Quantum Computing Fragmentation Functions

Universidad Complutense de Madrid & IPARCOS

Juan José Gálvez Viruet*

arxiv: 2510.18869

IPARCOS

MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES

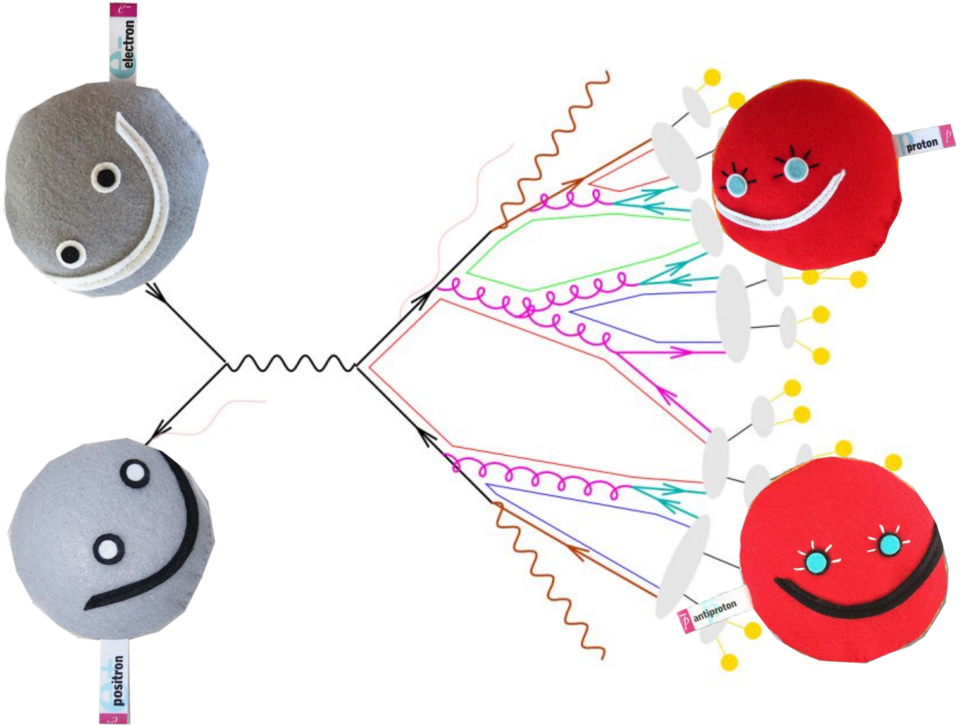
UNIÓN EUROPEA
FONDO EUROPEO DE DESARROLLO REGIONAL
"Una manera de hacer Europa"

AGENCIA ESTATAL DE INVESTIGACIÓN

*juagalve@ucm.es

QCD is confined: Hard interactions reconstructed from parton **fragmentation** into hadrons

- the cleanest process is **e+ e- annihilation**



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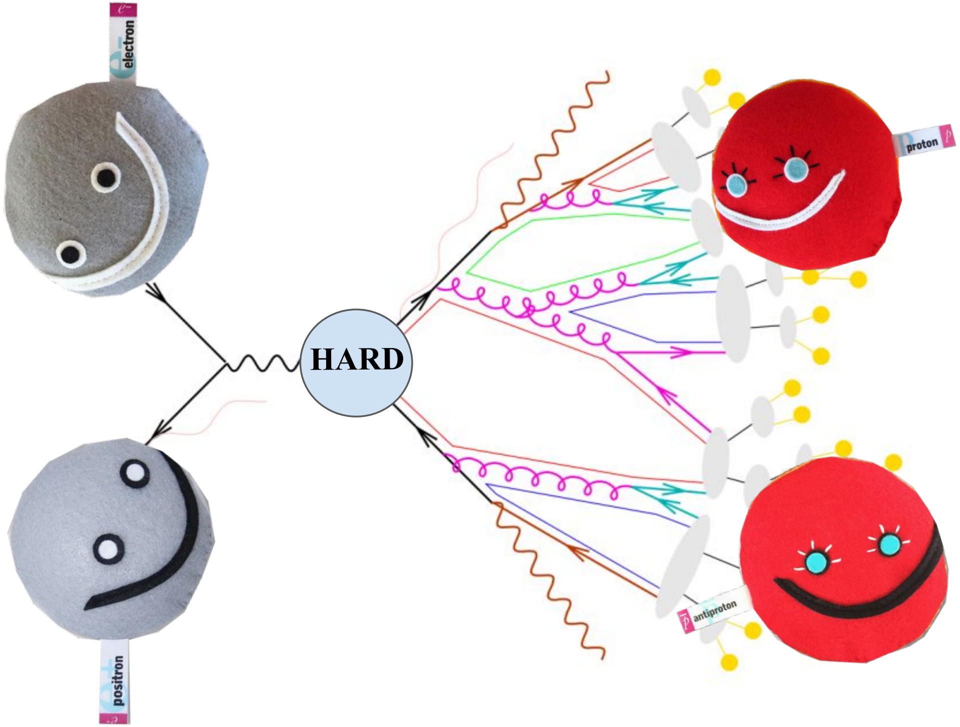

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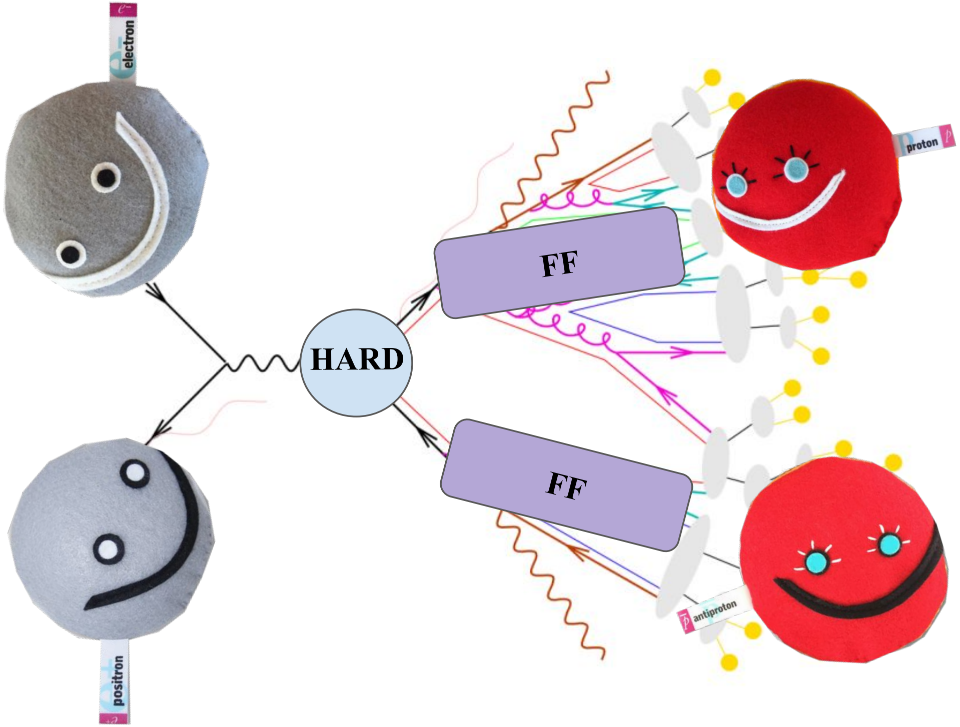

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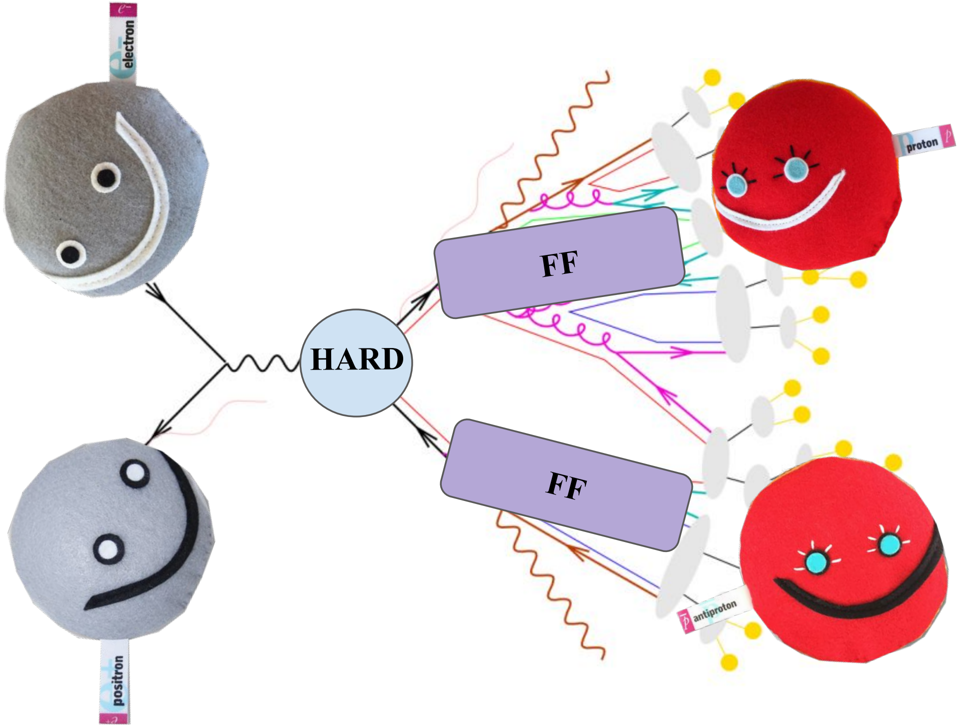
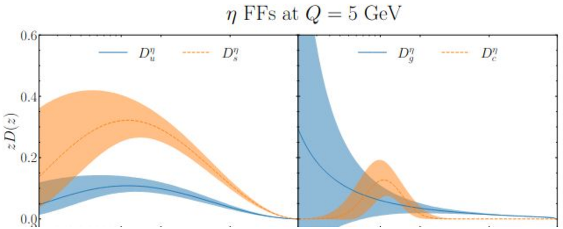


Obtained ...

