Quantum Colliders

Quantum Information and Computation for Particle Physics

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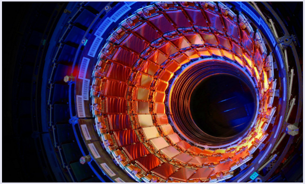
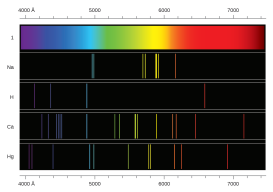
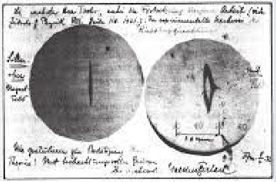


Quantum Theory: Quantization

- Quantum Mechanics originally named after quantized values:
  1. Electromagnetic radiation (Black-body/Photoelectric effect)
  2. Electron orbits (Atomic spectra)
  3. Angular momentum (Stern-Gerlach)





Quantum Theory: Copenhagen Interpretation

- Copenhagen interpretation of Quantum Mechanics:
  - Particles $\longleftrightarrow$ Waves $\rightarrow$ Superposition
  - Outcomes of measurements: Observable eigenvalues $\rightarrow$ Quantization
  - Probabilities of outcomes $\rightarrow$ Wave Intensity $|\Psi|^2$ $\rightarrow$ Interference

Quantum State

- **Pure state** $\rightarrow$ Wave function $|\Psi\rangle$
  1. Unit vector in Hilbert space $\mathcal{H}$
  2. $|\Psi\rangle = \sum_n \alpha_n \cdot |\phi_n\rangle\,, \ \langle\Psi|\Psi\rangle = \sum_n |\alpha_n|^2 = 1$
  3. Coherent mixture $\rightarrow$ α_n are complex amplitudes
  4. Expectation values: $\langle A\rangle = \langle\Psi|A|\Psi\rangle = \sum_{n,m} \alpha_m^* \alpha_n \langle\phi_m|A|\phi_n\rangle$
- **Mixed state** $\rightarrow$ Generalization to density matrix ρ
  1. Hermitian non-negative operator with unit trace in Hilbert space $\mathcal{H}$
  2. $\rho = \sum_n p_n \cdot |\phi_n\rangle\langle\phi_n|\,, \ \mathrm{Tr}\rho = \sum_n p_n = 1,\ p_n \geq 0$
  3. Incoherent mixture $\rightarrow$ p_n are probabilities
  4. Expectation values: $\langle A\rangle = \mathrm{Tr}(\rho A) = \sum_n p_n \langle\phi_n|A|\phi_n\rangle$

Quantum vs. Classical

- Quantum Mechanics: Superposition $\rightarrow$ *Fundamental* probabilistic description of measurements.
- Classical Mechanics: Random outputs using classical probability distributions resulting from *ignorance* (noise, experimental variations...)
- Is God *just* playing dice with the Universe? $\rightarrow$

Quantum vs. Classical

- Quantum Mechanics: Superposition $\rightarrow$ *Fundamental* probabilistic description of measurements.
- Classical Mechanics: Random outputs using classical probability distributions resulting from *ignorance* (noise, experimental variations...)
- Is God *just* playing dice with the Universe? $\rightarrow$ God well beyond a mere croupier!
- Quantum Correlations=Correlations not accounted by classical probabilistic theories.

Quantum Optics

- Light has motivated the major advancements in modern Physics:
  - EM radiation: Maxwell equations
  - Special Relativity: Michelson-Morley experiment
  - Quantum Mechanics: Black-body spectrum & Photoelectric effect
  - Quantum Field Theory: First quantized field

- Major paradigm in Quantum Information:
  - Discrete variables: Polarization, Orbital Angular Momentum
  - Continuous variables: Quantum amplitudes (Photons)

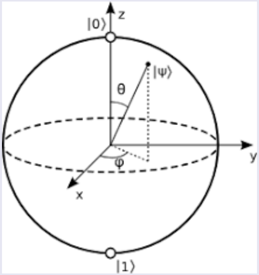
. . . and God said: "let there be light" . . .

Qubits: Pure state

- Qubit: Two-level quantum system $|0\rangle, |1\rangle \to$ Most simple!
- Paradigmatic example: Spin-1/2 particle. $|0\rangle \equiv |+\rangle$, $|1\rangle \equiv |-\rangle$
- General wave function: $|\Psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\phi}|1\rangle \equiv |\hat{\mathbf{n}}\rangle \to$ Eigenstate of spin projection: $\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}|\hat{\mathbf{n}}\rangle = |\hat{\mathbf{n}}\rangle$
- Projective measurements with ± 1 outcome:

$$\Pi_{\pm\hat{\mathbf{n}}} = |\pm\hat{\mathbf{n}}\rangle\langle\pm\hat{\mathbf{n}}| = \frac{1\pm\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}}{2}$$

- Unit vector $\hat{\mathbf{n}} = [\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta] \to$ Surface of Bloch sphere.



Qubits: Density matrix

- General density matrix (2×2) for 1 qubit $\rightarrow$ 3 parameters B_i (Bloch vector):

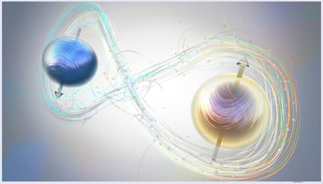
$$\rho = \frac{1 + \sum_i B_i \sigma^i}{2}, \ \mathbf{B} = \mathrm{Tr}[\boldsymbol{\sigma} \rho], \ |\mathbf{B}| \leq 1$$

- Two qubits $\rightarrow$ Most simple example of quantum correlations.
- General density matrix (4×4) for 2 qubits $\rightarrow$ 15 parameters $B_i^{\pm}$, C_{ij}

$$\rho = \frac{\mathbb{1} + \sum_i \left(B_i^+ \sigma^i \otimes \mathbb{1} + B_i^- \mathbb{1} \otimes \sigma^i \right) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}$$

- Polarization (Bloch) vectors $\mathbf{B}^{\pm}$ and correlation matrix $\mathbf{C}$:

$$B_i^+ = \langle \sigma^i \otimes \mathbb{1} \rangle, \ B_i^- = \langle \mathbb{1} \otimes \sigma^i \rangle, \ C_{ij} = \langle \sigma^i \otimes \sigma^j \rangle$$



