

The Cahn Effect and Equation of Motion Relations

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Topics Overview

- Introduce the Cahn effect and how it arises in the cross section of SIDIS
- What functions span this description?
- Introduce equation of motion relations
- LO computation
- NLO computation
- Conclusion



Literature on NLP

*First prelim SIDIS studies beyond tree level:

"Matches & Mis-matches" Bacchetta et al. JHEP 2008, Chen & Ma PLB 2017

Literature

Bacchetta et al. PLB 2019

MIT group, Gao, Ebert, Stewart JHEP 2022

Gamberg, Kang, Shao, Terry, Zhao arXiv:221.13209 ... & ...Page Gamberg Kang Stasto 2026...

Madrid group Vladimirov, Rodini, Scimemi, Piloñeta et al JHEP 2021, 2022, 2023, 2024 2025

Balitsky 2023 rapidity TMD evolution Balitsky & Prokudin 2026



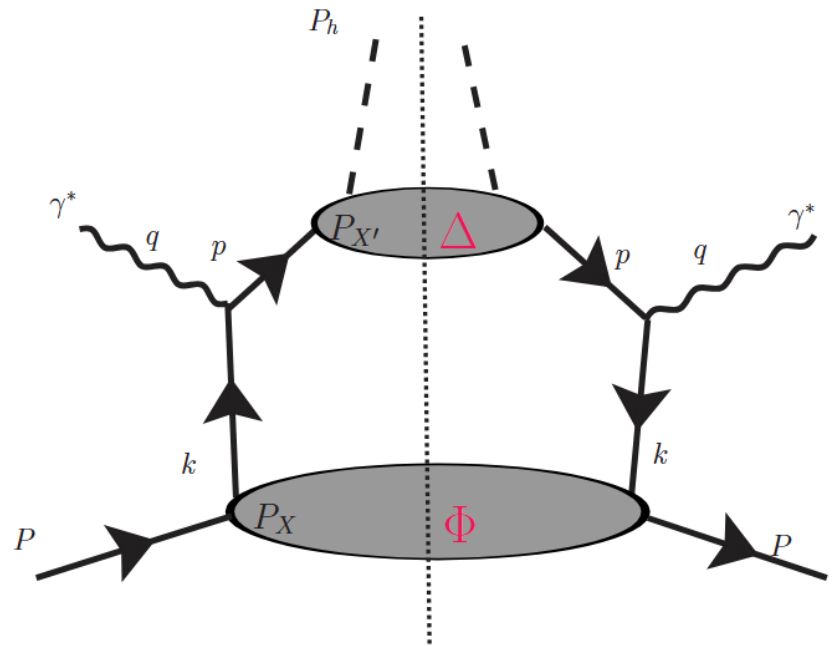
SIDIS Factorization

One factorizes the cross section into correlators that depend on separate transverse scales as depicted in the diagram to the right.

The schematic decomposition is written in terms of a hard partonic scattering cross section.

$$\sigma = \int d^2 k_T \int d^2 p_T \hat{\sigma}(Q) \otimes \Phi(x, k_T) \otimes \Delta(z, p_T)$$

Where Φ and Δ are the correlators for the PDF and Fragmentation regions, respectively.



SIDIS Kinematics

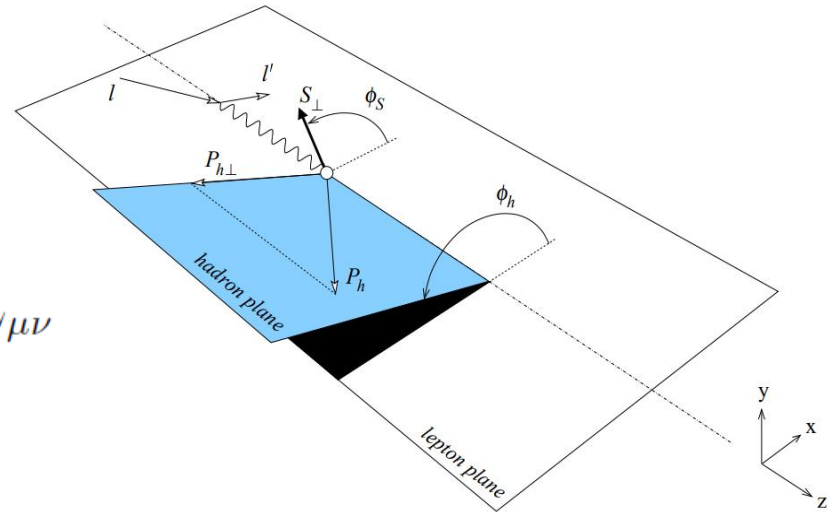
We work in Breit frame kinematics with virtual photon momentum along the z axis.

$$\frac{d\sigma^{SIDIS}}{dx dy dz d\phi_h dP_{h\perp}^2} = \frac{\alpha^2 y}{8zQ^4} 2MW^{\mu\nu} L_{\mu\nu}$$