By experimental fits



https://doi.org/10.1103/t5ds-vvc4

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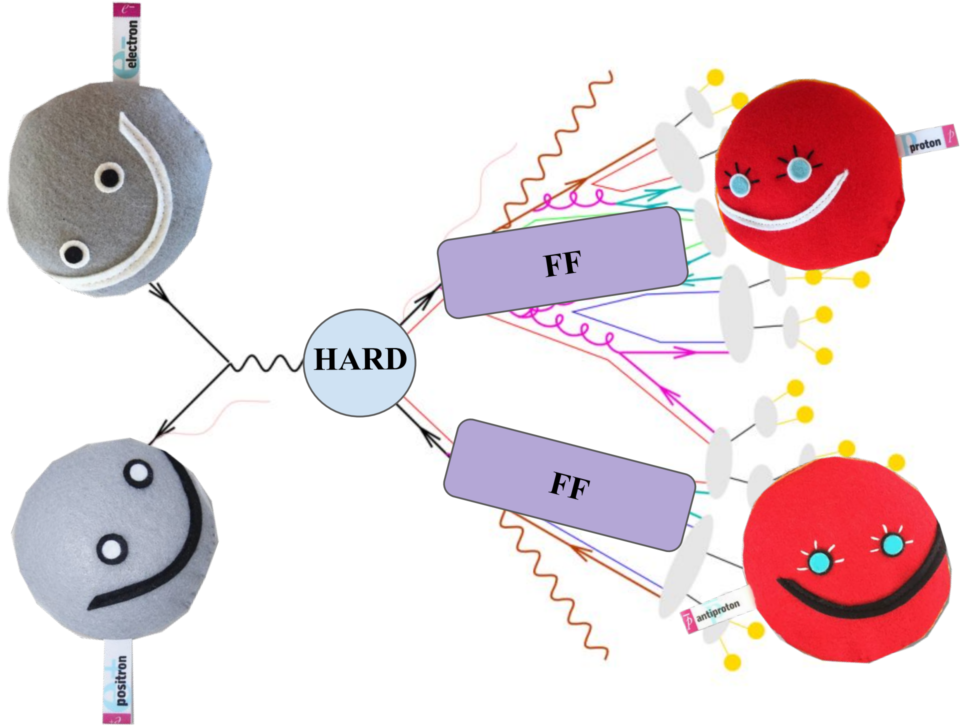

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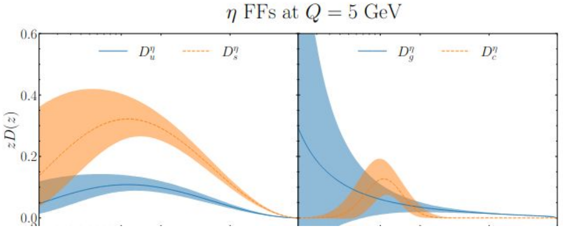

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On the Lattice?

$$D_j^h(z) \equiv \frac{\mathrm{Tr}_c}{N_{c,j}} \sum_X \langle j,p|h,X_{\mathrm{out}}\rangle\langle h,X_{\mathrm{out}}|j,p\rangle$$

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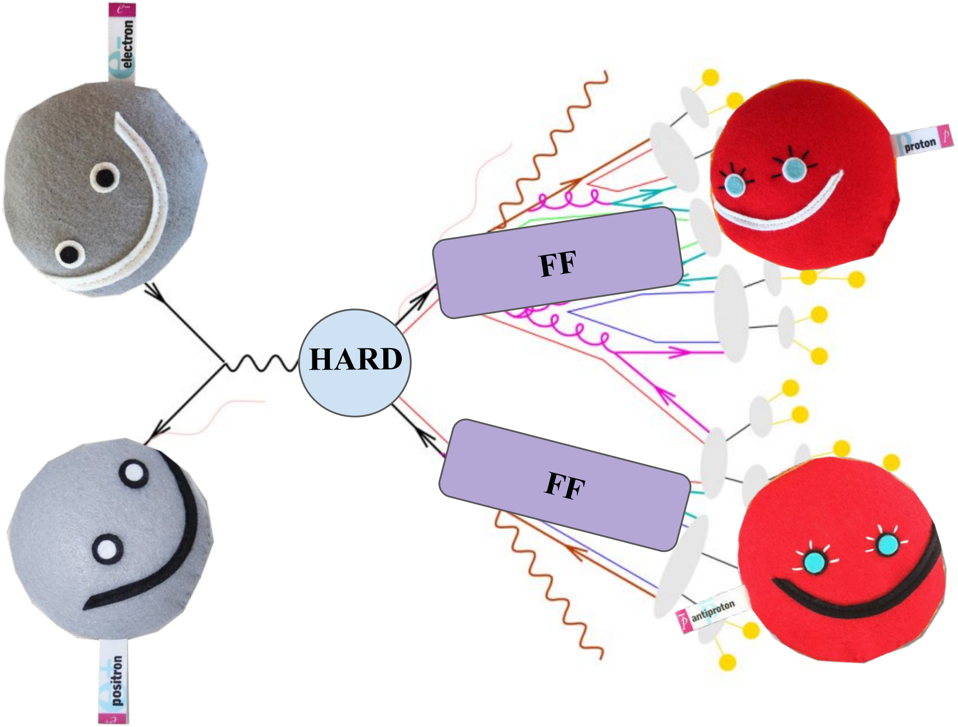

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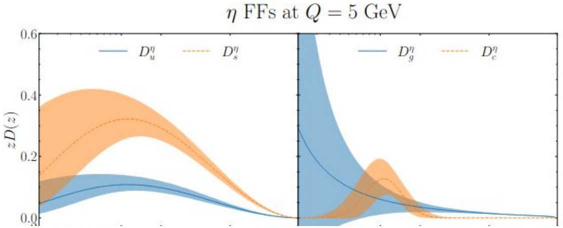

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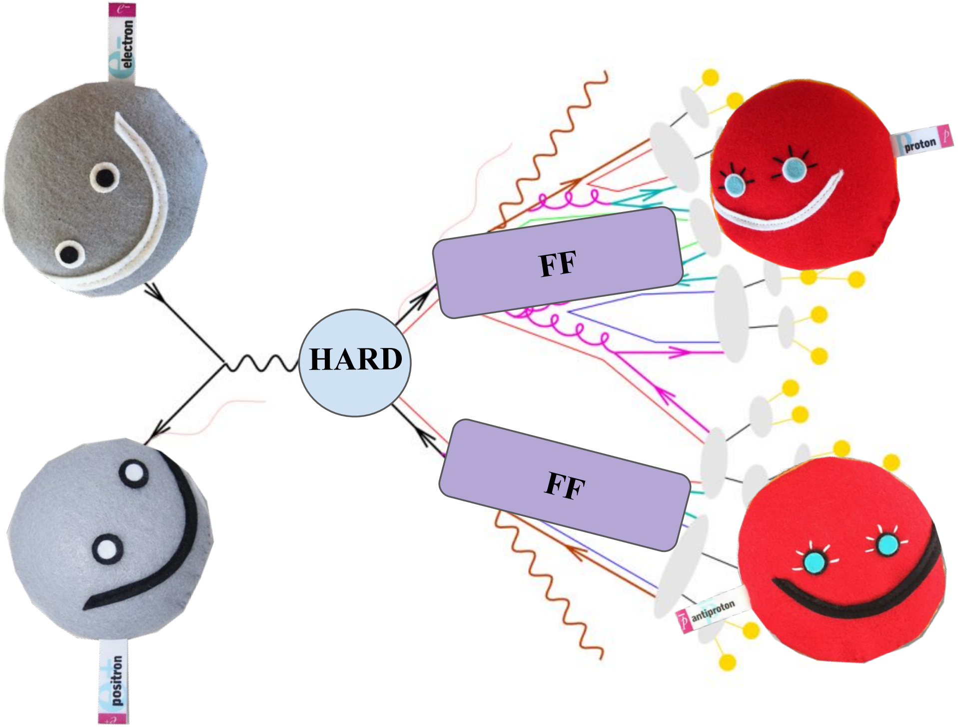

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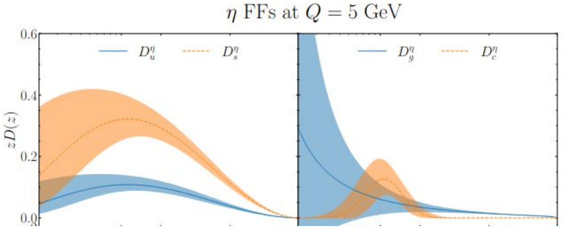

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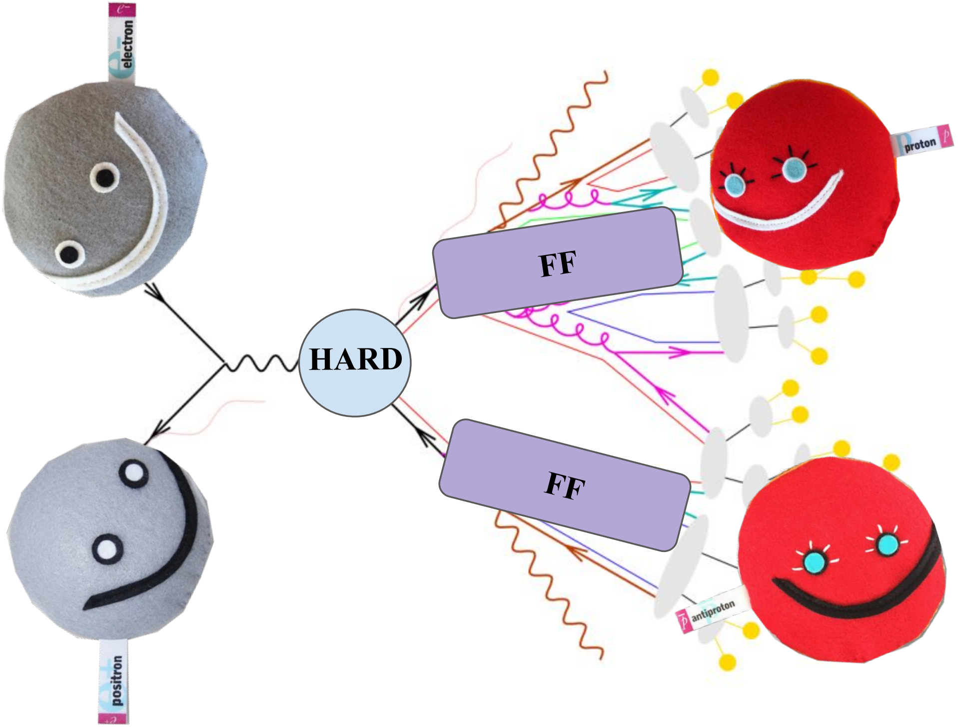

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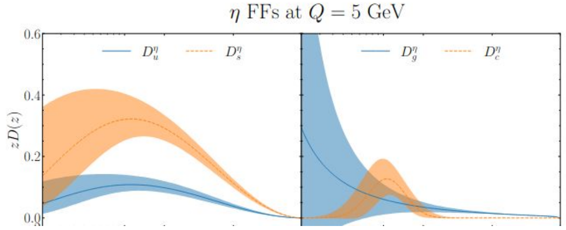

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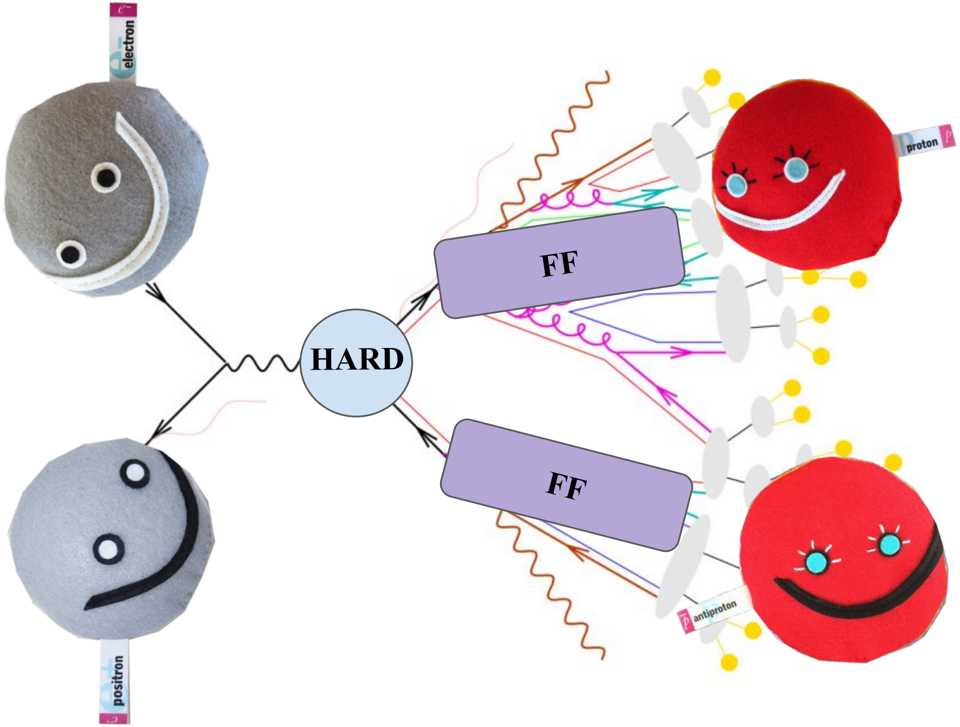