Quantum Tomography: Two qubits

- **Quantum Tomography**: Reconstruction of quantum state from measurement of a *quorum* of observables.
- Quantum tomography → Characterization of ALL quantum correlations.
- One-qubit tomography=3 parameters: polarization vector **B**

$$B_i = \langle \sigma^i \rangle$$

- Two-qubit tomography=15 parameters: polarization vectors $\mathbf{B}^{\pm}$ and correlation matrix **C**

$$B_i^+ = \langle \sigma^i \otimes \mathbb{1} \rangle\,,\ B_i^- = \langle \mathbb{1} \otimes \sigma^i \rangle\,,\ C_{ij} = \langle \sigma^i \otimes \sigma^j \rangle$$

Quantum Discord

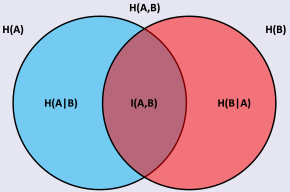
- Classically, two equivalent expressions for mutual information of bipartite system A and B (Alice and Bob):

$$
\begin{aligned}
I(A,B) &= H(A)+H(B)-H(A,B) = H(A)-H(A|B)\\
H(A,B) &= -\sum_{x,y} p(x,y)\log_2 p(x,y)\\
H(A|B) &= \sum_{y} p(y) H(A|B=y)
\end{aligned}
$$

- Quantum Mechanics can introduce a "discord" between both expressions:

PRL 88, 017901 (2001)

$$\mathcal{D}(A,B) \equiv H(B)-H(A,B)+H(A|B) \neq 0$$



- Most basic form of quantum correlations!
- Quantum discord is asymmetric: $\mathcal{D}(A,B) \neq \mathcal{D}(B,A)$

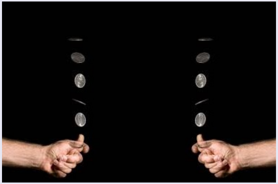
Quantum Discord: Classical states

- OK...but where is the Physics here? $\rightarrow$ Only classical states have zero discord!

$$\rho_{\text{class}} = \sum_{n,m} p_{n,m} \left|n\right\rangle \otimes \left|m\right\rangle \left\langle n\right| \otimes \left\langle m\right|$$

- $\left|n\right\rangle, \left|m\right\rangle$ form an *orthonormal* basis for A, B
- $p_{n,m}$: Classical probability of $\left|n\right\rangle \otimes \left|m\right\rangle$
- Qubits $\rightarrow$ Tails and heads with two coins!

$$\begin{aligned}\rho_{\text{class}} &= p_{++}\left|++\right\rangle\left\langle ++\right| + p_{+-}\left|+-\right\rangle\left\langle +-\right| \\ &+ p_{-+}\left|-+\right\rangle\left\langle -+\right| + p_{--}\left|--\right\rangle\left\langle --\right|\end{aligned}$$



Entanglement

- What if we generalize the previous idea? $\rightarrow$ Separability:

$$\rho_{\text{sep}} = \sum_{n,m} p_{n,m} \left|n\right\rangle \otimes \left|m\right\rangle \left\langle n\right| \otimes \left\langle m\right| = \sum_k p_k \rho_k^{(A)} \otimes \rho_k^{(B)}$$

- $\left|n\right\rangle, \left|m\right\rangle$ not necessarily orthonormal basis now $\rightarrow$ $p_{n,m}$ are *quasi-probabilities* (not disjoint events)
- Any classically correlated state (classical probability) is separable.
- **Entanglement**: Non-separability of a quantum state.

Entanglement: Two qubits

- Two qubits: Separability$=\exists$ Positive P-representation $P(\mathbf{n}_A, \mathbf{n}_B) \geq 0$:

$$\rho = \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B \, P(\mathbf{n}_A, \mathbf{n}_B) \left|\mathbf{n}_A \mathbf{n}_B\right\rangle \left\langle \mathbf{n}_A \mathbf{n}_B\right|, \quad \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B \, P(\mathbf{n}_A, \mathbf{n}_B) = 1$$

- $P(\mathbf{n}_A, \mathbf{n}_B)$ is a quasi-probability: Overlap $|\langle \mathbf{n}_A | \mathbf{n}_B \rangle|^2 \neq 0$
- Separability$=$Purely classical spins pointing at directions $\mathbf{n}_A, \mathbf{n}_B$

$$C_{ij} = \langle \sigma^i \otimes \sigma^j \rangle = \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B \; P(\mathbf{n}_A, \mathbf{n}_B) n_A^i n_B^j$$

- Entanglement$=$NO positive P-representation $\rightarrow$ Genuine non-classical!

Entanglement: Two qubits

- Practical entanglement criteria:
  - Peres-Horodecki criterion: ρ separable $\to$ Partial tranpose $\rho^{T_B} = \sum_n p_n \rho_n^A \otimes (\rho_n^B)^T$ non-negative. Negative-valued $\rho^{T_B} \to \rho$ entangled!
    Very powerful: 2×2, $2 \times 3 \to$ Peres-Horodecki criterion=Entanglement
    Peres, PRL 77, 1413 (1996), Horodecki, PLA 232, 333 (1997).
  - Concurrence $0 \leq C[\rho] \leq 1$, $C[\rho] > 0$ iff ρ entangled. Wootters, PRL 80, 2245 (1998)
  - Entanglement witness: Negative-valued observable W such that $\langle W \rangle_{\mathrm{sep}} = \mathrm{Tr}[W \rho_{\mathrm{sep}}] \geq 0 \Longrightarrow \langle W \rangle < 0$ implies entanglement. Terhal, PLA 271, 319 (2000)


Steering: Two qubits

- Steering: Original conception of Schrödinger of EPR paradox → Only well-defined in 2007! (Wiseman, Jones, Doherty, PRL 98, 140402 (2007))
- *Local* Quantum Mechanics: Alice post-measurement (un-normalized) state described by local-hidden states:

$$R_{\hat{\mathbf{n}}}=\Pi^B_{\hat{\mathbf{n}}}\rho\Pi^B_{\hat{\mathbf{n}}}=\int d\lambda\; p(1|\hat{\mathbf{n}}\lambda)p(\lambda)\rho_B(\lambda)$$

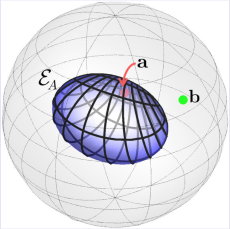
- If not, Bob measurements can "steer" Alice quantum state → **Steerability**.

