Graviton contribution to the muon anomalous magnetic moment in higher-derivative gravity

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IPARCOS

Overview

1. General Relativity and fourth-derivative models
2. The muon anomalous magnetic moment
3. Graviton contribution to the muon anomalous magnetic moment
4. Conclusions

Gravitons in General Relativity

General Relativity:

$$S_{HE} + S_{gf} = \int d^4x \left\{ \sqrt{-g} \left(-\frac{2}{\kappa^2} R \right) + \frac{f_\mu f^\mu}{\xi_{gf}} \right\}$$

- Gauge-fixing term: $f_\mu = \partial^\nu \left(h_{\mu\nu} - \frac{1}{2} c_{gf} \eta_{\mu\nu} h \right)$.
- Weak-field approximation: $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, $\quad \kappa = 2/\overline{M}_{pl}$.
- Massless gravitons.
- Not renormalizable: one-loop divergences contain higher-derivative terms R^2, $R^2_{\mu\nu}$.

Gravitons in fourth-derivative gravity

There are various models beyond GR that introduce spin 2 and spin 0 mediators of the gravitational interaction:

- Bigravity.
- Brane models such as ADD and RS.
- Fourth-derivative gravity.

Gravitons in fourth-derivative gravity

There are various models beyond GR that introduce spin 2 and spin 0 mediators of the gravitational interaction:

Stelle gravity:

$$S_{HE} + S_{ag} + S_{gf} = \int d^4x \left\{ \sqrt{-g} \left(-\frac{2}{\kappa^2} R + \frac{R^2}{6 f_0^2} + \frac{\frac{1}{3} R^2 - R_{\mu\nu}^2}{f_2^2} \right) - \frac{\kappa^2}{2\xi_{gf}} f_\mu \partial^2 f^\mu \right\}$$

[Stelle (1978), Salvio and Strumia (2017)]

- Gauge-fixing term: $f_\mu = \partial^\nu \left(h_{\mu\nu} - \frac{1}{2} c_{gf} \eta_{\mu\nu} h \right)$.
- Massive and ghost-like gravitons.
- Renormalizable.
- In the limit $f_0, f_2 \to \infty$, we should recover GR.

Muon anomalous magnetic moment

For a classical spin $1/2$ particle, we have:

$$\vec{\mu}_S = g\frac{q}{2m}\vec{S}\ , \quad \text{where} \ \ g = 2.$$

However, the experimental value of g differs from 2. For the muon, we have:

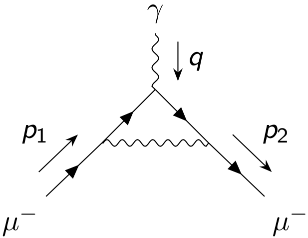
$$a_\mu^{exp} = \frac{g_\mu^{exp} - 2}{2} = 116592071.5(14.5) \times 10^{-11}$$

[Aguillard *et al.* (Muon $g-2$) (2025)]

We call $a_\mu = \frac{g_\mu - 2}{2}$ the **muon anomalous magnetic moment**.

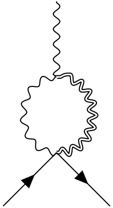
Muon anomalous magnetic moment

How do we compute $a_\mu = \frac{g_\mu - 2}{2}$?



$$\longrightarrow \qquad -ie\overline{u}(p_2)\Gamma^\mu(p_2, p_1)u(p_1)$$

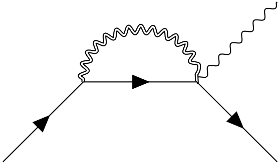
- Corrected vertex: $\Gamma^\mu(p_2, p_1) = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m} F_2(q^2)$.
- In the limit $q^2 \to 0$, we obtain: $\quad g_\mu = 2 + 2F_2(0) \quad \Rightarrow \quad a_\mu = F_2(0)$



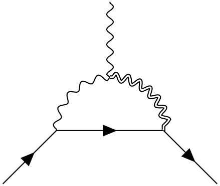
(a)

For a **minimally coupled graviton**, we obtain 4 diagrams.

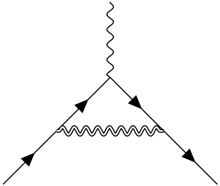
- Interaction vertices have the same structure in both GR and Stelle gravity, only the graviton propagator changes.
- Diagram (a) does not contribute to a_μ.