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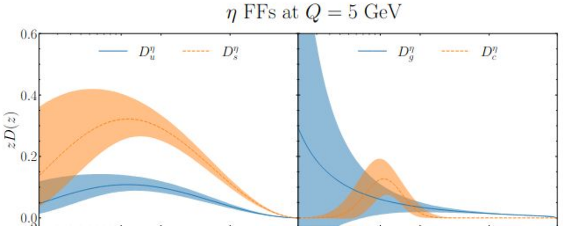

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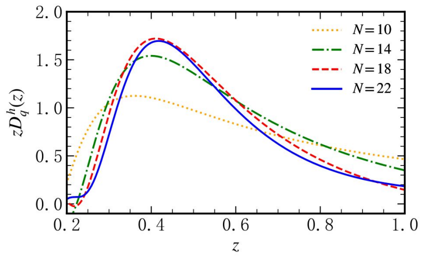

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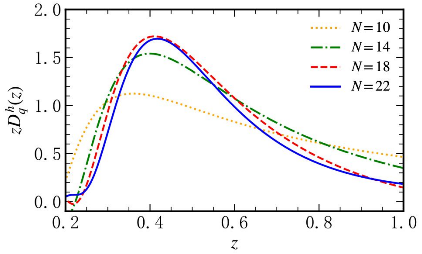
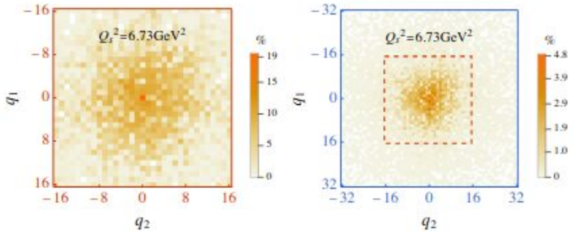


- Other works:

1. Medium induced jet broadening doi:10.1103/PhysRevD.106.074013



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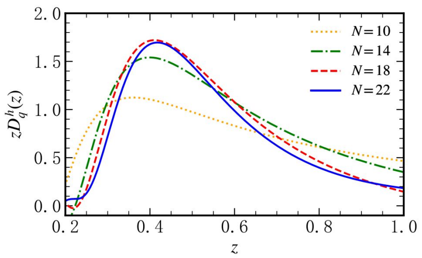

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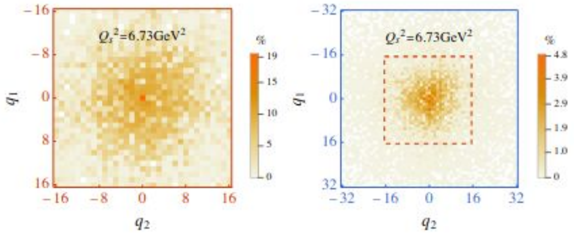
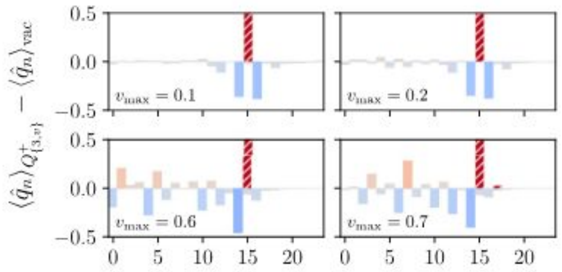


2. Energy loss on dense media doi:10.1103/PhysRevC.111.015202



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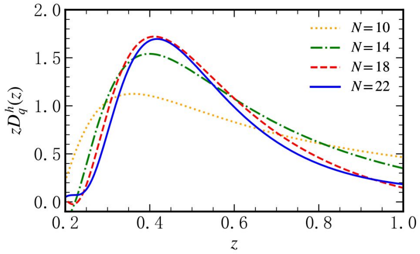

- See https://arxiv.org/abs/2510.26293 for (our) review

Preparations for Quantum Computing in Hadron Physics

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Dept. de Física Atómica, Molecular y Nuclear and Instituto Carlos I de Física Teórica y Computacional, Universidad de Granada, 18071 Granada, Spain

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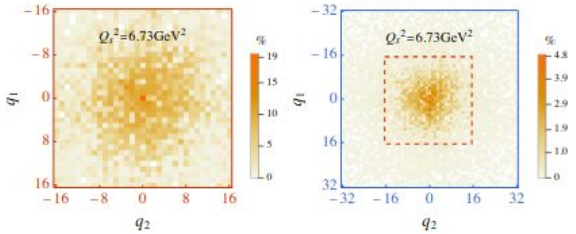

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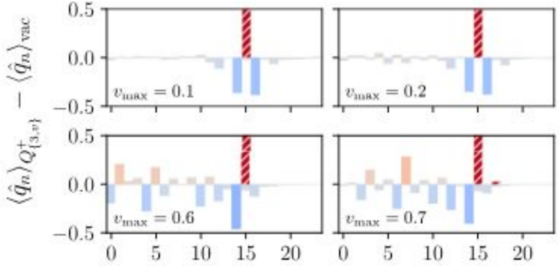
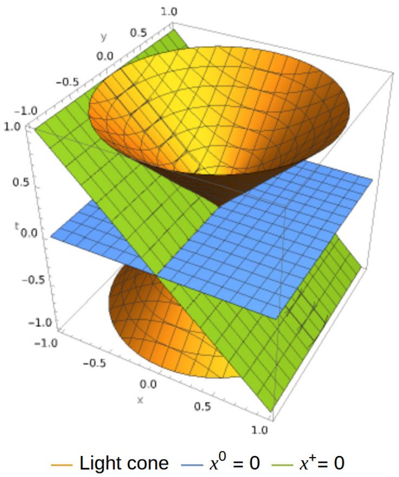


Ab initio QCD on the Light-Front gauge



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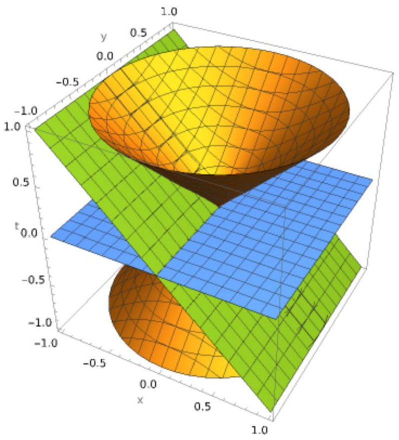

Coordinates

Instant Form (usual quantisation)	Light Front
(x^0, x^1, x^2, x^3)	$x^+ = x^3 + x^0$ $x^- = x^3 - x^0$ $x^1,\ x^2 \rightarrow x^\perp$

Energies:

$p^0 = \sqrt{m^2 + \|p\|^2}$	$p^- = \dfrac{m^2 + \|p^\perp\|^2}{p^+}$

Gauges:

$A^0 = 0$	$A^+ = 0$

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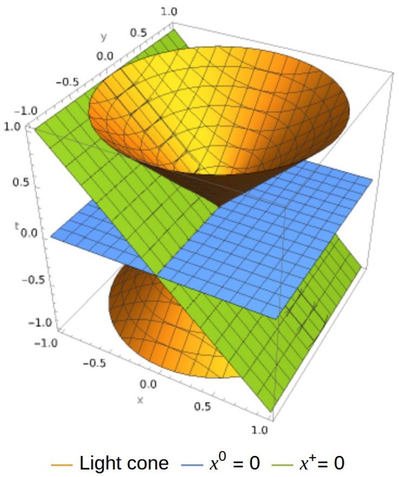

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$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi + g\, A^a_\mu J^{\mu,a}_D$$ define LF Hamiltonian by $$P^- = \sum \Pi\, \partial_+ A - \mathcal{L}$$

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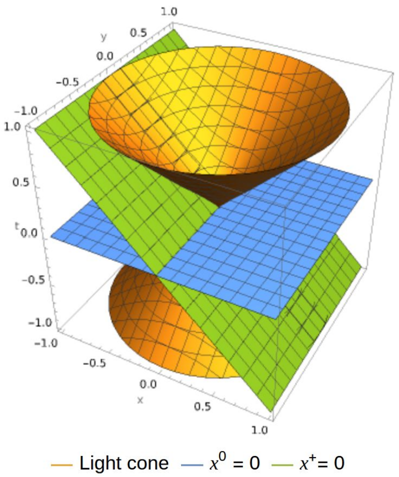

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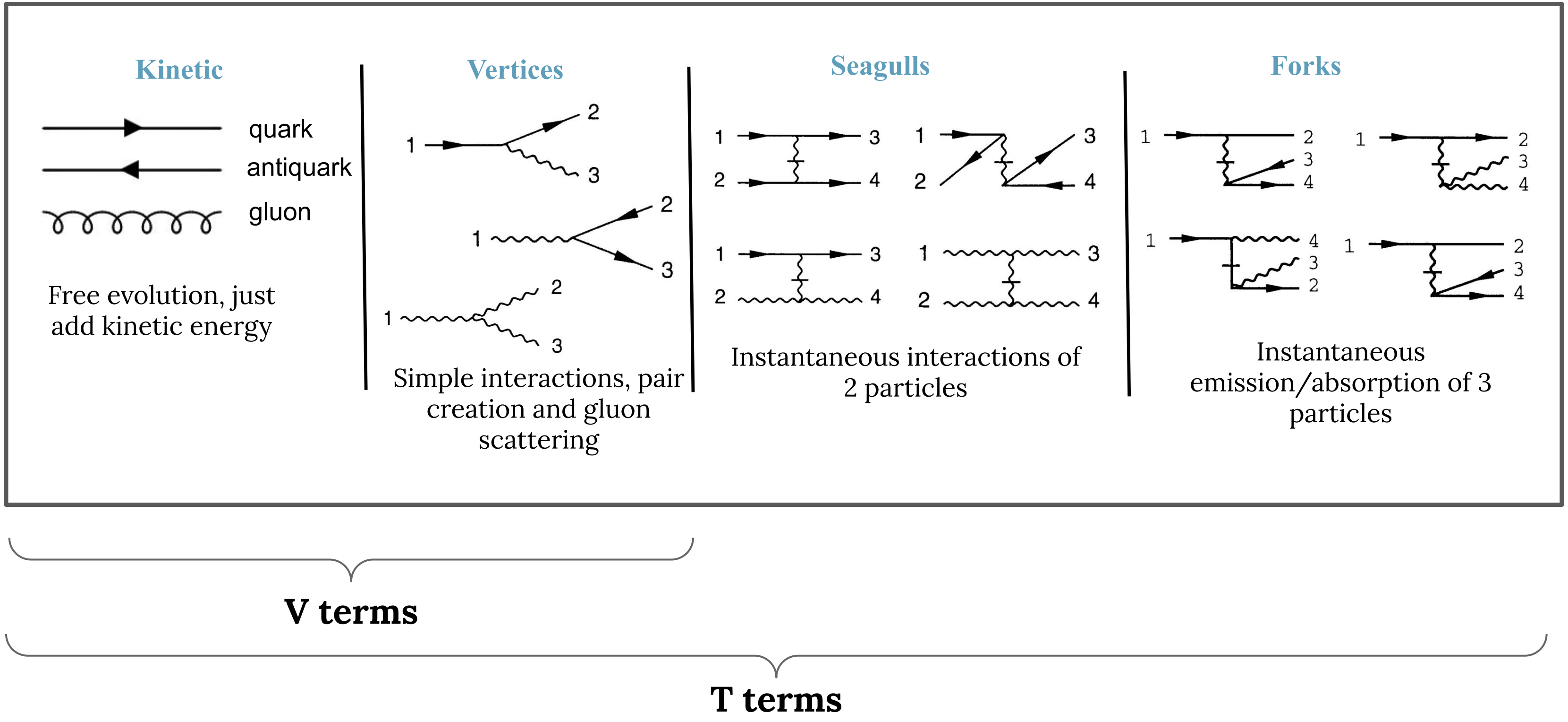
$$P_+ = \frac{1}{2}\int dx_+ d^2x_\perp \left(\bar{\tilde{\Psi}} \gamma^+ \frac{m^2 + (i\nabla_\perp)^2}{i\partial^+} \tilde{\Psi} + \tilde{A}^\mu_a (i\nabla_\perp)^2 \tilde{A}^a_\mu \right) + g \int dx_+ d^2x_\perp\, \tilde{J}^\mu_a \tilde{A}^a_\mu$$

$$+ \frac{g^2}{4}\int dx_+ d^2x_\perp\, \tilde{B}^{\mu\nu}_a \tilde{B}^a_{\mu\nu} + \frac{g^2}{2}\int dx_+ d^2x_\perp \tilde{J}^+_a \frac{1}{(i\partial^+)^2} \tilde{J}^+_a + \frac{g^2}{2}\int dx_+ d^2x_\perp\, \bar{\tilde{\Psi}} \gamma^\mu T^a \tilde{A}^a_\mu \frac{\gamma^+}{i\partial^+} (\gamma^\nu T^b \tilde{A}^b_\nu \tilde{\Psi})\,.$$

