In[ ]:=
```
SetOptions[$FrontEnd, "ClearEvaluationQueueOnKernelQuit" → False]; Quit[];
```

In[ ]:=
```
$LoadAddOns = {"FeynArts"};
Get["FeynCalc`"]
$FAVerbose = 0;
SetDirectory[NotebookDirectory[]];
FAPatch[PatchModelsOnly → True,
  FAModelsDirectory → FileNameJoin[{NotebookDirectory[], "SM"}]];
$KeepLogDivergentScalelessIntegrals = True;
Model["SM.fr"];
```

FeynCalc 10.2.0 (stable version). For help, use the online documentation, visit the forum and have a look at the supplied examples. The PDF–version of the manual can be downloaded here.

If you use FeynCalc in your research, please evaluate FeynCalcHowToCite[] to learn how to cite this software.

Please keep in mind that the proper academic attribution of our work is crucial to ensure the future development of this package!

FeynArts 3.12 (27 Mar 2025) patched for use with FeynCalc, for documentation see the manual or visit www.feynarts.de.

If you use FeynArts in your research, please cite

- T. Hahn, Comput. Phys. Commun., 140, 418–431, 2001, arXiv:hep–ph/0012260

```
Successfully patched FeynArts.
```

In[ ]:=
```
FCEnableTraditionalFormOutput[];
```

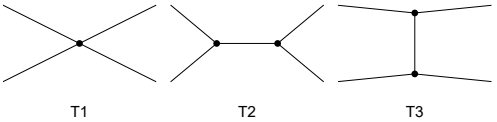
FeynArts

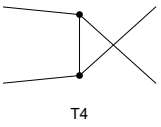
Topologies

In[ ]:=
```
top[n_, i_, j_] := CreateTopologies[n, i → j,
   ExcludeTopologies → {Tadpoles, WFCorrections}, Adjacencies → {3, 4, 5, 6}];
topCT[n_, i_, j_] := CreateCTTopologies[n, i → j,
   ExcludeTopologies → {TadpoleCTs, WFCorrectionCTs}, Adjacencies → {3, 4, 5, 6}];
```

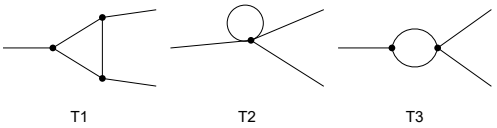
In[ ]:=
```
Paint[top[0, 2, 2], Sequence @@ {SheetHeader → "2->2 Tree Level"}];
Paint[top[1, 1, 2], Sequence @@ {SheetHeader → "1->2 One Loop"}];
Paint[topCT[1, 1, 2], Sequence @@ {SheetHeader → "1->2 One Loop CounterTerms"}];
```

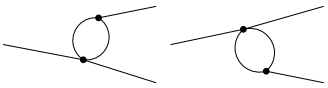
2 → 2 Tree Level



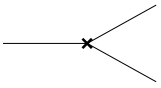


1 → 2 One Loop





1 → 2 One Loop CounterTerms



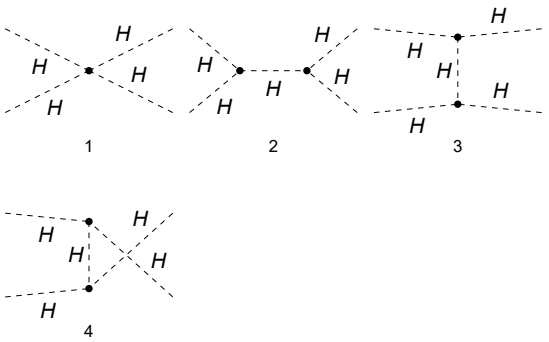
Diagrams

Tree Level

In[ ]:=
```
FAPatch[PatchModelsOnly → True,
  FAModelsDirectory → FileNameJoin[{NotebookDirectory[], "SM"}]];
```

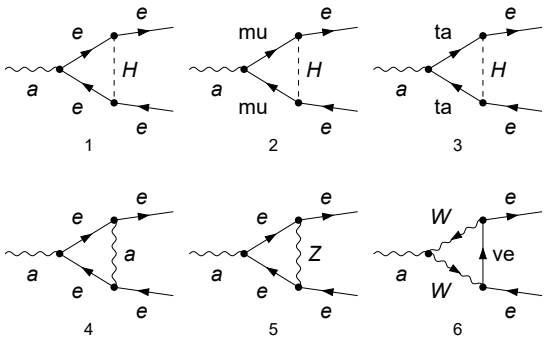
```
Successfully patched FeynArts.
```

In[ ]:=
```
hhhhdiag = InsertFields[top[0, 2, 2], {S[1], S[1]} → {S[1], S[1]},
    Model → FileNameJoin[{NotebookDirectory[], "SM/SM"}], GenericModel →
     FileNameJoin[{NotebookDirectory[], "SM/SM"}], InsertionLevel → {Particles}];
Paint[hhhhdiag, ColumnsXRows → {3, 3}, SheetHeader → None, Numbering → Simple];
```



