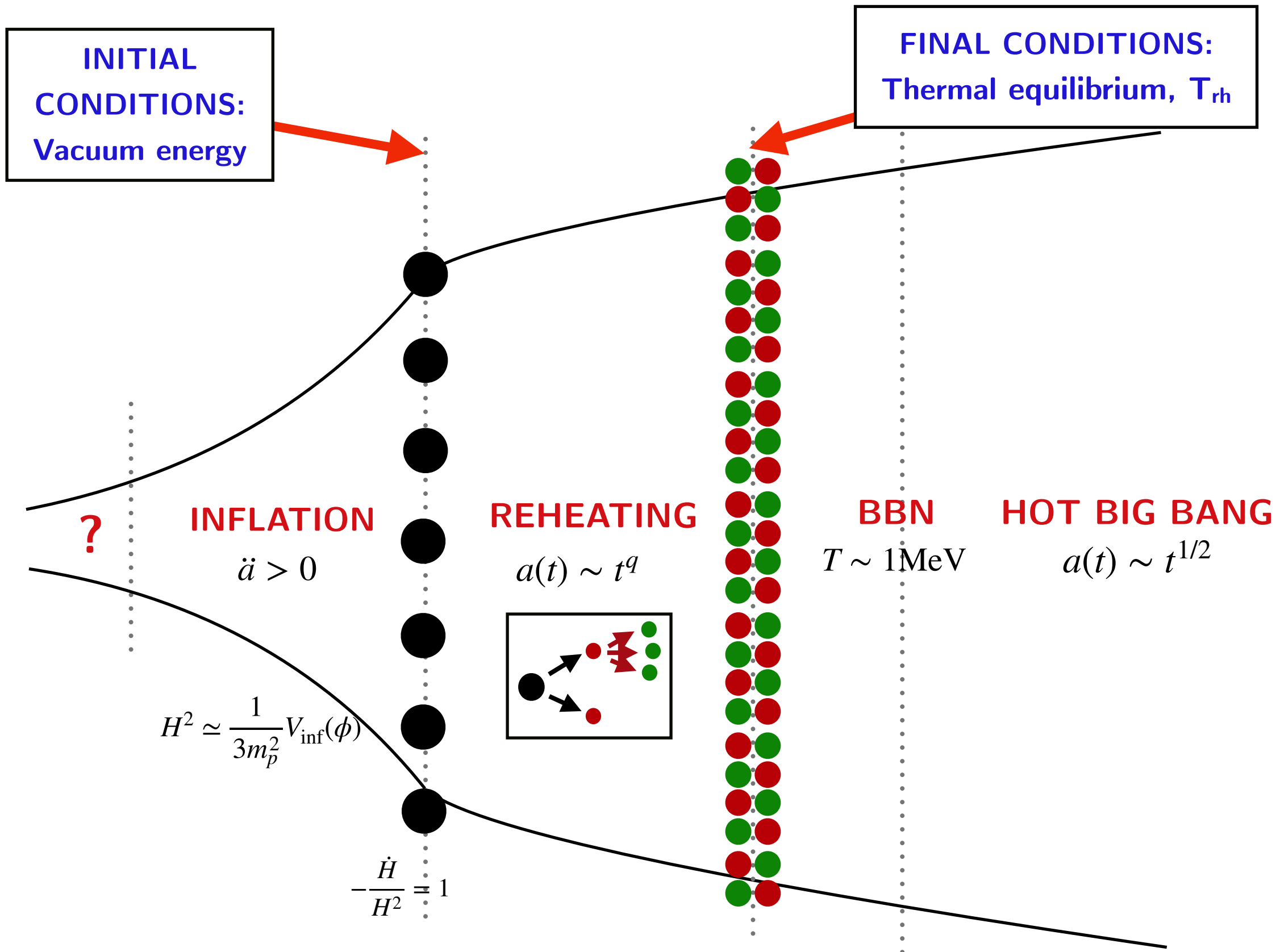
INFLATION

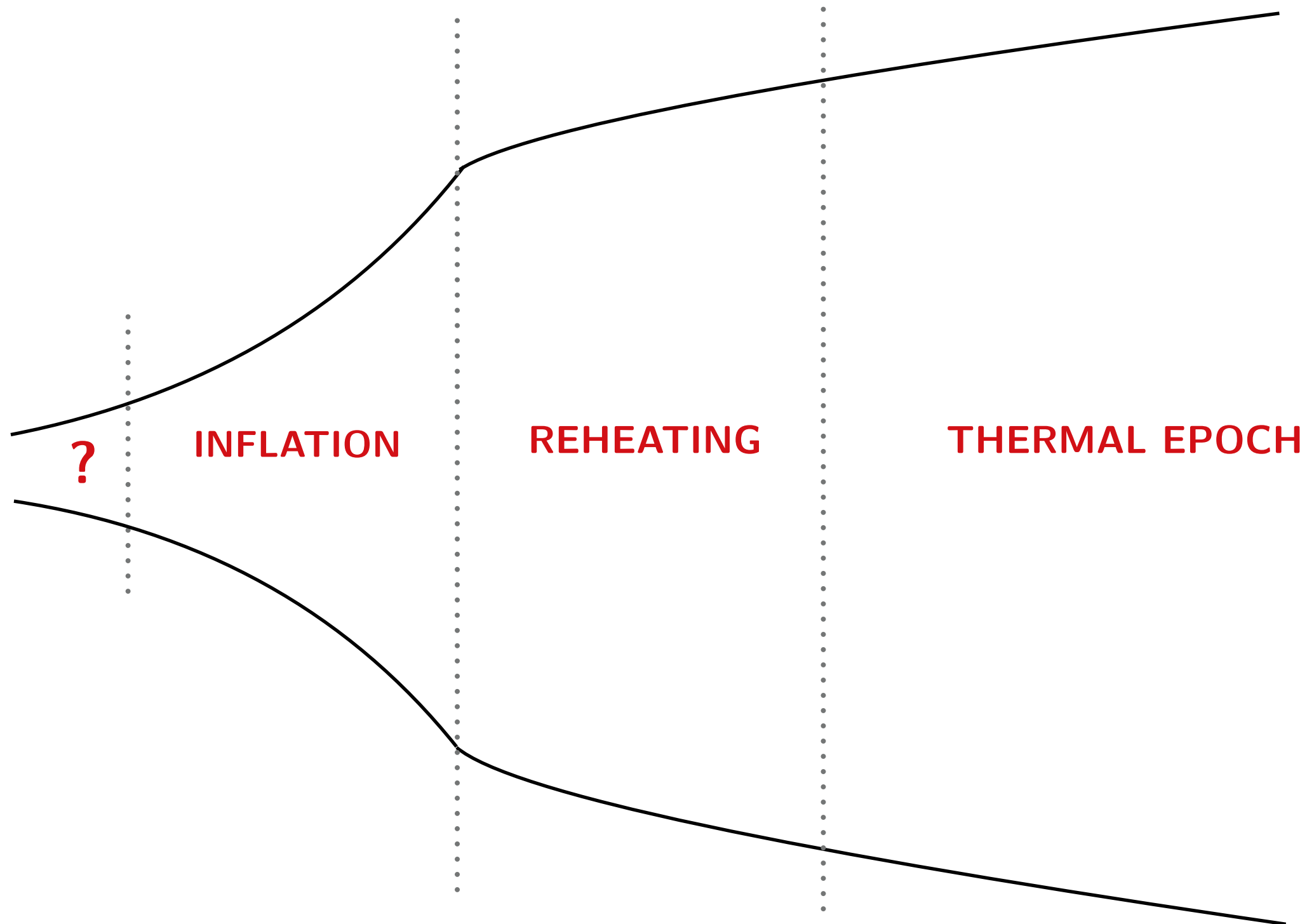
$$\ddot{a} > 0$$

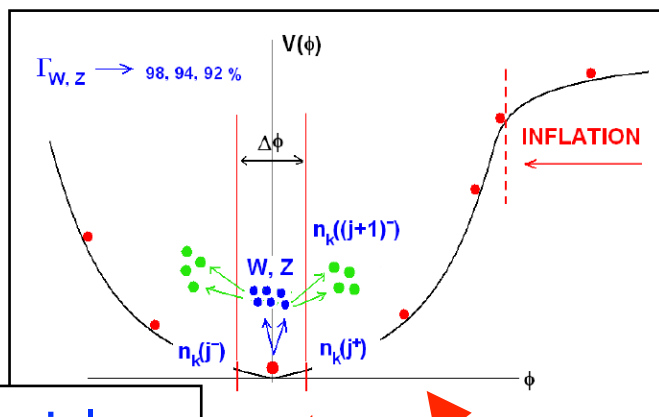
$$H^2 \simeq \frac{1}{3m_p^2} V_{\text{inf}}(\phi)$$