When decomposing into structure functions, one finds a term like,

$$\frac{d\sigma^{SIDIS}}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} \ni \frac{\alpha^2}{xyQ^2} \left(1 + \frac{\gamma^2}{2x}\right) (2-y) \sqrt{1-y \cos \phi_h} F_{UU}^{\cos \phi_h}$$

This modulation is the so-called Cahn Effect



TMD Decomposition

The TMDs are obtained by explicit projection with the correlator

$$\Phi_{ij}(x, k_T) = \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ik\xi} \langle P | \bar{\psi}_j(0) \psi_i(\xi) | P \rangle \Big|_{\xi^+=0}$$
$$\Delta_{ij}(z, p_T) = \frac{1}{2z} \int \frac{d\xi^+ d^2\xi_\perp}{(2\pi)^3} e^{ip\xi} \langle 0 | \psi_i(0) | h \rangle \langle h | \bar{\psi}_j(\xi) | 0 \rangle \Big|_{\xi^-=0}$$

The full sub leading basis gets contributions from 3 parton correlators as well. the TMDs arising from these are called **dynamical** functions.

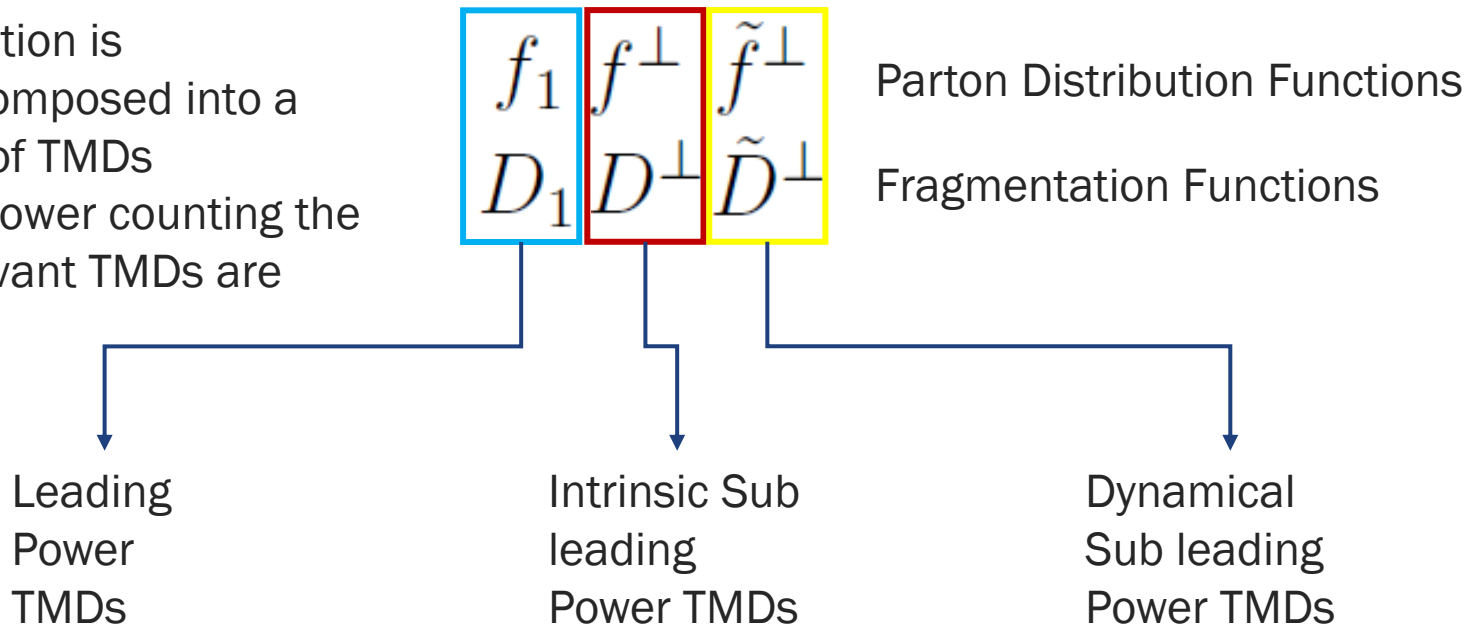
$$\tilde{\Phi}_{A,ij}^\alpha(x, k_T) = \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ik\xi} \langle P | \bar{\psi}_j(0) gA^\alpha(\xi) \psi_i(\xi) | P \rangle \Big|_{\xi^+=0}$$
$$\tilde{\Delta}_{A,ij}^\alpha(z, p_T) = \frac{1}{2z} \int \frac{d\xi^+ d^2\xi_\perp}{(2\pi)^3} e^{ip\xi} \langle 0 | \psi_i(0) | h \rangle \langle h | gA^\alpha(\xi) \bar{\psi}_j(\xi) | 0 \rangle \Big|_{\xi^-=0}$$