Loop Level

In[ ]:=
```
photoneediag = InsertFields[top[1, 1, 2], {V[1]} → {F[2, {1}], -F[2, {1}]},
    Model → FileNameJoin[{NotebookDirectory[], "SM/SM"}], GenericModel →
     FileNameJoin[{NotebookDirectory[], "SM/SM"}], InsertionLevel → {Particles}];
photoneediagClasses = InsertFields[top[1, 1, 2], {V[1]} → {F[2, {1}], -F[2, {1}]},
    Model → FileNameJoin[{NotebookDirectory[], "SM/SM"}], GenericModel →
     FileNameJoin[{NotebookDirectory[], "SM/SM"}], InsertionLevel → {Classes}];
photoneediagGeneric = InsertFields[top[1, 1, 2], {V[1]} → {F[2, {1}], -F[2, {1}]},
    Model → FileNameJoin[{NotebookDirectory[], "SM/SM"}], GenericModel →
     FileNameJoin[{NotebookDirectory[], "SM/SM"}], InsertionLevel → {Generic}];
Paint[photoneediag, ColumnsXRows → {3, 3}, SheetHeader → None, Numbering → Simple];
Paint[photoneediagClasses,
  ColumnsXRows → {3, 3}, SheetHeader → None, Numbering → Simple];
Paint[photoneediagGeneric,
  ColumnsXRows → {3, 3}, SheetHeader → None, Numbering → Simple];
```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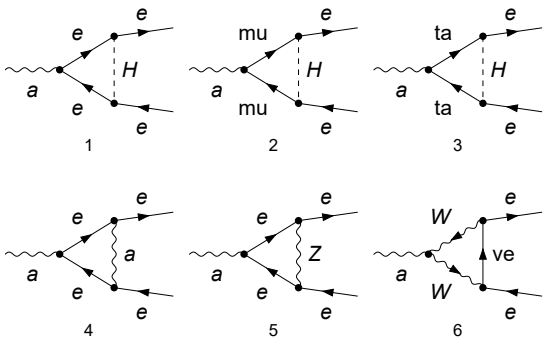
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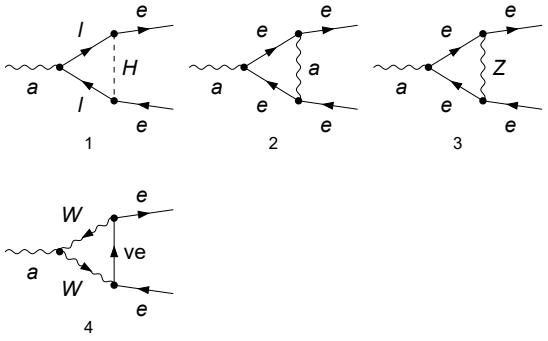
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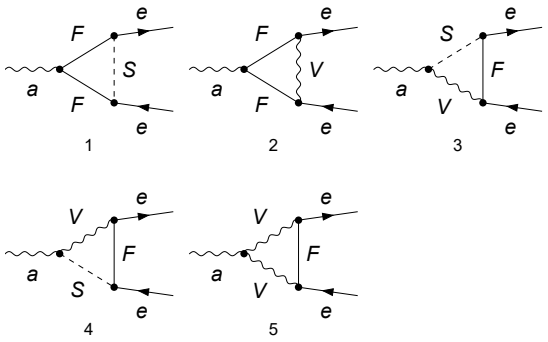
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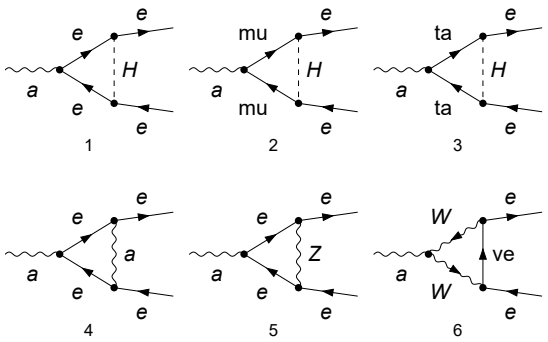
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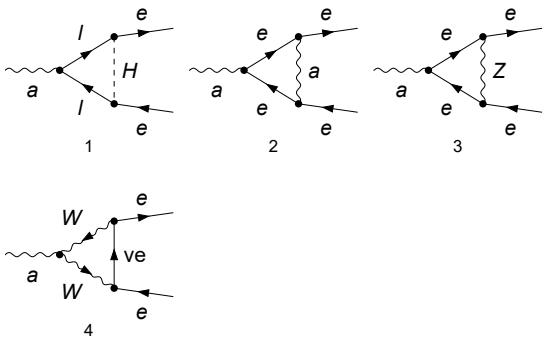
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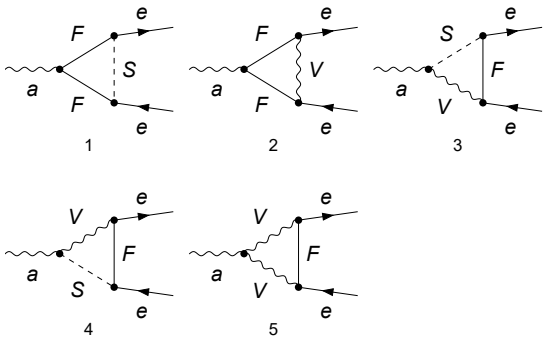
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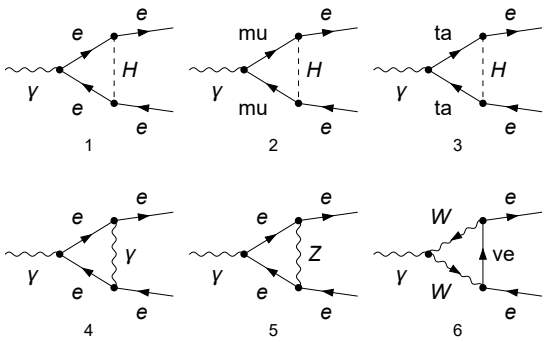
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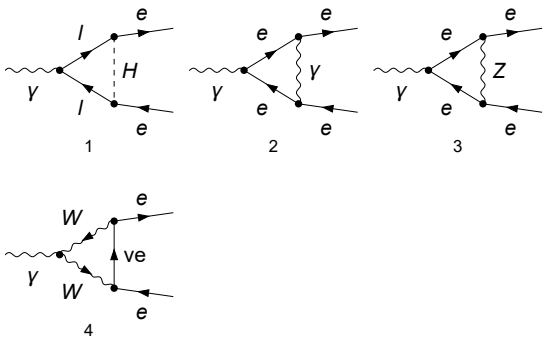
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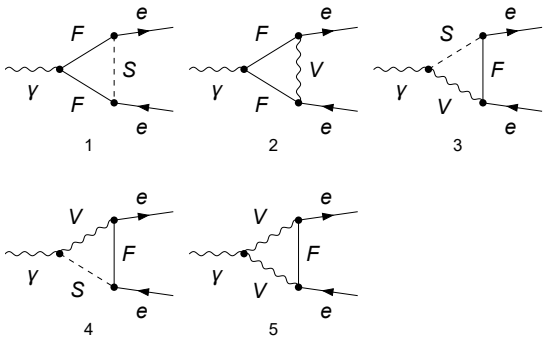
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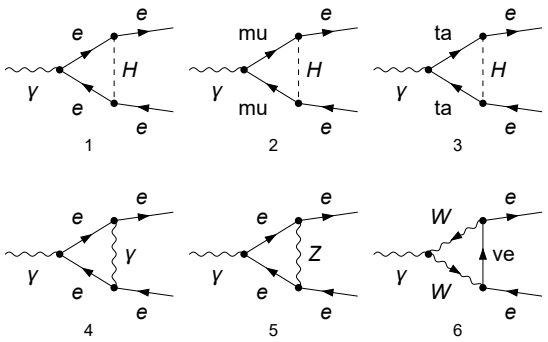
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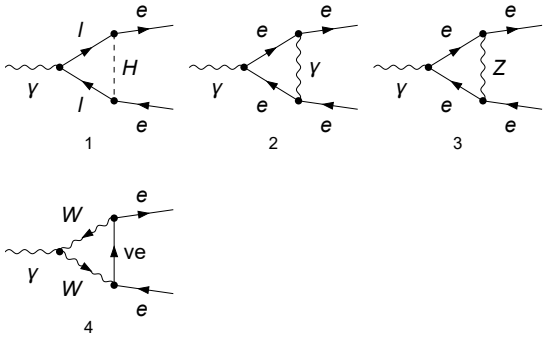
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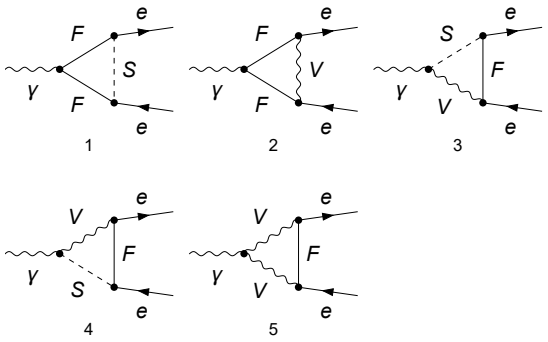
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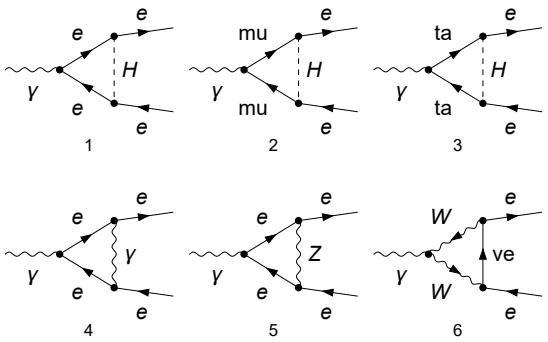
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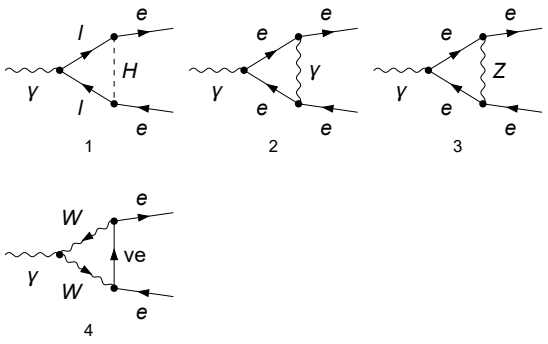
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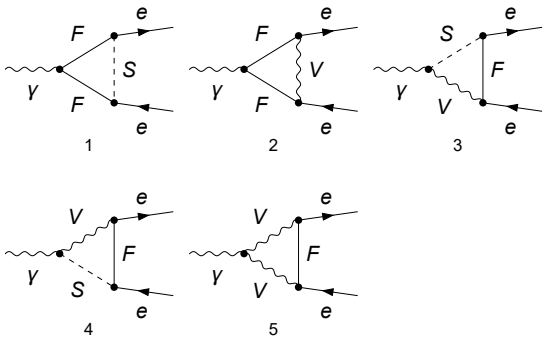
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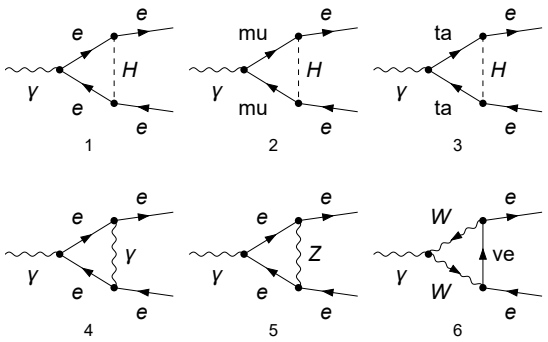
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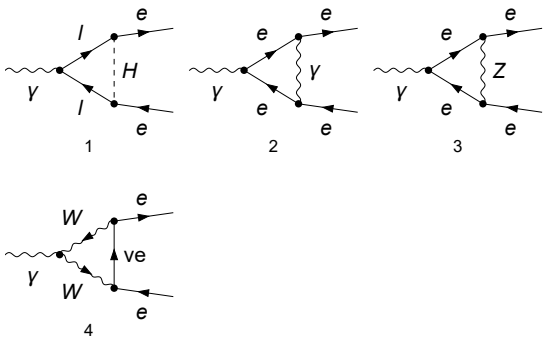
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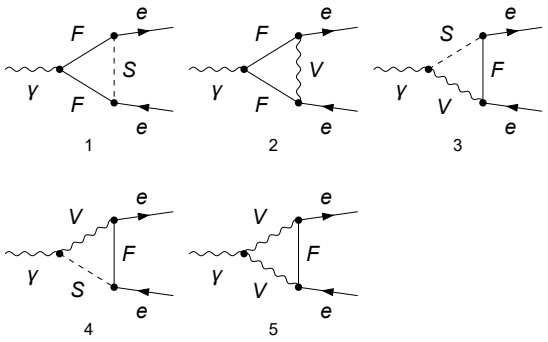
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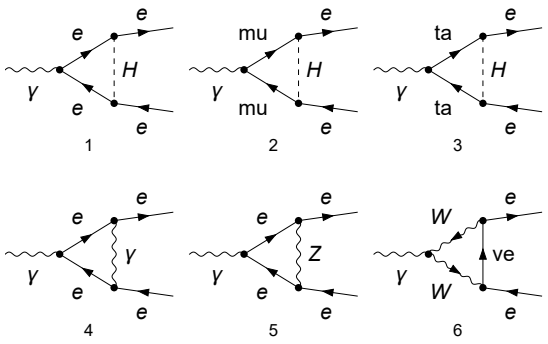
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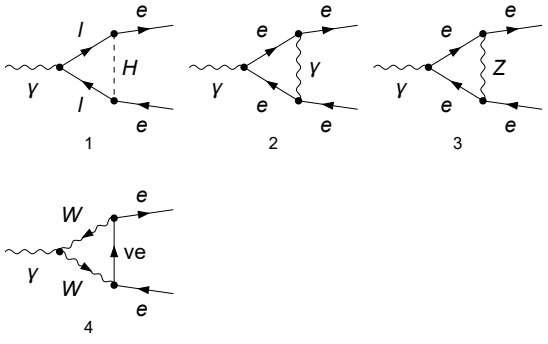
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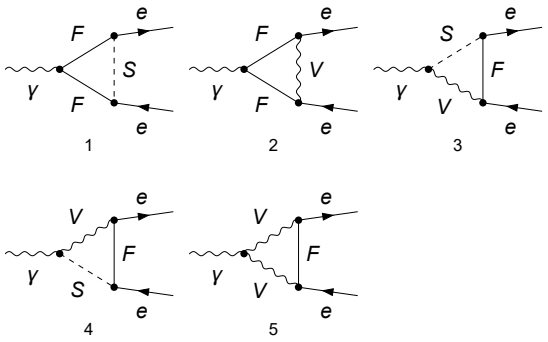
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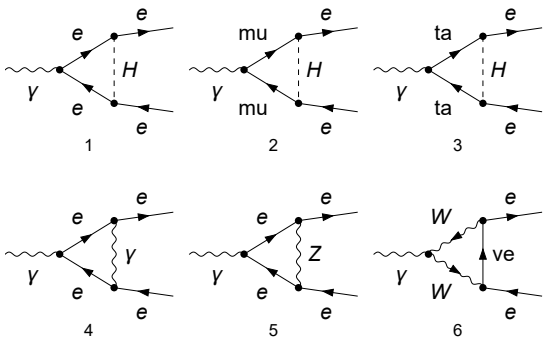
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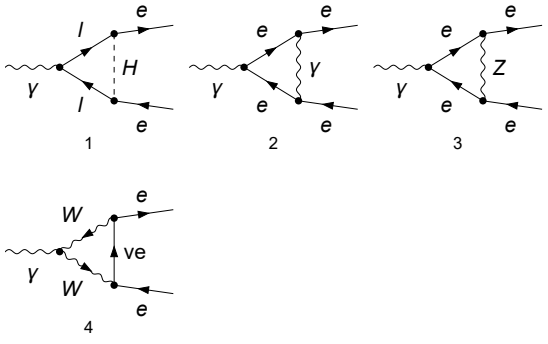
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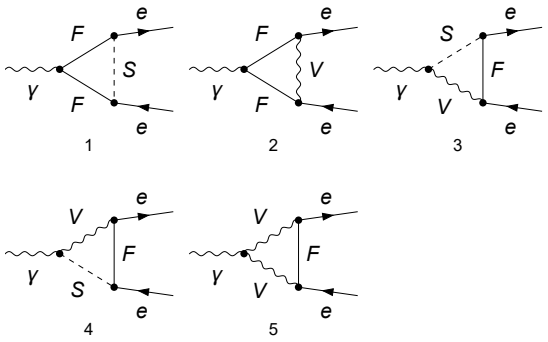
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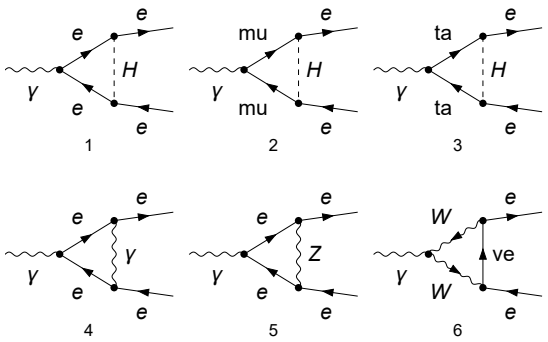
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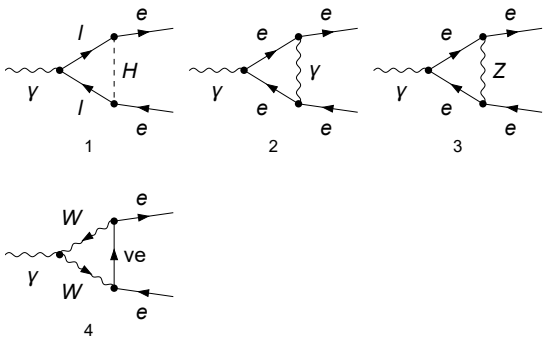
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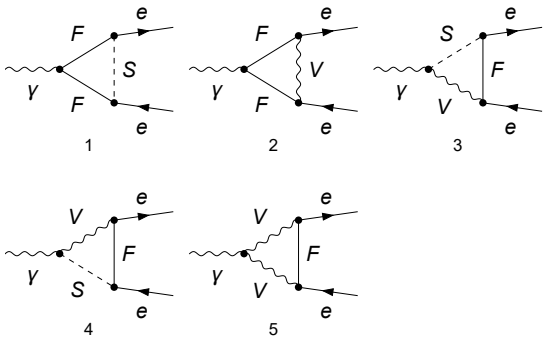
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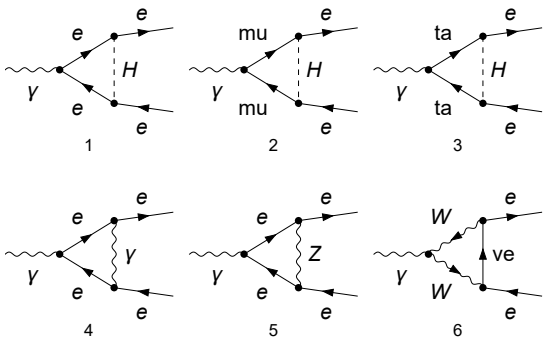
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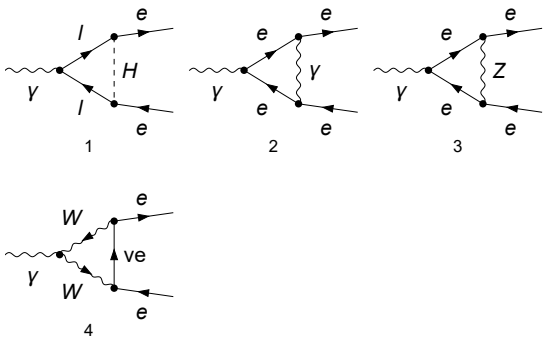
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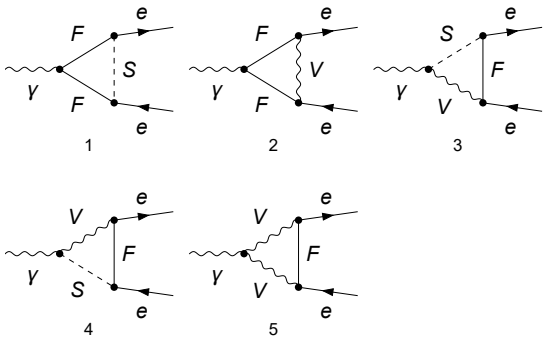
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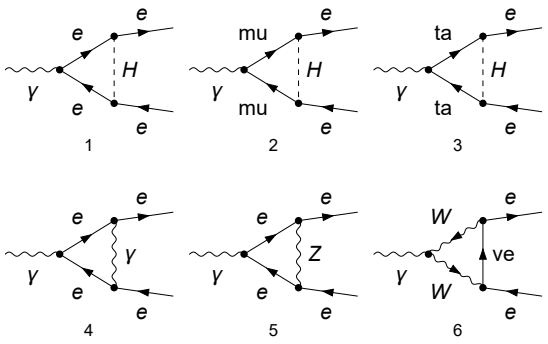
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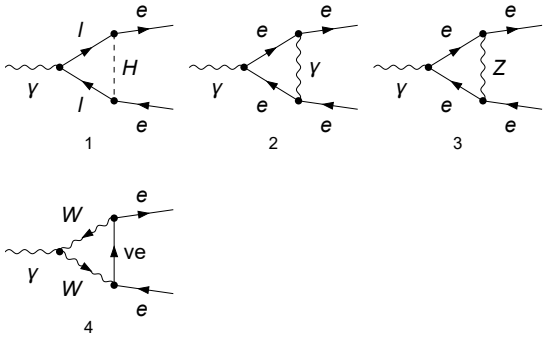
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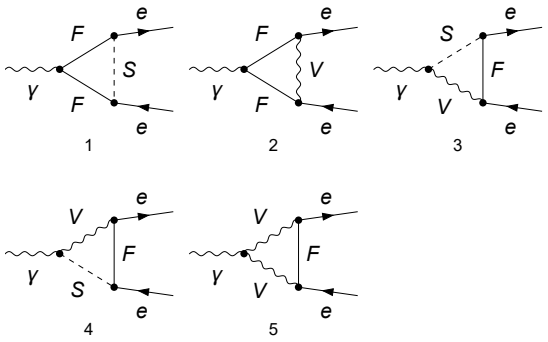
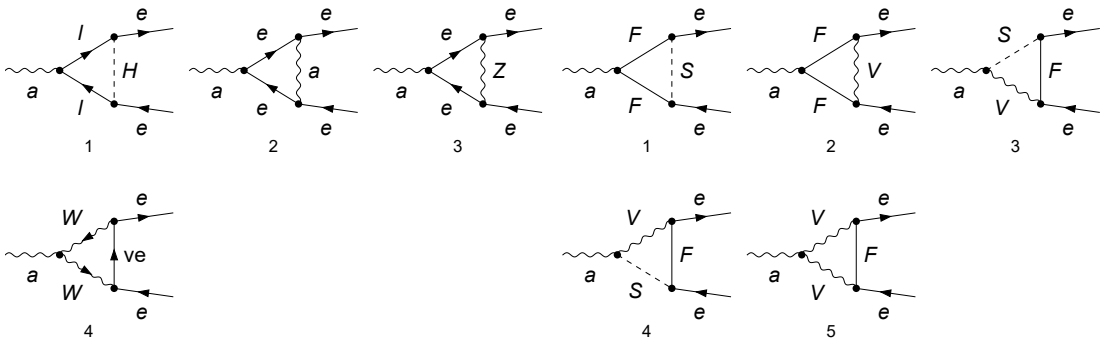




FeynCalc

Objects definitions

Four vectors

In[ ]:=
```
(* Use \[Name] to enter the greek letter "name" or Esc name Esc
 to enter greek letter "name" (with the aproppiate capital letter)*)
```

In[ ]:=
```
FV[p, μ]
```
Out[ ]=

$$\overline{p}^{\mu}$$

Scalar Product

In[ ]:=
```
SP[p, q]
```
Out[ ]=

$$\overline{p}\cdot\overline{q}$$

Metric Tensor

In[ ]:=
```
MT[μ, ν]
```
Out[ ]=

$$\overline{g}^{\mu\,\nu}$$

Dirac Matrix and Dirac slash

In[ ]:=
```
GA[μ]
```
Out[ ]=

$$\overline{\gamma}^{\mu}$$

In[ ]:=
```
GS[p]
```
Out[ ]=

$$\overline{\gamma}\cdot\overline{p}$$

Levi-Civita

In[ ]:=
```
LC[μ, ν, ρ, σ]
```
Out[ ]=

$$\overline{\epsilon}^{\mu\,\nu\,\rho\,\sigma}$$

FeynCalc has a different representation for some of them. This is relevant when one is trying to replace objects inside expressions, because sometimes one will need to use the internal representation. FCI converts the object to its internal representation. StandardForm writes the output as if it was an input.

