Ivan Vitev

Factorizing quarkonium production matrix elements using effective field theory

QCD Evolution 2026
11–15 May 2026, El Escorial, Spain

Los Alamos NATIONAL LABORATORY

Outline of the talk

- Quarkonia and their description in the NRQCD EFT
- pNRQCD and vNRQCD, pNRQCD LDME results and challenges
- Refactorization of matrix elements in vNRQCD, new LDME relations
- Quarkonia in the TMD formalism and refactorization for Soft Transition Functions
- Future directions for quarkonia in nuclear matter
- Conclusions

Credit for this work should go to Marston Copeland

LDRD LABORATORY DIRECTED RESEARCH & DEVELOPMENT

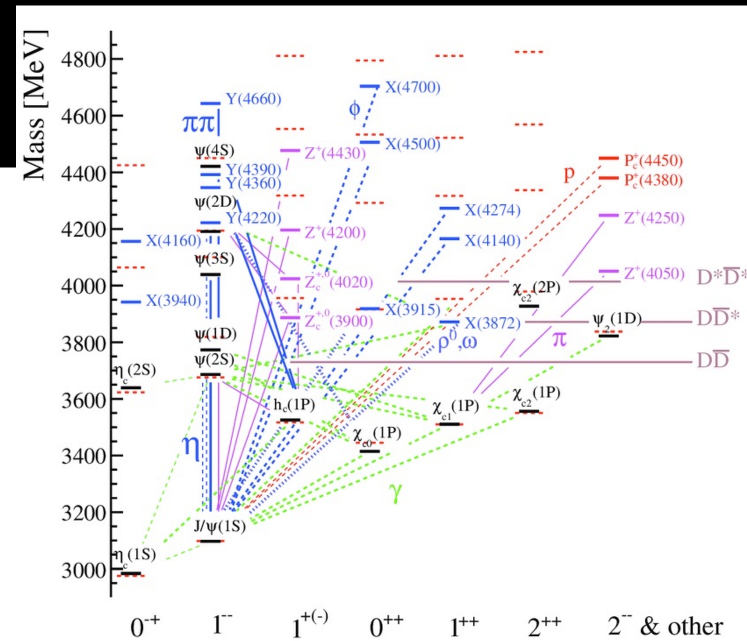
U.S. DEPARTMENT OF ENERGY

HEFTY COLLABORATION

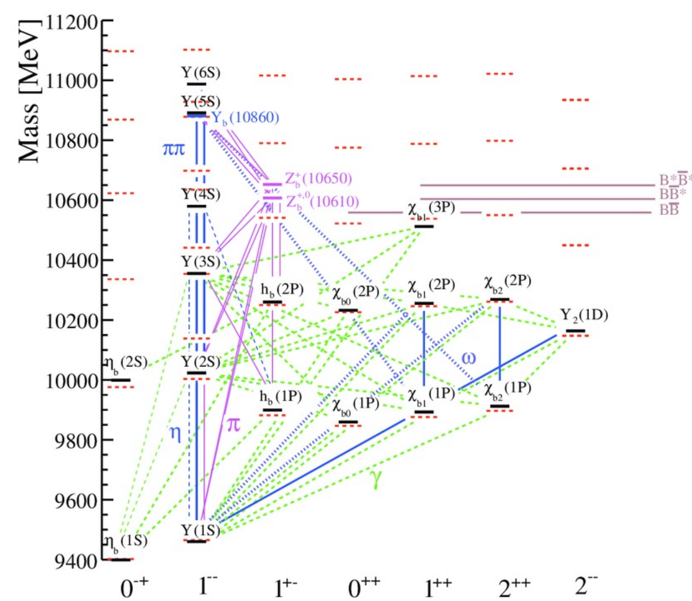
Quarkonia

Recall that both charm and bottom quarks were discovered in the form of quarkonia

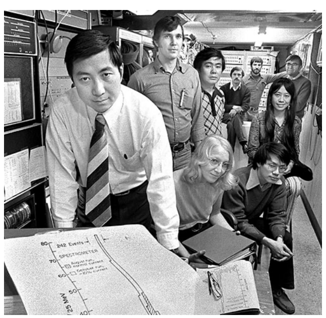
- J/psi in 1974: Samuel Ting at BNL and Burton Richter at SLAC
- Bottomonia in 1977: Leon Lederman at Fermilab