Hamiltonian as a dictionary of interactions among **on shell** partons:

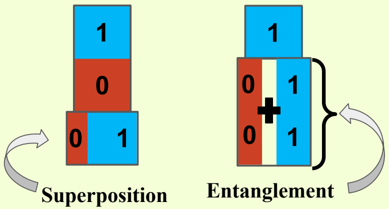
doi.org/10.1016/S0370-1573(97)00089-6



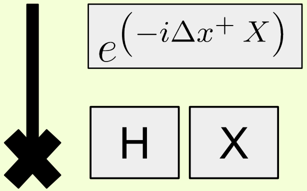
Quantum Computers

Quantum computers have two main components

Qubits featuring **superpositions** & **entanglement**



Logic **gates** to apply unitary evolution



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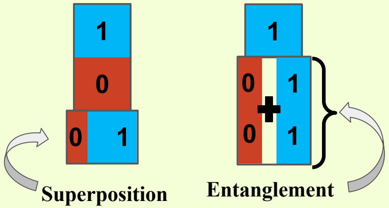

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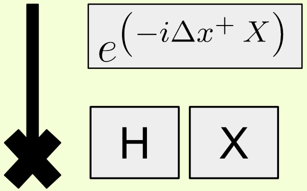
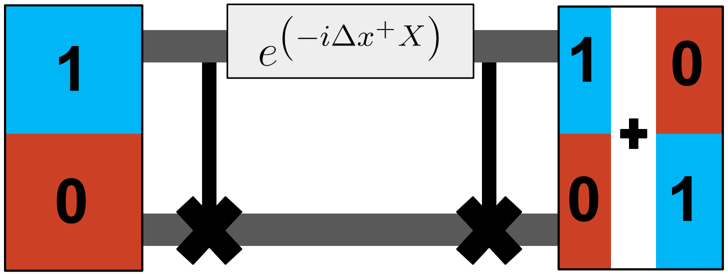


can we do the same with **particle states**?



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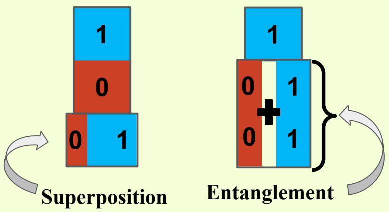

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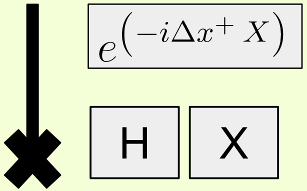

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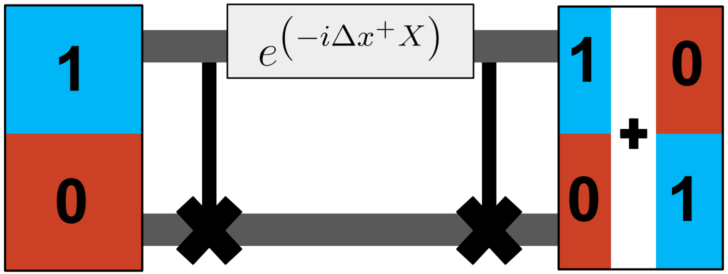

Four basic operations

Turn on

$$\mathfrak{C}_{10} = |1\rangle\langle 0|$$

Turn off

$$\mathfrak{C}_{01} = |0\rangle\langle 1|$$

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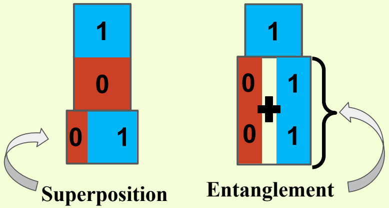

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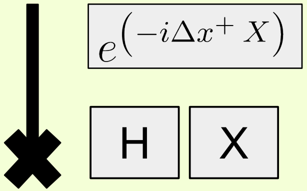

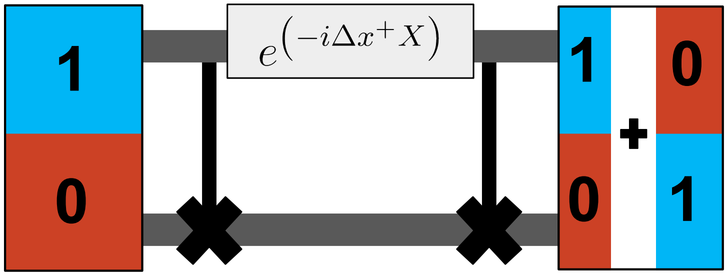

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Turn off

$$\mathfrak{C}_{01} = |0\rangle\langle 1|$$

Control

$$\mathfrak{C}_{11} = |1\rangle\langle 1|$$

Anticontrol

$$\mathfrak{C}_{00} = |0\rangle\langle 0|$$

Encoding I - Goal

Bosons

$$a_\rho^{(n)\dagger} = \sum_{i=1}^{n} \mathcal{S}_i \cdot \mathbb{P}_0^{(n-i)} \otimes \left(\mathfrak{C}_{10} \otimes \mathfrak{s}_\rho^\dagger\right)_i \otimes \mathbb{P}_{i-1}^{(i-1)}$$

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Commutation relations up to boundary term

$$\left[a_\rho^{(n)}, a_\eta^{(n)\dagger}\right] = \delta_{\rho\eta}\, \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)} - \mathcal{S}_n \cdot \left(\mathfrak{C}_{11} \otimes \mathfrak{s}_\rho^\dagger \mathfrak{s}_\eta\right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot \mathcal{S}_n$$

Fermions

$$b_\rho^{(n)\dagger} = \sum_{i=1}^{n} \mathcal{A}_i \cdot \mathbb{P}_0^{(n-i)} \otimes \left(\mathfrak{C}_{10} \otimes \mathfrak{s}_\rho^\dagger\right)_i \otimes \mathbb{P}_{i-1}^{(i-1)}$$

$$b_\rho^{(n)} = \sum_{i=1}^{n} \mathbb{P}_0^{(n-i)} \otimes \left(\mathfrak{C}_{01} \otimes \mathfrak{s}_\rho\right)_i \otimes \mathbb{P}_{i-1}^{(i-1)} \cdot \mathcal{A}_i$$

Anticommutation relations up to boundary term

$$\left\{b_\rho^{(n)}, b_\eta^{(n)\dagger}\right\} = \delta_{\rho\eta}\, \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)} + \mathcal{A}_n \cdot \left(\mathfrak{C}_{11} \otimes \mathfrak{s}_\rho^\dagger \mathfrak{s}_\eta\right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot \mathcal{A}_n$$

Details in PhysRevD.110.116018

Encoding II - Particle Registers

1. Encode each **single-particle** state as a binary number on **particle registers**

$|\Omega\rangle_G =$

$|(1, -1, 8)\rangle_Q =$

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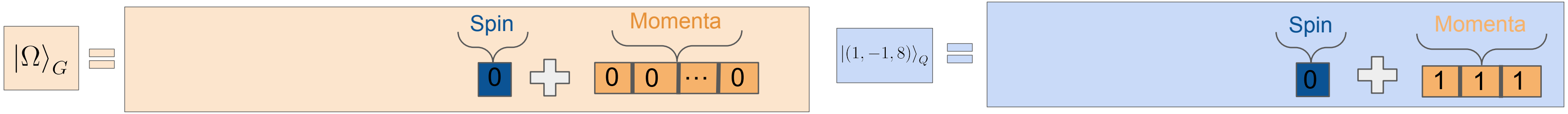

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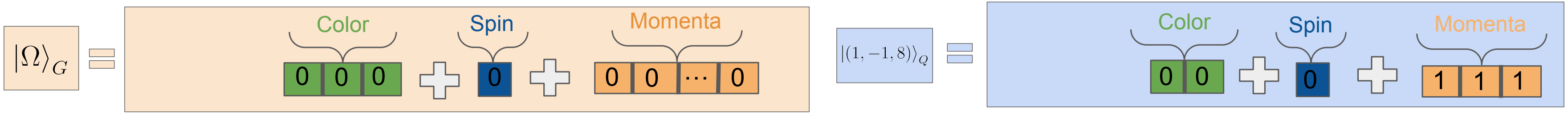

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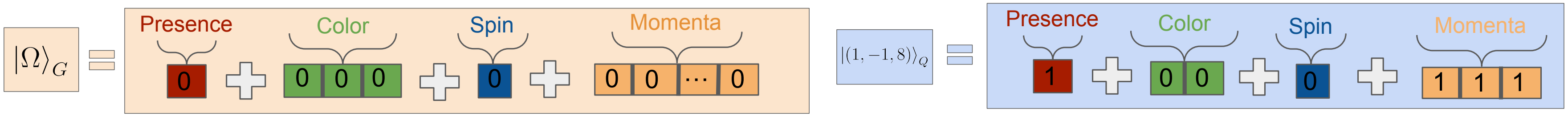

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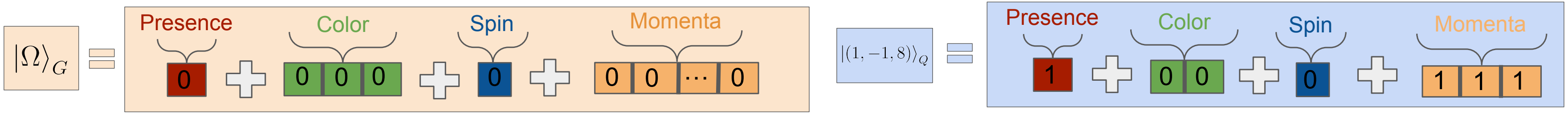

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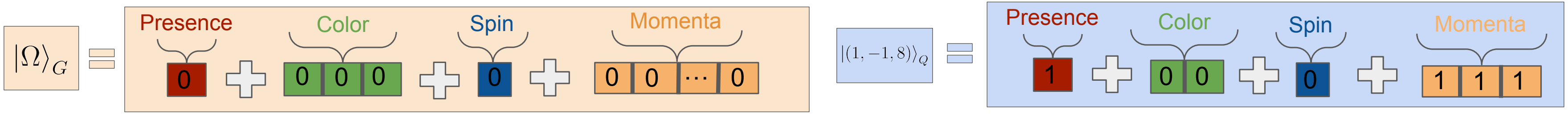

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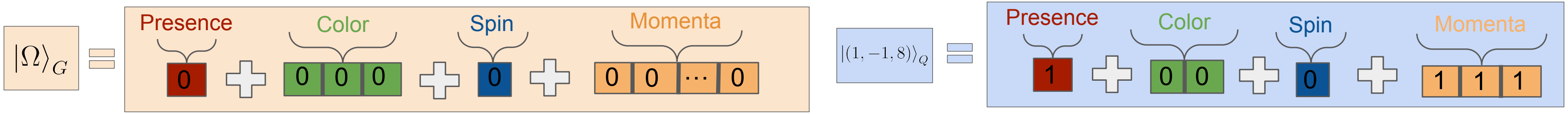

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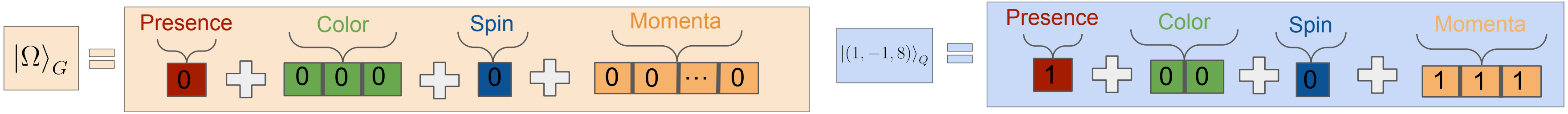

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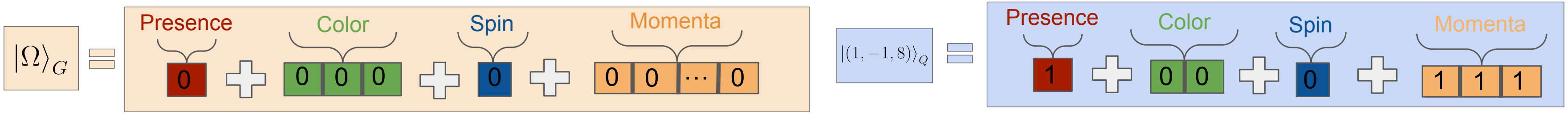

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... ...