One then makes perturbative expansions of these expressions

TMD Decomposition

$$\frac{d\sigma^{SIDIS}}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} \ni \frac{\alpha^2}{xyQ^2} \left(1 + \frac{\gamma^2}{2x}\right) (2-y) \sqrt{1-y \cos \phi_h} F_{UU}^{\cos \phi_h}$$

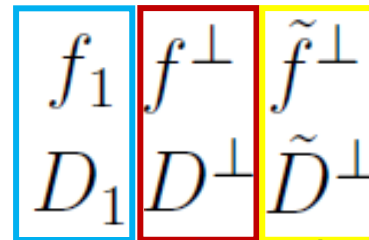
This structure function is decomposed into a set of TMDs
By power counting the relevant TMDs are



TMD Decomposition

$$\frac{d\sigma^{SIDIS}}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} \ni \frac{\alpha^2}{xyQ^2} \left(1 + \frac{\gamma^2}{2x}\right) (2-y)\sqrt{1-y} \cos\phi_h F_{UU}^{\cos\phi_h}$$

This structure function is decomposed into a set of TMDs
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Parton Distribution Functions

Fragmentation Functions

Leading power projections onto Q-Q correlators

Sub-leading power projections onto Q-Q correlators

Leading projections onto Q-G-Q correlators



Equation of motion relations

An all-orders statement relates these functions and reduces the basis down to a small set.

From the QCD Equation of motion one relates spinor components of the correlators and can write down the following nonperturbative expressions involving the TMDs.

$$\left. \begin{aligned} x f^\perp(x, k_T) &= f_1(x, k_T) + x \tilde{f}^\perp(x, k_T) \\ \frac{D^\perp}{z}(z, p_T) &= D_1(z, p_T) + \frac{\tilde{D}^\perp}{z}(z, p_T) \end{aligned} \right\} \begin{array}{l} \text{These are the} \\ \text{equation of} \\ \text{motion} \\ \text{relations} \end{array}$$

They are necessary, but not sufficient constraints on factorization

Mulders & Tangerman 1995 NPB

Boer and Mulders, 1998 PRD

Bacchetta et al. JHEP 2007



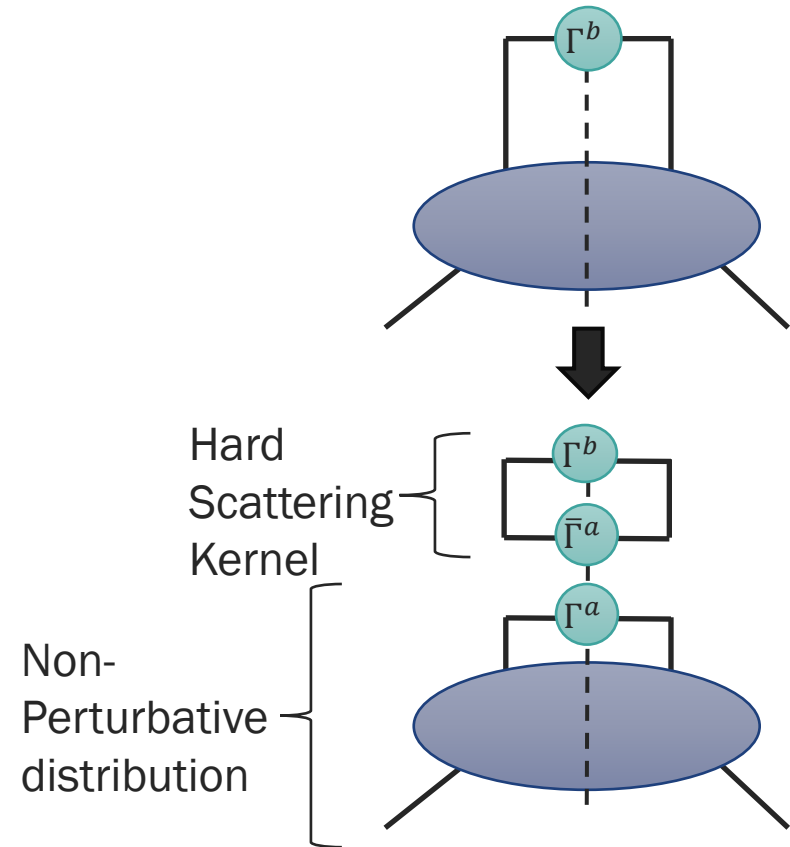
Perturbative computations

These functions can be computed perturbatively using **refactorization**.

This method demonstrates the mixing between TMDs at sub-leading power as the index a is summed over using a Fierz decomposition.

Ex. f^\perp mixes with f_1 (also with itself).

It turns out that the relevant TMDs all mix with the leading structures and this gives us a leading term in the factorization. By construction, the EOM must hold between these leading terms.



