Quantum Computing Fragmentation Functions

Universidad Complutense de Madrid & IPARCOS

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arxiv: 2510.18869

MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES

UNIÓN EUROPEA

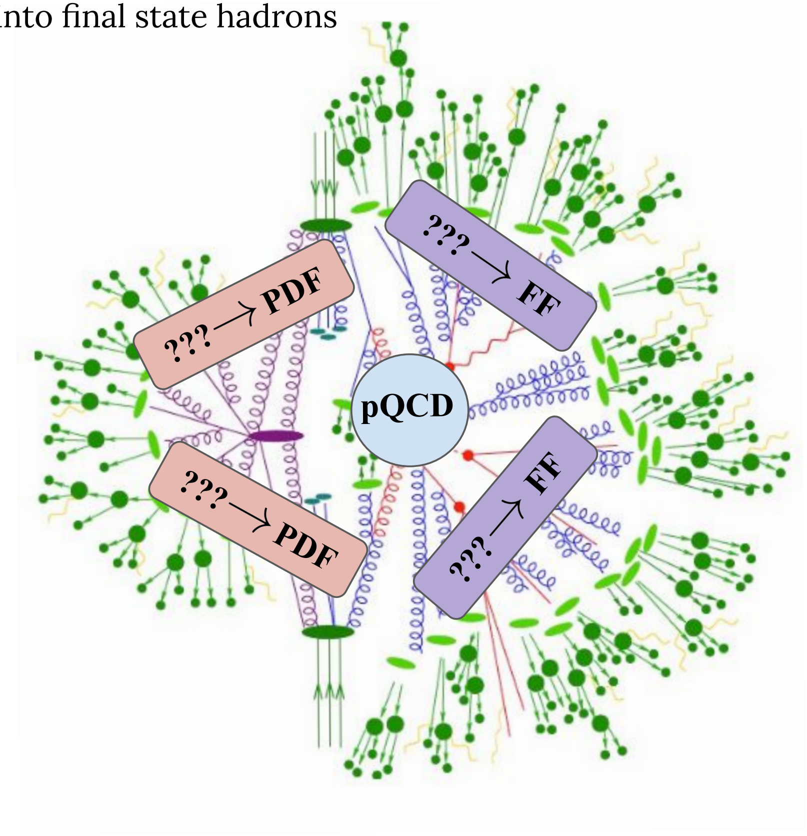
FONDO EUROPEO DE DESARROLLO REGIONAL

"Una manera de hacer Europa"

AGENCIA ESTATAL DE INVESTIGACIÓN

*juagalve@ucm.es

QCD is confined: Perturbative interactions between partons need to be reconstructed from their **fragmentation** into final state hadrons



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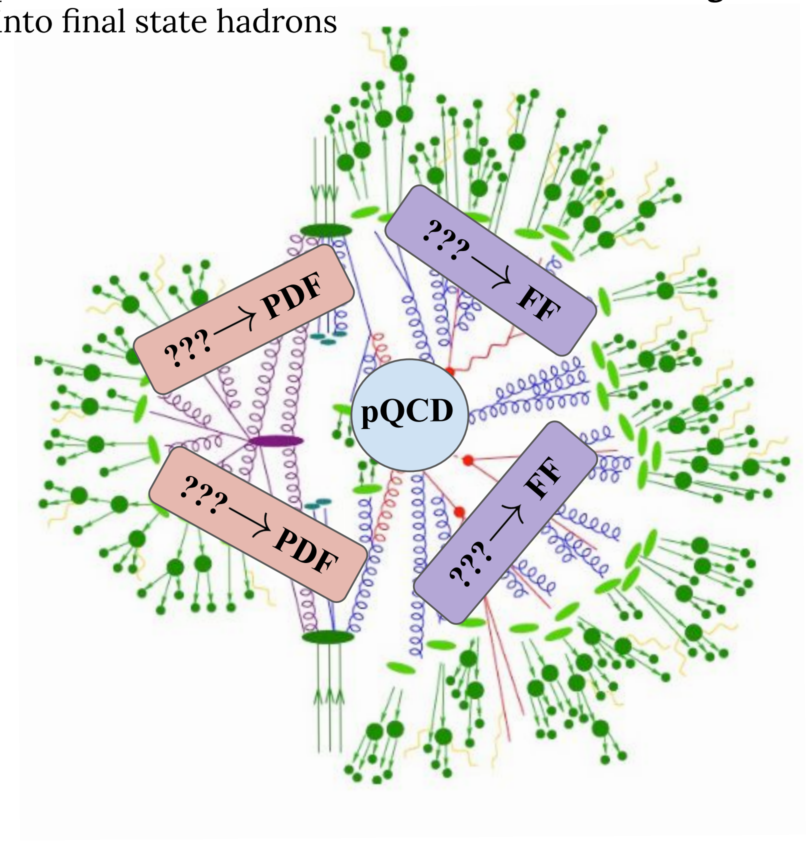
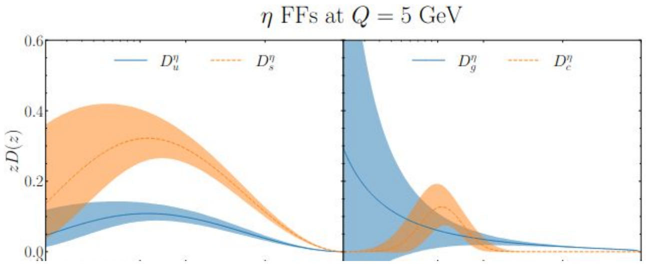