$$-\frac{\dot{H}}{H^2} = 1$$

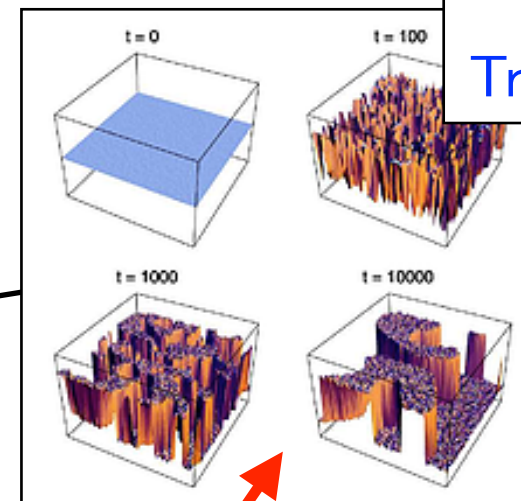








Particle
Production



Phase
Transitions

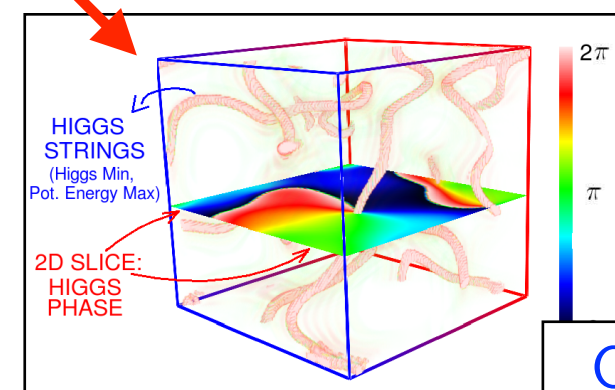
?