Leading Order subtleties

First, the QGQ correlators do not contribute at LO.

So the EOM implies that the intrinsic and leading power TMDs are equal to leading order. To prove this, we calculate them explicitly. Initially one finds that the trace in f^\perp vanishes at LO.

$$\text{Tr} \left\{ \frac{\gamma^-}{2} \frac{\gamma^\alpha}{2} \right\} = 0$$

This would prematurely imply that the EOM fails at leading order, but this is not the case

One instead finds that γ^- gets kinematic power corrections from bad lightcone components.



Leading Order subtleties

These power corrections take the following form.

$$\gamma^- \rightarrow \tilde{\gamma}^- = \gamma^- + \frac{\not{p}_\perp}{\tilde{p}^+}$$

When this power correction is implemented, we get a finite result for f^\perp

$$\text{Tr} \left\{ \left(\frac{\gamma^-}{2} + \frac{\not{p}_\perp}{2\tilde{p}^+} \right) \frac{\gamma^\alpha}{2} \right\} = \frac{\tilde{p}_\perp^\alpha}{\tilde{p}^+}$$

Implementing this into the LO expression for the refactorized TMD correlator, we obtain the simple form:

$$x f^\perp = f_1(x) \delta^{(2)}(k_T)$$

Which is exactly f_1 at LO.



Leading Order subtleties

The fragmentation function is similar at LO. The kinematic correction is implemented into the correlator by making the replacement,

$$\gamma^+ \rightarrow \tilde{\gamma}^+ = \gamma^+ + \frac{\tilde{k}_\perp}{\tilde{k}^+}$$

Inserting this into D^\perp nets the following expression.

$$\frac{1}{z} D^\perp(z, p_T) = \frac{1}{z^2} D_1(z) \delta^{(2)}(p_T)$$

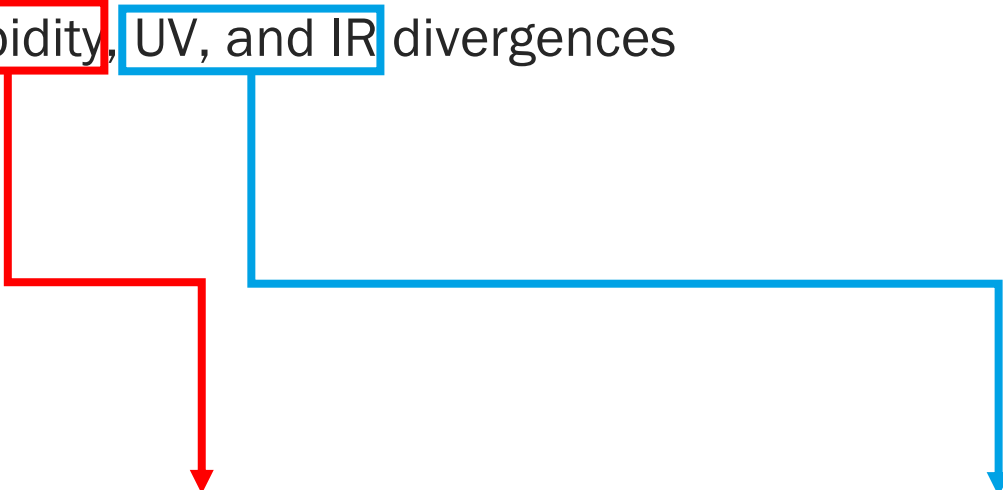
Which again, is what the unpolarized TMD D_1 becomes at LO.

Thus the EOM is satisfied at LO. Now to NLO...



At NLO

- Non-zero dynamical functions
- Rapidity, UV, and IR divergences



We use a novel rapidity regulator associated with the lightcone gluon momenta.

Dim. Reg. is used to regulate all other divergences.

The rapidity structure is what is most interesting and will be the primary focus.



Some Known Results

The leading power unpolarized functions are known, but in our regularization scheme the unsubtracted functions may be written as follows.

$$f_1^{R(1)}(x, k_T) = \frac{\alpha_s C_F}{2\pi^2} \int_x^1 \frac{dx'}{x'} f_1(x') \frac{\Gamma(1+\epsilon)(\mu^2 e^{\gamma_E})^\epsilon}{(k_T^2)^{1+\epsilon}} \times \left\{ (1-\hat{x})(1-\epsilon) - \delta(1-\hat{x}) \left(\frac{2}{\eta} + 2\text{Ln}_\nu [x'P^+] \right) + \frac{2\hat{x}}{[1-\hat{x}]_+} \right\}$$