In[ ]:=
```
ex = SP[p, q]
```
Out[ ]=

$$\overline{p}\cdot\overline{q}$$

In[ ]:=
```
ex // StandardForm
```
Out[ ]//StandardForm=
```
SP[p, q]
```

In[ ]:=
```
exI = FCI[ex]
```
Out[ ]=

$$\overline{p}\cdot\overline{q}$$

In[ ]:=
```
exI // StandardForm
```
Out[ ]//StandardForm=
```
Pair[Momentum[p], Momentum[q]]
```

The replacement will not work if it is not written in the precise form

In[ ]:=
```
exI /. SP[p, q] → 1
```
Out[ ]=

$$\overline{p}\cdot\overline{q}$$

In[ ]:=
```
exI /. FCI[SP[p, q]] → 1
```
Out[ ]=

$$1$$

We can also define D dimension objects

In[ ]:=
```
FVD[p, μ]
```
Out[ ]=

$$p^{\mu}$$

In[ ]:=
```
MTD[μ, ν]
```
Out[ ]=

$$g^{\mu\nu}$$

and D-4 objects

In[ ]:=
```
FVE[p, μ]
```
Out[ ]=

$$\hat{p}^{\mu}$$

In[ ]:=
```
MTE[μ, ν]
```
Out[ ]=

$$\hat{g}^{\mu\nu}$$

More on this on the FeynCalc tutorial

Contractions

We use Contract to make contractions of Lorentz indices.

In[ ]:=
```
FV[p, μ] * MT[μ, ν]
Contract[%]
```
Out[ ]=

$$\overline{p}^{\mu}\,\overline{g}^{\mu\nu}$$

Out[ ]=

$$\overline{p}^{\nu}$$

When we enter noncommutative objects, such as Dirac matrices, we use Dot (.) not Times (*)

In[ ]:=
```
FV[p, α] * MT[β, γ] * GA[α].GA[β].GA[γ]
Contract[%]
```
Out[ ]=

$$\overline{p}^{\alpha}\,\overline{\gamma}^{\alpha}.\overline{\gamma}^{\beta}.\overline{\gamma}^{\gamma}\,\overline{g}^{\beta\gamma}$$

Out[ ]=

$$(\overline{\gamma}\cdot\overline{p}).\overline{\gamma}^{\gamma}.\overline{\gamma}^{\gamma}$$

Note that FeynCalc assumes all expressions with Lorentz indices to be Lorentz covariant and respect Einstein's summation. Thus, we will not find lower indices. FeynCalc will not check if your expressions are mathematically sensible.

The products of Levi-Civita will by default apply the product formula with the determinant of metric

tensors when using Contract. To avoid this use the option EpsContract

In[ ]:=
```
LC[μ, ν][p, q] × LC[ρ, σ][r, s] × FV[x, μ]
Contract[%]
```

Out[ ]=

$$\bar{x}^{\mu}\,\bar{\epsilon}^{\mu\,\nu\,\bar{p}\,\bar{q}}\,\bar{\epsilon}^{\rho\,\sigma\,\bar{r}\,\bar{s}}$$

Out[ ]=

$$\begin{aligned}&-\bar{x}^{\rho}\,\bar{g}^{\nu\,\sigma}\,(\bar{p}\cdot\bar{r})\,(\bar{q}\cdot\bar{s})+\bar{x}^{\sigma}\,\bar{g}^{\nu\,\rho}\,(\bar{p}\cdot\bar{r})\,(\bar{q}\cdot\bar{s})+\bar{q}^{\rho}\,\bar{g}^{\nu\,\sigma}\,(\bar{p}\cdot\bar{r})\,(\bar{s}\cdot\bar{x})-\bar{q}^{\sigma}\,\bar{g}^{\nu\,\rho}\,(\bar{p}\cdot\bar{r})\,(\bar{s}\cdot\bar{x})+\\&\bar{x}^{\rho}\,\bar{g}^{\nu\,\sigma}\,(\bar{p}\cdot\bar{s})\,(\bar{q}\cdot\bar{r})-\bar{x}^{\sigma}\,\bar{g}^{\nu\,\rho}\,(\bar{p}\cdot\bar{s})\,(\bar{q}\cdot\bar{r})-\bar{q}^{\rho}\,\bar{g}^{\nu\,\sigma}\,(\bar{p}\cdot\bar{s})\,(\bar{r}\cdot\bar{x})+\bar{q}^{\sigma}\,\bar{g}^{\nu\,\rho}\,(\bar{p}\cdot\bar{s})\,(\bar{r}\cdot\bar{x})+\\&\bar{p}^{\rho}\,\bar{g}^{\nu\,\sigma}\,(\bar{q}\cdot\bar{s})\,(\bar{r}\cdot\bar{x})-\bar{p}^{\sigma}\,\bar{g}^{\nu\,\rho}\,(\bar{q}\cdot\bar{s})\,(\bar{r}\cdot\bar{x})-\bar{p}^{\rho}\,\bar{g}^{\nu\,\sigma}\,(\bar{q}\cdot\bar{r})\,(\bar{s}\cdot\bar{x})+\bar{p}^{\sigma}\,\bar{g}^{\nu\,\rho}\,(\bar{q}\cdot\bar{r})\,(\bar{s}\cdot\bar{x})+\\&\bar{q}^{\sigma}\,\bar{s}^{\nu}\,\bar{x}^{\rho}\,(\bar{p}\cdot\bar{r})-\bar{q}^{\rho}\,\bar{s}^{\nu}\,\bar{x}^{\sigma}\,(\bar{p}\cdot\bar{r})-\bar{q}^{\sigma}\,\bar{r}^{\nu}\,\bar{x}^{\rho}\,(\bar{p}\cdot\bar{s})+\bar{q}^{\rho}\,\bar{r}^{\nu}\,\bar{x}^{\sigma}\,(\bar{p}\cdot\bar{s})-\bar{p}^{\sigma}\,\bar{s}^{\nu}\,\bar{x}^{\rho}\,(\bar{q}\cdot\bar{r})+\bar{p}^{\rho}\,\bar{s}^{\nu}\,\bar{x}^{\sigma}\,(\bar{q}\cdot\bar{r})+\\&\bar{p}^{\sigma}\,\bar{r}^{\nu}\,\bar{x}^{\rho}\,(\bar{q}\cdot\bar{s})-\bar{p}^{\rho}\,\bar{r}^{\nu}\,\bar{x}^{\sigma}\,(\bar{q}\cdot\bar{s})+\bar{p}^{\sigma}\,\bar{q}^{\rho}\,\bar{s}^{\nu}\,(\bar{r}\cdot\bar{x})-\bar{p}^{\rho}\,\bar{q}^{\sigma}\,\bar{s}^{\nu}\,(\bar{r}\cdot\bar{x})-\bar{p}^{\sigma}\,\bar{q}^{\rho}\,\bar{r}^{\nu}\,(\bar{s}\cdot\bar{x})+\bar{p}^{\rho}\,\bar{q}^{\sigma}\,\bar{r}^{\nu}\,(\bar{s}\cdot\bar{x})\end{aligned}$$

In[ ]:=
```
LC[μ, ν][p, q] × LC[ρ, σ][r, s] × FV[x, μ]
Contract[%, EpsContract → False]
```

Out[ ]=

$$\bar{x}^{\mu}\,\bar{\epsilon}^{\mu\,\nu\,\bar{p}\,\bar{q}}\,\bar{\epsilon}^{\rho\,\sigma\,\bar{r}\,\bar{s}}$$

Out[ ]=

$$-\bar{\epsilon}^{\nu\,\bar{p}\,\bar{q}\,\bar{x}}\,\bar{\epsilon}^{\rho\,\sigma\,\bar{r}\,\bar{s}}$$

Kinematics

FeynCalc allows you to specify the values of scalar products before doing the calculation.

In[ ]:=
```
SP[p, q] = s;
SP[p, q]
```

Out[ ]=

$$s$$

In[ ]:=
```
FV[q, μ] × FV[q, ν] (FV[p, μ] × FV[p, ν] - MT[μ, ν] / SP[p, p])
Contract[%]
```

Out[ ]=

$$\bar{q}^{\mu}\,\bar{q}^{\nu}\left(\bar{p}^{\mu}\,\bar{p}^{\nu}-\frac{\bar{g}^{\mu\,\nu}}{\bar{p}^2}\right)$$

Out[ ]=

$$s^2-\frac{\bar{q}^2}{\bar{p}^2}$$

To clear the previously set values, use

In[ ]:=
```
FCClearScalarProducts[]
```

In[ ]:=
```
SP[p, q]
```

Out[ ]=

$$\bar{p}\cdot\bar{q}$$

```
In[ ]:= FV[q, μ] × FV[q, ν] (FV[p, μ] × FV[p, ν] - MT[μ, ν] / SP[p, p])
        Contract[%]
```

Out[ ]=

$$\overline{q}^{\mu}\,\overline{q}^{\nu}\left(\overline{p}^{\mu}\,\overline{p}^{\nu}-\frac{\overline{g}^{\mu\,\nu}}{\overline{p}^{2}}\right)$$

Out[ ]=

$$(\overline{p}\cdot\overline{q})^{2}-\frac{\overline{q}^{2}}{\overline{p}^{2}}$$

A good habit is to always apply FCClearScalarProducts[] before setting any new values

```
In[ ]:= FCClearScalarProducts[]
```

Expanding and undoing expansions

FeynCalc offers further useful functions for the manipulations of Lorentz tensors and Dirac matrices . To expand scalar products

```
In[ ]:= ex1 = SP[p + q + r, s + t]
        ex2 = FV[p + q + r, μ]
```

Out[ ]=

$$(\overline{p}+\overline{q}+\overline{r})\cdot(\overline{s}+\overline{t})$$

Out[ ]=

$$(\overline{p}+\overline{q}+\overline{r})^{\mu}$$

```
In[ ]:= ExpandScalarProduct[ex1]
        ExpandScalarProduct[ex2]
```

Out[ ]=

$$\overline{p}\cdot\overline{s}+\overline{p}\cdot\overline{t}+\overline{q}\cdot\overline{s}+\overline{q}\cdot\overline{t}+\overline{r}\cdot\overline{s}+\overline{r}\cdot\overline{t}$$

Out[ ]=

$$\overline{p}^{\mu}+\overline{q}^{\mu}+\overline{r}^{\mu}$$

Notice that ExpandScalarProduct can also do expansions only for the given momentum, while leaving the rest of the expression untouched, e.g.

```
In[ ]:= x SP[p1 + p2, q1 + q2] + y SP[p3 + p4, q3 + q4] + z SP[p5 + p6, q5 + q6]
        ExpandScalarProduct[%, Momentum → {p1}]
```

Out[ ]=

$$x\left((\overline{\mathrm{p1}}+\overline{\mathrm{p2}})\cdot(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})\right)+y\left((\overline{\mathrm{p3}}+\overline{\mathrm{p4}})\cdot(\overline{\mathrm{q3}}+\overline{\mathrm{q4}})\right)+z\left((\overline{\mathrm{p5}}+\overline{\mathrm{p6}})\cdot(\overline{\mathrm{q5}}+\overline{\mathrm{q6}})\right)$$

Out[ ]=

$$x\left(\overline{\mathrm{p1}}\cdot(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})+\overline{\mathrm{p2}}\cdot(\overline{\mathrm{q1}}+\overline{\mathrm{q2}})\right)+y\left((\overline{\mathrm{p3}}+\overline{\mathrm{p4}})\cdot(\overline{\mathrm{q3}}+\overline{\mathrm{q4}})\right)+z\left((\overline{\mathrm{p5}}+\overline{\mathrm{p6}})\cdot(\overline{\mathrm{q5}}+\overline{\mathrm{q6}})\right)$$

EpsEvaluate reorders the arguments of Eps according to its antisymmetric properties

```
In[ ]:= LC[μ, σ, ρ, ν]
        EpsEvaluate[%]
```

Out[ ]=

$$\overline{\epsilon}^{\mu\,\sigma\,\rho\,\nu}$$

Out[ ]=

$$-\overline{\epsilon}^{\mu\,\nu\,\rho\,\sigma}$$

The inverse of ExpandScalarProduct is called MomentumCombine. This also works for scalar

products, but the results may not be always optimal.

```
In[ ]:= 3 FV[p, μ] + 4 FV[q, μ]
        MomentumCombine[%]
```

Out[ ]=

$$3\,\overline{p}^{\mu} + 4\,\overline{q}^{\mu}$$

Out[ ]=

$$(3\,\overline{p} + 4\,\overline{q})^{\mu}$$

```
In[ ]:= SP[p + q + t, r + s]
        ExpandScalarProduct[%]
        MomentumCombine[%]
```

Out[ ]=

$$(\overline{r} + \overline{s}) \cdot (\overline{p} + \overline{q} + \overline{t})$$

Out[ ]=

$$\overline{p} \cdot \overline{r} + \overline{p} \cdot \overline{s} + \overline{q} \cdot \overline{r} + \overline{q} \cdot \overline{s} + \overline{r} \cdot \overline{t} + \overline{s} \cdot \overline{t}$$

Out[ ]=

$$(\overline{r} + \overline{s}) \cdot (\overline{p} + \overline{q} + \overline{t})$$

```
In[ ]:= SP[p + q + t, r + s] + SP[r, s]
        ExpandScalarProduct[%]
        MomentumCombine[%]
```

Out[ ]=

$$(\overline{r} + \overline{s}) \cdot (\overline{p} + \overline{q} + \overline{t}) + \overline{r} \cdot \overline{s}$$

Out[ ]=

$$\overline{p} \cdot \overline{r} + \overline{p} \cdot \overline{s} + \overline{q} \cdot \overline{r} + \overline{q} \cdot \overline{s} + \overline{r} \cdot \overline{s} + \overline{r} \cdot \overline{t} + \overline{s} \cdot \overline{t}$$

Out[ ]=

$$(\overline{p} + \overline{q}) \cdot (\overline{r} + \overline{s}) + \overline{r} \cdot (\overline{s} + \overline{t}) + \overline{s} \cdot \overline{t}$$

For Dirac matrices the corresponding functions are DiracGammaExpand and DiracGammaCombine

```
In[ ]:= GA[μ].GS[p + q].GA[ν].GS[r + s]
        DiracGammaExpand[%]
        DiracGammaCombine[%]
```

Out[ ]=

$$\overline{\gamma}^{\mu}.(\overline{\gamma}\cdot(\overline{p} + \overline{q})).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot(\overline{r} + \overline{s}))$$

Out[ ]=

$$\overline{\gamma}^{\mu}.(\overline{\gamma}\cdot\overline{p} + \overline{\gamma}\cdot\overline{q}).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot\overline{r} + \overline{\gamma}\cdot\overline{s})$$