INFLATION

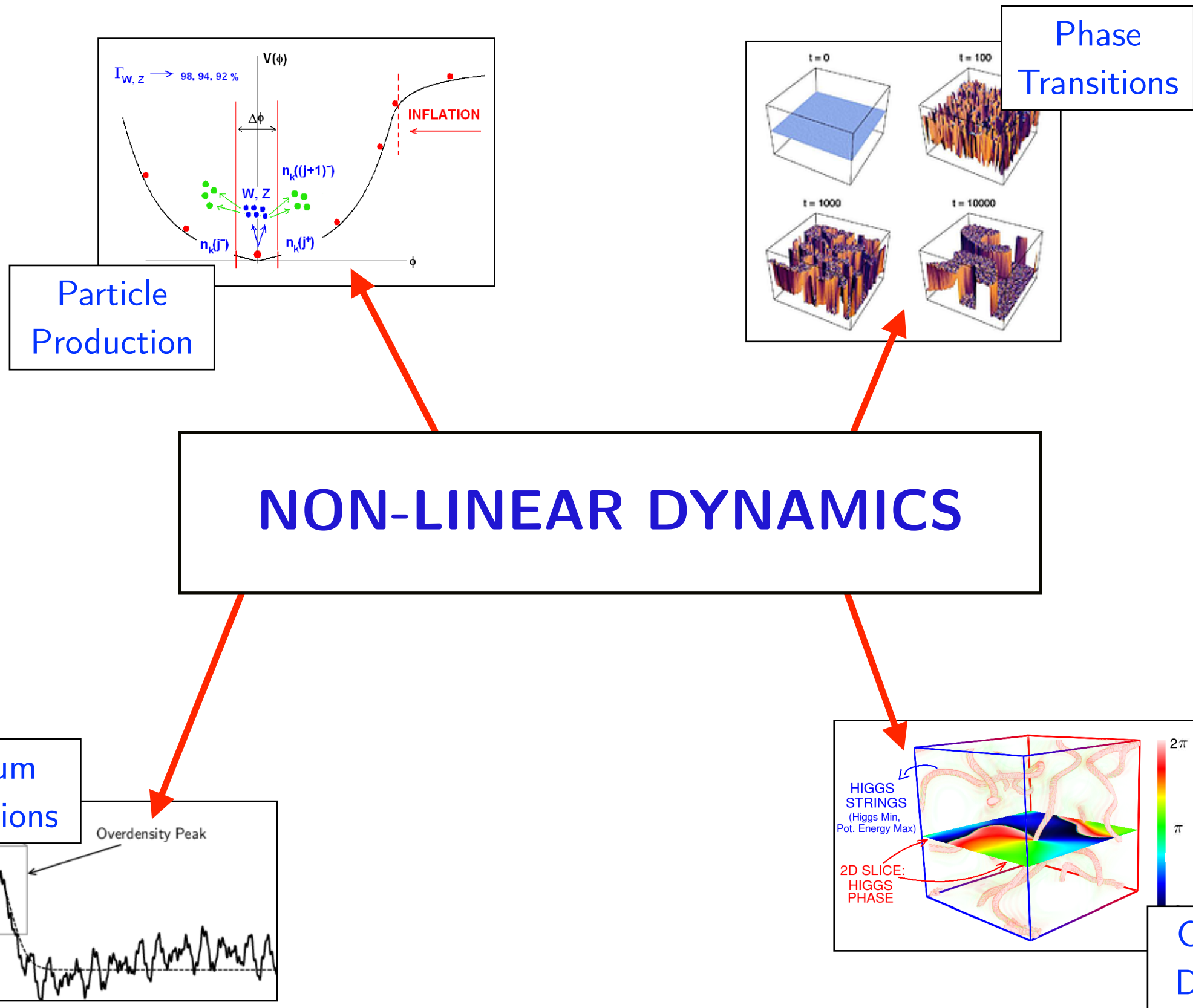
REHEATING

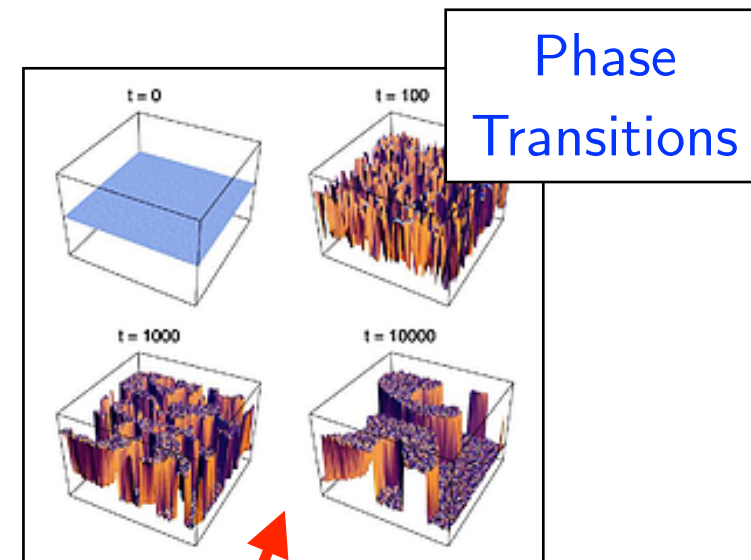
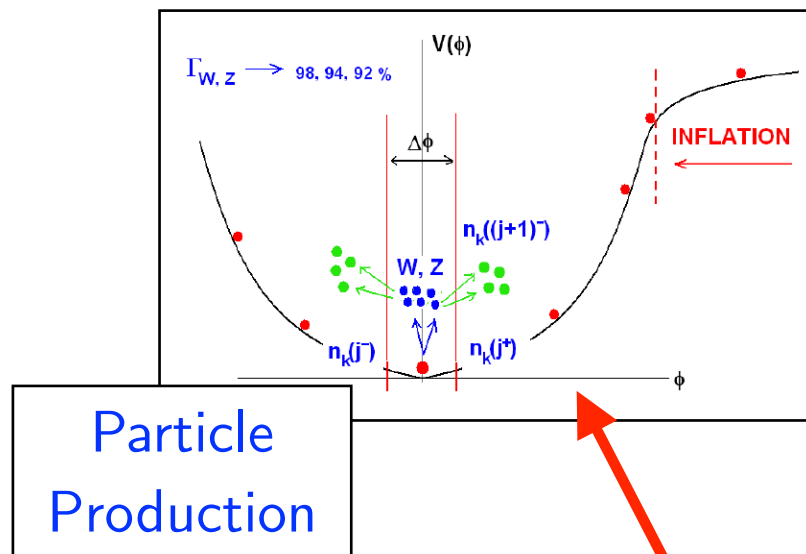
THERMAL EPOCH

Quantum
Fluctuations



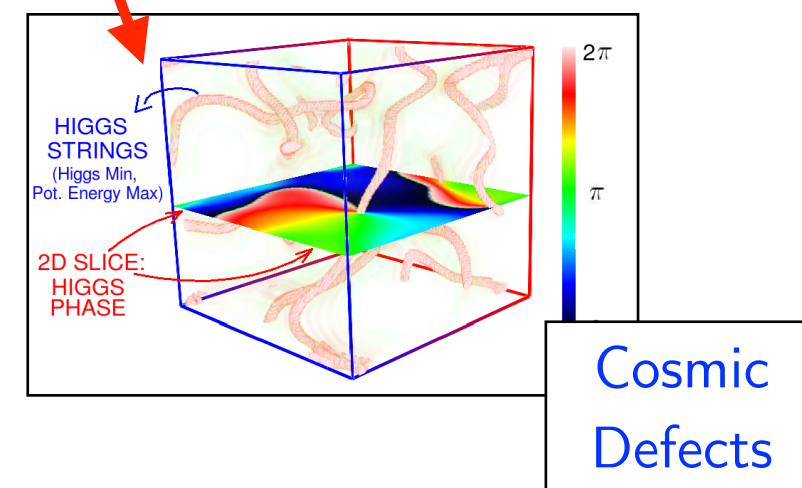
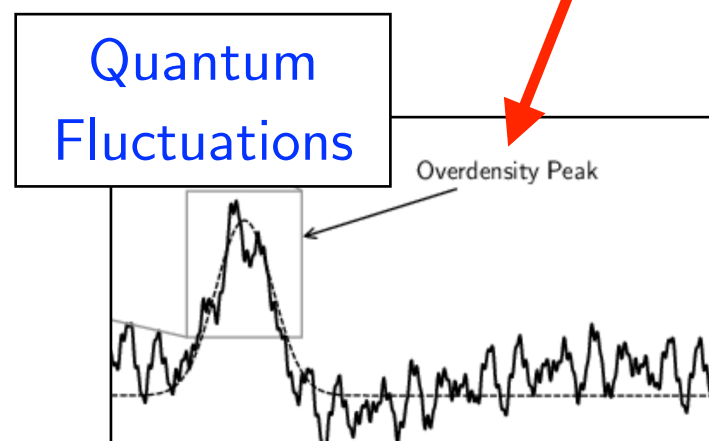
Cosmic
Defects