$$D_1^{R(1)}(z, p_T) = \frac{\alpha_s C_F}{2\pi^2} \frac{1}{z^2} \int_z^1 \frac{dz'}{z'} D_1(z') \frac{\Gamma(1+\epsilon)(\mu^2 e^{\gamma_E})^\epsilon}{(p_T^2)^{1+\epsilon}} \times \left\{ (1-\hat{z})(1-\epsilon) - \delta(1-\hat{z}) \left(\frac{2}{\eta} + 2\text{Ln}_\nu \left[\frac{P_h^-}{z'} \right] \right) + \frac{2\hat{z}}{[1-\hat{z}]_+} \right\}$$

$$f_1^{V(1)}(x, k_T) = \frac{\alpha_s C_F}{2\pi} \int \frac{dx'}{x'} \delta(1-\hat{x}) \delta^{(2)}(k_T) f_1(x') \left(\frac{3}{2} + \frac{2}{\eta} + 2\text{Ln}_\nu [x'P^+] \right) \left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right)$$

$$D_1^{V(1)}(z, p_T) = \frac{1}{z^2} \frac{\alpha_s C_F}{2\pi} \int \frac{dz'}{z'} \delta(1-\hat{z}) \delta^{(2)}(p_T) D_1(z') \left(\frac{3}{2} + \frac{2}{\eta} + 2\text{Ln}_\nu \left[\frac{P_h^-}{z'} \right] \right) \left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right)$$

Rapidity Divergences

Using lightcone gauge, one defines the propagator as follows.

$$d^{\mu\nu}(l) = -g^{\mu\nu} + \frac{n^\mu l^\nu + n^\nu l^\mu}{n \cdot l} \left(\frac{\nu}{n \cdot l} \right)^\eta$$

Which regulates the rapidity pole at $n \cdot l = 0$. ν is a scale parameter. For fragmentation functions, the pole appears at $\bar{n} \cdot l = 0$ so the regulator is modified in response to this.

One can perform an equivalent analysis in covariant gauge where the regulator is implemented in the vertex of the gauge link.

Thus while the number of diagrams needed is increased, the rapidity divergences are confined to diagrams with gauge links and don't appear in purely covariant diagrams.



Sub Leading Intrinsic Functions: Real Gluon Emission

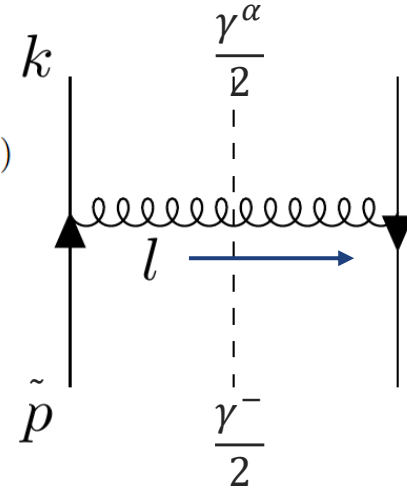
$$\hat{z} = \frac{z}{z'}$$

$$\hat{x} = \frac{x}{x'}$$

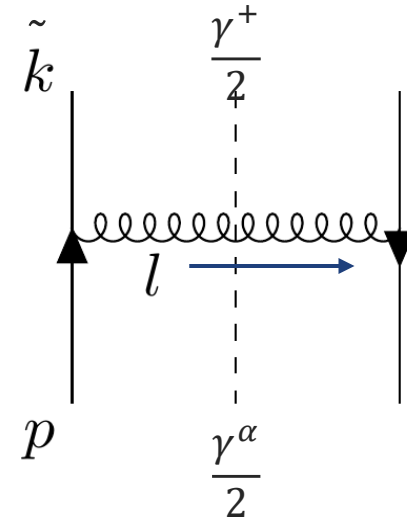
$$x' = p^+ / P^+$$

$$z' = P_h^- / k^-$$

$$\begin{aligned} \frac{k_T^\alpha}{P^+} x f_{intrinsic}^{\perp, R(1)}(x, k_T) &= g^2 C_F \left(\frac{\mu^2 e^{\gamma_E}}{4\pi} \right)^\epsilon \int_x^1 \frac{dx'}{x'} x x' P^+ \int d^2 \tilde{p}_T f_1(x', \tilde{p}_T) \\ &\times \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} (2\pi) \delta(l^2) \delta^{(2)}(l_\perp + k_T - p_\perp) \delta(l^+ + (x - x') P^+) \\ &\times d^{\mu\nu}(l) \text{Tr} \left\{ \frac{\gamma^\alpha}{2} \frac{\not{k}}{k^2} \gamma^\mu \frac{\gamma^-}{2} \gamma^\nu \frac{\not{k}}{k^2} \right\} \end{aligned}$$



$$\begin{aligned} \frac{p_T^\alpha}{P_h^-} \frac{1}{z} D_{intrinsic}^{\perp, R(1)}(z, p_T) &= g^2 C_F \left(\frac{\mu^2 e^{\gamma_E}}{4\pi} \right)^\epsilon \frac{1}{\hat{z}} \int_z^1 \frac{dz'}{z'} \frac{P_h^-}{z z'} \int d^2 \tilde{k}_T D_1(z', \tilde{k}_\perp) \\ &\times \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} (2\pi) \delta(l^2) \delta^{(2)}(l_\perp + k_\perp - p_T) \delta(l^- + P_h^- (\frac{1}{z'} - \frac{1}{z})) \\ &\times d^{\mu\nu}(l) \text{Tr} \left\{ \frac{\gamma^\alpha}{2} \frac{\not{p}}{p^2} \gamma^\mu \frac{\gamma^+}{2} \gamma^\nu \frac{\not{p}}{p^2} \right\} \end{aligned}$$