Experimental fits



https://doi.org/10.1103/t5ds-vvc4

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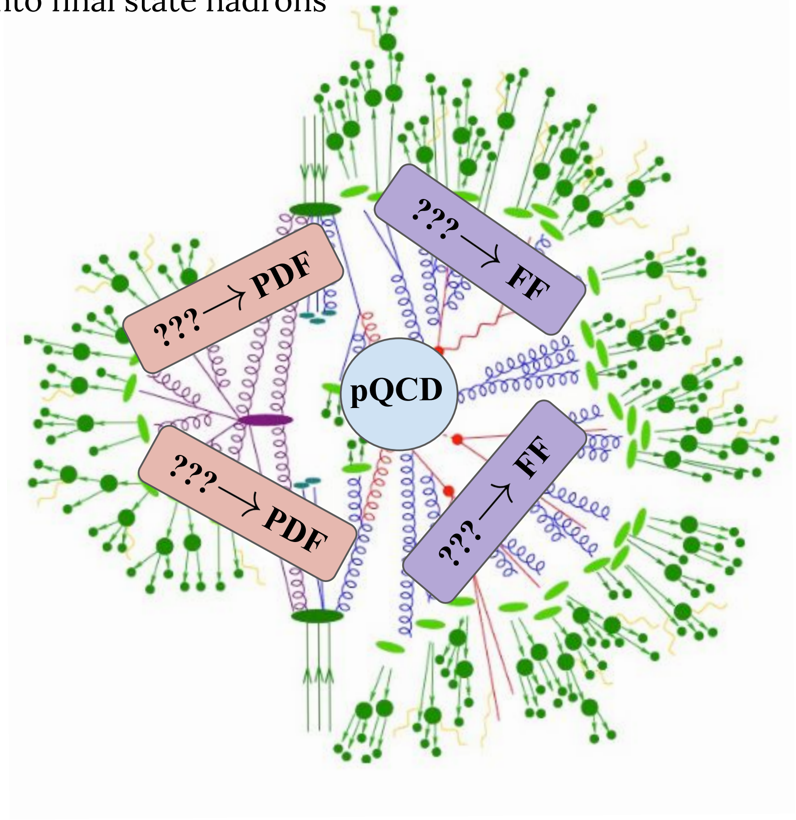

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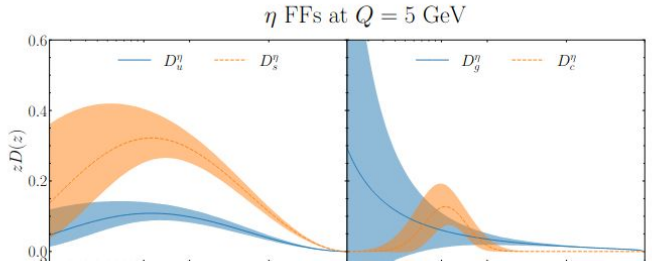

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On the Lattice?

$$D_j^h(z) \equiv \frac{\mathrm{Tr}_c}{N_{c,j}} \sum_X \langle j,p|h,X_{\mathrm{out}}\rangle\langle h,X_{\mathrm{out}}|j,p\rangle$$

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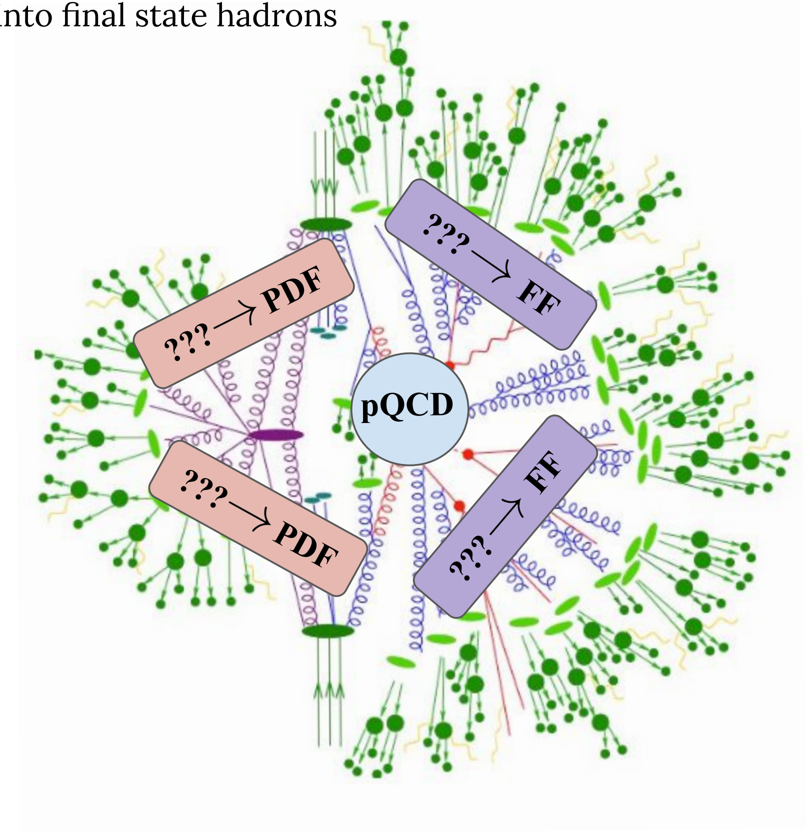

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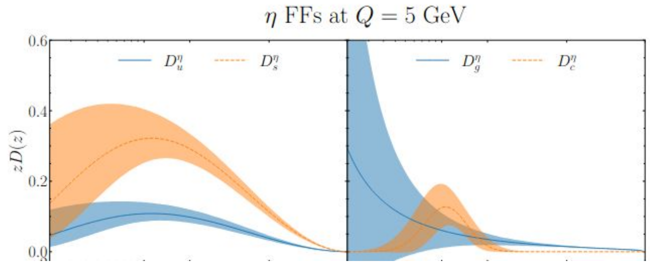

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doi:10.1103/PhysRevD.109.016006

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- Exp. value of initial parton j traced over color
- Light-Cone time evolution not possible on Lattice
- “Number of hadrons” operator

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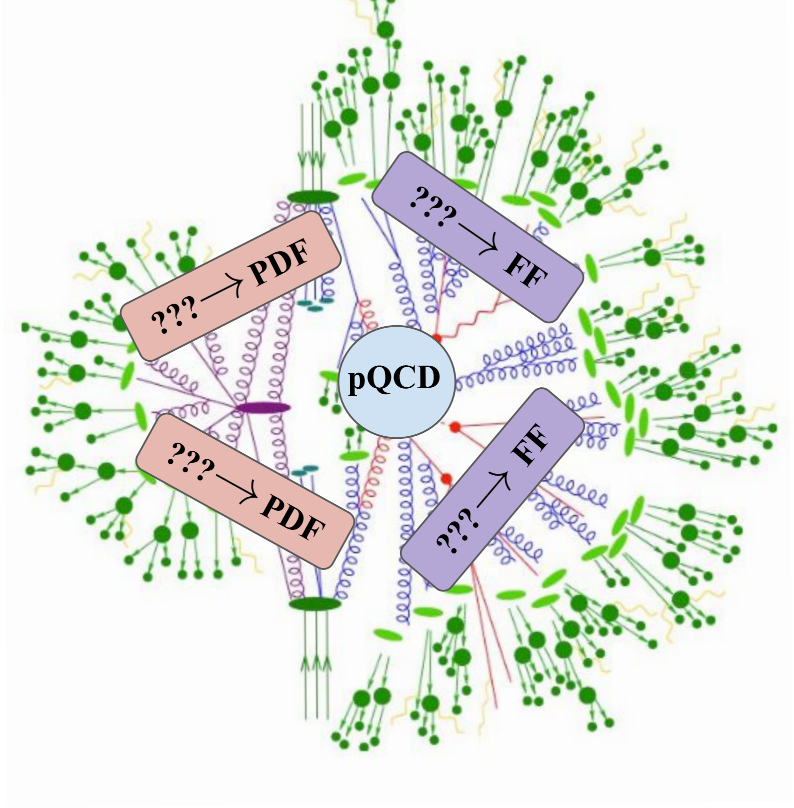

