Understanding large localized CP Violation in $B^\pm \to K^\pm\pi^+\pi^-$ using dispersive methods

IV CONGRESO
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FACULTAD DE CIENCIAS FÍSICAS

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ALBA REYES TORRECILLA

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In collaboration with L. A. Heuser, C. Hanhart, B. Kubis,
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IPARCOS

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WHAT IS CP VIOLATION?

$$C\,P$$

Charge conjugation

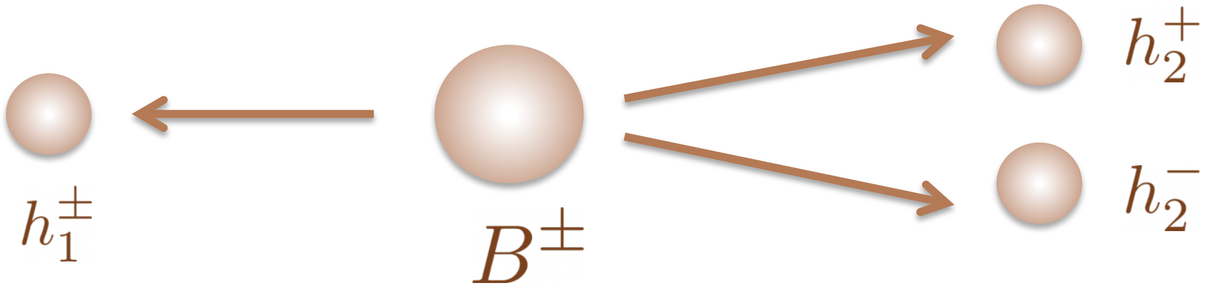
Parity

- A discrete symmetry composed of C and P (weak force violates both of them)
- This symmetry exchanges particles for antiparticles in physical processes
- It was believed to be a true conserved symmetry of nature for many decades, but in 1964 it was discovered that weak force also violates CP symmetry
- CP violation observed through CP conjugated decays

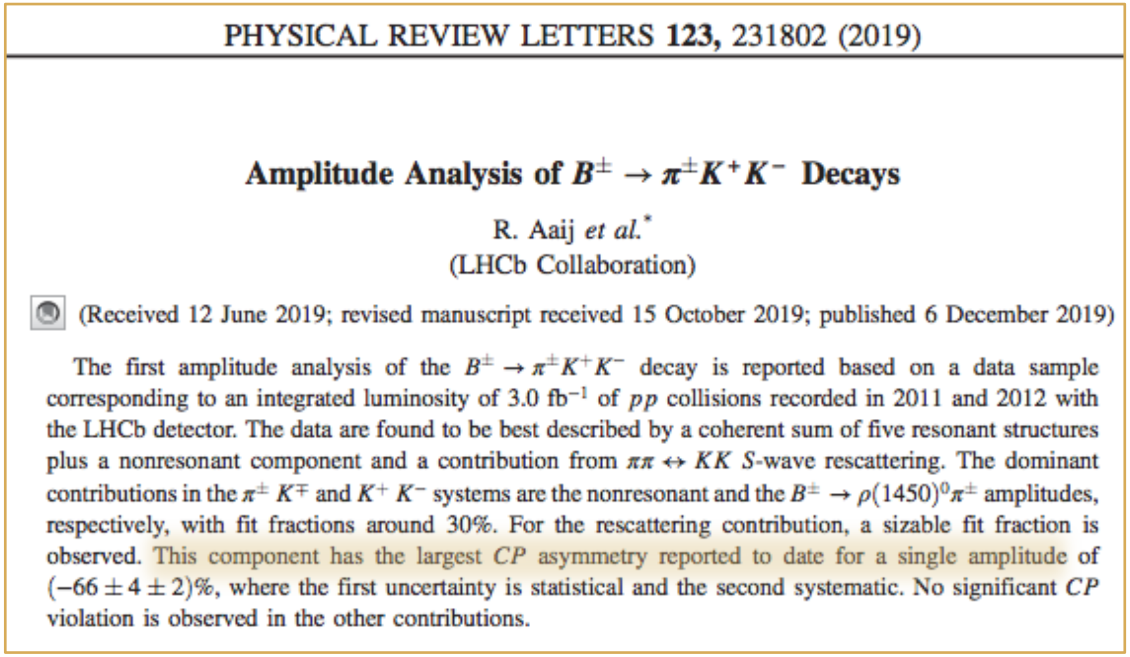
$$\mathcal{A}_{\mathrm{CP}} = \frac{\Delta\Gamma_{\mathrm{CP}}}{\Sigma\Gamma} = \frac{\Gamma^- - \Gamma^+}{\Gamma^- + \Gamma^+}$$

WHY ARE 3-BODY B DECAYS INTERESTING?

GOAL



- The **largest CP asymmetry reported to date for a single amplitude** has been found in B mesons decaying to three light mesons



PHYSICAL REVIEW LETTERS **123,** 231802 (2019)

Amplitude Analysis of $B^{\pm} \rightarrow \pi^{\pm} K^{+} K^{-}$ Decays

R. Aaij *et al.*[*]
(LHCb Collaboration)

(Received 12 June 2019; revised manuscript received 15 October 2019; published 6 December 2019)

The first amplitude analysis of the $B^{\pm} \rightarrow \pi^{\pm} K^{+} K^{-}$ decay is reported based on a data sample corresponding to an integrated luminosity of 3.0 fb^{-1} of pp collisions recorded in 2011 and 2012 with the LHCb detector. The data are found to be best described by a coherent sum of five resonant structures plus a nonresonant component and a contribution from $\pi\pi \leftrightarrow KK$ S-wave rescattering. The dominant contributions in the $\pi^{\pm} K^{\mp}$ and $K^{+} K^{-}$ systems are the nonresonant and the $B^{\pm} \rightarrow \rho(1450)^{0} \pi^{\pm}$ amplitudes, respectively, with fit fractions around 30%. For the rescattering contribution, a sizable fit fraction is observed. This component has the largest CP asymmetry reported to date for a single amplitude of $(-66 \pm 4 \pm 2)\%$, where the first uncertainty is statistical and the second systematic. No significant CP violation is observed in the other contributions.

WHY ARE 3-BODY B DECAYS INTERESTING?

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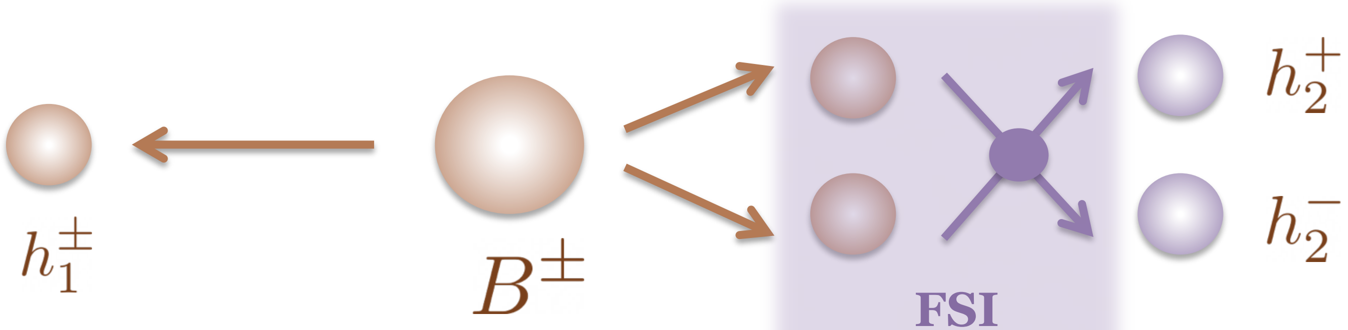

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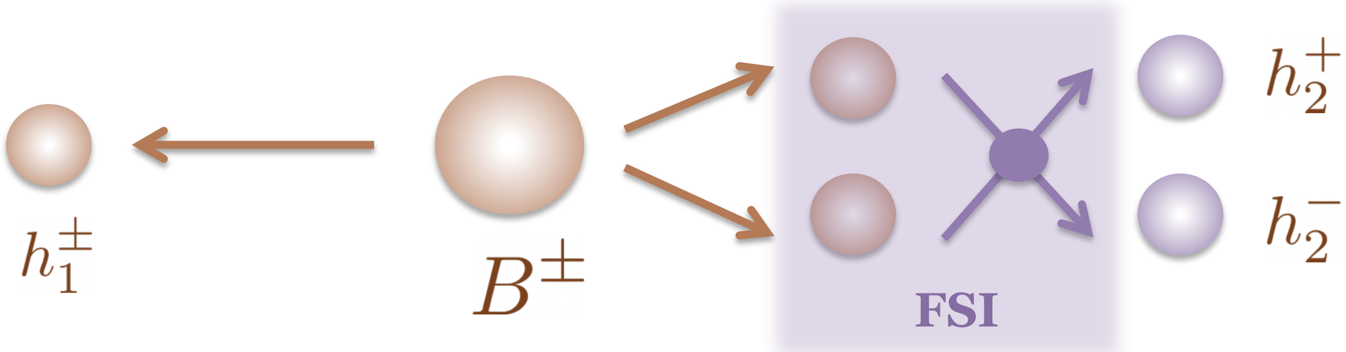
- **Final State Interactions** (FSI) are strongly believed to **amplify the CP violation** (CPV) that occurs in these hadron decays

To observe CPV in an asymmetry it is necessary an interference between strong and weak phases → **Strong phases can be generated at the HADRON level through FSI**

GOAL



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- Non-leptonic decays of heavy mesons are **difficult to describe** theoretically, three-body decays remain a challenge
- An approach that treats the **hadronic part** of the decays **systematically**

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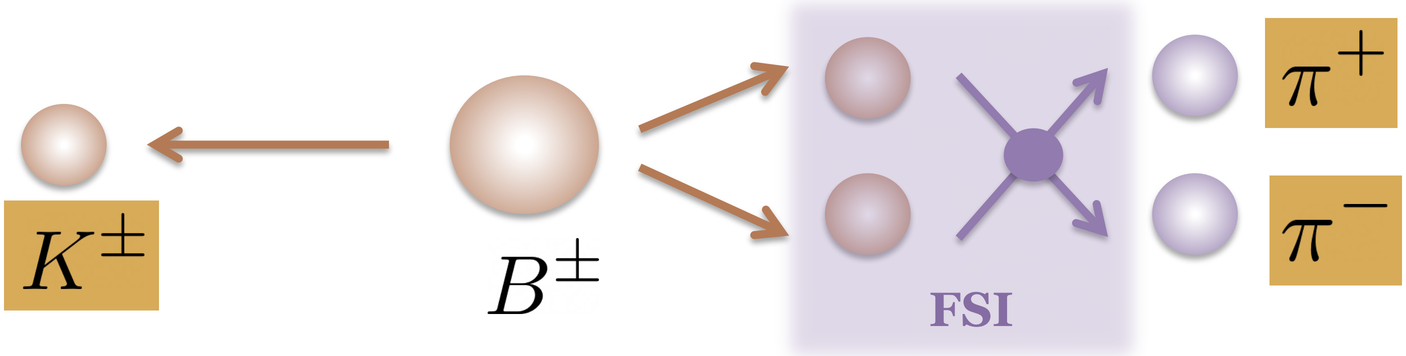

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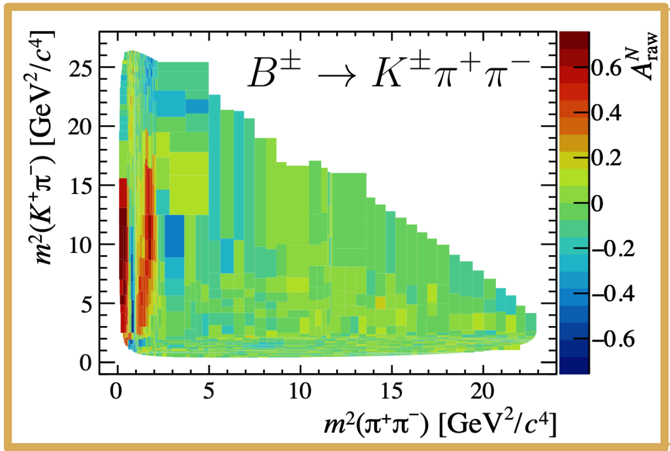
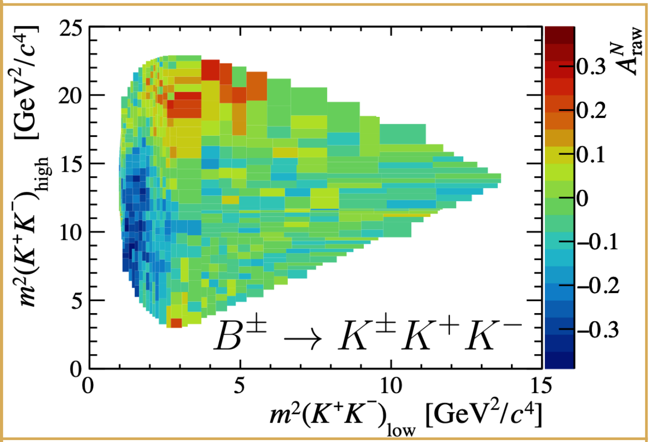
UNDERSTANDING: LARGE LOCALIZED CP VIOLATION

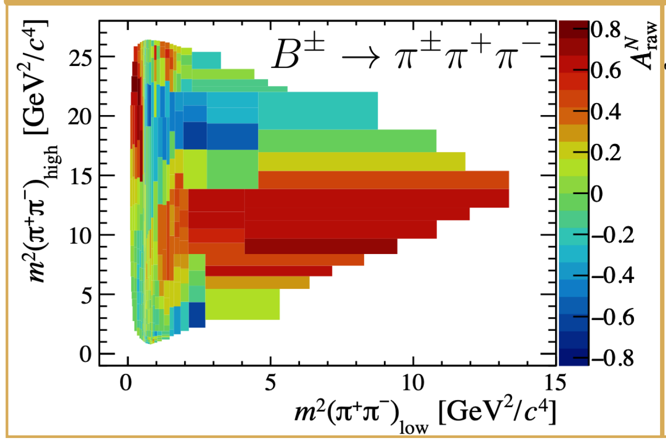
- Large **integrated** CPV asymmetries ⟶ 10%

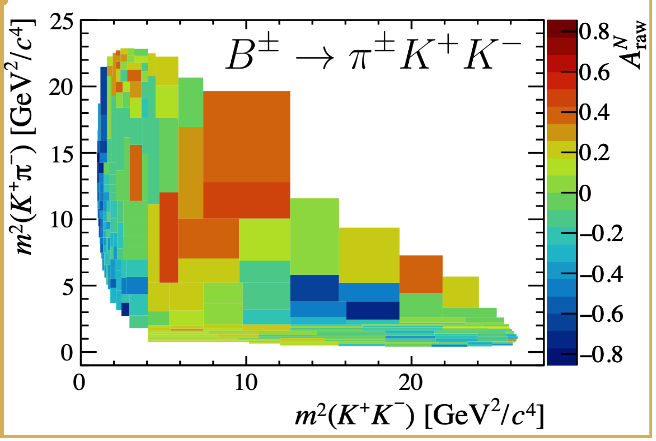
$$\mathcal{A}_{\mathrm{CP}}\left(B^{\pm}\to K^{\pm}\pi^{+}\pi^{-}\right)=+0.011\pm0.002\pm0.003\pm0.003,\quad \mathcal{A}_{\mathrm{CP}}\left(B^{\pm}\to \pi^{\pm}\pi^{+}\pi^{-}\right)=+0.080\pm0.004\pm0.003\pm0.003,$$

$$\mathcal{A}_{\mathrm{CP}}\left(B^{\pm}\to K^{\pm}K^{+}K^{-}\right)=-0.037\pm0.002\pm0.002\pm0.003,\ \mathcal{A}_{\mathrm{CP}}\left(B^{\pm}\to \pi^{\pm}K^{+}K^{-}\right)=-0.114\pm0.007\pm0.003\pm0.003,$$









RUN2 $5.9\ fb^{-1}$

Asymmetries:

$$\mathcal{A}_{\mathrm{CP}}=\frac{\Delta\Gamma_{\mathrm{CP}}}{\Sigma\Gamma}=\frac{\Gamma^{-}-\Gamma^{+}}{\Gamma^{-}+\Gamma^{+}}$$

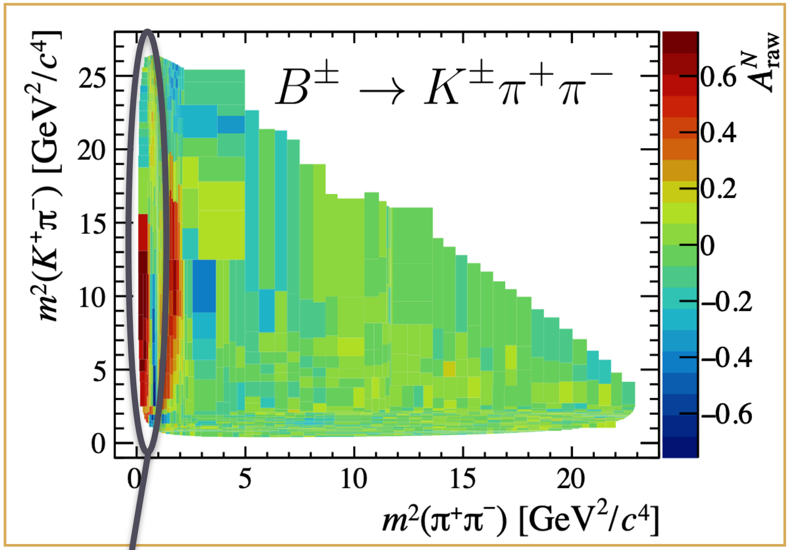
- **GIANT localized** CPV asymmetries

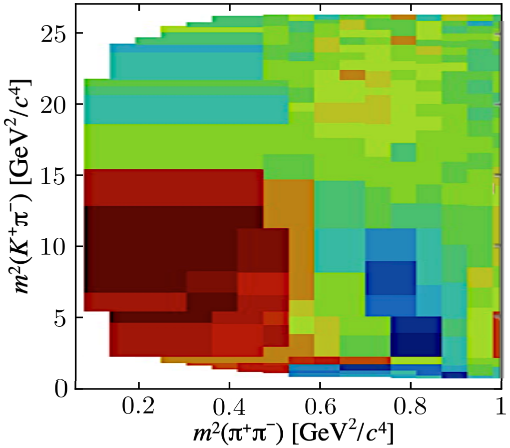
↓

60% – 80%

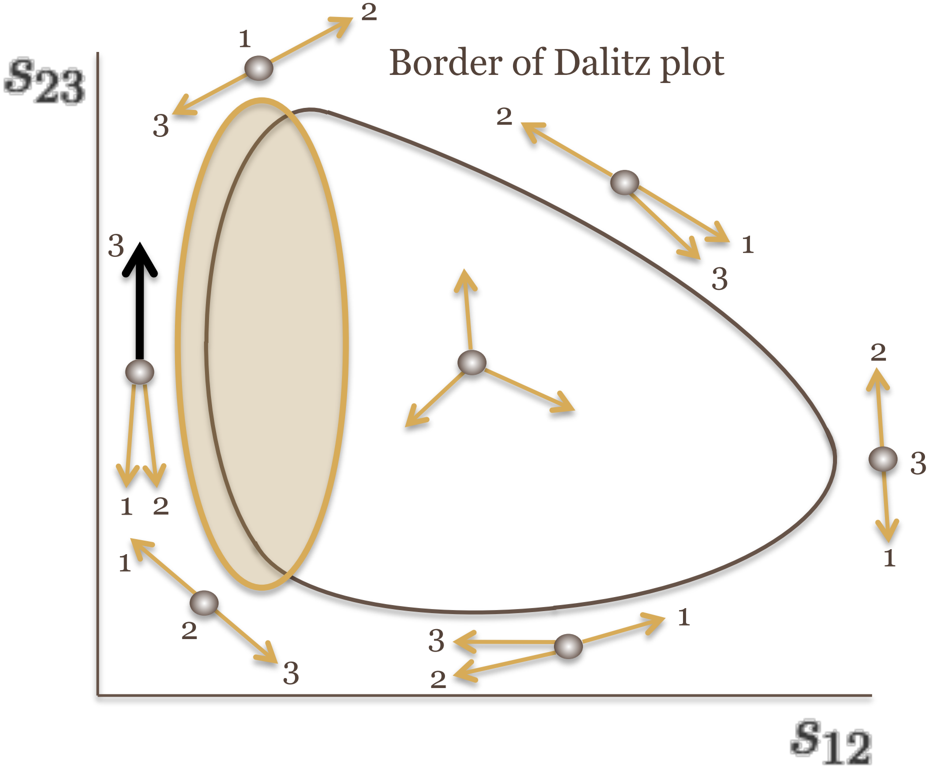
UNDERSTANDING: LARGE LOCALIZED CP VIOLATION

- Most of the observed giant CPV lies at $m^2_{\pi\pi} \leq 1$ GeV invariant mass region, where the $\pi\pi$ scattering is elastic



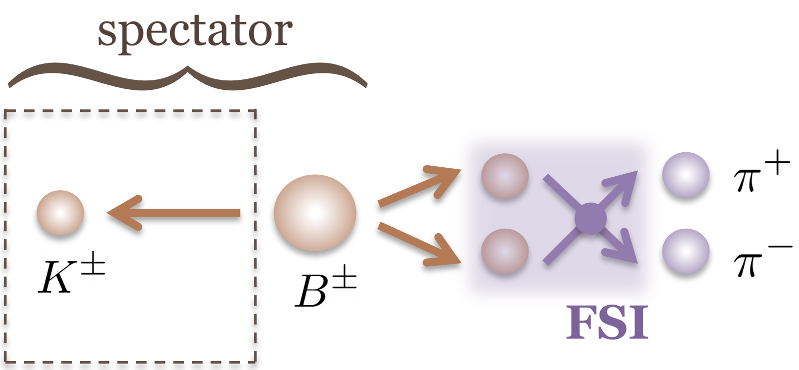


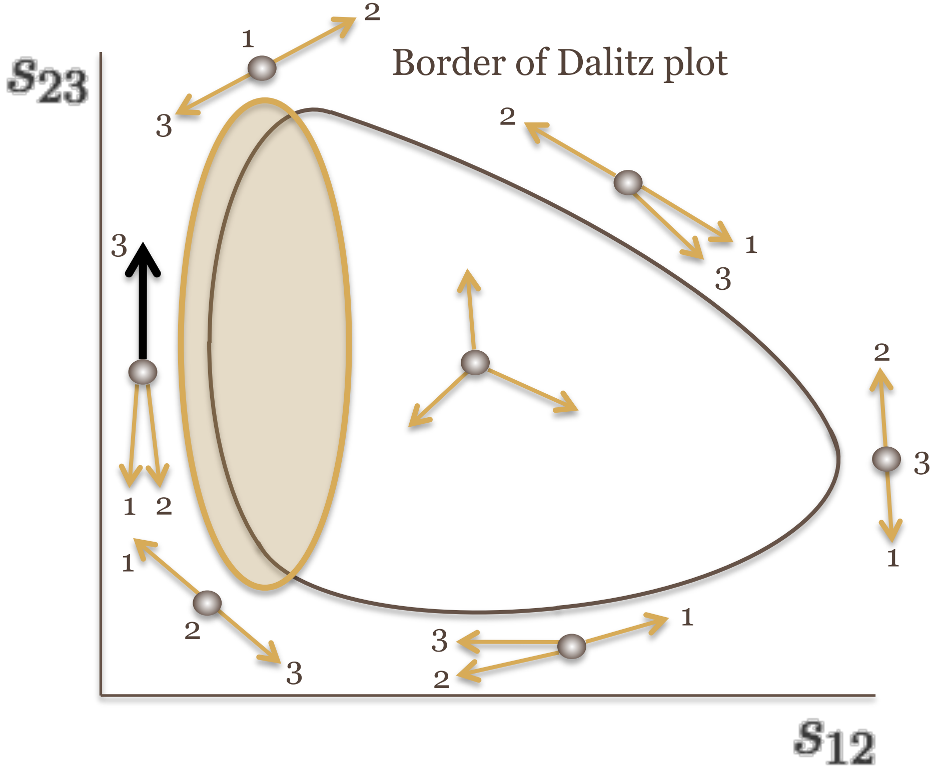
Elastic $\pi\pi$ scattering



We will make some assumptions in our method:

ASSUMPTIONS OF THE METHOD





- For small $m^2_{\pi\pi}$, the kaon final-state rescattering is negligible

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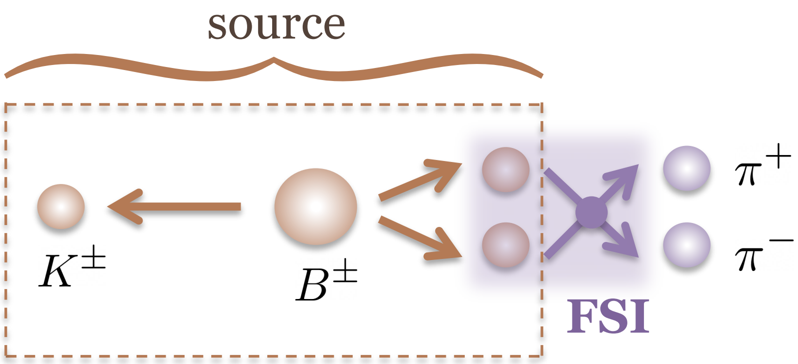
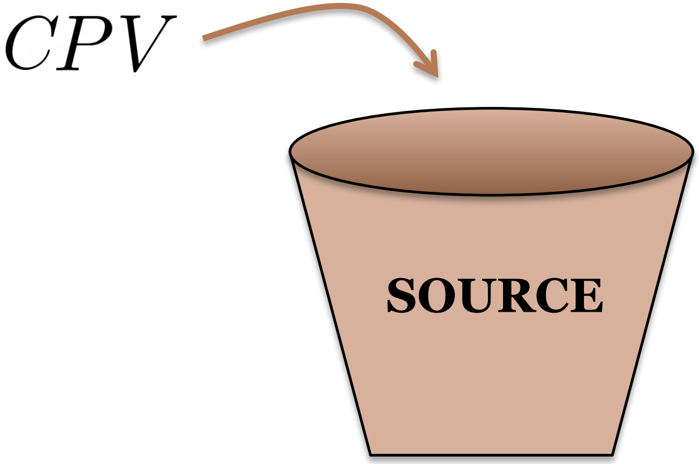


- For small $m^2_{\pi\pi}$, the kaon final-state rescattering is negligible
- The weak interaction produces hadrons on a small distance scale, forming a source for the outgoing hadrons



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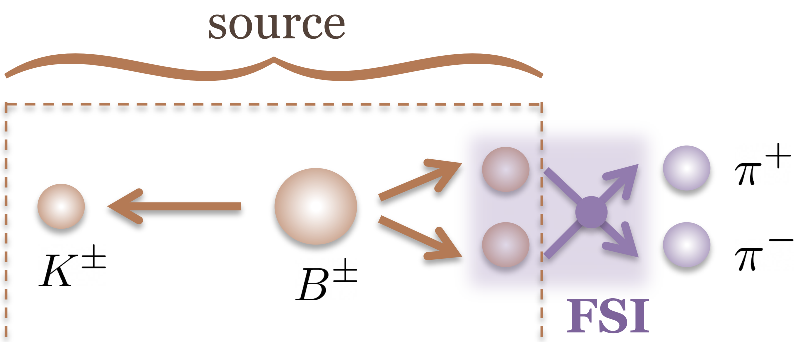

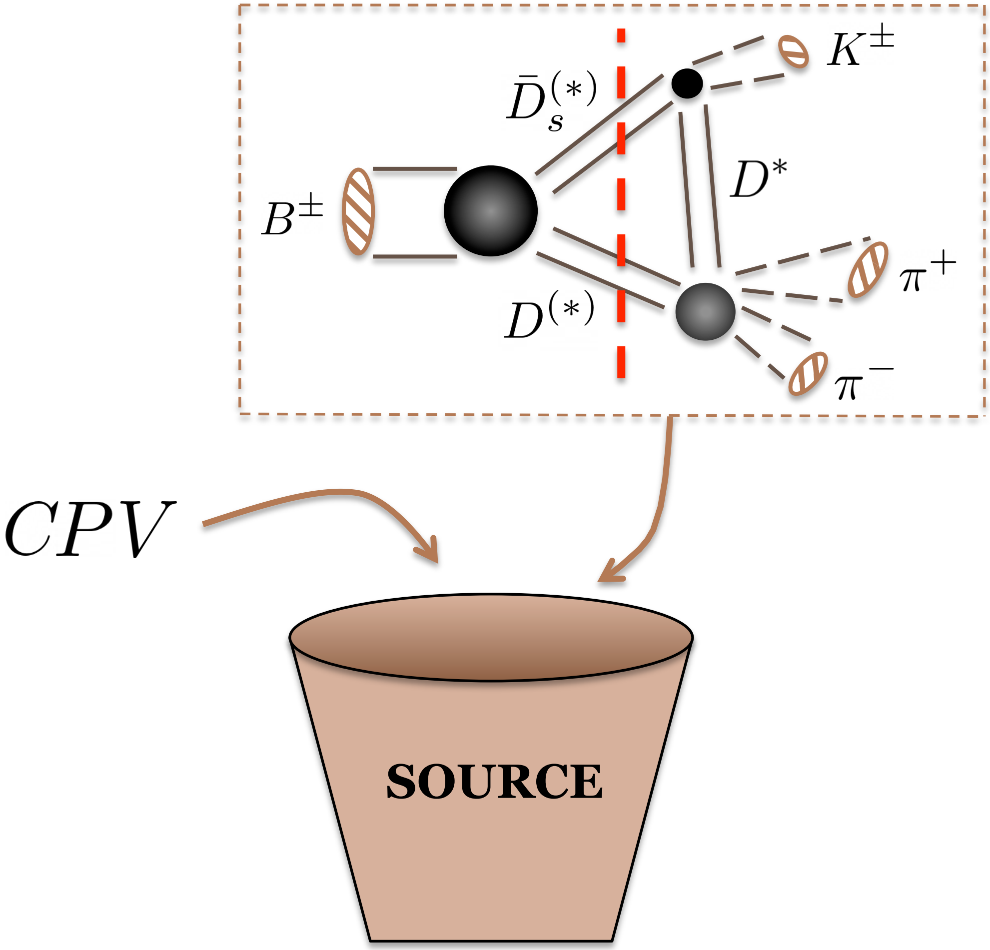

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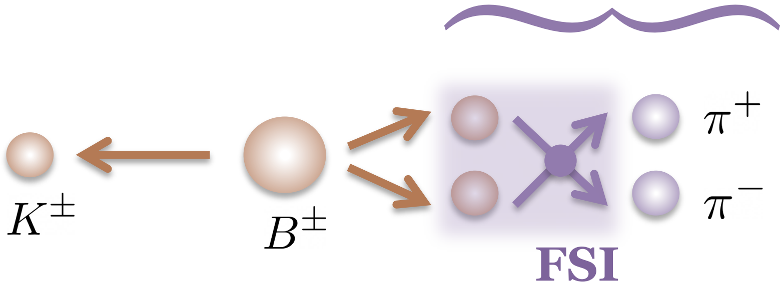

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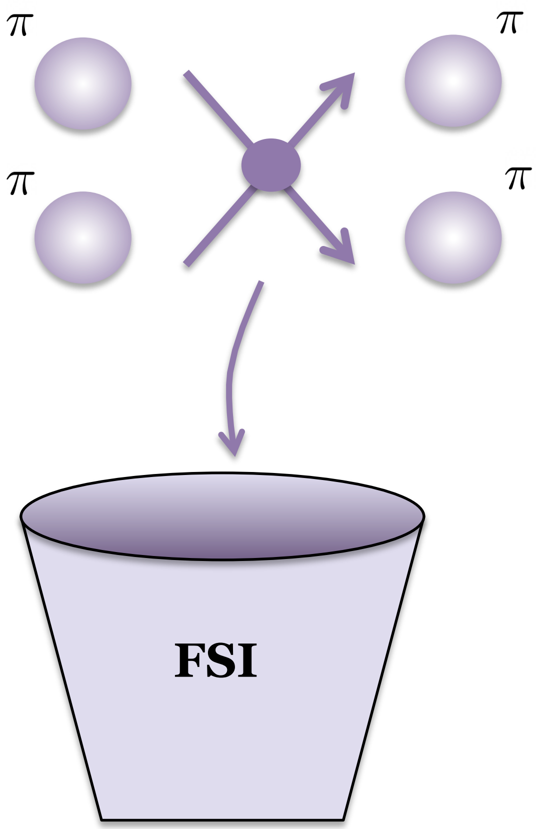
Elastic rescattering under $1\ \mathrm{GeV}$