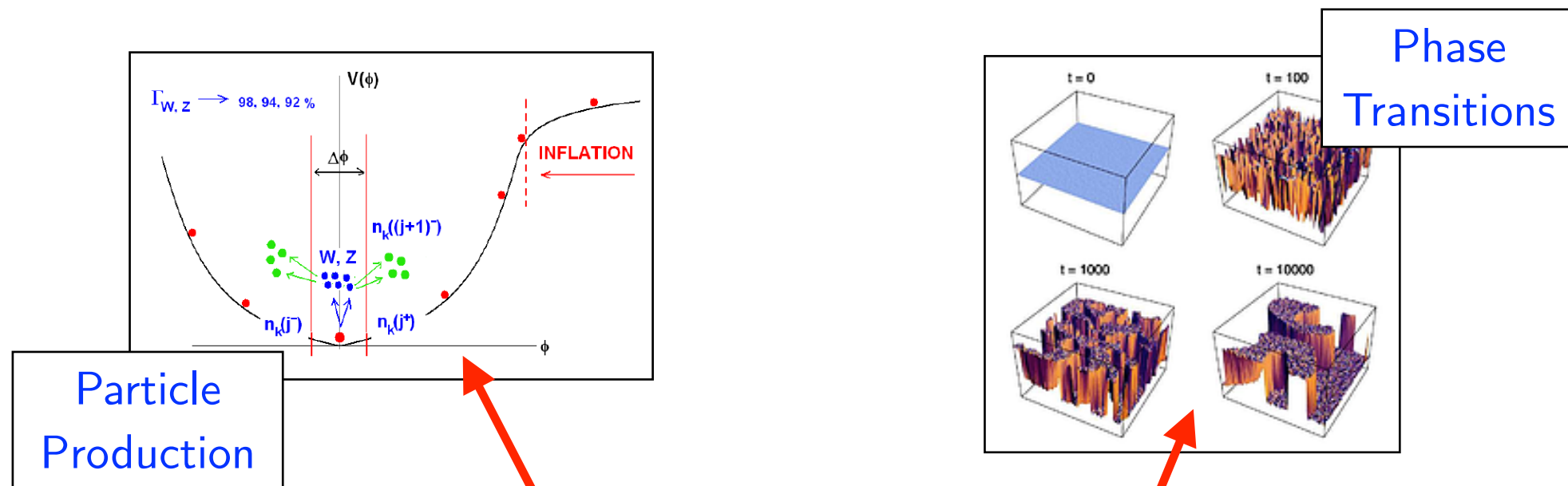




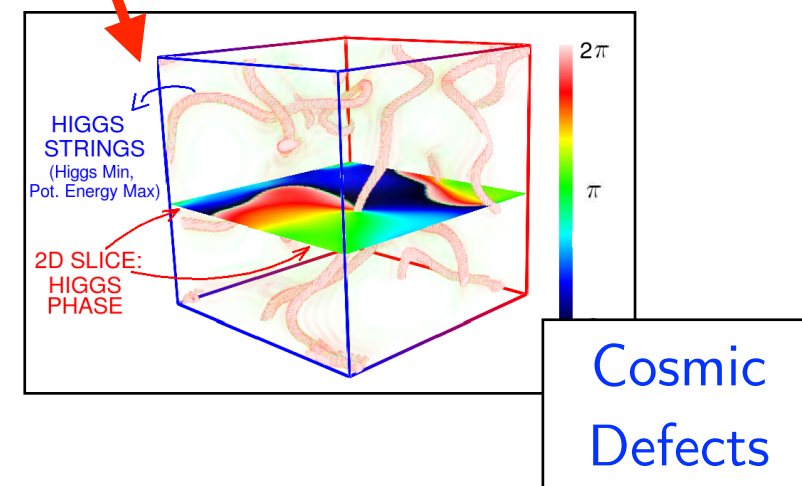
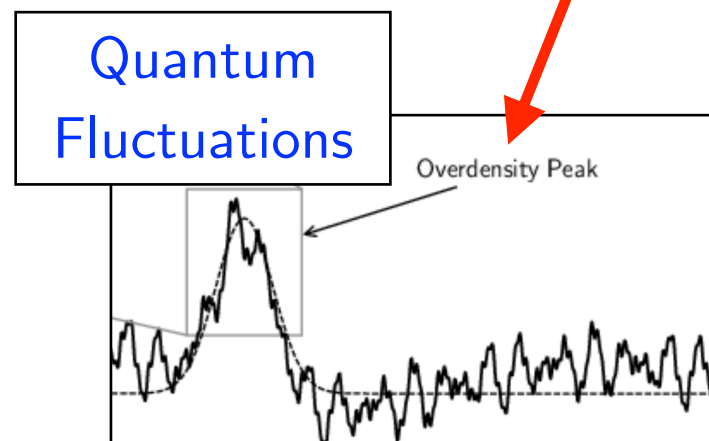
(CLASSICAL)

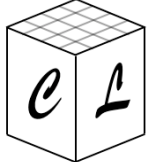
LATTICE SIMULATIONS



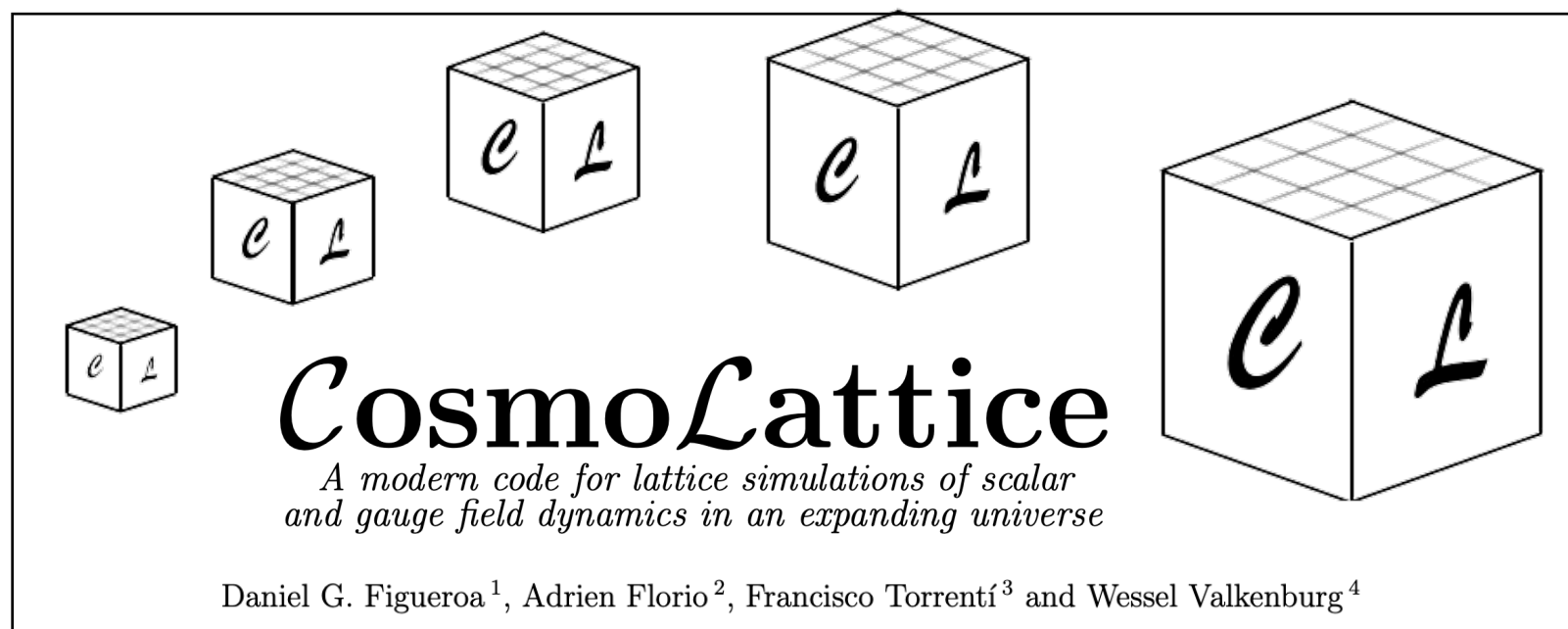


CosmoLattice





CosmoLattice: original release



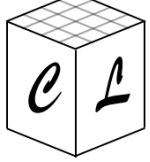
What is CosmoLattice?

CosmoLattice is a modern package for **lattice simulations of field dynamics in an expanding universe**. We have developed CosmoLattice to provide a new up-to-date, relevant numerical tool for the scientific community working in the **physics of the early universe**.

[arXiv: 2102.01031](https://arxiv.org/abs/2102.01031)

100-page-long user manual!

<http://www.cosmolattice.net>



CosmoLattice: versions

CosmoLattice - Version 1.2

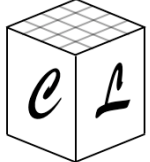
Latest version

- It simulates **scalars**, **U(1)** and **SU(2)** gauge fields.
- It simulates **gravitational waves** sourced from scalar and U(1) gauge fields.
- Written in **C++**, with a modular structure separating physics (**CosmoInterface**) and technical details (**TempLat**).
- Parallelized with **MPI**.
- Implements **symplectic evolution algorithms** with various degrees of accuracy.