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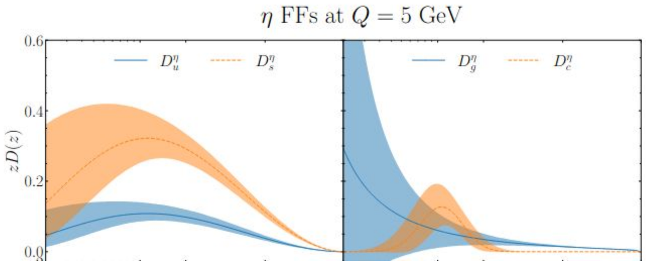

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On a Quantum Computer ????

Algorithms for **quantum simulations** of fragmentation:

- (2011) Real-time scattering of scalar QFT efficient by the JLP algorithm
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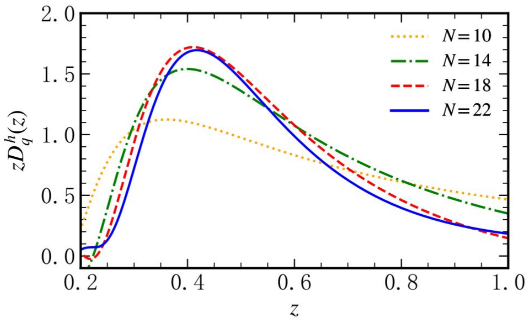

- Also jet quenching and evolution on a media

arXiv:2502.17558 doi:10.1103/PhysRevD.106.074013 doi:10.1103/PhysRevC.111.015202

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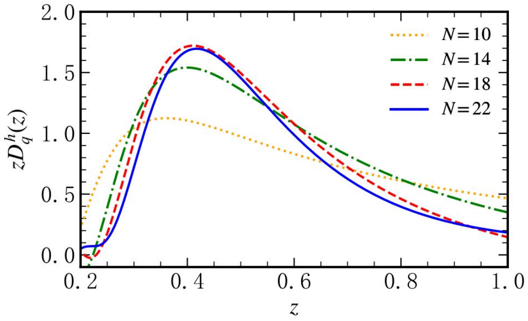

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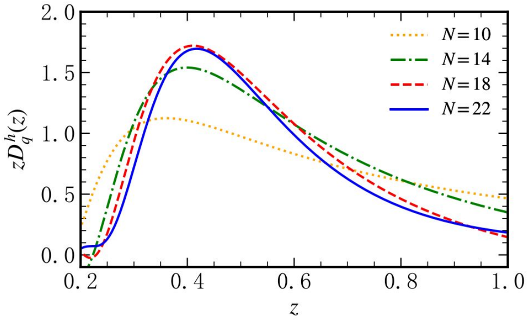

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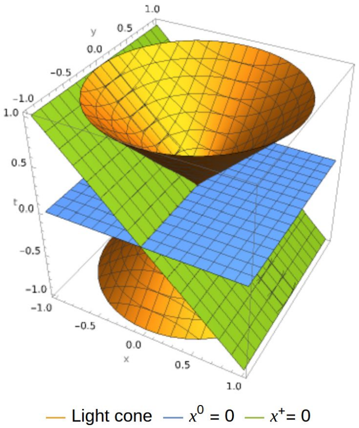
Our approach: ab initio QCD on the Light-Cone gauge



Coordinates

Instant Form (usual quantisation)	**Light Cone**
(x^0, x^1, x^2, x^3)	$x^+ = x^3 + x^0$ $x^- = x^3 - x^0$ $x^1,\ x^2 \to x^\perp$
Energies:	
$p^0 = \sqrt{m^2 + \|p\|^2}$	$p^- = \dfrac{m^2 + \|p^\perp\|^2}{p^+}$
Gauges:	
$A^0 = 0$	$A^+ = 0$

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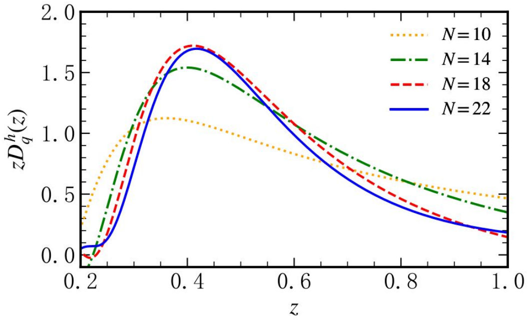

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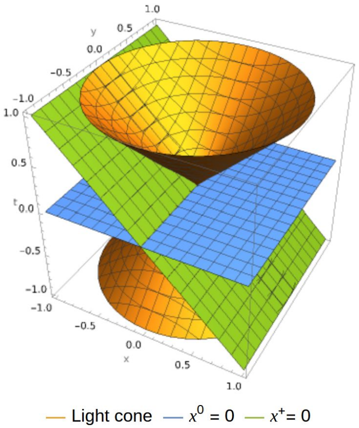

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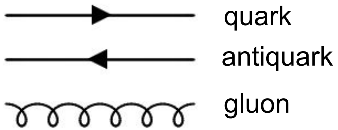
From

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + \bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi + g\, A^a_\mu J^{\mu,a}_D$$

define LF Hamiltonian by $P^- = \sum \Pi\, \partial_+ A - \mathcal{L}$

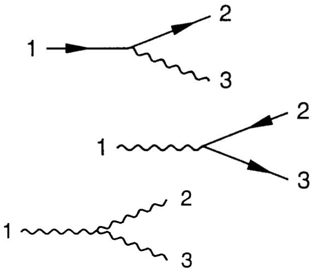
Hamiltonian as a dictionary of interactions among **on shell** partons:

Kinetic



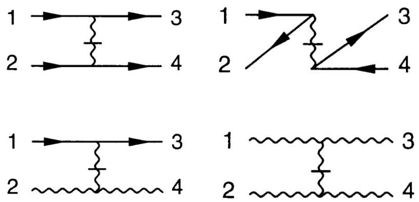
Free evolution, just add kinetic energy

Vertices



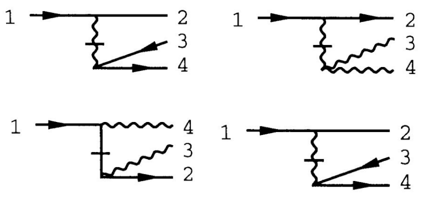
Simple interactions, pair creation and gluon scattering

Seagulls



Instantaneous interactions of 2 particles

Forks



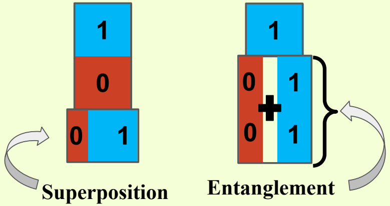
Instantaneous emission/absorption of 3 particles

doi.org/10.1016/S0370-1573(97)00089-6

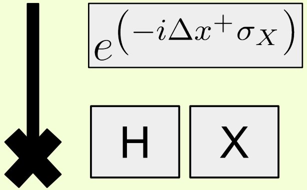
Quantum Computers & encoding

Quantum computers have two main components

Qubits featuring **superpositions** & **entanglement**



Logic **gates** to apply unitary evolution



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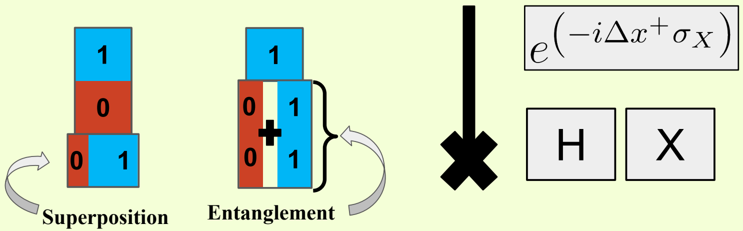
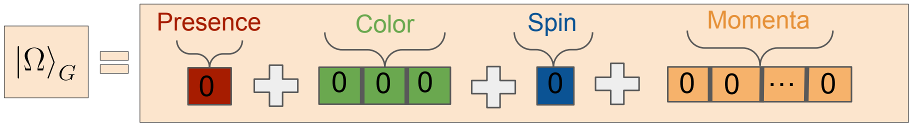
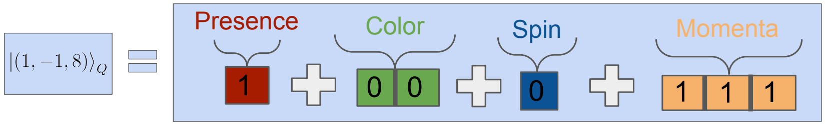


1. Encode each **single-particle** state as a binary number on **particle registers**





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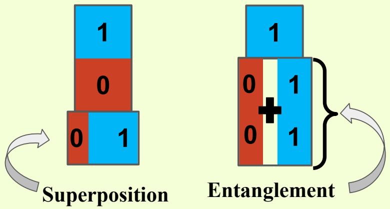

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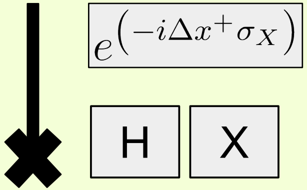

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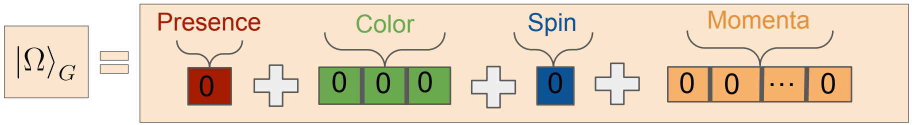

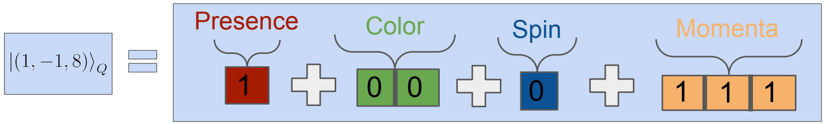

2. Define multi-particle states combining registers

No particles

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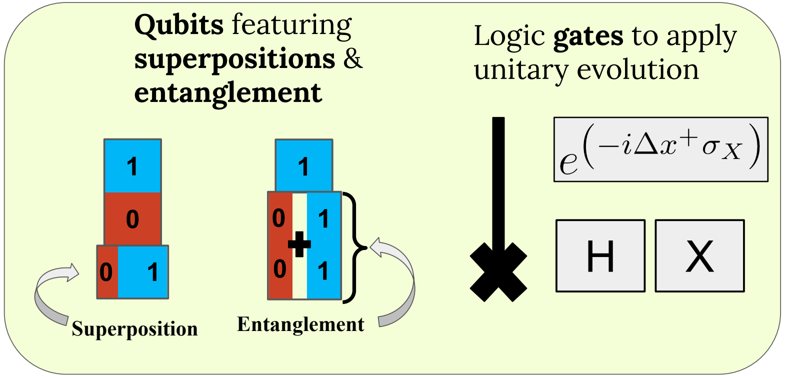

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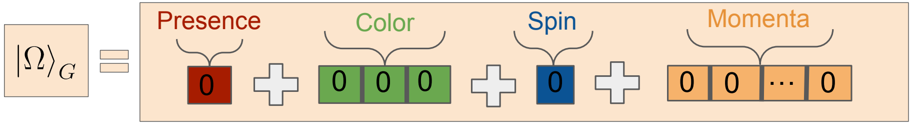