- Universal $\pi\pi$ FSI → dispersive approach

UNDERSTANDING: DISPERSIVE METHOD FOR FSI

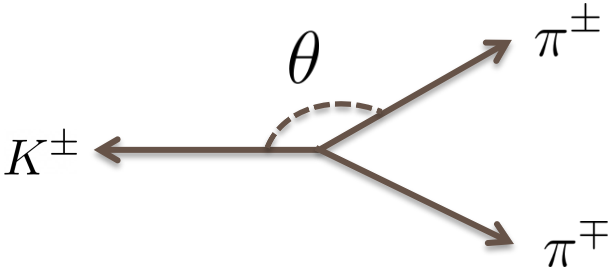
Notation

$$\mathcal{A}^{\pm}(s,t) = \sum_i f_i(s,t)\,\boxed{\mathcal{A}_i^{\pm}(s)}$$

angular dependence

source

rescattering



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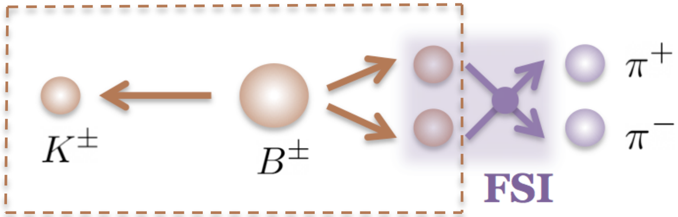

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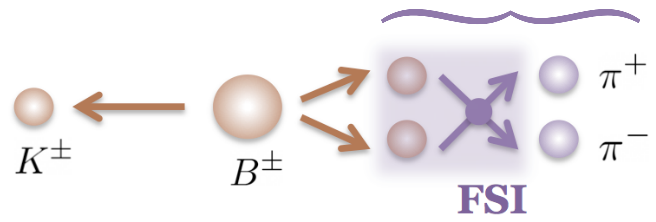

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angular dependence

- In this kinematic range we are dealing with $LI = S0, S2, P1$ waves

$$f_i(s,t) = \begin{cases} 1 & \text{for } i \in \{S0, S2\} \\ g(s)\cos\theta & \text{for } i = P1 \end{cases}$$

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Non-resonant partial wave, but **essential** to reproduce CPV data and the localized giant CPV with accuracy

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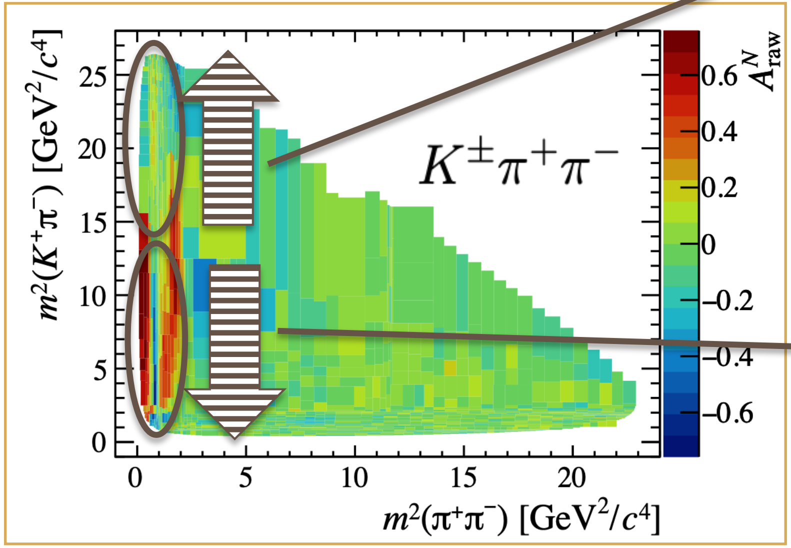
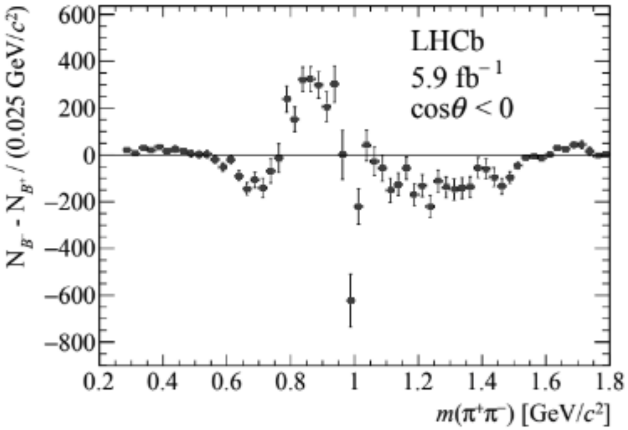
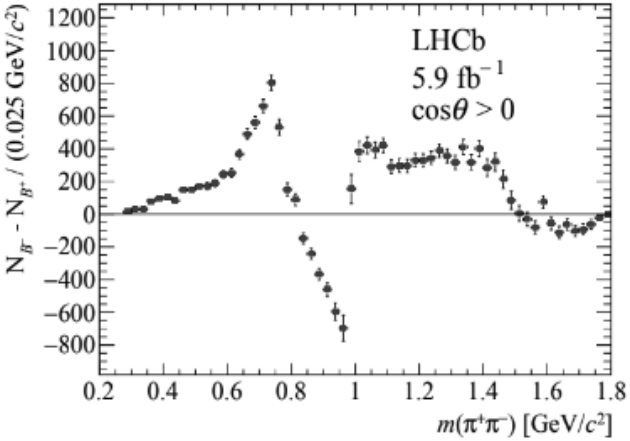




Backward
$\cos\theta < 0$



Forward
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UNDERSTANDING: DISPERSIVE METHOD FOR FSI

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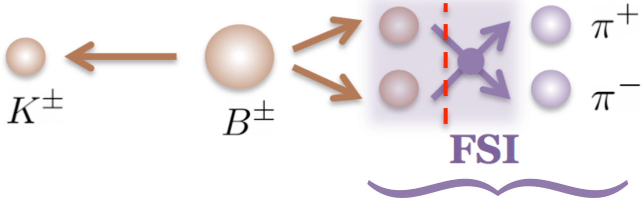

- Discontinuity relation for the production partial-wave amplitudes along $\pi\pi$ threshold

$$\underline{\text{Disc}\,\mathcal{A}_i^{\pm}(s)} = 2i\rho_\pi(s)\mathcal{M}_i(s)^*\mathcal{A}_i^{\pm}(s)$$

Elastic !

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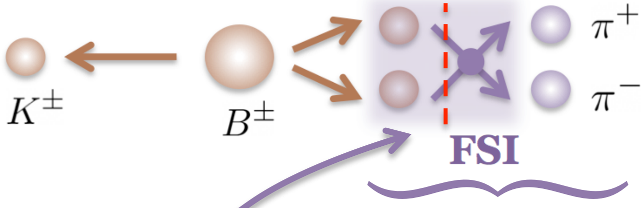

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$\pi\pi$ scattering partial-wave amplitude

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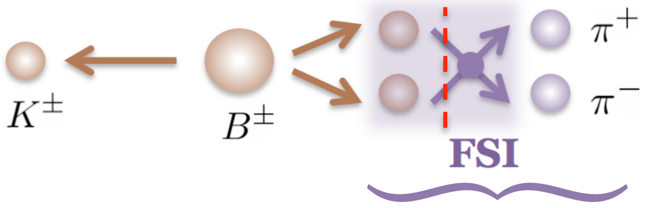

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$$\boxed{\mathcal{A}_i^{\pm}(s) = P_i(s)\Omega_i(s)\bar{\mathcal{A}}_i^{\pm}}$$

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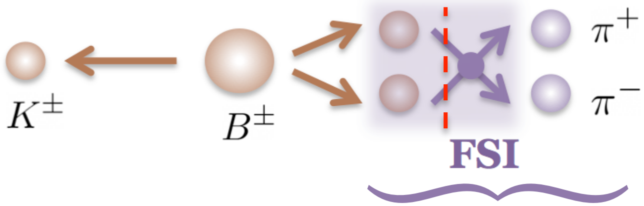

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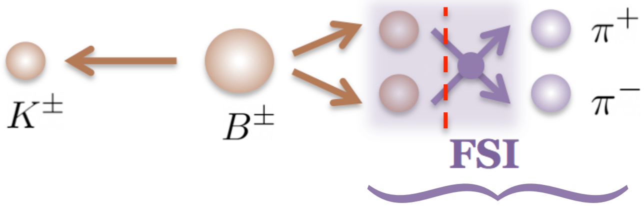

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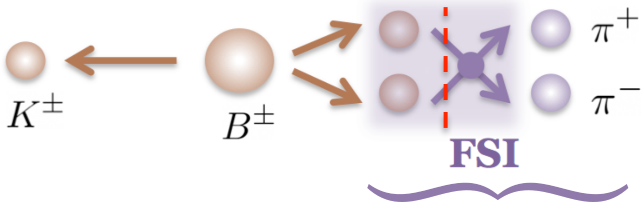

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phase-shift

polynomial

$$\Omega_i(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} \mathrm{d}s' \frac{\delta_i(s')}{s'\left(s'-s-i\epsilon\right)}\right\}$$

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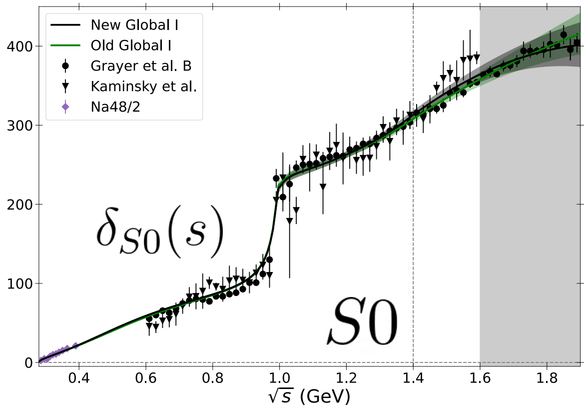
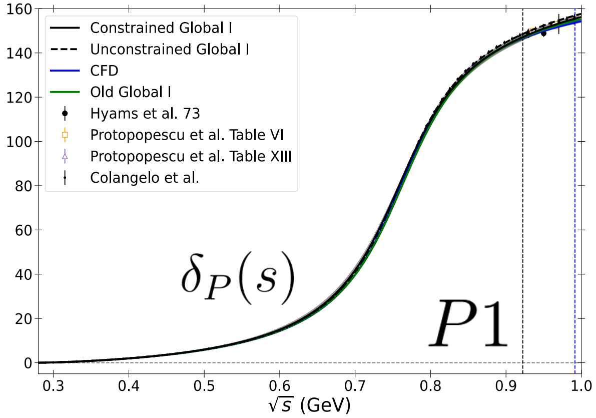
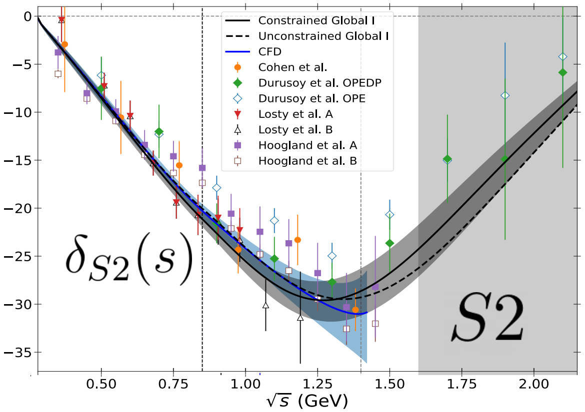
UNDERSTANDING: DISPERSIVE METHOD FOR FSI

Omnès functions

The phase-shifts are very well known!







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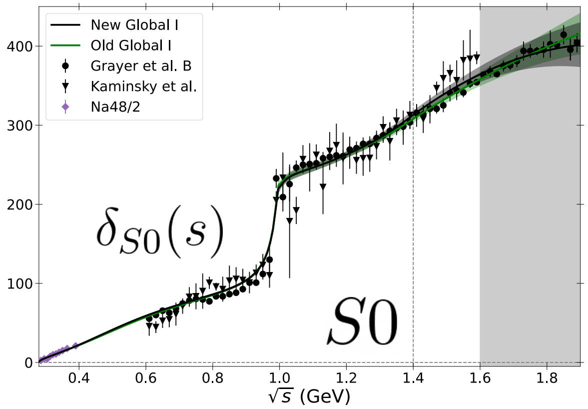
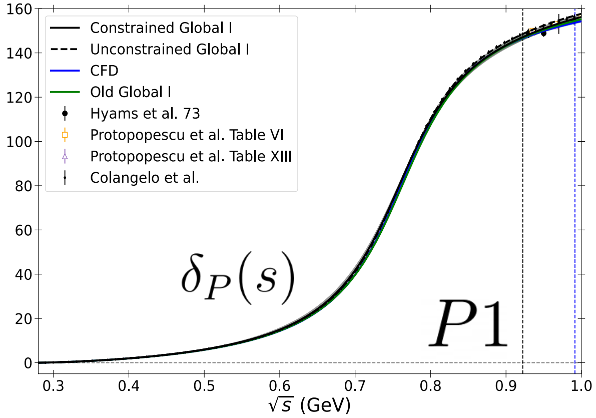
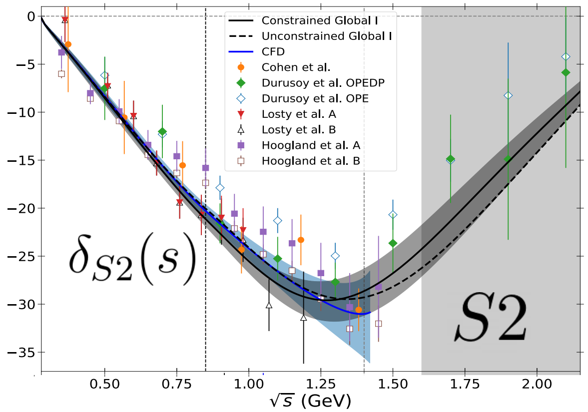
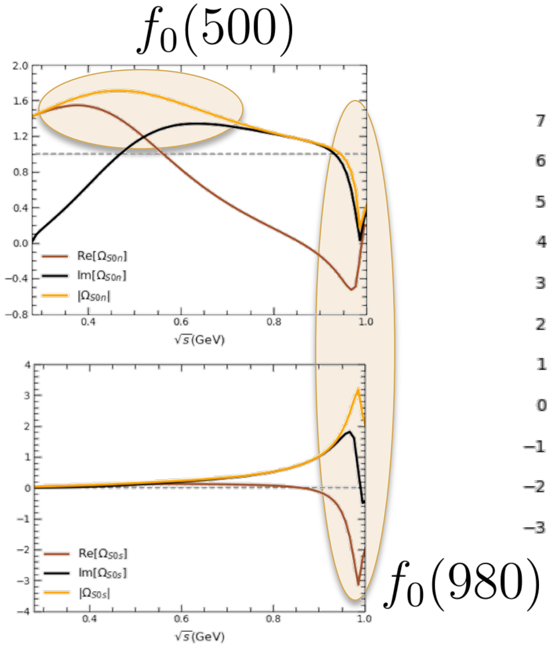
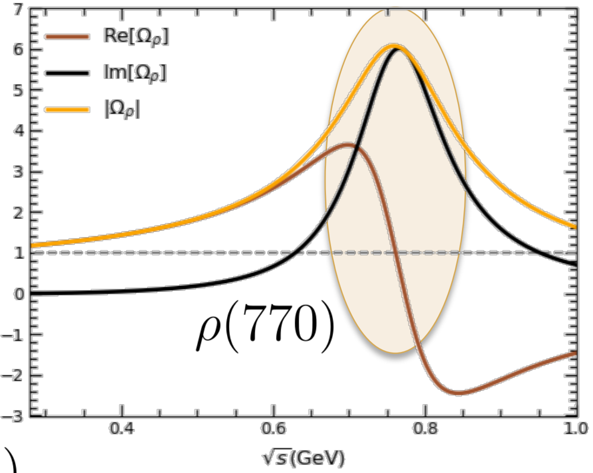


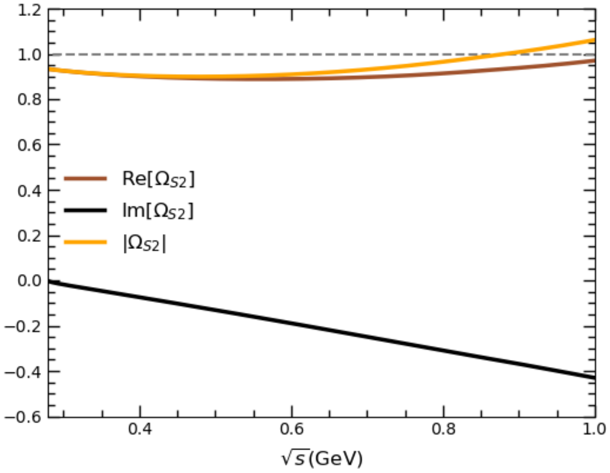






Omnès funtions

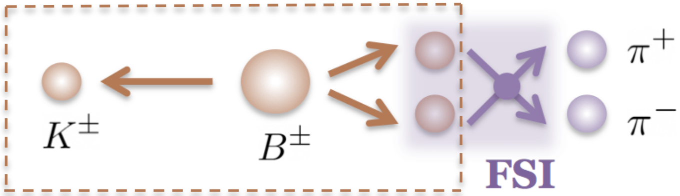




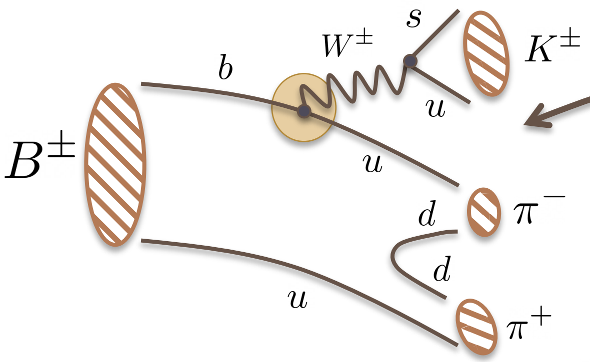
UNDERSTANDING: DISPERSIVE METHOD FOR FSI

Source parameterization

$$\mathcal{A}^{\pm}(s,t) = \sum_i f_i(s,t) P_i(s) \Omega_i(s) \boxed{\bar{A}_i^{\pm}}$$

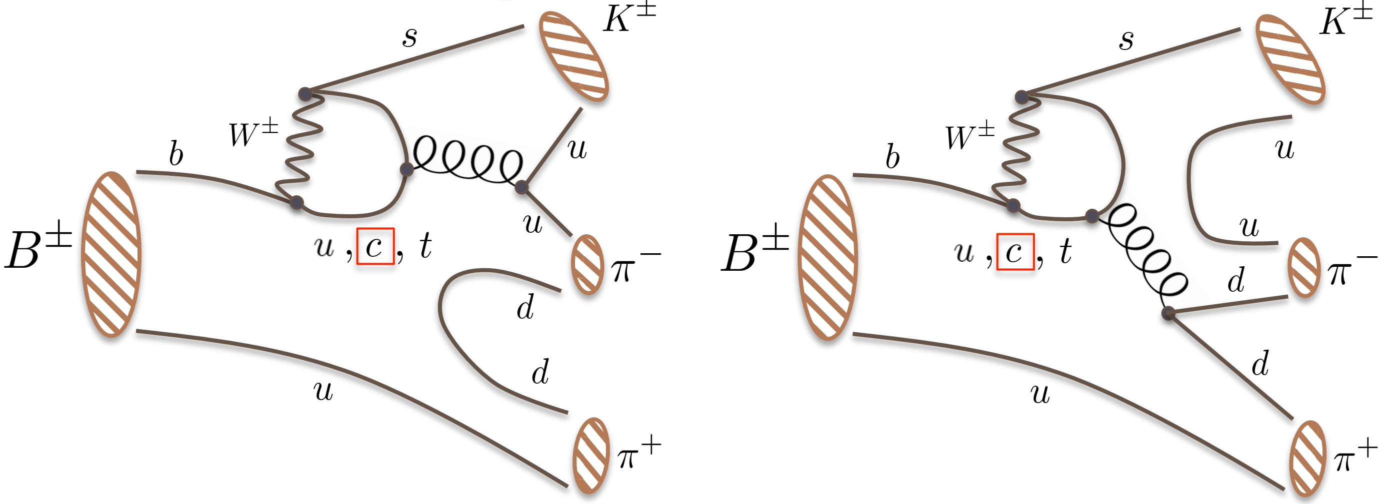


$\sim \lambda^4$



$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$\boxed{\sim \lambda^2}$



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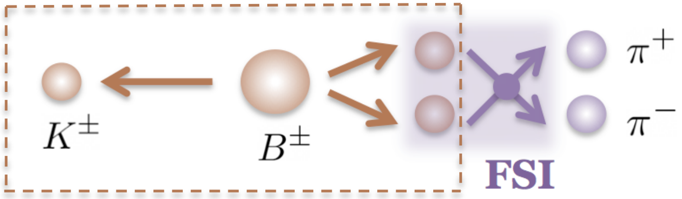
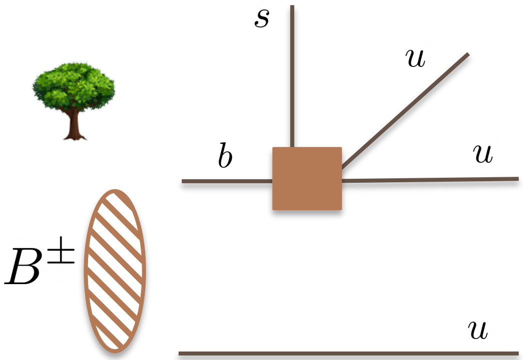
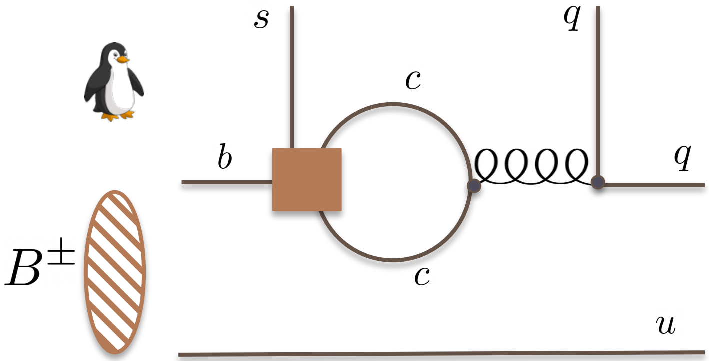




$u\bar{u}$ source

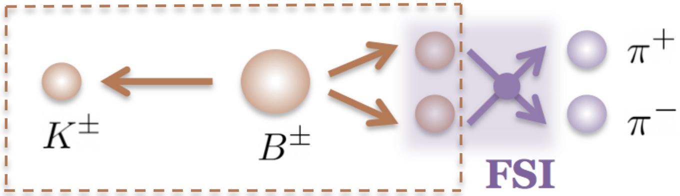


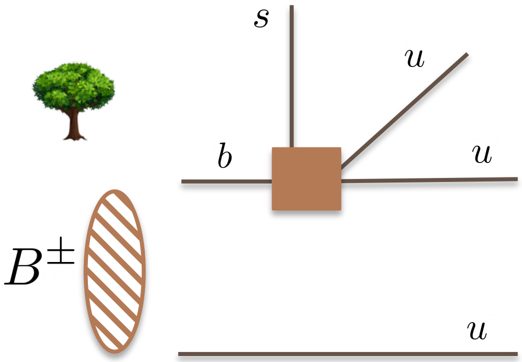
$q\bar{q}$ source

UNDERSTANDING: DISPERSIVE METHOD FOR FSI

Source parameterization

$$\mathcal{A}^{\pm}(s,t)=\sum_i f_i(s,t)P_i(s)\Omega_i(s)\bar{\mathcal{A}}_i^{\pm}$$





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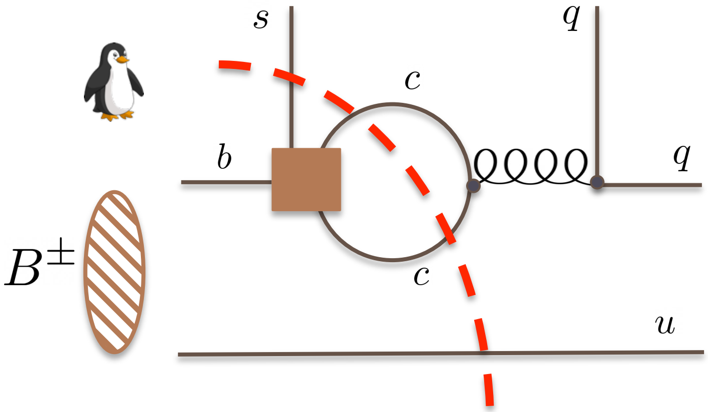
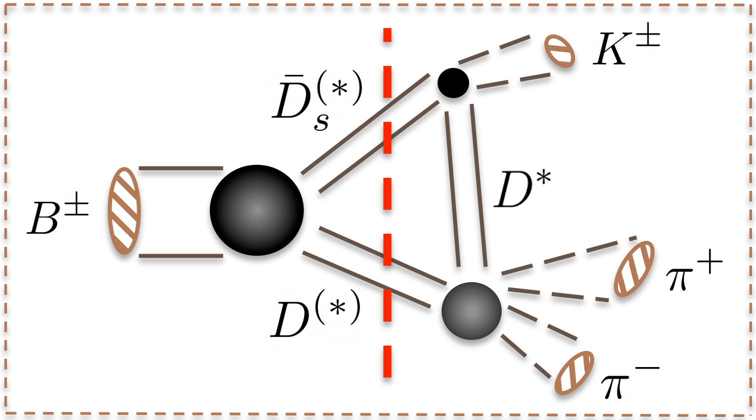


$q\bar{q}$ source

- The $D\bar{D}$ hadronization occurs at much higher scales than our $\pi\pi$ scattering at 1 GeV
- This allows us to approximate the source as a **COMPLEX** constant



UNDERSTANDING: DISPERSIVE METHOD FOR FSI

Source parameterization

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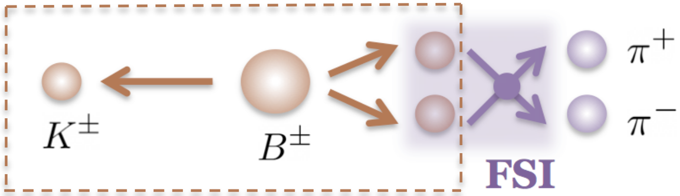

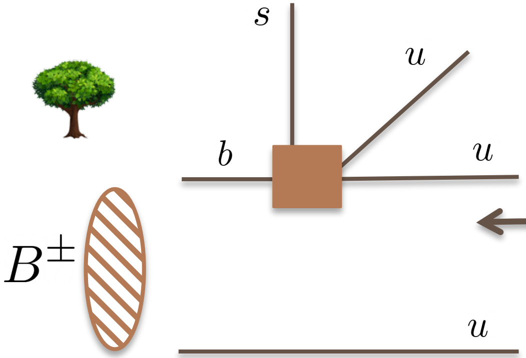

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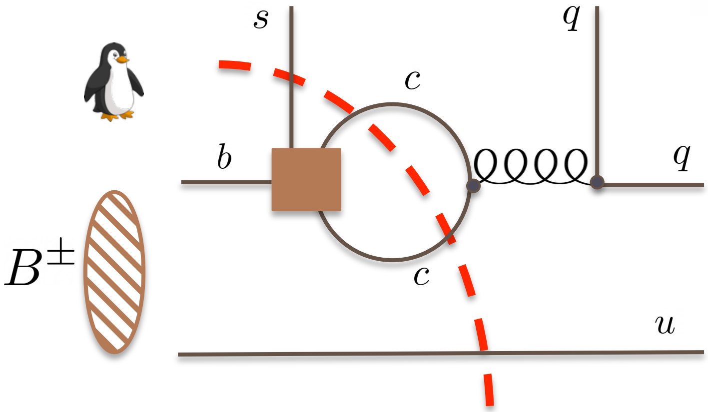

$q\bar{q}$ source

- The relevant effective weak Hamiltonian

$$H_{eff}=\frac{G_F}{\sqrt{2}}\left(e^{i\gamma}\left|V_{ub}^*V_{us}\right|(\bar{b}u)(\bar{u}s)+\left|V_{cb}^*V_{cs}\right|(\bar{b}c)(\bar{c}s)\right)$$

UNDERSTANDING: DISPERSIVE METHOD FOR FSI

Source parameterization

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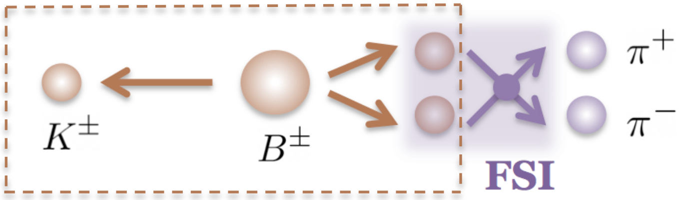

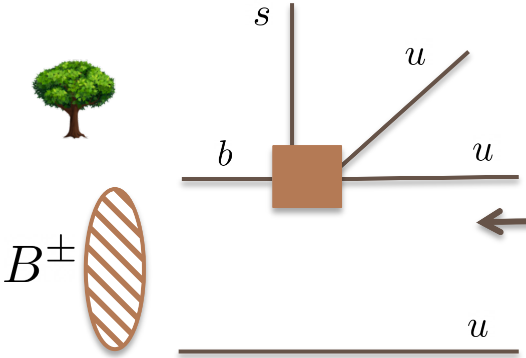

$u\bar{u}$ source

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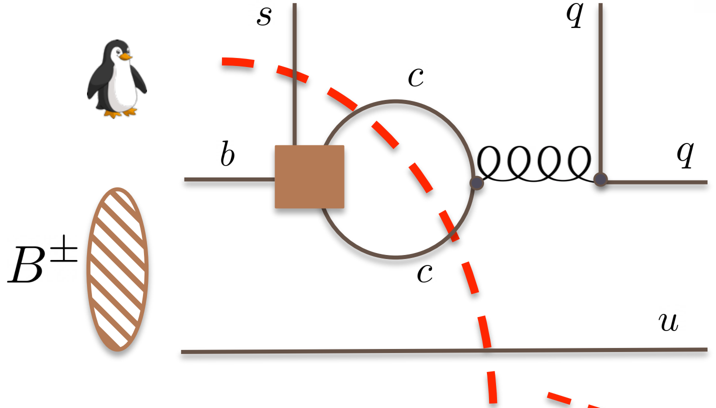


