Towards quantum simulation of strongly interacting matter in neutron stars

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18th December, 2025

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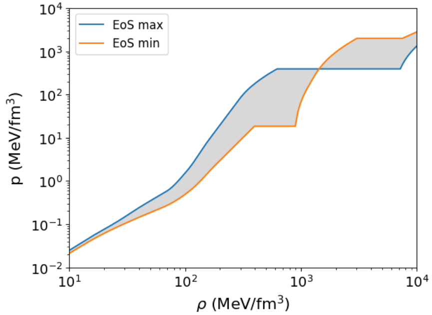
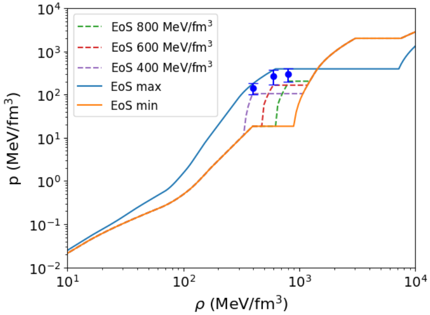
EoS



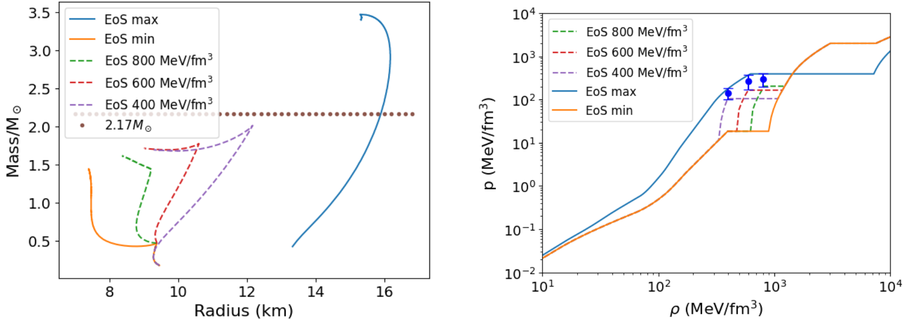
Initial band [a]



Eventual improvement. $0 < c_s < 1$

[a] EoS by E. Lope-Oter arXiv 2410.14776

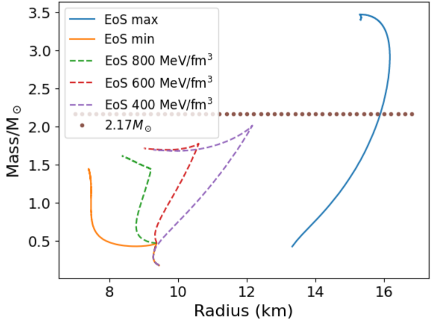
Mass-radius diagram

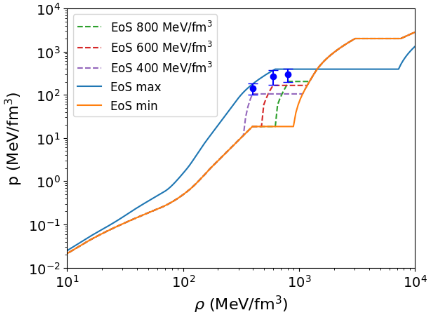


Discontinuous line: constrained EoS (quantum computer). Band between blue curve and discontinuous ones.

WORK IN PROGRESS!

Mass-radius diagram





Discontinuous line: constrained EoS (quantum computer). Band between blue curve and discontinuous ones.

WORK IN PROGRESS!

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QCD Lagrangian and Hamiltonian

QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} + \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - g A^a_\mu \bar{\psi} T^a \gamma^\mu \psi$$

QCD Hamiltonian

$$H_{QCD} = H_g + H_q + H_{int}$$

$$H_{QCD} = \int d^3x \{ \underbrace{\frac{1}{2}\left[(E_i^a)^2 + (B_i^a)^2\right]}_{H_g} + \underbrace{g\,\vec{A}^a \cdot (\psi_l^\dagger \gamma^0 \vec{\gamma} T^a \psi_l)}_{H_{int}}$$

$$+ \underbrace{\psi_l^\dagger \left(-i\gamma^0 \vec{\gamma} \cdot \vec{\nabla} + m\gamma^0\right) \psi_l}_{H_q} \}$$