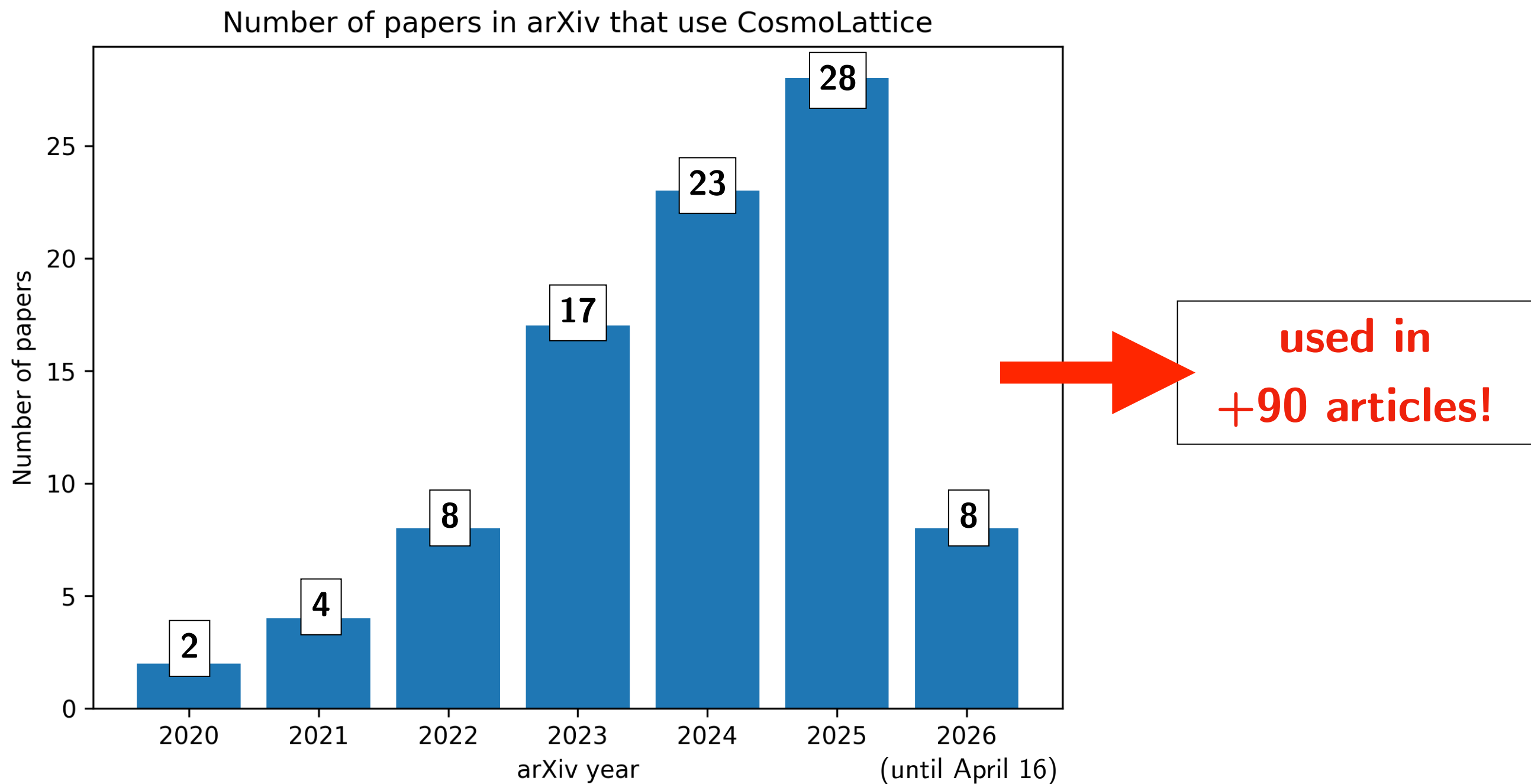
CosmoLattice - Version 2.0

Summer 2026!

- **New matter sectors** (axion-gauge interactions, no-minimal couplings to curvature, topological defects).
- **GPU implementation**. Improvements in efficiency and memory consumption (for CPU).
- **Other new features** (Runge-Kutta solvers, 2+1D and 1+1D, external power spectra, single precision support...)



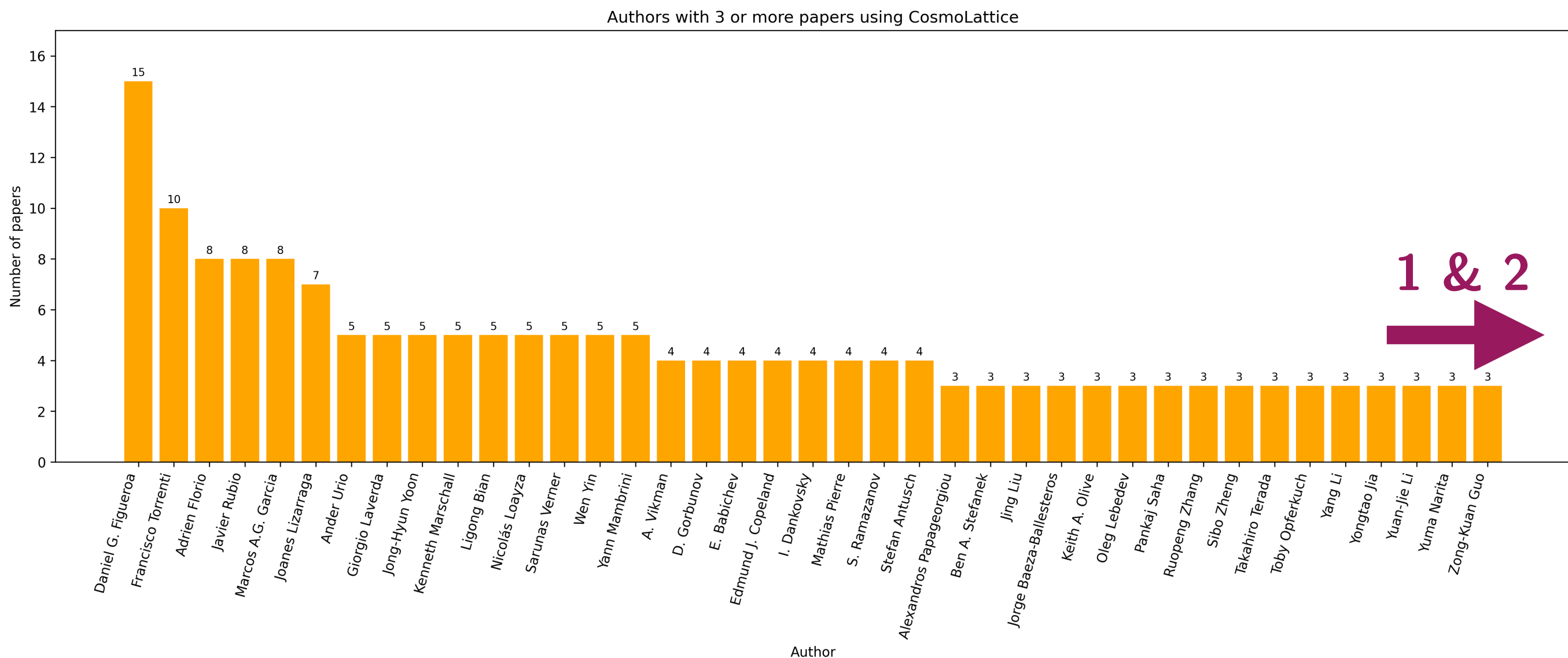
Total # of papers using CL



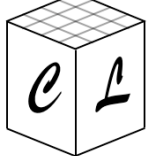
<http://www.cosmolattice.net/publications>



Papers using CL per author



+ 100 researchers have authored papers using CL



CosmoLattice developers



Jorge BAEZA-BALLESTEROS
(DESY, Zeuthen, Germany)



Daniel FIGUEROA
(IFIC, Valencia, Spain)



Adrien FLORIO
(Bielefeld Univ., Germany)



Nicolás LOAYZA
(CEICO-FZU, Prague, Czechia)



Kenneth MARSCHALL
(IFIC, Valencia, Spain)



Antonino S. MIDIRI
(UNIGE, Geneva, Switzerland)



Alberto ROPER POL
(UNIGE, Geneva, Switzerland)



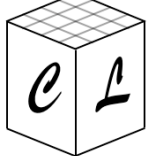
Franz SATTLER
(Bielefeld Univ., Germany)



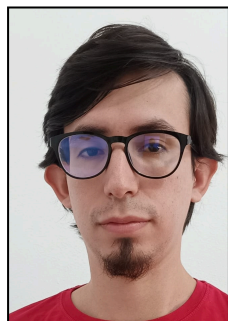
Francisco TORRENTÍ
(U. Carlos III, Madrid, Spain)