Out[ ]=

$$\overline{\gamma}^{\mu}.(\overline{\gamma}\cdot(\overline{p} + \overline{q})).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot(\overline{r} + \overline{s}))$$

Notice the DiracGammaExpand does not expand the whole noncommutative product . If you need that, use DotSimplify

```
In[ ]:= GA[μ].GS[p + q].GA[ν].GS[r + s]
        % // DiracGammaExpand // DotSimplify
```

Out[ ]=

$$\overline{\gamma}^{\mu}.(\overline{\gamma}\cdot(\overline{p} + \overline{q})).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot(\overline{r} + \overline{s}))$$

Out[ ]=

$$\overline{\gamma}^{\mu}.(\overline{\gamma}\cdot\overline{p}).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot\overline{r}) + \overline{\gamma}^{\mu}.(\overline{\gamma}\cdot\overline{p}).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot\overline{s}) + \overline{\gamma}^{\mu}.(\overline{\gamma}\cdot\overline{q}).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot\overline{r}) + \overline{\gamma}^{\mu}.(\overline{\gamma}\cdot\overline{q}).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot\overline{s})$$

Handling indices

When you square an expression with dummy indices, you must rename them first . People often do this by hand, e . g . as in

In[ ]:=
```
ex1 = (FV[p, μ] + FV[q, μ]) FV[r, μ] × FV[r, ν]
```

Out[ ]=

$$\bar{r}^{\mu}\,\bar{r}^{\nu}\,(\bar{p}^{\mu}+\bar{q}^{\mu})$$

In[ ]:=
```
ex1 (ex1 /. μ → ρ)
Contract[%]
```

Out[ ]=

$$\bar{r}^{\mu}\,(\bar{r}^{\nu})^2\,\bar{r}^{\rho}\,(\bar{p}^{\mu}+\bar{q}^{\mu})\,(\bar{p}^{\rho}+\bar{q}^{\rho})$$

Out[ ]=

$$\bar{r}^2\,(\bar{p}\cdot\bar{r}+\bar{q}\cdot\bar{r})^2$$

However, FeynCalc offers a function for that

In[ ]:=
```
FCRenameDummyIndices[ex1]
```

Out[ ]=

$$\bar{r}^{\nu}\,\bar{r}^{\$\mathrm{AL}(\$19)}\left(\bar{p}^{\$\mathrm{AL}(\$19)}+\bar{q}^{\$\mathrm{AL}(\$19)}\right)$$

In[ ]:=
```
ex1 FCRenameDummyIndices[ex1]
Contract[%]
```

Out[ ]=

$$\bar{r}^{\mu}\,\bar{r}^{\nu}\,\bar{r}^{\nu}\,(\bar{p}^{\mu}+\bar{q}^{\mu})\,\bar{r}^{\$\mathrm{AL}(\$20)}\left(\bar{p}^{\$\mathrm{AL}(\$20)}+\bar{q}^{\$\mathrm{AL}(\$20)}\right)$$

Out[ ]=

$$\bar{r}^2\,(\bar{p}\cdot\bar{r}+\bar{q}\cdot\bar{r})^2$$

Often we also need to uncontract already contracted indices. This is done by Uncontract. By default, it handles only contractions with Dirac matrices and Levi-Civita tensors

In[ ]:=
```
LC[][p, q, r, s]
Uncontract[%, p]
Uncontract[%%, p, q]
```

Out[ ]=

$$\bar{\epsilon}^{\bar{p}\,\bar{q}\,\bar{r}\,\bar{s}}$$

Out[ ]=

$$\bar{p}^{\$\mathrm{AL}(\$21)}\,\bar{\epsilon}^{\$\mathrm{AL}(\$21)\,\bar{q}\,\bar{r}\,\bar{s}}$$

Out[ ]=

$$\bar{p}^{\$\mathrm{AL}(\$23)}\,\bar{q}^{\$\mathrm{AL}(\$22)}\left(-\bar{\epsilon}^{\$\mathrm{AL}(\$22)\,\$\mathrm{AL}(\$23)\,\bar{r}\,\bar{s}}\right)$$

To uncontract scalar products as well, use the option Pair -> All

In[ ]:=
```
SP[p, q]
Uncontract[%, p, Pair → All]
```

Out[ ]=

$$\overline{p}\cdot\overline{q}$$

Out[ ]=

$$\overline{p}^{\$AL(\$24)}\,\overline{q}^{\$AL(\$24)}$$

In[ ]:=

Dirac algebra

The two most relevant functions for the manipulations of Dirac matrices are DiracSimplify and DiracTrace.

The goal of DiracSimplify is to eliminate all pairs of Dirac matrices with the equal indices or contracted with the same 4 - vectors

In[ ]:=
```
GA[μ].GS[p + m].GA[μ]
DiracSimplify[%]
```

Out[ ]=

$$\overline{\gamma}^{\mu}.(\overline{\gamma}\cdot(\overline{m}+\overline{p})).\overline{\gamma}^{\mu}$$

Out[ ]=

$$-2\,\overline{\gamma}\cdot\overline{m}-2\,\overline{\gamma}\cdot\overline{p}$$

In[ ]:=
```
GA[μ].GS[p + m1].GA[ν].GS[p + m2]
DiracSimplify[%]
```

Out[ ]=

$$\overline{\gamma}^{\mu}.\left(\overline{\gamma}\cdot\left(\overline{\text{m1}}+\overline{p}\right)\right).\overline{\gamma}^{\nu}.\left(\overline{\gamma}\cdot\left(\overline{\text{m2}}+\overline{p}\right)\right)$$

Out[ ]=

$$\overline{\gamma}^{\mu}.\left(\overline{\gamma}\cdot\overline{\text{m1}}\right).\overline{\gamma}^{\nu}.\left(\overline{\gamma}\cdot\overline{\text{m2}}\right)+\overline{\gamma}^{\mu}.\left(\overline{\gamma}\cdot\overline{\text{m1}}\right).\overline{\gamma}^{\nu}.(\overline{\gamma}\cdot\overline{p})+\overline{\gamma}^{\mu}.(\overline{\gamma}\cdot\overline{p}).\overline{\gamma}^{\nu}.\left(\overline{\gamma}\cdot\overline{\text{m2}}\right)-\overline{p}^{2}\,\overline{\gamma}^{\mu}.\overline{\gamma}^{\nu}+2\,\overline{p}^{\nu}\,\overline{\gamma}^{\mu}.(\overline{\gamma}\cdot\overline{p})$$

DiracTrace is used for the evaluation of Dirac traces . The trace is not evaluated by default. To obtain the result we can either use the option DiracTraceEvaluate or use DiracSimplify instead.

In[ ]:=
```
DiracTrace[GA[μ, ν]]
DiracTrace[GA[μ, ν], DiracTraceEvaluate → True]
DiracTrace[GA[μ, ν]] // DiracSimplify
```

Out[ ]=

$$\text{tr}(\overline{\gamma}^{\mu}.\overline{\gamma}^{\nu})$$

Out[ ]=

$$4\,\overline{g}^{\mu\nu}$$

Out[ ]=

$$4\,\overline{g}^{\mu\nu}$$

By default FeynCalc refuses to compute a D-dimensional trace that contains γ^5. This is because by default FeynCalc is using anticommuting γ^5 in D-dimensions, a scheme known as Naive Dimensional Regularization (NDR). Check the tutorial web for the implemented solution (changing the scheme).

Gordon' s identities are implemented via GordonSimplify

In[ ]:=
```
SpinorUBar[p1, m1].GA[μ].SpinorU[p2, m2]
GordonSimplify[%]
```

Out[ ]=

$$\overline{u}(\text{p1},\ \text{m1}).\overline{\gamma}^{\mu}.u(\text{p2},\ \text{m2})$$

Out[ ]=

$$\frac{\left(\overline{\text{p1}}+\overline{\text{p2}}\right)^{\mu}\left(\varphi\left(\overline{\text{p1}},\ \text{m1}\right)\right).\left(\varphi\left(\overline{\text{p2}},\ \text{m2}\right)\right)}{\text{m1}+\text{m2}}+\frac{i\left(\varphi\left(\overline{\text{p1}},\ \text{m1}\right)\right).\sigma^{\mu\,\overline{\text{p1}}-\overline{\text{p2}}}.\left(\varphi\left(\overline{\text{p2}},\ \text{m2}\right)\right)}{\text{m1}+\text{m2}}$$

Color algebra

FeynCalc objects relevant for the color algebra are
The SU(N) T^a generator in the fundamental representation. The fundamental indices are implicit.

In[ ]:=
```
SUNT[a]
```

Out[ ]=

$$T^{a}$$

The structure constants of SU(N) . The arguments a, b, c should be of symbolic type.

In[ ]:=
```
SUNF[a, b, c]
```

Out[ ]=

$$f^{a\,b\,c}$$

The symmetric SU(N) d_{abc}.

In[ ]:=
```
SUND[a, b, c]
SUND[a, b, c, Explicit → True]
```

Out[ ]=

$$d^{a\,b\,c}$$

Out[ ]=

$$2\ \text{tr}\left(T^{a}.T^{b}.T^{c}\right)+2\ \text{tr}\left(T^{b}.T^{a}.T^{c}\right)$$

The SU(N) Kronecker delta with color indices a and b in the adjoint representation.

In[ ]:=
```
SUNDelta[a, b]
```

Out[ ]=

$$\delta^{a\,b}$$

The number of colors. Trick[SUNDelta[a, a]] yields n_c^2 -1.

In[ ]:=
```
SUNN
```

Out[ ]=

$$N$$

CA is one of the Casimir operator eigenvalues of SU(N) (CA = N) .
CF is one of the Casimir operator eigenvalues of SU(N) (CF =\frac{N^2-1}{2N})

In[ ]:=
```
CA
CF
```

Out[ ]=

$$C_A$$

Out[ ]=

$$C_F$$

There are two main functions to deal with colored objects : SUNSimplify and SUNTrace . In general, SUNSimplify will also simplify color traces when possible

In[ ]:=
```
SUNT[a, a]
SUNSimplify[%]
```

Out[ ]=

$$T^a.T^a$$

Out[ ]=

$$C_F$$

In[ ]:=
```
SUNT[a, b, a, b]
SUNSimplify[%]
```

Out[ ]=

$$T^a.T^b.T^a.T^b$$

Out[ ]=

$$-\frac{1}{2} C_F (C_A - 2 C_F)$$

In[ ]:=
```
SUNT[b, d, a, b, d]
SUNSimplify[%]
```

Out[ ]=

$$T^b.T^d.T^a.T^b.T^d$$

Out[ ]=

$$\frac{T^a \left(C_A^2 + 1\right)}{4 C_A^2}$$

In[ ]:=
```
SUNF[a, r, s] × SUNF[b, r, s]
SUNSimplify[%]
```

Out[ ]=

$$f^{a\,r\,s} f^{b\,r\,s}$$

Out[ ]=

$$C_A \delta^{a\,b}$$

In[ ]:=
```
SUNF[a, b, c] × SUNF[a, b, c]
SUNSimplify[%]
```

Out[ ]=

$$\left(f^{a\,b\,c}\right)^2$$

Out[ ]=

$$2 C_A^2 C_F$$

In[ ]:=
```
SUNF[a, b, c] × SUND[d, b, c]
SUNSimplify[%]
```

Out[ ]=

$$d^{b\,c\,d} f^{a\,b\,c}$$

Out[ ]=

$$0$$

In[ ]:=
```
SUND[a, b, c] × SUND[a, b, c]
SUNSimplify[%]
```

Out[ ]=

$$(d^{a\,b\,c})^2$$

Out[ ]=

$$-2\,(4 - C_A^2)\,C_F$$

The color factors C_A and C_F are reconstructed from N_c using heuristics . The reconstruction can be disabled by setting the option SUNNToCACF to False

In[ ]:=
```
SUNSimplify[SUNT[b, d, a, b, d], SUNNToCACF → False]
```

Out[ ]=

$$\frac{(N^2 + 1)\,T^a}{4\,N^2}$$

In[ ]:=
```
CA
SUNSimplify[CA, SUNNToCACF → False]
CF
SUNSimplify[CF, SUNNToCACF → False]
```

Out[ ]=

$$C_A$$

Out[ ]=

$$N$$

Out[ ]=

$$C_F$$

Out[ ]=

$$-\frac{1 - N^2}{2\,N}$$

The color traces are not evaluated by default . The evaluation can be forced either by applying SUNSimplify or setting the option SUNTraceEvaluate to True

In[ ]:=
```
SUNTrace[SUNT[a, b]]
SUNTrace[SUNT[a, b]] // SUNSimplify
SUNTrace[SUNT[a, b], SUNTraceEvaluate → True]
```

Out[ ]=

$$\mathrm{tr}\left(T^a.T^b\right)$$

Out[ ]=

$$\frac{\delta^{a\,b}}{2}$$

Out[ ]=

$$\frac{\delta^{a\,b}}{2}$$

In[ ]:=
```
SUNTrace[SUNT[a, b, b, a]]
SUNTrace[SUNT[a, b, b, a]] // SUNSimplify
```

Out[ ]=

$$\mathrm{tr}\left(T^a.T^b.T^b.T^a\right)$$

Out[ ]=

$$C_A\,C_F^2$$

Use SUNTF to get color matrices with explicit fundamental indices

In[ ]:=
```
SUNTF[{a, b, c}, i, j] × SUNTrace[SUNT[b, a]]
% // SUNSimplify
```

Out[ ]=

$$\text{tr}\left(T^b.T^a\right)\left(T^a\ T^b\ T^c\right)_{ij}$$

Out[ ]=

$$\frac{1}{2}\ C_F\ T^c_{ij}$$

In[ ]:=
```
SUNDelta[a, b] × SUNTF[{a, b}, i, j] × SUNTF[{c, d}, j, i]
SUNSimplify[%]
```

Out[ ]=

$$\delta^{a\,b}\left(T^a\ T^b\right)_{ij}\left(T^c\ T^d\right)_{j\,i}$$

Out[ ]=

$$\frac{1}{2}\ C_F\ \delta^{c\,d}$$

Color traces with more than 3 distinct matrices are not evaluated by default (assuming that no other simplifications are possible) . The evaluation can be forced using the option SUNTraceEvaluate set to True.
One can automatically rename dummy indices using the SUNIndexNames option.