QCD Lagrangian and Hamiltonian

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QCD Lagrangian and Hamiltonian

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Gauge condition

- Static star: **temporal gauge** $A^0 = 0$

Gauss condition

$$\mathcal{G}^a = \frac{1}{g}(\delta^a_b \nabla_i + g f^{abc} A^c_i) E^{ib} + \psi^\dagger T^a \psi$$

$$\mathcal{G}^a(x) \left|\psi\right\rangle = 0$$

Expansion in normal modes[a]

[a] Juan José Gálvez's slides

Pure glue QCD Hamiltonian in Weyl gauge

$a^{\dagger}\backslash a$		a	a^2	a^3	a^4
			$\frac{1}{4}\frac{k_i k_j}{k^0} - 3\frac{g^2}{(2\pi)^3}\int\frac{d^3p'}{E_{p'}}\delta_{ij}$	$-igf^{abc}q_i\delta_{jl}$	$\frac{1}{4}g^2 f^{abc}f^{aef}\delta_{jl}\delta_{im}$
$a^{\dagger}$		$k^0\left(\delta_{ij} - \frac{1}{2}\frac{k^i k^j}{(k^0)^2}\right) - 6\frac{g^2}{(2\pi)^3}\int\frac{d^3p'}{E_{p'}}\delta_{ij}$	$igf^{abc}[-q_j\delta_{il} + q_i\delta_{jl} - p_j\delta_{il}]$	$\frac{1}{2}g^2[f^{abc}f^{aef}\delta_{jl}\delta_{im} + f^{acf}f^{abe}\delta_{jl}\delta_{im}]$	
$a^{\dagger 2}$	$\frac{1}{4}\frac{k_i k_j}{k^0} - 3\frac{g^2}{(2\pi)^3}\int\frac{d^3p'}{E_{p'}}\delta_{ij}$	$-igf^{abc}[-q_j\delta_{il} + q_i\delta_{jl} - p_j\delta_{il}]$	$\frac{1}{4}g^2[f^{abc}f^{aef}\delta_{im}\delta_{jl} + f^{abe}f^{acf}\delta_{im}\delta_{jl} - f^{abe}f^{acf}\delta_{ij}\delta_{lm}]$		
$a^{\dagger 3}$	$igf^{abc}q_i\delta_{jl}$	$\frac{1}{2}g^2[f^{abc}f^{aef}\delta_{jl}\delta_{im} + f^{acf}f^{abe}\delta_{jl}\delta_{im}]$			
$a^{\dagger 4}$	$\frac{1}{4}g^2 f^{abc}f^{aef}\delta_{jl}\delta_{im}$				

Gauss operator

$a^{\dagger} \backslash a$		a	a^2
		$E_p p^i$	$igf^{abc} E_p \delta_{ij}$
$a^{\dagger}$	$E_p p^i$	$igf^{abc} \left(E_p - E_k\right) \delta_{ij}$	
$a^{\dagger 2}$	$-igf^{abc} E_p \delta_{ij}$		

