Quantum algorithms for bosonic Hamiltonians and applications to lattice field theory

Gloria Tejedor García

Stony Brook University

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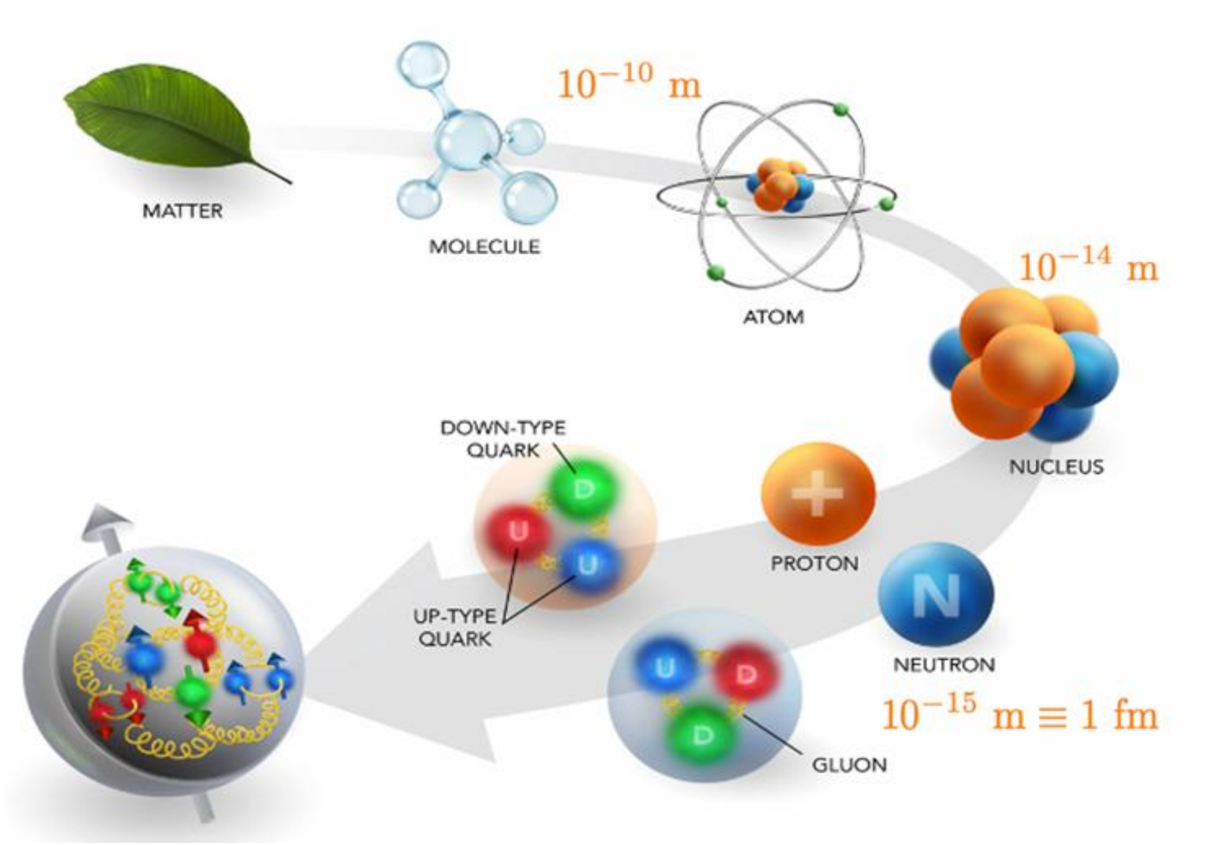
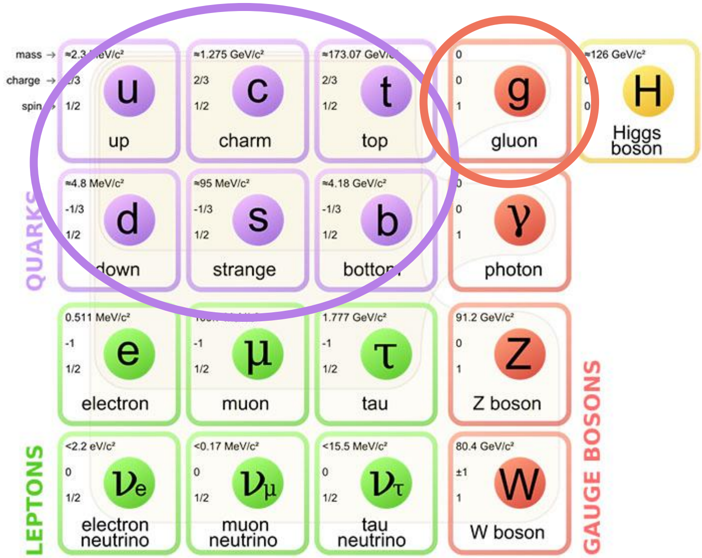
Nucleon Structure and the Origin of QCD

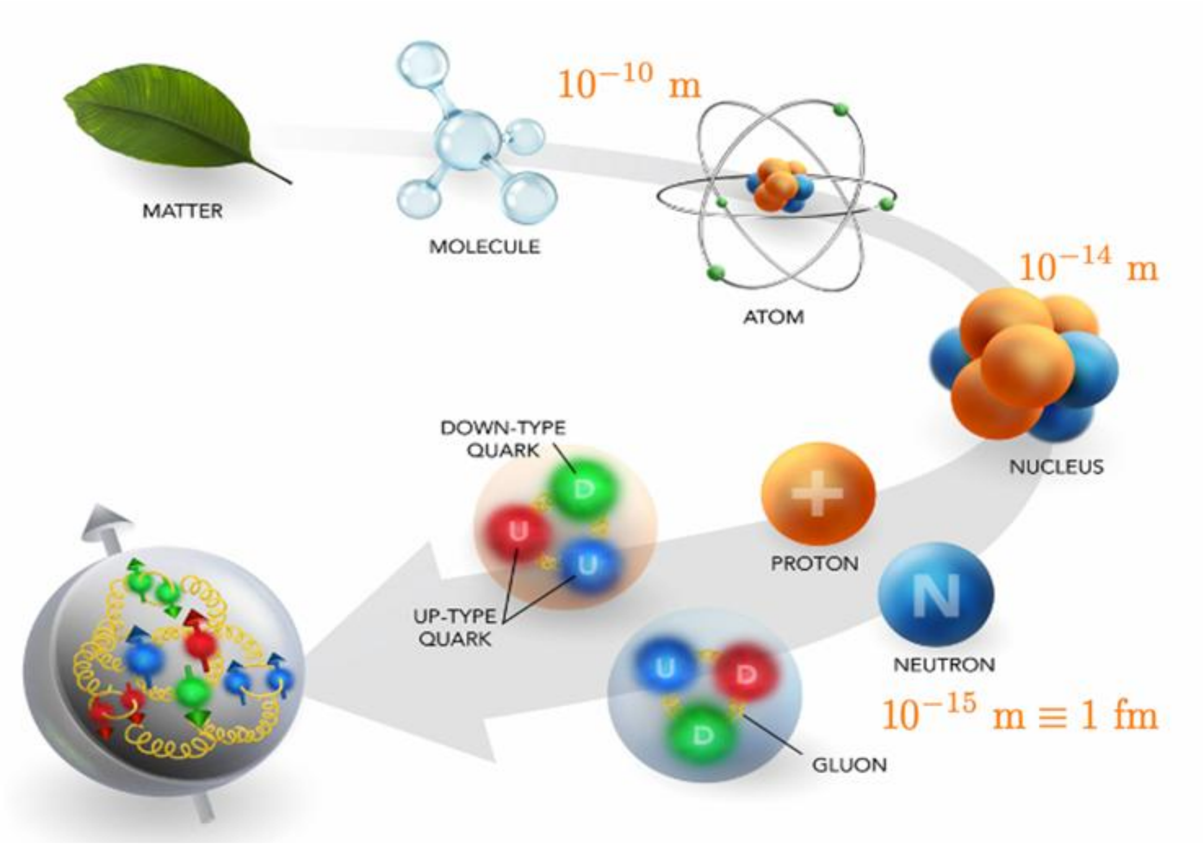


Nucleon Structure and the Origin of QCD

Quarks: Fundamental particles inside protons and neutrons.

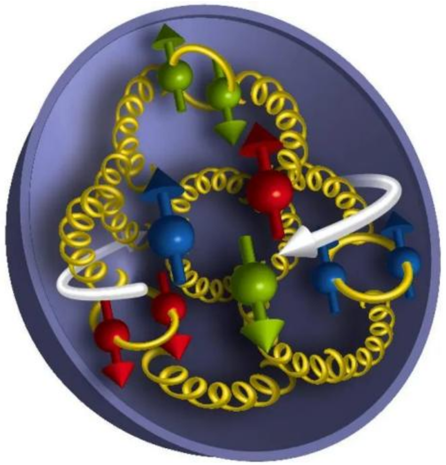
Gluons: Force carriers of the strong interaction, mediating quark binding.



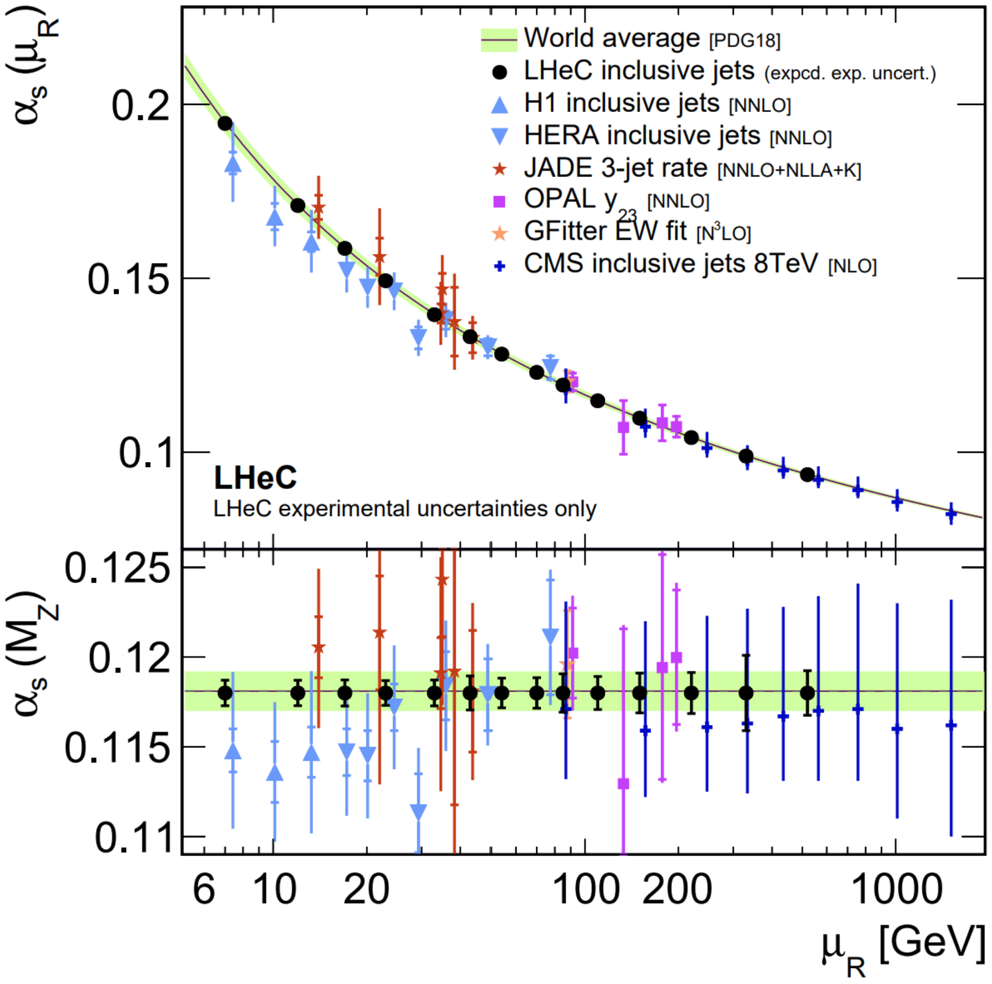


Unresolved Issues in QCD

- **ASYMPTOTIC FREEDOM**
  - → **Weak** interaction at high energies
  - → Perturbative
- **COLOR CONFINEMENT**
  - → **Strong** interaction at low energies
  - → **Non**-perturbative