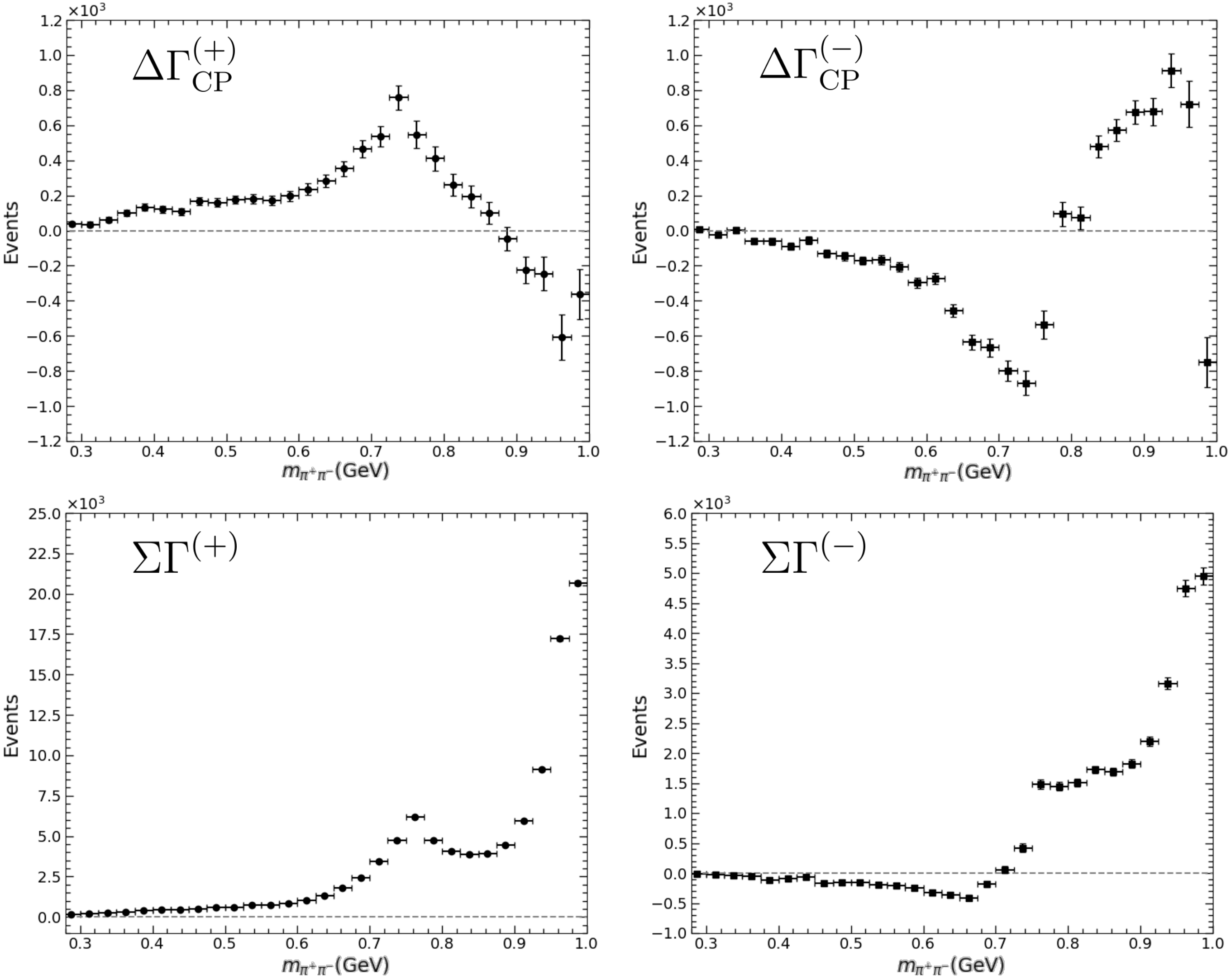
$q\bar{q}$ source

- Therefore, we parameterize the source as:

$$\bar{\mathcal{A}}_i^{\pm} = \hat{A}_i + e^{\pm i\gamma}\hat{B}_i$$
$$= a_i + ic_i \pm ib_i$$

RESULTS

Fitting LHCb data

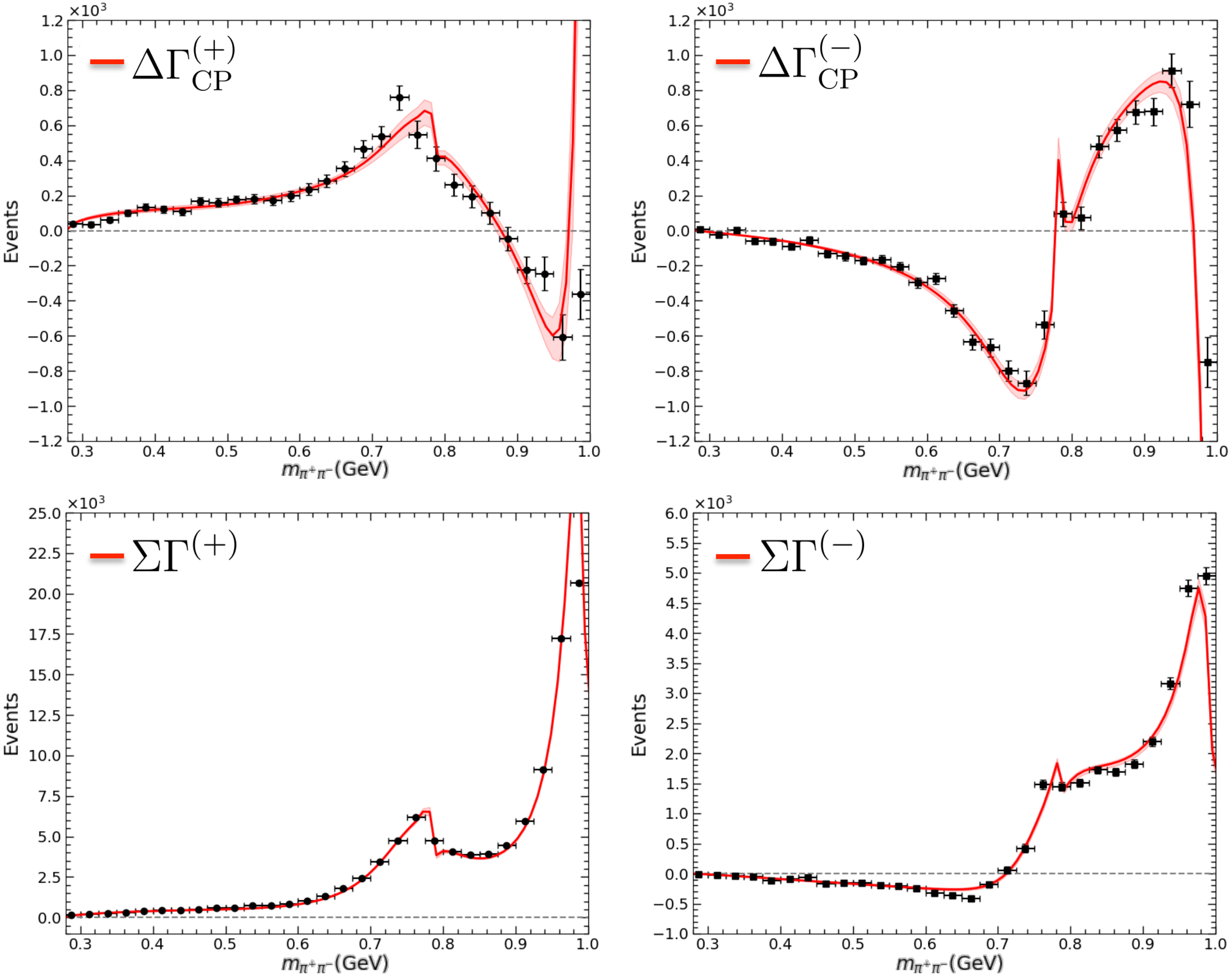


- Our parameters:

Partial Wave	Parameter
$S0$	a_{S0n} b_{S0n} c_{S0n}
	a_{S0s} c_{S0s}
	p_{S0}
$S2$	a_{S2} b_{S2} p_{S2}
$P1$	a_{P1} b_{P1} c_{P1} p_{P1}

RESULTS

Fitting LHCb data



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Partial Wave	Parameter
$S0$	a_{S0n}
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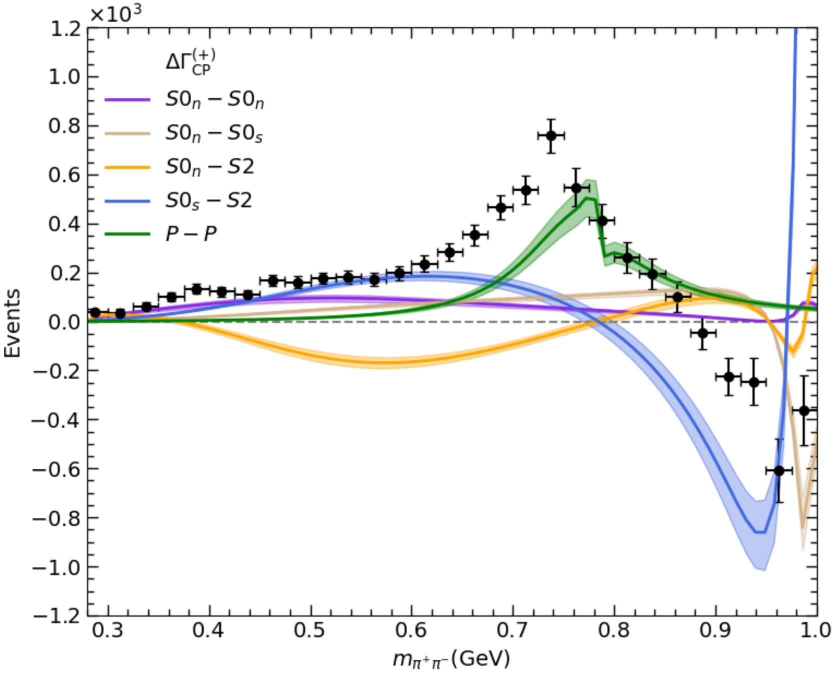
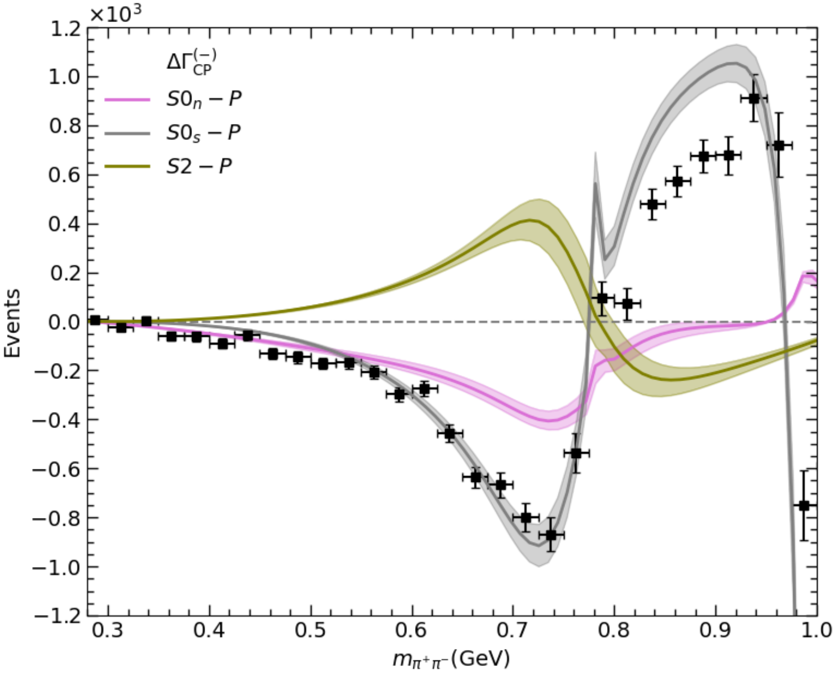
	a_{S0s}
	c_{S0s}
	p_{S0}
$S2$	a_{S2}
	b_{S2}
	p_{S2}
$P1$	a_{P1}
	b_{P1}
	c_{P1}
	p_{P1}

RESULTS

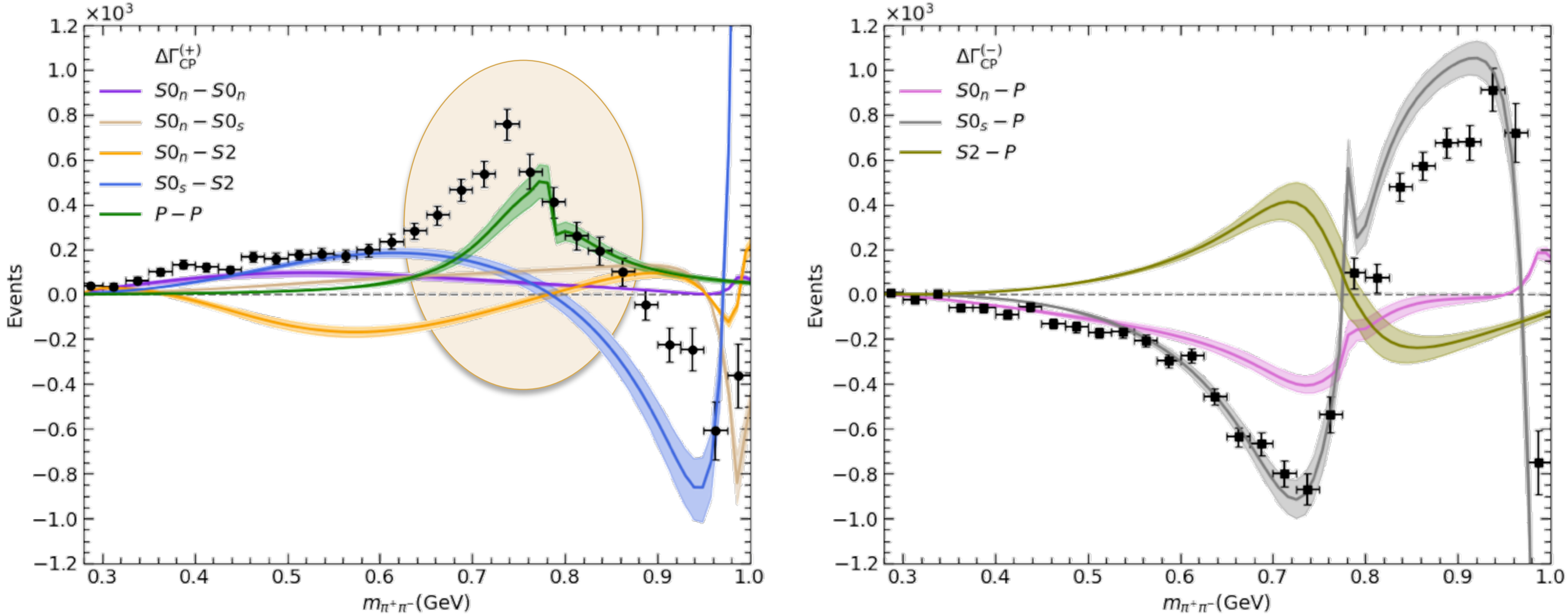
Individual contributions





RESULTS

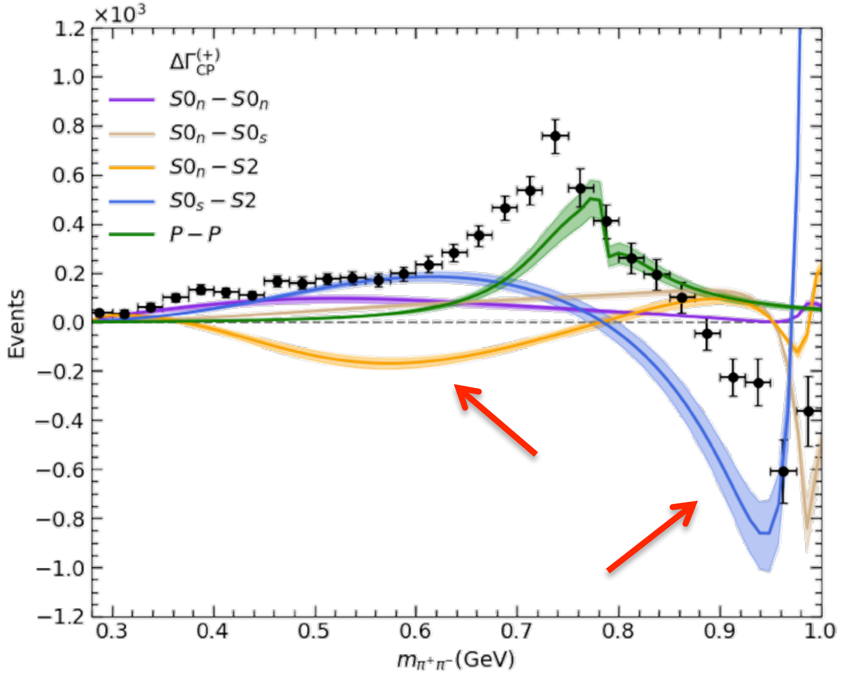
Individual contributions

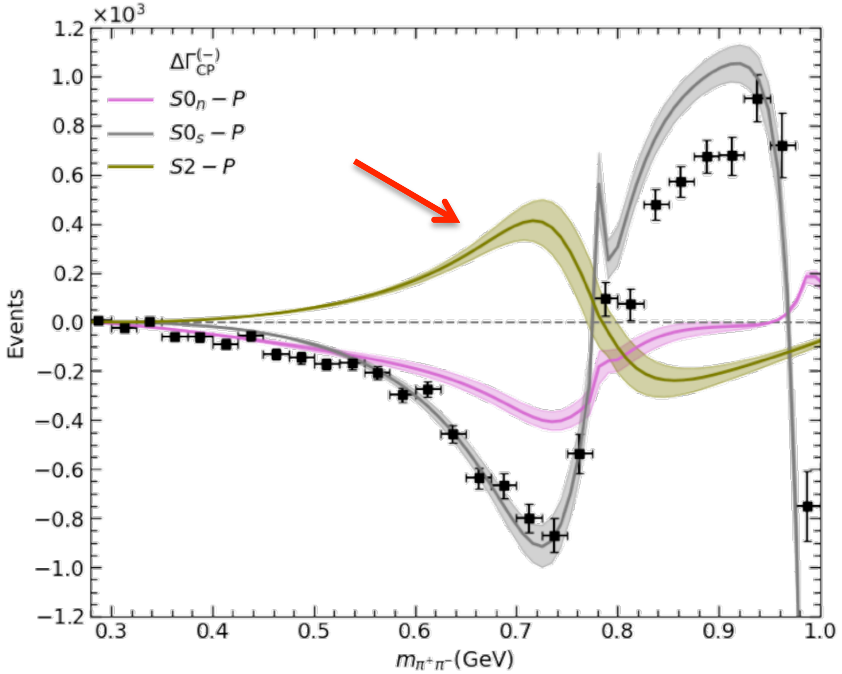


- P-P is only possible if we consider the phase from the $c\bar{c}$ loop

RESULTS

Individual contributions

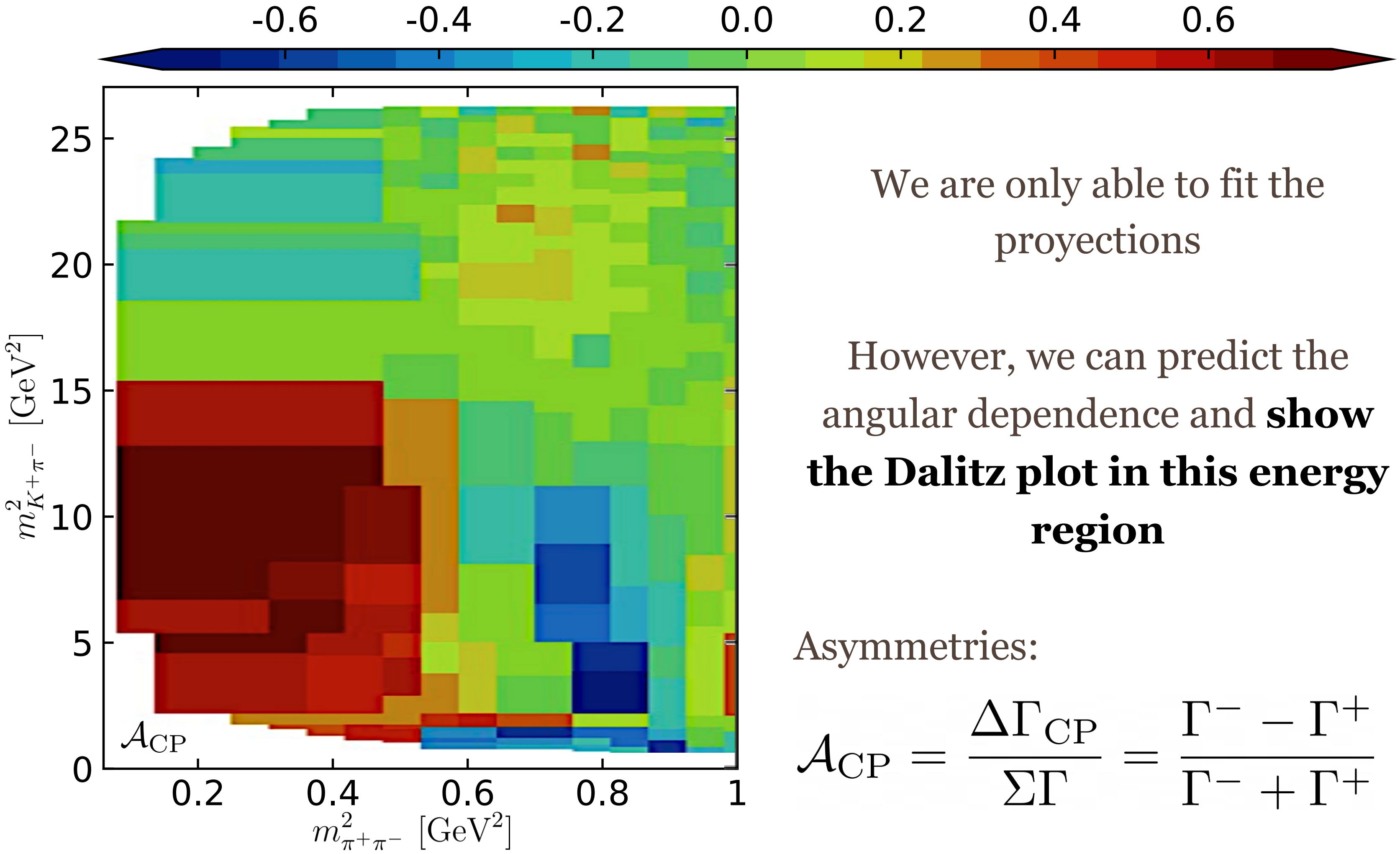




- P-P is only possible if we consider the phase from the $c\bar{c}$ loop
- **Crucial** role of isospin 2 to reproduce the data in this regime

RESULTS

Giant localized CPV



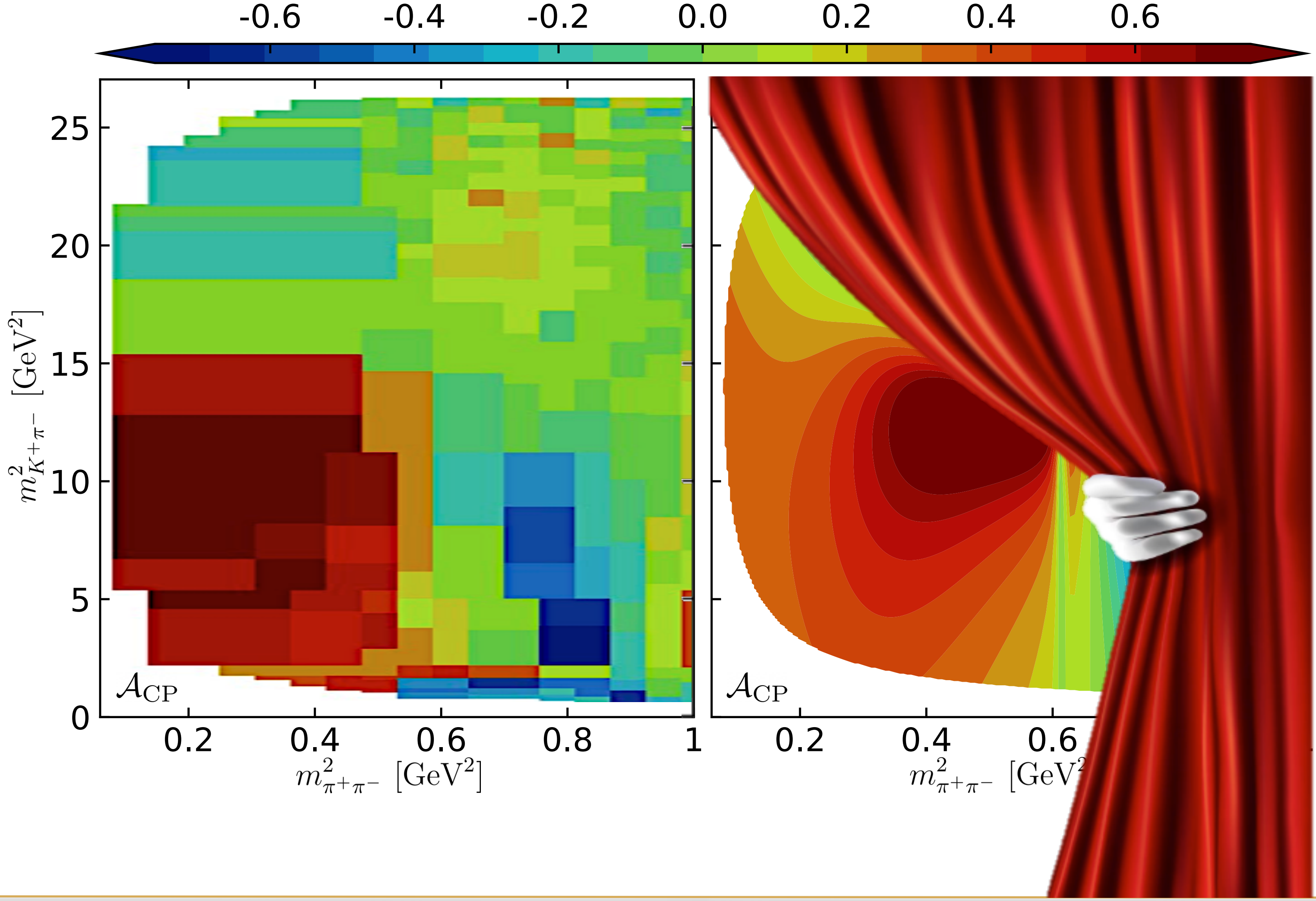
We are only able to fit the proyections

However, we can predict the angular dependence and **show the Dalitz plot in this energy region**

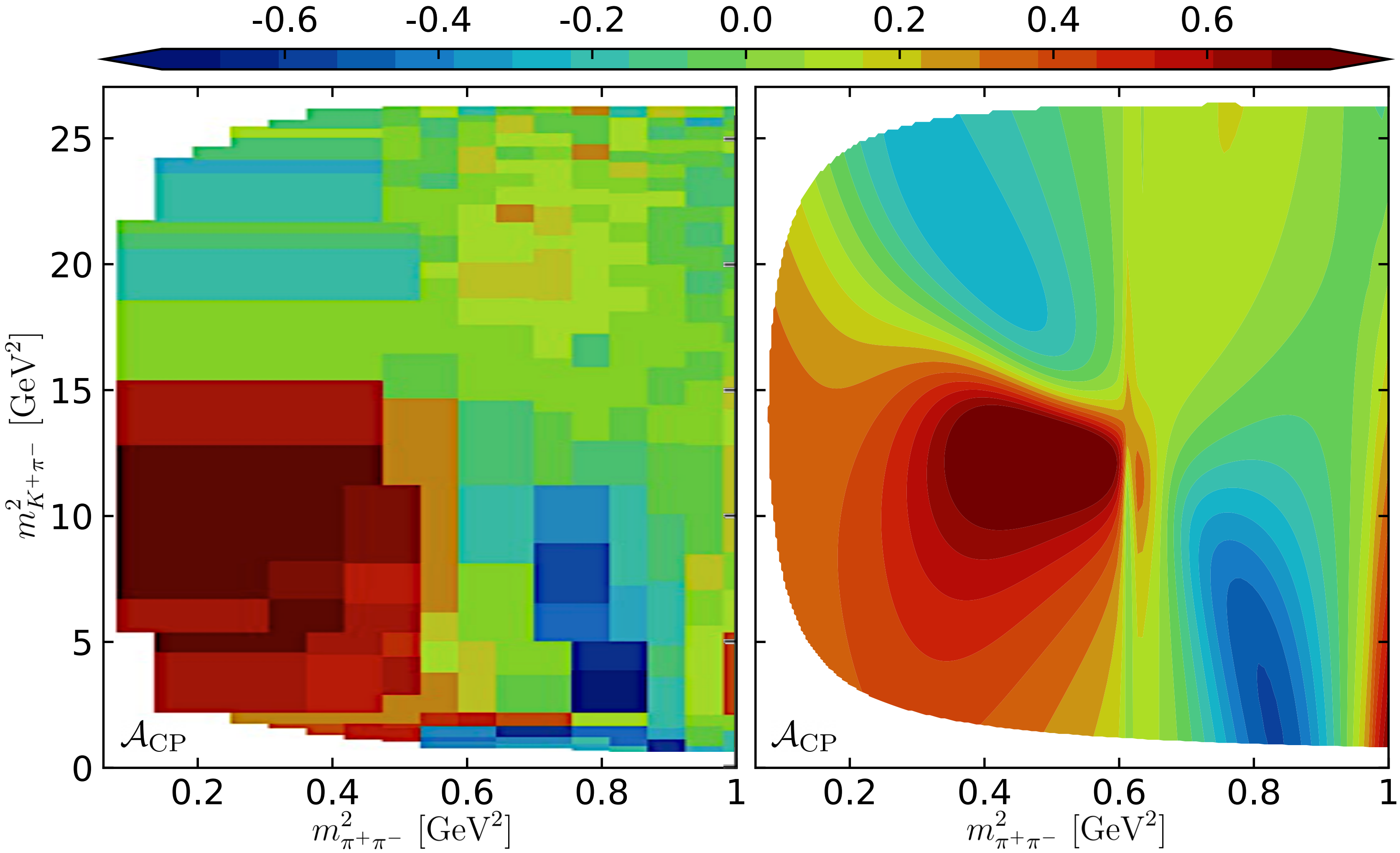
Asymmetries:

$$\mathcal{A}_{\rm CP} = \frac{\Delta\Gamma_{\rm CP}}{\Sigma\Gamma} = \frac{\Gamma^- - \Gamma^+}{\Gamma^- + \Gamma^+}$$

Giant localized CPV



Giant localized CPV



CONCLUSIONS

- The LHCb has measured **the biggest CP violation observed to date** in B decays to three light mesons ⟶ We have developed a dispersive method **able to describe the hadronic FSI that enhance CP violation in that decays**

- The universal $\pi\pi$ scattering allows us to **include systematically every partial wave, even the non resonant waves** ⟶ The **isospin-2 S2 wave is found to be crucial** in this kinematic region to explain the data with accuracy. It is not included in other analyses

- **Our dispersive approach respects the analytic structure** of the light-meson production **and our parameters have physical meaning**

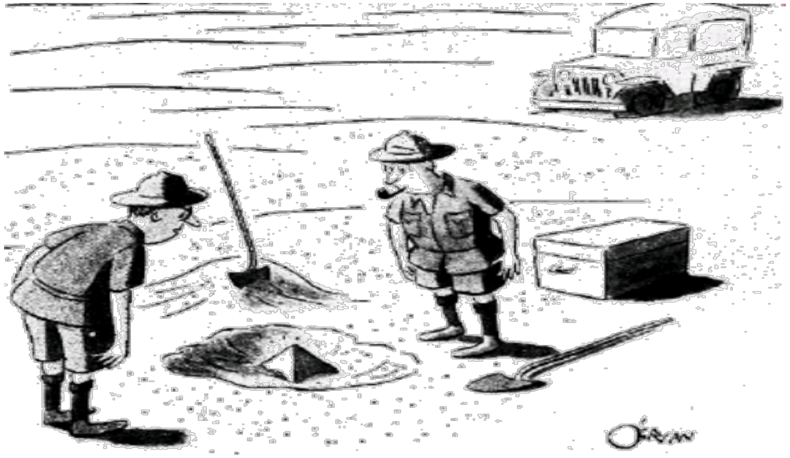
- We have been able to **reproduce the main features in the data sets** and provide a **prediction of the giant localized CP asymmetry** in $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$

THANK YOU!

HAPPY IPARCOS DAY!

WHAT IS CP VIOLATION?