In[ ]:=

In[ ]:=
```
SUNTrace[SUNT[a, b, c, d]] // SUNSimplify
SUNTrace[SUNT[a, b, c, d]] // SUNSimplify[#, SUNTraceEvaluate → True] &
SUNTrace[SUNT[a, b, c, d]] //
 SUNSimplify[#, SUNTraceEvaluate → True, SUNIndexNames → {j}] &
```

Out[ ]=

$$\text{tr}\left(T^a.T^b.T^c.T^d\right)$$

Out[ ]=

$$\frac{1}{4}\ \delta^{a\,d}\ (C_A - 2\ C_F)\ \delta^{b\,c} - \frac{1}{4}\ \delta^{a\,c}\ (C_A - 2\ C_F)\ \delta^{b\,d} + \frac{1}{4}\ \delta^{a\,b}\ (C_A - 2\ C_F)\ \delta^{c\,d} -$$
$$\frac{1}{8}\ i\ f^{a\,d\,\text{FCGV(sun1751)}}\ d^{b\,c\,\text{FCGV(sun1751)}} + \frac{1}{8}\ i\ d^{a\,d\,\text{FCGV(sun1751)}}\ f^{b\,c\,\text{FCGV(sun1751)}} +$$
$$\frac{1}{8}\ d^{a\,d\,\text{FCGV(sun1751)}}\ d^{b\,c\,\text{FCGV(sun1751)}} - \frac{1}{8}\ d^{b\,d\,\text{FCGV(sun1751)}}\ d^{a\,c\,\text{FCGV(sun1751)}} + \frac{1}{8}\ d^{c\,d\,\text{FCGV(sun1751)}}\ d^{a\,b\,\text{FCGV(sun1751)}}$$

Out[ ]=

$$\frac{1}{4}\ \delta^{a\,d}\ (C_A - 2\ C_F)\ \delta^{b\,c} - \frac{1}{4}\ \delta^{a\,c}\ (C_A - 2\ C_F)\ \delta^{b\,d} + \frac{1}{4}\ \delta^{a\,b}\ (C_A - 2\ C_F)\ \delta^{c\,d} -$$
$$\frac{1}{8}\ i\ f^{a\,d\,j}\ d^{b\,c\,j} + \frac{1}{8}\ i\ d^{a\,d\,j}\ f^{b\,c\,j} + \frac{1}{8}\ d^{a\,d\,j}\ d^{b\,c\,j} - \frac{1}{8}\ d^{b\,d\,j}\ d^{a\,c\,j} + \frac{1}{8}\ d^{c\,d\,j}\ d^{a\,b\,j}$$

Propagators

All propagators (and products thereof) appearing in loop diagrams and integrals are represented via FeynAmpDenominator

This container is capable of representing different propagator types, where the familiar quadratic propagators of the form 1/(q 2 – m 2 + iη) are described using PropagatorDenominator

```
In[ ]:= FAD[{q, m}]
```

Out[ ]=

$$\frac{1}{q^2-m^2}$$

```
In[ ]:= FCI[%] // StandardForm
```

```
Out[ ]//StandardForm=
FeynAmpDenominator[PropagatorDenominator[Momentum[q, D], m]]
```

```
In[ ]:= FAD[{q, m0}, {q + p1, m1}, {q + p2, m1}]
```

Out[ ]=

$$\frac{1}{\left(q^2-\text{m0}^2\right).\left((\text{p1}+q)^2-\text{m1}^2\right).\left((\text{p2}+q)^2-\text{m1}^2\right)}$$

The presence of FeynAmpDenominator in an expression does not automatically mean that it is a loop amplitude. FeynAmpDenominator can equally appear in tree level amplitudes, where it stands for the usual 4-dimensional propagator.

In FeynCalc there is no explicit way to distinguish between loop amplitudes and tree - level amplitudes . When you use functions that manipulate loop integrals, you need to tell them explicitly what is your loop momentum .

There are several functions, that are useful both for tree - and loop - level amplitudes, depending on what we want to do
For example, one can split one FeynAmpDenominator into many

```
In[ ]:= FAD[{k1 – k2}, {k1 – p2, m}, {k2 + p2, m}]
        FeynAmpDenominatorSplit[%]
        % // FCE // StandardForm
```

Out[ ]=

$$\frac{1}{(\text{k1}-\text{k2})^2.\left((\text{k1}-\text{p2})^2-m^2\right).\left((\text{k2}+\text{p2})^2-m^2\right)}$$

Out[ ]=

$$\frac{1}{(\text{k1}-\text{k2})^2\left((\text{k1}-\text{p2})^2-m^2\right)\left((\text{k2}+\text{p2})^2-m^2\right)}$$

```
Out[ ]//StandardForm=
FAD[k1 - k2] × FAD[{k1 - p2, m}] × FAD[{k2 + p2, m}]
```

or combine several into one

```
In[ ]:= FeynAmpDenominatorCombine[FAD[k1 – k2] × FAD[{k1 – p2, m}] × FAD[{k2 + p2, m}]]
        % // FCE // StandardForm
```

Out[ ]=

$$\frac{1}{(\text{k1}-\text{k2})^2.\left((\text{k1}-\text{p2})^2-m^2\right).\left((\text{k2}+\text{p2})^2-m^2\right)}$$

```
Out[ ]//StandardForm=
FAD[k1 - k2, {k1 - p2, m}, {k2 + p2, m}]
```

At the tree - level we often do not need the FeynAmpDenominators but rather want to express

everything in terms of explicit scalar products, in order to exploit kinematic simplifications . This is handled by FeynAmpDenominatorExplicit

In[ ]:=
```
FeynAmpDenominatorExplicit[FAD[{k2 + p2, m}, k1 - k2, {k1 - p2, m}]]
```

Out[ ]=

$$\frac{1}{\left(-2\,(\text{k1}\cdot\text{k2})+\text{k1}^2+\text{k2}^2\right)\left(-2\,(\text{k1}\cdot\text{p2})+\text{k1}^2-m^2+\text{p2}^2\right)\left(2\,(\text{k2}\cdot\text{p2})+\text{k2}^2-m^2+\text{p2}^2\right)}$$

Loops

1 - loop tensor reduction using Passarino - Veltman method is handled by TID

In[ ]:=
```
FVD[q, μ] × FVD[q, ν] × FAD[{q, m}]
TID[%, q]
```

Out[ ]=

$$\frac{q^{\mu}\,q^{\nu}}{q^2-m^2}$$

Out[ ]=

$$\frac{m^2\,g^{\mu\,\nu}}{D\left(q^2-m^2\right)}$$

In[ ]:=
```
int = FVD[q, μ] × SPD[q, p] × FAD[{q, m0}, {q + p, m1}]
TID[%, q]
```

Out[ ]=

$$\frac{q^{\mu}\,(p\cdot q)}{\left(q^2-\text{m0}^2\right).\left((\,p+q\,)^2-\text{m1}^2\right)}$$

Out[ ]=

$$\frac{p^{\mu}\left(\text{m0}^2-\text{m1}^2+p^2\right)^2}{4\,p^2\left(q^2-\text{m0}^2\right).\left((\,q-p\,)^2-\text{m1}^2\right)}-\frac{p^{\mu}\left(\text{m0}^2-\text{m1}^2+p^2\right)}{4\,p^2\left(q^2-\text{m0}^2\right)}+\frac{p^{\mu}\left(\text{m0}^2-\text{m1}^2+3\,p^2\right)}{4\,p^2\left(q^2-\text{m1}^2\right)}$$

By default, TID tries to reduce everything to scalar integrals with unit denominators . However, if it encounters zero Gram determinants, it automatically switches to the coefficient functions

In[ ]:=
```
FCClearScalarProducts[]
SPD[p, p] = 0;
TID[int, q]
```

TID: Warning! Following sets of external momenta are linearly dependent {{-p}}. To avoid singularities due to zero Gram determinants, TID will automatically switch to the reduction in terms of Passarino–Veltman coefficient functions. If you want to have the reduction to scalar integrals, please find a set of linearly independent momenta using FCLoopFindTensorBasis and supply it to the function via the option TensorReductionBasisChange.

Out[ ]=

$$\frac{p^{\mu}}{2\left(q^2-\text{m1}^2\right)}-\frac{1}{2}\,i\,\pi^2\left(\text{m0}^2-\text{m1}^2\right)p^{\mu}\,\text{B}_1\left(0,\,\text{m0}^2,\,\text{m1}^2\right)$$

If we want the result to be express entirely in terms of Passarino - Veltman function, i . e . without FADs, we can use ToPaVe

In[ ]:=
```
TID[int, q, ToPaVe → True]
```

TID: Warning! Following sets of external momenta are linearly dependent {{-p}}. To avoid singularities due to zero Gram determinants, TID will automatically switch to the reduction in terms of Passarino–Veltman coefficient functions. If you want to have the reduction to scalar integrals, please find a set of linearly independent momenta using FCLoopFindTensorBasis and supply it to the function via the option TensorReductionBasisChange.

Out[ ]=

$$\frac{1}{2} i \pi^2 p^\mu \, \mathrm{A}_0\left(\mathrm{m1}^2\right) - \frac{1}{2} i \pi^2 \left(\mathrm{m0}^2 - \mathrm{m1}^2\right) p^\mu \, \mathrm{B}_1\left(0, \mathrm{m0}^2, \mathrm{m1}^2\right)$$

ToPaVe is actually also a standalone function, so it can be used independently of TID

In[ ]:=
```
FCClearScalarProducts[]
FAD[q, {q + p1}, {q + p2}]
ToPaVe[%, q]
```

Out[ ]=

$$\frac{1}{q^2.(\mathrm{p1} + q)^2.(\mathrm{p2} + q)^2}$$

Out[ ]=

$$i \pi^2 \, \mathrm{C}_0\left(\mathrm{p1}^2, \mathrm{p2}^2, \mathrm{p1}^2 - 2\,(\mathrm{p1}\cdot\mathrm{p2}) + \mathrm{p2}^2, 0, 0, 0\right)$$

Even if there are no Gram determinants, for some tensor integrals the full result in terms of scalar integrals is just too large

In[ ]:=
```
int = FVD[q, μ] × FVD[q, ν] × FAD[q, {q + p1}, {q + p2}]
res = TID[int, q]
```

Out[ ]=

$$\frac{q^\mu \, q^\nu}{q^2.(\mathrm{p1} + q)^2.(\mathrm{p2} + q)^2}$$

Out[ ]=

$$\begin{aligned}
&-\Big(\big(2\,(D-1)\,\mathrm{p2}^\mu\,\mathrm{p2}^\nu\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p1}^4 - (D-1)\,\mathrm{p2}^\mu\,\mathrm{p2}^\nu\left(\mathrm{p1}\cdot\mathrm{p2} + \mathrm{p2}^2\right)\mathrm{p1}^4 + 2\,(D-1)\,\mathrm{p1}^\mu\,\mathrm{p1}^\nu\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^2\,\mathrm{p1}^2 + \\
&\quad (D-1)\,\mathrm{p2}^\mu\,\mathrm{p1}^\nu\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p1}\cdot\mathrm{p2} + \mathrm{p2}^2\right)\mathrm{p1}^2 + (D-1)\,\mathrm{p1}^\mu\,\mathrm{p2}^\nu\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p1}\cdot\mathrm{p2} + \mathrm{p2}^2\right)\mathrm{p1}^2 + \\
&\quad 2\,g^{\mu\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\left((\mathrm{p1}\cdot\mathrm{p2})^2 - \mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p1}^2 - g^{\mu\nu}\left(\mathrm{p1}\cdot\mathrm{p2} + \mathrm{p2}^2\right)\left((\mathrm{p1}\cdot\mathrm{p2})^2 - \mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p1}^2 - \\
&\quad \mathrm{p2}^\mu\,\mathrm{p1}^\nu\left(D\,(\mathrm{p1}\cdot\mathrm{p2})^2 + D\,\mathrm{p1}^2\,\mathrm{p2}^2 - 2\,\mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p1}^2 - \mathrm{p1}^\mu\,\mathrm{p2}^\nu\left(D\,(\mathrm{p1}\cdot\mathrm{p2})^2 + D\,\mathrm{p1}^2\,\mathrm{p2}^2 - 2\,\mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p1}^2 + \\
&\quad \mathrm{p1}^\mu\,\mathrm{p1}^\nu\left(\mathrm{p1}\cdot\mathrm{p2} + \mathrm{p2}^2\right)\left(-D\,(\mathrm{p1}\cdot\mathrm{p2})^2 + 2\,(\mathrm{p1}\cdot\mathrm{p2})^2 - \mathrm{p1}^2\,\mathrm{p2}^2\right)\big)\Big/ \\
&\quad \left(4\,(2-D)\,q^2.(q - \mathrm{p1})^2\left((\mathrm{p1}\cdot\mathrm{p2})^2 - \mathrm{p1}^2\,\mathrm{p2}^2\right)^2\right)\Big) - \\
&\big(2\,(D-1)\,\mathrm{p1}^\mu\,\mathrm{p1}^\nu\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^4 - (D-1)\,\mathrm{p1}^\mu\,\mathrm{p1}^\nu\left(\mathrm{p1}^2 + \mathrm{p1}\cdot\mathrm{p2}\right)\mathrm{p2}^4 + 2\,(D-1)\,\mathrm{p2}^\mu\,\mathrm{p2}^\nu\,\mathrm{p1}^2\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^2 + \\
&\quad (D-1)\,\mathrm{p2}^\mu\,\mathrm{p1}^\nu\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p1}^2 + \mathrm{p1}\cdot\mathrm{p2}\right)\mathrm{p2}^2 + (D-1)\,\mathrm{p1}^\mu\,\mathrm{p2}^\nu\,(\mathrm{p1}\cdot\mathrm{p2})\left(\mathrm{p1}^2 + \mathrm{p1}\cdot\mathrm{p2}\right)\mathrm{p2}^2 + \\
&\quad 2\,g^{\mu\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\left((\mathrm{p1}\cdot\mathrm{p2})^2 - \mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p2}^2 - g^{\mu\nu}\left(\mathrm{p1}^2 + \mathrm{p1}\cdot\mathrm{p2}\right)\left((\mathrm{p1}\cdot\mathrm{p2})^2 - \mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p2}^2 - \\
&\quad \mathrm{p2}^\mu\,\mathrm{p1}^\nu\left(D\,(\mathrm{p1}\cdot\mathrm{p2})^2 + D\,\mathrm{p1}^2\,\mathrm{p2}^2 - 2\,\mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p2}^2 - \mathrm{p1}^\mu\,\mathrm{p2}^\nu\left(D\,(\mathrm{p1}\cdot\mathrm{p2})^2 + D\,\mathrm{p1}^2\,\mathrm{p2}^2 - 2\,\mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p2}^2 + \\
&\quad \mathrm{p2}^\mu\,\mathrm{p2}^\nu\left(\mathrm{p1}^2 + \mathrm{p1}\cdot\mathrm{p2}\right)\left(-D\,(\mathrm{p1}\cdot\mathrm{p2})^2 + 2\,(\mathrm{p1}\cdot\mathrm{p2})^2 - \mathrm{p1}^2\,\mathrm{p2}^2\right)\big)\Big/ \\
&\quad \left(4\,(2-D)\,q^2.(q - \mathrm{p2})^2\left((\mathrm{p1}\cdot\mathrm{p2})^2 - \mathrm{p1}^2\,\mathrm{p2}^2\right)^2\right) + \\
&\big(-\big((D-1)\,\mathrm{p1}^\mu\,\mathrm{p1}^\nu\,\mathrm{p2}^4\,\mathrm{p1}^4\big) - (D-1)\,\mathrm{p2}^\mu\,\mathrm{p2}^\nu\,\mathrm{p2}^4\,\mathrm{p1}^4 + (D-1)\,\mathrm{p2}^\mu\,\mathrm{p1}^\nu\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^2\,\mathrm{p1}^4 + \\
&\quad (D-1)\,\mathrm{p1}^\mu\,\mathrm{p2}^\nu\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^2\,\mathrm{p1}^4 + 2\,(D-1)\,\mathrm{p2}^\mu\,\mathrm{p2}^\nu\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^2\,\mathrm{p1}^4 - g^{\mu\nu}\,\mathrm{p2}^2\left((\mathrm{p1}\cdot\mathrm{p2})^2 - \mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p1}^4 + \\
&\quad \mathrm{p2}^\mu\,\mathrm{p2}^\nu\left(-D\,(\mathrm{p1}\cdot\mathrm{p2})^2 + 2\,(\mathrm{p1}\cdot\mathrm{p2})^2 - \mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p1}^4 + 2\,(D-1)\,\mathrm{p1}^\mu\,\mathrm{p1}^\nu\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^4\,\mathrm{p1}^2 + \\
&\quad (D-1)\,\mathrm{p2}^\mu\,\mathrm{p1}^\nu\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^4\,\mathrm{p1}^2 + (D-1)\,\mathrm{p1}^\mu\,\mathrm{p2}^\nu\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^4\,\mathrm{p1}^2 - g^{\mu\nu}\,\mathrm{p2}^4\left((\mathrm{p1}\cdot\mathrm{p2})^2 - \mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p1}^2 + \\
&\quad 2\,g^{\mu\nu}\,(\mathrm{p1}\cdot\mathrm{p2})\,\mathrm{p2}^2\left((\mathrm{p1}\cdot\mathrm{p2})^2 - \mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p1}^2 - \mathrm{p2}^\mu\,\mathrm{p1}^\nu\,\mathrm{p2}^2\left(D\,(\mathrm{p1}\cdot\mathrm{p2})^2 + D\,\mathrm{p1}^2\,\mathrm{p2}^2 - 2\,\mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p1}^2 - \\
&\quad \mathrm{p1}^\mu\,\mathrm{p2}^\nu\,\mathrm{p2}^2\left(D\,(\mathrm{p1}\cdot\mathrm{p2})^2 + D\,\mathrm{p1}^2\,\mathrm{p2}^2 - 2\,\mathrm{p1}^2\,\mathrm{p2}^2\right)\mathrm{p1}^2 + \mathrm{p1}^\mu\,\mathrm{p1}^\nu\,\mathrm{p2}^4\left(-D\,(\mathrm{p1}\cdot\mathrm{p2})^2 + 2\,(\mathrm{p1}\cdot\mathrm{p2})^2 - \mathrm{p1}^2\,\mathrm{p2}^2\right)\big)\Big/
\end{aligned}$$