(b)



(c)



(d)

Graviton propagator

$$G_{\mu\nu;\rho\sigma} = it_j P^{(j)}_{\mu\nu;\rho\sigma}, \; j = 2, 1, 0s, 0w, 0sw, 0ws$$

[Nieuwenhuizen (1973)]

Model	t_2	t_1	t_{0s}	t_{0w}	$t_{0ws} = t_{0sw}$
$S_{HE} + S_{gf}$	$\frac{1}{p^2}$	$\frac{\xi_{gf}}{p^2}$	$-\frac{1}{2p^2}$	$\frac{4\xi_{gf} - 3c_{gf}^2}{2(c_{gf}-2)^2}\frac{1}{p^2}$	$\frac{\sqrt{3}c_{gf}}{2(c_{gf}-2)}\frac{1}{p^2}$
$S_{HE} + S_{ag} + S_{gf}$	$\frac{1}{p^2} - \frac{1}{p^2 - M_2^2}$	$\frac{2\xi_{gf}}{\kappa^2 p^4}$	$-\frac{1}{2p^2} + \frac{1}{2(p^2 - M_0^2)}$	$\frac{\left(\frac{3\kappa^2 M_0^2 p^2}{p^2 - M_0^2} + 8\xi_{gf}\right)}{2(c_{gf}-2)^2 \kappa^2 p^4}$	$\frac{-\frac{\sqrt{3}c_{gf} M_0^2}{2(c_{gf}-2)}}{p^2(p^2 - M_0^2)}$

- Ghost-like mode with mass $M_2^2 = \frac{f_2^2 \overline{M}_{pl}^2}{2}$ and scalar mode with mass $M_0^2 = \frac{f_0^2 \overline{M}_{pl}^2}{2}$.
- Only spin 2 and spin 0s contributions to a_μ.

Graviton contribution for large masses

Spin 2 contribution to a_μ.

Diagram	**General Relativity**	**Stelle gravity** $(M_2^2 \gg m^2)$
(b)	$-\frac{53\kappa^2 m^2}{576\pi^2} - \frac{5\kappa^2 m^2}{96\pi^2}\frac{1}{\hat{\varepsilon}} + \frac{5\kappa^2 m^2}{96\pi^2}\log\frac{m^2}{\mu^2}$	$-\frac{35\kappa^2 m^2}{576\pi^2} - \frac{5\kappa^2 m^2}{96\pi^2}\log c_2$
(c)	$\frac{49\kappa^2 m^2}{288\pi^2} + \frac{5\kappa^2 m^2}{96\pi^2}\frac{1}{\hat{\varepsilon}} - \frac{5\kappa^2 m^2}{96\pi^2}\log\frac{m^2}{\mu^2}$	$\frac{5\kappa^2 m^2}{64\pi^2} + \frac{5\kappa^2 m^2}{96\pi^2}\log c_2$
(d)	$-\frac{5\kappa^2 m^2}{192\pi^2}$	$-\frac{25\kappa^2 m^2}{576\pi^2}$
$(b)+(c)+(d)$	$\frac{5\kappa^2 m^2}{96\pi^2}$	$-\frac{5\kappa^2 m^2}{192\pi^2}$

$$\frac{1}{\hat{\varepsilon}} = -\frac{2}{D-4} - \gamma_E + \log 4\pi, \qquad c_2 = \frac{{M_2}^2}{m^2}$$

Graviton contribution for large masses

Spin 0s contribution to a_μ.

Diagram	General Relativity	Stelle gravity $(M_0^2 \gg m^2)$
(b)	$\frac{7\kappa^2 m^2}{576\pi^2} + \frac{\kappa^2 m^2}{384\pi^2}\frac{1}{\hat{\varepsilon}} - \frac{\kappa^2 m^2}{384\pi^2}\log\frac{m^2}{\mu^2}$	$\frac{\kappa^2 m^2}{256\pi^2} + \frac{\kappa^2 m^2}{384\pi^2}\log c_0$
(c)	$-\frac{\kappa^2 m^2}{72\pi^2} - \frac{\kappa^2 m^2}{192\pi^2}\frac{1}{\hat{\varepsilon}} + \frac{\kappa^2 m^2}{192\pi^2}\log\frac{m^2}{\mu^2}$	$-\frac{\kappa^2 m^2}{256\pi^2} + \frac{\kappa^2 m^2}{384\pi^2}\log c_0$
(d)	$\frac{5\kappa^2 m^2}{1152\pi^2} + \frac{\kappa^2 m^2}{384\pi^2}\frac{1}{\hat{\varepsilon}} - \frac{\kappa^2 m^2}{384\pi^2}\log\frac{m^2}{\mu^2}$	$-\frac{\kappa^2 m^2}{256\pi^2} + \frac{\kappa^2 m^2}{384\pi^2}\log c_0$
$(b)+(c)+(d)$	$\frac{\kappa^2 m^2}{384\pi^2}$	$-\frac{\kappa^2 m^2}{128\pi^2}$

$$\frac{1}{\hat{\varepsilon}} = -\frac{2}{D-4} - \gamma_E + \log 4\pi, \qquad c_0 = \frac{{M_0}^2}{m^2}$$

Graviton contribution for small masses

Stelle graviton contribution to a_μ.