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Encoding II - Particle Registers

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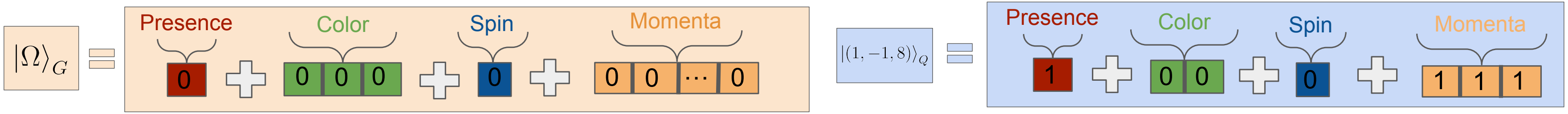

Gluons have 8 colors (Gell-Mann basis):

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...		...	...
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To initiate a register, change modes and presence

$$|(1,\uparrow,(1,0,0))\rangle_G = \mathfrak{C}_{10} \otimes \mathfrak{s}^\dagger_{(1,\uparrow,(1,0,0))} |\Omega\rangle_G$$

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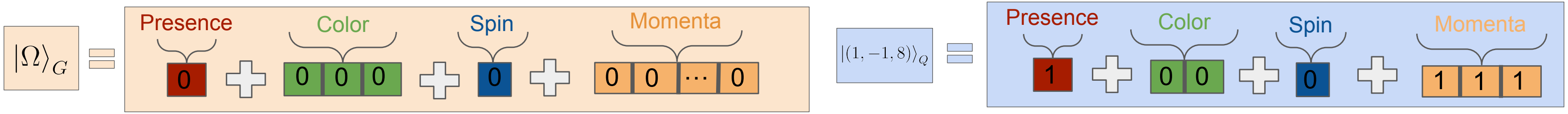

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Encoding III - Multi-particle memories

2. Define multi-particle states combining registers

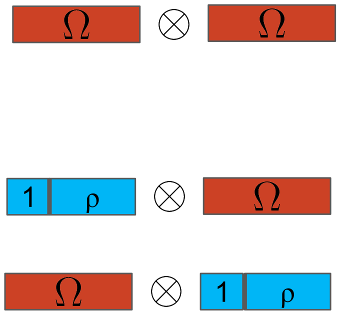
No particles $\Omega \otimes \Omega$

Encoding III - Multi-particle memories

2. Define multi-particle states combining registers

No particles

1 particle



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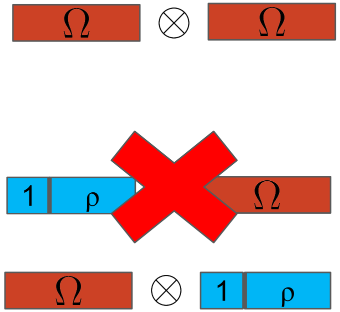

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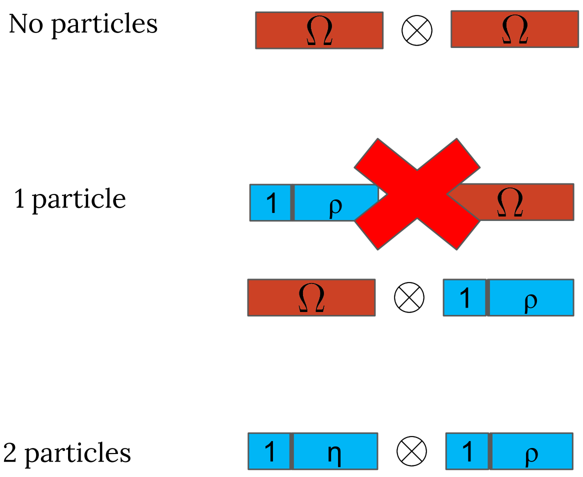


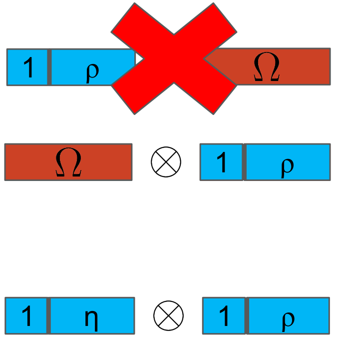
Encoding III - Multi-particle memories

2. Define multi-particle states combining registers

No particles

1 particle

2 particles



Introduce projectors to select registers:

$$\mathbb{P}_i^{(n)} = (\mathfrak{C}_{00} \otimes \mathfrak{i})_n \otimes \cdots \otimes (\mathfrak{C}_{00} \otimes \mathfrak{i})_{i+1} \otimes (\mathfrak{C}_{11} \otimes \mathfrak{i})_i \otimes \cdots \otimes (\mathfrak{C}_{11} \otimes \mathfrak{i})_1$$

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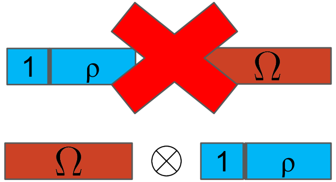
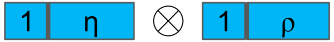


2 particles



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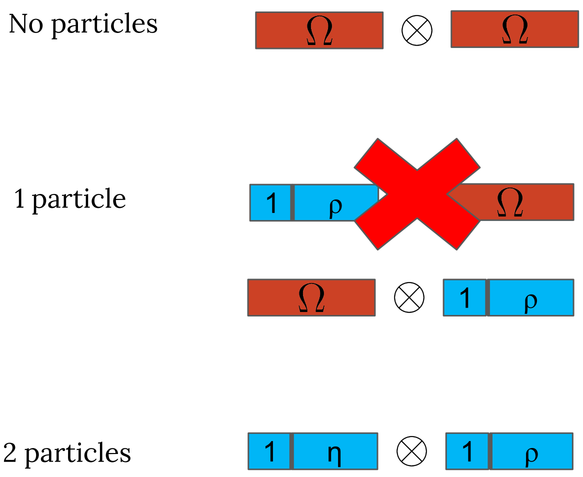

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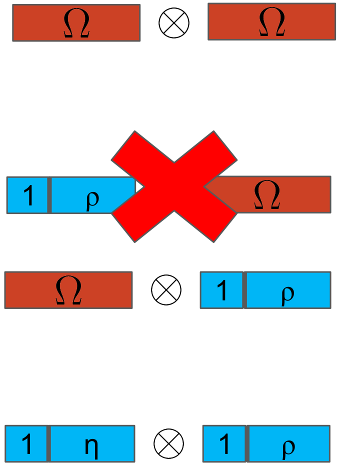

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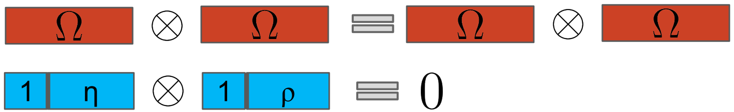
$\mathbb{P}_0^{(2)}$ 

$\mathbb{P}_0^{(2)}$

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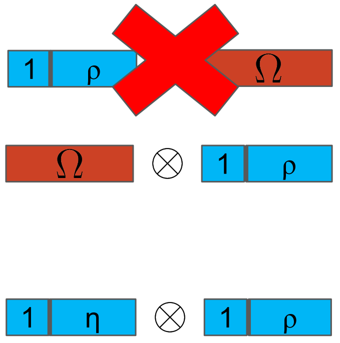

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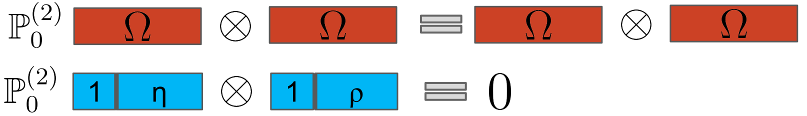
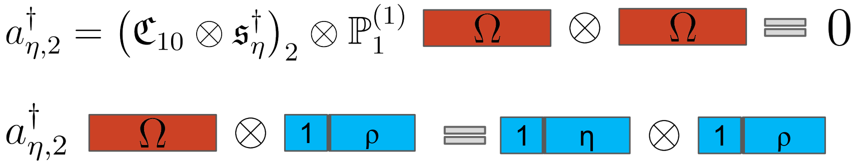


Combine projectors and set operators to create particles on specific registers



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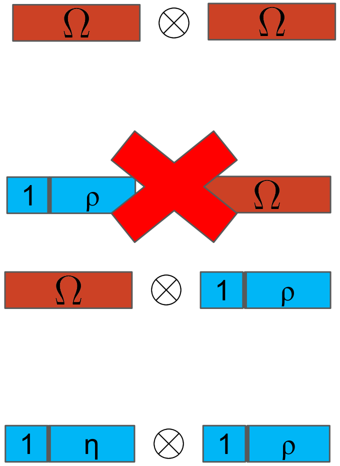

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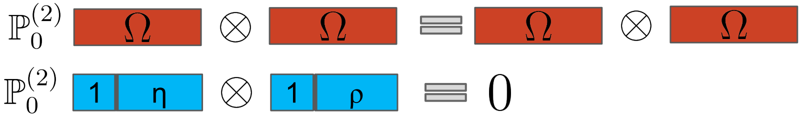
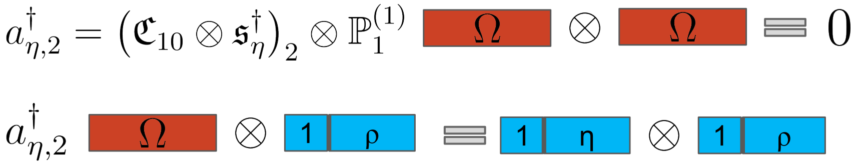


Combine projectors and set operators to create particles on specific registers

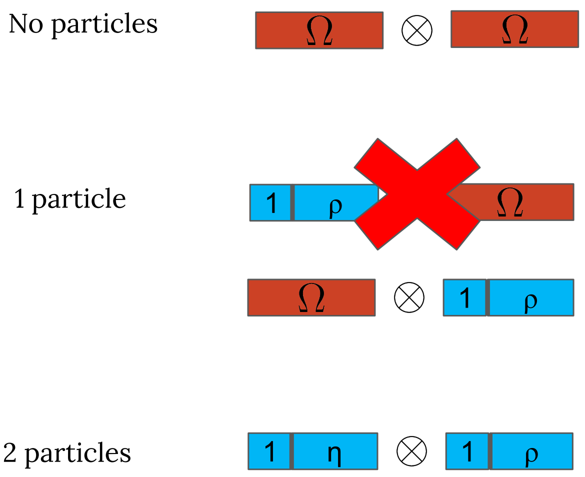
$$a^\dagger_{\eta,2} = \left(\mathfrak{C}_{10} \otimes \mathfrak{s}^\dagger_\eta\right)_2 \otimes \mathbb{P}_1^{(1)}$$