○ Matter-antimatter asymmetry in the universe ⟶ The three necessary Sakharov conditions:



- Baryon number violation
- **CP violation**
- Out of thermal equilibrium interactions (phase transition)

Primitive Universe	Today Universe
$\eta = \frac{n_B}{n_\gamma} = \frac{n_{\bar{B}}}{n_\gamma} \approx 10^{-19}$	$\eta \approx 10^{-9} >> \frac{n_{\bar{B}}}{n_\gamma}$

→ **The CP violation magnitude found to date is not enough to explain the cosmological matter-antimatter asymmetry**

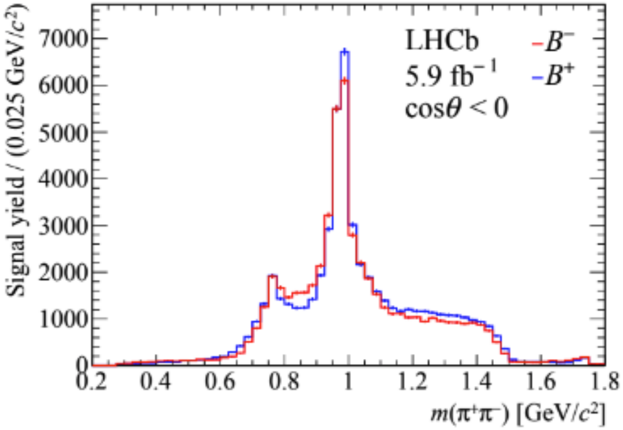
LHCb data

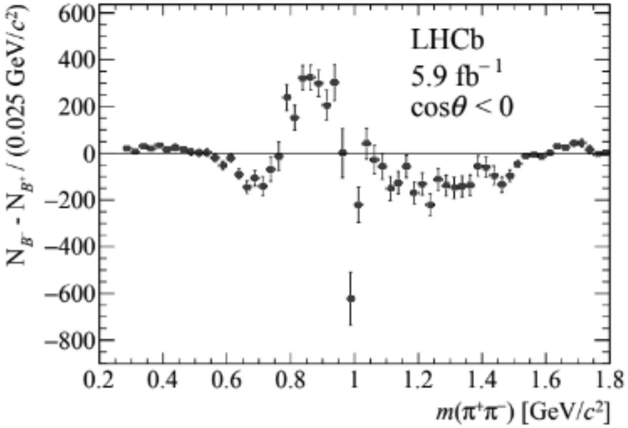
[R. Aaij et al. (LHCb), Phys. Rev. D 108 (2023)]

These are the available data sets:

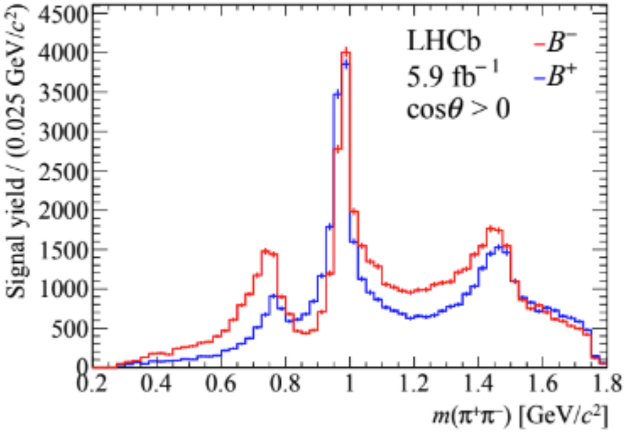
Yields:

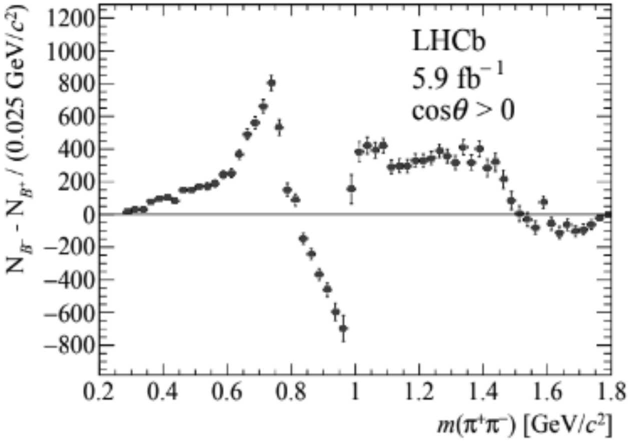
Differences:





Backward
$\cos\theta < 0$





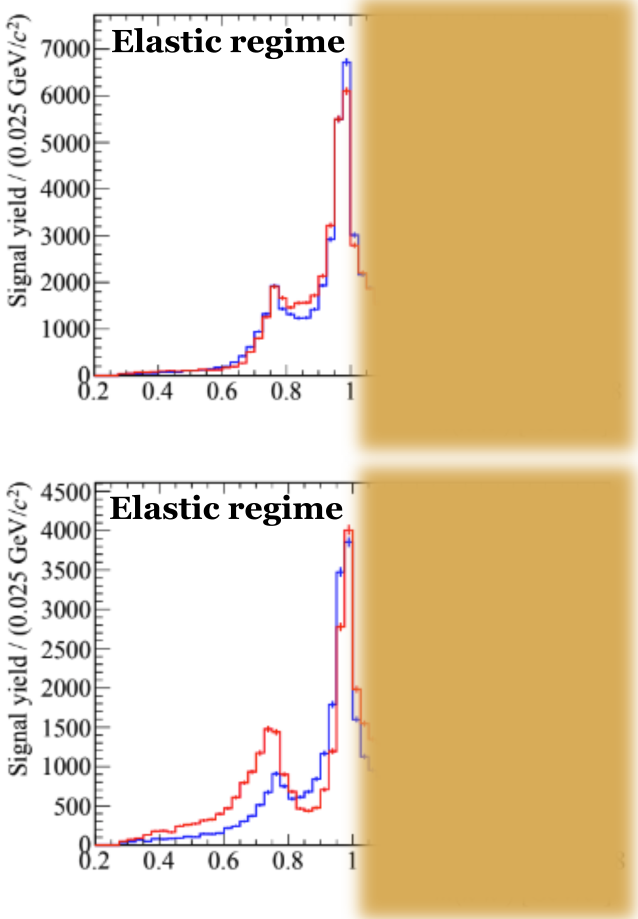
Forward
$\cos\theta > 0$

LHCb data

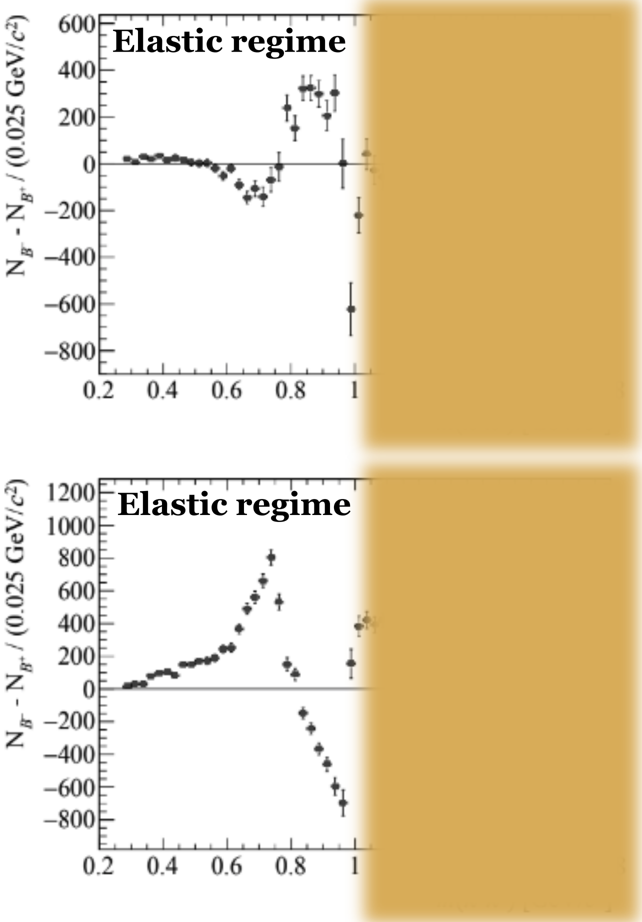
[R. Aaij et al. (LHCb), Phys. Rev. D 108 (2023)]

But we want to describe the elastic regime:

Yields:



Differences:



Backward
$\cos\theta < 0$

Forward
$\cos\theta > 0$

Notation

$$\mathcal{A}^{\pm}(s,t) = \sum_i f_i(s,t)\mathcal{A}_i^{\pm}(s)$$

angular dependence

- In this kinematic range we are dealing with $LI = S0, S2, P1$ waves

$$f_i(s,t) = \begin{cases} 1 & \text{for } i \in \{S0, S2\} \\ g(s)\cos\theta & \text{for } i = P1 \end{cases}$$

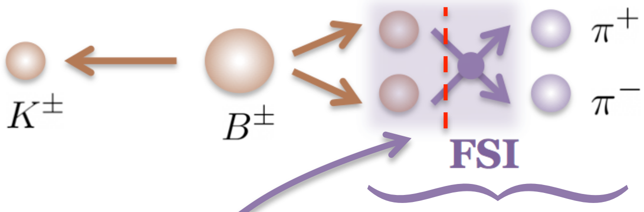
$$g(s) \;=\; -\sigma_\pi(s)\lambda^{1/2}(s, M_K^2, M_B^2)$$

$$\sigma_h(s) \;=\; \sqrt{1 - 4M_h^2/s}$$

$$\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$$

Dispersive method through Omnès functions

$$\mathcal{A}^{\pm}(s,t) = \sum_i f_i(s,t)\boxed{\mathcal{A}_i^{\pm}(s)}$$



- Discontinuity relation for the production partial-wave amplitudes along $\pi\pi$ threshold

$$\underline{\text{Disc}\,\mathcal{A}_i^{\pm}(s)} = 2i\boxed{\rho_\pi(s)\mathcal{M}_i(s)^*}\mathcal{A}_i^{\pm}(s)$$

Elastic !

UNIVERSAL

$\pi\pi$ scattering partial-wave amplitude

[R. García-Martín, R. Kaminski, J. R. Peláez, J. Ruiz de Elvira, and F. J. Ynduráin, Phys. Rev. D 83 (2011)]

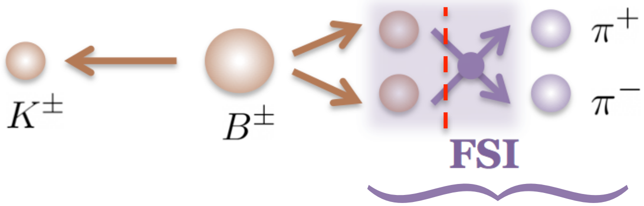
$$\rho_h(s) = \sigma_h(s)/16\pi$$

$$\sigma_h(s) = \sqrt{1 - 4M_h^2/s}$$

2. FORMALISM FOR THE FSI

Dispersive method through Omnès functions

$$\mathcal{A}^{\pm}(s,t) = \sum_i f_i(s,t)\boxed{\mathcal{A}_i^{\pm}(s)}$$



- Discontinuity relation for the production partial-wave amplitudes along $\pi\pi$ threshold

$$\underline{\text{Disc}\,\mathcal{A}_i^{\pm}(s)} = 2i\rho_{\pi}(s)\mathcal{M}_i(s)^{*}\mathcal{A}_i^{\pm}(s)$$

Elastic !

- We refer to this quantity as the discontinuity of the amplitude rather than simply its imaginary part because, in the context of dispersion relations, what enters the formalism is not the imaginary part per se, but the discontinuity across the cut in the complex plane.

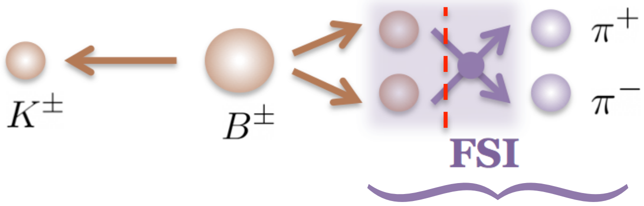
Why Discontinuity and not Imaginary part?

- The phase of our amplitude doesn’t come only from meson-meson scattering, which would generate a clean imaginary part via unitarity

2. FORMALISM FOR THE FSI

Dispersive method through Omnès functions

$$\mathcal{A}^{\pm}(s,t) = \sum_i f_i(s,t)\boxed{\mathcal{A}_i^{\pm}(s)}$$



- Our amplitude includes strong phases from $\pi\pi$ scattering, CP violation weak phase and potentially strong phases from intermediate $D\bar{D}$ hadronization

- What we are computing is not just the imaginary part due to rescattering, but the net discontinuity across the cut in the complex plane

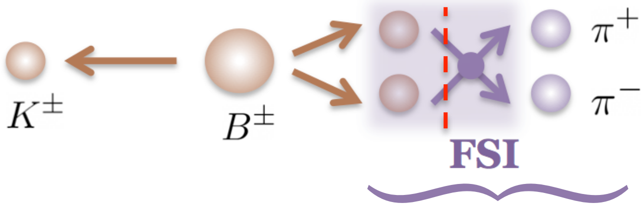
$$\mathrm{Disc}\mathcal{A}(s) = \mathcal{A}(s+i\epsilon) - \mathcal{A}(s-i\epsilon)$$

- If the amplitudes were analytic and only had meson-meson scattering contributions, it would reduce to

$$\mathrm{Disc}\mathcal{A}(s) = 2i\mathrm{Im}\mathcal{A}(s)$$

Hints for solving the discontinuity relation. Omnès function

$$\mathcal{A}^{\pm}(s,t) = \sum_i f_i(s,t)\boxed{\mathcal{A}_i^{\pm}(s)}$$



$$\underline{\text{Disc}\,\mathcal{A}_i^{\pm}(s)} = 2i\rho_\pi(s)\mathcal{M}_i(s)^*\mathcal{A}_i^{\pm}(s)$$

- Partial wave elastic $\pi\pi$ scattering amplitude:

$$\mathcal{M}_i(s) = e^{i\delta_i(s)}\frac{\sin\delta_i(s)}{\rho_\pi(s)}$$

$$\underline{\text{Disc}\,\mathcal{A}_i^{\pm}(s)} = 2ie^{-i\delta_i(s)}\sin\delta_i(s)\mathcal{A}_i^{\pm}(s)$$

$$\boxed{\text{Disc}\,\mathcal{A}_i^{\pm}(s) = \mathcal{A}_i^{\pm}(s+i\epsilon) - \mathcal{A}_i^{\pm}(s-i\epsilon)}$$

$$\begin{aligned}\mathcal{A}_i^{\pm}(s-i\epsilon) &= \mathcal{A}_i^{\pm}(s+i\epsilon)\left(1-2ie^{-i\delta_i(s)}\sin\delta_i(s)\right)\\ &= \mathcal{A}_i^{\pm}(s+i\epsilon)e^{-2i\delta_i(s)}\end{aligned}$$

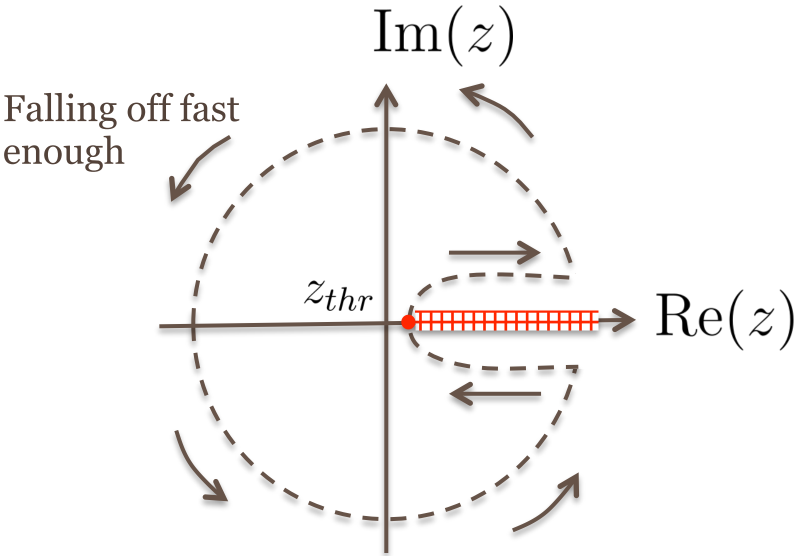
2. FORMALISM FOR THE FSI

Hints for solving the discontinuity relation. Omnès function

$$\mathcal{A}_i^{\pm}(s-i\epsilon)=\mathcal{A}_i^{\pm}(s+i\epsilon)e^{-2i\delta_i(s)}$$

$$\left\{\begin{aligned}&\ln\frac{\mathcal{A}_i^{\pm}(s-i\epsilon)}{\mathcal{A}_i^{\pm}(s+i\epsilon)}=-2i\delta_i(s) \qquad\qquad \text{Disc }\ln\mathcal{A}_i^{\pm}(s)=2i\delta_i(s)\\ &\ln\frac{\mathcal{A}_i^{\pm}(s-i\epsilon)}{\mathcal{A}_i^{\pm}(s+i\epsilon)}=\ln\mathcal{A}_i^{\pm}(s-i\epsilon)-\ln\mathcal{A}_i^{\pm}(s+i\epsilon)=-\text{Disc}\ln\mathcal{A}_i^{\pm}(s)\end{aligned}\right.$$

- For every holomorphic function:



Falling off fast enough

$$f(s+i\epsilon)=\frac{1}{2\pi i}\int_{z_{thr}}^{\infty}\mathrm{d}z\,\frac{\text{Disc }f(z)}{z-s-i\epsilon}$$

$$f\rightarrow\text{Disc }\ln\mathcal{A}_i^{\pm}(s)$$

2. FORMALISM FOR THE FSI

Hints for solving the discontinuity relation. Omnès function

$$\begin{aligned}\ln \mathcal{A}_i^{\pm}(s+i\epsilon) &= \frac{1}{2\pi i}\int_{z_{thr}}^{\infty} \mathrm{d}z \frac{\text{Disc }\ln \mathcal{A}_i^{\pm}(z)}{z-s-i\epsilon} \\ &= \frac{1}{2\pi i}\int_{z_{thr}}^{\infty} \mathrm{d}z \frac{2i\delta_i(z)}{z-s-i\epsilon}\end{aligned}$$

$$\mathcal{A}_i^{\pm}(0) = 0$$

$$s_0 = 0$$

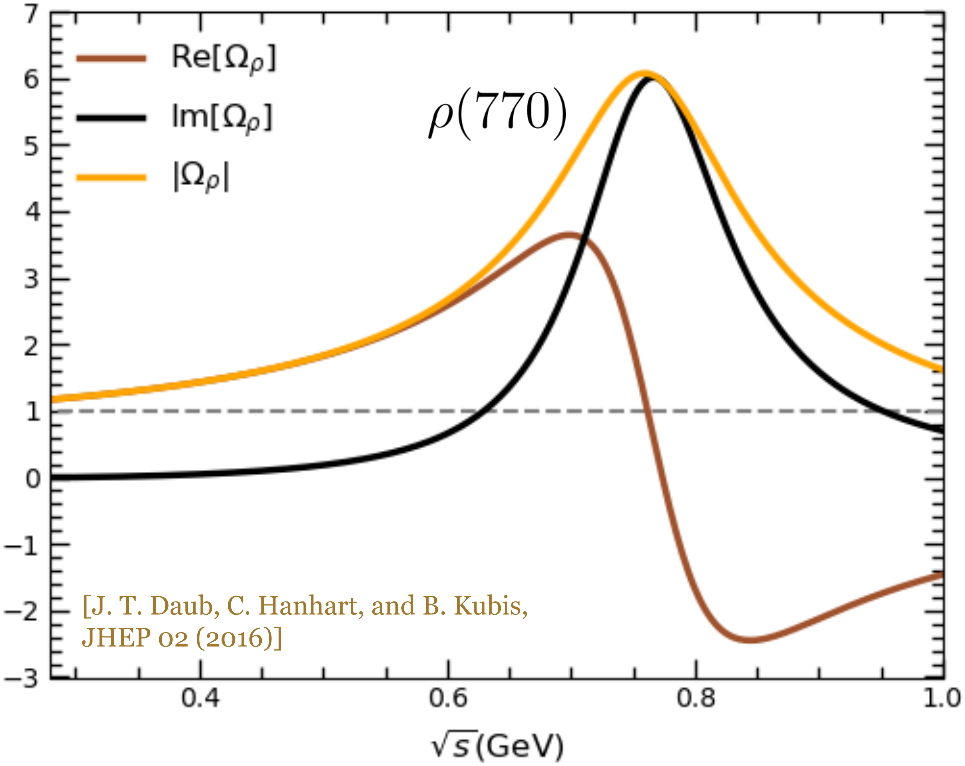
- One subtraction to help convergence

$$\ln \mathcal{A}_i^{\pm}(s+i\epsilon) = \frac{s}{\pi}\int_{z_{thr}}^{\infty} \mathrm{d}z \frac{\delta_i(z)}{x(z-s-i\epsilon)}$$

$$\mathcal{A}_i^{\pm}(s) = \exp\left\{\frac{s}{\pi}\int_{z_{thr}}^{\infty} \mathrm{d}z \frac{\delta_i(z)}{x(z-s-i\epsilon)}\right\}$$

$$\Omega_i(s)$$

Omnès functions: P1 channel



[J. T. Daub, C. Hanhart, and B. Kubis, JHEP 02 (2016)]

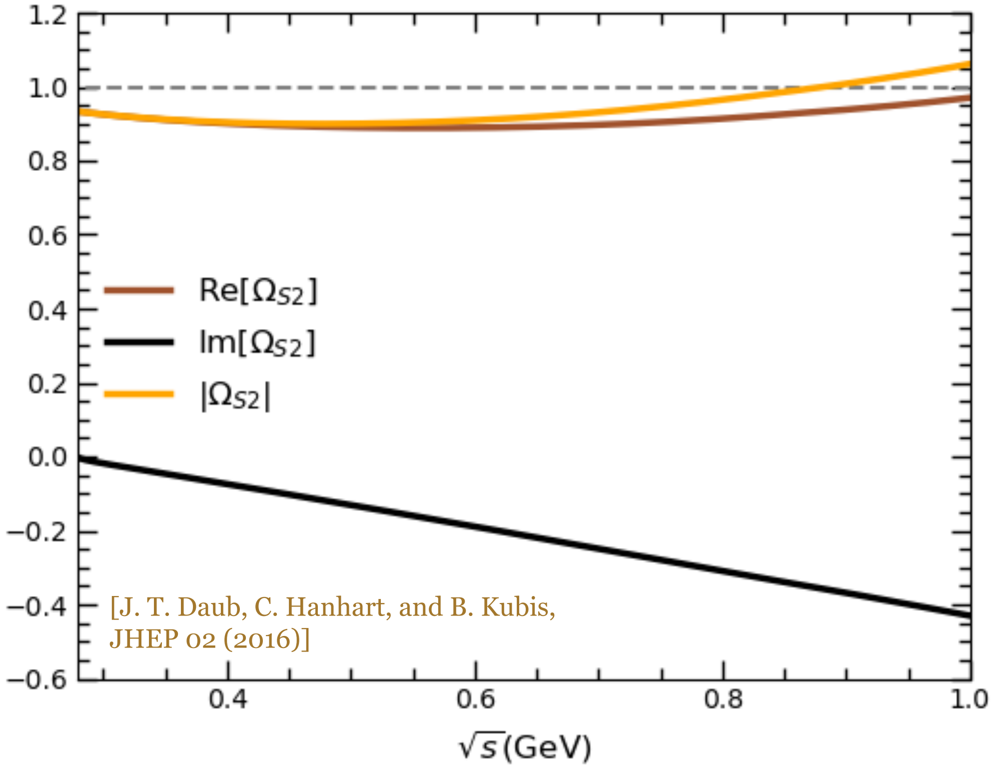
- Nothing to fit here (model independent)
- Correct pole positions and analytic structure

[K. M. Watson, Phys. Rev. 95 (1954).]

- Correct threshold behavior
- Compatible with Watson's theorem

- Allow us to disentangle the strong phases from the weak phases

Omnès functions: S2 channel



[J. T. Daub, C. Hanhart, and B. Kubis, JHEP 02 (2016)]

- Nothing to fit here (model independent)
- Correct pole positions and analytic structure

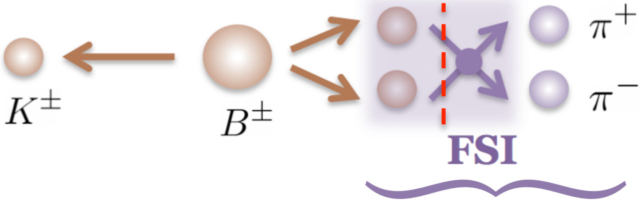
[K. M. Watson, Phys. Rev. 95 (1954).]

- Correct threshold behavior
- Compatible with Watson's theorem

- Allow us to disentangle the strong phases from the weak phases
- No Breit-Wigner description needed
- We can include non-resonant waves, like the S2 (not treated as a background)

Dispersive method through Omnès functions - SO wave, special case

$$\mathcal{A}^{\pm}(s,t) = \sum_i f_i(s,t)\boxed{\mathcal{A}_i^{\pm}(s)}$$



- The SO wave needs a special treatment

2. FORMALISM FOR THE FSI

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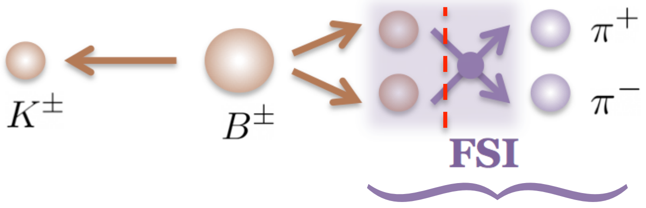

- The SO wave needs a special treatment

- The scalar $f_0(980)$ resonance lies almost at $K\bar{K}$ threshold and couples strongly to the two kaons → $\pi\pi - K\bar{K}$ **coupled-channel** formalism

$s\bar{s}$ source

$$\mathcal{A}^{\pm}_{S0} \longrightarrow \mathcal{A}^{\pm}_{S0n}, \mathcal{A}^{\pm}_{S0s}$$

$n\bar{n}$ source

$$\nearrow \ \text{Disc}\, \mathcal{A}^{\pm}_{S0n} = 2i\left(\mathcal{M}_{11}\rho_\pi \mathcal{A}^{\pm}_{S0n} + \mathcal{M}^{*}_{12}\rho_K \mathcal{B}^{\pm}_{S0n}\right)$$

$$\searrow \ \text{Disc}\, \mathcal{A}^{\pm}_{S0s} = 2i\left(\mathcal{M}_{11}\rho_\pi \mathcal{A}^{\pm}_{S0s} + \mathcal{M}^{*}_{12}\rho_K \mathcal{B}^{\pm}_{S0s}\right)$$

$K\bar{K}$ is virtual

$1 \equiv \pi\pi$

$2 \equiv K\bar{K}$

Dispersive method through Omnès functions - SO wave, special case

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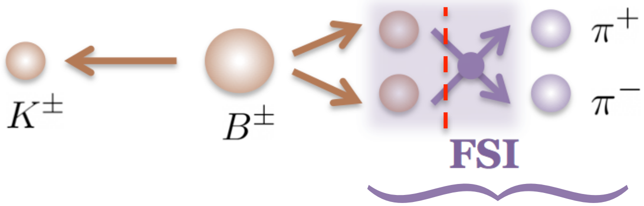

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$K\bar{K}$ is virtual

$\pi\pi \to \pi\pi, K\bar{K}$ scalar scattering partial-wave amplitude

[R. García-Martín, R. Kaminski, J. R. Peláez, J. Ruiz de Elvira, and F. J. Ynduráin, Phys. Rev. D 83 (2011)]

[J. R. Peláez and A. Rodas, Phys. Rept. 969 (2022)]

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2. FORMALISM FOR THE FSI

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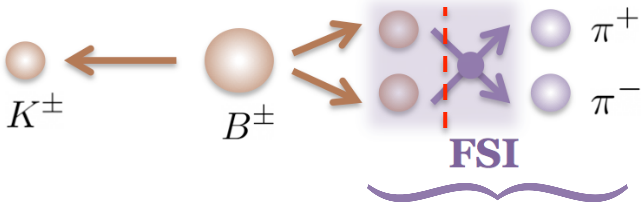

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$$\text{Disc}\,\mathcal{A}_{S0s}^{\pm} = 2i\left(\mathcal{M}_{11}\rho_{\pi}\mathcal{A}_{S0s}^{\pm} + \mathcal{M}_{12}^{*}\rho_{K}\mathcal{B}_{S0s}^{\pm}\right)$$

$K\bar{K}$ is virtual

- Numerical solution ⟶ Ω_{ij}

$$\Omega_{S0n} = \Omega_{11} + \frac{1}{2}\Omega_{12}$$

$$\Omega_{S0s} = \Omega_{12}$$

$1 \equiv \pi\pi$
$2 \equiv K\bar{K}$

[S. Ropertz, C. Hanhart, and B. Kubis, Eur. Phys. J. C 78 (2018)]

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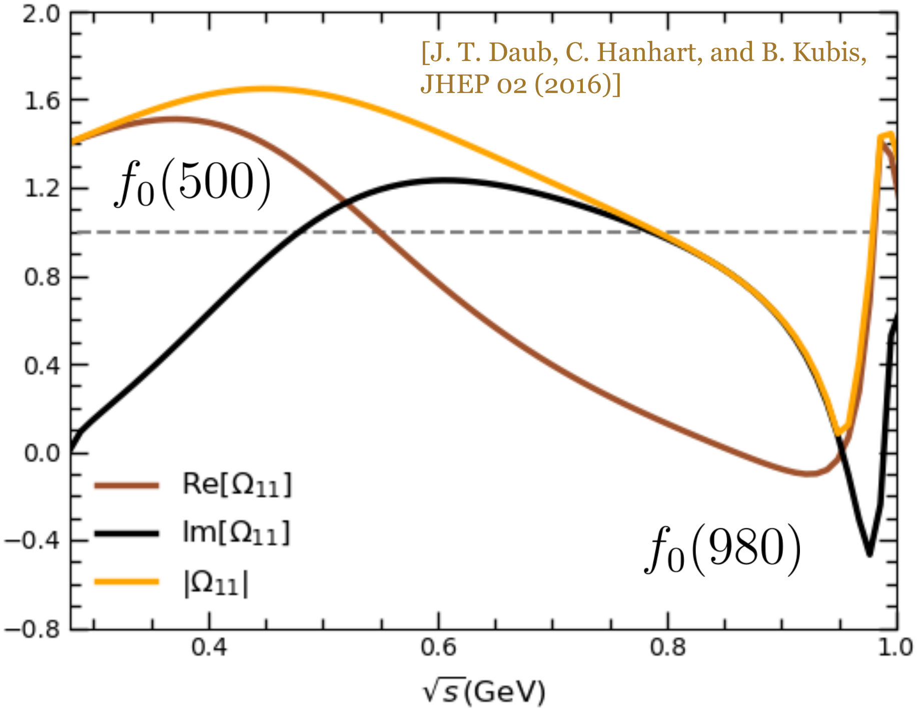
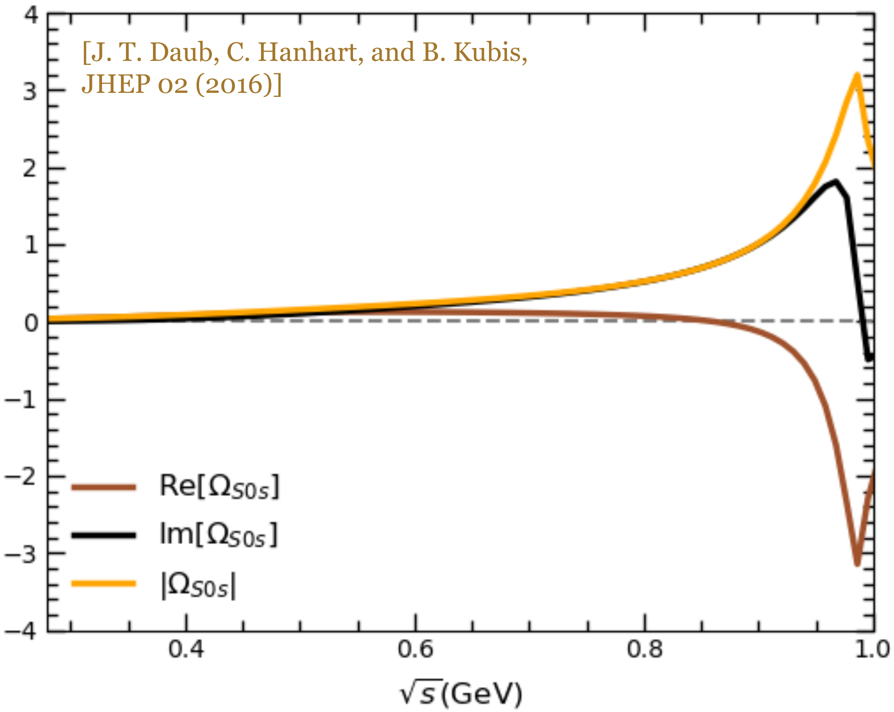
Omnès functions: SO channel



[J. T. Daub, C. Hanhart, and B. Kubis, JHEP 02 (2016)]



[J. T. Daub, C. Hanhart, and B. Kubis, JHEP 02 (2016)]