LHeC and FCC-he Study Group, 2020



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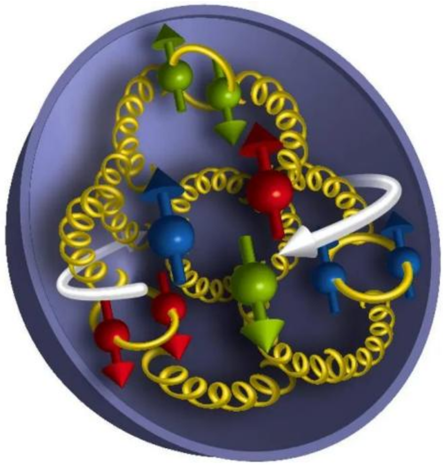

* Lattice QCD
* Effective Field Theories
* Quantum Simulations

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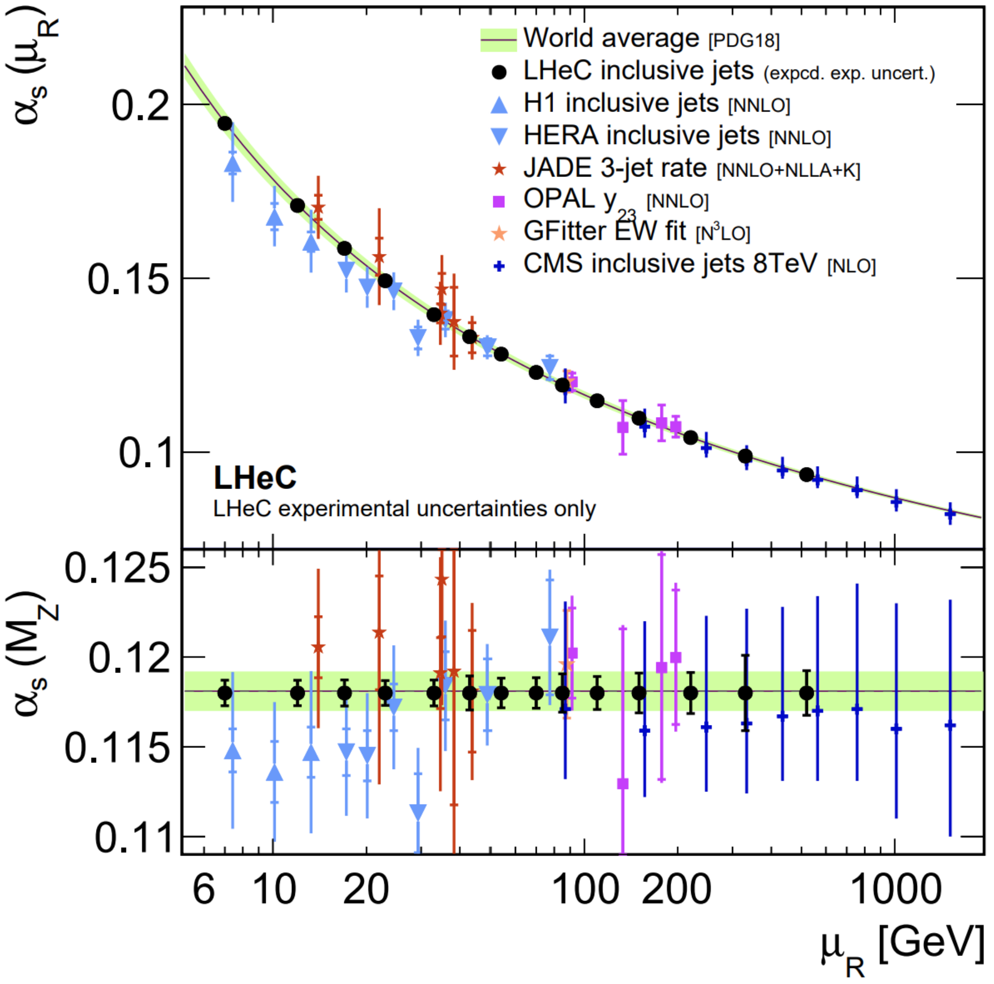

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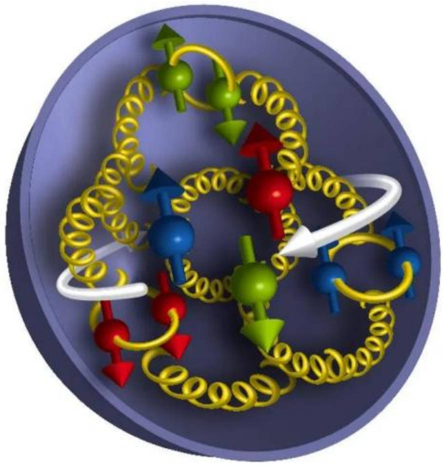

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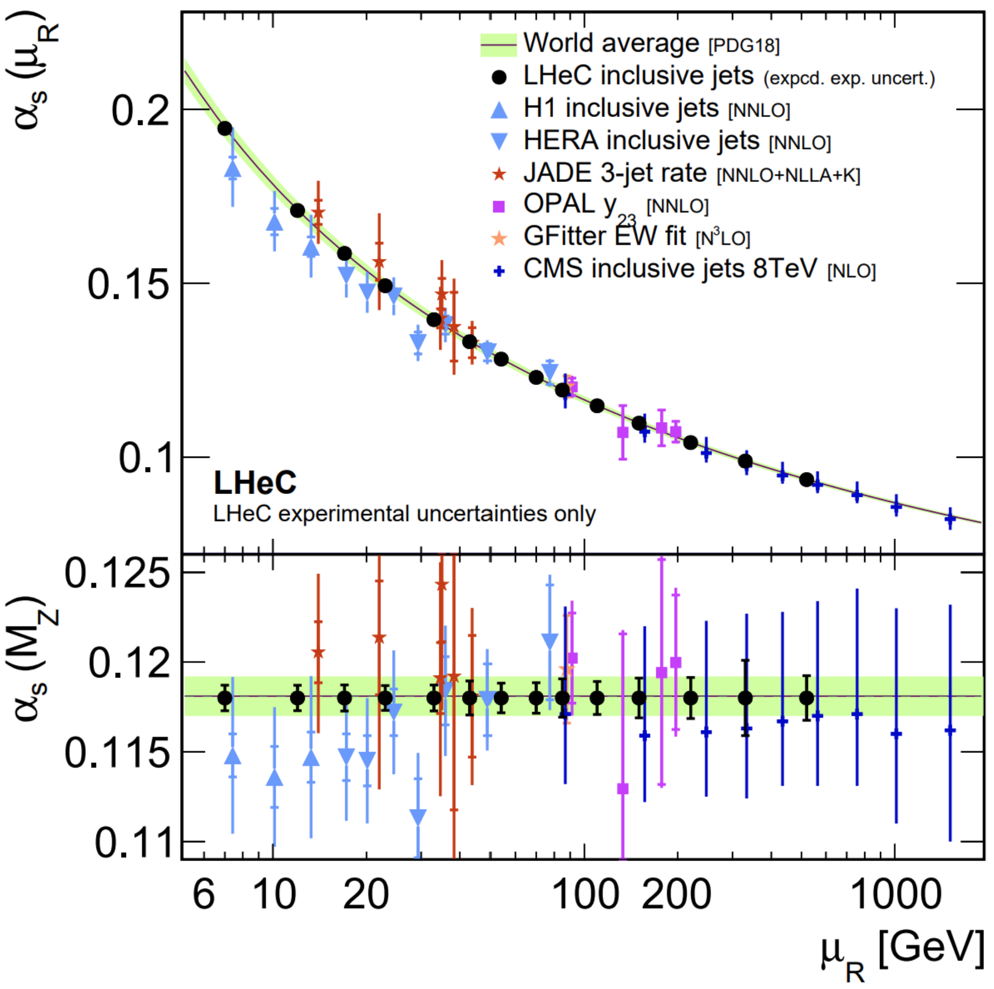

COMPLEXITY CLASSES AND QCD

How difficult are QCD simulations?

- **P/BPP** → **Classical** computers <u>solve</u> in polynomial time

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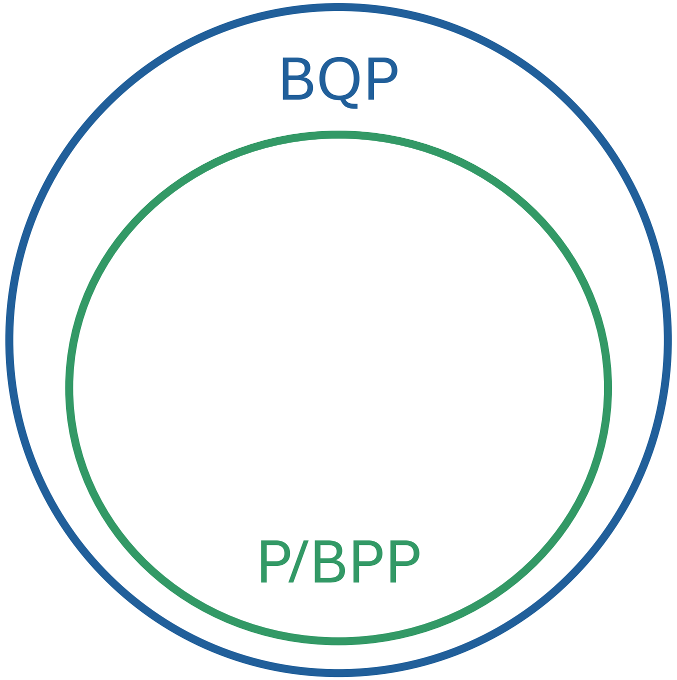

- **BQP** → **Quantum** computers solve in polynomial time
- **P/BPP** → **Classical** computers solve in polynomial time

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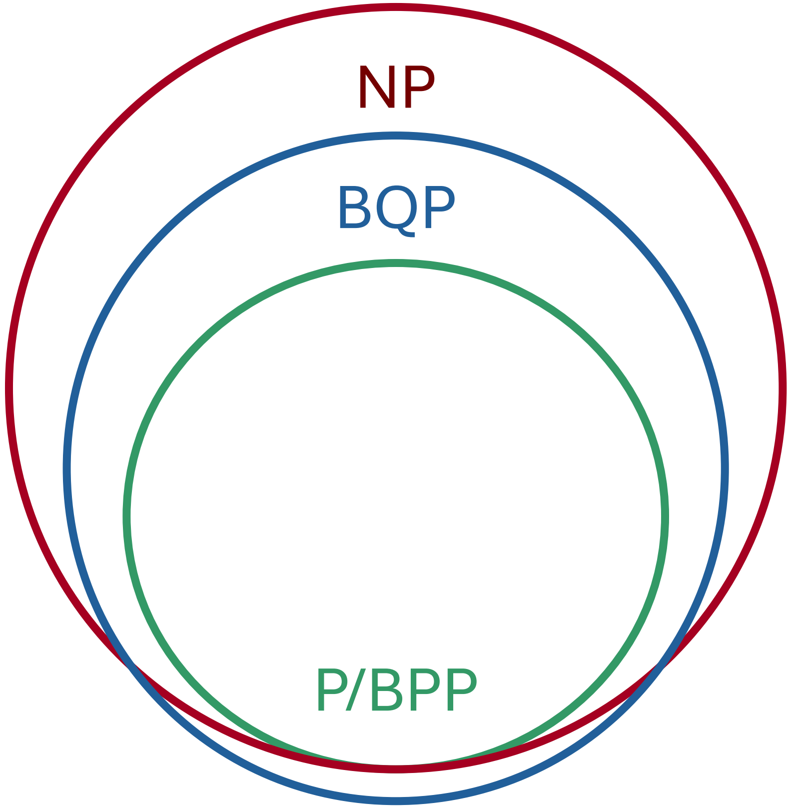