3. Add (anti)symmetrizers for (fermion)bosons

Encoding III - Multi-particle memories

2. Define multi-particle states combining registers



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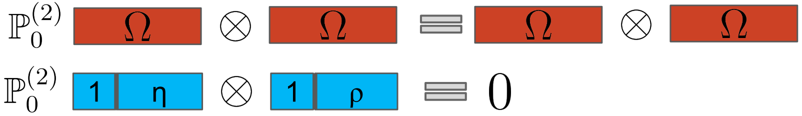

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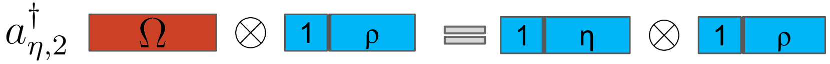

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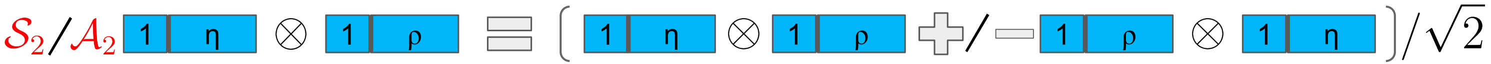

$$\mathcal{S}_j = \left(\mathcal{I} + \mathcal{P}_{(j)(j-1)} + \cdots + \mathcal{P}_{(j)(1)}\right) / \sqrt{j} \qquad \mathcal{A}_j = \left(\mathcal{I} - \mathcal{P}_{(j)(j-1)} - \cdots - \mathcal{P}_{(j)(1)}\right) / \sqrt{j}$$

Encoding IV - Commutation relations revisited

Bosons

$$a_\rho^{(n)\dagger} = \sum_{i=1}^{n} \mathcal{S}_i \cdot \mathbb{P}_0^{(n-i)} \otimes \left(\mathfrak{C}_{10} \otimes \mathfrak{s}_\rho^\dagger\right)_i \otimes \mathbb{P}_{i-1}^{(i-1)}$$

$$a_\rho^{(n)} = \sum_{i=1}^{n} \mathbb{P}_0^{(n-i)} \otimes \left(\mathfrak{C}_{01} \otimes \mathfrak{s}_\rho\right)_i \otimes \mathbb{P}_{i-1}^{(i-1)} \cdot \mathcal{S}_i$$

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Commutation relations

$$\left[a_\rho^{(n)}, a_\eta^{(n)\dagger}\right] = \delta_{\rho\eta}\, \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)}$$

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Commutation relations up to boundary term

$$\left[a_\rho^{(n)}, a_\eta^{(n)\dagger}\right] = \delta_{\rho\eta}\, \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)} - \mathcal{S}_n \cdot \left(\mathfrak{C}_{11} \otimes \mathfrak{s}_\rho^\dagger \mathfrak{s}_\eta\right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot \mathcal{S}_n$$

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Fermions

$$b_\rho^{(n)\dagger} = \sum_{i=1}^{n} \mathcal{A}_i \cdot \mathbb{P}_0^{(n-i)} \otimes \left(\mathfrak{C}_{10} \otimes \mathfrak{s}_\rho^\dagger\right)_i \otimes \mathbb{P}_{i-1}^{(i-1)}$$

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Anticommutation relations up to boundary term

$$\left\{b_\rho^{(n)}, b_\eta^{(n)\dagger}\right\} = \delta_{\rho\eta}\, \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)} + \mathcal{A}_n \cdot \left(\mathfrak{C}_{11} \otimes \mathfrak{s}_\rho^\dagger \mathfrak{s}_\eta\right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot \mathcal{A}_n$$

Details in PhysRevD.110.116018

Encoding IV - Commutation relations revisited

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Commutation relations up to boundary term

$$\left[a_\rho^{(n)}, a_\eta^{(n)\dagger}\right] = \delta_{\rho\eta}\, \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)} - \mathcal{S}_n \cdot \left(\mathfrak{C}_{11} \otimes \mathfrak{s}_\rho^\dagger \mathfrak{s}_\eta\right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot \mathcal{S}_n$$

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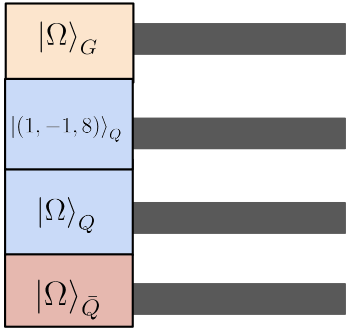
Details in PhysRevD.110.116018

Algorithm I - Set up and exponentiation

1. Start with quark with max momenta

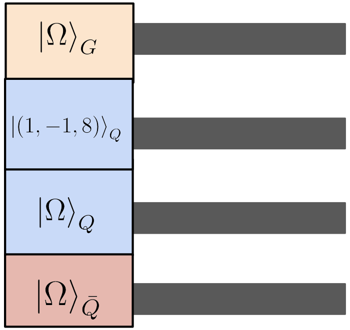


Algorithm I - Set up and exponentiation

1. Start with quark with max momenta
2. Apply codification

$$V_1 = \int V(1\,;\,2,\,3)\, a_3^\dagger\, b_2^\dagger\, b_1 + h.c.$$

$$= \underbrace{\mathcal{S}_{j_b} \cdot \mathbb{P}_0^{(n_b - j_b)} \cdot \left(\mathfrak{C}_{10} \otimes \mathfrak{s}_3^\dagger\right)_{j_b} \otimes \mathbb{P}_{j_b - 1}^{(j_b - 1)}}_{\text{gluons}} \bar{\otimes} \underbrace{\mathcal{A}_{j_f} \cdot \mathbb{P}_0^{(n_f - j_f)} \cdot \left(\mathfrak{C}_{11} \otimes \mathfrak{s}_2^\dagger \mathfrak{s}_1\right)_{j_f} \otimes \mathbb{P}_{j_f - 1}^{(j_f - 1)} \cdot \mathcal{A}_{j_f}}_{\text{fermions}} + h.c.$$

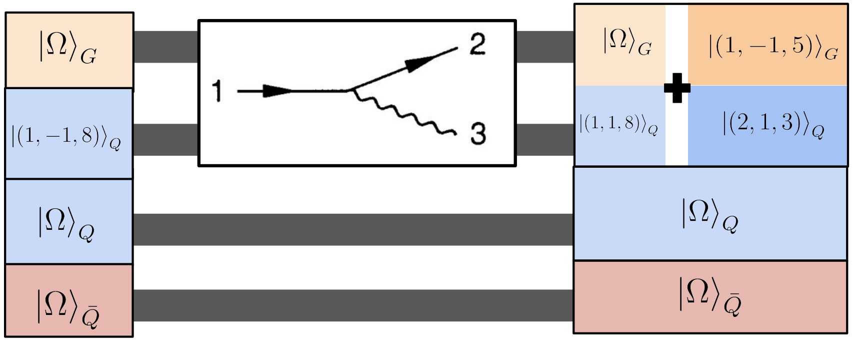


Algorithm I - Set up and exponentiation

1. Start with quark with max momenta
2. Apply codification and exponentiate using Trotter decomposition

$$V_1 = \int V(1\,;\,2,\,3)\, a_3^\dagger\, b_2^\dagger\, b_1 + h.c.$$

$$= \underbrace{\mathcal{S}_{j_b} \cdot \mathbb{P}_0^{(n_b-j_b)} \cdot \left(\mathfrak{C}_{10} \otimes \mathfrak{s}_3^\dagger\right)_{j_b} \otimes \mathbb{P}_{j_b-1}^{(j_b-1)}}_{\text{gluons}} \bar{\otimes} \underbrace{\mathcal{A}_{j_f} \cdot \mathbb{P}_0^{(n_f-j_f)} \cdot \left(\mathfrak{C}_{11} \otimes \mathfrak{s}_2^\dagger \mathfrak{s}_1\right)_{j_f} \otimes \mathbb{P}_{j_f-1}^{(j_f-1)} \cdot \mathcal{A}_{j_f}}_{\text{fermions}} + h.c.$$

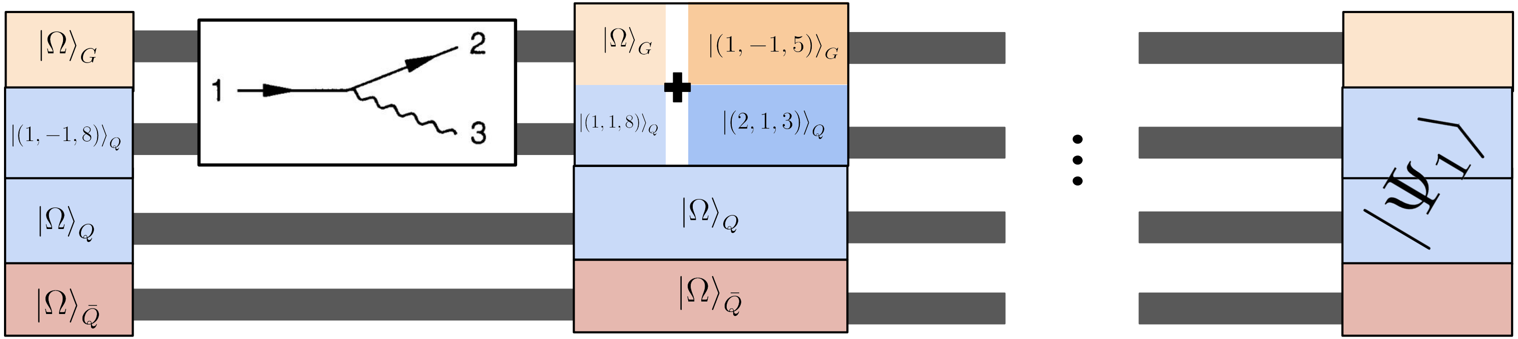


Algorithm I - Set up and exponentiation

1. Start with quark with max momenta
2. Apply codification and exponentiate using Trotter decomposition
3. Repeat for all Hamiltonian terms to evolve **1 Trotter step**

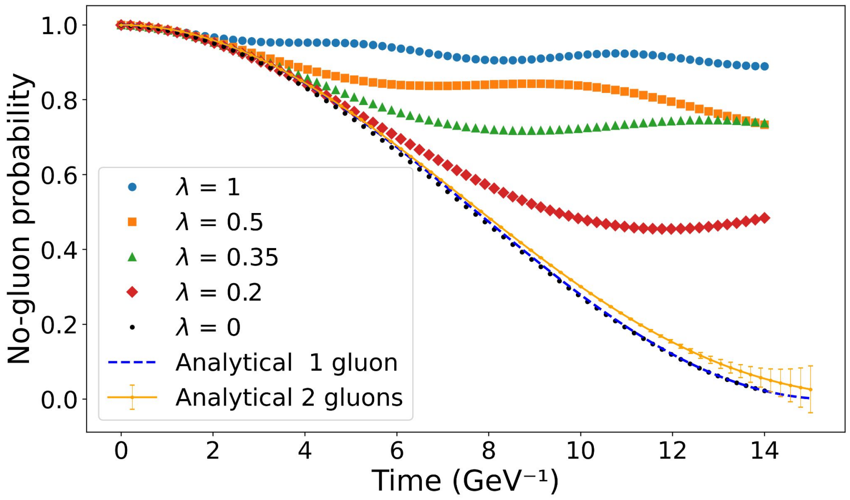
$$V_1 = \int V(1\,;\,2,\,3)\, a_3^\dagger\, b_2^\dagger\, b_1 + h.c.$$

$$= \underbrace{\mathcal{S}_{j_b} \cdot \mathbb{P}_0^{(n_b - j_b)} \cdot \left(\mathfrak{C}_{10} \otimes \mathfrak{s}_3^\dagger\right)_{j_b} \otimes \mathbb{P}_{j_b-1}^{(j_b-1)}}_{\text{gluons}} \bar{\otimes} \underbrace{\mathcal{A}_{j_f} \cdot \mathbb{P}_0^{(n_f - j_f)} \cdot \left(\mathfrak{C}_{11} \otimes \mathfrak{s}_2^\dagger \mathfrak{s}_1\right)_{j_f} \otimes \mathbb{P}_{j_f-1}^{(j_f-1)} \cdot \mathcal{A}_{j_f}}_{\text{fermions}} + h.c.$$