$$\Big(4\,(2-D)\,q^2.(q-\text{p1})^2.(q-\text{p2})^2\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)^2\Big)-$$
$$\Big(-4\,\text{p2}^\mu\,\text{p2}^\nu\,(\text{p1}^2\,\text{p2}^2-(\text{p1}\cdot\text{p2})^2)\,\text{p1}^4-(D-1)\,\text{p2}^\mu\,\text{p2}^\nu$$
$$(-3\,\text{p2}^2\,(\text{p1}^2-2\,(\text{p1}\cdot\text{p2})+\text{p2}^2)+\text{p1}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2})-2\,(\text{p1}\cdot\text{p2})\,(\text{p2}^2-\text{p1}\cdot\text{p2})+\text{p2}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2}))\,\text{p1}^4+$$
$$4\,g^{\mu\nu}\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)^2\,\text{p1}^2-4\,\text{p2}^\mu\,\text{p1}^\nu\,(\text{p1}\cdot\text{p2})\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)\,\text{p1}^2-$$
$$4\,\text{p1}^\mu\,\text{p2}^\nu\,(\text{p1}\cdot\text{p2})\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)\,\text{p1}^2+$$
$$4\,\text{p1}^\mu\,\text{p1}^\nu\,\text{p2}^2\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)\,\text{p1}^2+8\,\text{p2}^\mu\,\text{p2}^\nu\,(\text{p1}\cdot\text{p2})\,(\text{p1}^2\,\text{p2}^2-(\text{p1}\cdot\text{p2})^2)\,\text{p1}^2-$$
$$4\,\text{p2}^\mu\,\text{p2}^\nu\,\text{p2}^2\,(\text{p1}^2\,\text{p2}^2-(\text{p1}\cdot\text{p2})^2)\,\text{p1}^2+(D-1)\,\text{p2}^\mu\,\text{p1}^\nu\,(\text{p1}\cdot\text{p2})$$
$$(-3\,\text{p2}^2\,(\text{p1}^2-2\,(\text{p1}\cdot\text{p2})+\text{p2}^2)+\text{p1}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2})-2\,(\text{p1}\cdot\text{p2})\,(\text{p2}^2-\text{p1}\cdot\text{p2})+\text{p2}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2}))\,\text{p1}^2+$$
$$(D-1)\,\text{p1}^\mu\,\text{p2}^\nu\,(\text{p1}\cdot\text{p2})\,(-3\,\text{p2}^2\,(\text{p1}^2-2\,(\text{p1}\cdot\text{p2})+\text{p2}^2)+\text{p1}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2})-$$
$$2\,(\text{p1}\cdot\text{p2})\,(\text{p2}^2-\text{p1}\cdot\text{p2})+\text{p2}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2}))\,\text{p1}^2-g^{\mu\nu}\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)$$
$$(-3\,\text{p2}^2\,(\text{p1}^2-2\,(\text{p1}\cdot\text{p2})+\text{p2}^2)+\text{p1}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2})-2\,(\text{p1}\cdot\text{p2})\,(\text{p2}^2-\text{p1}\cdot\text{p2})+\text{p2}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2}))\,\text{p1}^2-$$
$$2\,(D-1)\,\text{p2}^\mu\,\text{p2}^\nu\,(\text{p1}\cdot\text{p2})\,(-\text{p1}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2})+2\,(\text{p1}\cdot\text{p2})\,(\text{p2}^2-\text{p1}\cdot\text{p2})-\text{p2}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2})+$$
$$(\text{p1}^2-2\,(\text{p1}\cdot\text{p2})+\text{p2}^2)\,(\text{p1}^2+2\,\text{p2}^2))\,\text{p1}^2-8\,g^{\mu\nu}\,(\text{p1}\cdot\text{p2})\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)^2+$$
$$4\,g^{\mu\nu}\,\text{p2}^2\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)^2+8\,\text{p2}^\mu\,\text{p1}^\nu\,(\text{p1}\cdot\text{p2})^2\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)+$$
$$8\,\text{p1}^\mu\,\text{p2}^\nu\,(\text{p1}\cdot\text{p2})^2\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)+4\,\text{p1}^\mu\,\text{p1}^\nu\,\text{p2}^4\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)-$$
$$8\,\text{p1}^\mu\,\text{p1}^\nu\,(\text{p1}\cdot\text{p2})\,\text{p2}^2\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)-4\,\text{p2}^\mu\,\text{p1}^\nu\,(\text{p1}\cdot\text{p2})\,\text{p2}^2\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)-$$
$$4\,\text{p1}^\mu\,\text{p2}^\nu\,(\text{p1}\cdot\text{p2})\,\text{p2}^2\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)+(D-1)\,\text{p1}^\mu\,\text{p1}^\nu\,\text{p2}^4\,(-\text{p1}^2\,(\text{p1}\cdot\text{p2}-\text{p1}^2)+$$
$$2\,(\text{p1}\cdot\text{p2})\,(\text{p1}\cdot\text{p2}-\text{p1}^2)-\text{p2}^2\,(\text{p1}\cdot\text{p2}-\text{p1}^2)+(\text{p1}^2+2\,(\text{p1}\cdot\text{p2}))\,(\text{p1}^2-2\,(\text{p1}\cdot\text{p2})+\text{p2}^2))-$$
$$(D-1)\,\text{p2}^\mu\,\text{p1}^\nu\,(\text{p1}\cdot\text{p2})\,\text{p2}^2\,(-\text{p1}^2\,(\text{p1}\cdot\text{p2}-\text{p1}^2)+2\,(\text{p1}\cdot\text{p2})\,(\text{p1}\cdot\text{p2}-\text{p1}^2)-\text{p2}^2\,(\text{p1}\cdot\text{p2}-\text{p1}^2)+$$
$$(\text{p1}^2+2\,(\text{p1}\cdot\text{p2}))\,(\text{p1}^2-2\,(\text{p1}\cdot\text{p2})+\text{p2}^2))-(D-1)\,\text{p1}^\mu\,\text{p2}^\nu\,(\text{p1}\cdot\text{p2})\,\text{p2}^2\,(-\text{p1}^2\,(\text{p1}\cdot\text{p2}-\text{p1}^2)+$$
$$2\,(\text{p1}\cdot\text{p2})\,(\text{p1}\cdot\text{p2}-\text{p1}^2)-\text{p2}^2\,(\text{p1}\cdot\text{p2}-\text{p1}^2)+(\text{p1}^2+2\,(\text{p1}\cdot\text{p2}))\,(\text{p1}^2-2\,(\text{p1}\cdot\text{p2})+\text{p2}^2))+$$
$$g^{\mu\nu}\,\text{p2}^2\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)\,(-\text{p1}^2\,(\text{p1}\cdot\text{p2}-\text{p1}^2)+2\,(\text{p1}\cdot\text{p2})\,(\text{p1}\cdot\text{p2}-\text{p1}^2)-\text{p2}^2\,(\text{p1}\cdot\text{p2}-\text{p1}^2)+$$
$$(\text{p1}^2+2\,(\text{p1}\cdot\text{p2}))\,(\text{p1}^2-2\,(\text{p1}\cdot\text{p2})+\text{p2}^2))-\text{p2}^\mu\,\text{p2}^\nu\,(-D\,(\text{p1}\cdot\text{p2})^2+2\,(\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)$$
$$(-\text{p1}^2\,(\text{p1}\cdot\text{p2}-\text{p1}^2)+2\,(\text{p1}\cdot\text{p2})\,(\text{p1}\cdot\text{p2}-\text{p1}^2)-\text{p2}^2\,(\text{p1}\cdot\text{p2}-\text{p1}^2)+$$
$$(\text{p1}^2+2\,(\text{p1}\cdot\text{p2}))\,(\text{p1}^2-2\,(\text{p1}\cdot\text{p2})+\text{p2}^2))+\text{p1}^\mu\,\text{p1}^\nu\,(-D\,(\text{p1}\cdot\text{p2})^2+2\,(\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)$$
$$(-3\,\text{p2}^2\,(\text{p1}^2-2\,(\text{p1}\cdot\text{p2})+\text{p2}^2)+\text{p1}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2})-2\,(\text{p1}\cdot\text{p2})\,(\text{p2}^2-\text{p1}\cdot\text{p2})+\text{p2}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2}))-$$
$$2\,(D-1)\,\text{p1}^\mu\,\text{p1}^\nu\,(\text{p1}\cdot\text{p2})\,\text{p2}^2\,(-\text{p1}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2})+2\,(\text{p1}\cdot\text{p2})\,(\text{p2}^2-\text{p1}\cdot\text{p2})-\text{p2}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2})+$$
$$(\text{p1}^2-2\,(\text{p1}\cdot\text{p2})+\text{p2}^2)\,(\text{p1}^2+2\,\text{p2}^2))-2\,g^{\mu\nu}\,(\text{p1}\cdot\text{p2})\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)\,(-\text{p1}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2})+$$
$$2\,(\text{p1}\cdot\text{p2})\,(\text{p2}^2-\text{p1}\cdot\text{p2})-\text{p2}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2})+(\text{p1}^2-2\,(\text{p1}\cdot\text{p2})+\text{p2}^2)\,(\text{p1}^2+2\,\text{p2}^2))+$$
$$\text{p2}^\mu\,\text{p1}^\nu\,(D\,(\text{p1}\cdot\text{p2})^2+D\,\text{p1}^2\,\text{p2}^2-2\,\text{p1}^2\,\text{p2}^2)\,(-\text{p1}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2})+2\,(\text{p1}\cdot\text{p2})\,(\text{p2}^2-\text{p1}\cdot\text{p2})-$$
$$\text{p2}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2})+(\text{p1}^2-2\,(\text{p1}\cdot\text{p2})+\text{p2}^2)\,(\text{p1}^2+2\,\text{p2}^2))+$$
$$\text{p1}^\mu\,\text{p2}^\nu\,(D\,(\text{p1}\cdot\text{p2})^2+D\,\text{p1}^2\,\text{p2}^2-2\,\text{p1}^2\,\text{p2}^2)\,(-\text{p1}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2})+2\,(\text{p1}\cdot\text{p2})\,(\text{p2}^2-\text{p1}\cdot\text{p2})-$$
$$\text{p2}^2\,(\text{p2}^2-\text{p1}\cdot\text{p2})+(\text{p1}^2-2\,(\text{p1}\cdot\text{p2})+\text{p2}^2)\,(\text{p1}^2+2\,\text{p2}^2))\Big)\Big/$$
$$\Big(4\,(2-D)\,q^2.(-\text{p1}+\text{p2}+q)^2\,(\text{p1}^2-2\,(\text{p1}\cdot\text{p2})+\text{p2}^2)\,((\text{p1}\cdot\text{p2})^2-\text{p1}^2\,\text{p2}^2)^2\Big)$$

Of course we collect with respect to FAD and isolate the prefactors, but the full result still remains messy

In[ ]:=
```
Collect2[res, FeynAmpDenominator, IsolateNames → KK]
```

Out[ ]=

$$\frac{\text{KK}(1037)}{4\,q^2.(q-\text{p1})^2.(q-\text{p2})^2}-\frac{\text{KK}(1040)}{4\,q^2.(-\text{p1}+\text{p2}+q)^2}-\frac{\text{KK}(1042)}{4\,q^2.(q-\text{p1})^2}-\frac{\text{KK}(1044)}{4\,q^2.(q-\text{p2})^2}$$

In such cases, we can get a much more compact results, if we stick to coefficient functions and do not demand the full reduction to scalars . To do so, use the option UsePaVeBasis

In[ ]:=
```
res = TID[int, q, UsePaVeBasis → True]
```

Out[ ]=

$$i\pi^2\, g^{\mu\nu}\, C_{00}\left(p1^2,\, p2^2,\, -2\,(p1\cdot p2)+p1^2+p2^2,\, 0,\, 0,\, 0\right)+i\pi^2\, p1^\mu\, p1^\nu\, C_{11}\left(p1^2,\, -2\,(p1\cdot p2)+p1^2+p2^2,\, p2^2,\, 0,\, 0,\, 0\right)+$$
$$i\pi^2\, p2^\mu\, p2^\nu\, C_{11}\left(p2^2,\, -2\,(p1\cdot p2)+p1^2+p2^2,\, p1^2,\, 0,\, 0,\, 0\right)+$$
$$i\pi^2\,(p1^\nu\, p2^\mu+p1^\mu\, p2^\nu)\, C_{12}\left(p1^2,\, -2\,(p1\cdot p2)+p1^2+p2^2,\, p2^2,\, 0,\, 0,\, 0\right)$$