Gauss Operator squared

$a^{\dagger}\backslash a$		a	a^2	a^3	a^4
	$\left\{9\int \frac{d^3k}{(2\pi)^3}\frac{(E_k-E_p)^2}{E_kE_p} + \frac{E_p^2}{2g^2}\right\}\delta(0)$		$\frac{3}{4}\int \frac{d^3k}{(2\pi)^3}\frac{E_k^2-E_p^2}{E_kE_p}\delta_{ij} - \frac{E_p^2}{2g^2}p^ip^j$	$-\frac{2i}{g}f^{abc}E_kE_pp^i\delta_{lj}$	$f^{abc}f^{ade}E_kE_l$
$a^{\dagger}$		$\frac{3}{4}\int \frac{d^3k}{(2\pi)^3}\frac{E_k^2+E_p^2}{E_kE_p}\delta_{ij} + \frac{E_p^2}{g^2}p^ip^j$	$-\frac{2i}{g}f^{abc}(E_kE_pp^i\delta_{lj} + E_qE_pq^l\delta_{ij} + E_qE_kp^i\delta_{lj})$	$2f^{abc}f^{ade}E_k(E_q+E_l)$	
$a^{\dagger 2}$	$\frac{3}{4}\int \frac{d^3k}{(2\pi)^3}\frac{E_k^2-E_p^2}{E_kE_p}\delta_{ij} - \frac{E_p^2}{2g^2}p^ip^j$	$\frac{2i}{g}f^{abc}(E_kE_pp^i\delta_{lj} + E_qE_pq^l\delta_{ij} + E_qE_kp^i\delta_{lj})$	$2f^{abc}f^{ade}E_kE_q + f^{abe}f^{adc}E_kE_p + f^{abe}f^{adc}E_lE_q$		
$a^{\dagger 3}$	$\frac{2i}{g}f^{abc}E_kE_pp^i\delta_{lj}$	$2f^{abc}f^{ade}E_k(E_q+E_l)$			
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$a^{\dagger}$		$\frac{3}{4}\int \frac{d^3k}{(2\pi)^3}\frac{E_k^2+E_p^2}{E_kE_p}\delta_{ij} + \frac{E_p^2}{g^2}p^ip^j$	$-\frac{2i}{g}f^{abc}(E_kE_pp^i\delta_{lj} + E_qE_pq^l\delta_{ij} + E_qE_kp^i\delta_{lj})$	$2f^{abc}f^{ade}E_k(E_q + E_l)$	
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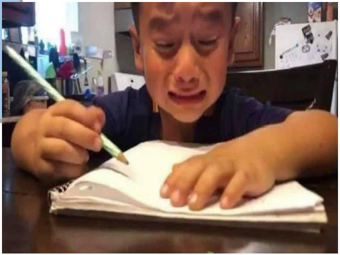

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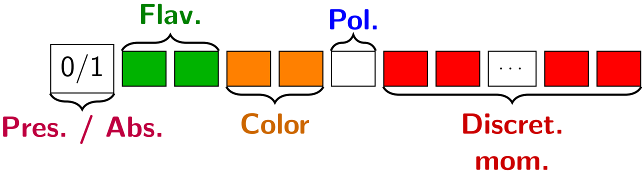
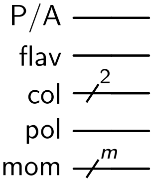
Particle degrees of freedom

$$N \text{ values } \rightarrow \log_2 N \text{ qubits}$$



extra Qubit: presence/absence (P/A)



Quark encoding

Number of qubits: color(2), polarization(1), flavor(1), momentum(n)

$$\text{Mom} = \begin{cases} -2 & \xrightarrow{\text{enc.}} & 00 \\ -1 & \rightarrow & 01 \\ 1 & \rightarrow & 10 \\ 2 & \rightarrow & 11 \end{cases} \qquad \text{Col} = \begin{cases} 1 \text{ (red)} & \xrightarrow{\text{enc.}} & 01 \\ 2 \text{ (green)} & \rightarrow & 10 \\ 3 \text{ (blue)} & \rightarrow & 11 \\ & \text{vacío:} & 00 \end{cases}$$

$$\text{Pol} = \begin{cases} -1/2 & \xrightarrow{\text{enc}} & 0 \\ +1/2 & \rightarrow & 1 \end{cases} \qquad \text{Flav} = \begin{cases} 1 \quad \text{(up)} & \xrightarrow{\text{enc}} & 0 \\ 2 \quad \text{(down)} & \rightarrow & 1 \end{cases}$$

mom $= 1$, pol $= 1/2$, col $= 3$ and flav$=1 \rightarrow$ ' $\underbrace{10}_{\text{mom.}}\underbrace{1}_{\text{pol.}}\underbrace{11}_{\text{color}}\underbrace{0}_{\text{flav.}}\underbrace{1}_{\text{Present}}$ '

Creation and annihilation operators

Creation operators. (Annihilation: adjoints)

$$a^{(n)}_{j,\,c,s,p}{}^{\dagger} = \mathcal{S}_{j\leftarrow(j-1)} \cdot \mathbb{P}_0^{(n-j)} \otimes \left(\mathbb{C}_{10} \otimes s^{\dagger}_{c,s,p}\right)_j \otimes \mathbb{P}_{j-1}^{(j-1)}$$

$$b^{(n)}_{j,\,f,c,s,p}{}^{\dagger} = \mathcal{A}_{j\leftarrow(j-1)} \cdot \mathbb{P}_0^{(n-j)} \otimes \left(\mathbb{C}_{10} \otimes s^{\dagger}_{f,c,s,p}\right)_j \otimes \mathbb{P}_{j-1}^{(j-1)}$$