- **NP** → **Classical** computers verify in polynomial time.
- **BQP** → **Quantum** computers solve in polynomial time
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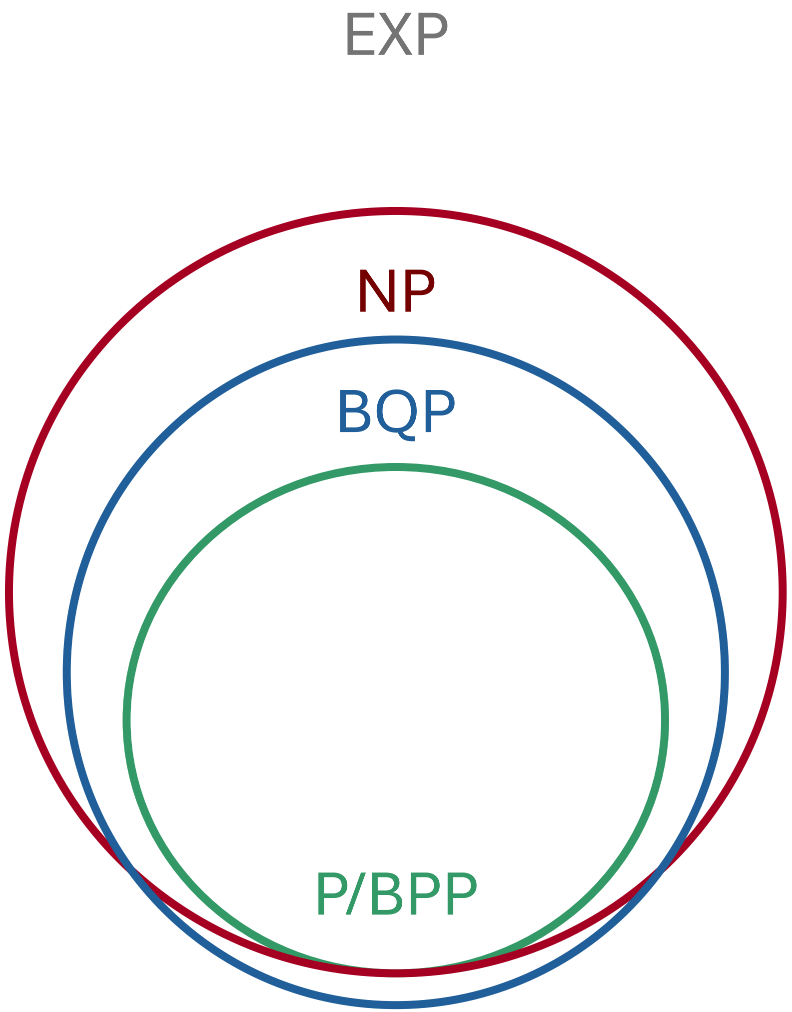
How difficult are QCD simulations?



- **EXP** → Beyond NP.
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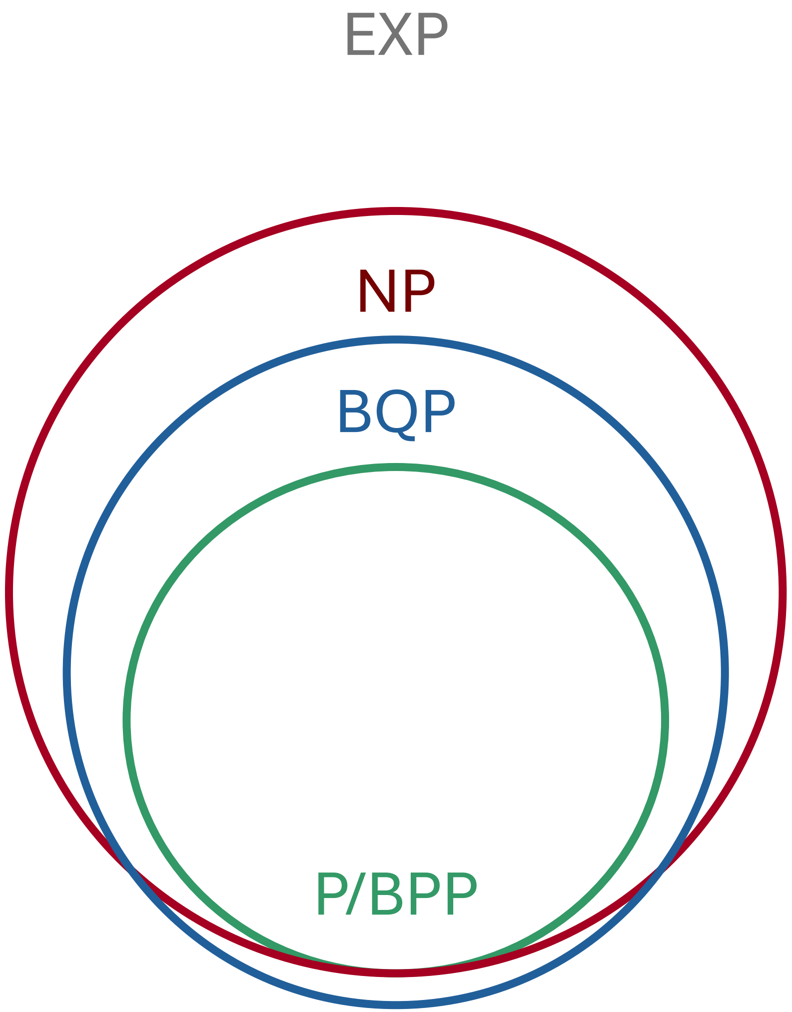

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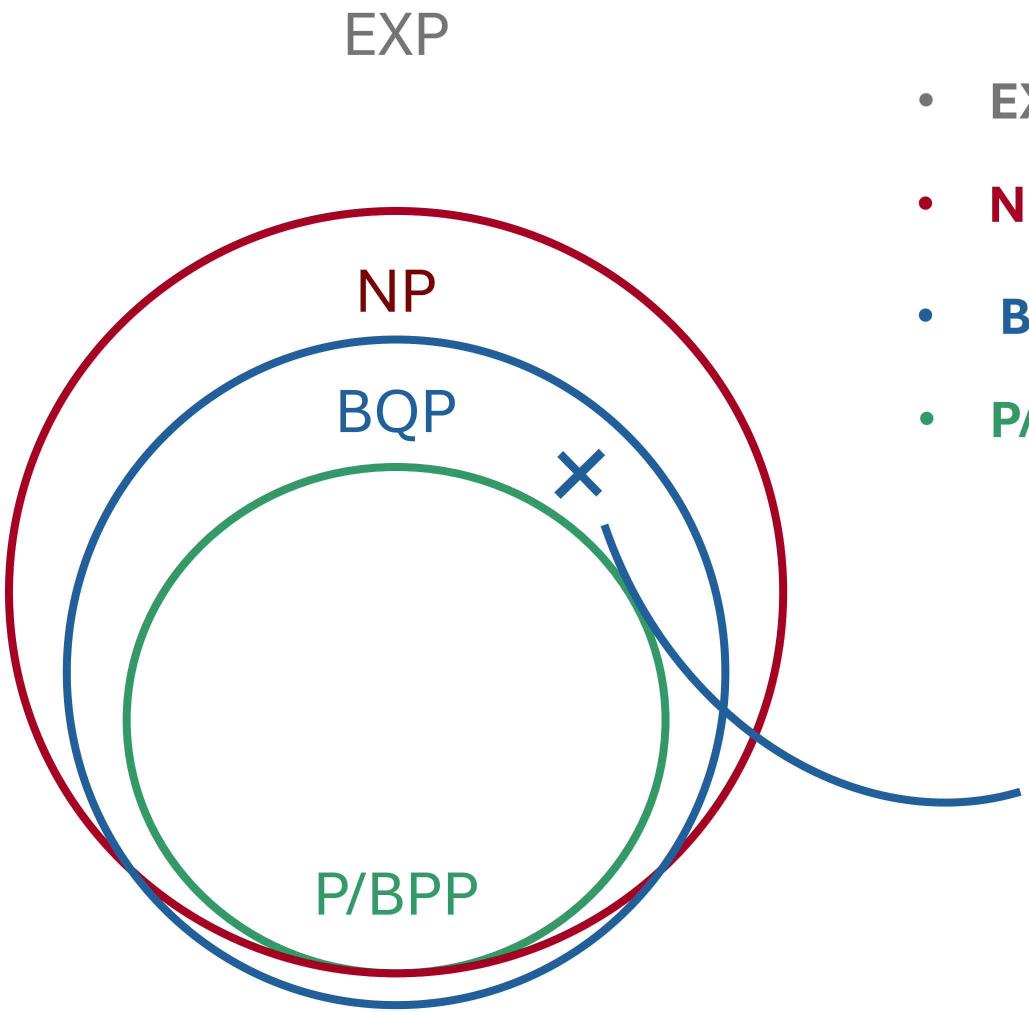
WHERE IS QCD?

How difficult are QCD simulations?



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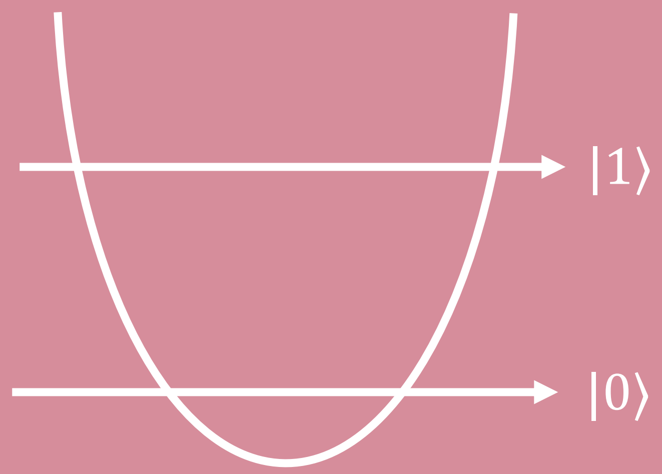
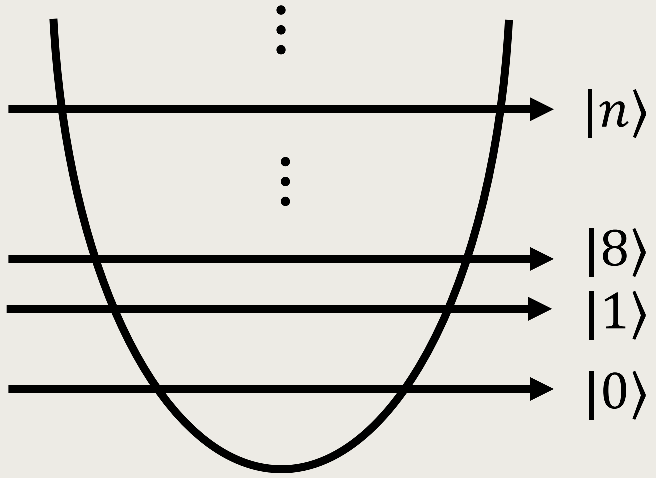
- **Scalar field scattering** is BQP (Jordan–Lee–Preskill)
- **Real-time** or **high-density** (?) QCD simulations

FROM COLLIDER PHYSICS TO QUANTUM SIMULATIONS

Qubits and Qumodes

	QUBIT	QUMODE
Nature of information	• **Discrete** → Two-level system	• **Continuous** → Harmonic oscillator with infinite many levels