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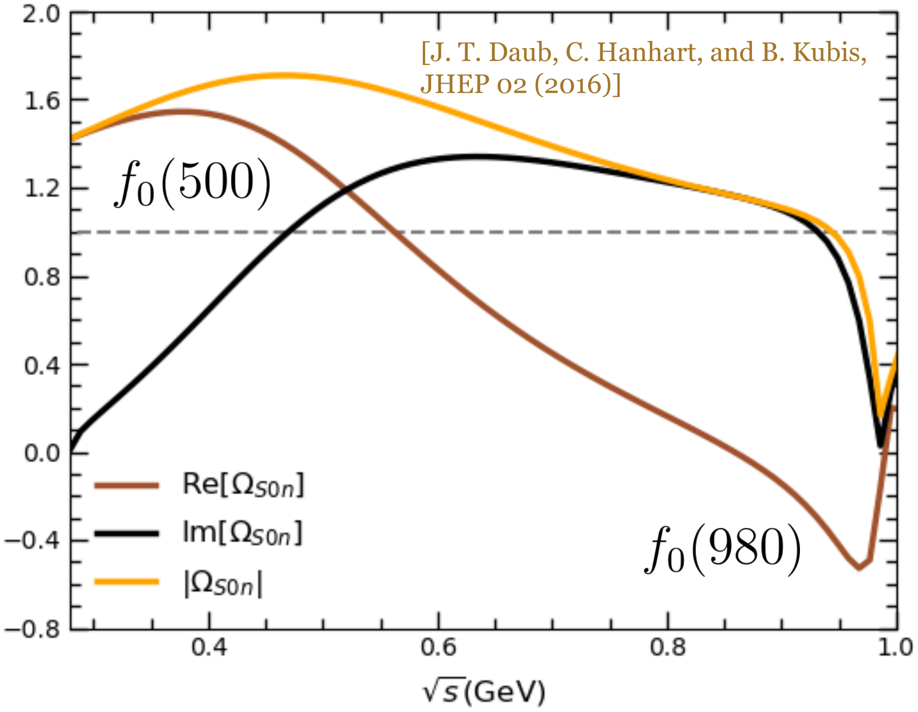
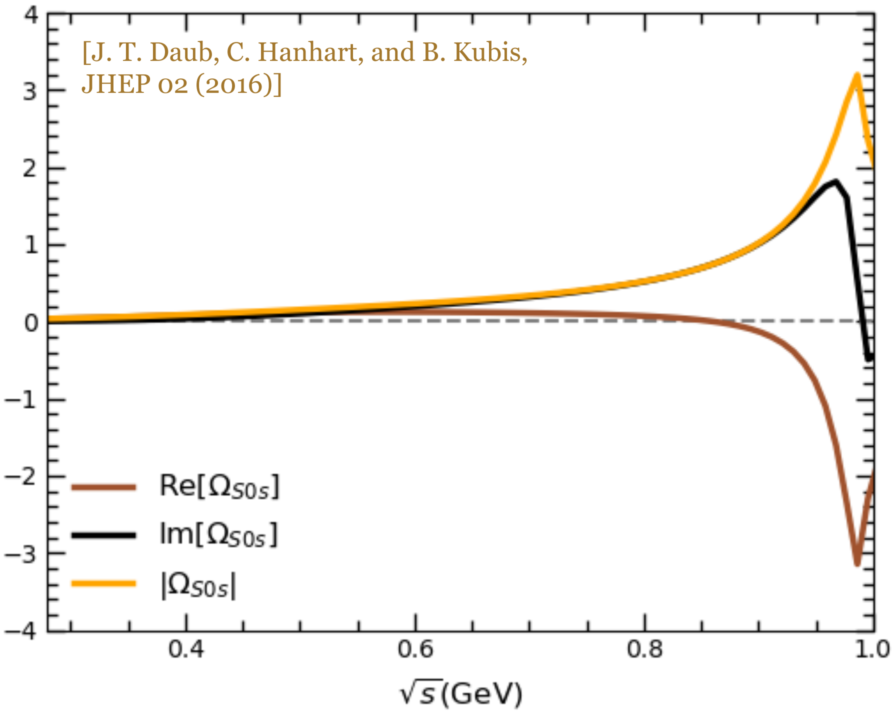


[J. T. Daub, C. Hanhart, and B. Kubis, JHEP 02 (2016)]



[J. T. Daub, C. Hanhart, and B. Kubis, JHEP 02 (2016)]

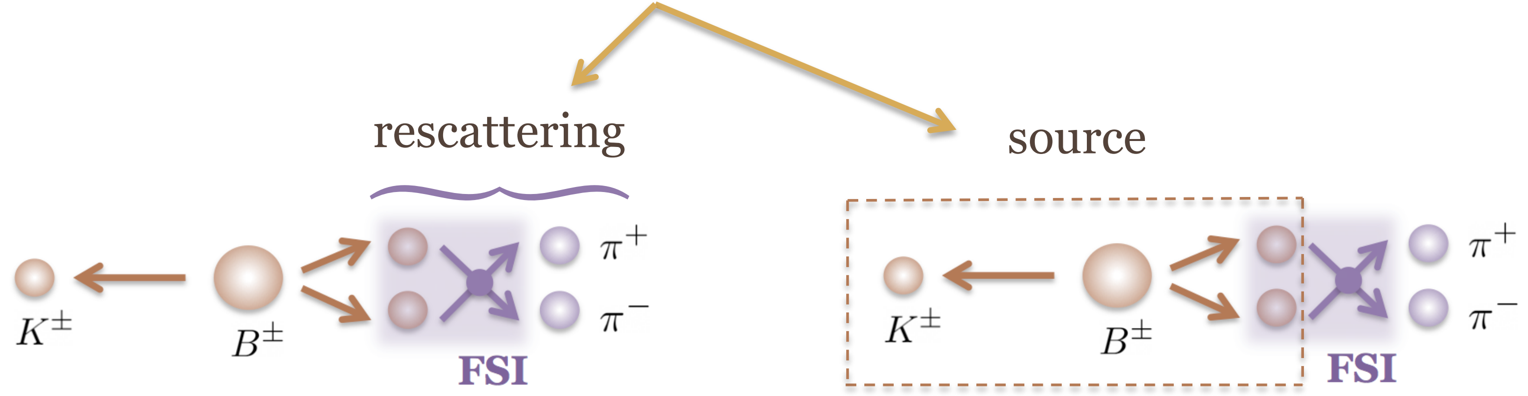
- No Breit-Wigner needed, $f_0(500)$ and $f_0(980)$ resonances are **not** Breit-Wigners

U.-G. Meißner and S. Gardner, Eur. Phys. J. A 18 (2003).

Dispersive method through Omnès functions

To **summarize** the method:

$$\mathcal{A}^{\pm}(s,t) = \sum_i f_i(s,t)\boxed{\mathcal{A}_i^{\pm}(s)}$$



Partial wave content

$$\mathcal{A}^{\pm}(s,t) = \sum_i f_i(s,t)\boxed{P_i(s)\Omega_i(s)\boxed{\bar{\mathcal{A}}_i^{\pm}}} \qquad i \in \{S0_n, S0_s, S2, P\}$$

Universal $\pi\pi$ FSI

Not fitted

2. FORMALISM FOR THE FSI

Omnès functions: P channel

- Our P wave will show the $\rho(770)$ resonance together with the $\omega(782)$ contribution
- The $\pi\pi$ P-wave spectrum can be significantly affected by $\rho-\omega$ interference in reactions that show strong production of ω
- The smallness of the violation of the isospin symmetry is compensated for by an enhancement of $\mathcal{O}(M_\omega/\Gamma_\omega)\approx 90$ through the narrow ω propagator

[J. T. Daub, C. Hanhart, and B. Kubis, JHEP 02 (2016)]

- The strength of the $\rho-\omega$ mixing can be obtained from the electromagnetic pion vector form factor if we know the production strength of the ω relative to the ρ from the flavor structure of the source ⟶ $\epsilon_{\rho\omega}=1.7\cdot 10^{-3}$
- Our pure $u\bar{u}$ favors the ω production by a factor of 3 compared to electromagnetic production

3 factor

$\rho-\omega$ mixing strength

ρ contribution

$$\left(1+\underbrace{\frac{3\epsilon_{\rho\omega}s}{M_\omega^2-s-iM_\omega\Gamma_\omega}}_{\omega\ \text{contribution}}\right)\Omega_P$$

[S. Holz, C. Hanhart, M. Hoferichter, and B. Kubis, Eur. Phys. J. C 82 (2022)]
[G. Colangelo, M. Hoferichter, B. Kubis, and P. Stoffer, JHEP 10 (2022)]
[J. M. Dias, T. Ji, X.-K. Dong, F.-K. Guo, C. Hanhart, U.-G. Meißner, Y. Zhang, and Z.-H. Zhang, Phys. Rev. D 111 (2025)]

Omnès functions: P channel

- Electromagnetic current:

$$J_\mu^{EM} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d$$

- Quark content:

$$\rho \to \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \qquad I = 1$$

$$\omega \to \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \qquad I = 0$$

$$J_\mu^{EM} = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) + \frac{1}{6}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d)$$

$$\frac{\mathcal{A}_\omega^{EM}}{\mathcal{A}_\rho^{EM}} = \frac{1}{3}$$

- Just looking at the flavor structure, both productions would be equal

$$\langle\rho|u\bar{u}\rangle = \frac{1}{\sqrt{2}}$$

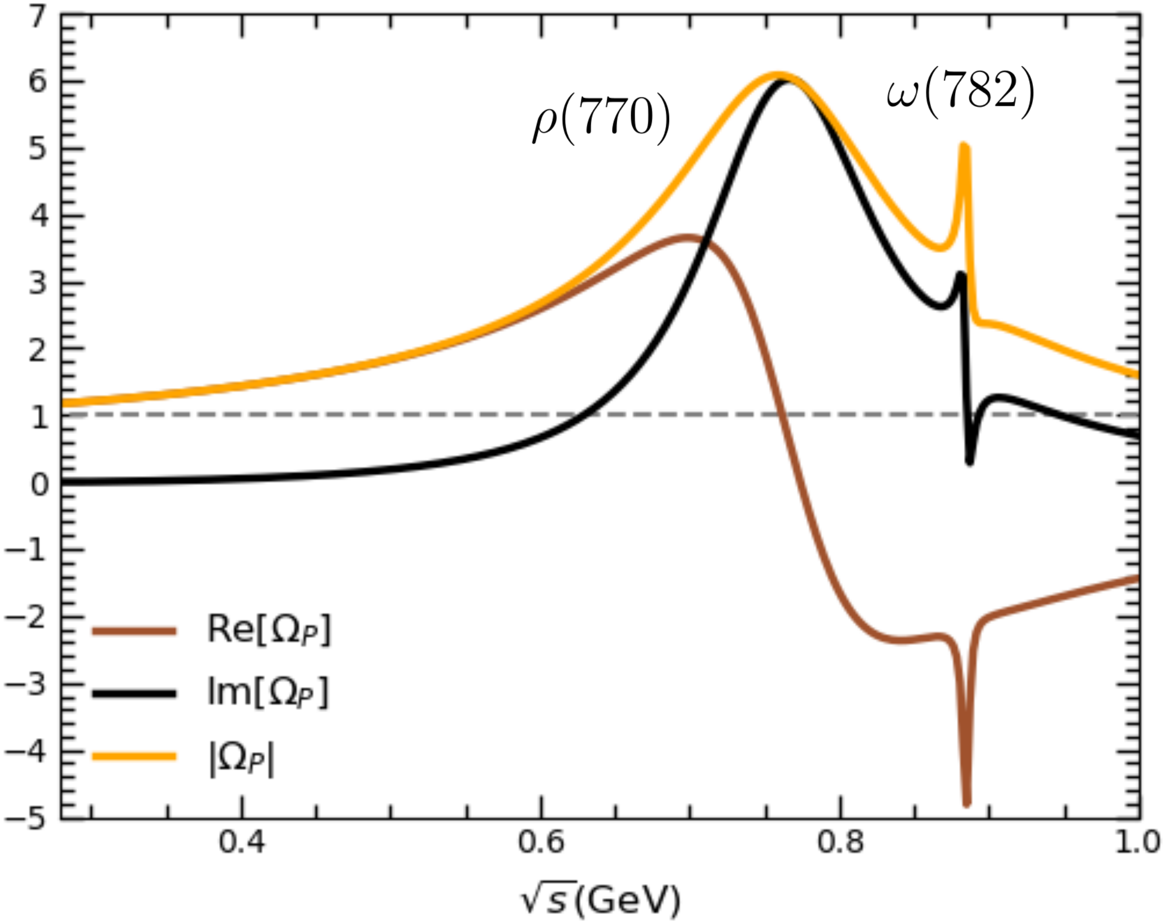
$$\langle\omega|u\bar{u}\rangle = \frac{1}{\sqrt{2}}$$

$$\frac{\mathcal{A}_\omega^{flav}}{\mathcal{A}_\rho^{flav}} = 1$$

$$\boxed{R^{flav} = 3R^{EM}}$$

2. FORMALISM FOR THE FSI

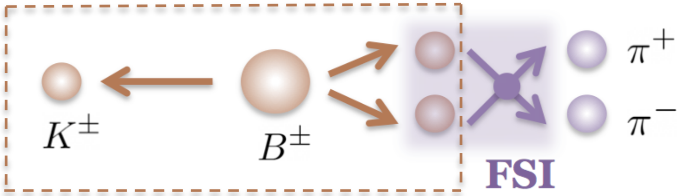
Omnès functions: P channel



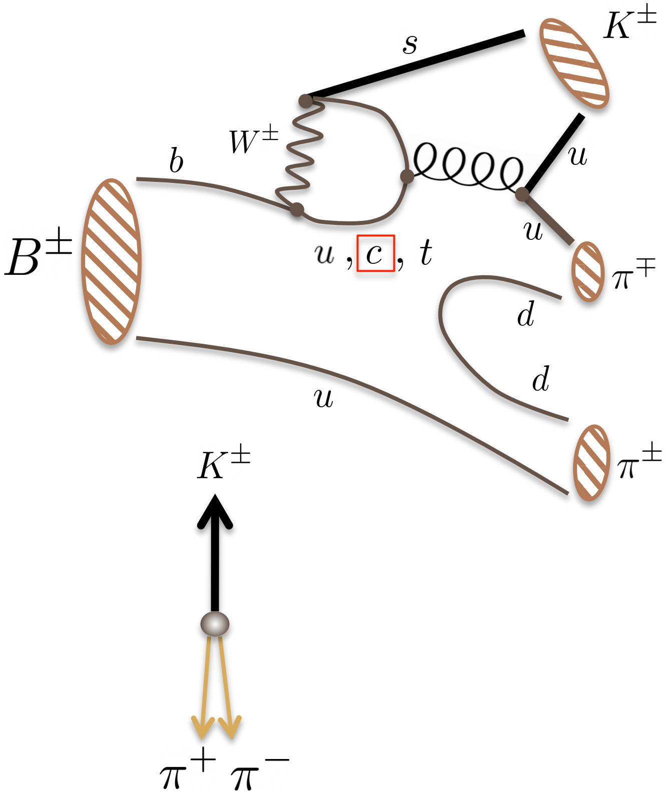
2. FORMALISM FOR THE FSI

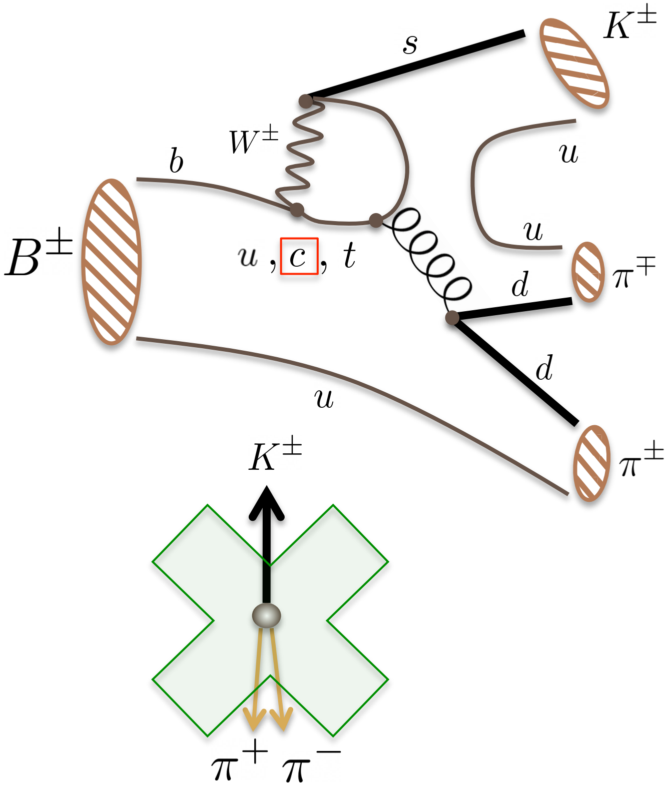
Source parameterization

$$\mathcal{A}^{\pm}(s,t) = \sum_i f_i(s,t)P_i(s)\Omega_i(s)\boxed{\bar{A}_i^{\pm}}$$



$\boxed{\sim \lambda^2}$





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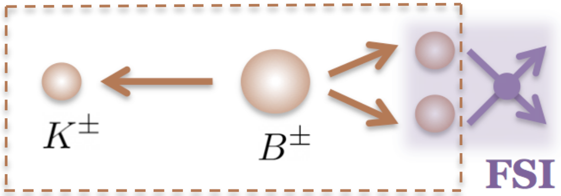

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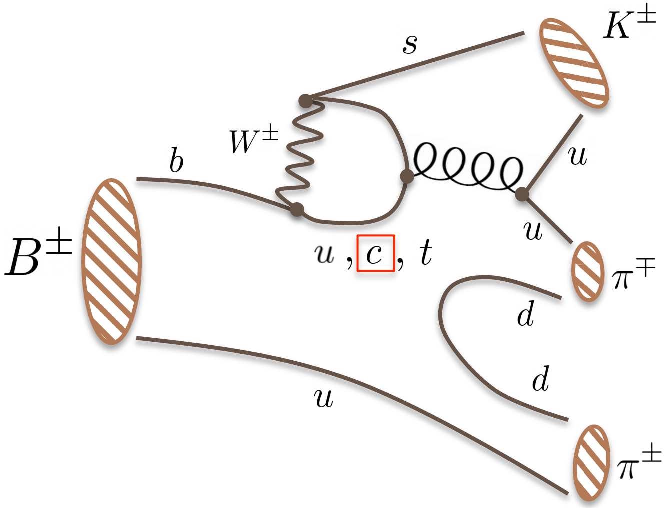

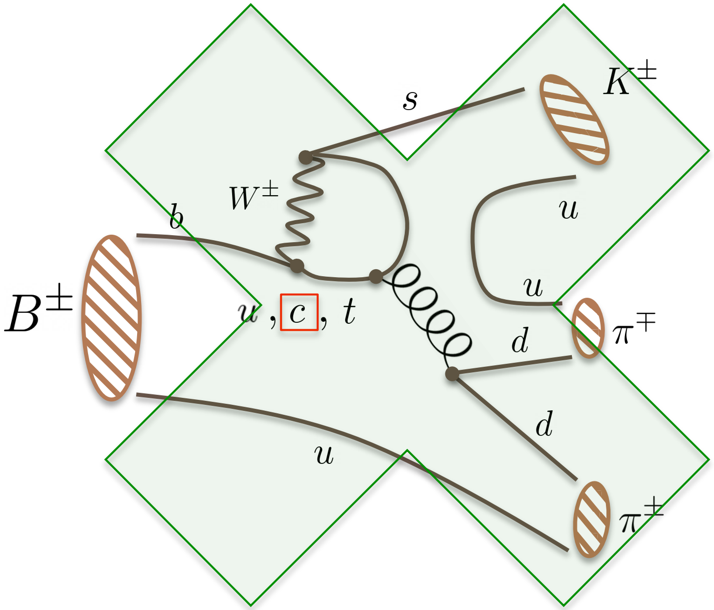

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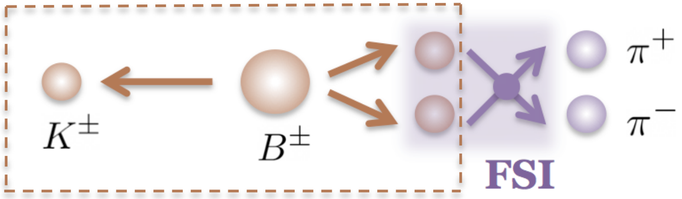

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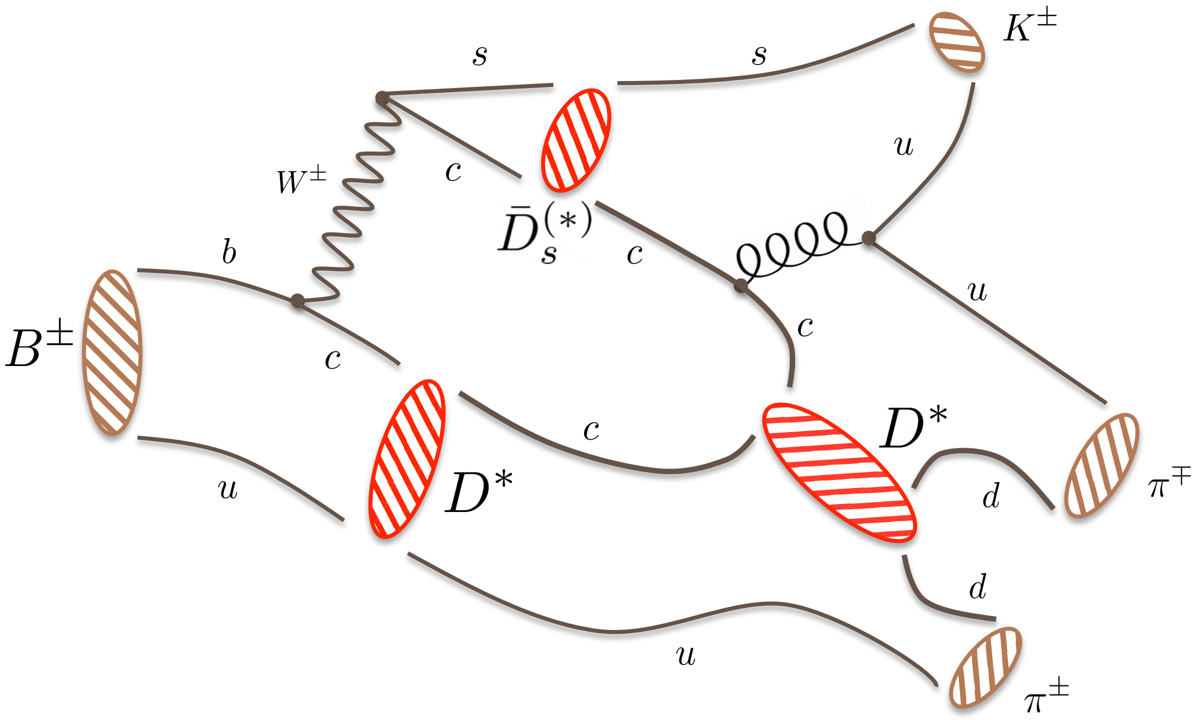

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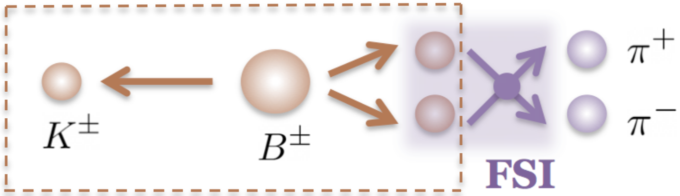

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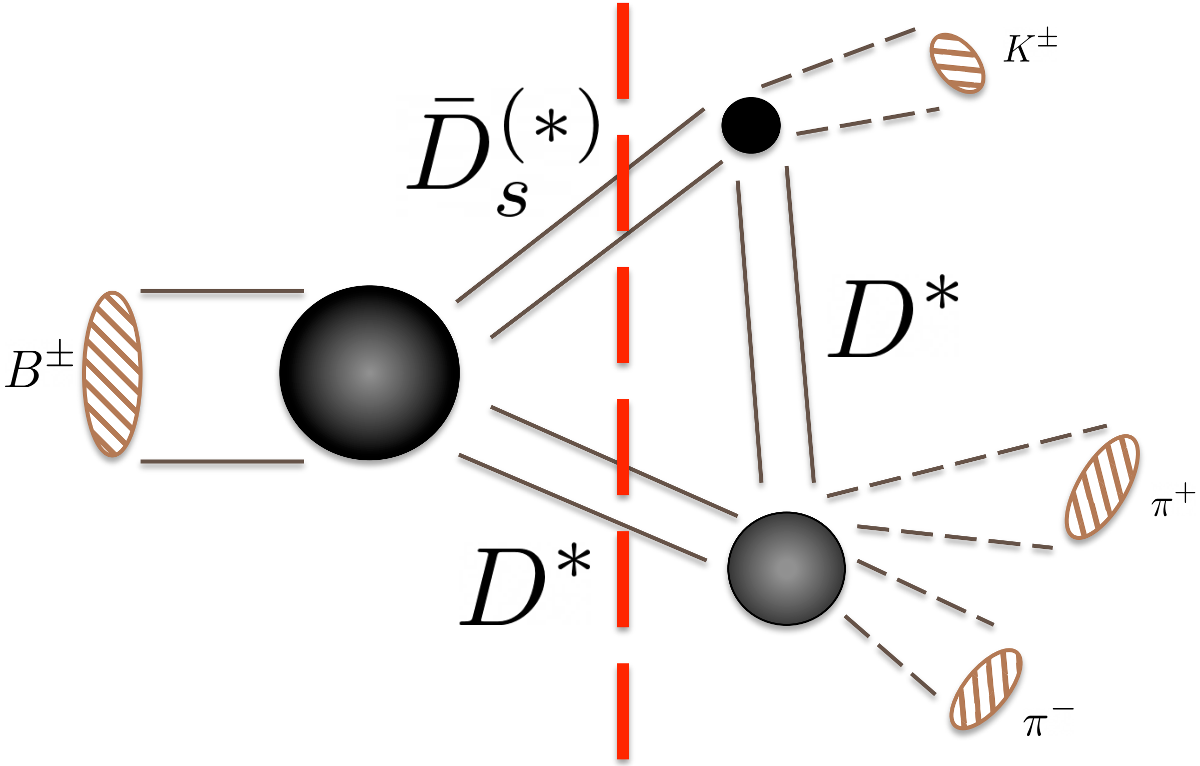

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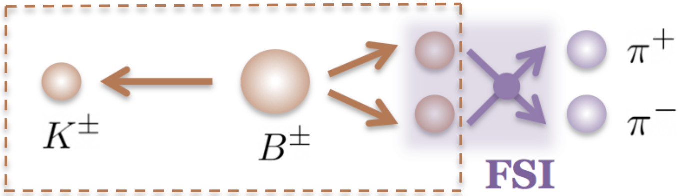
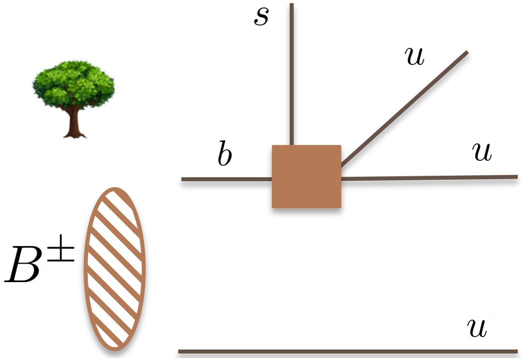
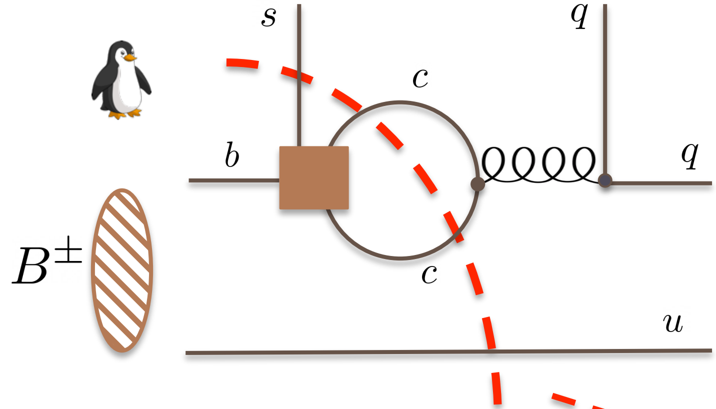




$u\bar{u}$ source



$q\bar{q}$ source

- Definitions:

$$\begin{aligned} a_i &= \mathrm{Re}\hat{A}_i + \hat{B}_i \cos\gamma \\ b_i &= \hat{B}_i \sin\gamma \\ c_i &= \mathrm{Im}\hat{A}_i \end{aligned}$$

- Therefore, we parameterize the source as:

$$\begin{aligned} \bar{\mathcal{A}}_i^{\pm} &= \hat{A}_i + e^{\pm i\gamma}\hat{B}_i \\ &= a_i + ic_i \pm ib_i \end{aligned}$$

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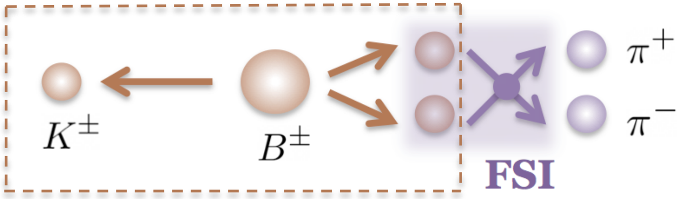

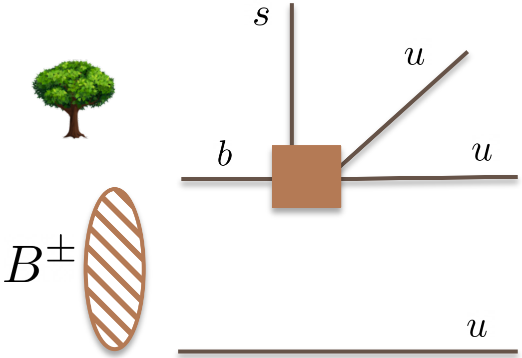

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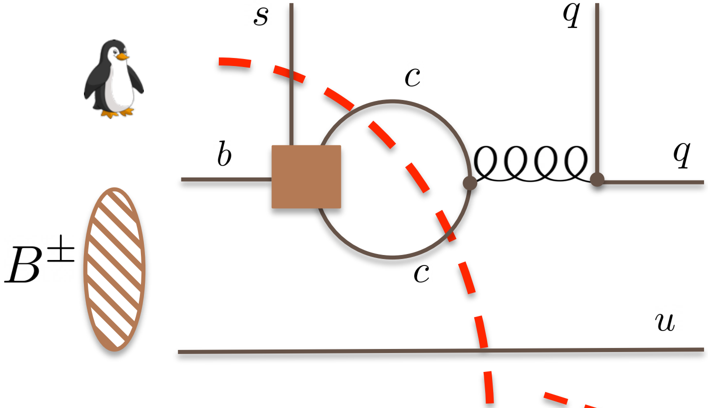

$q\bar{q}$ source

- Our parameters have **physical meaning**

$\cancel{c_{S2}}$ No isospin 2 from $q\bar{q}$ source

$\cancel{b_{S0s}}$ No CPV phase from $s\bar{s}$ source

- Therefore, we parameterize the source as:

$$\begin{aligned}\bar{\mathcal{A}}_i^{\pm}&=\hat{A}_i+e^{\pm i\gamma}\hat{B}_i\\&=a_i+ic_i\pm ib_i\end{aligned}$$

3. RESULTS

- Our expression for the CP-differences

$$\Delta\Gamma_{\mathrm{CP}}(s,\cos\theta) = \Gamma^{-}(s,\cos\theta) - \Gamma^{+}(s,\cos\theta) =$$

$$= 4g(s)\sqrt{s}\sum_{i,j} f_i f_j P_i P_j |\Omega_i||\Omega_j| \Big[a_i b_j \sin(\delta_i - \delta_j) + c_i b_j \cos(\delta_i - \delta_j)\Big]$$

