Ghosts in Higher Derivative Quantum Field Theories

Based on arXiv:2511.03013

(Jose A. R. Cembranos, Eric G. Hemon, Juan J. Sanz-Cillero)

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1. Why Higher Derivative Theories?

- Motivation $\longrightarrow$ Renormalizability of General Relativity as a QFT

$$S_{HE} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\,R$$

Non-renormalizable!

1. Why Higher Derivative Theories?

- Motivation ⟶ Renormalizability of General Relativity as a QFT

$$S_{HE} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\, R$$

Non-renormalizable!

- Renormalizable generalizations of the HE-action may be considered

Quadratic Gravity: $\mathcal{L} = \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + \gamma R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - \frac{\bar{M}_p{}^2}{2} R - \Lambda + \Box R$

2. Higher Derivative Classical Mechanics Oscillator

- Quadratic Gravity inserts higher derivative terms ⟶ Ghost modes appear

- Classical Mechanics Toy Model: $L = -\frac{a}{2}\ddot{x}^2 + \frac{b}{2}\dot{x}^2 - \frac{c}{2}x^2$

$\underbrace{\frac{b}{2}\dot{x}^2 - \frac{c}{2}x^2}$ Harmonic oscillator

$\underbrace{-\frac{a}{2}\ddot{x}^2 + \frac{b}{2}\dot{x}^2 - \frac{c}{2}x^2}$ Pais-Uhlenbeck oscillator

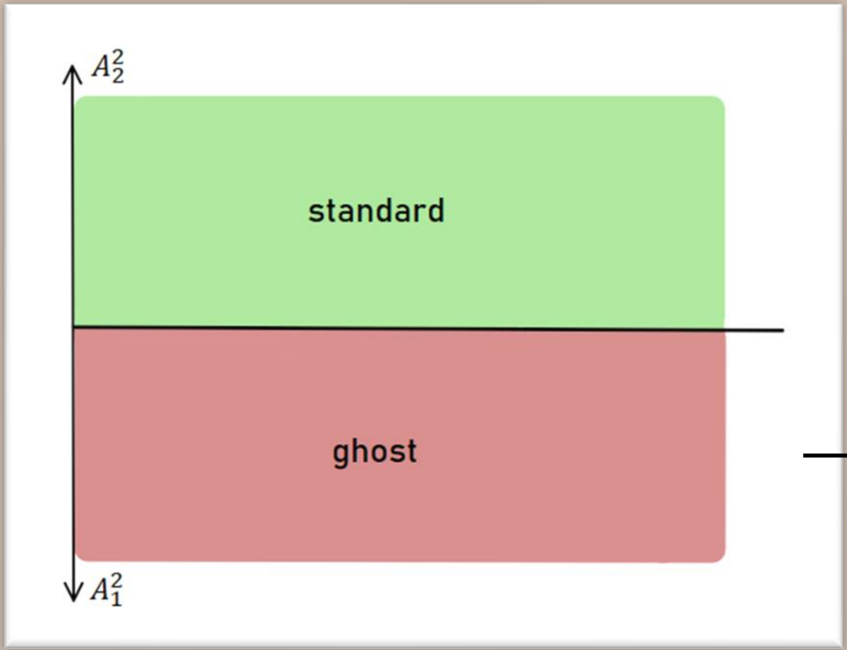
EOM ⟹

$$x(t) = A_1 \cos(\omega_1 t + \delta_1) + A_2 \cos(\omega_2 t + \delta_2)\ ;\ \ {\omega_{1,2}}^2 \equiv \frac{b \pm \sqrt{b^2 - 4ac}}{2a} \geq 0$$

2. Higher Derivative Classical Mechanics Oscillator

Legendre →

$$H = \frac{a\Delta\omega^2}{2}[\omega_2^2 A_2^2 - \omega_1^2 A_1^2] \quad ; \quad \Delta\omega^2 \equiv \omega_1^2 - \omega_2^2$$



Unbounded!

→ Ostrogradsky inestability

3. Higher Derivative Quantum Field Theory

- Lagrangian density: $\mathcal{L} = -\frac{a}{2}\phi(\Box + {m_1}^2)(\Box + {m_2}^2)\phi$

!!!

- Propagator: $\tilde{G}(\boldsymbol{p}) = \frac{-i/a}{(\boldsymbol{p}^2 - {m_1}^2)(\boldsymbol{p}^2 - {m_2}^2)} = \frac{1}{a\Delta m^2}\left(\frac{i}{\boldsymbol{p}^2 - {m_2}^2} + \frac{-i}{\boldsymbol{p}^2 - {m_1}^2}\right)$

Better UV convergence!!

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!!!

Better UV convergence!!

- Hawking-Hertog canonical coordinates:

$$\psi_{1(2)} = \sqrt{\frac{a}{\Delta m^2}}(\Box + {m_{2(1)}}^2)\phi \longrightarrow \mathcal{L} = -\frac{1}{2}\psi_2(\Box + m_2^2)\psi_2 + \frac{1}{2}\psi_1(\Box + m_1^2)\psi_1$$

!!!