Ander URIO
(UPV/EHU, Bilbao, Spain)



Workshop lecturers



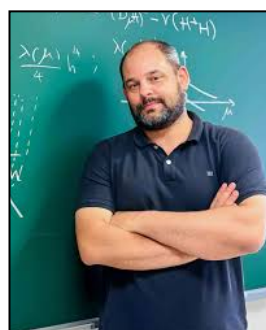
Nicolás LOAYZA
(CEICO-FZU, Prague, Czechia)

Inflation & reheating
Axion field dynamics & GWs



Kenneth MARSCHALL
(IFIC, Valencia, Spain)

Inflation & reheating
Magnetohydrodynamics



Javier RUBIO
(UCM, Madrid, Spain)

Inflation & reheating
Higgs cosmology

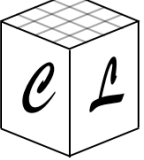


Francisco TORRENTÍ
(U. Carlos III, Madrid, Spain)

Inflation & reheating
Topological defects

CosmoLattice

in five minutes



CosmoLattice: matter content

- Matter content [version 1.2]:

$$S = - \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + (D_\mu^A \phi)^* (D^\mu_A \phi) + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \Phi)^\dagger (D^\mu \Phi) + \frac{1}{2} \text{Tr}\{G_{\mu\nu} G^{\mu\nu}\} + V(\phi, |\phi|, |\Phi|) \right\}$$

$$\phi \in \mathcal{R}e$$

Scalar sector

$$\begin{aligned} \varphi &\equiv \frac{1}{\sqrt{2}}(\varphi_0 + i\varphi_1) \\ D_\mu^A &\equiv \partial_\mu - iQ_A^{(\varphi)} g_A A_\mu \\ F_{\mu\nu} &\equiv \partial_\mu A_\nu - \partial_\nu A_\mu \end{aligned}$$

U(1) gauge sector

$$\begin{aligned} \Phi &= \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_0 + i\varphi_1 \\ \varphi_2 + i\varphi_3 \end{pmatrix} \\ D_\mu &\equiv \mathcal{J} D_\mu^A - ig_B Q_B B_\mu^a T_a \\ G_{\mu\nu} &\equiv \partial_\mu B_\nu - \partial_\nu B_\mu - i[B_\mu, B_\nu] \end{aligned}$$

SU(2) gauge sector

Potential
(Interactions)

- Metric:

$$ds^2 = -a^{2\alpha}(\eta) d\eta^2 + a^2(\eta) \delta_{ij} dx^i dx^j$$

- Self-consistent expansion (**Friedmann equations**)
- Fixed power-law background $a(t) \sim t^{\frac{2}{3(1+w)}}$

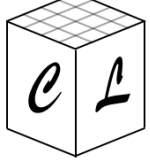
- Gravitational waves:

$$ds^2 = -a^{2\alpha}(\eta) d\eta^2 + a^2(\eta) (\delta_{ij} + h_{ij}) dx^i dx^j$$



$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\nabla^2}{a^2} h_{ij} = \frac{2}{m_p^2 a^2} \Pi_{ij}^{\text{TT}}$$

$$\Pi_{ij}^{\text{TT}} = (\partial_i \phi \partial_j \phi)^{\text{TT}} + \dots$$



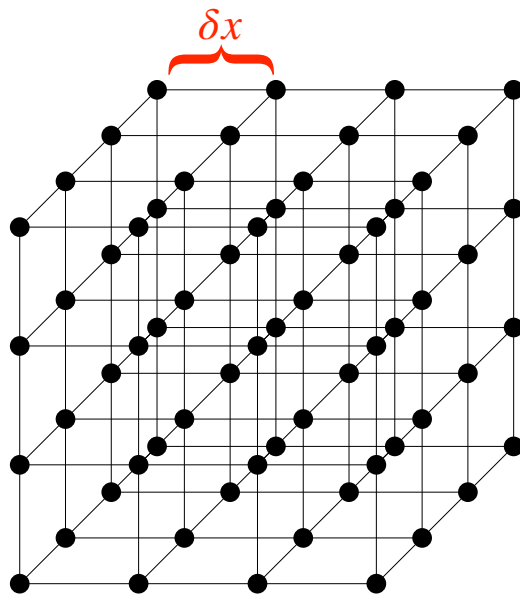
CosmoLattice: lattice formulation

- Equations are written as a set of **coupled first-order differential equations**, which are solved with a **Hamiltonian scheme**:

Example: scalar field

$$\phi'' - \frac{1}{a^2} \nabla^2 \phi + \frac{3}{a} \frac{da}{dt} \frac{d\phi}{dt} = - \frac{\partial V}{\partial \phi} \xrightarrow{\pi_\phi \equiv \phi' a^{3-\alpha}} \begin{array}{ll} \text{KICK:} & (\pi_\phi)' = - a^{3+\alpha} \frac{\partial V}{\partial \phi} + a^{1+\alpha} \nabla^2 \phi \\ \text{DRIFT:} & \phi' \equiv \pi_\phi a^{\alpha-3} \end{array}$$

- Scalar fields and momenta are defined in the **lattice sites**:



N : number of points/dimension

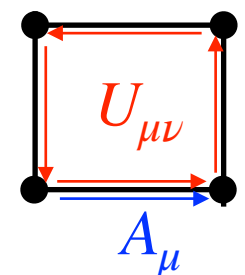
$L = N \cdot \delta x$: length side

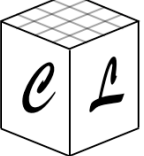
δt : time step

Minimum and maximum momenta:

$$k_{\min} = \frac{2\pi}{L} \quad k_{\max} = \frac{\sqrt{3}}{2} N k_{\min}$$

- Gauge fields** introduced via **links** and **plaquettes** (like in **lattice-QCD**)





CosmoLattice: lattice formulation

- Algorithms use second Friedmann equation to evolve the scale factor.

$$\frac{a''}{a} = \frac{a^{2\alpha}}{3m_p^2} \langle (\alpha - 2)(K_\phi + K_\varphi + K_\Phi) + \alpha(G_\phi + G_\varphi + G_\Phi) + (\alpha + 1)V + (\alpha - 1)(K_{U(1)} + G_{U(1)} + K_{SU(2)} + G_{SU(2)}) \rangle$$

- The first Friedmann equation is used to check the accuracy of the simulation.

$$\left(\frac{a'}{a}\right)^2 = \frac{a^{2\alpha}}{3m_p^2} \langle K_\phi + K_\varphi + K_\Phi + G_\phi + G_\varphi + G_\Phi + K_{U(1)} + G_{U(1)} + K_{SU(2)} + G_{SU(2)} + V \rangle$$

$\langle \dots \rangle$ represents volume averaging

$$K_\phi = \frac{1}{2a^{2\alpha}} \phi'^2$$

$$K_\varphi = \frac{1}{a^{2\alpha}} (D_0^A \varphi)^* (D_0^A \varphi) ;$$

$$K_\Phi = \frac{1}{a^{2\alpha}} (D_0 \Phi)^\dagger (D_0 \Phi)$$

(Kinetic-Scalar)

$$G_\phi = \frac{1}{2a^2} \sum_i (\partial_i \phi)^2$$

$$G_\varphi = \frac{1}{a^2} \sum_i (D_i^A \varphi)^* (D_i^A \varphi) ;$$

$$G_\Phi = \frac{1}{a^2} \sum_i (D_i \Phi)^\dagger (D_i \Phi)$$

(Gradient-Scalar)