Algorithm II - Halting protocol

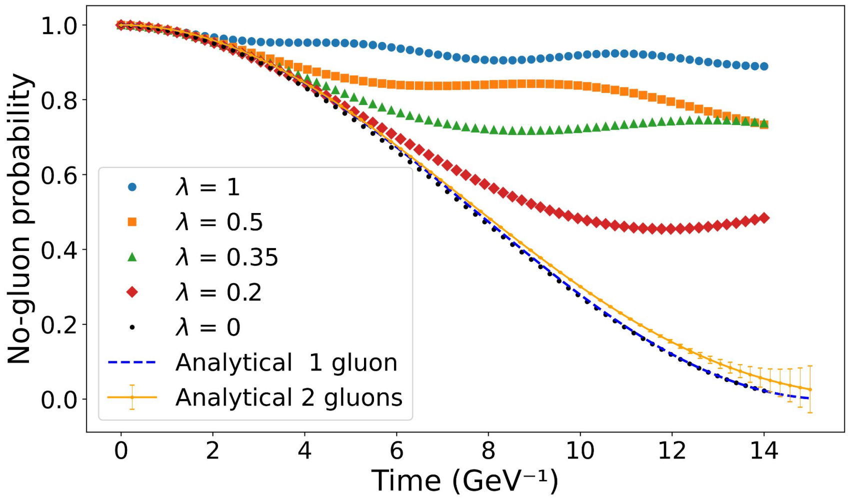
$$\exp\{-i\Delta t(V_1 + \lambda E_c)\}$$



Algorithm II - Halting protocol

$$\exp\{-i\Delta t(V_1 + \lambda E_c)\}$$

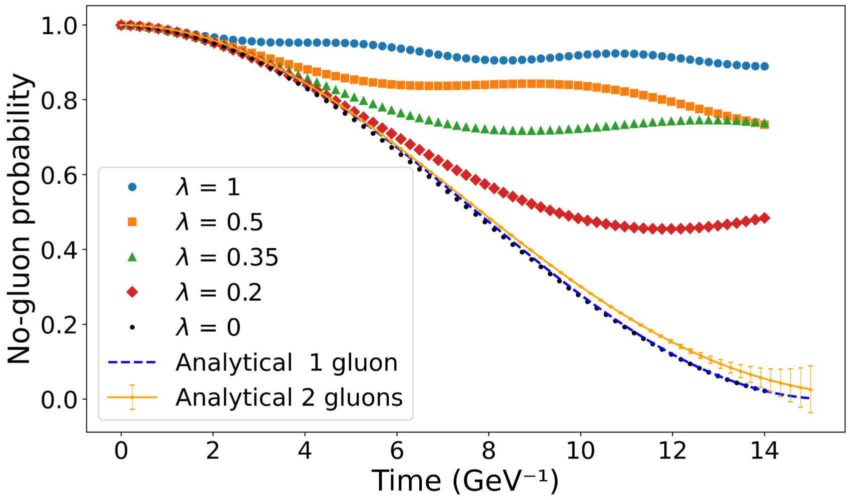
$$S = \sum_i -p_i \log(p_i)$$



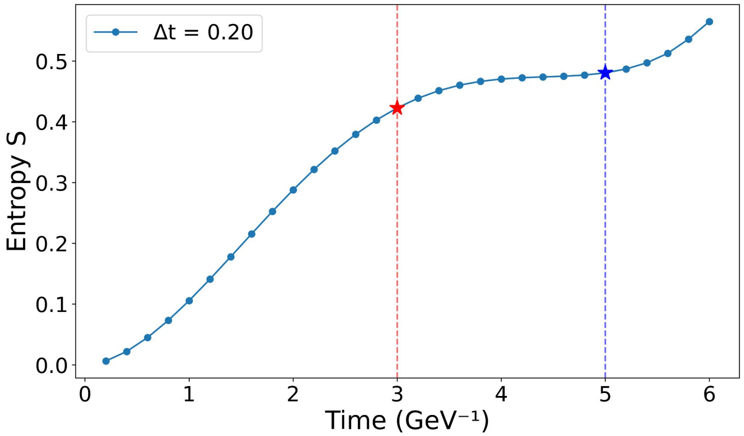
Algorithm II - Halting protocol

4. Reach saturation

$$\exp\{-i\Delta t(V_1 + \lambda E_c)\}$$



$$S = \sum_i -p_i \log(p_i)$$



30 Trotter Steps for a total evolution time of 6 GeV^{-1}

Algorithm III - Measurement

5. Measure J/Ψ wavefunction

An ansatz

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5. Measure J/Ψ wavefunction

An ansatz
$$|J/\Psi\rangle = \sum \frac{\chi_0(x)}{\sqrt{x\,(z-x)}}\,\delta_{c_q c_{\bar{q}}}\;\sigma_{ij}\;|x\;i\,c_q,(z-x)\;j\,c_{\bar{q}}\rangle$$

doi.org/10.1140/epjc/s10052-022-10988-5

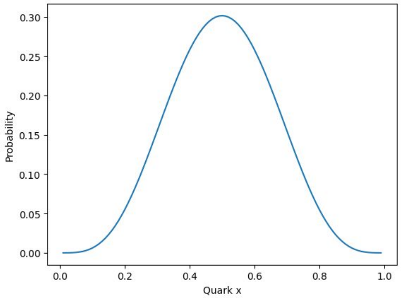
Algorithm III - Measurement

5. Measure J/Ψ wavefunction

An ansatz $$|J/\Psi\rangle = \sum \frac{\chi_0(x)}{\sqrt{x\,(z-x)}} \delta_{c_q c_{\bar{q}}} \; \sigma_{ij} \; |x \; i \, c_q, (z-x) \; j \, c_{\bar{q}}\rangle$$

doi.org/10.1140/epjc/s10052-022-10988-5

- Simple momentum fraction dependence



Algorithm III - Measurement

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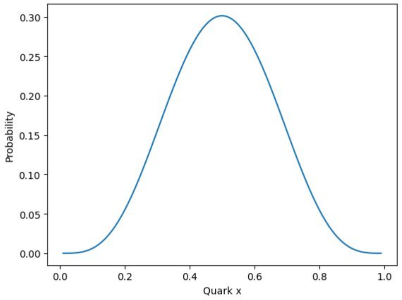
- Simple momentum fraction dependence



- Color singlet

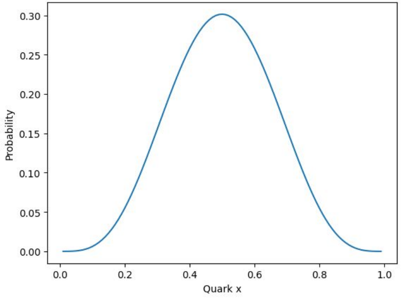
Algorithm III - Measurement

5. Measure J/Ψ wavefunction

An ansatz $$|J/\Psi\rangle = \sum \frac{\chi_0(x)}{\sqrt{x\,(z-x)}} \delta_{c_q c_{\bar{q}}} \sigma_{ij} |x\ i\ c_q, (z-x)\ j\ c_{\bar{q}}\rangle$$

doi.org/10.1140/epjc/s10052-022-10988-5

- Simple momentum fraction dependence



- Color singlet
- Spin singlet

Algorithm III - Measurement

5. Measure J/Ψ wavefunction

An ansatz $$|J/\Psi\rangle = \sum \frac{\chi_0(x)}{\sqrt{x\,(z-x)}}\,\delta_{c_q c_{\bar{q}}}\,\sigma_{ij}\,|x\ i\ c_q, (z-x)\ j\ c_{\bar{q}}\rangle$$

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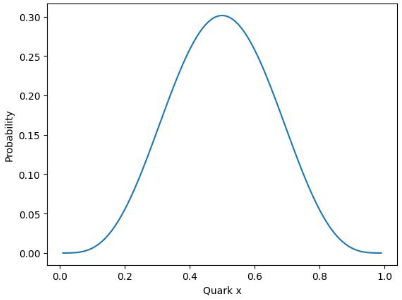
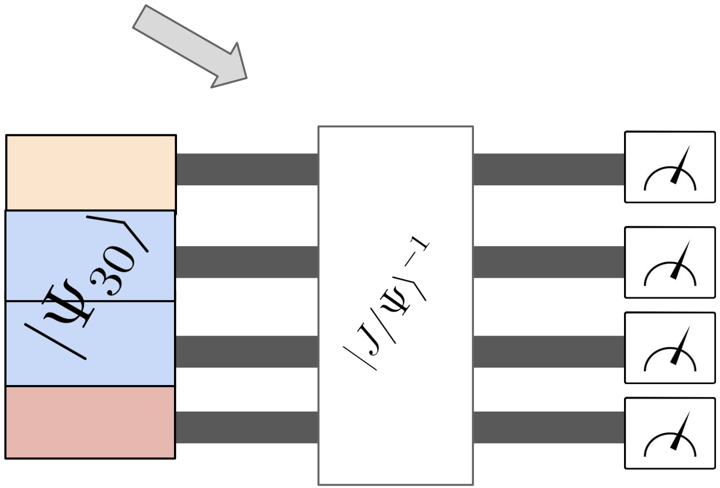
- Simple momentum fraction dependence



- Color singlet

- Spin singlet



Algorithm III - Measurement

5. Measure J/Ψ wavefunction

An ansatz $|J/\Psi\rangle$ Should be obtained from Hamiltonian! $x)\ j\ c_{\bar{q}}\rangle$

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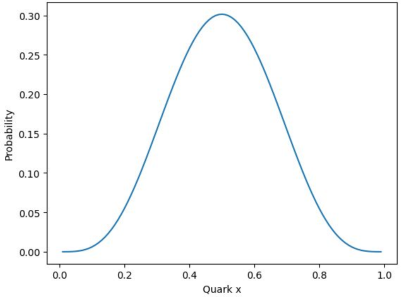
- Simple momentum fraction dependence