3. Higher Derivative Quantum Field Theory

- Path integral analysis: Canonical Coordinate Insertion

$$\mathcal{L} = -\frac{a}{2}\phi(\Box + {m_1}^2)(\Box + {m_2}^2)\phi \longrightarrow S_E = \int \frac{d^4p_E}{(2\pi)^4}\frac{a}{2}(p_E^2 + m_1^2)(p_E^2 + m_2^2)\left|\tilde{\phi}(p_E)\right|^2$$

Bounded

3. Higher Derivative Quantum Field Theory

- Path integral analysis: Canonical Coordinate Insertion

Bounded

$$\mathcal{L} = -\frac{a}{2}\phi(\Box + {m_1}^2)(\Box + {m_2}^2)\phi \longrightarrow S_E = \int \frac{d^4p_E}{(2\pi)^4}\frac{a}{2}(p_E^2 + m_1^2)(p_E^2 + m_2^2)\left|\tilde{\phi}(p_E)\right|^2$$

$$\mathcal{L} = -\frac{1}{2}\psi_2(\Box + m_2^2)\psi_2 + \frac{1}{2}\psi_1(\Box + m_1^2)\psi_1 \longrightarrow S_E = \int \frac{d^4p_E}{(2\pi)^4}\left(\frac{1}{2}(p_E^2 + m_2^2)\left|\tilde{\psi}_2(p_E)\right|^2 - \frac{1}{2}(p_E^2 + m_1^2)\left|\tilde{\psi}_1(p_E)\right|^2\right)$$

Unbounded

What is happening? We focus on the path integral of the theory

3. Higher Derivative Quantum Field Theory

- Path integral analysis: Canonical Coordinate Insertion

$$W[J] = \int D\phi e^{i\int d^4\boldsymbol{x}\frac{-a}{2}\phi(\Box+m_1{}^2)(\Box+m_2{}^2)\phi+i\int d^4\boldsymbol{x}\, J\phi}$$

$$1 = \int D\psi_1 D\psi_2 \delta\left(\psi_1 - \sqrt{\frac{a}{\Delta m^2}}(\Box+m_2{}^2)\phi\right)\delta\left(\psi_2 - \sqrt{\frac{a}{\Delta m^2}}(\Box+m_1{}^2)\phi\right)$$

$$W[J] = \int D\psi_1 D\psi_2 D\lambda e^{i\int d^4\boldsymbol{x}\left(-\frac{1}{2}\psi_2(\Box+m_2^2)\psi_2+\frac{1}{2}\psi_1(\Box+m_1^2)\psi_1+\lambda\left((\Box+m_2^2)\psi_2-(\Box+m_1^2)\psi_1\right)\right)+i\int d^4x\, J\,\frac{\psi_2-\psi_1}{\sqrt{a\Delta m^2}}}$$

Also unbounded

Lagrange multiplier

3. Higher Derivative Quantum Field Theory

- Path integral analysis: Canonical Coordinate Insertion

$$W[J] = \int D\psi_1 D\psi_2 D\lambda e^{i\int d^4\boldsymbol{x}\left(-\frac{1}{2}\psi_2(\Box+m_2^2)\psi_2+\frac{1}{2}\psi_1(\Box+m_1^2)\psi_1+\lambda\left((\Box+m_2^2)\psi_2-(\Box+m_1^2)\psi_1\right)\right)+i\int d^4x\, J\,\frac{\psi_2-\psi_1}{\sqrt{a\Delta m^2}}}$$

→ The fields $\psi_2 + \psi_1$ and λ can be integrated out by virtue of not being in the source terms, resulting in a bounded Euclidean action

→ We have explicitely shown that these two fields do not contribute in either canonical quantization and in the perturbative expansión of the Green functions

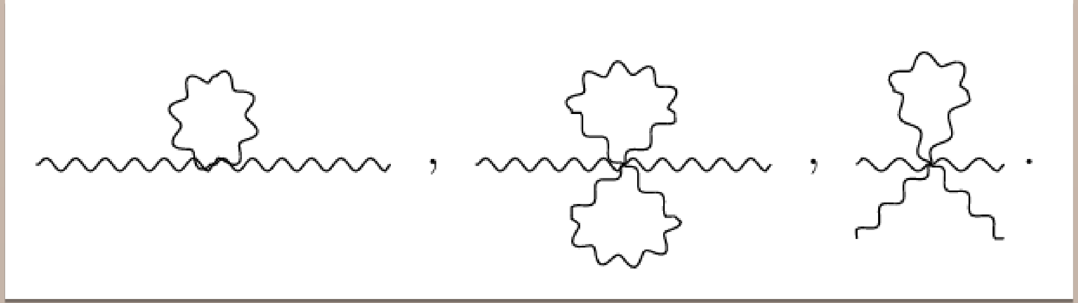
4. Renormalizability of Higher Derivative QFT

- Improved Ultraviolet Convergence of the Green Functions

→ We have proven the renormalizability of any ϕ^{2n} self-interaction

→ Key point: only diagrams with one vertex are superficially divergent

Ej: $\lambda_4\phi^4 + \lambda_6\phi^6$



4. Renormalizability of Higher Derivative QFT

- $\lambda_4\phi^4 + \lambda_6\phi^6$ all-order renormalization ($\overline{MS}$-scheme)

$$\delta Z_\phi = 0$$

$$\delta m_1^2 = -\delta m_2^2 \equiv \delta M^2$$

Due to momentum independence of UV divergences

$$\delta\lambda_6 = 0$$

$$\delta\lambda_4 = -\frac{15\lambda_6}{(4\pi)^2 a}\frac{1}{\hat{\varepsilon}}$$

→ Necesary to cancel out the 4-point Green function UV divergence

$$(\delta M^2)^2 + \Delta m^2 \delta M^2 = \frac{3\lambda_4}{4\pi^2 a^2}\frac{1}{\hat{\varepsilon}} - \frac{90\lambda_6}{(4\pi)^4 a^3}\frac{1}{\hat{\varepsilon}^2}$$

5. Conclusion

- Rigorous canonical quantization of the theory. Path integral consistency of the interacting theory.

- Proof of the renormalizability of a general set of interactions for higher derivative theories. Several explicit examples of renormalized theories are provided.

- Open problems: Dealing with negative energy/squared norm states, cosmological applications, etc.

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