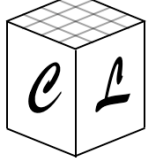
$$K_{U(1)} = \frac{1}{2a^{2+2\alpha}} \sum_i F_{0i}^2$$

$$K_{SU(2)} = \frac{1}{2a^{2+2\alpha}} \sum_{a,i} (G_{0i}^a)^2$$

$$G_{U(1)} = \frac{1}{2a^4} \sum_{i,j < i} F_{ij}^2$$

$$G_{SU(2)} = \frac{1}{2a^4} \sum_{a,i,j < i} (G_{ij}^a)^2$$

(Electric & Magnetic)



CosmoLattice: simulating a model

- Equations solved in (dimensionless) **program variables**:

Choose:
 $\{\alpha, \omega_*, f_*\}$



$$\begin{aligned} d\tilde{\eta} &\equiv a^{-\alpha} \omega_* dt \\ d\tilde{x}^i &\equiv \omega_* dx^i \end{aligned}$$

Space and time

$$\tilde{\phi} = \frac{\phi}{f_*} \quad \tilde{\varphi} = \frac{\varphi}{f_*} \quad \tilde{\Phi} = \frac{\Phi}{f_*}$$

$$\widetilde{A}_\mu = \frac{A_\mu}{\omega_*} \quad \widetilde{B}_\mu^a = \frac{B_\mu^a}{\omega_*}$$

- Write **scalar potential** and first and second derivatives in **one file** (*model_name.h*)

$$\tilde{V}(\tilde{\phi}, |\tilde{\varphi}|, |\widetilde{\Phi}|) \equiv \frac{1}{f_*^2 \omega_*^2} V(f_* \tilde{\phi}, f_* |\tilde{\varphi}|, f_* |\widetilde{\Phi}|)$$



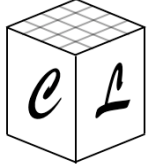
$$\frac{\partial \tilde{V}}{\partial \tilde{\phi}}, \quad \frac{\partial \tilde{V}}{\partial |\tilde{\varphi}|}, \quad \frac{\partial \tilde{V}}{\partial |\widetilde{\Phi}|}, \quad \frac{\partial^2 \tilde{V}}{\partial \tilde{\phi}^2}, \quad \frac{\partial^2 \tilde{V}}{\partial |\tilde{\varphi}|^2},$$

- Parameters passed via **one file** (input.txt)
(no need to re-compile!)



```

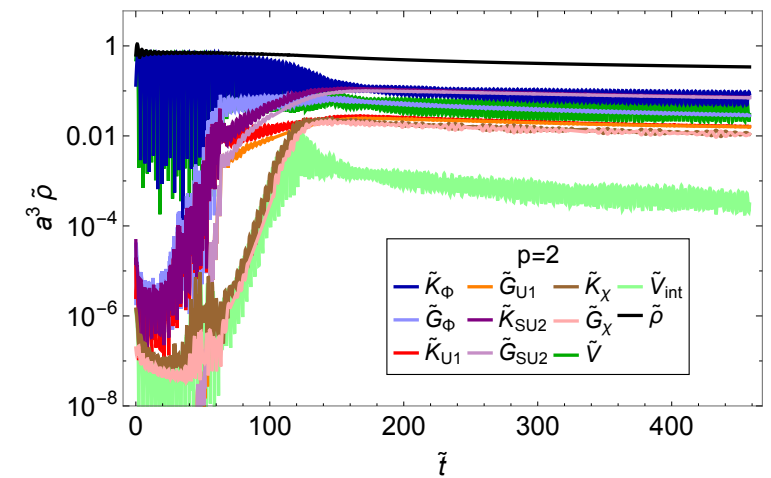
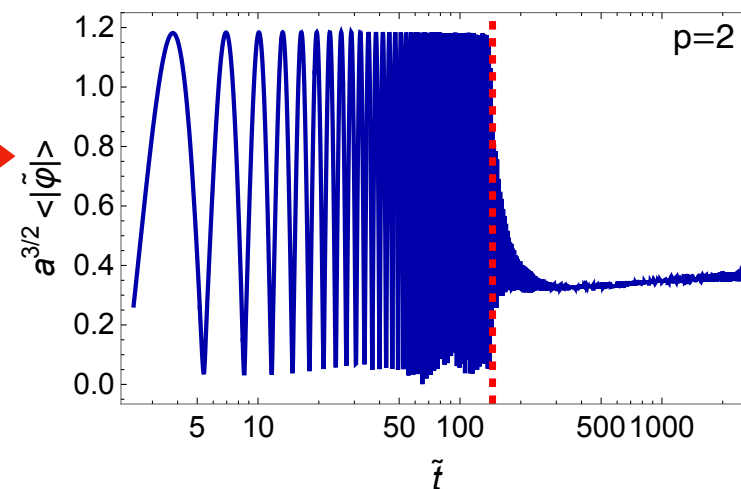
1 #Output
2 outputfile = ./
3
4 #Evolution
5 expansion = true
6 evolver = VV2
7
8 #Lattice
9 N = 32
10 dt = 0.01
11 kIR = 0.75
12 nBinsSpectra = 55
13
14 #Times
15 tOutputFreq = 0.1
16 tOutputInfreq = 1
17 tMax = 300
18
19 #IC
20 kCutOff = 1.75
21 initial_amplitudes = 7.42675e18 0 # homogeneous amplitudes in GeV
22 initial_momenta = -6.2969e30 0 # homogeneous amplitudes in GeV2
23
24 #Model Parameters
25 lambda = 9e-14
26 q = 100
    
```