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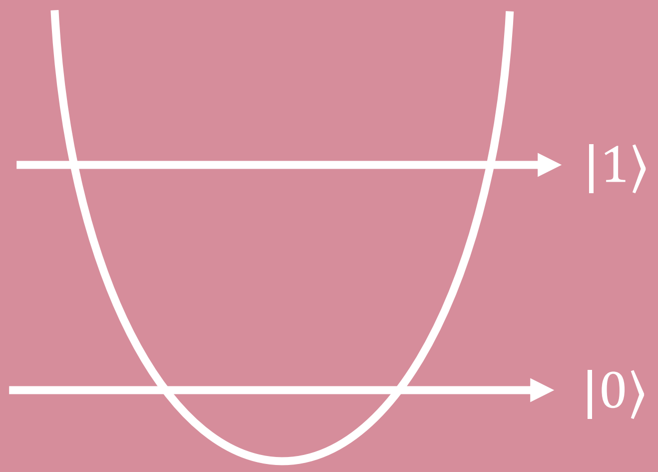

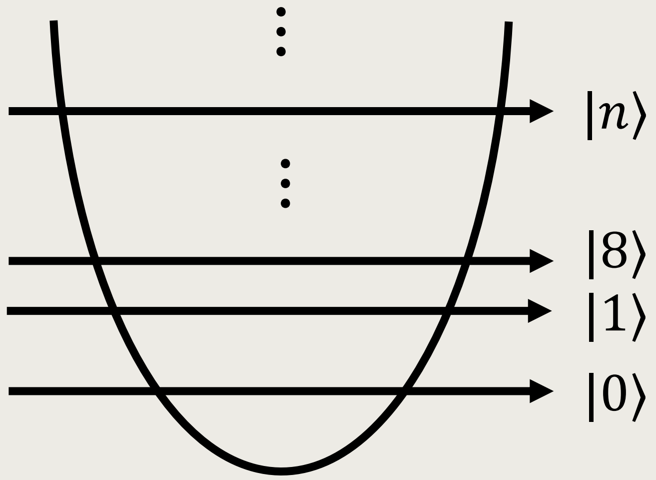

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Physical implementation	• Superconducting Circuits • Ion traps • Neutral Atoms	• Superconducting Circuits • Ion traps • Neutral atoms

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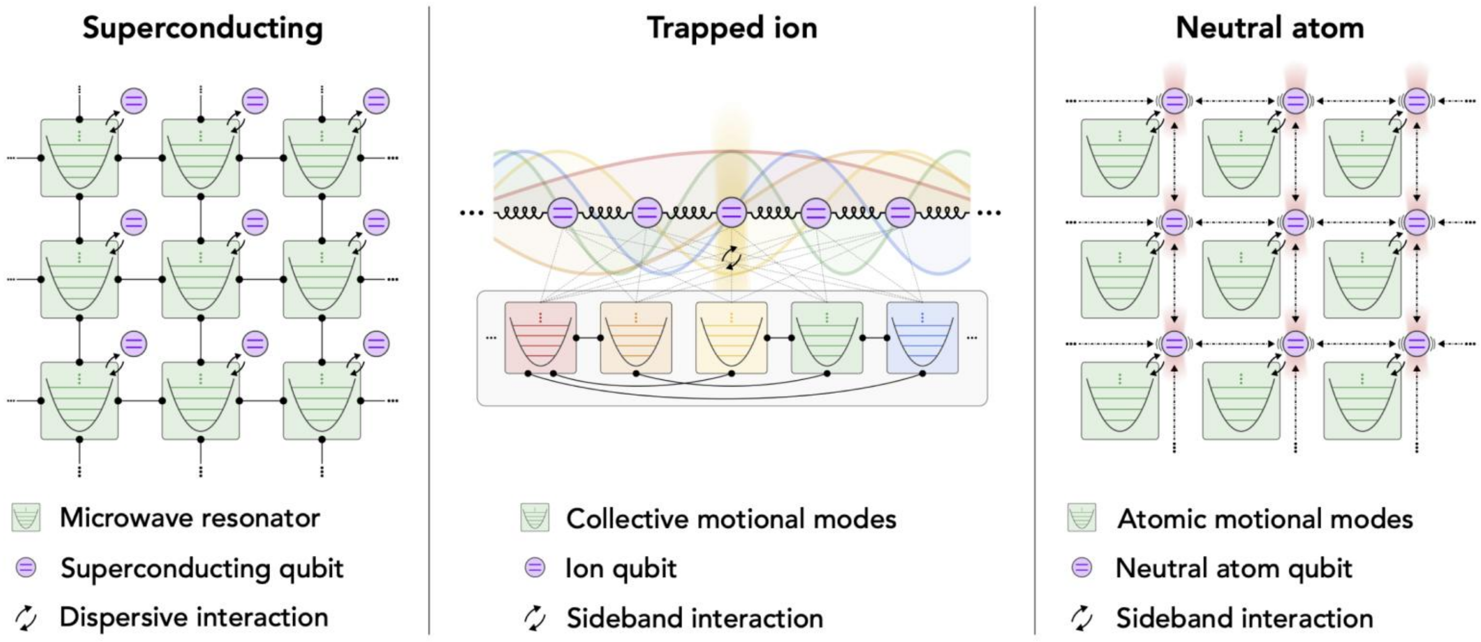

S. Girvin *et al*, 2025

Physical implementation

- Superconducting Circuits
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	SPIN	**BOSONS**

Qubits and Qumodes (Gates)

Type	Operation	Short	Operator
Qubit gates	Pauli operators		σ^i
	Rotation	$\mathrm{R}_i(\theta)$	$e^{i\theta\sigma^i/2}$
	Controlled NOT	CNOT	$e^{i\frac{\pi}{4}(\mathbb{I}_1-\sigma_1^z)(\mathbb{I}_2-\sigma_2^x)}$
Qumode gates	Rotation	$\mathrm{R}(\theta)$	$e^{i\theta\hat{a}^\dagger\hat{a}}$
	Displacement	$\mathrm{D}(z)$	$e^{z\hat{a}^\dagger-z^*\hat{a}}$
	Single-mode squeezing	$\mathrm{S}(z)$	$e^{(z^*\hat{a}\hat{a}-z\hat{a}^\dagger\hat{a}^\dagger)/2}$
	Beam splitter	$\mathrm{BS}(z)$	$e^{z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger}$
	Kerr	$\mathrm{K}(z)$	$e^{i\theta(\hat{a}^\dagger\hat{a})^2}$
	Cross-Kerr	$\mathrm{CK}(z)$	$e^{i\theta\hat{a}^\dagger\hat{a}\,\hat{b}^\dagger\hat{b}}$
Hybrid gates	Red sideband	$\mathrm{RSB}(z)$	$e^{iz\hat{a}\sigma^+ + iz^*\hat{a}^\dagger\sigma^-}$
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	Controlled rotation	$\mathrm{CR}(\theta)$	$e^{i\theta\sigma^z\hat{a}^\dagger\hat{a}}$
	Controlled displacement	$\mathrm{CD}(z)$	$e^{\sigma^z(z\hat{a}^\dagger-z^*\hat{a})}$
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Measurements	Qubit Pauli strings		σ^i
	Average phonon number		$\hat{N}$
	Qumode PNR		$\|n\rangle\langle n\|$
	Qumode homodyne		$\hat{X}, \hat{P}$
	Hybrid		$\sigma^i\hat{X}, \sigma^i\hat{P}$

→

PAULI OPERATORS

$$[\sigma_j, \sigma_k] = 2\, i\, \varepsilon_{jkl}\, \sigma_l$$

J. Y. Araz, M. Grau, J. Montgomery & F. Ringer, 2025

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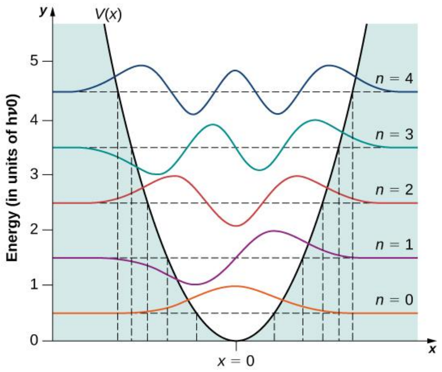
QHO SPECTRUM



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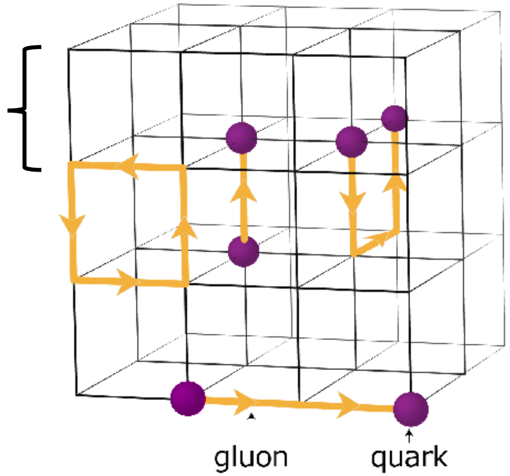
J. Y. Araz, M. Grau, J. Montgomery & F. Ringer, 2025

1. Use the **Hamiltonian** notation $\mathcal{L} = \bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi + g_s\,\bar{\psi}\,\gamma^\mu T^a\,\psi\,A^a_\mu - \frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} \rightarrow \mathcal{H} = \ldots$

1 Use the **Hamiltonian** notation

8 **Discretize** spatial component

$$x = an \quad (n = 0, \ldots, L-1)$$



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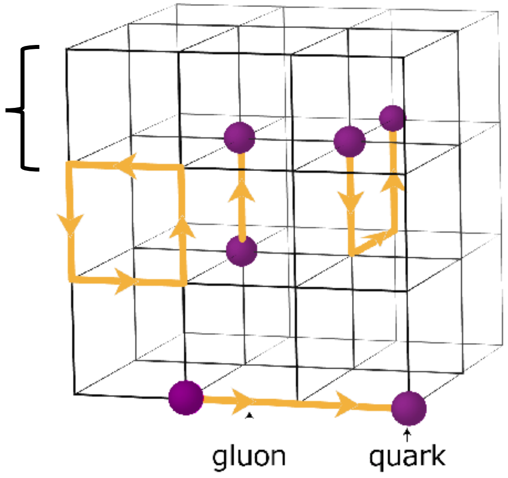

Take **limits** $L \to \infty$, $a \to 0$ + finite volume formalism

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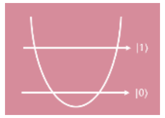
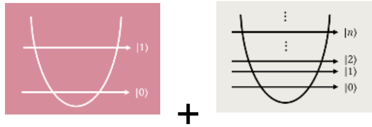
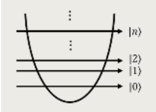
3 Map to **qubits** and **qumodes**



QUBIT



+
QUBIT-QUMODE
COUPLING



QUMODE

$$\mathcal{L} = \bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi + g_s\,\bar{\psi}\,\overbrace{\gamma^\mu T^a}\,\psi\,A^a_\mu - \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}$$

Quantum electrodynamics in 2+1

$$H = H_E + H_B + H_M + H_K$$

* $Q_n = \Psi_n^\dagger \Psi_n - \frac{1-(-)^{n_x+n_y}}{2} \mathbf{1}$
* $J_n = q_n^0 p_n^1 - q_n^1 p_n^0$

$$H_E^{(Q)} = g^2 \sum_{n,n'} \left(\mathcal{H}_{nn'}^{(2)} J_n J_{n'} + \mathcal{H}_{nn'}^{(1)} Q_n J_{n'} + \mathcal{H}_{nn'}^{(0)} Q_n Q_{n'} \right),$$

$$H_B^{(Q)} = \frac{1}{2g^2} \sum_n \left(q_{n+e_y} - q_n \right)^2,$$

$$H_M^{(Q)} = m_0 \sum_n (-)^{n_x+n_y} Q_n,$$

$$H_K^{(Q)} = \frac{1}{2} \sum_n \left(q_n^0 - i\, q_n^1 \right) \sigma_n^+ \sigma_{n+e_x}^- + \frac{1}{2} \sum_n P_{L(n,n+e_y)} \sigma_n^+ \sigma_{n+e_y}^- + \text{h.c.}$$