Steering: Two qubits

- 1-qubit Alice post-measurement state → Conditional Bloch vector:

$$\rho_{\hat{\mathbf{n}}} = \frac{R_{\hat{\mathbf{n}}}}{\mathrm{Tr}[R_{\hat{\mathbf{n}}}]} = \frac{1 + \mathbf{B}^{+}_{\hat{\mathbf{n}}} \cdot \sigma}{2}, \quad \mathbf{B}^{+}_{\hat{\mathbf{n}}} = \frac{\mathbf{B}^{+} + \mathbf{C} \cdot \hat{\mathbf{n}}}{1 + \hat{\mathbf{n}} \cdot \mathbf{B}^{-}}$$

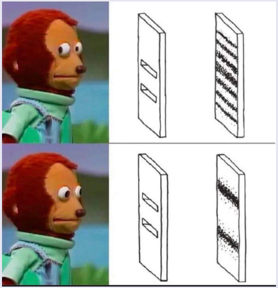
- Set of conditional polarizations $\mathbf{B}^{+}_{\hat{\mathbf{n}}}$ describes the...
- **Steering ellipsoid**: Fundamental QI object, containing all information about the system.
- Similar for Bob → Steering: also asymmetric between Alice and Bob.

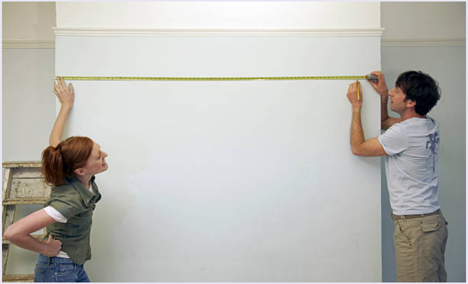


Jevtic, Pusey, Jennings, Rudolph
PRL 113, 020402 (2014)

Bell nonlocality

- EPR Paradox: Quantum Mechanics challenges local realism!
- Bell test: Joint Alice and Bob perform causally disconnected measurements in alternate freely chosen setups M_A, M'_A, M_B, M'_B
- Bell Theorem: Local realistic theories satisfy certain (Bell) inequalities
- Beauty: NO underlying model, just two people enjoying their measurements!





Bell nonlocality: Two qubits

- Two qubits: M_A, M_B projective spin measurements along axes $\mathbf{a}_i, \mathbf{b}_i$

$$C(M_A, M_B) = \sum_{a,b=\pm 1} (a \cdot b) p(a, b | M_A M_B)$$

- Local realistic theory $\to$ Local hidden-variable model (LHVM):

$$p(a, b | M_A M_B) = \int d\lambda \; p(a | M_A \lambda) p(b | M_B \lambda) p(\lambda)$$

- **Bell (CHSH) inequality** $\to$ LHVM satisfy:

$$|C(M_A, M_B) - C(M_A, M'_B) + C(M'_A, M_B) + C(M'_A, M'_B)| \leq 2$$

- QM prediction:

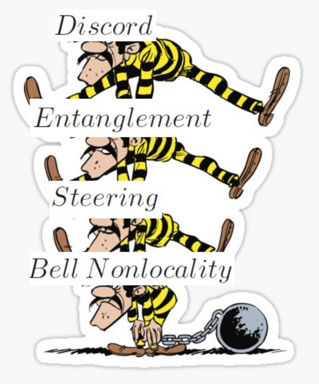
$$|\mathbf{a}_1^{\mathrm{T}} \mathbf{C} \left(\mathbf{b}_1 - \mathbf{b}_2 \right) + \mathbf{a}_2^{\mathrm{T}} \mathbf{C} \left(\mathbf{b}_1 + \mathbf{b}_2 \right) | \leq 2$$

- CHSH violation *iff* $\sqrt{\mu_1 + \mu_2} > 1$, $\mu_{1,2}$ largest eigenvalues of $\mathbf{C}^{\mathrm{T}} \mathbf{C}$
- Bell nonlocal states: Quantum states able to violate CHSH inequality

Hierarchy of Quantum Correlations

- Steering and Discord: asymmetric.
- Bell Nonlocality and Entanglement: symmetric.
- Quantum Hierarchy:

$$\textit{Bell Nonlocality} \subset \textit{Steering} \subset \textit{Entanglement} \subset \textit{Discord}$$



Harmonic oscillators

- EM field quantization $\sim$ Massless KG quantization:

$$\hat{\Phi}(\mathbf{x}) = \int \mathrm{d}^3\mathbf{k}\ [\hat{a}(\mathbf{k})\Phi_{\mathbf{k}}(\mathbf{x}) + \hat{a}^\dagger(\mathbf{k})\Phi^*_{\mathbf{k}}(\mathbf{x})]$$

- Canonical commutation rules $\rightarrow [\hat{a}(\mathbf{k}), \hat{a}^\dagger(\mathbf{k}')] = \delta(\mathbf{k} - \mathbf{k}')$
- QFT=Harmonic Oscillator ensemble $\rightarrow$ HO Textbook:
  - Phase space variables: $\hat{X} = \dfrac{\hat{a} + \hat{a}^\dagger}{\sqrt{2}}$, $\hat{P} = i\dfrac{\hat{a}^\dagger - \hat{a}}{\sqrt{2}}$
  - Number states: $\hat{a}^\dagger\hat{a}\,|n\rangle = n\,|n\rangle$
  - Coherent states: $\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle$, $|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}}\sum_{n=0}^{\infty}\dfrac{\alpha^n}{\sqrt{n!}}\,|n\rangle$, $\alpha \in \mathbb{C}$
  - Squeezed coherent states: Coherent states of Bogoliubov transformation

$$U = e^{\lambda\frac{(\hat{a}^\dagger)^2 - \hat{a}^2}{2}},\ \hat{b} = U^\dagger\hat{a}U = (\cosh\lambda)\hat{a} + (\sinh\lambda)\hat{a}^\dagger$$