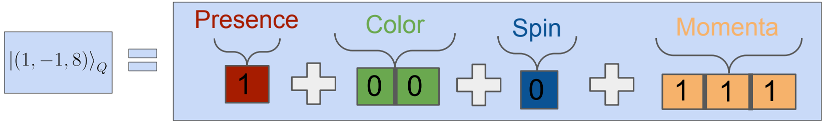

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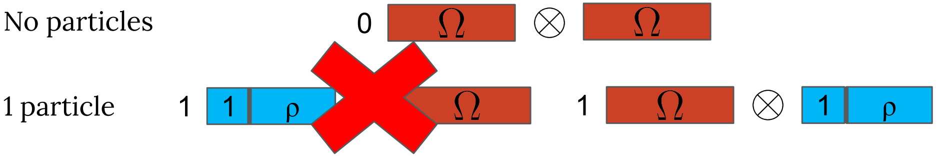

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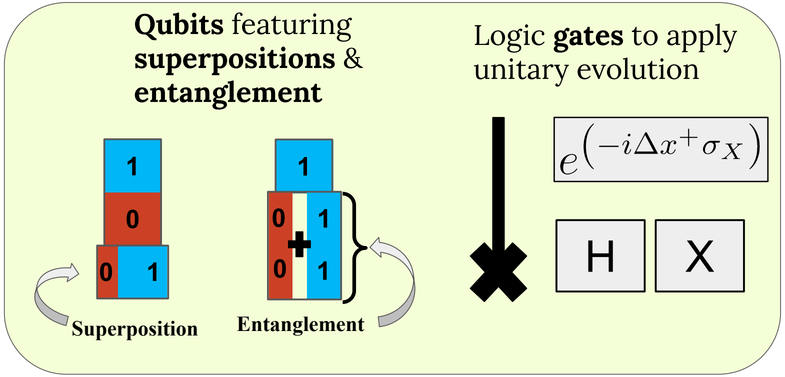

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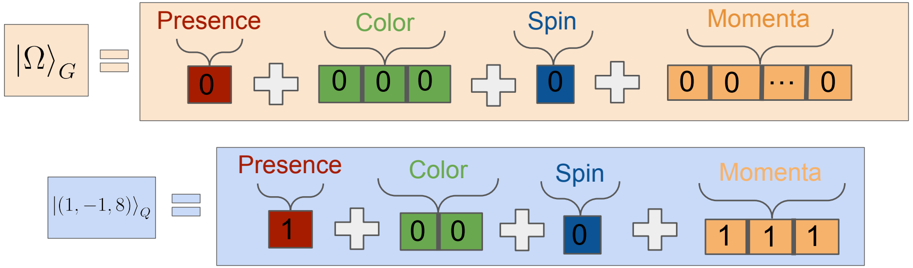

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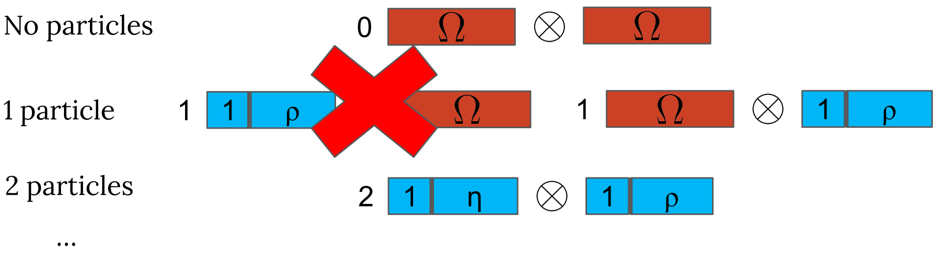

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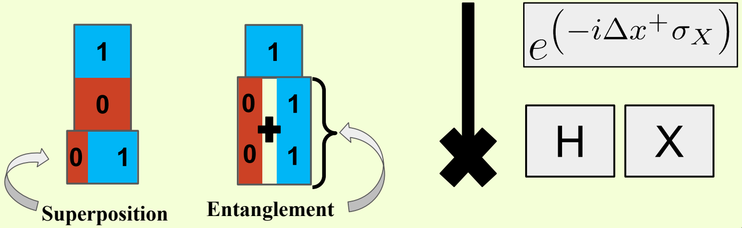

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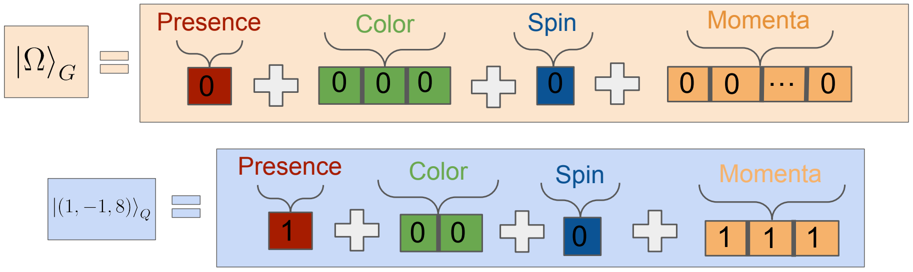

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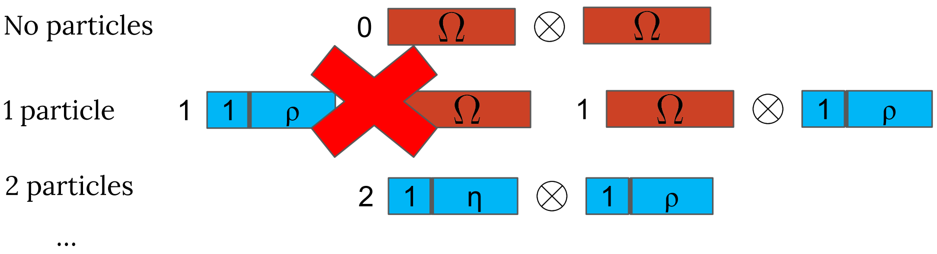
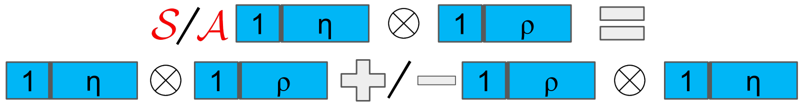