V. Ale, T. Rainaldi, E. Rico, F. Ringer & G. Siopsis, 2025

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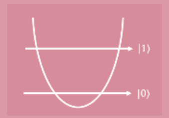
$$H_K^{(Q)} = \frac{1}{2} \sum_n \left(q_n^0 - i\, q_n^1 \right) \sigma_n^+ \sigma_{n+e_x}^- + \frac{1}{2} \sum_n P_{L(n,n+e_y)} \sigma_n^+ \sigma_{n+e_y}^- + \text{h.c.}$$



QUBIT

V. Ale, T. Rainaldi, E. Rico, F. Ringer & G. Siopsis, 2025

Quantum electrodynamics in 8+1

$$H = H_E + H_B + H_M + H_K$$

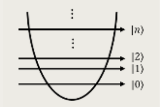
* $Q_n = \Psi_n^\dagger \Psi_n - \frac{1-(-)^{n_x+n_y}}{2}\mathbb{1}$
* $J_n = q_n^0 p_n^1 - q_n^1 p_n^0$

$$H_E^{(Q)} = g^2 \sum_{n,n'} \left(\mathcal{H}^{(2)}_{nn'} J_n J_{n'} + \mathcal{H}^{(1)}_{nn'} Q_n J_{n'} + \mathcal{H}^{(0)}_{nn'} Q_n Q_{n'} \right),$$

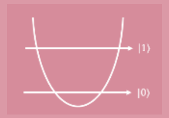
$$H_B^{(Q)} = \frac{1}{2g^2} \sum_n \left(q_{n+e_y} - q_n\right)^2,$$

$$H_M^{(Q)} = m_0 \sum_n (-)^{n_x+n_y} Q_n,$$

$$H_K^{(Q)} = \frac{1}{2} \sum_n \left(q_n^0 - i\, q_n^1\right) \sigma_n^+ \sigma_{n+e_x}^- + \frac{1}{2} \sum_n P_{L(n,n+e_y)} \sigma_n^+ \sigma_{n+e_y}^- + \text{h.c.}$$



QUMODE



QUBIT

V. Ale, T. Rainaldi, E. Rico, F. Ringer & G. Siopsis, 2025

$$H = H_E + H_B + H_M + H_K$$

$$H_E^{(Q)} = g^2 \sum_{n,n'} \left(\mathcal{H}^{(2)}_{nn'} J_n J_{n'} + \mathcal{H}^{(1)}_{nn'} Q_n J_{n'} + \mathcal{H}^{(0)}_{nn'} Q_n Q_{n'} \right),$$

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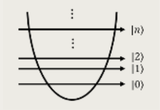
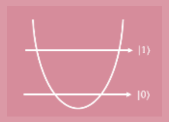
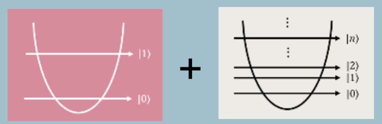


QUMODE



QUBIT



QUBIT + QUMODE

V. Ale, T. Rainaldi, E. Rico, F. Ringer & G. Siopsis, 2025

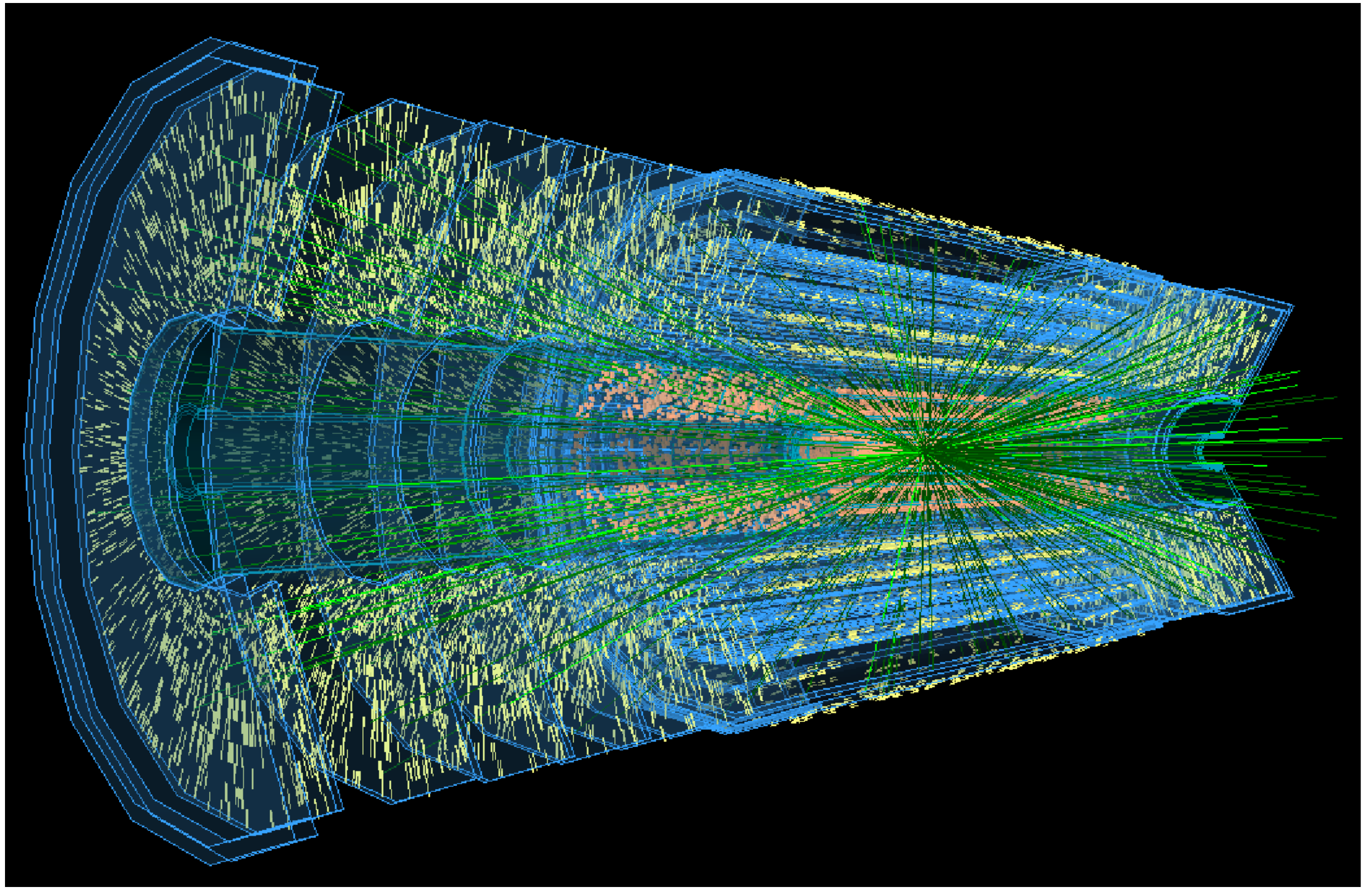
PDFs, TMDs and Ground State Preparation

PDFs and **TMDs** are expectation values of QCD operators in a **single-hadron eigenstate**

$$\langle P|\mathcal{O}(t,x)\mathcal{O}(0)|P\rangle,\ \ t^2 - x^2 = 0$$

LOWEST-ENERGY EIGENSTATE WITH THE HADRON'S QUANTUM NUMBERS

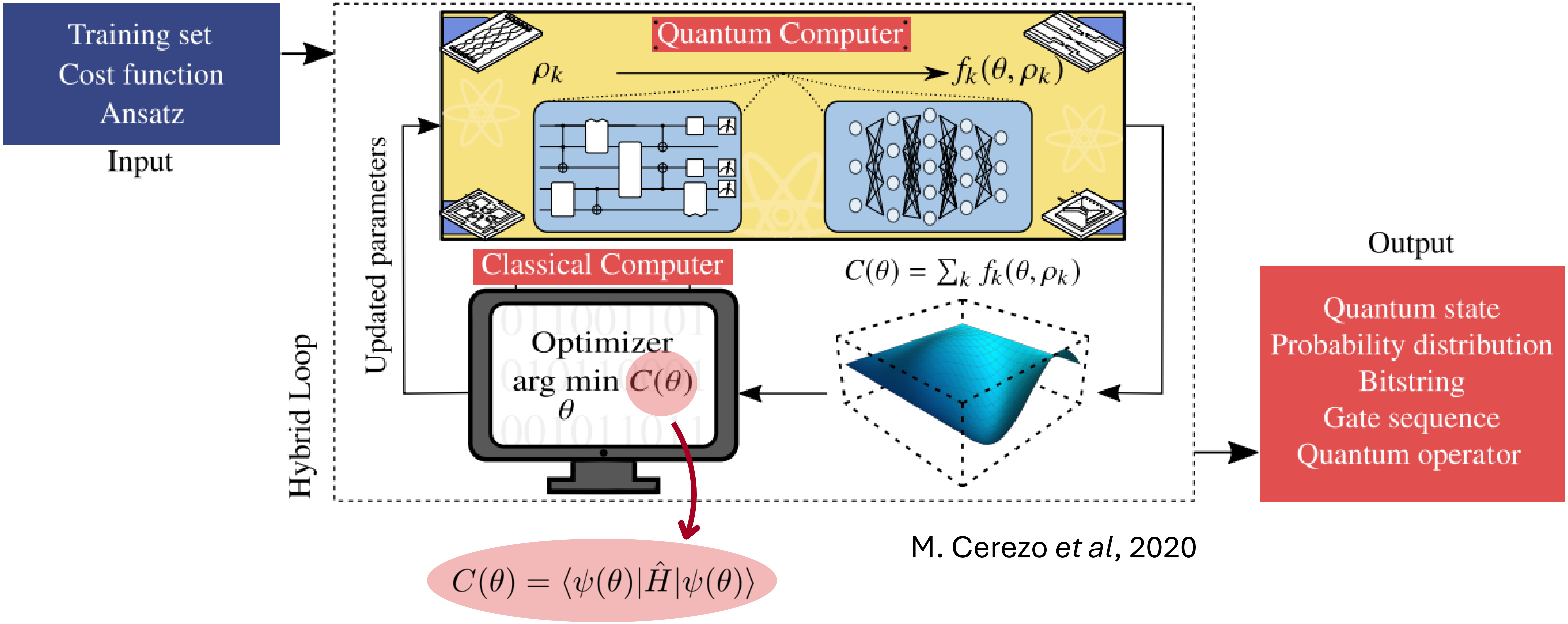
Wrong eigenstate → measured correlator becomes a **mixture of different hadronic structures**



IFIC (CSIC-UV)