Fock, P and Wigner representations

- Number states: complete orthonormal basis $I = \sum_n |n\rangle\langle n|$

$$\rho = \sum_{n,m} \rho_{nm} |n\rangle\langle m|$$

- P-representation: Normal-ordered expectation values

$$\rho = \int \mathrm{d}^2\alpha\ P(\alpha)\,|\alpha\rangle\langle\alpha|\,,\ \int \mathrm{d}^2\alpha\ |\alpha|^2 P(\alpha) = \langle \hat{a}^\dagger \hat{a}\rangle\ \int \mathrm{d}^2\alpha\ P(\alpha) = 1,$$

- Wigner representation ($\alpha = \frac{X+iP}{\sqrt{2}}$): *Symmetric* expectation values $\rightarrow$ Quantum Boltzmann distribution:

$$W(X,P) = \frac{1}{2\pi}\int \mathrm{d}x\ \langle X+\frac{x}{2}|\rho|X-\frac{x}{2}\rangle\, e^{-iPx}$$

$$\int \mathrm{d}^2\alpha\ |\alpha|^2 W(\alpha) = \langle\frac{\hat{a}^\dagger\hat{a}+\hat{a}\hat{a}^\dagger}{2}\rangle$$

Quantum Correlations

- *Quasi-probability* distributions: $I = \frac{1}{\pi}\int \mathrm{d}^2\alpha \ |\alpha\rangle\langle\alpha|\,, \ |\langle\alpha|\beta\rangle|^2 \neq 0$
- General quantum state of EM field:

$$\rho = \int \mathrm{d}^2\{\alpha_{\mathbf{k}}\}\ P(\{\alpha_{\mathbf{k}}\}) \prod_{\mathbf{k}} \otimes |\alpha_{\mathbf{k}}\rangle\langle\alpha_{\mathbf{k}}|$$

- Classical EM field $\sim P(\alpha_{\mathbf{k}}) \geq 0, \ |\alpha_{\mathbf{k}}| \gg 1, |\Delta\alpha_{\mathbf{k}}/\alpha_{\mathbf{k}}| \ll 1$
- Measurements=Normal-ordered expectation values
  - Positive P-representation $\Longrightarrow$ Classical expectation value
  - Quantum correlations $\Longrightarrow$ Negative P-representations!
- Bipartite Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$: Product space of Fock spaces of two modes $\hat{a}_i, \hat{a}_j \rightarrow$ Infinite dimension!

$$\rho = \int \mathrm{d}^2\alpha_i \mathrm{d}^2\alpha_j \ P(\alpha_i, \alpha_j) |\alpha_i\alpha_j\rangle\langle\alpha_i\alpha_j|$$

Continuous Variable Entanglement

- Continuous system: Physical density matrix $\Longrightarrow$ Non-negative covariance matrix M (uncertainty principle)

$$
\begin{aligned}
M_{\alpha\beta} &= \langle \Delta\xi_\alpha \Delta\xi_\beta \rangle = \frac{\langle \{\Delta\xi_\alpha \Delta\xi_\beta\} \rangle}{2} + \frac{\langle [\Delta\xi_\alpha \Delta\xi_\beta] \rangle}{2} \equiv V_{\alpha\beta} + i\frac{\Omega_{\alpha\beta}}{2} \\
\Delta\xi_\alpha &= \xi_\alpha - \langle \xi_\alpha \rangle\,,\ \xi_\alpha = [X_1, X_2, P_1, P_2],
\end{aligned}
$$

- Transpose operation via Wigner representation is time inversion!

$$
\begin{aligned}
\rho \quad &: \quad W(X_1, X_2, P_1, P_2) \to \rho^{T_2} : \ W(X_1, X_2, P_1, -P_2) \\
&\to \quad V' = \Lambda V \Lambda,\ \Lambda \equiv \mathrm{diag}[1, 1, 1, -1]
\end{aligned}
$$

- Peres-Horodecki: ρ^{T_2} covariance matrix $M' = V' + i\frac{\Omega}{2}$ not non-negative $\to$ Entanglement!
- Gaussian state$\to$ Peres-Horodecki$=$Entanglement

Simon, PRL 84, 2726 (2000)

Cauchy-Schwarz violation

- Classical light (non-negative P-representation) satisfies

$$|\langle \hat{a}^2 \rangle| = \left| \int \mathrm{d}^2\alpha \ P(\alpha)\alpha^2 \right| \leq \int \mathrm{d}^2\alpha \ P(\alpha)|\alpha|^2 = \langle \hat{a}^\dagger \hat{a} \rangle$$

- Proof explicitly based on positivity of P $\rightarrow$ Effective scalar product $\rightarrow$ Cauchy-Schwarz inequalities!
- Operators A, B do satisfy mathematical Cauchy-Schwarz inequality:

$$\langle A|B \rangle \equiv \mathrm{Tr}(\rho A^\dagger B) = \langle A^\dagger B \rangle$$

- *Quantum* CS inequality: $|\langle \hat{a}^2 \rangle| \leq \langle \hat{a}\hat{a}^\dagger \rangle \langle \hat{a}^\dagger \hat{a} \rangle = \langle \hat{a}^\dagger \hat{a} \rangle (\langle \hat{a}^\dagger \hat{a} \rangle + 1)$
- There is room for violation of *classical* CS inequality!
- Explicit example: Squeezed vacuum $|\lambda\rangle = e^{\lambda \frac{(\hat{a}^\dagger)^2 - \hat{a}^2}{2}} |0\rangle$:

$$|\langle \hat{a}^2 \rangle| = \cosh\lambda |\sinh\lambda| > \sinh^2\lambda = \langle \hat{a}^\dagger \hat{a} \rangle, \ \lambda \neq 0 \in \mathbb{R}$$

Cauchy-Schwarz violation: Bipartite case

- Quantum correlations tested via

$$g_{ij} = \langle \hat{a}_i^\dagger \hat{a}_j \rangle\,,\ c_{ij} = \langle \hat{a}_i \hat{a}_j \rangle\,,\ \Gamma_{ij} = \langle \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_i \rangle$$

- Classical vs. Quantum CS inequality
  - $|c_{ij}|^2 \leq g_{ii} g_{jj}$ **vs.** $|c_{ij}|^2 \leq g_{ii}(g_{jj}+1)$
  - $\Gamma_{ij} \leq \sqrt{\Gamma_{ii}\Gamma_{jj}}$ **vs.** $\Gamma_{ij} \leq \sqrt{(\Gamma_{ii}+g_{ii})(\Gamma_{jj}+g_{jj})}$
- CS violation *independent* from entanglement $\rightarrow$ Complementary test of quantum behavior!
- Explicit example: Product number state $\rho = |nm\rangle\langle nm|$ with $n, m \neq 0$

$$\Gamma_{ij} = nm > \sqrt{n(n-1)m(m-1)} = \sqrt{\Gamma_{ii}\Gamma_{jj}}$$

- Number states do have negative Wigner representations, strongest requirement than negative P-representation $\rightarrow$ Entanglement is not the whole story...