Sub Leading Intrinsic Functions: Real Gluon Emission

An explicit computation of these diagrams lead to the following results. We have expanded in the rapidity regulator to see the explicit pole.

$$\hat{z} = \frac{z}{z'}$$

$$\hat{x} = \frac{x}{x'}$$

$$x' = p^+ / P^+$$

$$z' = P_h^- / k^-$$

$$x f_{intrinsic}^{\perp, R(1)}(x, k_T) = \frac{\alpha_s C_F}{2\pi^2} (\mu^2 e^{\gamma_E})^\epsilon \int_x^1 \frac{dx'}{x'} f_1(x') \frac{\Gamma(1 + \epsilon)}{(k_T^2)^{1+\epsilon}} \times \left\{ \hat{x}(\epsilon - 1) - \delta(1 - \hat{x}) \left(\frac{1}{\eta} + \text{Ln}_\nu [x' P^+] \right) + \frac{\hat{x}}{[1 - \hat{x}]_+} \right\}$$

$$\frac{D_{intrinsic}^{\perp, R(1)}(z, p_T)}{z} = \frac{1}{z^2} \frac{\alpha_s C_F}{2\pi^2} (\mu^2 e^{\gamma_E})^\epsilon \int_z^1 \frac{dz'}{z'} D_1(z') \frac{\Gamma(1 + \epsilon)}{(p_T^2)^{1+\epsilon}} \times \left\{ (1 - \epsilon) - \delta(1 - \hat{z}) \left(\frac{1}{\eta} + \text{Ln}_\nu \left[\frac{P_h^-}{z'} \right] \right) + \frac{\hat{z}}{[1 - \hat{z}]_+} \right\}$$



Sub Leading Kinematic Functions: Real Gluon Emission

Performing an identical calculation of the sub leading TMDs with the kinematic insertion, we find that there are no contributions from the kinematic sector at NLO when matching onto Collinear PDFs/FFs.

The kinematic term, proportional to \tilde{p}_\perp , is integrated alongside the TMD $f_1(x', \tilde{p}_\perp)$, which is even in the transverse argument. Thus, integrating to zero. Recall that unlike the LO calculation, $k_T \neq \tilde{p}_\perp$.

Equivalently for FFs. Therefore,

$$x f_{kinematic}^{\perp, R(1)} = 0$$

$$\frac{1}{z} D_{kinematic}^{\perp, R(1)} = 0$$

Now onto the virtual diagrams.

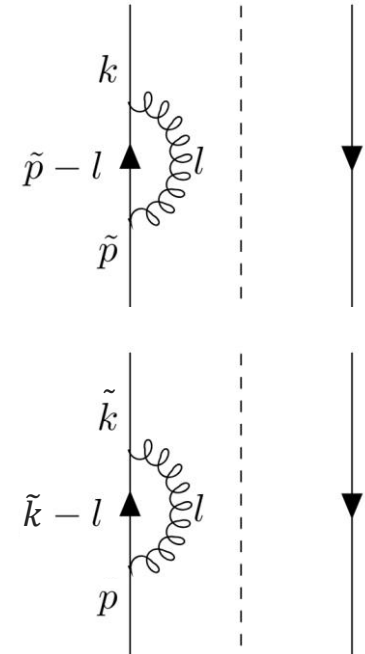


Sub Leading Intrinsic and Kinematic functions: Virtual Corrections

$$\frac{k_T^\alpha}{P^+} x f^{\perp, V(1)}(x, k_T) = \int_x^1 \frac{dx'}{x'} x \delta(1 - \hat{x}) f_1(x') \text{Tr} \left\{ \frac{\gamma^\alpha \not{\tilde{p}}}{2 \tilde{p}^2} \Sigma \frac{\tilde{\gamma}^-}{2} \right\}$$

$$\frac{p_T^\alpha}{P_h^-} \frac{1}{z} D^{\perp, V(1)}(z, p_T) = \int_z^1 \frac{dz'}{z'} \frac{1}{\hat{z} z} \delta(1 - \hat{z}) D_1(z') \text{Tr} \left\{ \frac{\gamma^\alpha \not{\tilde{k}}}{2 \tilde{k}^2} \Sigma \frac{\tilde{\gamma}^+}{2} \right\}$$

Σ is the quark self energy insertion. It is independent of Dirac structure.