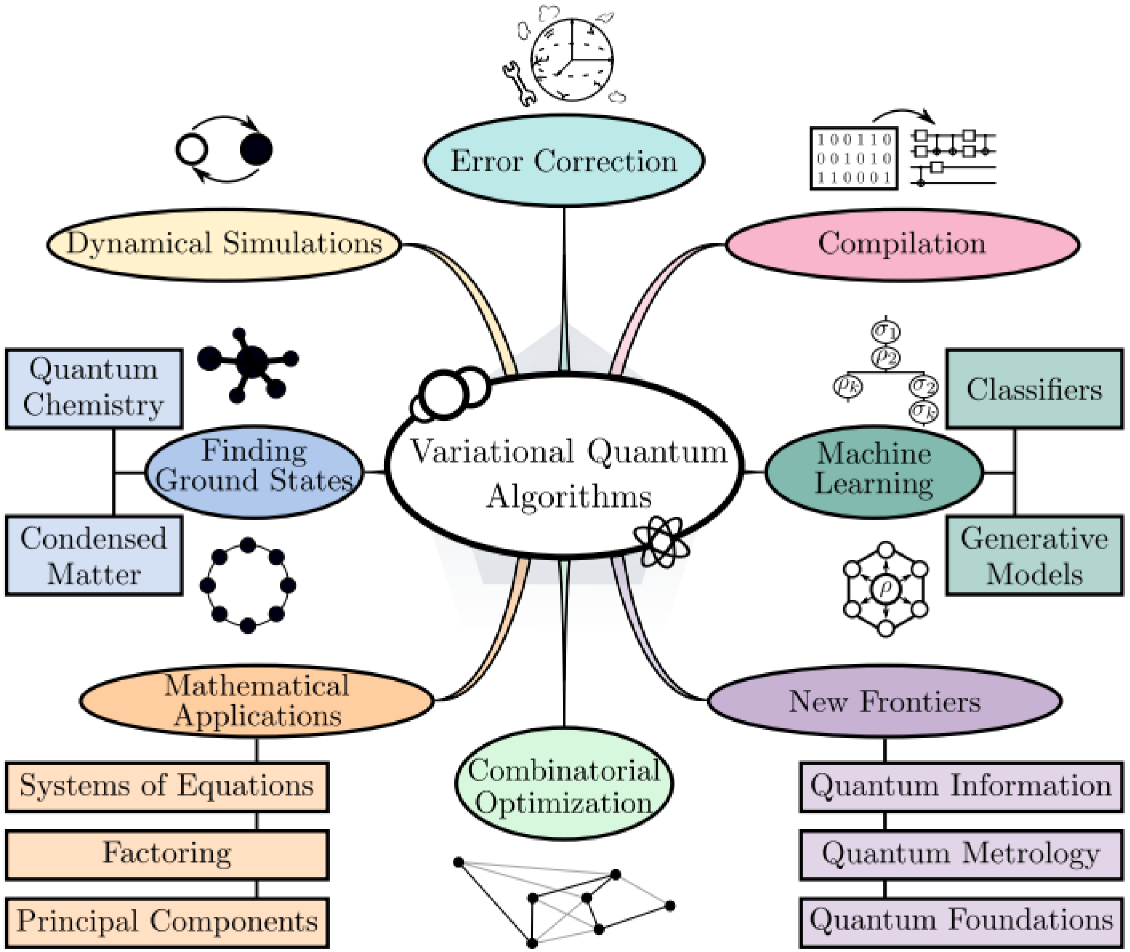
VARIATIONAL QUANTUM ALGORITHMS (VQA)

What is a Variational Quantum Algorithm (VQA)?



M. Cerezo *et al*, 2020

Why VQA?



M. Cerezo *et al*, 2020

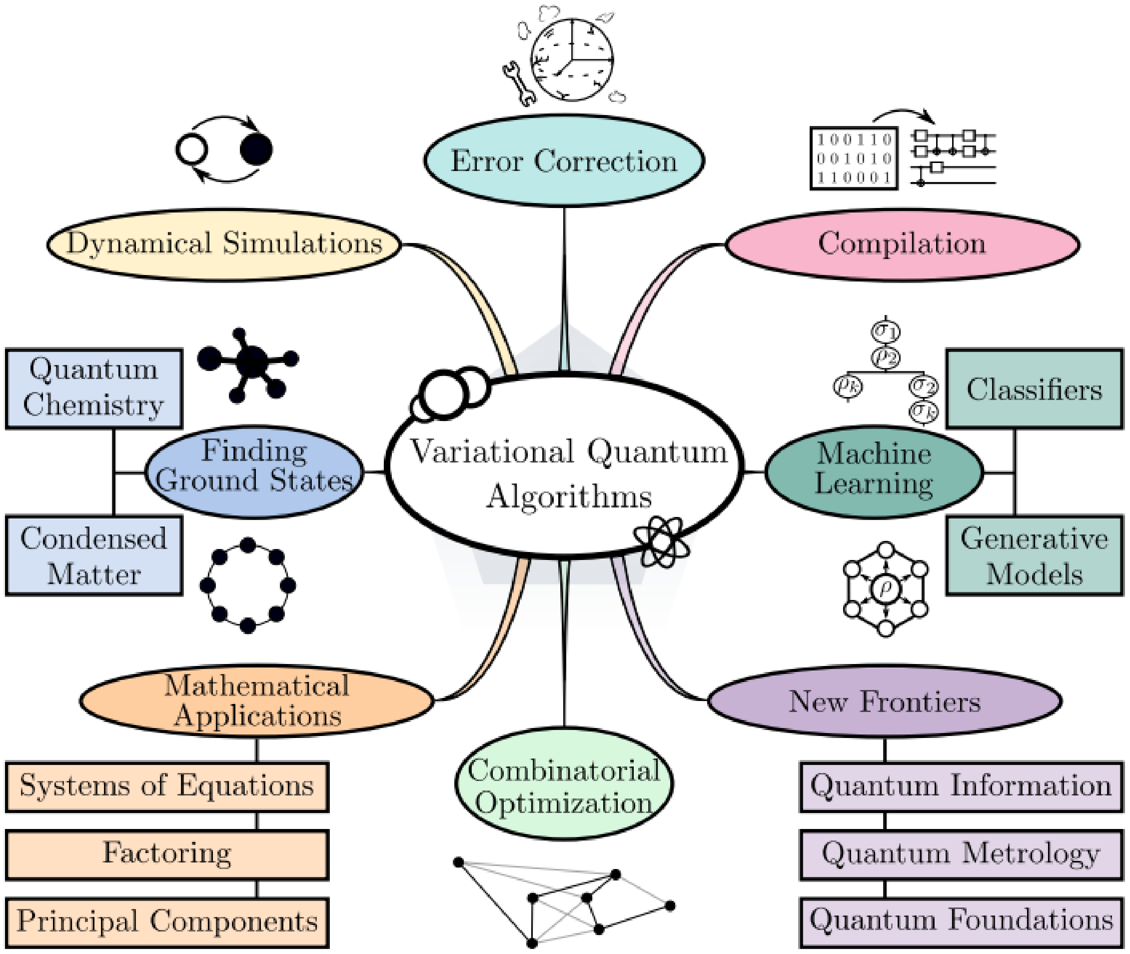
BOSE-HUBBARD (BH) MODEL

$$\hat{H}_{\mathrm{BH}} = -J\sum_r \left(\hat{a}_r^\dagger \hat{a}_{r+1} + \mathrm{h.c.}\right) - \frac{U}{2}\sum_r \hat{n}_r\left(\hat{n}_r - 1\right) - \mu\sum_r \hat{n}_r$$

- $J \rightarrow$ Hopping amplitude ($J = 1$)
- $U \rightarrow$ Strength of the on-site interaction

$$\Lambda_{BH} = \frac{N_B U}{J}$$

- $\mu \rightarrow$ Chemical potential ($\mu = 0$)

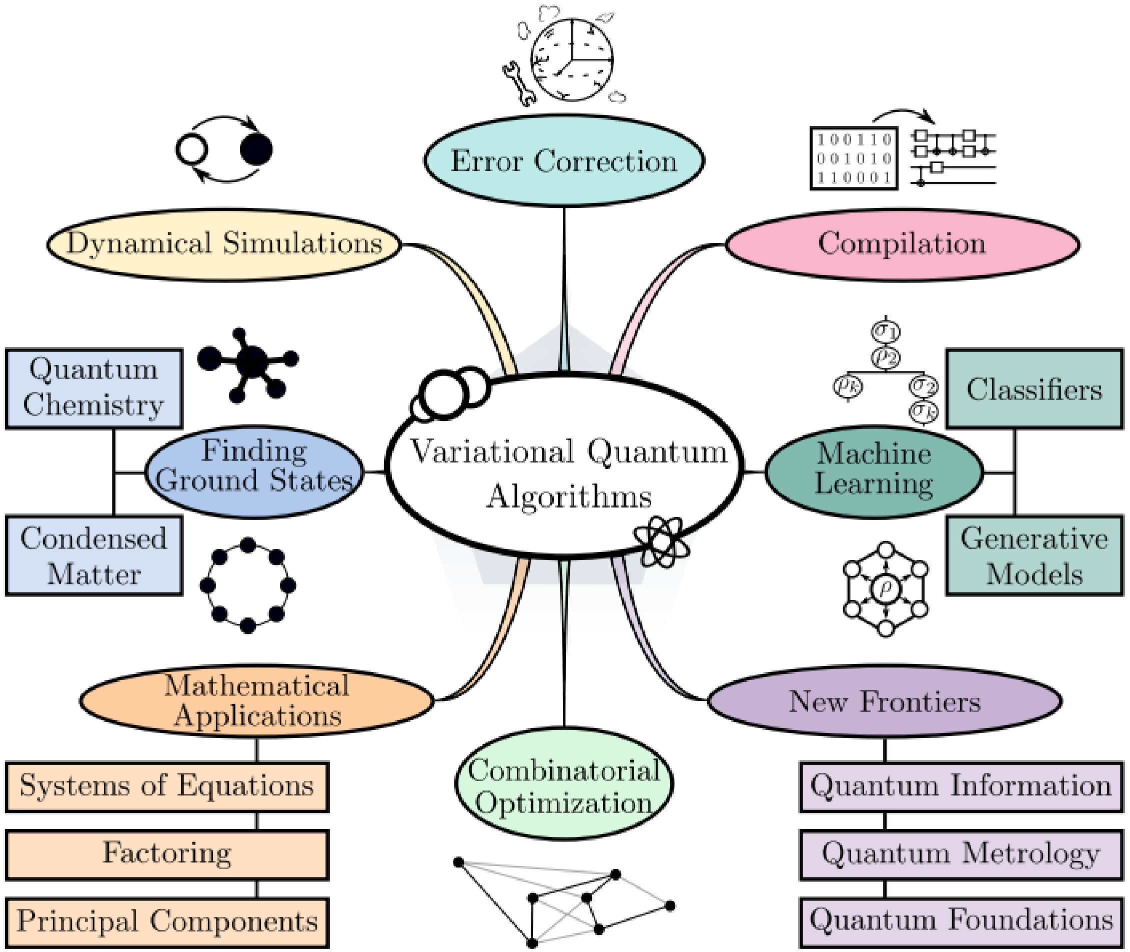


M. Cerezo *et al*, 2020

BOSE-HUBBARD (BH) MODEL

$$\hat{H}_{\mathrm{BH}} = -J\sum_r \left(\hat{a}_r^\dagger \hat{a}_{r+1} + \mathrm{h.c.}\right) - \frac{U}{2}\sum_r \hat{n}_r\left(\hat{n}_r - 1\right) - \mu\sum_r \hat{n}_r$$

- **Fixed** number of bosons, N_B.
- **Periodic** boundary conditions (PBC).
- Atractive case, $U > 0$.