This can be further reduce (as with the option ToPaVe)

In[ ]:=
```
pvRes = PaVeReduce[res]
```

Out[ ]=

$$\Big(i\pi^2\, B_0\left(p1^2,\, 0,\, 0\right)$$
$$\big(D\, p1^\mu\, p1^\nu\,(p1\cdot p2)^3-2\, p1^\mu\, p1^\nu\,(p1\cdot p2)^3-g^{\mu\nu}\, p1^2\,(p1\cdot p2)^3+p1^\nu\, p2^\mu\, p1^2\,(p1\cdot p2)^2+p1^\mu\, p2^\nu\, p1^2\,(p1\cdot p2)^2+$$
$$D\, p1^\mu\, p1^\nu\, p2^2\,(p1\cdot p2)^2-2\, p1^\mu\, p1^\nu\, p2^2\,(p1\cdot p2)^2+g^{\mu\nu}\, p1^2\, p2^2\,(p1\cdot p2)^2-D\, p2^\mu\, p2^\nu\, p1^4\,(p1\cdot p2)+$$
$$p2^\mu\, p2^\nu\, p1^4\,(p1\cdot p2)+g^{\mu\nu}\, p1^4\, p2^2\,(p1\cdot p2)-2\, D\, p1^\mu\, p1^\nu\, p1^2\, p2^2\,(p1\cdot p2)+3\, p1^\mu\, p1^\nu\, p1^2\, p2^2\,(p1\cdot p2)-$$
$$D\, p1^\nu\, p2^\mu\, p1^2\, p2^2\,(p1\cdot p2)+p1^\nu\, p2^\mu\, p1^2\, p2^2\,(p1\cdot p2)-D\, p1^\mu\, p2^\nu\, p1^2\, p2^2\,(p1\cdot p2)+$$
$$p1^\mu\, p2^\nu\, p1^2\, p2^2\,(p1\cdot p2)-g^{\mu\nu}\, p1^4\, p2^4+p1^\mu\, p1^\nu\, p1^2\, p2^4+D\, p1^\nu\, p2^\mu\, p1^4\, p2^2-2\, p1^\nu\, p2^\mu\, p1^4\, p2^2+$$
$$D\, p1^\mu\, p2^\nu\, p1^4\, p2^2-2\, p1^\mu\, p2^\nu\, p1^4\, p2^2+D\, p2^\mu\, p2^\nu\, p1^4\, p2^2-p2^\mu\, p2^\nu\, p1^4\, p2^2\big)\Big)\Big/$$
$$\Big(4\,(2-D)\left((p1\cdot p2)^2-p1^2\, p2^2\right)^2\Big)-\Big(i\pi^2\, B_0\left(p2^2,\, 0,\, 0\right)\big(-D\, p2^\mu\, p2^\nu\,(p1\cdot p2)^3+2\, p2^\mu\, p2^\nu\,(p1\cdot p2)^3+$$
$$g^{\mu\nu}\, p2^2\,(p1\cdot p2)^3-D\, p2^\mu\, p2^\nu\, p1^2\,(p1\cdot p2)^2+2\, p2^\mu\, p2^\nu\, p1^2\,(p1\cdot p2)^2-p1^\nu\, p2^\mu\, p2^2\,(p1\cdot p2)^2-$$
$$p1^\mu\, p2^\nu\, p2^2\,(p1\cdot p2)^2-g^{\mu\nu}\, p1^2\, p2^2\,(p1\cdot p2)^2+D\, p1^\mu\, p1^\nu\, p2^4\,(p1\cdot p2)-p1^\mu\, p1^\nu\, p2^4\,(p1\cdot p2)-$$
$$g^{\mu\nu}\, p1^2\, p2^4\,(p1\cdot p2)+D\, p1^\nu\, p2^\mu\, p1^2\, p2^2\,(p1\cdot p2)-p1^\nu\, p2^\mu\, p1^2\, p2^2\,(p1\cdot p2)+D\, p1^\mu\, p2^\nu\, p1^2\, p2^2\,(p1\cdot p2)-$$
$$p1^\mu\, p2^\nu\, p1^2\, p2^2\,(p1\cdot p2)+2\, D\, p2^\mu\, p2^\nu\, p1^2\, p2^2\,(p1\cdot p2)-3\, p2^\mu\, p2^\nu\, p1^2\, p2^2\,(p1\cdot p2)+$$
$$g^{\mu\nu}\, p1^4\, p2^4-D\, p1^\mu\, p1^\nu\, p1^2\, p2^4+p1^\mu\, p1^\nu\, p1^2\, p2^4-D\, p1^\nu\, p2^\mu\, p1^2\, p2^4+2\, p1^\nu\, p2^\mu\, p1^2\, p2^4-$$
$$D\, p1^\mu\, p2^\nu\, p1^2\, p2^4+2\, p1^\mu\, p2^\nu\, p1^2\, p2^4-p2^\mu\, p2^\nu\, p1^4\, p2^2\big)\Big)\Big/\Big(4\,(2-D)\left((p1\cdot p2)^2-p1^2\, p2^2\right)^2\Big)-$$
$$\Big(i\pi^2\, B_0\left(p1^2-2\,(p1\cdot p2)+p2^2,\, 0,\, 0\right)\big(2\, g^{\mu\nu}\,(p1\cdot p2)^4+D\, p1^\mu\, p1^\nu\,(p1\cdot p2)^3-2\, p1^\mu\, p1^\nu\,(p1\cdot p2)^3+$$
$$D\, p1^\nu\, p2^\mu\,(p1\cdot p2)^3-4\, p1^\nu\, p2^\mu\,(p1\cdot p2)^3+D\, p1^\mu\, p2^\nu\,(p1\cdot p2)^3-4\, p1^\mu\, p2^\nu\,(p1\cdot p2)^3+$$
$$D\, p2^\mu\, p2^\nu\,(p1\cdot p2)^3-2\, p2^\mu\, p2^\nu\,(p1\cdot p2)^3-g^{\mu\nu}\, p1^2\,(p1\cdot p2)^3-g^{\mu\nu}\, p2^2\,(p1\cdot p2)^3+p1^\nu\, p2^\mu\, p1^2\,(p1\cdot p2)^2+$$
$$p1^\mu\, p2^\nu\, p1^2\,(p1\cdot p2)^2+2\, p2^\mu\, p2^\nu\, p1^2\,(p1\cdot p2)^2+2\, p1^\mu\, p1^\nu\, p2^2\,(p1\cdot p2)^2+p1^\nu\, p2^\mu\, p2^2\,(p1\cdot p2)^2+$$
$$p1^\mu\, p2^\nu\, p2^2\,(p1\cdot p2)^2-2\, g^{\mu\nu}\, p1^2\, p2^2\,(p1\cdot p2)^2-D\, p2^\mu\, p2^\nu\, p1^4\,(p1\cdot p2)+p2^\mu\, p2^\nu\, p1^4\,(p1\cdot p2)-$$
$$D\, p1^\mu\, p1^\nu\, p2^4\,(p1\cdot p2)+p1^\mu\, p1^\nu\, p2^4\,(p1\cdot p2)+g^{\mu\nu}\, p1^2\, p2^4\,(p1\cdot p2)+g^{\mu\nu}\, p1^4\, p2^2\,(p1\cdot p2)-$$
$$2\, D\, p1^\mu\, p1^\nu\, p1^2\, p2^2\,(p1\cdot p2)+3\, p1^\mu\, p1^\nu\, p1^2\, p2^2\,(p1\cdot p2)-3\, D\, p1^\nu\, p2^\mu\, p1^2\, p2^2\,(p1\cdot p2)+$$
$$6\, p1^\nu\, p2^\mu\, p1^2\, p2^2\,(p1\cdot p2)-3\, D\, p1^\mu\, p2^\nu\, p1^2\, p2^2\,(p1\cdot p2)+6\, p1^\mu\, p2^\nu\, p1^2\, p2^2\,(p1\cdot p2)-$$
$$2\, D\, p2^\mu\, p2^\nu\, p1^2\, p2^2\,(p1\cdot p2)+3\, p2^\mu\, p2^\nu\, p1^2\, p2^2\,(p1\cdot p2)+2\, D\, p1^\mu\, p1^\nu\, p1^2\, p2^4-4\, p1^\mu\, p1^\nu\, p1^2\, p2^4+$$
$$D\, p1^\nu\, p2^\mu\, p1^2\, p2^4-2\, p1^\nu\, p2^\mu\, p1^2\, p2^4+D\, p1^\mu\, p2^\nu\, p1^2\, p2^4-2\, p1^\mu\, p2^\nu\, p1^2\, p2^4+D\, p1^\nu\, p2^\mu\, p1^4\, p2^2-$$
$$2\, p1^\nu\, p2^\mu\, p1^4\, p2^2+D\, p1^\mu\, p2^\nu\, p1^4\, p2^2-2\, p1^\mu\, p2^\nu\, p1^4\, p2^2+2\, D\, p2^\mu\, p2^\nu\, p1^4\, p2^2-4\, p2^\mu\, p2^\nu\, p1^4\, p2^2\big)\Big)\Big/$$
$$\Big(4\,(2-D)\left((p1\cdot p2)^2-p1^2\, p2^2\right)^2\Big)+\Big(i\pi^2\, C_0\left(p1^2,\, p2^2,\, p1^2-2\,(p1\cdot p2)+p2^2,\, 0,\, 0,\, 0\right)$$
$$\big(g^{\mu\nu}\, p2^4\, p1^6-p2^\mu\, p2^\nu\, p2^2\, p1^6+g^{\mu\nu}\, p2^6\, p1^4-D\, p2^\mu\, p2^\nu\,(p1\cdot p2)^2\, p1^4+2\, p2^\mu\, p2^\nu\,(p1\cdot p2)^2\, p1^4-$$
$$D\, p1^\mu\, p1^\nu\, p2^4\, p1^4+p1^\mu\, p1^\nu\, p2^4\, p1^4-D\, p1^\nu\, p2^\mu\, p2^4\, p1^4+2\, p1^\nu\, p2^\mu\, p2^4\, p1^4-$$
$$D\, p1^\mu\, p2^\nu\, p2^4\, p1^4+2\, p1^\mu\, p2^\nu\, p2^4\, p1^4-D\, p2^\mu\, p2^\nu\, p2^4\, p1^4+p2^\mu\, p2^\nu\, p2^4\, p1^4-$$
$$2\, g^{\mu\nu}\,(p1\cdot p2)\, p2^4\, p1^4-g^{\mu\nu}\,(p1\cdot p2)^2\, p2^2\, p1^4+D\, p1^\nu\, p2^\mu\,(p1\cdot p2)\, p2^2\, p1^4-$$
$$p1^\nu\, p2^\mu\,(p1\cdot p2)\, p2^2\, p1^4+D\, p1^\mu\, p2^\nu\,(p1\cdot p2)\, p2^2\, p1^4-p1^\mu\, p2^\nu\,(p1\cdot p2)\, p2^2\, p1^4+$$
$$2\, D\, p2^\mu\, p2^\nu\,(p1\cdot p2)\, p2^2\, p1^4-2\, p2^\mu\, p2^\nu\,(p1\cdot p2)\, p2^2\, p1^4-p1^\mu\, p1^\nu\, p2^6\, p1^2-g^{\mu\nu}\,(p1\cdot p2)^2\, p2^4\, p1^2+$$
$$2\, D\, p1^\mu\, p1^\nu\,(p1\cdot p2)\, p2^4\, p1^2-2\, p1^\mu\, p1^\nu\,(p1\cdot p2)\, p2^4\, p1^2+D\, p1^\nu\, p2^\mu\,(p1\cdot p2)\, p2^4\, p1^2-$$
$$p1^\nu\, p2^\mu\,(p1\cdot p2)\, p2^4\, p1^2+D\, p1^\mu\, p2^\nu\,(p1\cdot p2)\, p2^4\, p1^2-p1^\mu\, p2^\nu\,(p1\cdot p2)\, p2^4\, p1^2+$$
$$2\, g^{\mu\nu}\,(p1\cdot p2)^3\, p2^2\, p1^2-D\, p1^\nu\, p2^\mu\,(p1\cdot p2)^2\, p2^2\, p1^2-D\, p1^\mu\, p2^\nu\,(p1\cdot p2)^2\, p2^2\, p1^2-$$
$$D\, p1^\mu\, p1^\nu\,(p1\cdot p2)^2\, p2^4+2\, p1^\mu\, p1^\nu\,(p1\cdot p2)^2\, p2^4\big)\Big)\Big/\Big(4\,(2-D)\left((p1\cdot p2)^2-p1^2\, p2^2\right)^2\Big)$$

Amplitudes

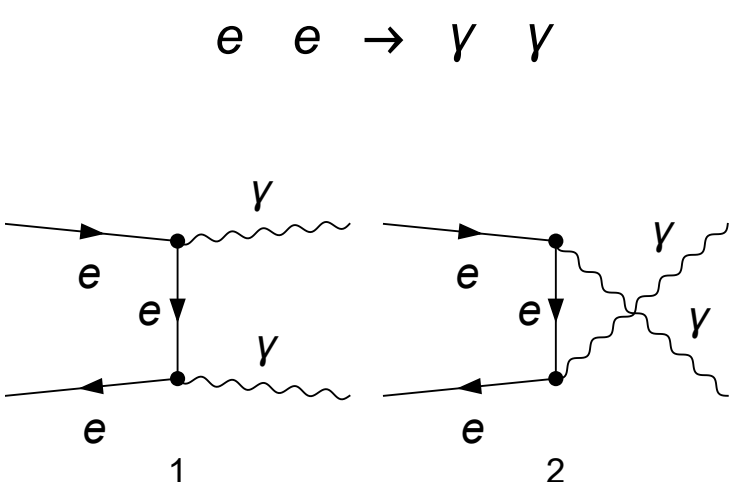
Tree Level

e+ e- -> a a

Generate Feynman diagrams

In[ ]:=
```
eeaaDiag =
  InsertFields[CreateTopologies[0, 2 → 2], {F[2, {1}], -F[2, {1}]} → {V[1], V[1]},
   Model → FileNameJoin[{NotebookDirectory[], "SM/SM"}], GenericModel →
    FileNameJoin[{NotebookDirectory[], "SM/SM"}], InsertionLevel → {Particles}];
```

In[ ]:=
```
Paint[eeaaDiag, ColumnsXRows → {2, 1}, Numbering → Simple, ImageSize → {512, 256}];
```