3. Add (anti)symmetrizers for (fermion)bosons



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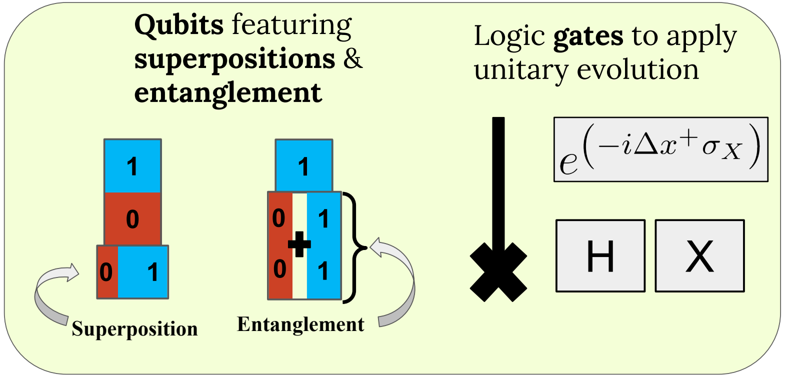

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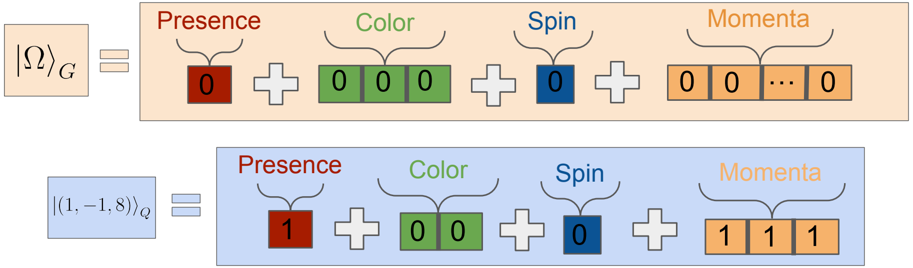

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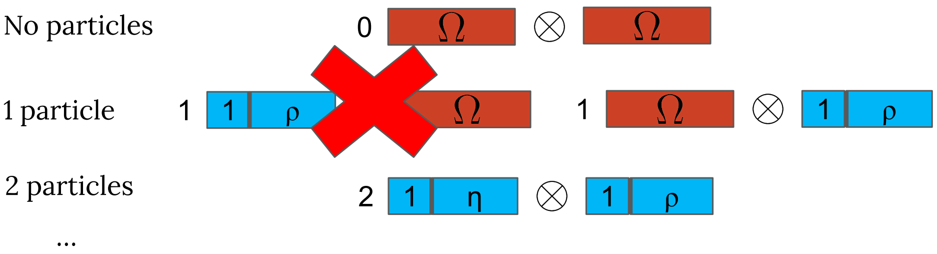

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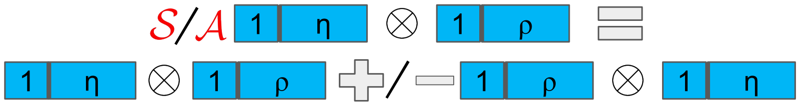

From steps 1, 2 and 3

$$a_{\rho}^{(n)\dagger} = \sum_{i=1}^{n} \mathcal{S}_{i\leftarrow(i-1)} \mathbb{P}_0^{(n-i)} \otimes \left(\mathfrak{C}_{10} \otimes \mathfrak{s}_{\rho}^{\dagger}\right)_i \otimes \mathbb{P}_{i-1}^{(i-1)}$$

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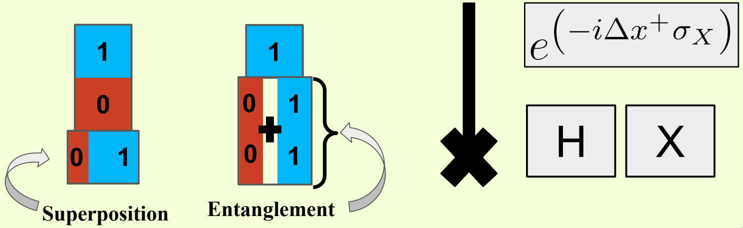

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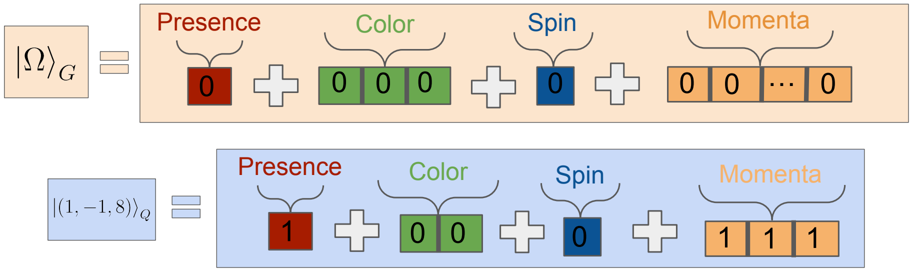

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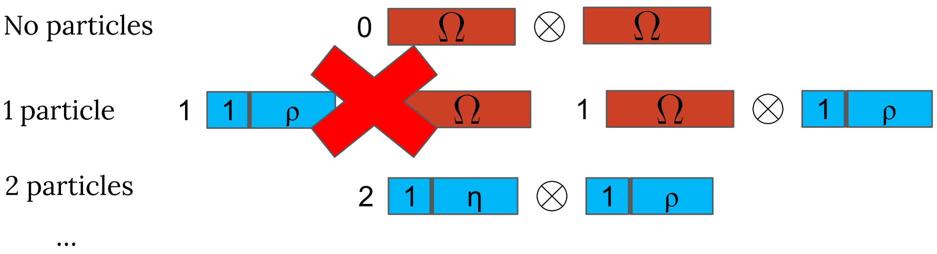

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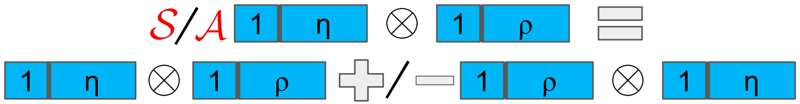

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Fulfil commutation relations up to boundary term

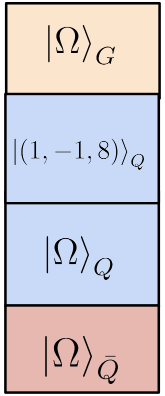
$$\left[a_\rho^{(n)}, a_\eta^{(n)\dagger}\right] = \delta_{\rho\eta} \left(\mathfrak{C}_{00} \otimes \mathfrak{i}\right)_n \otimes \mathbb{I}^{(n-1)} - S_{n\leftarrow n-1} \cdot \left(\mathfrak{C}_{11} \otimes \mathfrak{s}_\rho^\dagger \mathfrak{s}_\eta\right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot S_{n\leftarrow n-1}$$