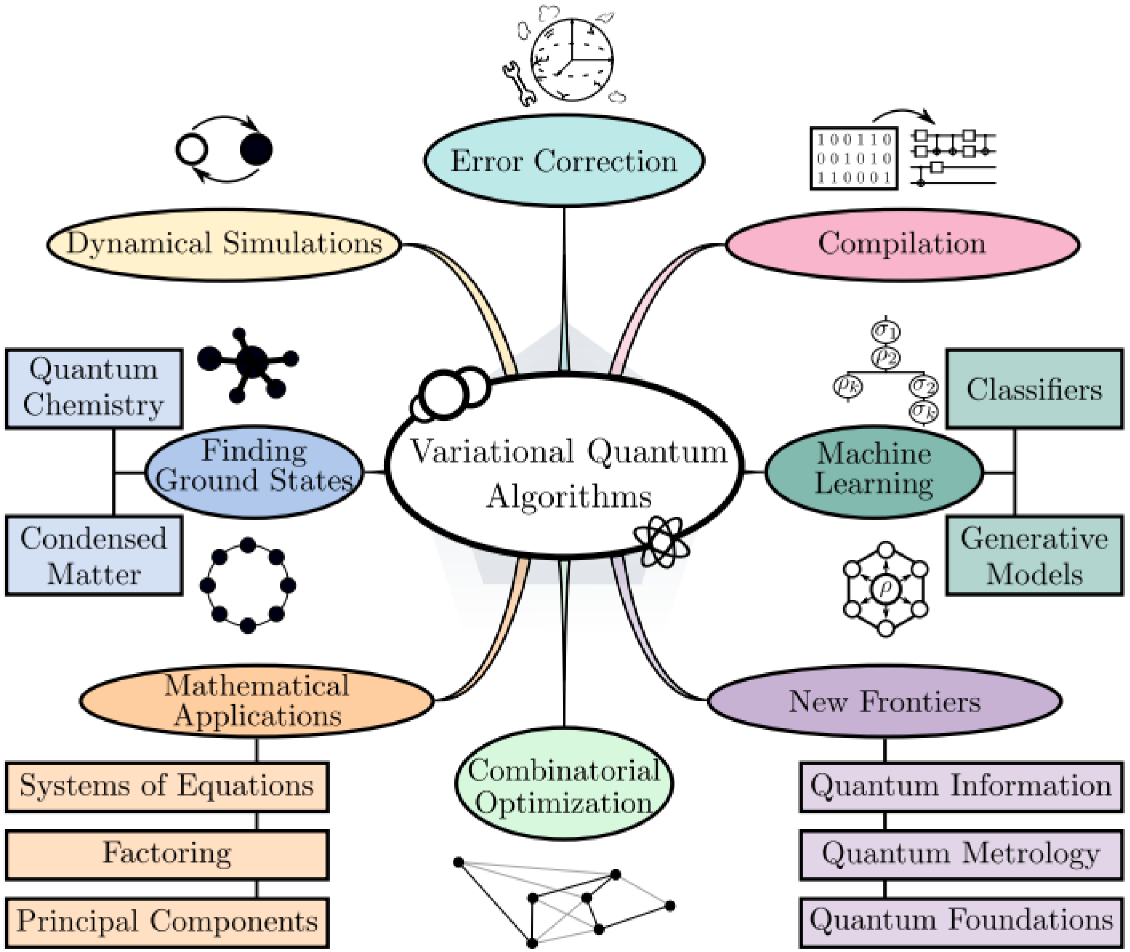


M. Cerezo *et al*, 2020

BOSONIC KITAEV CHAIN (BKC) MODEL

$$\hat{H}_{\mathrm{BKC}} = \sum_r \left(-Je^{i\theta}\hat{a}_r^\dagger \hat{a}_{r+1} - e^{i\phi}\Delta \hat{a}_r \hat{a}_{r+1} + \mathrm{h.c.}\right) - \mu \sum_r \hat{n}_r$$

- J → Hopping amplitude ($J = 1$)
- Δ → Pairing strength amplitude
- μ → Chemical potential ($\mu = -8$)



M. Cerezo *et al*, 2020

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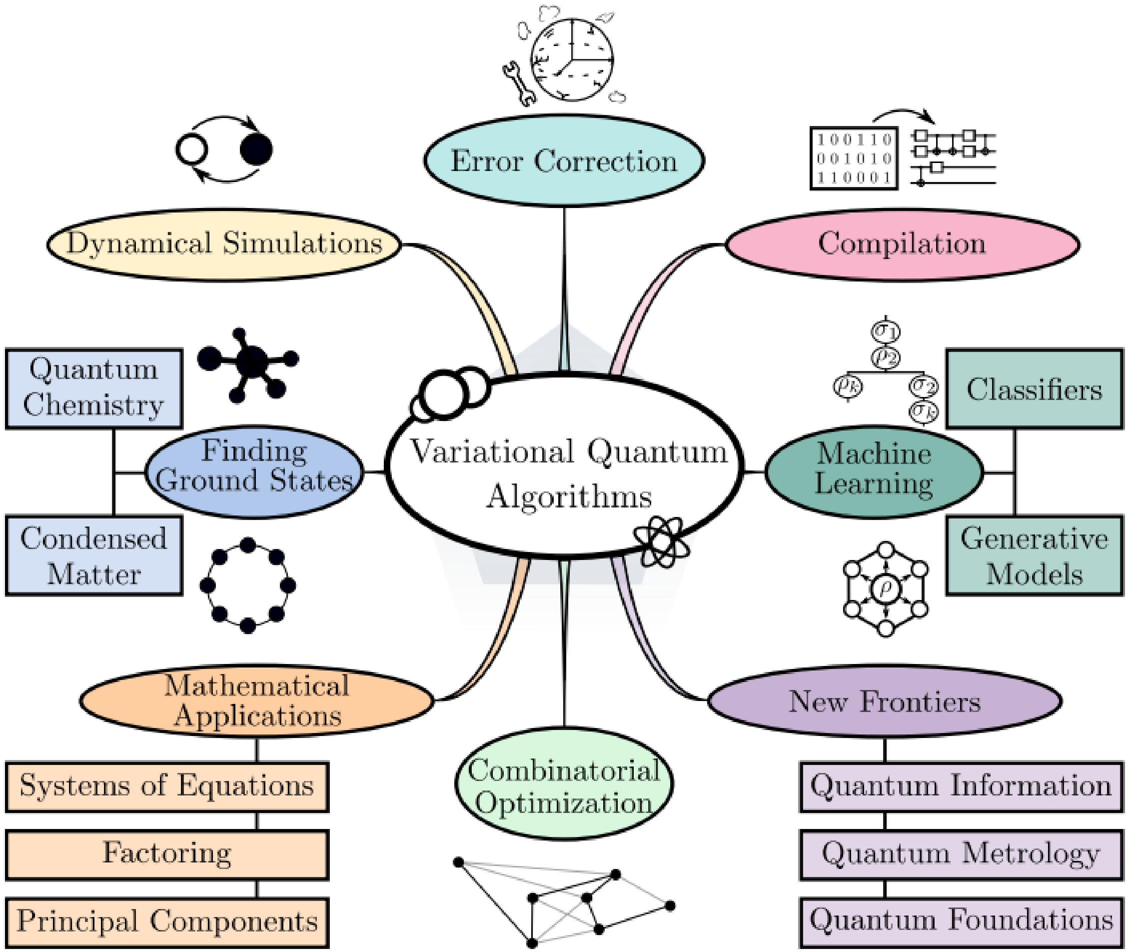
$$\hat{H}_{\mathrm{BKC}} = \sum_r \left(-Je^{i\theta}\hat{a}_r^\dagger \hat{a}_{r+1} - e^{i\phi}\Delta \hat{a}_r \hat{a}_{r+1} + \mathrm{h.c.}\right) - \mu \sum_r \hat{n}_r$$

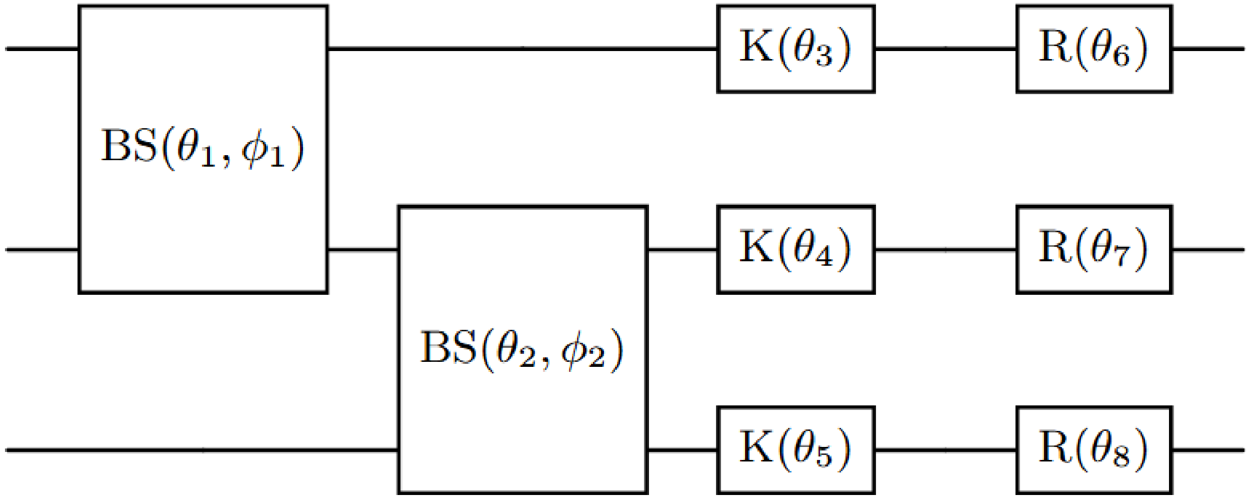
- **Variable** number of bosons, N_B.
- **Periodic** boundary conditions (PBC).



M. Cerezo *et al*, 2020

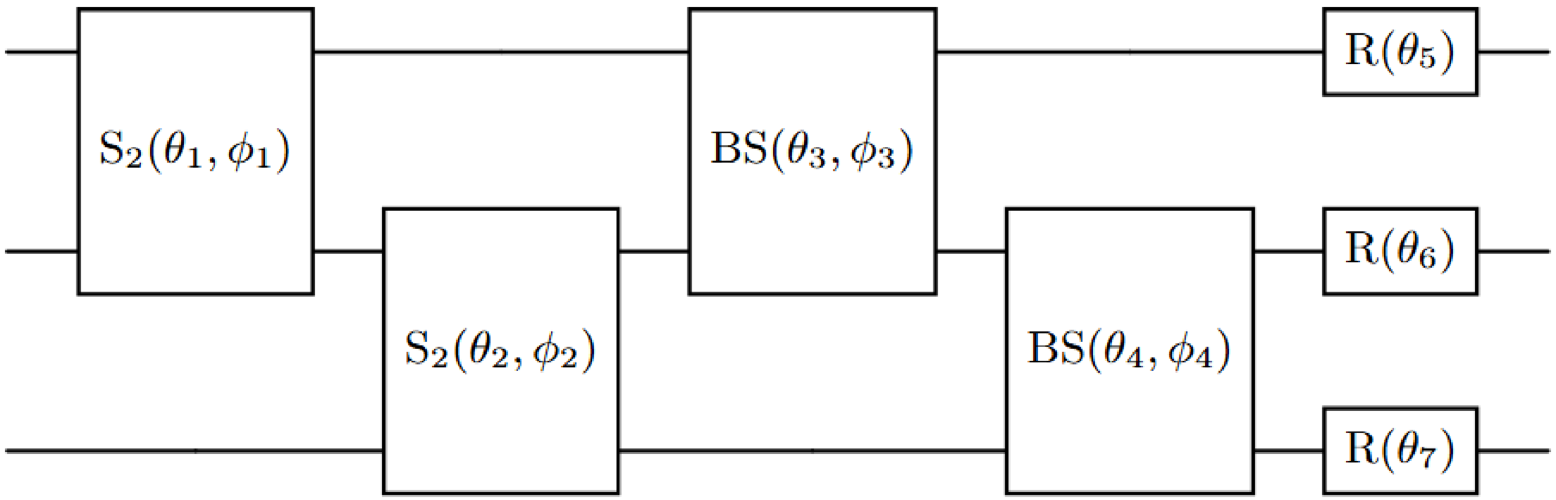
STANDARD VQE ($N_S = 3$)

BH model layer



- $\mathrm{BS}(\theta, \phi) = \exp(\theta(e^{i\phi}\hat{a}_1\hat{a}_2^\dagger - e^{-i\phi}\hat{a}_1^\dagger\hat{a}_2))$
- $\mathrm{K}(\theta) = \exp(i\theta\hat{N}^2)$
- $\mathrm{R}(\theta) = \exp(i\theta\hat{N})$