With $\mathbb{C}_{10} = |1\rangle\langle 0|$

$$\underbrace{|\Omega\rangle_n \cdots |\Omega\rangle_{j+1}}_{\text{vacuum}} \; \underbrace{|j\rangle}_{\text{encode here}} \; \underbrace{|\psi_{j-1}\rangle \cdots |\psi_1\rangle}_{\text{already particles}}$$

Expectation values → already (anti)symmetrized states → drop (anti)symmetrizers

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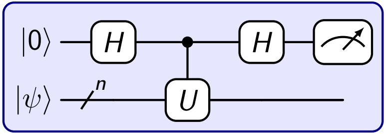
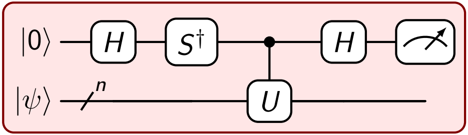
Hadamard Test

Hadamard test

Algorithm for expectation values



Circuit A: $\mathrm{Re}\langle\psi|U|\psi\rangle$



Circuit B: $\mathrm{Im}\langle\psi|U|\psi\rangle$

$$P_0^A - P_1^A = \mathrm{Re}\,\langle\psi|\,U\,|\psi\rangle\,.$$

$$P_0^B - P_1^B = \mathrm{Im}\,\langle\psi|\,U\,|\psi\rangle\,.$$

Observables, Hermitian operators $\longrightarrow$ No imaginary part!

EoS, Low temperature

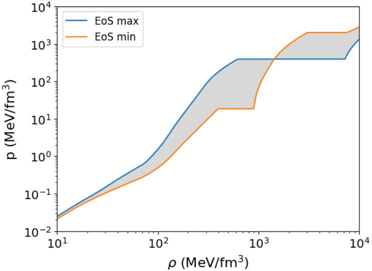
EoS

$$p(T \to 0, \mu, \rho) \sim \rho \left[\frac{\partial}{\partial \rho} \left(\langle H_{\text{phys}} \rangle - \mu \langle N \rangle \right)_{\text{min}} \right]$$

Gauge fixing

$$\langle \psi | \, H - \lambda \mathcal{G}^2 \, | \psi \rangle$$

λ Lagrange multiplier



Wavefunction. Variational method

Example

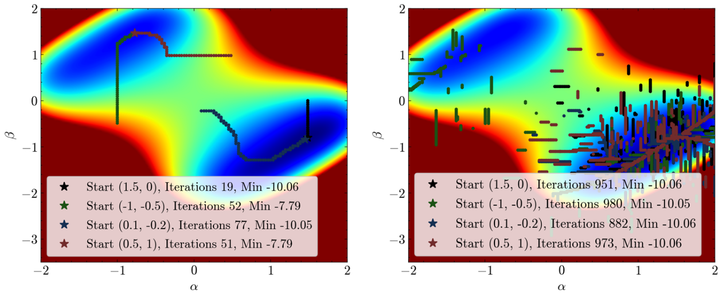
$$|\psi\rangle = ?$$

- 1 quark, 2 gluons
- Simplest function: $|q\rangle + |qg\rangle + |qgg\rangle$
- Color states in triplet irreps.
- Entanglement between momenta, spin and symmetrization of gluons
- $\sim$20 params. and $\sim$20 qubits

Maximum gradient method

Grid of combination of the n parameter values, target function evaluation. Initial position→ **local** minimum. Solution: genetic algorithm (random shift)

Challenge: state function. Number of parameters.



[1]

[1] Figure from Adrián Castaño

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Conclusions

Status of the project

1. Test Einstein's Equations in matter
2. Einstein Telescope (GW): model and observation comparison
3. Target densities: $\rho \sim 400\text{MeV}/\text{fm}^3$ and $\Delta p \sim 30\,\%$
4. Hardware improvement (noise)