Obtain the amplitude

In[ ]:=
```
eeaaAmp = FCFAConvert[CreateFeynAmp[eeaaDiag], IncomingMomenta → {p1, p2},
  OutgoingMomenta → {k1, k2}, UndoChiralSplittings → True, ChangeDimension → 4,
  TransversePolarizationVectors → {k1, k2}, List → False, SMP → True, Contract → True]
```

Out[ ]=

$$-\frac{e^2\,(\varphi(-\overline{p2}, Me)).(\overline{\gamma}\cdot\overline{\varepsilon^*}(k1)).(\overline{\gamma}\cdot(\overline{k1}-\overline{p2})+Me).(\overline{\gamma}\cdot\overline{\varepsilon^*}(k2)).(\varphi(\overline{p1}, Me))}{(\overline{p2}-\overline{k1})^2-Me^2} -$$

$$\frac{e^2\,(\varphi(-\overline{p2}, Me)).(\overline{\gamma}\cdot\overline{\varepsilon^*}(k2)).(\overline{\gamma}\cdot(\overline{k2}-\overline{p2})+Me).(\overline{\gamma}\cdot\overline{\varepsilon^*}(k1)).(\varphi(\overline{p1}, Me))}{(\overline{p2}-\overline{k2})^2-Me^2}$$

Fix the kinematics

In[ ]:=
```
FCClearScalarProducts[];
SetMandelstam[s, t, u, p1, p2, -k1, -k2, Me, Me, 0, 0];
```

Square the amplitude

In[ ]:=
```
eeaaAmpSquared = (eeaaAmp ComplexConjugate[eeaaAmp]) // FeynAmpDenominatorExplicit //
    DoPolarizationSums[#, k1, 0] & // DoPolarizationSums[#, k2, 0] & //
  FermionSpinSum[#, ExtraFactor → 1 / 2^2] & // DiracSimplify //
 TrickMandelstam[#, {s, t, u, 2 Me^2}] & // Simplify
```

Out[ ]=

$$-\frac{2\,e^4\left(6\,Me^8-Me^4\left(3\,t^2+14\,t\,u+3\,u^2\right)+Me^2\left(t^3+7\,t^2\,u+7\,t\,u^2+u^3\right)-t\,u\left(t^2+u^2\right)\right)}{\left(Me^2-t\right)^2\left(Me^2-u\right)^2}$$

In[ ]:=
```
eeaaAmpSquaredMassless = eeaaAmpSquared /. Me → 0
```

Out[ ]=

$$\frac{2\,e^4\left(t^2+u^2\right)}{t\,u}$$

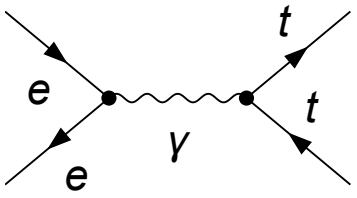
e+ e- -> a ->t tbar

Generate Feynman diagrams

In[ ]:=
```
eettDiag = InsertFields[CreateTopologies[0, 2 → 2],
   {F[2, {1}], -F[2, {1}]} → {F[3, {3}], -F[3, {3}]},
   Model → FileNameJoin[{NotebookDirectory[], "SM/SM"}],
   GenericModel → FileNameJoin[{NotebookDirectory[], "SM/SM"}],
   InsertionLevel → {Particles}, ExcludeParticles → {S[1], V[2]}];
```

In[ ]:=
```
Paint[eettDiag, ColumnsXRows → {1, 1}, Numbering → Simple, ImageSize → {512, 256}];
```

$$e\quad e\rightarrow\quad t\quad t$$



1

Obtain the amplitude

In[ ]:=
```
eettAmp = FCFAConvert[CreateFeynAmp[eettDiag],
  IncomingMomenta → {p1, p2}, OutgoingMomenta → {k1, k2},
  UndoChiralSplittings → True, ChangeDimension → 4, List → False, SMP → True,
  Contract → True, DropSumOver → True, Prefactor → 3 / 2 SMP["e_Q"]]
```

Out[ ]=

$$\frac{e^2\, e_Q\, \delta_{\text{Col3 Col4}}\, (\varphi(-\overline{\text{p2}}, \text{Me}))\,.\,\bar{\gamma}^{\text{Lor1}}\,.\,(\varphi(\overline{\text{p1}}, \text{Me}))\,(\varphi(\overline{\text{k1}}, m_t))\,.\,\bar{\gamma}^{\text{Lor1}}\,.\,(\varphi(-\overline{\text{k2}}, m_t))}{(\overline{\text{k1}} + \overline{\text{k2}})^2}$$

Fix the kinematics

In[ ]:=
```
FCClearScalarProducts[];
SetMandelstam[s, t, u, p1, p2, -k1, -k2, Me, Me, SMP["m_t"], SMP["m_t"]];
```

Square the amplitude

In[ ]:=
```
eettAmpSquared = (eettAmp ComplexConjugate[eettAmp]) // FeynAmpDenominatorExplicit //
    SUNSimplify[#, Explicit → True, SUNNToCACF → False] & //
   FermionSpinSum[#, ExtraFactor → 1 / 2^2] & // DiracSimplify //
  TrickMandelstam[#, {s, t, u, 2 Me^2 + 2 SMP["m_t"]^2}] & // Simplify
```

Out[ ]=

$$\frac{2\, e^4\, N\, e_Q^2\, \left(4\, m_t^2\, \left(\text{Me}^2 - u\right) + 2\, m_t^4 + 2\, \text{Me}^4 - 4\, \text{Me}^2\, u + s^2 + 2\, s\, u + 2\, u^2\right)}{s^2}$$

In[ ]:=
```
eettAmpSquaredMassless =
 eettAmpSquared /. {Me → 0, SMP["m_t"] → 0} // TrickMandelstam[#, {s, t, u, 0}] &
```

Out[ ]=

$$\frac{2\, e^4\, N\, e_Q^2\, \left(t^2 + u^2\right)}{s^2}$$

In[ ]:=
```
eettAmpSquaredMasslessSUNN3 = eettAmpSquaredMassless /. SUNN → 3
```

Out[ ]=

$$\frac{6\, e^4\, e_Q^2\, \left(t^2 + u^2\right)}{s^2}$$

In[ ]:=

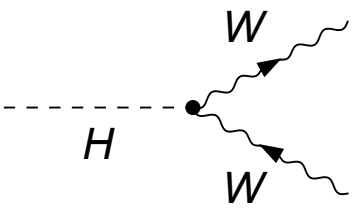
H -> W+ W-

Generate Feynman diagrams

In[ ]:=
```
hWWDiag = InsertFields[CreateTopologies[0, 1 → 2], {S[1]} → {V[3], -V[3]},
    Model → FileNameJoin[{NotebookDirectory[], "SM/SM"}], GenericModel →
     FileNameJoin[{NotebookDirectory[], "SM/SM"}], InsertionLevel → {Particles}];
```

In[ ]:=
```
Paint[hWWDiag, ColumnsXRows → {1, 1}, Numbering → Simple, ImageSize → {512, 256}];
```

$$H \to W\ W$$



1

Obtain the amplitude

In[ ]:=
```
hWWAmp = FCFAConvert[CreateFeynAmp[hWWDiag], IncomingMomenta → {p},
  OutgoingMomenta → {k1, k2}, UndoChiralSplittings → True, ChangeDimension → 4,
  TransversePolarizationVectors → {k1, k2}, List → False, SMP → True, Contract → True]
```

Out[ ]=

$$\frac{e^2\ \text{vev}\ (\bar{\varepsilon}^*(\text{k1}) \cdot \bar{\varepsilon}^*(\text{k2}))}{2\ \text{sw}^2}$$

Fix the kinematics

In[ ]:=
```
FCClearScalarProducts[];
SP[k1, k1] = MW^2;
SP[k2, k2] = MW^2;
SP[p, p] = MH^2;
SP[k1, k2] = MH^2 / 2 - MW^2;
```

Square the amplitude

In[ ]:=
```
hWWAmpSquared = (hWWAmp ComplexConjugate[hWWAmp]) // FeynAmpDenominatorExplicit //
    DoPolarizationSums[#, k1] & // DoPolarizationSums[#, k2] & // Simplify
```

Out[ ]=

$$\frac{e^4\ \text{vev}^2\ \left(\text{MH}^4 - 4\ \text{MH}^2\ \text{MW}^2 + 12\ \text{MW}^4\right)}{16\ \text{MW}^4\ \text{sw}^4}$$

In[ ]:=
```
hWWAmpSquaredAlt = hWWAmpSquared /. vev → 2 MW sw / SMP["e"] // Simplify
```

Out[ ]=

$$\frac{e^2\ \left(\text{MH}^4 - 4\ \text{MH}^2\ \text{MW}^2 + 12\ \text{MW}^4\right)}{4\ \text{MW}^2\ \text{sw}^2}$$

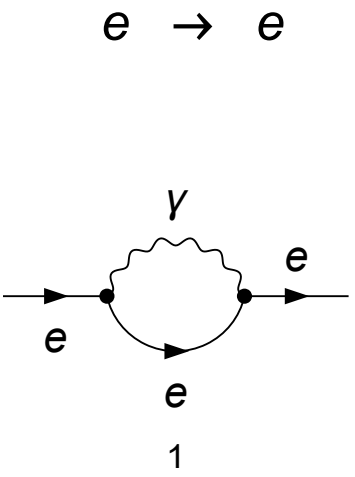
One Loop

e- -> e-

Generate Feynman diagrams

In[ ]:=
```
eeDiag = InsertFields[CreateTopologies[1, 1 → 1, ExcludeTopologies → Tadpoles],
    {F[2, {1}]} → {F[2, {1}]}, Model → FileNameJoin[{NotebookDirectory[], "SM/SM"}],
    GenericModel → FileNameJoin[{NotebookDirectory[], "SM/SM"}],
    InsertionLevel → {Particles}, ExcludeParticles → {S[_], V[2], V[3]}];
```

In[ ]:=
```
Paint[eeDiag, ColumnsXRows → {1, 1}, Numbering → Simple, ImageSize → {512, 256}];
```



Obtain the amplitude

In[ ]:=
```
(* The 1/(2*Pi)^D prefactor is implicit. We keep the full gauge dependence *)
```

In[ ]:=
```
eeAmp =
 FCFAConvert[CreateFeynAmp[eeDiag, Truncated → True, PreFactor → 1, GaugeRules → {}],
  IncomingMomenta → {p}, OutgoingMomenta → {p}, LoopMomenta → {q},
  UndoChiralSplittings → True, ChangeDimension → D, List → False, SMP → True]
```

Out[ ]=

$$(-i\,\mathrm{e}\,\gamma^{\mathrm{Lor2}}).(\mathrm{Me}+\gamma\cdot q).(-i\,\mathrm{e}\,\gamma^{\mathrm{Lor1}})\left(\frac{(1-\xi_A)\,(q-p)^{\mathrm{Lor1}}\,(p-q)^{\mathrm{Lor2}}}{(q^2-\mathrm{Me}^2).(q-p)^2}+\frac{g^{\mathrm{Lor1\,Lor2}}}{(q^2-\mathrm{Me}^2).(q-p)^2}\right)$$

Calculate the amplitude

In[ ]:=
```
eeAmp1 = TID[eeAmp, q, ToPaVe → True]
```

Out[ ]=

$$-\frac{1}{2\,p^2}\,i\,\pi^2\,\mathrm{e}^2\,\mathrm{B}_0\left(p^2,\,0,\,\mathrm{Me}^2\right)\left(-\mathrm{Me}^2\,\xi_A\,\gamma\cdot p+2\,\mathrm{Me}\,\xi_A\,(\gamma\cdot p).(\gamma\cdot p)-p^2\,\xi_A\,\gamma\cdot p-D\left(\mathrm{Me}^2-p^2\right)\gamma\cdot p+\right.$$
$$\left.2\,D\,\mathrm{Me}\,p^2-2\,D\,p^2\,\gamma\cdot p+\mathrm{Me}^2\,\gamma\cdot p+2\left(\mathrm{Me}^2-p^2\right)\gamma\cdot p-2\,\mathrm{Me}\,(\gamma\cdot p).(\gamma\cdot p)+5\,p^2\,\gamma\cdot p\right)+$$
$$\frac{i\,\pi^2\,\mathrm{e}^2\,(1-\xi_A)\,\mathrm{B}_0(0,\,0,\,0)\left(\mathrm{Me}^2\,\gamma\cdot p-2\,\mathrm{Me}\,(\gamma\cdot p).(\gamma\cdot p)+2\,\mathrm{Me}\,p^2-p^2\,\gamma\cdot p\right)}{2\,p^2}+$$
$$\frac{1}{2\,p^2}\,i\,\pi^2\,\mathrm{e}^2\,(1-\xi_A)\left(2\,\mathrm{Me}^3\,p^2+\mathrm{Me}^2\left(\mathrm{Me}^2-p^2\right)\gamma\cdot p-2\,\mathrm{Me}\left(\mathrm{Me}^2-p^2\right)(\gamma\cdot p).(\gamma\cdot p)-p^2\left(\mathrm{Me}^2-p^2\right)\gamma\cdot p-\right.$$
$$\left.4\,\mathrm{Me}\,p^2\,(\gamma\cdot p).(\gamma\cdot p)+2\,\mathrm{Me}\,p^4\right)\mathrm{C}_0\left(0,\,p^2,\,p^2,\,0,\,0,\,\mathrm{Me}^2\right)+\frac{i\,\pi^2\,(2-D)\,\mathrm{e}^2\,\mathrm{A}_0\left(\mathrm{Me}^2\right)\gamma\cdot p}{2\,p^2}$$

In[ ]:=
```
eeAmpDiv = PaVeUVPart[eeAmp1, Prefactor → 1 / (2 Pi)^D] /. D → 4 - 2 Epsilon //
    Series[#, {Epsilon, 0, 0}] & // Normal // SelectNotFree2[#, Epsilon] & // Simplify
```

Out[ ]=

$$-\frac{i\,\mathrm{e}^2\,(\xi_A\,(\mathrm{Me}-\overline{\gamma}\cdot\overline{p})+3\,\mathrm{Me})}{16\,\pi^2\,\varepsilon}$$

In[ ]:=
```
sigma = I eeAmpDiv
```

Out[ ]=

$$\frac{\mathrm{e}^2\,(\xi_A\,(\mathrm{Me}-\overline{\gamma}\cdot\overline{p})+3\,\mathrm{Me})}{16\,\pi^2\,\varepsilon}$$

In[ ]:=
```
sigmaFeynmanGauge = sigma /. GaugeXi[A] → 1
```

Out[ ]=

$$\frac{\mathrm{e}^2\,(4\,\mathrm{Me}-\overline{\gamma}\cdot\overline{p})}{16\,\pi^2\,\varepsilon}$$

Some Extras

We can now refer to the full FeynCalc Manual and learn some extra things, and try working with everything we have seen.

Some extra features are nonrelativistic calculations, Pauli algebra, γ^5 to chiral projectors and viceversa, LaTeX exports...