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- LHCb data are projections with $\cos\theta$ integrated

$$\cos\theta > 0 \qquad \cos\theta < 0$$

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- LHCb data are projections with $\cos\theta$ integrated

$$\begin{tikzcd}
\cos\theta > 0 \arrow[d] & \cos\theta < 0 \arrow[d] \\
\Delta\Gamma^{>}_{\rm CP}(s) & \Delta\Gamma^{<}_{\rm CP}(s)
\end{tikzcd}$$

3. RESULTS

- Our expression for the CP-differences

$$\Delta\Gamma_{\rm CP}(s, \cos\theta) = \Gamma^{-}(s, \cos\theta) - \Gamma^{+}(s, \cos\theta) =$$

$$= 4g(s)\sqrt{s}\sum_{i,j} f_i f_j P_i P_j |\Omega_i||\Omega_j| \Big[a_i b_j \sin(\delta_i - \delta_j) + c_i b_j \cos(\delta_i - \delta_j)\Big]$$

- LHCb data are projections with $\cos\theta$ integrated

$$\begin{tikzcd}
\cos\theta > 0 \arrow[d] \arrow[dd, bend right] & \cos\theta < 0 \arrow[d] \arrow[dd, bend left] \\
\Delta\Gamma^{>}_{\rm CP}(s) & \Delta\Gamma^{<}_{\rm CP}(s) \\
\Sigma\Gamma^{>}(s) & \Sigma\Gamma^{<}(s)
\end{tikzcd}$$

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- LHCb data are projections with $\cos\theta$ integrated

$$\begin{aligned}\Delta\Gamma^{(\pm)}_{\mathrm{CP}}(s) &= \overset{\cos\theta>0}{\Delta\Gamma^{>}_{\mathrm{CP}}}(s) \pm \overset{\cos\theta<0}{\Delta\Gamma^{<}_{\mathrm{CP}}}(s)\\ \Sigma\Gamma^{(\pm)}(s) &= \Sigma\Gamma^{>}(s) \pm \Sigma\Gamma^{<}(s)\end{aligned}$$

3. RESULTS

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- LHCb data are projections with $\cos\theta$ integrated

$$\cos\theta > 0 \qquad \cos\theta < 0$$

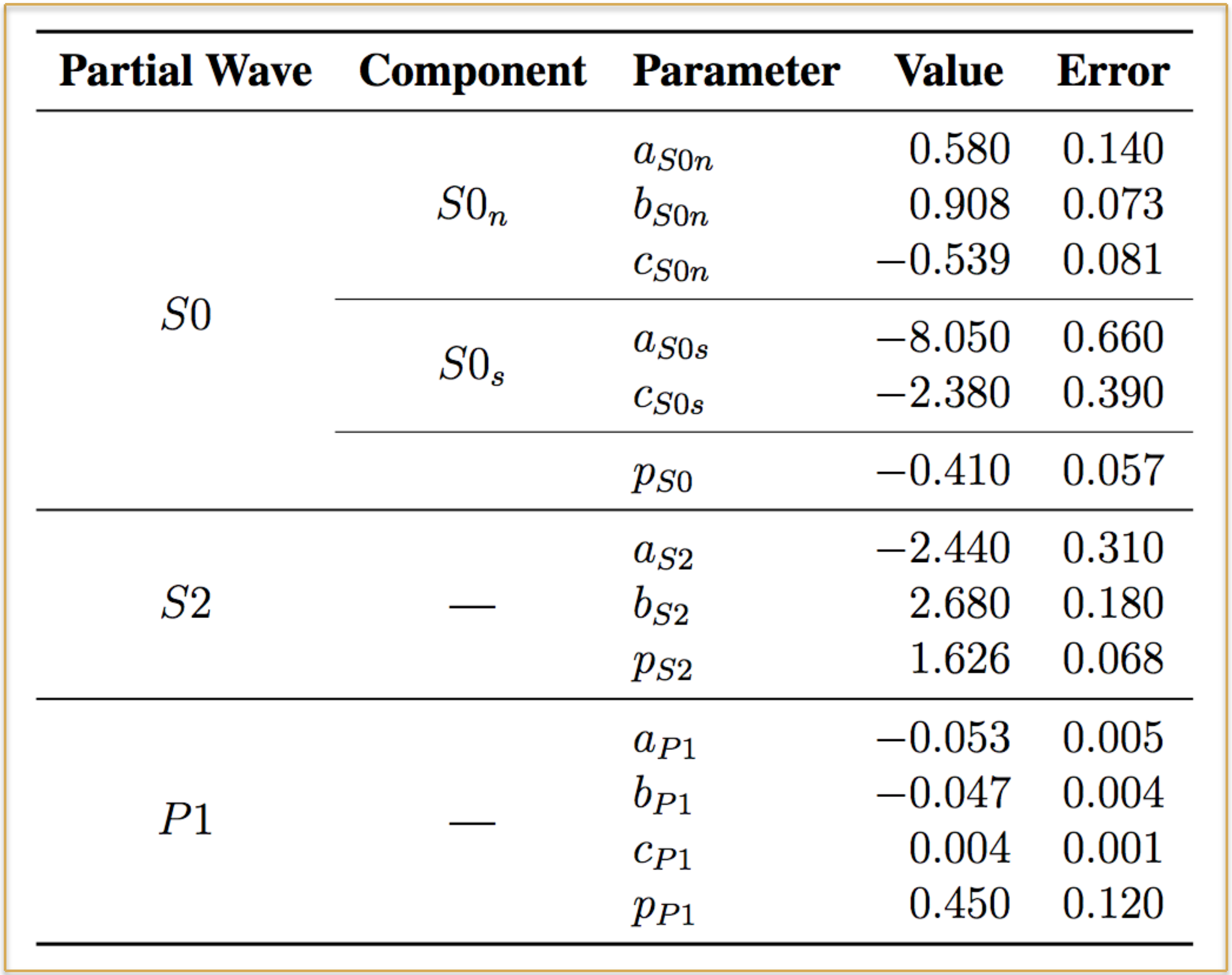
$$\Delta\Gamma^{(\pm)}_{\rm CP}(s) = \Delta\Gamma^{>}_{\rm CP}(s) \pm \Delta\Gamma^{<}_{\rm CP}(s)$$

$$\Sigma\Gamma^{(\pm)}(s) = \Sigma\Gamma^{>}(s) \pm \Sigma\Gamma^{<}(s)$$

$+ \longrightarrow$ To "+" contribute S-S and P-P self-interferences

$- \longrightarrow$ To "–" contribute S-P interferences

- Best fit parameters

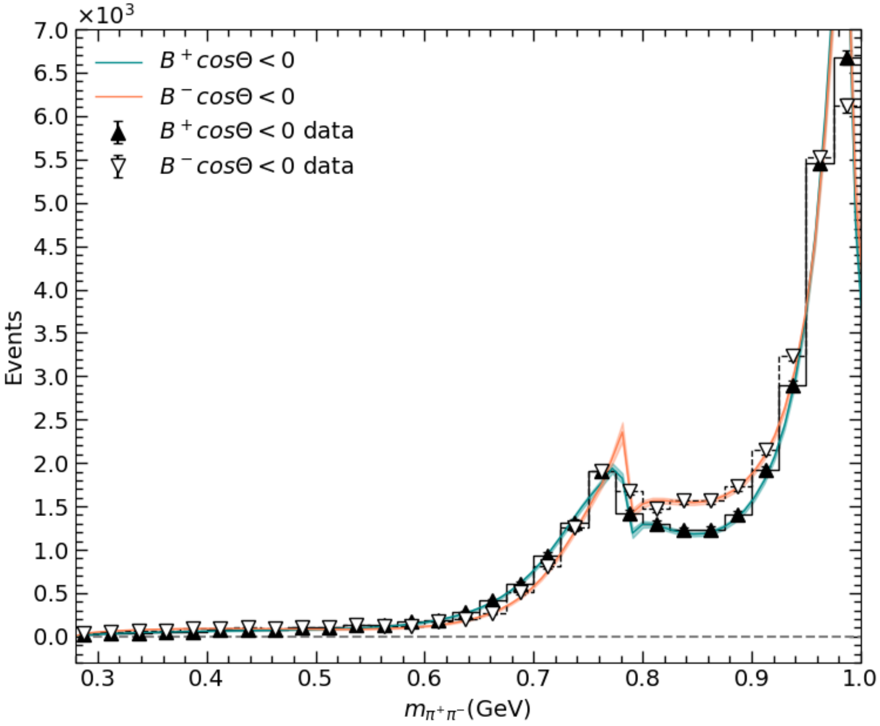


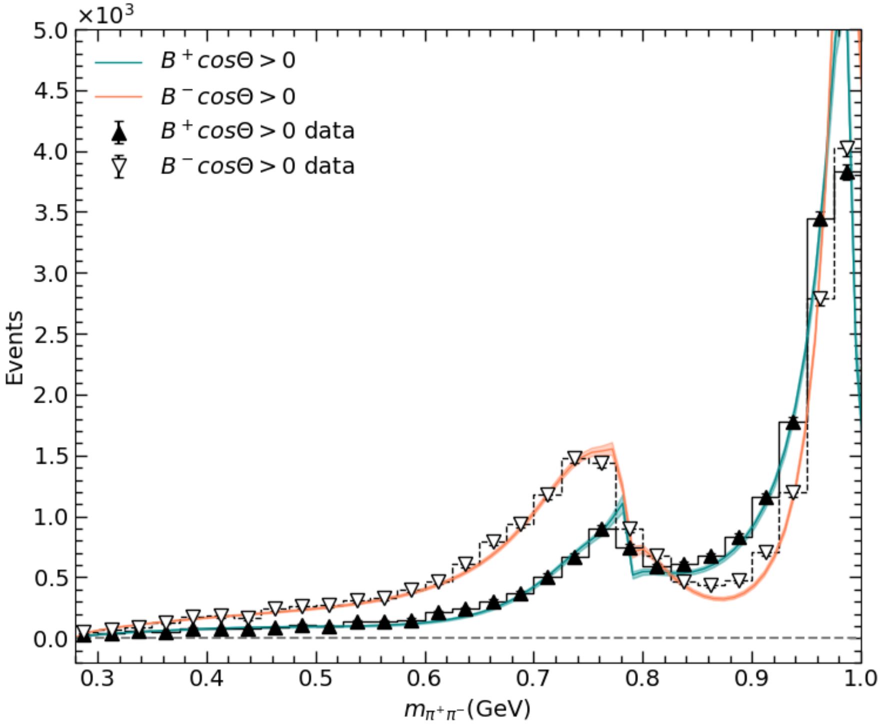
Partial Wave	Component	Parameter	Value	Error
$S0$	$S0_n$	a_{S0n}	0.580	0.140
		b_{S0n}	0.908	0.073
		c_{S0n}	−0.539	0.081
	$S0_s$	a_{S0s}	−8.050	0.660
		c_{S0s}	−2.380	0.390
		p_{S0}	−0.410	0.057
$S2$	—	a_{S2}	−2.440	0.310
		b_{S2}	2.680	0.180
		p_{S2}	1.626	0.068
$P1$	—	a_{P1}	−0.053	0.005
		b_{P1}	−0.047	0.004
		c_{P1}	0.004	0.001
		p_{P1}	0.450	0.120

3. RESULTS

Fitting the LCHb data

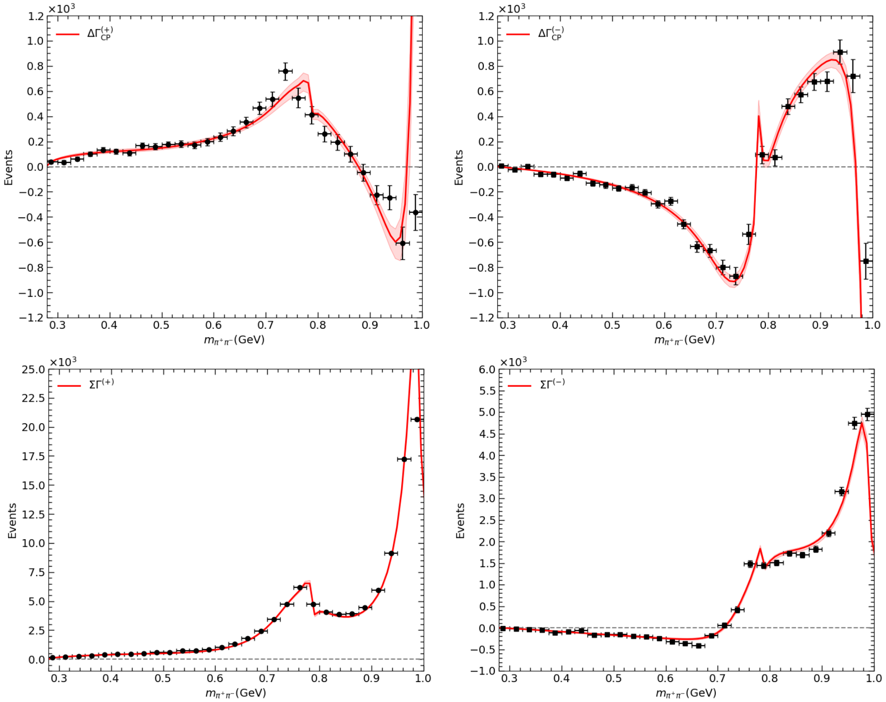
Our best fit can also reproduce the data of the $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$ yields with good accuracy





3. RESULTS

Fitting strategy



- In the minimization process, the energy range in each bin is averaged
- Checked that the minimum is stable
- We obtain a great χ^2 by adding an extra systematic error of a 5%

$$\chi^2/d.o.f = 3.4 \rightarrow \chi^2/d.o.f \approx 1$$

- Got no improvements by adding higher partial waves or extra parameters
- Data sets are not background subtracted ⟶ We modeled a naïve background

$$\Gamma^{\pm}(s, \cos\theta) + B \cdot \left(\sqrt{s} - 2M_\pi\right)$$

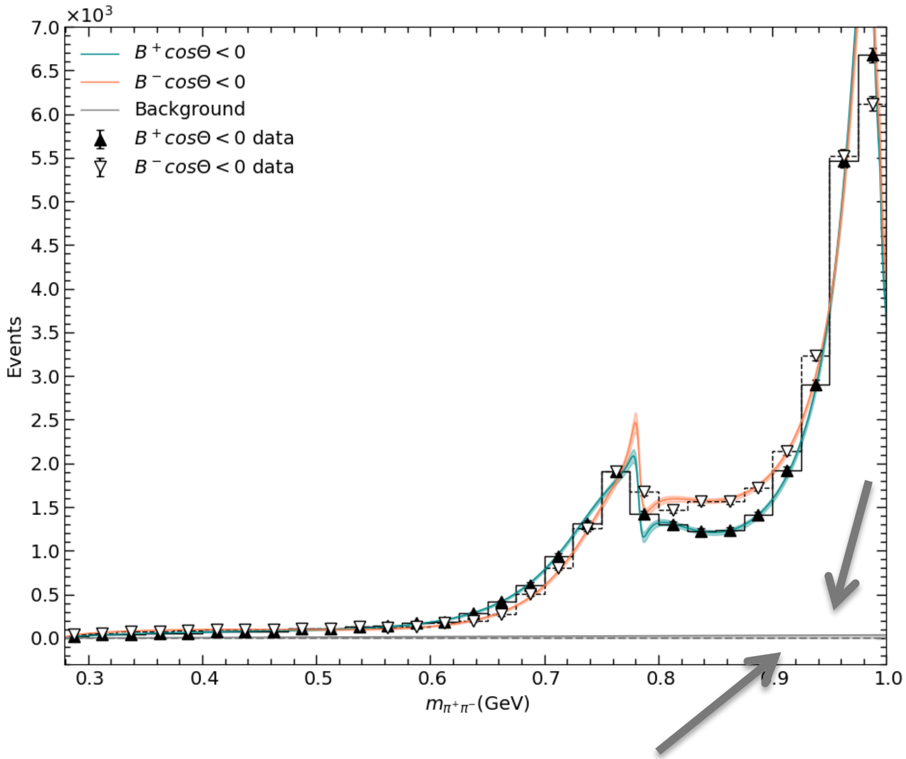
3. RESULTS

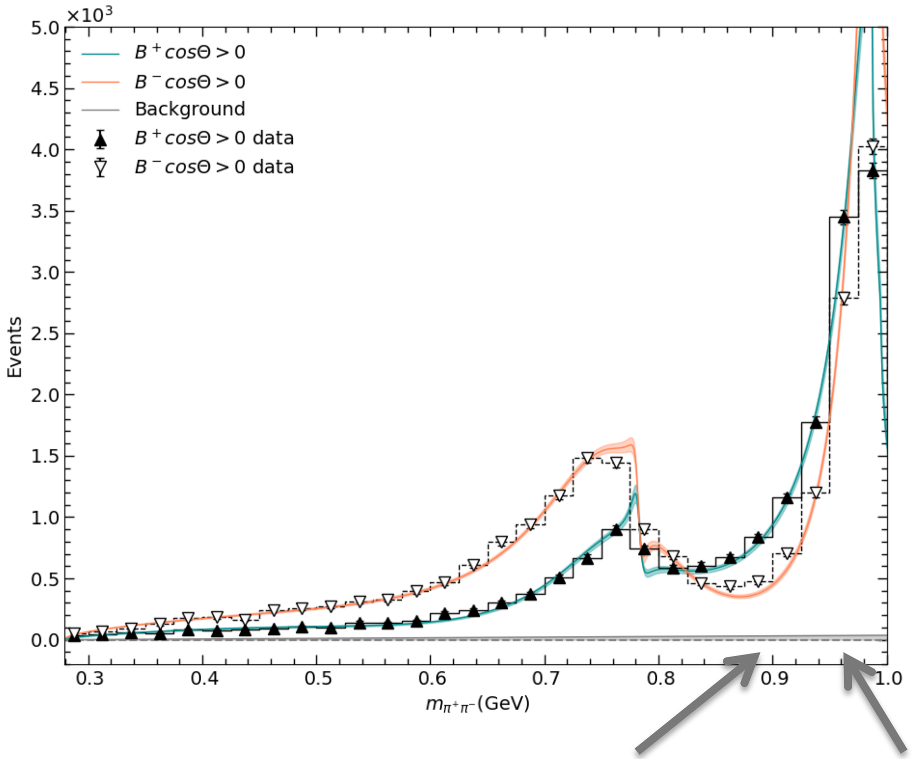
Background

- Data sets are not background subtracted → We modeled a naïve background

$$\Gamma^{\pm}\left(s,\cos\theta\right)+B\cdot\left(\sqrt{s}-2M_{\pi}\right)$$

$$B=50\pm 20$$





3. RESULTS

Background

- Data sets are not background subtracted ⟶ We modeled a naïve background

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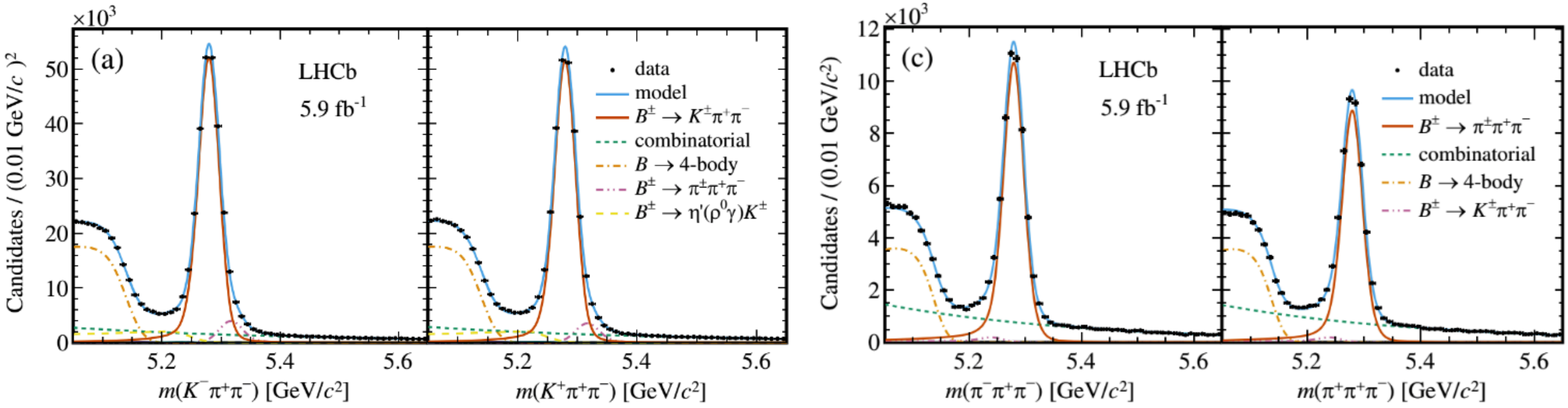
$$B = 50 \pm 20$$



[R. Aaij et al. (LHCb), Phys. Rev. D 108 (2023)]

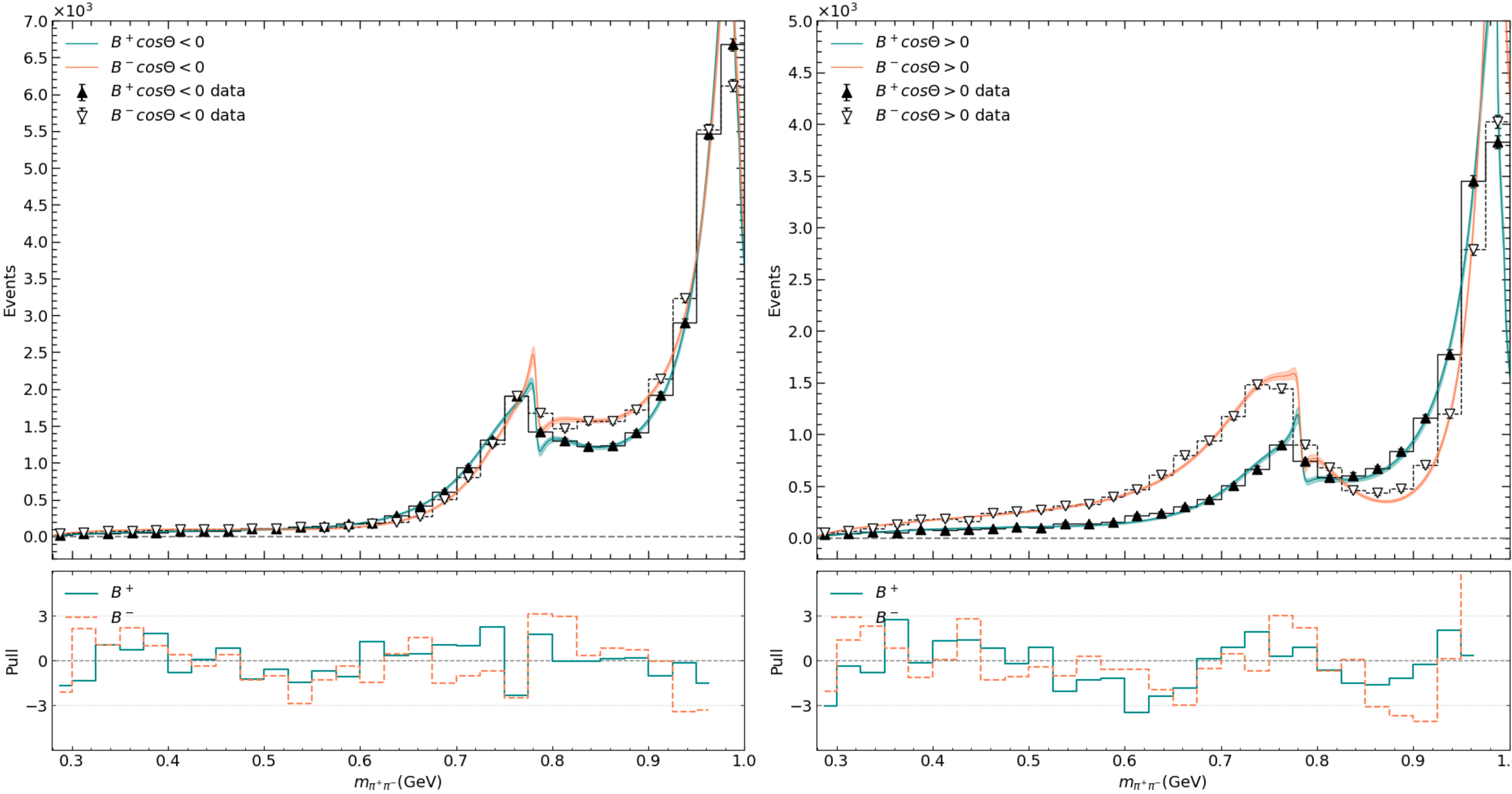
Statistical interpretation

- We believe this $\chi^2/d.o.f$ has **no strict statistical interpretation**
- The projection **data is not corrected for acceptance** (efficiency, production asymmetry…) **nor background subtracted**
- The acceptance correction **is only provided for the integrated yields but we do not know its energy dependence**, we have corrected all bins with that very same average correction
- We also **corrected for background with a very naive estimate** (private communication)
- **Not even LHCb is providing** $\chi^2/d.o.f$ in their Dalitz-plot amplitude analyses nor in their projections. Moreover, previous theoretical analysis of Belle data neither provide these $\chi^2/d.o.f$
- Still, we know from private communications that **the goodness of our description is better or at least comparable** to the present amplitude analyses of LHCb

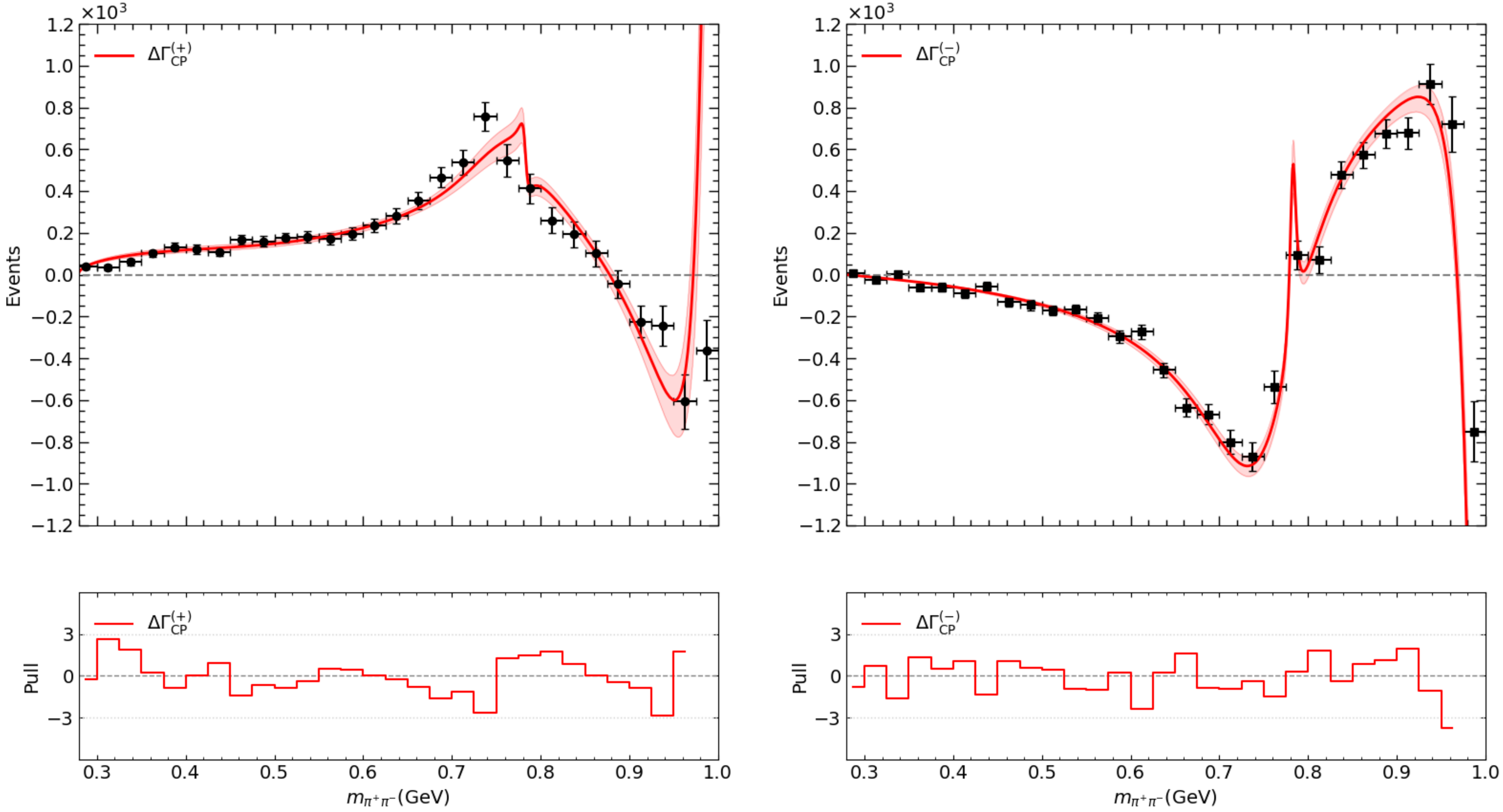
3. RESULTS

Pulls

- We have calculated the pulls of our fits, as LHCb does

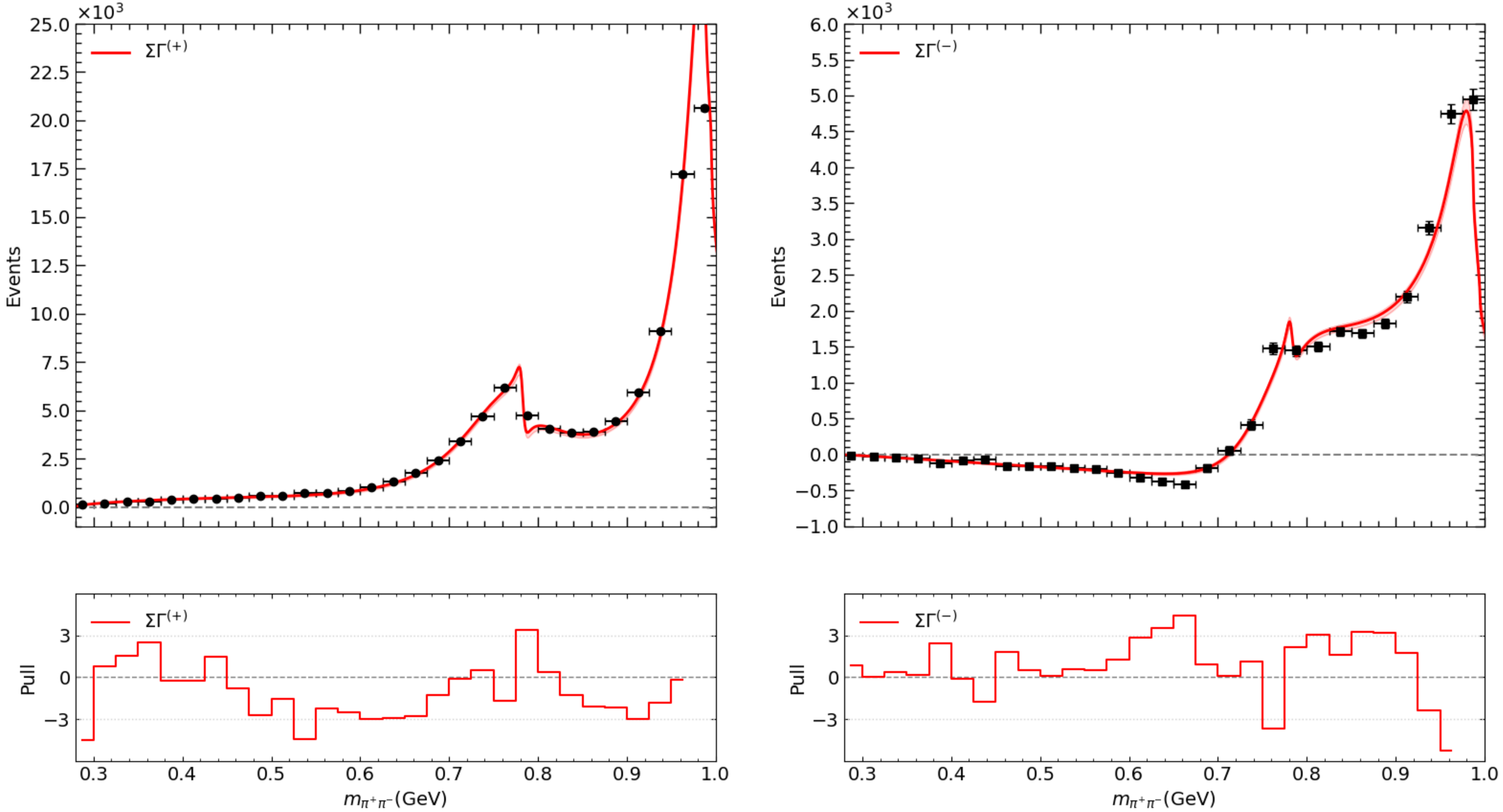


Pulls


×10^3
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
Events
$\Delta\Gamma^{(+)}_{\mathrm{CP}}$
$\Delta\Gamma^{(-)}_{\mathrm{CP}}$
Pull
3
0
−3
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
$m_{\pi^+\pi^-}$(GeV)