BKC model layer



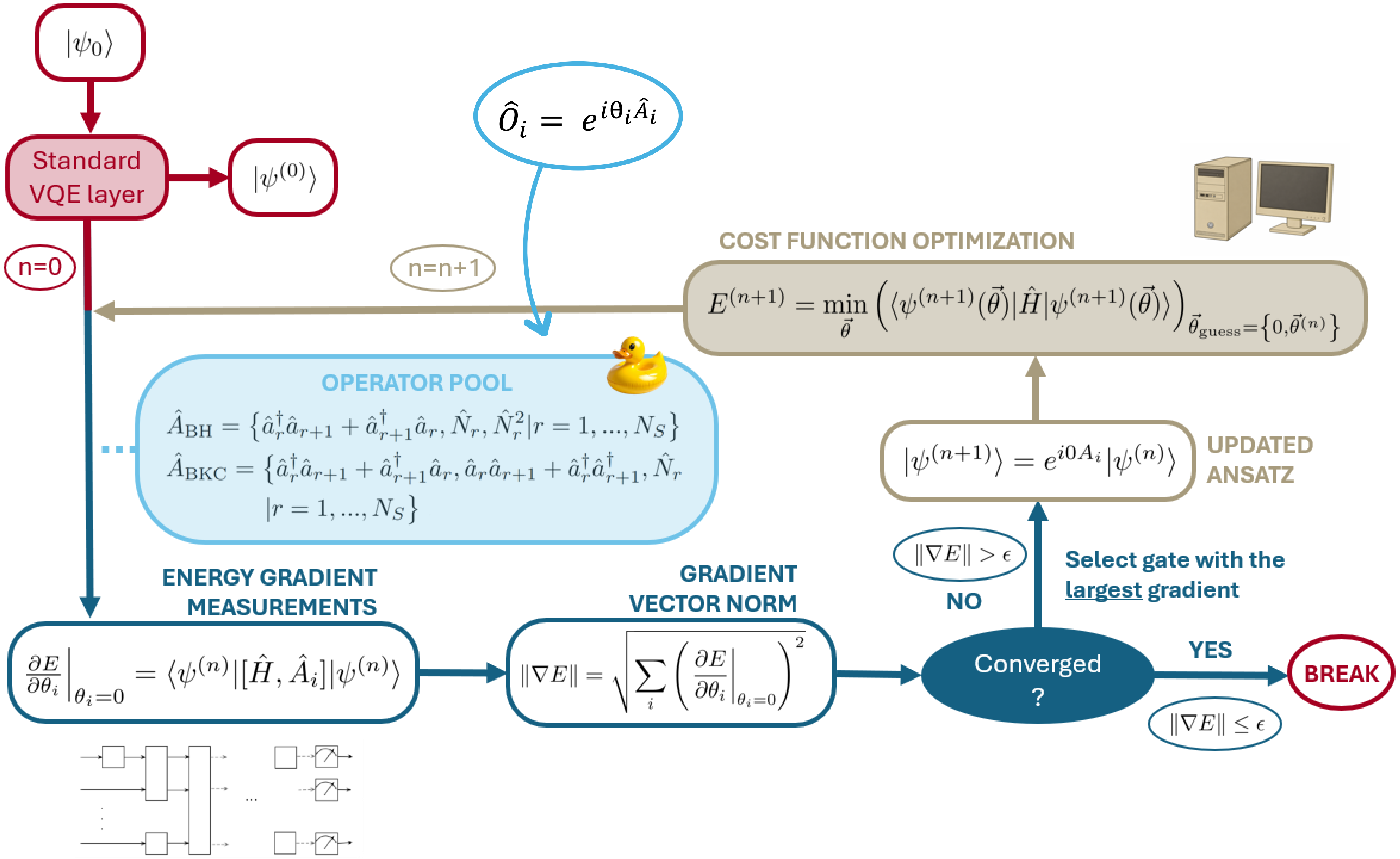
- $\mathrm{S}_2(\theta,\phi) = \exp(\theta(e^{i\phi}\hat{a}_1\hat{a}_2 - e^{-i\phi}\hat{a}_1^\dagger\hat{a}_2^\dagger))$
- $\mathrm{BS}(\theta,\phi) = \exp(\theta(e^{i\phi}\hat{a}_1\hat{a}_2^\dagger - e^{-i\phi}\hat{a}_1^\dagger\hat{a}_2))$
- $\mathrm{R}(\theta) = \exp(i\theta\hat{N})$

Adaptive Derivative-Assembled Pseudo-Trotter ansatz Variational Quantum Eigensolver

ADAPT-VQE

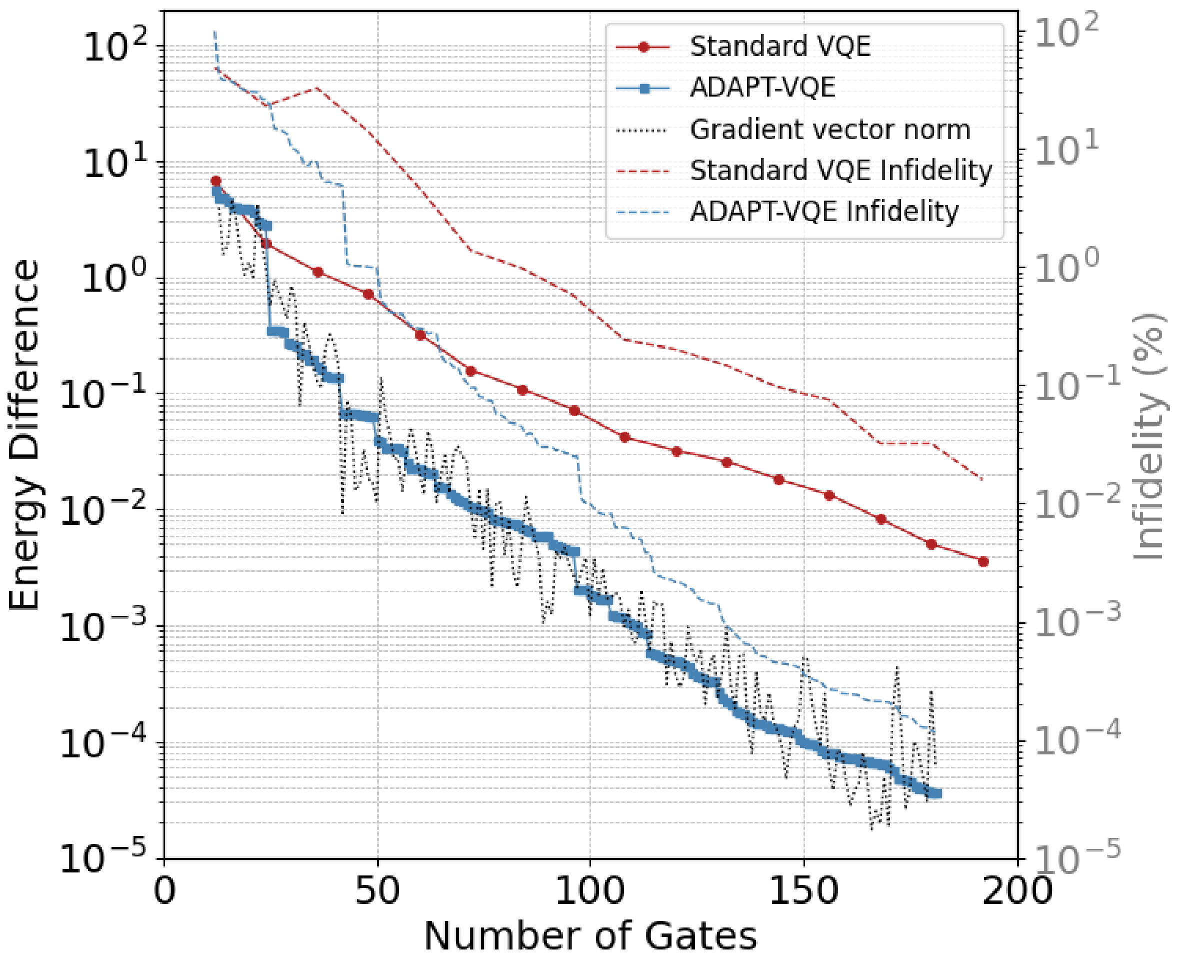
H. R. Grimsley, S. E. Economou, E. Barnes & N. J. Mayhall, 2019

ADAPT-VQE steps



ADAPT-VQE — BH model

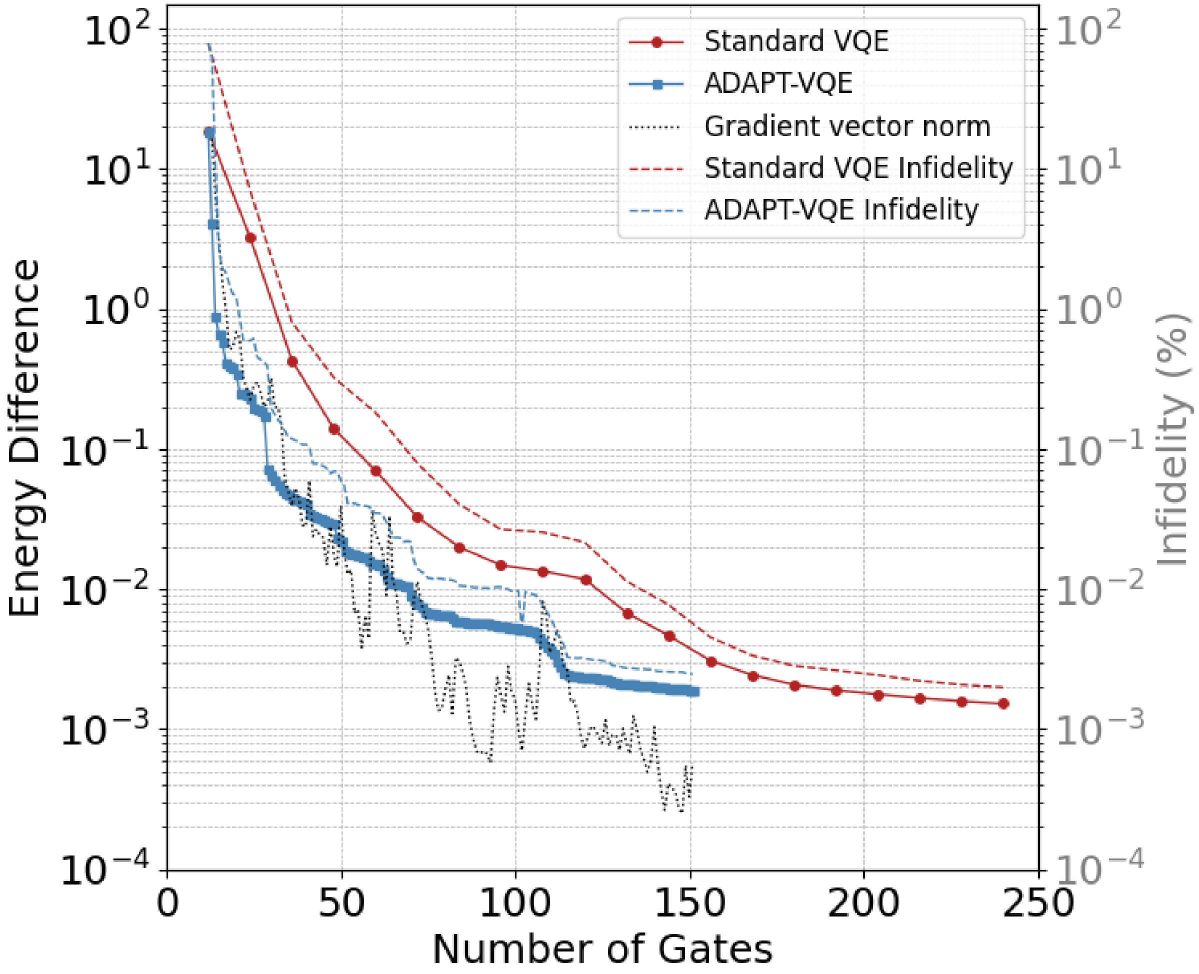
$$N_S = 4 \quad N_B = 12 \quad \Lambda_{BH} = 4.0$$



Work in progress!

ADAPT-VQE — BKC model

$$N_S = 4 \quad \Delta = 2.5 \quad \mu = -8.0$$



Work in progress!

Outlook

Stony Brook University | Quantum Information Science and Technology

* Improve operator pools
* Scalable-circuits ADAPT-VQE
* Hybrid qubit-quomode architectures

Outlook

* Improve operator pools
* Scalable-circuits ADAPT-VQE
* Hybrid qubit-quomode architectures
* **Excitations** and **correlation** functions
* Real-time dynamics and scattering