Details in PhysRevD.110.116018

Light-Front time evolution

Fragmentation functions:

1. Start with quark with max momenta



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Fragmentation functions:

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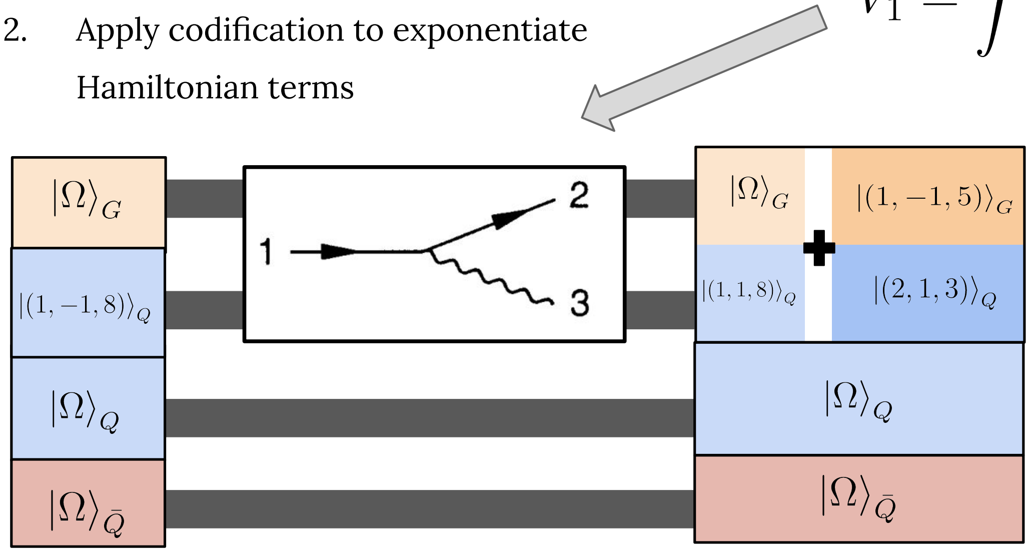
$$V_1 = \int V(1\,;\,2,\,3)\, a_3^\dagger\, b_2^\dagger\, b_1 + h.c.$$



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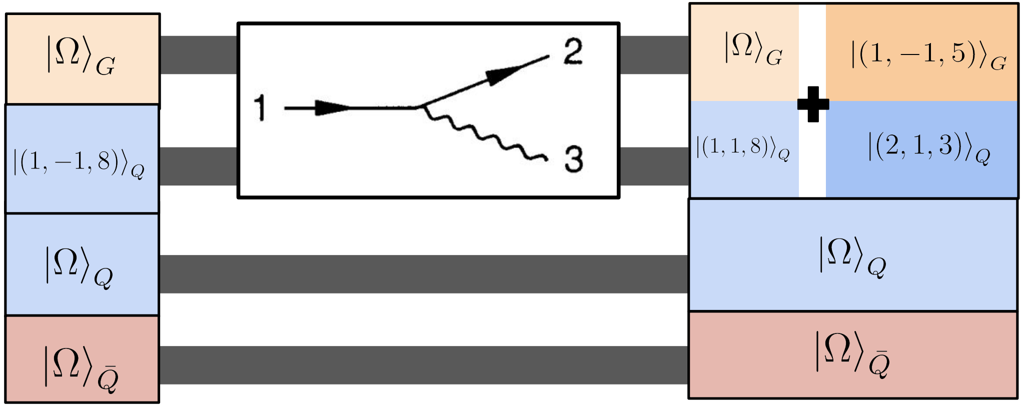
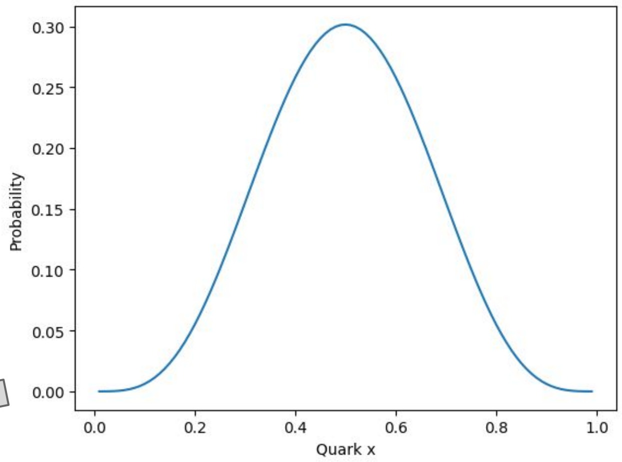
2. Apply codification to exponentiate Hamiltonian terms