Entanglement: practical resource

- PhD question: What makes Hawking radiation quantum? $\rightarrow$ Experimental competition: Zero-point quantum effect **vs.** Thermal or coherent stimulation
- PhD answer: Quantum correlations=Smoking gun of genuinely quantum Hawking effect!

PHYSICAL REVIEW A **89**, 043808 (2014)

Violation of Cauchy-Schwarz inequalities by spontaneous Hawking radiation in resonant boson structures

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PAPER

Entanglement and violation of classical inequalities in the Hawking radiation of flowing atom condensates

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- Eventual experimental observations of Hawking radiation for the first (and sole!) time:

LETTER
https://doi.org/10.1038/s41586-019-1241-0

Observation of thermal Hawking radiation and its temperature in an analogue black hole

Juan Ramón Muñoz de Nova[1], Katrine Golubkov[1], Victor I. Kolobov[1] & Jeff Steinhauer[1]*

nature physics — ARTICLES
https://doi.org/10.1038/s41567-020-01076-0

Observation of stationary spontaneous Hawking radiation and the time evolution of an analogue black hole

Victor I. Kolobov, Katrine Golubkov, Juan Ramón Muñoz de Nova and Jeff Steinhauer

How did I end up here????

- So many analogies...why not studying quantum problems in High-Energy Physics, a relativistic QFT?
- One day, after years of coffee breaks at the Technion with my friend Yoav Afik...

- Juan: Mmmm...Could you measure a CS violation in a collider? It is like entanglement but not the same and...
- Yoav: This Cauchy-Schwarz thing is weird...Entanglement is interesting. Tell me more. With fermions.
- Juan: Oh, well, then there is the spin, it is a qubit you see, there are products of Pauli matrices...
- Yoav: Better. Give me a sec.

Dramatization

Top Drosophila

- Yoav eventually came to my office and showed me one single equation on top quarks...

$$R^I_{\alpha_1\alpha_2,\ \beta_1\beta_2} = \overline{\sum} \langle t(k_1,\alpha_2), \bar{t}(k_2,\beta_2)|\mathcal{T}|a(p_1), b(p_2)\rangle^* \times \langle t(k_1,\alpha_1), \bar{t}(k_2,\beta_1)|\mathcal{T}|a(p_1), b(p_2)\rangle \qquad (2.3)$$

where $I \equiv ab = gg, q\bar{q}$ and the bar denotes averaging over the spins and colors of I and summing over the colors of $t, \bar{t}$. Moreover, α, β are spin labels referring to t and $\bar{t}$, respectively. The matrices R^I can be decomposed in the spin spaces of t and $\bar{t}$ as follows:

$$R^I = f_I \left[A^I \mathbb{1} \otimes \mathbb{1} + \tilde{B}^{I+}_i \sigma^i \otimes \mathbb{1} + \tilde{B}^{I-}_i \mathbb{1} \otimes \sigma^i + \tilde{C}^I_{ij} \sigma^i \otimes \sigma^j \right], \qquad (2.4)$$

$$f_{gg} = \frac{(4\pi\alpha_s)^2}{N_c(N_c^2-1)}, \qquad f_{q\bar{q}} = \frac{(N_c^2-1)(4\pi\alpha_s)^2}{N_c^2},$$

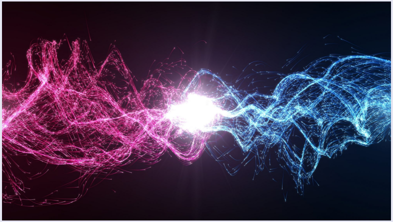
where N_c denotes the number of colors. The first (second) factor in the tensor products of the 2×2 unit matrix $\mathbb{1}$ and of the Pauli matrices σ^i refers to the t ($\bar{t}$) spin space.

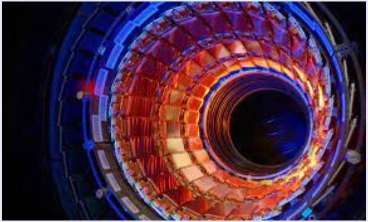
W. Bernreuther, D. Heisler, Z.-G. Si, JHEP 12, 026 (2015)

- Top quarks are a drosophila of relativistic two-qubit system $\rightarrow$ Huge potential!

Quantum High-Energy Colliders?

- Naively, Quantum Correlations should easily emerge in colliders...right? $\rightarrow$ Not so fast!
  - Momentum measurement $\rightarrow$ Decoherence
  - Lack of control of internal d.o.f. in initial state $\rightarrow$ Decoherence
  - Most relevant observables in colliders: cross-sections, lifetimes... $\rightarrow$ Classical probabilistic objects
  - QFT cares about expectation values, not quantum states!





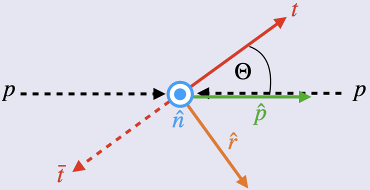
Top-pair production: Kinematics

- $t\bar{t}$ production: Kinematics determined by invariant mass $M_{t\bar{t}}$ and top direction $\hat{k}$ in c.m. frame

$$\begin{aligned} k^\mu &= (k^0, \mathbf{k}), \bar{k}^\mu = (k^0, -\mathbf{k}) \\ M_{t\bar{t}}^2 &\equiv s \equiv (k+\bar{k})^2 \end{aligned}$$

- Invariant mass $M_{t\bar{t}}$ is simply related to top c. m. velocity β

$$M_{t\bar{t}} = \frac{2m_t}{\sqrt{1-\beta^2}} \Longrightarrow \beta = 0 \to M_{t\bar{t}} = 2m_t \simeq 346\ \mathrm{GeV}$$