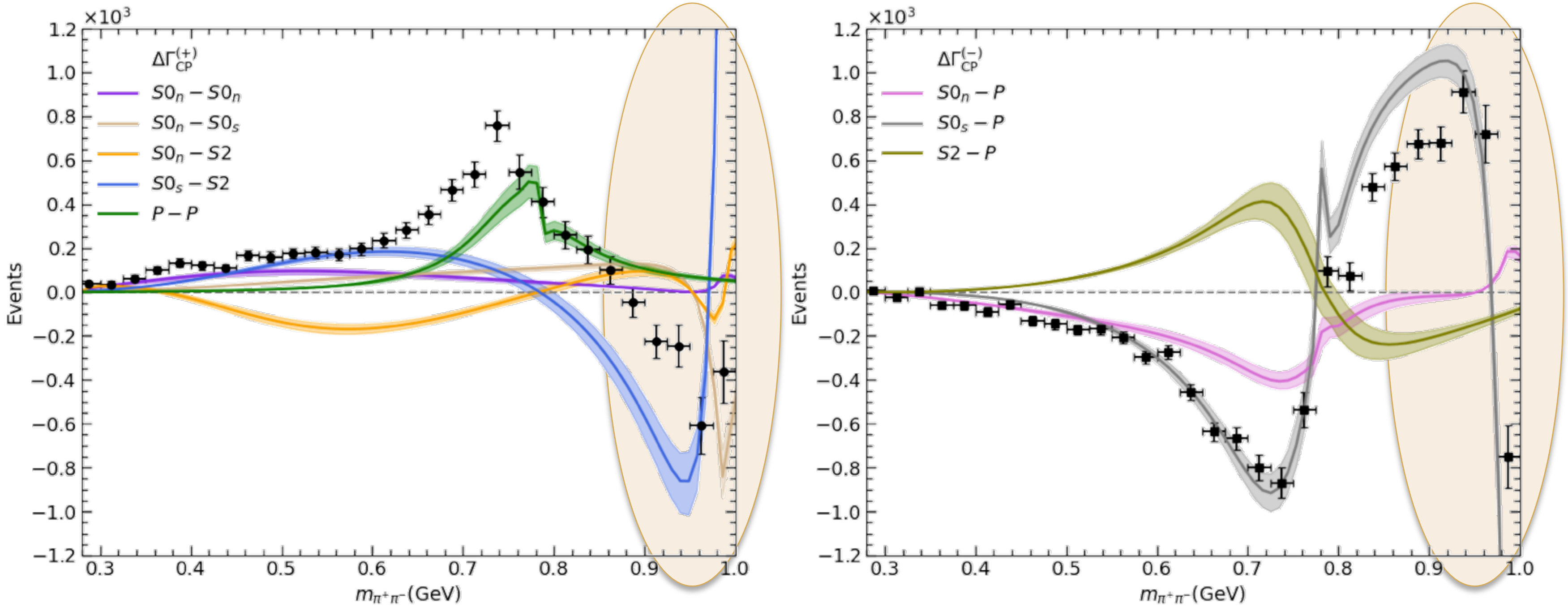
3. RESULTS

Pulls



3. RESULTS

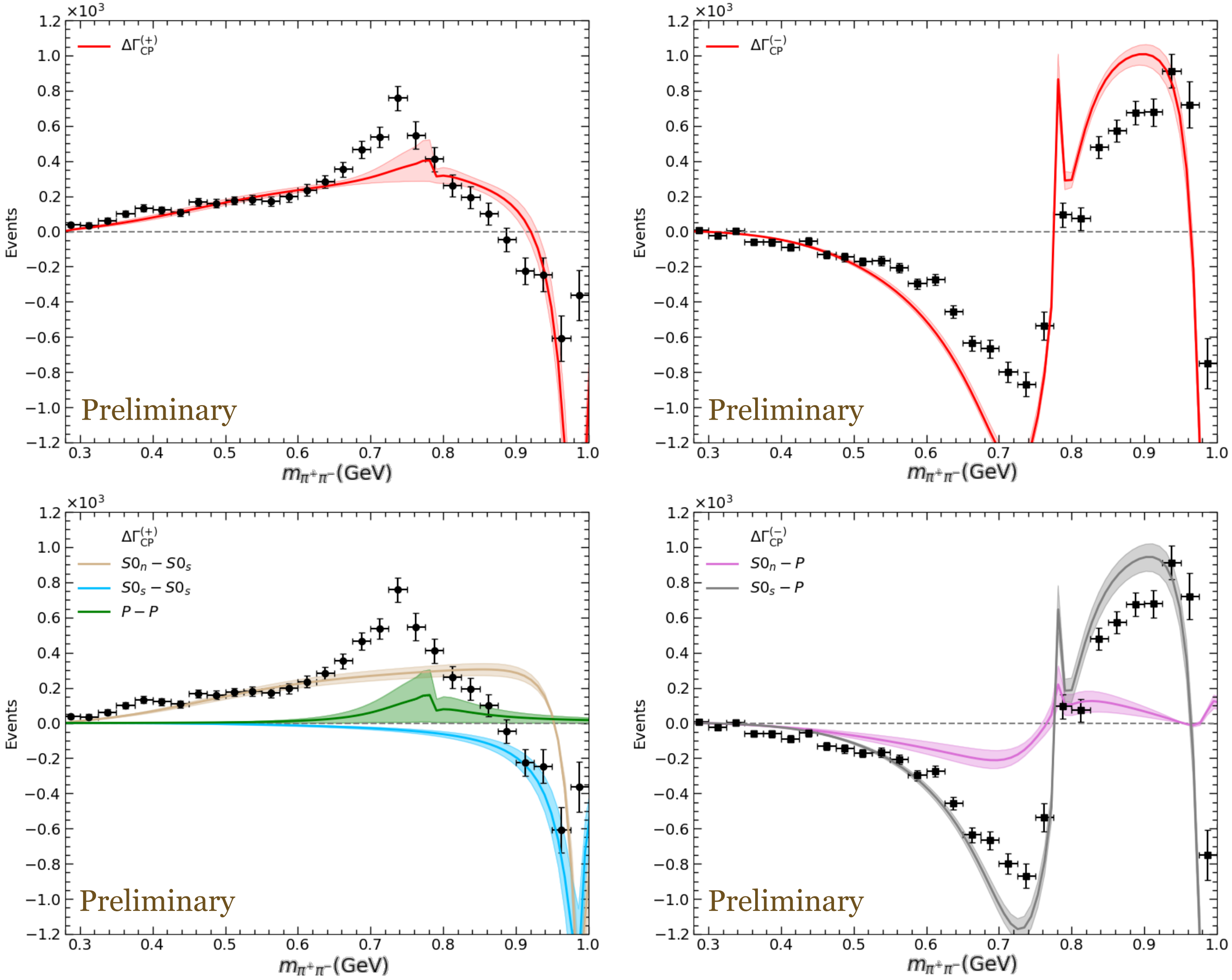
Individual contributions



- We found big contributions from Ω_{S0s} (coupled channel treatment)

3. RESULTS

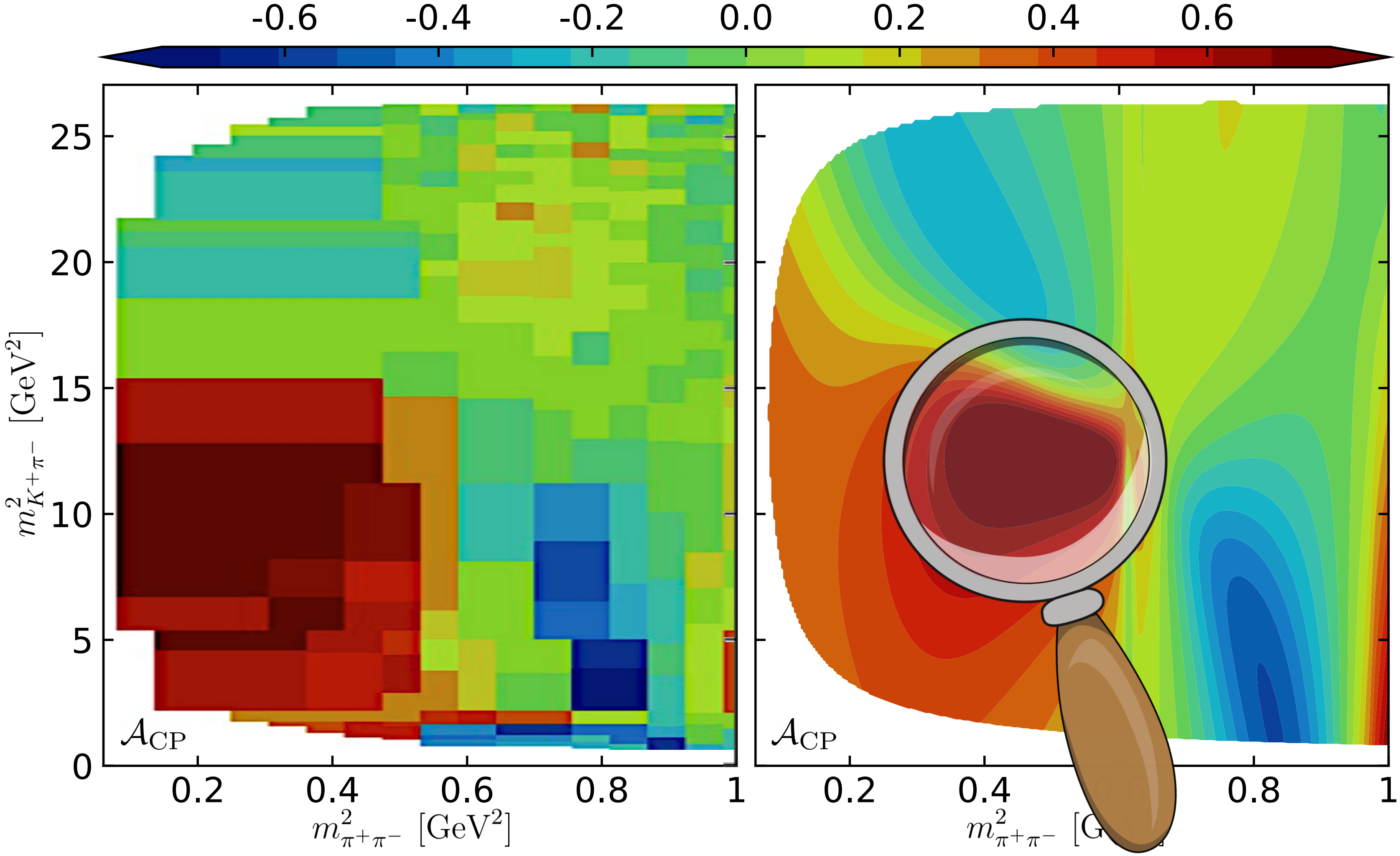
Individual contributions wihout isospin 2 S2 wave



If we remove the isospin 2 component the fit is much worse, therefore it is crucial. So far it is not included in LHCb analyses

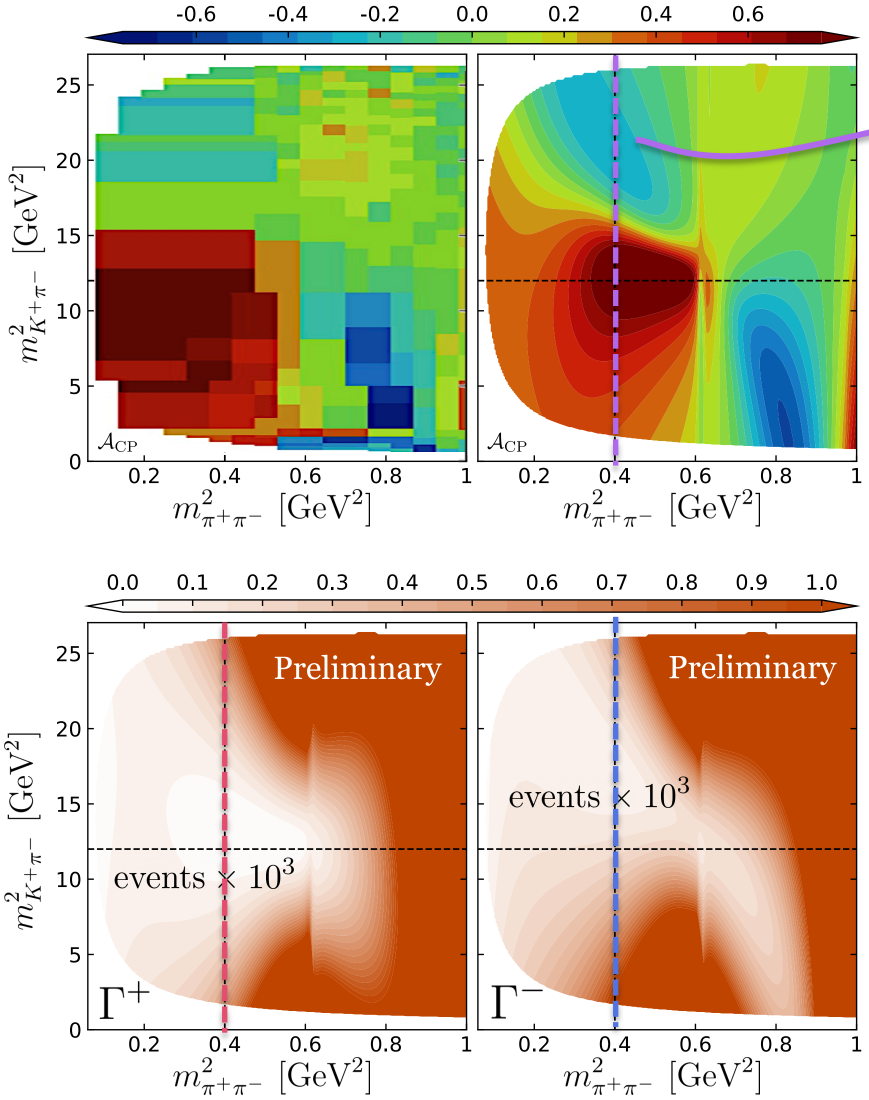
- **Crucial** role of isospin 2 to reproduce the data in this regime

Giant localized CPV

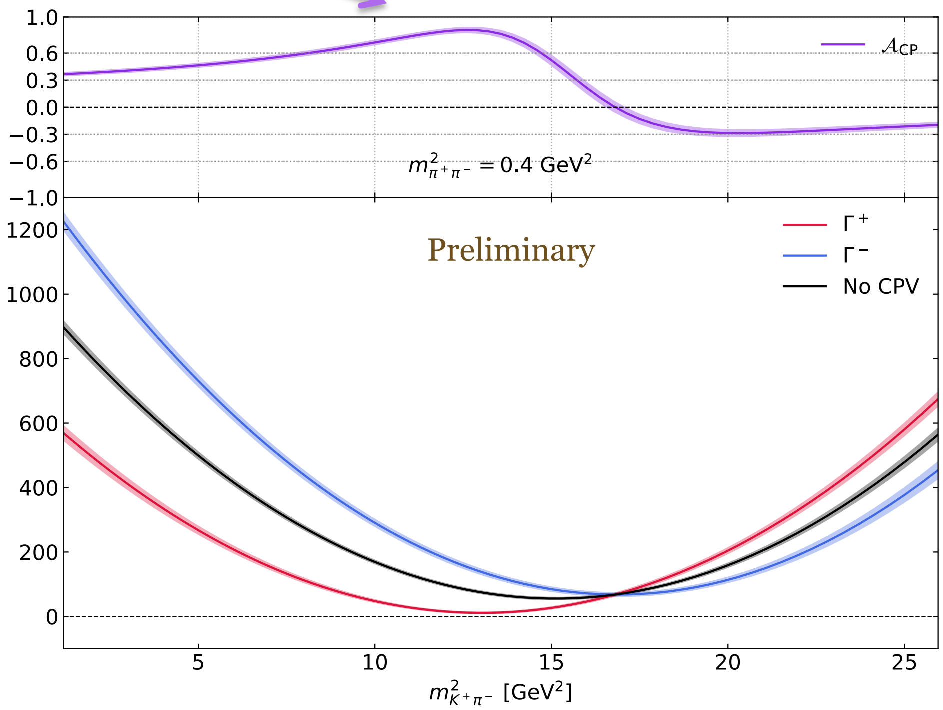


[R. Aaij et al. (LHCb), Phys. Rev. D 108 (2023)]

Giant local CPV



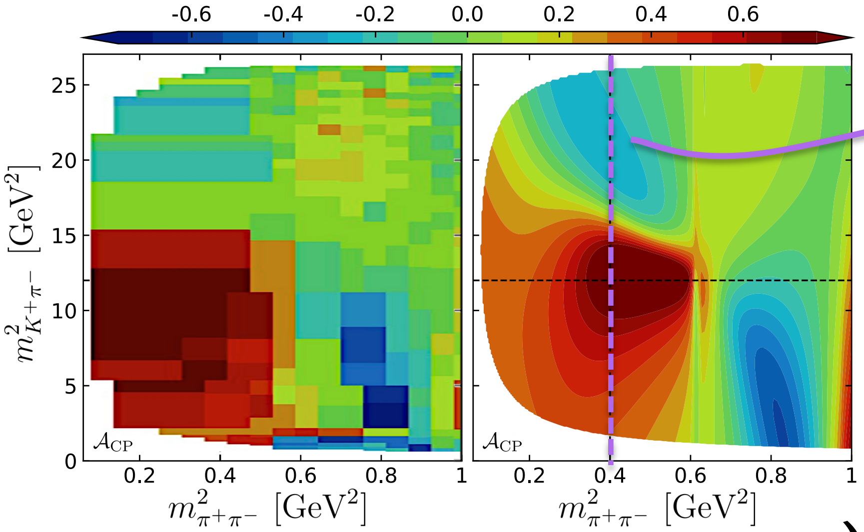
If we cut our Dalitz plots through the dashed lines:



Asymmetries:

$$\mathcal{A}_{\rm CP} = \frac{\Delta\Gamma_{\rm CP}}{\Sigma\Gamma} = \frac{\Gamma^- - \Gamma^+}{\Gamma^- + \Gamma^+}$$

Giant localized CPV



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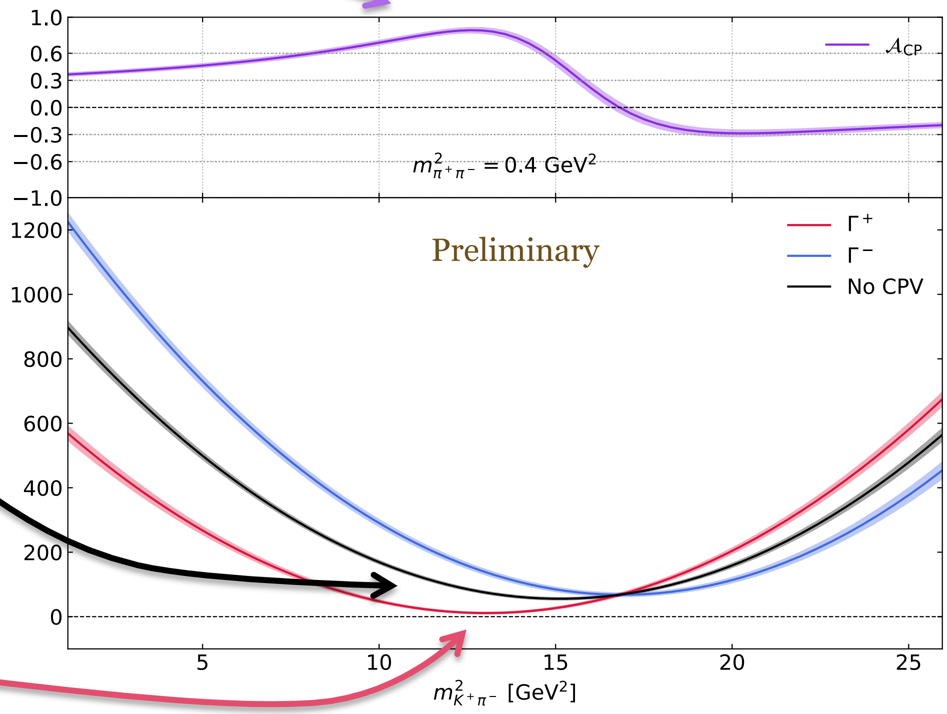


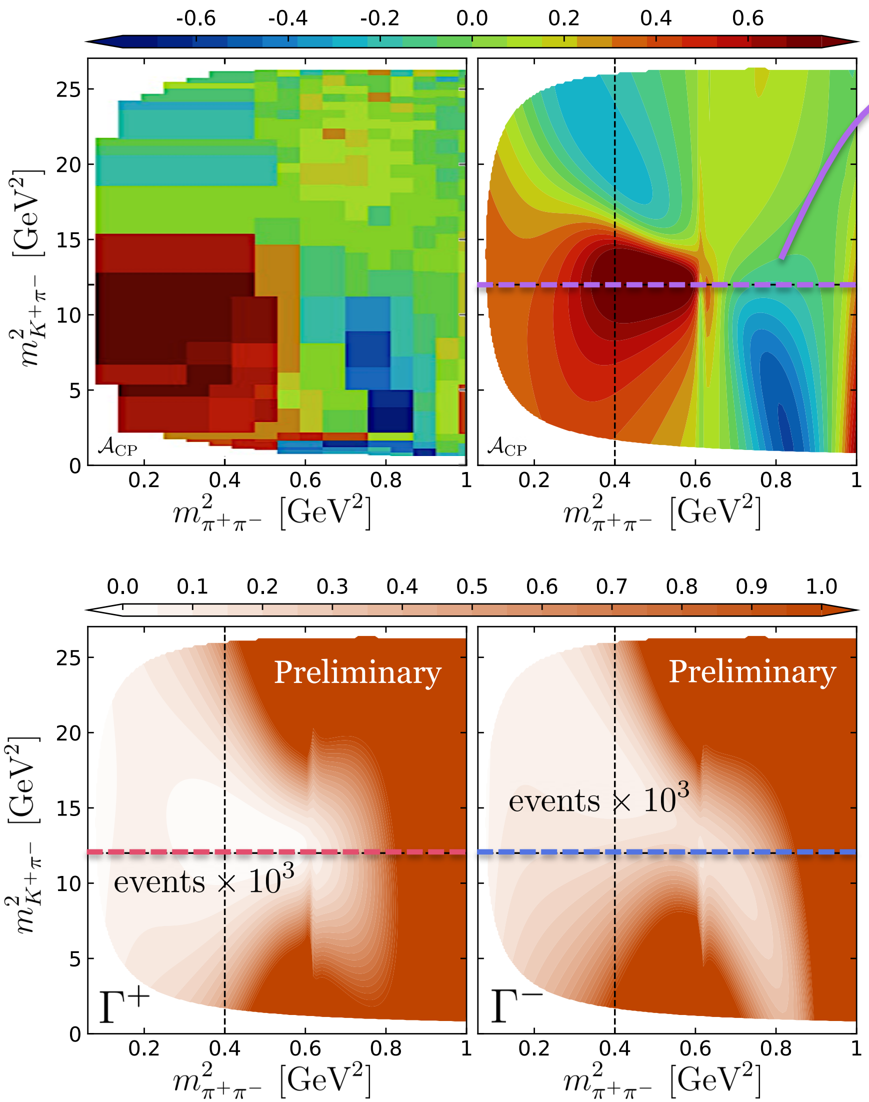
- The CP symmetric part of the decay width is already suppressed and comparable to CPV magnitude
- In this region Γ^+ is almost zero
- Not a big absolute CPV difference, but a **huge relative CPV asymmetry**

Asymmetries:

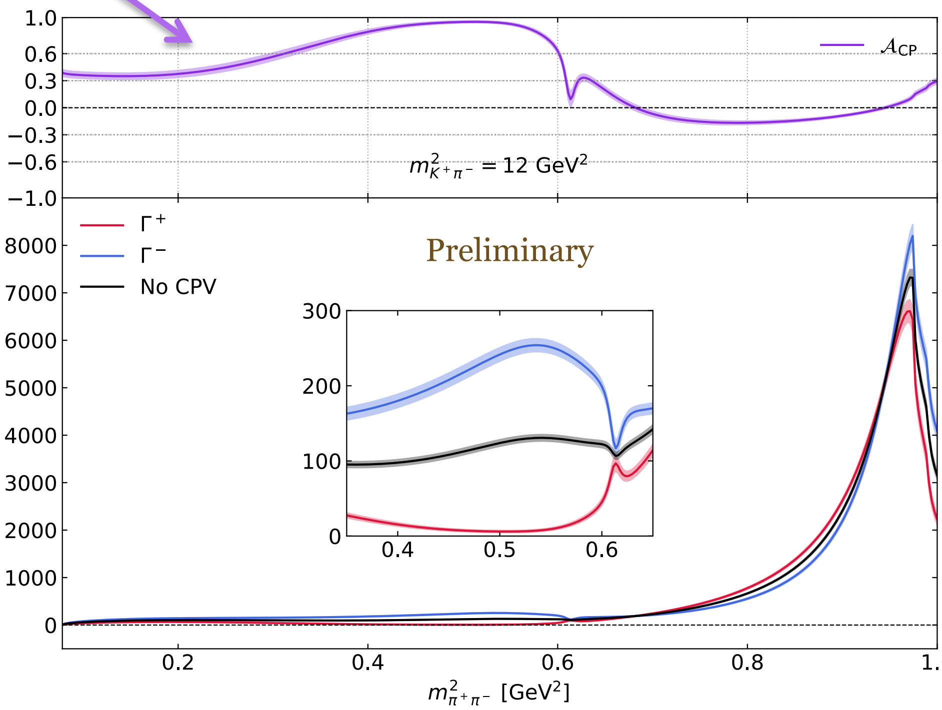
$$\mathcal{A}_{\rm CP} = \frac{\Delta\Gamma_{\rm CP}}{\Sigma\Gamma} = \frac{\Gamma^- - \Gamma^+}{\Gamma^- + \Gamma^+}$$

3. RESULTS

Giant local CPV



If we cut our Dalitz plots through the dashed lines:



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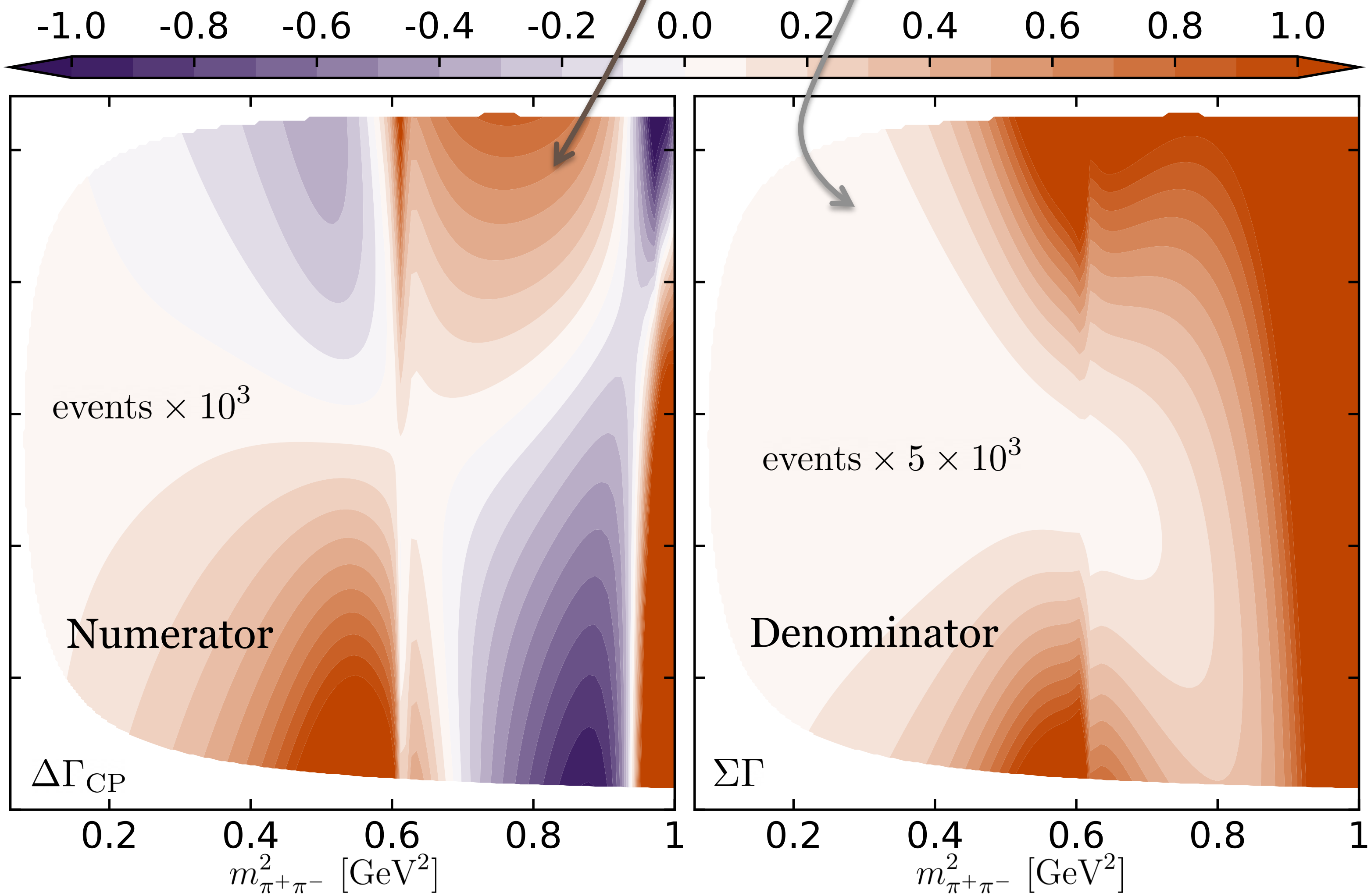
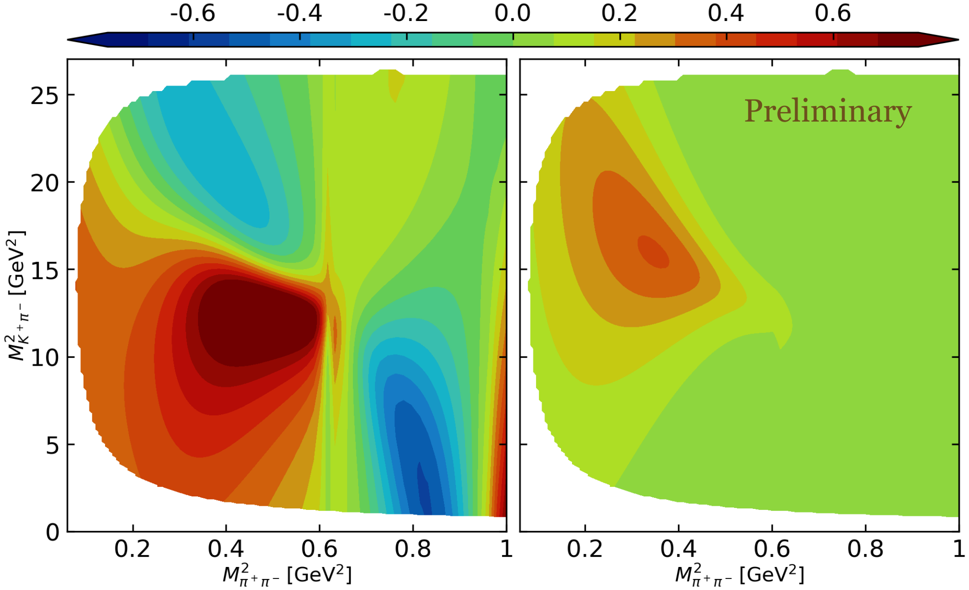
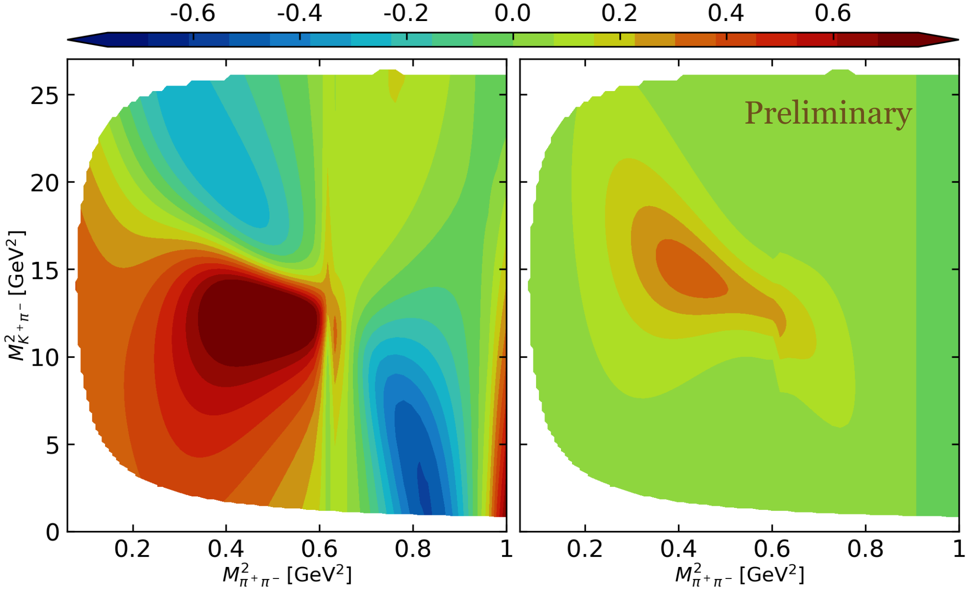