$$V_1 = \int V(1\,;\,2,\,3)\, a_3^\dagger\, b_2^\dagger\, b_1 + h.c.$$



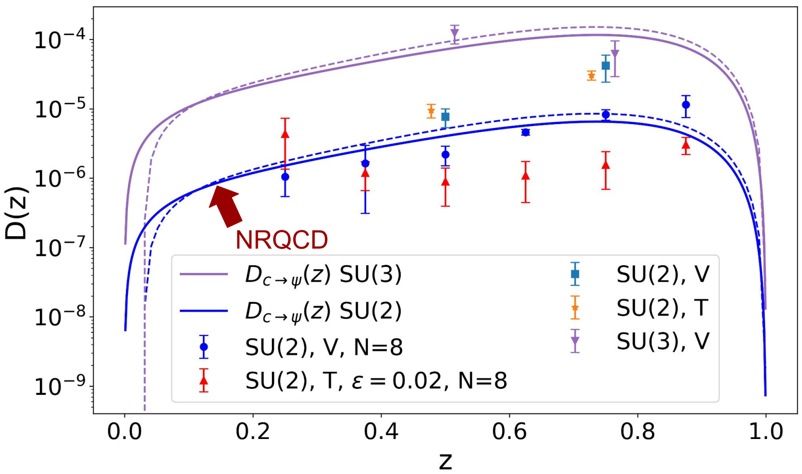


3. Measure "meson" wavefunction

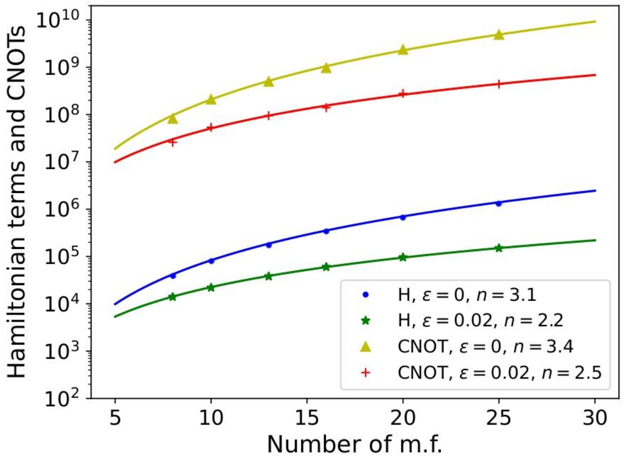
$$|J/\Psi\rangle = \sum \frac{\chi_0(x)}{\sqrt{x\,(z-x)}}\, \delta_{c_q c_{\bar{q}}}\ \sigma_{ij}\ |x\ i\, c_q, (z-x)\ j\, c_{\bar{q}}\rangle$$

doi.org/10.1140/epjc/s10052-022-10988-5

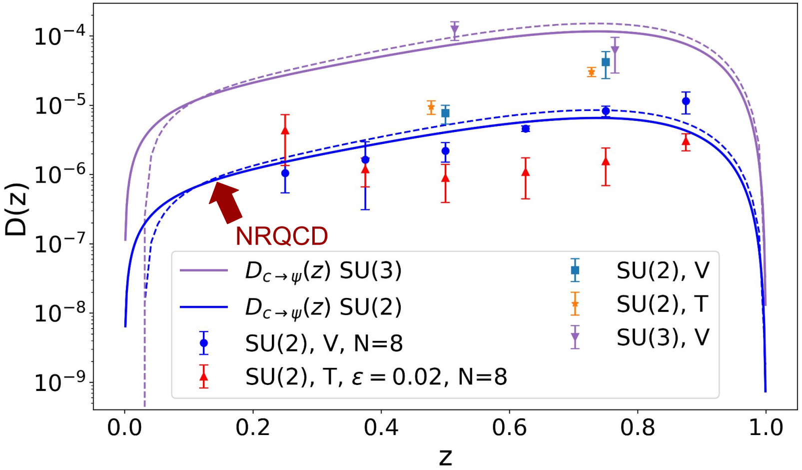
Results & conclusions



$$C(N; a, n) = a \cdot N^n$$



Results & conclusions



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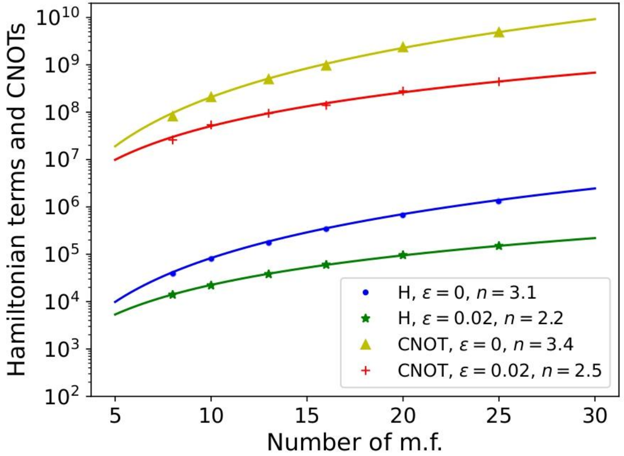
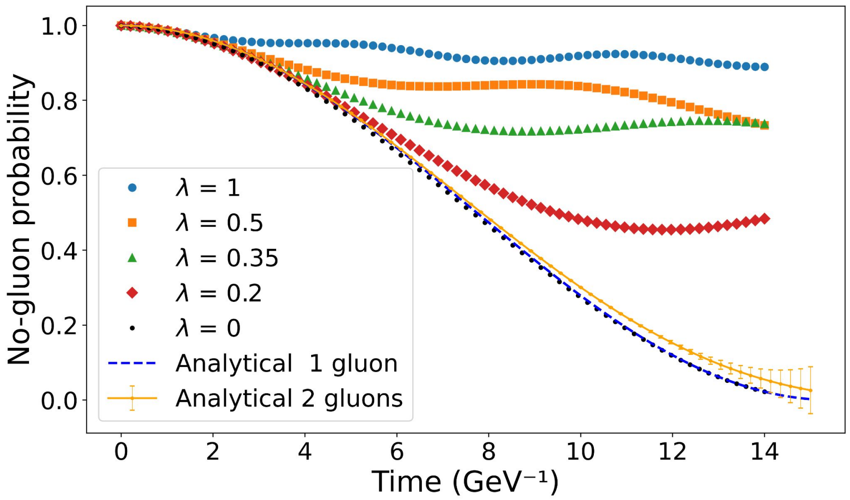


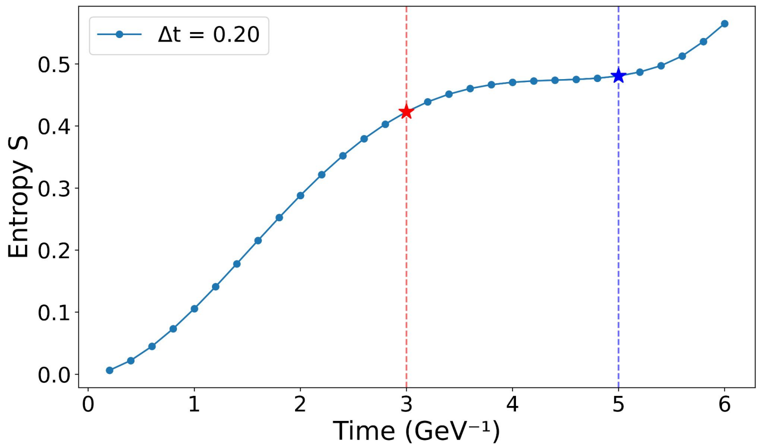
- **End to end** simulation of fragmentation in 1+1 LF QCD with 2 quarks, 1 antiquark, and 1 gluon.
- 29 qubits $\implies$ 17 GB of wavefunction
- Total number of gates $\sim 10^8$ far from today $\sim 10^3$ gates, but improving fast!

Halting protocol

$$\exp\{-i\Delta t(V_1 + \lambda E_c)\}$$



$$S = \sum_i -p_i \log(p_i)$$



More about scaling