Sub Leading Intrinsic and Kinematic functions: Virtual Corrections

The resulting form for the TMDs including the kinematic corrections are as follows:

$$x f^{\perp, V(1)}(x, k_T) = \frac{\alpha_s C_F}{2\pi} \int \frac{dx'}{x'} \delta(1-\hat{x}) \delta^{(2)}(k_T) f_1(x') \left(\frac{3}{2} + \frac{2}{\eta} + 2\text{Ln}_\nu [x' P^+] \right) \left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right)$$

$$\frac{1}{z} D^{\perp, V(1)}(z, p_T) = \frac{1}{z^2} \frac{\alpha_s C_F}{2\pi} \int \frac{dz'}{z'} \delta(1-\hat{z}) \delta^{(2)}(p_T) D_1(z') \left(\frac{3}{2} + \frac{2}{\eta} + 2\text{Ln}_\nu \left[\frac{P_h^-}{z'} \right] \right) \left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right)$$

Notice the differing rapidity structure compared with the real emission diagrams. The rapidity pole is different by a factor of 2.

It can also be noted that these two functions match what is obtained by the leading power functions f_1 and D_1 , which are well known.



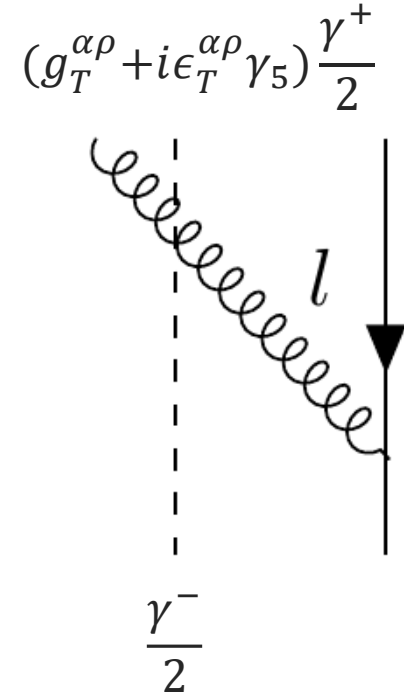
Sub Leading Dynamical Functions

The final set of functions to calculate (aside from the known leading power functions) are the dynamical TMDs.

These TMDs are $\mathcal{O}(\alpha_s)$ and are effectively tree level in the following computations.

$$k_T^\alpha x \tilde{f}^{\perp, R(1)}(x, k_T) = -g^2 C_F \left(\frac{\mu^2 e^{\gamma_E}}{4\pi} \right)^\epsilon \int \frac{dx'}{x'} x' P^+ \int d^2 \tilde{p}_\perp f_1(x', \tilde{p}_\perp) \\ \times \int \frac{d^D l}{(2\pi)^{D-1}} \delta(l^2) \delta(l^+ - P^+(x' - x)) \delta^{(2)}(l_\perp - \tilde{p}_\perp + k_T) d^{\nu\rho}(l) \\ \times \text{Tr} \left\{ \frac{\gamma^-}{2} \gamma^\nu \frac{\not{\tilde{p}} - \not{l}}{(\tilde{p} - l)^2} (g_T^{\alpha\rho} + i\epsilon_T^{\alpha\rho} \gamma_5) \frac{\gamma^+}{2} \right\}$$

$$p_T^\alpha \frac{1}{z} \tilde{D}^{\perp, R(1)}(z, p_T) = -g^2 C_F \left(\frac{\mu^2 e^{\gamma_E}}{4\pi} \right)^\epsilon \int \frac{dz'}{z'} \frac{P_h^-}{z'} \frac{1}{\hat{z}} \int d^2 \tilde{p}_\perp D_1(z', \tilde{k}_\perp) \\ \times \int \frac{d^D l}{(2\pi)^{D-1}} \delta(l^2) \delta(l^- - P_h^-(\frac{1}{z'} - \frac{1}{z})) \delta^{(2)}(l_\perp - \tilde{k}_\perp + p_T) d^{\nu\rho}(l) \\ \times \text{Tr} \left\{ \frac{\gamma^+}{2} \gamma^\nu \frac{\not{\tilde{k}} - \not{l}}{(\tilde{k} - l)^2} (g_T^{\alpha\rho} - i\epsilon_T^{\alpha\rho} \gamma_5) \frac{\gamma^-}{2} \right\}$$