Diagram	**Spin 2** $(M_2^2 \ll m^2)$	**Spin 0s** $(M_0^2 \ll m^2)$
(b)	$-\frac{\sqrt{c_2}\kappa^2 m^2}{18\pi} + \frac{25 c_2 \kappa^2 m^2}{384\pi^2} - \frac{5 c_2 \kappa^2 m^2 \log c_2}{192\pi^2}$	$\frac{\sqrt{c_0}\kappa^2 m^2}{288\pi} - \frac{c_0 \kappa^2 m^2}{384\pi^2} + \frac{c_0 \kappa^2 m^2 \log c_0}{384\pi^2}$
(c)	$\frac{5\sqrt{c_2}\kappa^2 m^2}{72\pi} - \frac{5 c_2 \kappa^2 m^2}{96\pi^2} + \frac{5 c_2 \kappa^2 m^2 \log c_2}{96\pi^2}$	$-\frac{\sqrt{c_0}\kappa^2 m^2}{144\pi} + \frac{c_0 \kappa^2 m^2}{192\pi^2} - \frac{c_0 \kappa^2 m^2 \log c_0}{192\pi^2}$
(d)	$-\frac{5\sqrt{c_2}\kappa^2 m^2}{144\pi} - \frac{25 c_2 \kappa^2 m^2}{768\pi^2} - \frac{25 c_2 \kappa^2 m^2 \log c_2}{384\pi^2}$	$-\frac{\sqrt{c_0}\kappa^2 m^2}{576\pi} - \frac{c_0 \kappa^2 m^2}{384\pi^2} - \frac{c_0 \kappa^2 m^2 \log c_0}{192\pi^2}$
$(b)+(c)+(d)$	$-\frac{\sqrt{c_2}\kappa^2 m^2}{48\pi} - \frac{5 c_2 \kappa^2 m^2}{256\pi^2} - \frac{5 c_2 \kappa^2 m^2 \log c_2}{128\pi^2}$	$-\frac{\sqrt{c_0}\kappa^2 m^2}{192\pi} - \frac{c_0 \kappa^2 m^2 \log c_0}{128\pi^2}$

$$c_2 = \frac{{M_2}^2}{m^2}, \qquad c_0 = \frac{{M_0}^2}{m^2}$$

Final remarks

The Planck mass M_{pl} as the coupling constant makes the graviton contribution negligible in both models:

- $a_\mu^{SM} = 116592033(62) \times 10^{-11}$. [Aliberti *et al.* (2025)]
- $a_\mu^{exp} = 116592071.5(14.5) \times 10^{-11}$. [Aguillard *et al.* (Muon $g-2$) (2025)]
- Our results are proportional to $\kappa^2 m^2 = \frac{4m^2}{\overline{M}_{pl}^2} \sim 10^{-40}$.

Our calculations can be extended to spin 2 massive particles coupled to muons at another scale λ instead of $\overline{M}_{pl}$.
To ensure experimental compatibility of results: $\lambda \gtrsim 4.8 \times 10^3 \ GeV/c^2$.

Conclusions

- We have made use of the muon anomalous magnetic moment to show the behavior of models of quadratic gravity.
- The graviton contribution is negligible, as expected.
- The same calculations can be applied to other models of spin 2 particles, imposing restrictions on their parameters.
- Further work is needed in order to fully recover the GR limit of Stelle gravity.

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Backup slides

Minimal coupling of gravitons

$$S_{Maxwell} + S_{muon} = \int d^4x \sqrt{-g} \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \overline{\psi} \left(\frac{i}{2} e_a^\mu \left\{ \gamma^a, D_\mu \right\} - m \right) \psi \right\}$$

Nieuwenhuizen spin projectors and transfer operators

$$P^{(2)}_{\mu\nu;\rho\sigma} = \frac{1}{2}\left(T_{\mu\rho}T_{\nu\sigma} + T_{\mu\sigma}T_{\nu\rho}\right) - \frac{1}{D-1}T_{\mu\nu}T_{\rho\sigma},$$

$$P^{(1)}_{\mu\nu;\rho\sigma} = \frac{1}{2}\left(T_{\mu\rho}L_{\nu\sigma} + T_{\mu\sigma}L_{\nu\rho} + T_{\nu\rho}L_{\mu\sigma} + T_{\nu\sigma}L_{\mu\rho}\right),$$

$$P^{(0s)}_{\mu\nu;\rho\sigma} = \frac{1}{D-1}T_{\mu\nu}T_{\rho\sigma},$$

$$P^{(0w)}_{\mu\nu;\rho\sigma} = L_{\mu\nu}L_{\rho\sigma},$$

$$P^{(0sw)}_{\mu\nu;\rho\sigma} = \frac{1}{\sqrt{D-1}}T_{\mu\nu}L_{\rho\sigma},$$

$$P^{(0ws)}_{\mu\nu;\rho\sigma} = \frac{1}{\sqrt{D-1}}L_{\mu\nu}T_{\rho\sigma}.$$

$$T_{\mu\nu} = \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}, \qquad L_{\mu\nu} = \frac{p_\mu p_\nu}{p^2}.$$