Top-pair production: Quantum State

- Production process from initial state I with internal degrees of freedom λ: $|I\lambda\rangle \to t + \bar{t}$
- $t\bar{t}$ spins described by production spin density matrix (R-matrix):

$$R^{I\lambda}_{\alpha\beta,\alpha'\beta'}(M_{t\bar{t}},\hat{k}) \equiv \langle M_{t\bar{t}}\hat{k}\alpha\beta|\, T\,|I\lambda\rangle\,\langle I\lambda|\, T^{\dagger}\,|M_{t\bar{t}}\hat{k}\alpha'\beta'\rangle$$

- Experiment: Momentum measurements+Average over events $\to$ Genuine density-matrix description!

$$R^{I}(M_{t\bar{t}},\hat{k}) = \frac{1}{N_\lambda}\sum_{\lambda} R^{I\lambda}(M_{t\bar{t}},\hat{k})$$

$$\rho^{I}(M_{t\bar{t}},\hat{k}) = \frac{R^{I}(M_{t\bar{t}},\hat{k})}{\mathrm{Tr}\left[R^{I}(M_{t\bar{t}},\hat{k})\right]}$$

LO QCD $t\bar{t}$ Quantum state

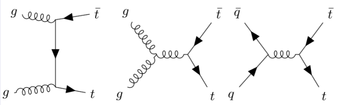
- $t\bar{t}$ production from most elementary QCD processes:

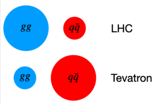
$$\begin{aligned} q+\bar{q} &\rightarrow t+\bar{t},\ q=u,d\dots \\ g+g &\rightarrow t+\bar{t} \end{aligned}$$

- Initial state $I = q\bar{q}, gg \rightarrow R^I(M_{t\bar{t}}, \hat{k}) \rightarrow \rho^I(M_{t\bar{t}}, \hat{k}) = \frac{R^I(M_{t\bar{t}},\hat{k})}{\mathrm{Tr}[R^I(M_{t\bar{t}},\hat{k})]}$
- LHC $\rightarrow$ Total quantum state: *Incoherent* mixture of $I = q\bar{q}, gg$ processes with (PDF-dependent) probability w_I

$$\rho(M_{t\bar{t}}, \hat{k}) = \sum_{I=q\bar{q},gg} w_I(M_{t\bar{t}})\rho^I(M_{t\bar{t}}, \hat{k})$$

- QCD Input: $w_I(M_{t\bar{t}}), \rho^I(M_{t\bar{t}}, \hat{k}) \rightarrow$ QI Output: Textbook problem of *convex sum* of quantum states!





Y. Afik, JRMdN, EPJ Plus 136, 907 (2021), Quantum 6, 820 (2022)

$t\bar{t}$ Quantum Correlations

- Quantum state $\rho(M_{t\bar{t}}, \hat{k})$: function of $M_{t\bar{t}}$ and production angle Θ.
- Two main regions of quantumness:
  - Ultrarelativistic high-p_T for both $q\bar{q}$ and gg (spin triplet)
  - Threshold for gg (spin singlet).

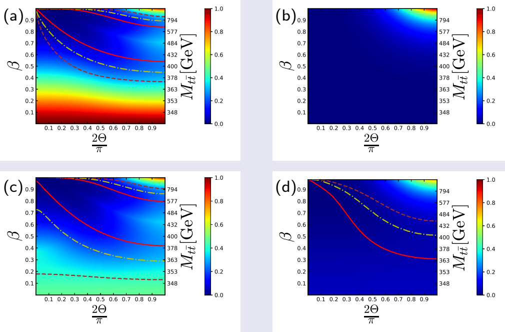
- Colorbar: Discord.
- Solid, dashed-dotted, dashed: Boundaries of Entanglement, Steering, Bell Nonlocality $\rightarrow$ Hierarchy!

a) $gg \rightarrow t\bar{t}$

b) $q\bar{q} \rightarrow t\bar{t}$

c) Run 2 LHC $\sqrt{s} = 13$ TeV

d) Tevatron $\sqrt{s} = 1.96$ TeV



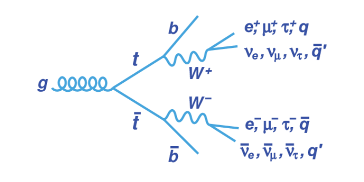
Y. Afik, JRMdN, PRL 130, 221801 (2023)

Quantum Tomography: Two qubits, two tops

- Top quarks: Spin polarizations $\mathbf{B}^{\pm}$ and spin correlation matrix $\mathbf{C}$ extracted from fitting differential cross-section of dileptonic decay:

$$\frac{1}{\sigma_{\ell\bar{\ell}}}\frac{d\sigma_{\ell\bar{\ell}}}{d\Omega_{+}d\Omega_{-}} = \frac{1}{(4\pi)^2}\left[1 + \mathbf{B}^{+}\cdot\hat{\ell}_{+} - \mathbf{B}^{-}\cdot\hat{\ell}_{-} - \hat{\ell}_{+}\cdot\mathbf{C}\cdot\hat{\ell}_{-}\right]$$

- $\hat{\ell}_{\pm}$: antilepton (lepton) directions in top (antitop) rest frames → Spin well-defined in parent rest frame
- Whole spin-density matrix can be reconstructed! → Quantum tomography in a high-energy collider!



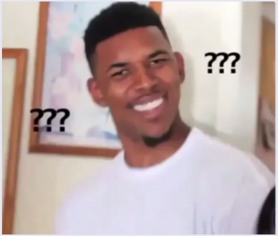
Cauchy-Schwarz violation in spin systems

- Simple criterion of entanglement: Cauchy-Schwarz violation

$$|\mathrm{Tr}\ \mathbf{C}| = |\langle \boldsymbol{\sigma} \cdot \boldsymbol{\sigma} \rangle| = \left| \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B\ P(\mathbf{n}_A, \mathbf{n}_B) \mathbf{n}_A \cdot \mathbf{n}_B \right|$$

$$\leq \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B\ P(\mathbf{n}_A, \mathbf{n}_B) \left| \mathbf{n}_A \cdot \mathbf{n}_B \right| \leq \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B\ P(\mathbf{n}_A, \mathbf{n}_B) = 1$$