Sub Leading Dynamical Functions

The results for both calculations give:

$$\begin{aligned}
 x \tilde{f}^{\perp, R(1)}(x, k_T) &= \frac{\alpha_s C_F}{2\pi^2} \int_x^1 \frac{dx'}{x'} f_1(x') \frac{\Gamma(1 + \epsilon)(\mu^2 e^{\gamma_E})^\epsilon}{(k_T^2)^{1+\epsilon}} \\
 &\times \left\{ (\epsilon - 1) + \delta(1 - \hat{x}) \left(\frac{1}{\eta} + \text{Ln}_\nu [x' P^+] \right) - \frac{\hat{x}}{[1 - \hat{x}]_+} \right\} \\
 \frac{1}{z} \tilde{D}^{\perp, R(1)}(z, p_T) &= \frac{\alpha_s C_F}{2\pi^2} \frac{1}{z^2} \int_z^1 \frac{dz'}{z'} D_1(z') \frac{\Gamma(1 + \epsilon)(\mu^2 e^{\gamma_E})^\epsilon}{(p_T^2)^{1+\epsilon}} \\
 &\times \left\{ \hat{z}(1 - \epsilon) + \delta(1 - \hat{z}) \left(\frac{1}{\eta} + \text{Ln}_\nu \left[\frac{P_h^-}{z'} \right] \right) - \frac{\hat{z}}{[1 - \hat{z}]_+} \right\}
 \end{aligned}$$

Comment: These functions carry an special rapidity divergence that was first identified by A. Vladimirov and S. Rodini. The way they should be defined is with good lightcone components only, that is with a leading power rapidity divergence. However, this is only expected at higher orders as this is tree level.



Sub Leading dynamical functions: Virtual Terms

The virtual correction to the dynamical functions are defined as the diagram where the gluon attaches on the same side as its insertion. There is no cut gluon in this diagram, much like the virtual intrinsic diagrams.

When computed using the methods from the previous calculation, one finds that it integrates to zero independent of regulator.

$$x \tilde{f}^{\perp, V(1)} = 0$$

$$\frac{1}{z} \tilde{D}^{\perp, V(1)} = 0$$

This is much like our statement regarding kinematic terms in the intrinsic functions.



Verification of Equation of Motion Relations at NLO

Putting together the previous results one can check that when constructed in the following way, one recovers the leading power unpolarized functions.

$$x f^{\perp, R(1)}(x, k_T) - x \tilde{f}^{\perp, R(1)}(x, k_T) = \frac{\alpha_s C_F}{2\pi^2} \int_x^1 \frac{dx'}{x'} f_1(x') \frac{\Gamma(1+\epsilon)(\mu^2 e^{\gamma_E})^\epsilon}{(k_T^2)^{1+\epsilon}} \\ \times \left\{ (1-\hat{x})(1-\epsilon) - \delta(1-\hat{x}) \left(\frac{2}{\eta} + 2\text{Ln}_\nu [x' P^+] \right) + \frac{2\hat{x}}{[1-\hat{x}]_+} \right\} = f_1^{R(1)}(x, k_T)$$

$$\frac{1}{z} D^{\perp, R(1)}(z, p_T) - \frac{1}{z} \tilde{D}^{\perp, R(1)}(z, p_T) = \frac{\alpha_s C_F}{2\pi^2} \frac{1}{z^2} \int_z^1 \frac{dz'}{z'} D_1(z') \frac{\Gamma(1+\epsilon)(\mu^2 e^{\gamma_E})^\epsilon}{(p_T^2)^{1+\epsilon}} \\ \times \left\{ (1-\hat{z})(1-\epsilon) - \delta(1-\hat{z}) \left(\frac{2}{\eta} + 2\text{Ln}_\nu \left[\frac{P_h^-}{z'} \right] \right) + \frac{2\hat{z}}{[1-\hat{z}]_+} \right\} = D_1^{R(1)}(z, p_T)$$

The real diagrams satisfy the equation of motion relations independent of the virtual terms. It is noted that the epsilon divergence appearing here is in fact an IR pole.



Equation of Motion for Virtual Diagrams

It turns out that since the virtual diagram associated with the dynamical function is zero, the intrinsic f^\perp virtual correction is equal to the leading power virtual correction. We found this earlier by direct calculation.

I will not repeat myself here. But this is a direct verification of the EOM for both virtual and real diagrams and both are satisfied independently.

So, the EOMs are satisfied to NLO. This in principle should also be true beyond NLO since the EOM is an operator relation. However, we have no direct computation of this.



Important Point

The intrinsic TMD carries half the rapidity divergence of the leading power TMD ([Gamberg, Kang, Shao, Terry, Zhao arXiv:221.13209](#)).

In the EOM it is combined with the special rapidity divergence in the dynamical function, which comes from a leading order calculation.

As mentioned before the dynamical function must carry the leading power rapidity divergence, but this would only appear beyond NLO.

This implies that some other basis is more precise when dealing with renormalization of these sub-leading TMDs.



Conclusion

- Defined Observable and TMD basis.
- Equation of Motion as an operator relation.
- Expansion of TMD correlators to NLO.
- Shown the consistency of the EOM by direct computation of TMD basis.
- Discussed special rapidity divergence and its association with intrinsic functions.

Thanks For Your Attention!