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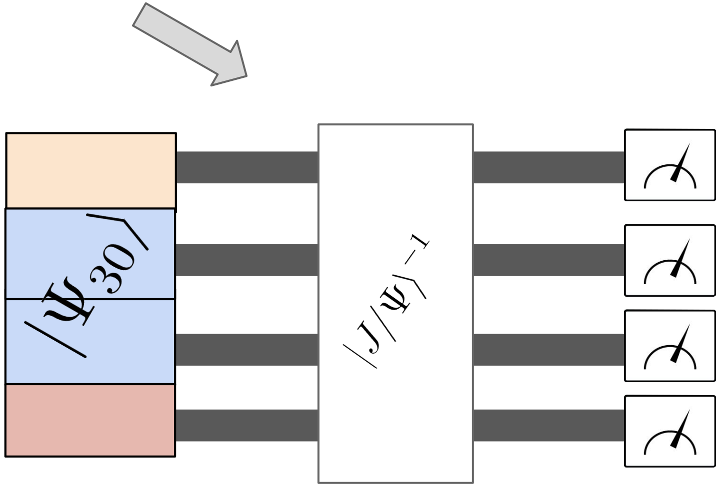
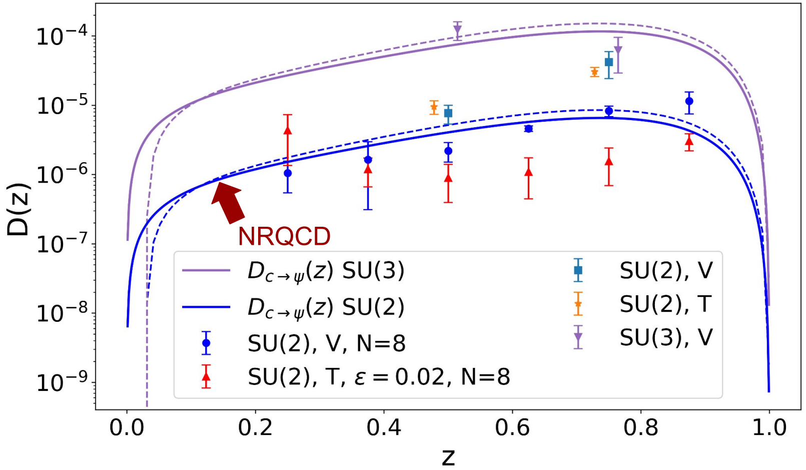
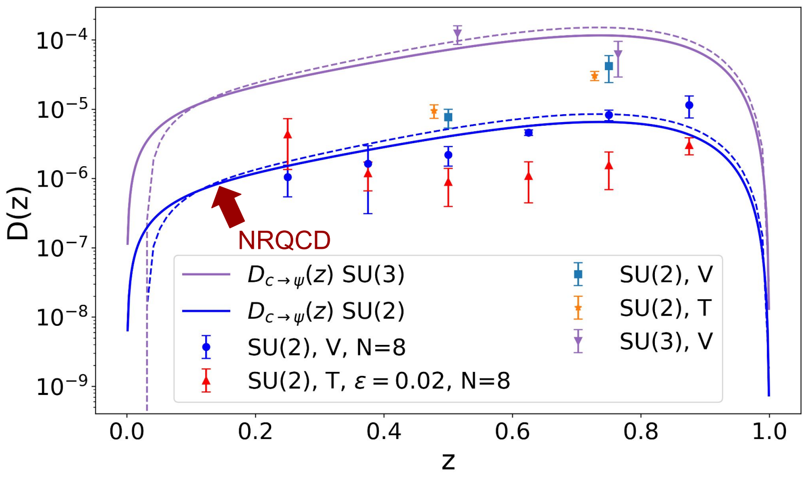


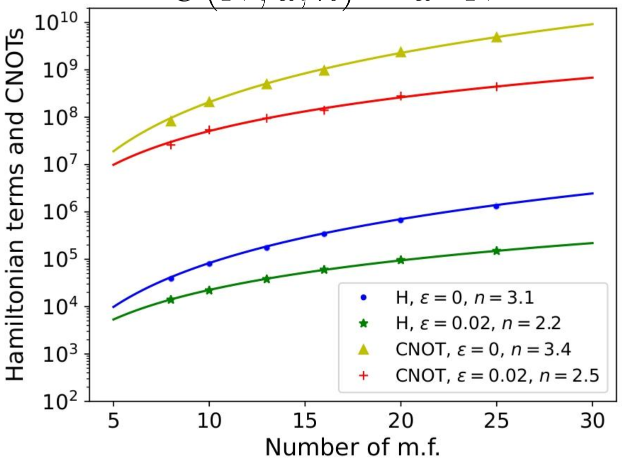
Results & conclusions



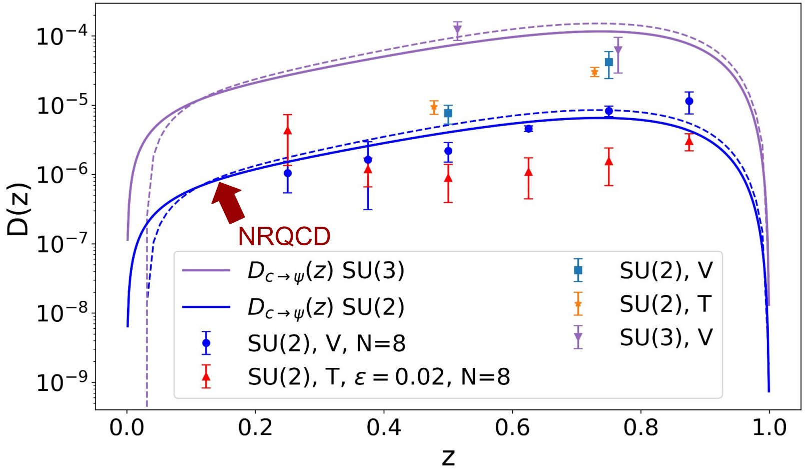
Results & conclusions

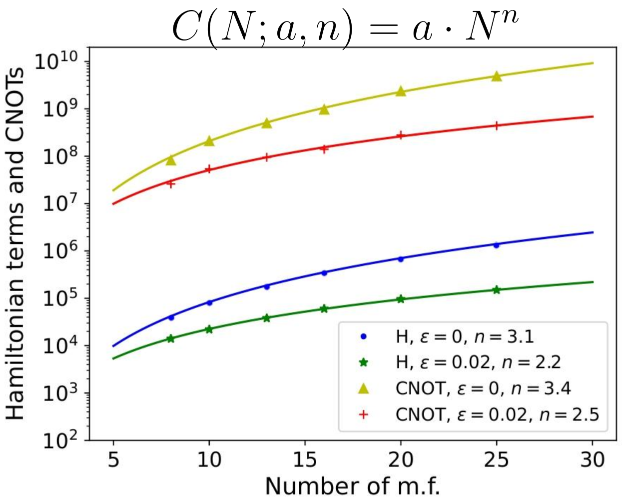


$C(N; a, n) = a \cdot N^n$



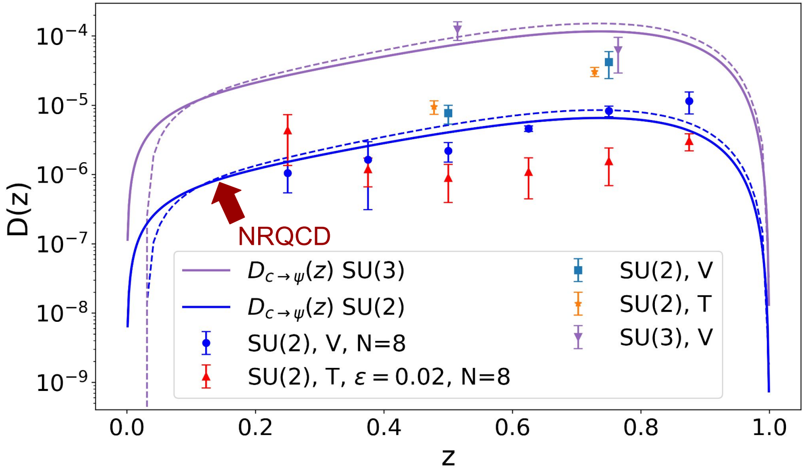
Results & conclusions

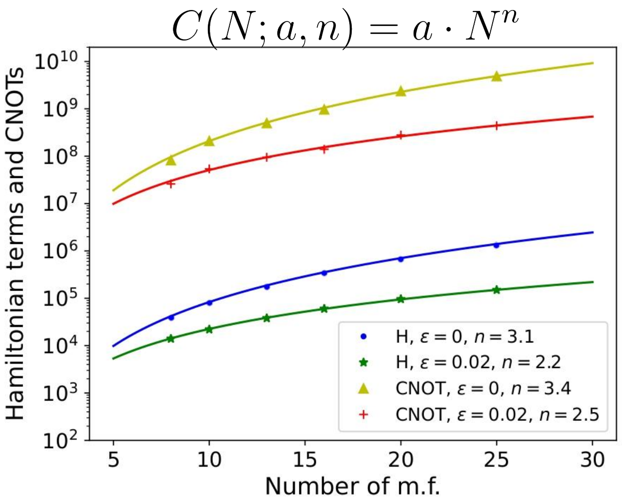




- **End to end** simulation of fragmentation in LF QCD with 2 quarks, 1 antiquark, and 1 gluon.

Results & conclusions





- **End to end** simulation of fragmentation in LF QCD with 2 quarks, 1 antiquark, and 1 gluon.
- 29 qubits $\Longrightarrow$ 17 GB of wavefunction

Run configuration	V/T	N_c	N grid	N_{qubits}	Δt (GeV^{-1})	ϵ	Runtime (min)
1	V	2	4	25	0.2	0	1.5
2	T	2	4	25	0.1	0	22
3	T	2	4	25	0.2	0.02	15
4	V	2	8	29	0.2	0	110
5	T	2	8	29	0.2	0.02	1800
6	V	3	4	29	0.25	0	120

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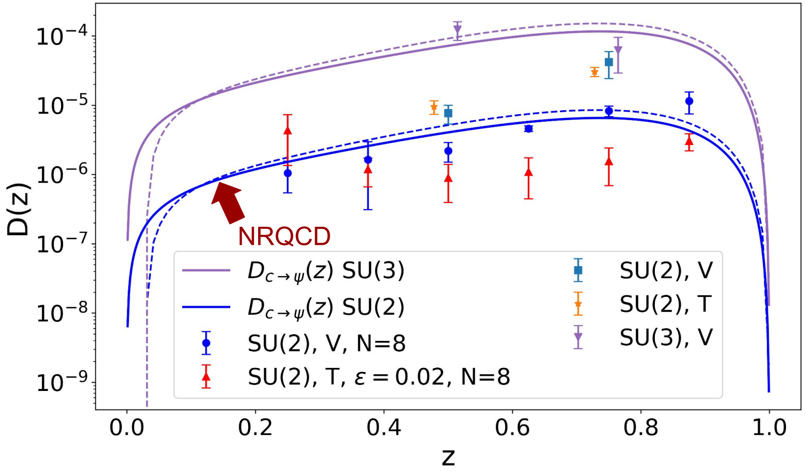
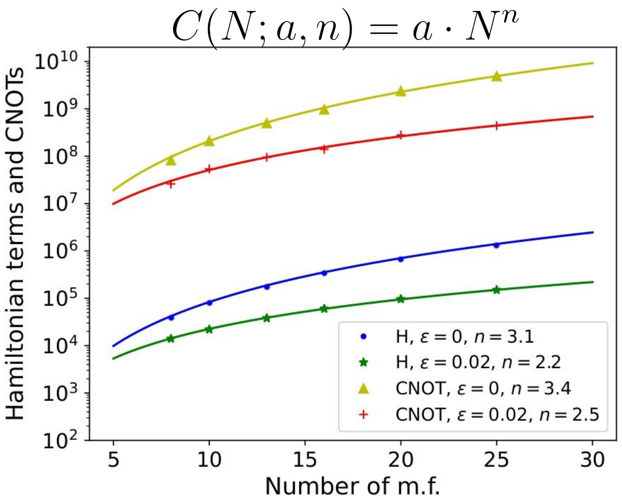
Results & conclusions





- **End to end** simulation of fragmentation in LF QCD with 2 quarks, 1 antiquark, and 1 gluon.
- 29 qubits $\Longrightarrow$ 17 GB of wavefunction
- Total number of gates $\sim 10^8$ far from today$\sim 10^3$ gates, but improving fast!

Run configuration	V/T	N_c	N grid	N_{qubits}	Δt (GeV^{-1})	ϵ	Runtime (min)
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Appendix: More about scaling