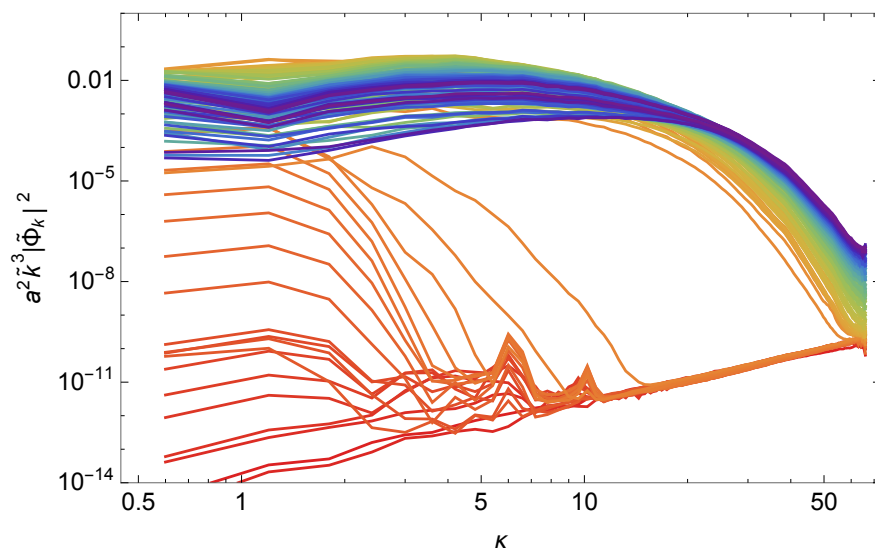
CosmoLattice: Output

Three
kinds of
output

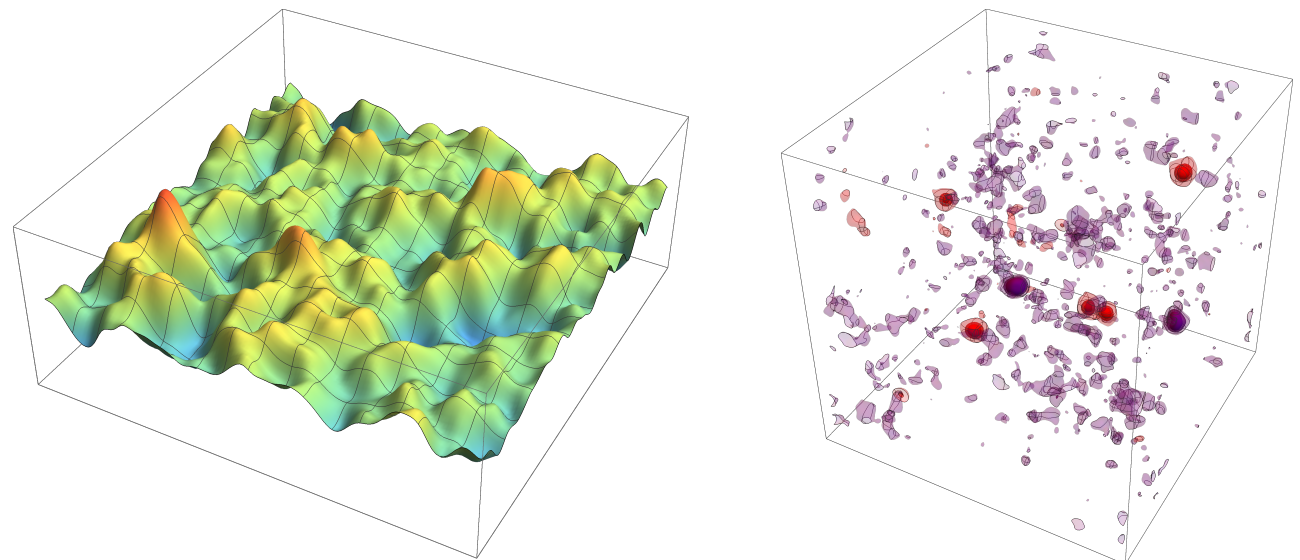
Volume averages: Spatial averages of certain quantities, such as field amplitudes or energies

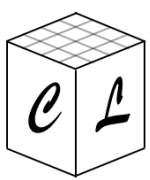


Spectra: Binned spectra
in momentum space



Snapshots: Values of a certain
quantity at all points of the lattice.





“Lattice cosmology” reviews

The Art of Simulating the Early Universe

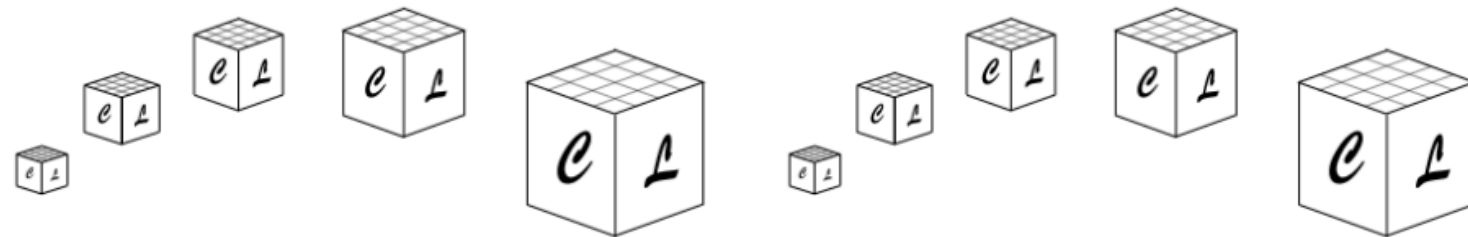
A dissertation on lattice techniques for the simulation of scalar and gauge field dynamics in an expanding Universe

Part I: Integration Techniques and Canonical Cases

Daniel G. Figueroa¹, Adrien Florio², Francisco Torrenti³ and Wessel Valkenburg⁴

- Interacting scalar fields
- Gauge fields: U(1) interactions
- Gauge fields: SU(N) interactions

arXiv: 2006.15122



The art of simulating the early Universe

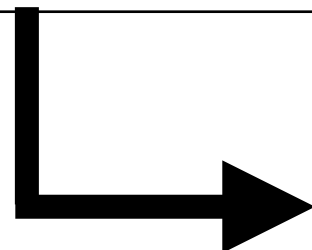
(A dissertation series about lattice techniques for simulating the dynamics of interacting fields in an expanding Universe)

Part II. Non-Canonical Cases & Gravitational Waves

Jorge Baeza-Ballesteros¹, Daniel G. Figueroa², Adrien Florio³, Joanes Lizarraga⁴,
Nicolás Loayza⁸, Kenneth Marschall², Toby Opferkuch^{5,6}, Ben A. Stefanek²,
Francisco Torrentí⁷, Ander Urio⁴

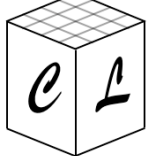
- Scalars with couplings ξR^2
- Scalar with non-minimal kinetic terms.
- Axion-U(1) interactions
- Cosmic defects
- Different initial conditions
- Scalar dynamics in d+1 dimensions
- Gravitational waves

arXiv: 2512.15627



THEORETICAL BASIS FOR COSMOLATTICE 2.0 (2026)

Baeza-Ballesteros, Figueroa, Florio, Loayza, Sattler, F.T. & Urio



CosmoLattice schools

5-8 September 2022



IFIC, Valencia, Spain

25-29 September 2023



Online

25-29 September 2025

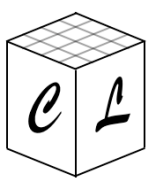


IBS, Daejeon, Korea



Recorded lectures:

<https://www.youtube.com/@CosmoLattice>



CosmoLattice schools

5-8 September 2022



IFIC, Valencia, Spain

25-29 September 2023



Online

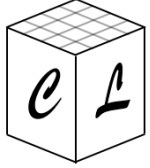
25-29 September 2025



IBS, Daejeon, Korea

4th school in 2027!

(probably, somewhere in the world)



Workshop schedule

Day 1 (Thursday)

Lecture 1: Inflation and post-inflationary dynamics [JAVIER]

Coffee break

Lecture 2: Introduction to lattice field theory [KEN]

Lunch break

Practice 1: My first run in CosmoLattice

Coffee break

Practice 2: Let's play with scalar fields!

Day 2 (Friday)

Lecture 3: Abelian and non-Abelian gauge fields [PACO]

Coffee break

Practice 3: Running gauge preheating

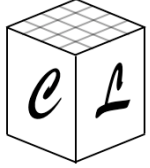
Lunch break

Lecture 4: Gravitational wave emission from scalar and gauge fields [NICO]

Coffee break

Practice 4: Simulating GW emission from scalar and gauge fields

New features on CosmoLattice v2.0 [PACO]



Workshop schedule

Day 1

(Thursday)

Lecture 1: Inflation and post-inflationary dynamics [JAVIER]

Coffee break

Lecture 2: Introduction to lattice field theory [KEN]

Lunch break

Practice 1: My first run in CosmoLatti

Coffee break

Practice 2: Let's play with scalar fields!

Let's start!

Day 2

(Friday)

Lecture 3: Abelian and non-Abelian gauge fields [PACO]

Coffee break

Practice 3: Running gauge preheating

Lunch break

Lecture 4: Gravitational wave emission from scalar and gauge fields [NICO]

Coffee break

Practice 4: Simulating GW emission from scalar and gauge fields

New features on CosmoLattice v2.0 [PACO]

Investigación cofinanciada por la Comunidad de Madrid en el marco de la convocatoria correspondiente al año 2025 de las ayudas a la atracción de talento investigador "César Nombela" (2025-T1/COM-36104).