- Wait a minute...A cosine average larger than one??? $\rightarrow$ CS Violation $\rightarrow$ Entanglement
- Entanglement witness: $D \equiv \dfrac{\mathrm{Tr}[\mathbf{C}]}{3}$
- Genuine quantum range: $-1 \leq D < -1/3$

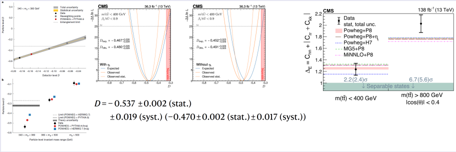


Experimental entanglement observation

- D directly measurable from the distribution of angular separation:

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi}=\frac{1}{2}(1-D\cos\varphi),\ \cos\varphi\equiv\hat{\ell}_+\cdot\hat{\ell}_-$$

- Entanglement detection from one single magnitude → No need for Quantum Tomography!
- Experimental milestones:
  1. Threshold entanglement observed by ATLAS and CMS with 5σ!
  2. Toponium signatures? → Toponium observation! ATLAS, ATLAS-CONF-2025-008 (2025), CMS, ROPP 88, 087801 (2025)
  3. Ultrarelativistic entanglement observed by CMS with 5σ!



ATLAS, Nature 633, 542 (2024)
CMS, ROPP 87, 117801 (2024)
CMS, PRD 110, 112016 (2024)

Steering ellipsoid

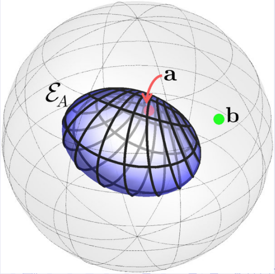
- Conditional quantum states:

$$\rho_{\hat{\mathbf{n}}}^{(\pm)} = \frac{1 + \mathbf{B}_{\hat{\mathbf{n}}}^{\pm} \cdot \boldsymbol{\sigma}_{\pm}}{2}, \ \mathbf{B}_{\hat{\mathbf{n}}}^{\pm} = \frac{\mathbf{B}^{\pm} + \mathbf{C}^{\pm} \cdot \hat{\mathbf{n}}}{1 + \mathbf{B}^{\mp} \cdot \hat{\mathbf{n}}}, \ \mathbf{C}^{+} = \mathbf{C}, \ \mathbf{C}^{-} = \mathbf{C}^{T}$$

- Direct conditional quantum tomography:

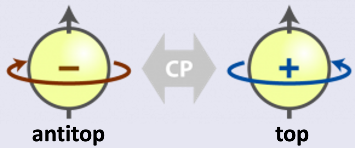
$$p(\hat{\ell}_{\pm}|\hat{\ell}_{\mp} = \mp\hat{\mathbf{n}}) = \frac{p(\hat{\ell}_{\pm}, \hat{\ell}_{\mp} = \mp\hat{\mathbf{n}})}{p(\hat{\ell}_{\mp} = \mp\hat{\mathbf{n}})} = \frac{1 \pm \mathbf{B}_{\hat{\mathbf{n}}}^{\pm} \cdot \hat{\ell}_{\pm}}{4\pi}$$

- $\mathbf{B}_{\hat{\mathbf{n}}}^{\pm} \rightarrow$ Steering ellipsoid.
- Discord $\rightarrow$ Minimization of conditional entropy over surface.
- Highly-challenging QI measurements in conventional setups $\rightarrow$ Natural implementation in colliders!



New Physics Witnesses

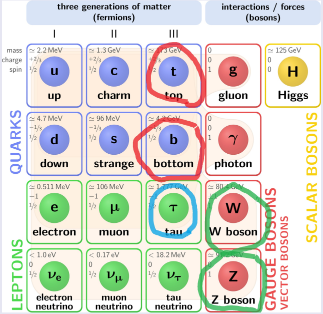
- Approximate *CP*-invariance of Standard Model $\rightarrow$ $\mathbf{C} = \mathbf{C}^{\mathrm{T}}$, $\mathbf{B}^{+} = \mathbf{B}^{-}$ $\rightarrow$ Symmetric Discord and Steering!
- Discord and/or Steering asymmetry $\Longrightarrow$ New Physics!
- New Physics witnesses: Symmetry protected observables by SM, only non-zero for New Physics $\rightarrow$ No SM contribution to New Physics witnesses!



Y. Afik, JRMdN, PRL 130, 221801 (2023)

Alternative and complementary approaches

- Quantum spin-correlations in alternative qubits and qutrits:

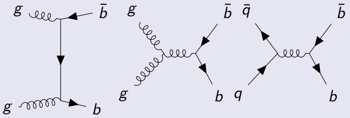


1. t (EPJ Plus 136, 907 (2021))
2. b (PRD 111, L111902 (2025))
3. τ (EPJC 83, 162 (2023))
4. $W^{\pm}$(PLB 825, 136866 (2022))
5. Z^0(PRD 107, 016012 (2023))

- Complementary approaches QI-HEP:
  - Flavor entanglement in mesons: PRL 99, 131802 (2007)
  - Neutrino oscillations: PRL 117, 050402 (2016)
  - QI techniques to study QCD interactions: PRL 124, 062001 (2020)
  - Other fundamental quantum features: interference, indistinguishability... PLB 868, 139639 (2025)

Quantum bottoms

- *Mutatis mutandis*: Quantum information with $b\bar{b}$!
- $b\bar{b}$ quantum tomography: $\Lambda_b(udb), \bar{\Lambda}_b(\bar{u}\bar{d}\bar{b})$ decays retain $b\bar{b}$ spin information Y. Kats, D. Uzan, JHEP 03 (2024) 063.
- Experimentally challenging $\longleftrightarrow$ Theoretically interesting:
  - Spin correlations in $b\bar{b}$ not measured yet $\rightarrow$ Uncharted territory!
  - Ultrarelativistic $b\bar{b}$ at LHC
  - ATLAS, CMS and also LHCb can play the game!
  - Paves the way to study quantum correlations in hadronizing systems $\rightarrow$ Quark-Gluon Plasma STAR Collaboration, Nature 548, 62 (2017).