Dalitz plot contributions

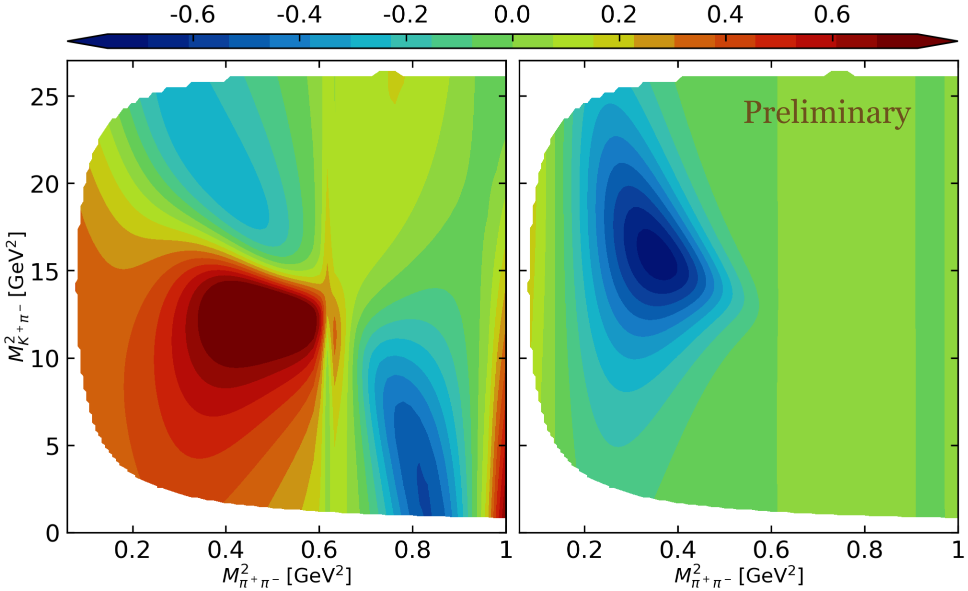


$$S0_n - S0_n$$

$$S0_n - S0_s$$

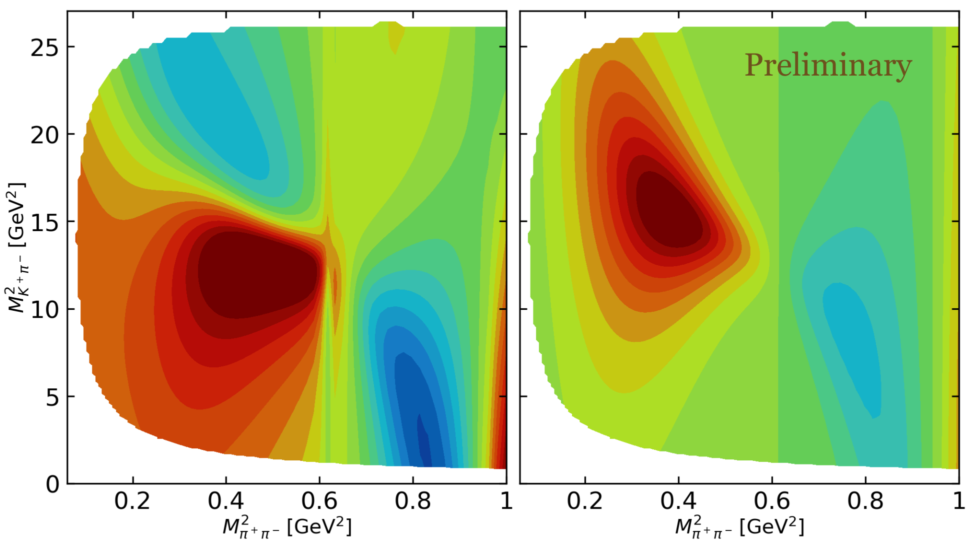


Dalitz plot contributions



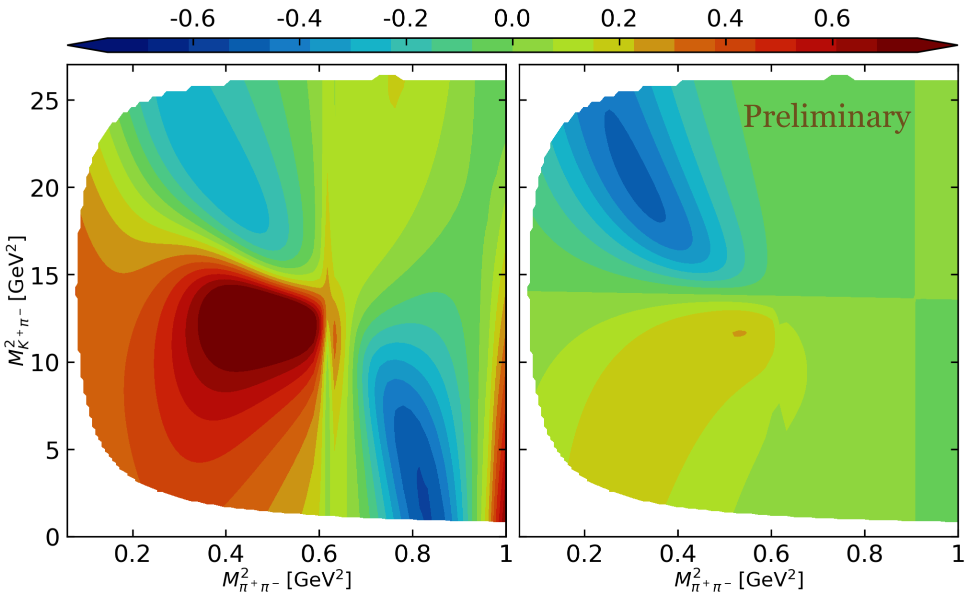
$$S0_n - S2$$

$$S0_s - S2$$



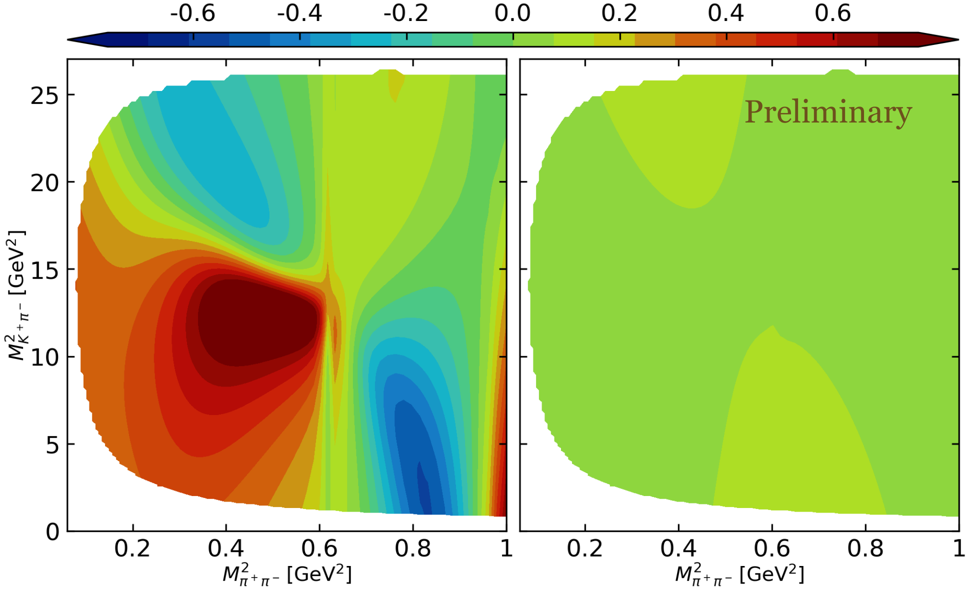
3. RESULTS

Dalitz plot contributions

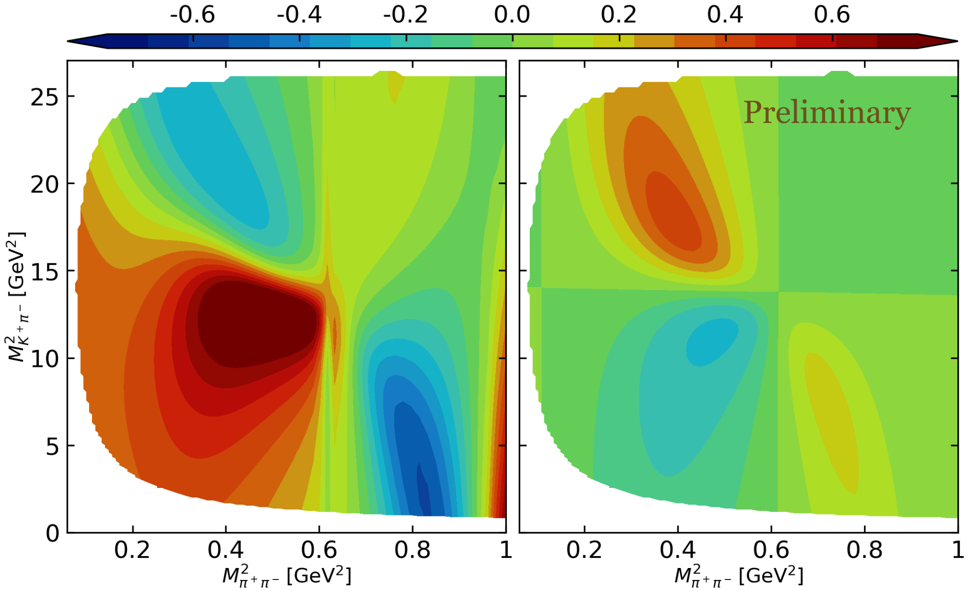


$S0_n - P$

$S0_s - P$

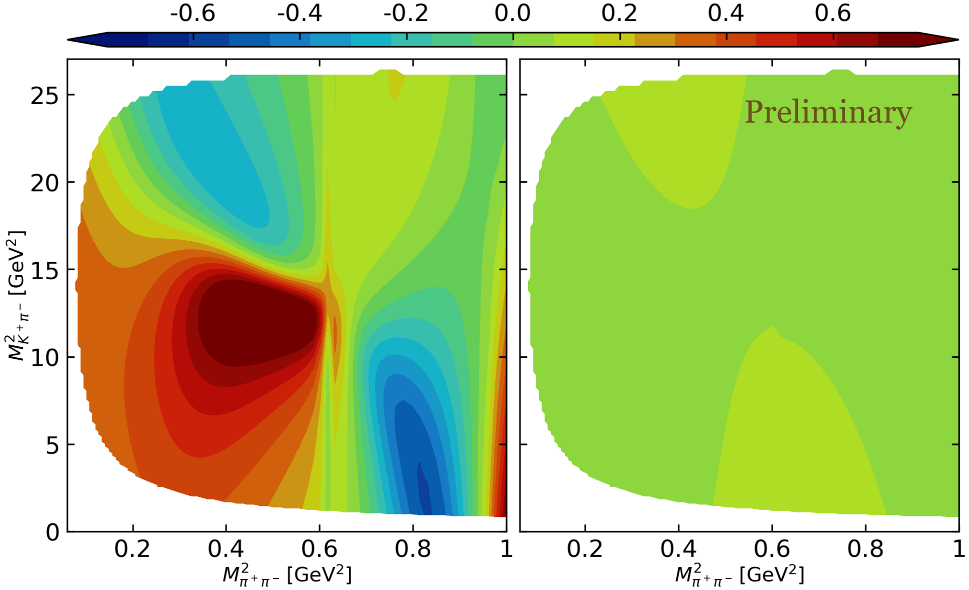


Dalitz plot contributions

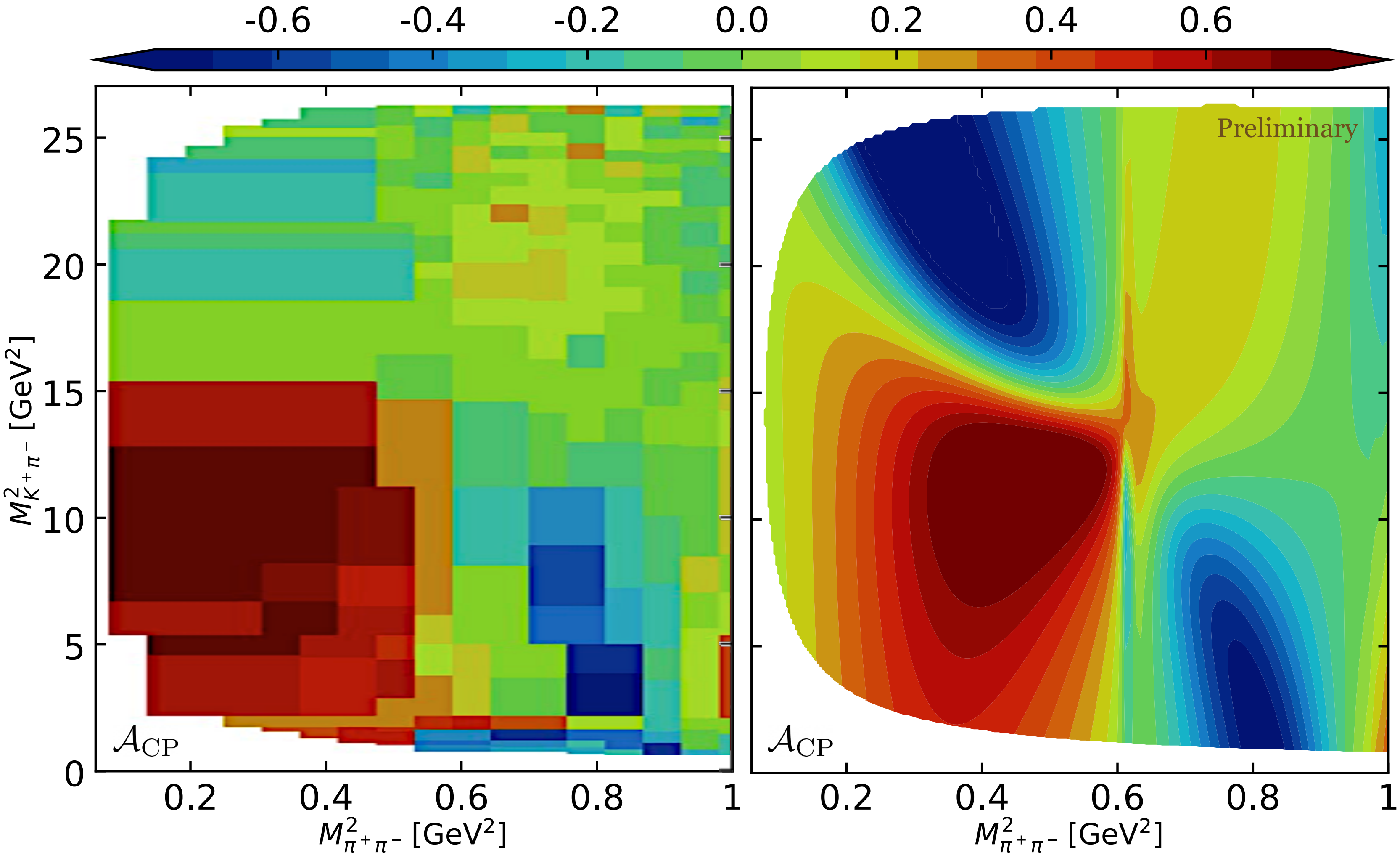


$S2 - P$

$P - P$

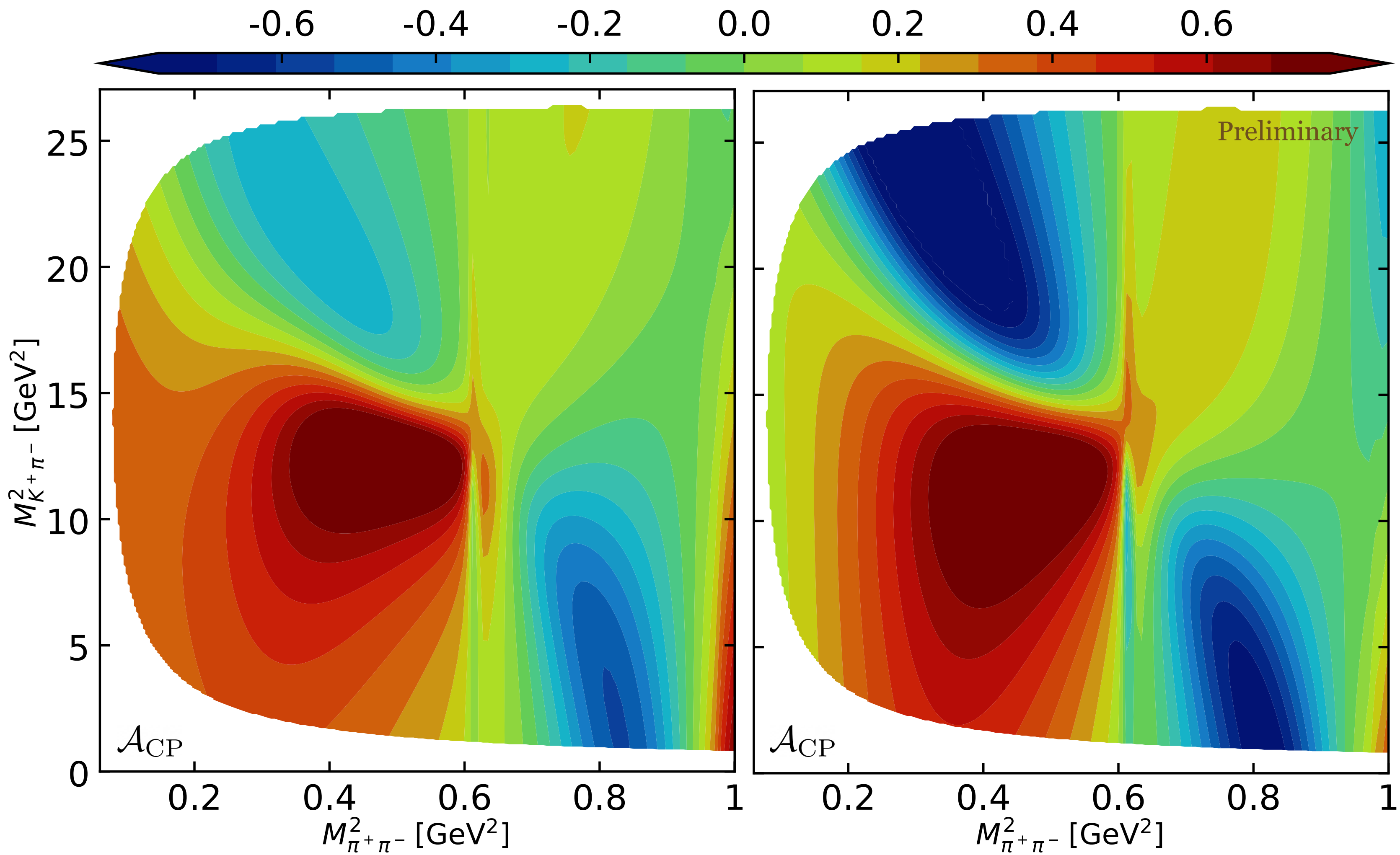


Dalitz plot wihout isospin 2 S2 wave



[R. Aaij et al. (LHCb), Phys. Rev. D 108 (2023)]

Dalitz plot wihout isospin 2 S2 wave



4. CONCLUSIONS

- We have developed a dispersive method **able to describe the hadronic FSI** that **enhance CP violation in charmless three-body decays**
- The universal $\pi\pi$ scattering allows us to **include systematically every partial wave, even the non resonant waves** like the isospin-2 S2 wave
- This **dispersive approach respects the analytic structure** of the light-meson production and it is preferred over rough approximations such as Breit-Wigners

4. CONCLUSIONS

- Fitting our formalism to the projected CP-violation data of the $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$ decay below 1 GeV, $m_{\pi\pi} < 1$ GeV, we have been able to **reproduce the main features in the data sets** and **provide a prediction of the giant localized CP asymmetry** found by the LHCb

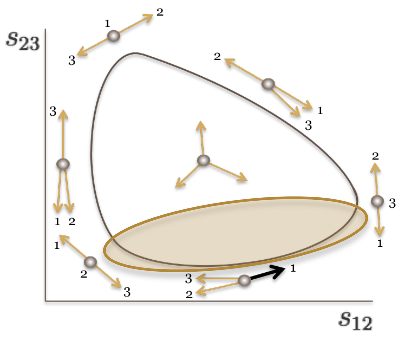
- Our **parameters have physical meaning**, e.g., the phase from the $c\bar{c}$ -loop is essential to get a reasonable fit

- The non resonant **isospin-2 S2 wave is found to be crucial** in this kinematic region to explain the data, and it is not included in LHCb analyses

- This formalism has the potential to **describe other data sets and other processes** with reasonable accuracy

- This is a promising path so **background subtracted data** would improve and help our conclusions

5. FUTURE PROSPECTS

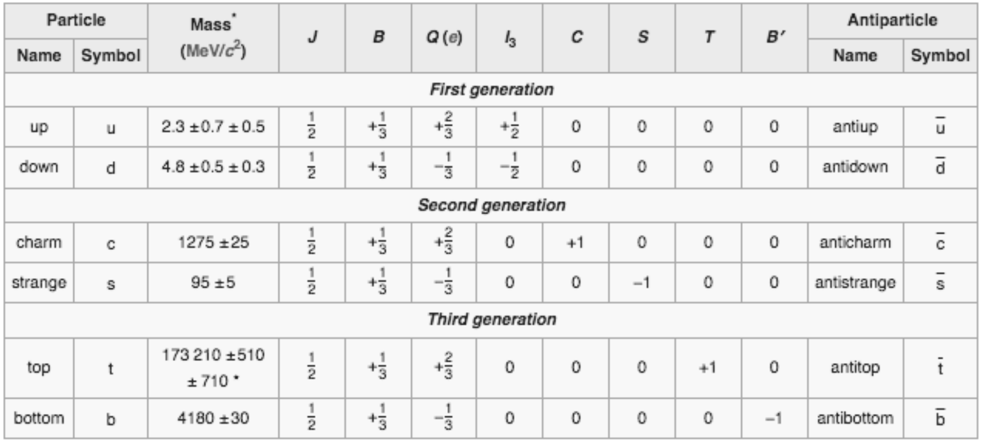


- Show other regions of the Dalitz plot
- Describe other CPV processes with relevant two-body FSI
- Include spectator interactions → Khuri-Treiman equations

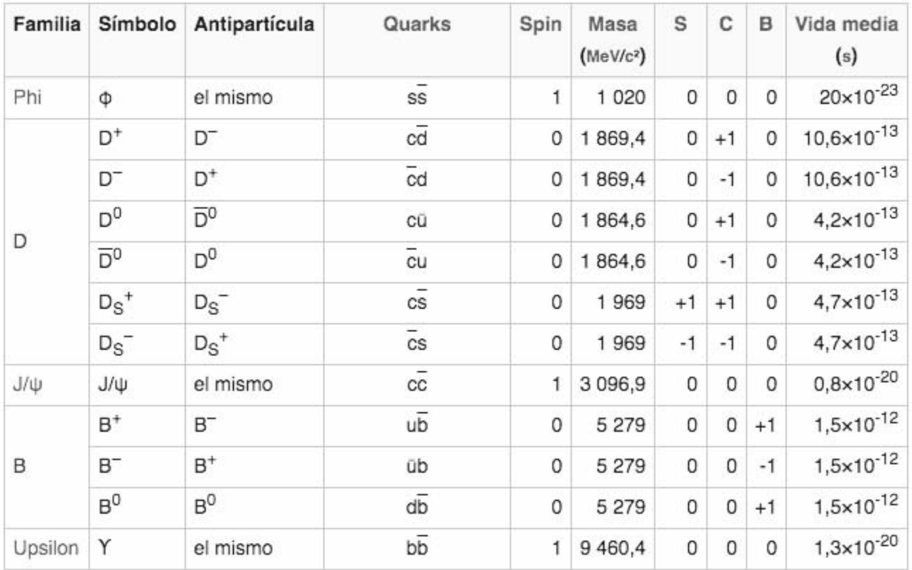
[N. N. Khuri and S. B. Treiman, Phys. Rev. 119 (1960)]
[D. Stamen, T. Isken, B. Kubis, M. Mikhasenko, and M. Niehus, Eur. Phys. J. C 83 (2023)]

- Study the $\pi\pi$ inelastic regime (coupled-channel formalism)
- Connect our source parameters in the production with quark-level physics
- Improve our results by using background-subtracted data

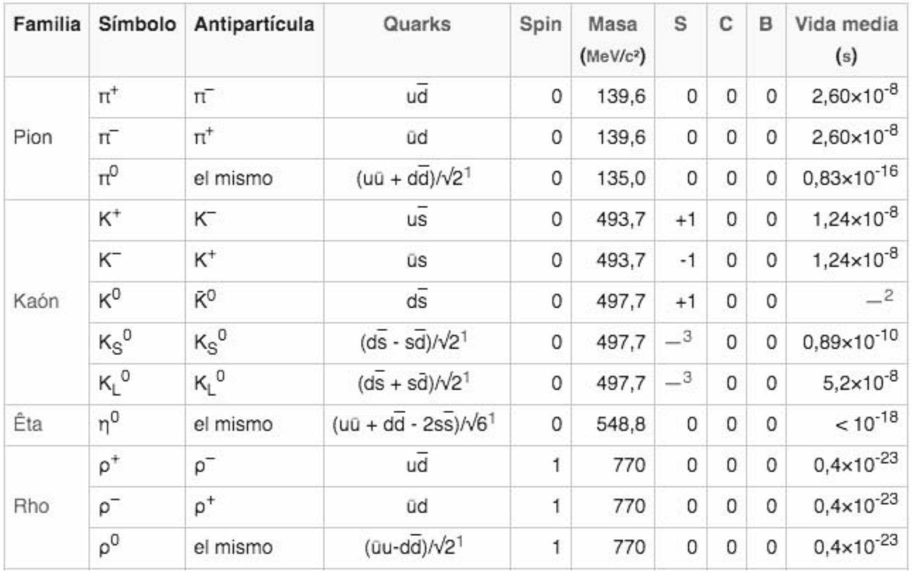
- Quark content



Particle		Mass* (MeV/c^2)	J	B	Q (e)	I_3	C	S	T	B'	Antiparticle	
Name	Symbol										Name	Symbol
First generation												
up	u	2.3 ±0.7 ± 0.5	$\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{2}{3}$	$+\frac{1}{2}$	0	0	0	0	antiup	$\bar{u}$
down	d	4.8 ±0.5 ± 0.3	$\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	0	0	0	0	antidown	$\bar{d}$
Second generation												
charm	c	1275 ±25	$\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{2}{3}$	0	+1	0	0	0	anticharm	$\bar{c}$
strange	s	95 ±5	$\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{1}{3}$	0	0	−1	0	0	antistrange	$\bar{s}$
Third generation												
top	t	173 210 ±510 ± 710 *	$\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{2}{3}$	0	0	0	+1	0	antitop	$\bar{t}$
bottom	b	4180 ±30	$\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{1}{3}$	0	0	0	0	−1	antibottom	$\bar{b}$



Familia	Símbolo	Antipartícula	Quarks	Spin	Masa (MeV/c²)	S	C	B	Vida media (s)
Phi	φ	el mismo	$s\bar{s}$	1	1 020	0	0	0	20×10^{-23}
D	D^+	D^-	$c\bar{d}$	0	1 869,4	0	+1	0	$10{,}6\times10^{-13}$
	D^-	D^+	$\bar{c}d$	0	1 869,4	0	-1	0	$10{,}6\times10^{-13}$
	D^0	$\bar{D}^0$	$c\bar{u}$	0	1 864,6	0	+1	0	$4{,}2\times10^{-13}$
	$\bar{D}^0$	D^0	$\bar{c}u$	0	1 864,6	0	-1	0	$4{,}2\times10^{-13}$
	$D_S{}^+$	$D_S{}^-$	$c\bar{s}$	0	1 969	+1	+1	0	$4{,}7\times10^{-13}$
	$D_S{}^-$	$D_S{}^+$	$\bar{c}s$	0	1 969	-1	-1	0	$4{,}7\times10^{-13}$
J/ψ	J/ψ	el mismo	$c\bar{c}$	1	3 096,9	0	0	0	$0{,}8\times10^{-20}$
B	B^+	B^-	$u\bar{b}$	0	5 279	0	0	+1	$1{,}5\times10^{-12}$
	B^-	B^+	$\bar{u}b$	0	5 279	0	0	-1	$1{,}5\times10^{-12}$
	B^0	B^0	$d\bar{b}$	0	5 279	0	0	+1	$1{,}5\times10^{-12}$
Upsilon	Υ	el mismo	$b\bar{b}$	1	9 460,4	0	0	0	$1{,}3\times10^{-20}$



Familia	Símbolo	Antipartícula	Quarks	Spin	Masa (MeV/c²)	S	C	B	Vida media (s)
Pion	π^+	π^-	$u\bar{d}$	0	139,6	0	0	0	$2{,}60\times10^{-8}$
	π^-	π^+	$\bar{u}d$	0	139,6	0	0	0	$2{,}60\times10^{-8}$
	π^0	el mismo	$(u\bar{u} + d\bar{d})/\sqrt{2}$[1]	0	135,0	0	0	0	$0{,}83\times10^{-16}$
Kaón	K^+	K^-	$u\bar{s}$	0	493,7	+1	0	0	$1{,}24\times10^{-8}$
	K^-	K^+	$\bar{u}s$	0	493,7	-1	0	0	$1{,}24\times10^{-8}$
	K^0	$\bar{K}^0$	$d\bar{s}$	0	497,7	+1	0	0	—[2]
	$K_S{}^0$	$K_S{}^0$	$(d\bar{s} - s\bar{d})/\sqrt{2}$[1]	0	497,7	—[3]	0	0	$0{,}89\times10^{-10}$
	$K_L{}^0$	$K_L{}^0$	$(d\bar{s} + s\bar{d})/\sqrt{2}$[1]	0	497,7	—[3]	0	0	$5{,}2\times10^{-8}$
Êta	η^0	el mismo	$(u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$[1]	0	548,8	0	0	0	$< 10^{-18}$
Rho	ρ^+	ρ^-	$u\bar{d}$	1	770	0	0	0	$0{,}4\times10^{-23}$
	ρ^-	ρ^+	$\bar{u}d$	1	770	0	0	0	$0{,}4\times10^{-23}$
	ρ^0	el mismo	$(\bar{u}u - d\bar{d})/\sqrt{2}$[1]	1	770	0	0	0	$0{,}4\times10^{-23}$

IV CONGRESO

IPARCOS

2025