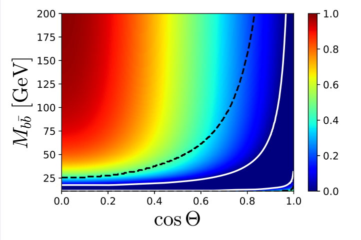
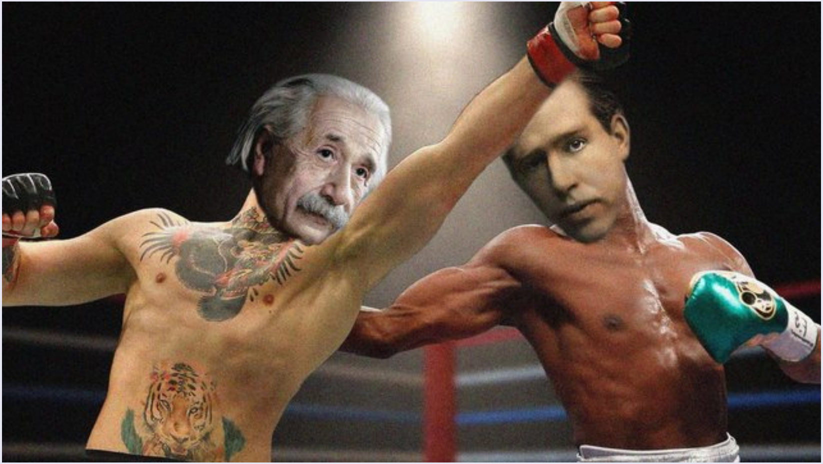


Figure: $b\bar{b}$ concurrence. Y. Afik, Y. Kats, JRMdN, A. Soffer, D. Uzan PRD 111, L111902 (2025)

Conclusions and outlook

- Quantum Information theory $\longleftrightarrow$ High-Energy Physics. Interdisciplinary, huge potential and great interest!
- QI perspective:
  1. Highest-energy observation of entanglement ever!
  2. Genuinely relativistic, exotic symmetries and interactions, fundamental nature $\rightarrow$ Frontier of known Physics!
  3. Highly-demanding measurements naturally implemented at LHC.
- HEP perspective:
  1. Quantum Tomography: Novel experimental tool.
  2. QI measurements can be used to understand HEP (e.g., toponium, hadronization).
  3. QI techniques can inspire new approaches for searching New Physics: PRD 106, 055007 (2022), JHEP 148, (2023), EPJC 83, 162 (2023)
- Already first measurements of $t\bar{t}$ entanglement by ATLAS and CMS. Highest-energy entanglement ever! $\rightarrow$ Many more on its way!
- QIHEP community $\rightarrow$ *Quantum manifesto* (over 70 authors from theory-experiment, QI-HEP): EPJ Plus 140, 855 (2025)

Thank You



Backup

Quantum optics representations

- Quantum state completely determined by characteristic function

$$\chi(\eta) \equiv \langle e^{\eta \hat{a}^\dagger - \eta^* \hat{a}} \rangle = \mathrm{Tr}[e^{\eta \hat{a}^\dagger - \eta^* \hat{a}} \hat{\rho}].$$

- Normal and antinormal versions

$$\begin{aligned} \chi_N(\eta) &\equiv \langle e^{\eta \hat{a}^\dagger} e^{-\eta^* \hat{a}} \rangle , \\ \chi_A(\eta) &\equiv \langle e^{-\eta^* \hat{a}} e^{\eta \hat{a}^\dagger} \rangle . \end{aligned}$$

- Fourier transforms $\rightarrow P, Q, W$ representations for computing normal, antinormal or symmetric expectation values:

$$\begin{aligned} P(\alpha) &\equiv \int \frac{\mathrm{d}^2\eta}{\pi^2} e^{(\eta^*\alpha - \eta\alpha^*)} \chi_N(\eta) \rightarrow \int \mathrm{d}^2\alpha \; |\alpha|^2 P(\alpha) = \langle \hat{a}^\dagger \hat{a} \rangle \\ Q(\alpha) &\equiv \int \frac{\mathrm{d}^2\eta}{\pi^2} e^{(\eta^*\alpha - \eta\alpha^*)} \chi_A(\eta) \rightarrow \int \mathrm{d}^2\alpha \; |\alpha|^2 Q(\alpha) = \langle \hat{a} \hat{a}^\dagger \rangle \\ W(\alpha) &\equiv \int \frac{\mathrm{d}^2\eta}{\pi^2} \; e^{(\eta^*\alpha - \eta\alpha^*)} \chi(\eta) \rightarrow \int \mathrm{d}^2\alpha \; |\alpha|^2 W(\alpha) = \langle \frac{\hat{a}^\dagger \hat{a} + \hat{a} \hat{a}^\dagger}{2} \rangle \end{aligned}$$

Quantum Discord: Two qubits

- How do we translate classical into quantum?
- Shannon entropy → Von Neumann entropy ($p_n \geq 0$, ρ eigenvalues)

$$H(A,B) \;\rightarrow\; H(\rho) = -\sum_n p_n \log_2 p_n$$

$$H(A) \;\rightarrow\; H(\rho_A),\; H(B) \rightarrow H(\rho_B),\; \rho_{A,B} = \mathrm{Tr}_{B,A}\rho$$

- Conditional probability → Conditional state $\rho_{A|B}$ = One-qubit state after Bob's spin measurement along $\hat{\mathbf{n}}$:

$$H(A|B) = p_{\hat{\mathbf{n}}} H(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} H(\rho_{-\hat{\mathbf{n}}})$$

$$\rho_{\hat{\mathbf{n}}} = \frac{\Pi^B_{\hat{\mathbf{n}}}\rho\Pi^B_{\hat{\mathbf{n}}}}{p_{\hat{\mathbf{n}}}} = \frac{1 + \mathbf{B}^+_{\hat{\mathbf{n}}} \cdot \sigma}{2},\;\; \mathbf{B}^+_{\hat{\mathbf{n}}} = \frac{\mathbf{B}^+ + \mathbf{C}\cdot\hat{\mathbf{n}}}{1 + \hat{\mathbf{n}}\cdot\mathbf{B}^-},\;\; p_{\hat{\mathbf{n}}} = \frac{1 + \hat{\mathbf{n}}\cdot\mathbf{B}^-}{2}$$

- Genuine quantumness → Minimization over all spin directions to exclude quantization effects:

$$\mathcal{D}(A,B) = H(\rho_B) - H(\rho) + \min_{\hat{\mathbf{n}}} p_{\hat{\mathbf{n}}} H(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} H(\rho_{-\hat{\mathbf{n}}}) \;\neq 0$$