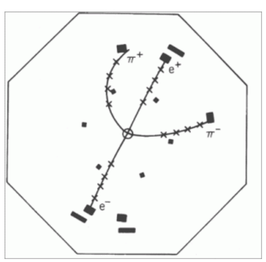
Charmonium states



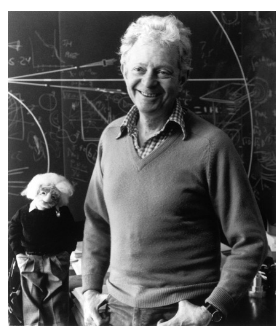
Bottomonium states



S. Ting and the J particle



Ψ' to $J/\Psi + \pi^+ + \pi^-$

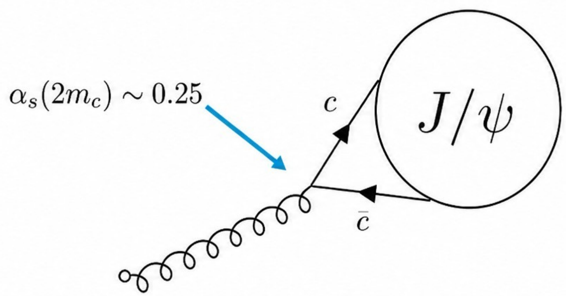


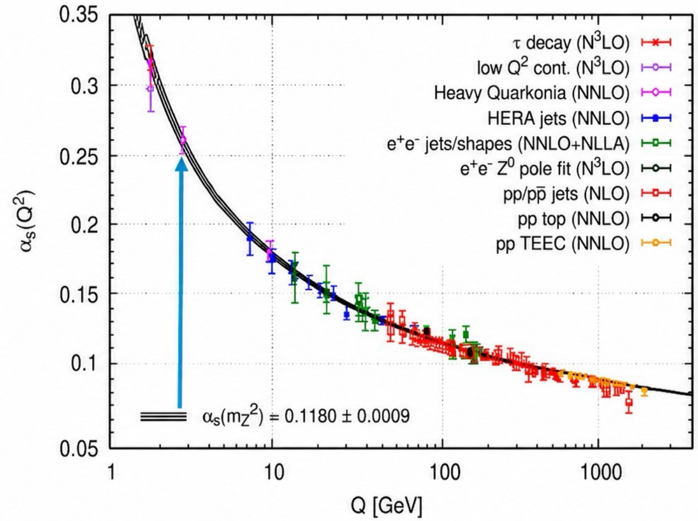
L. Lederman, 1996

Production of quarkonia

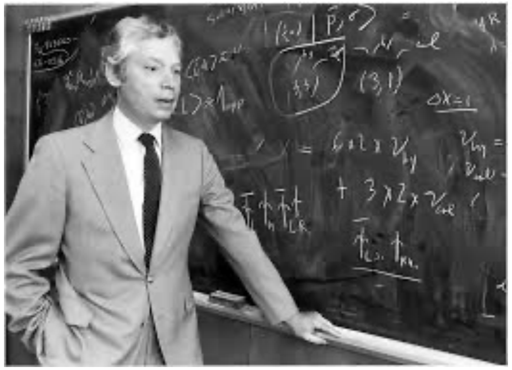
• Heavy flavor, at least in principle, provides a hard scale scale that allows the short distance part in production and decay to be treated perturbatively

In practice, the mass scales still are not very large. Even Perturbative corrections can be substantial.





The hierarchy of scales allow for an effective field theory (EFT) treatment



S. Weinberg

"Phenomenological Lagrangians"

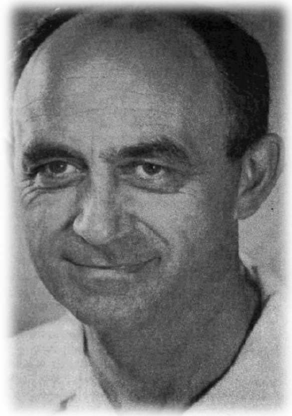
S. Wineberg *et al. (1979)*

Many different scales :

- Collision energy (Q), large mass of quarks ($2m_Q$), relative momentum of quarks (mv), binding energy (mv^2)

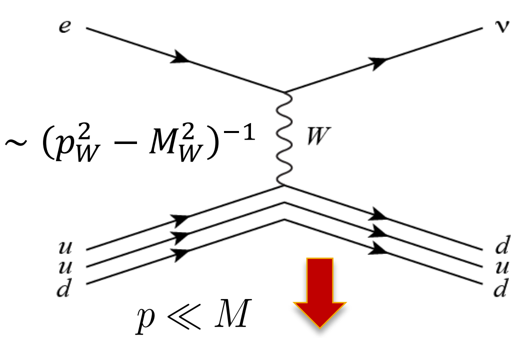
A word about Effective Field Theories [EFTs]

- The first, probably best known, effective theory is the Fermi interaction

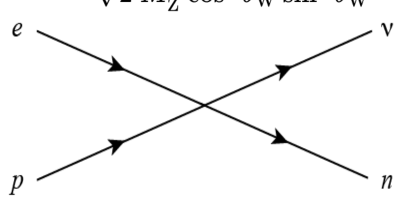


E. Fermi

Framework based on exploiting symmetries and controlled expansions for problems with a natural separation of energy/momentum or distance scales



$$G_F^0 \simeq \frac{\pi\alpha}{\sqrt{2}\, M_Z^2 \cos^2\theta_W \sin^2\theta_W}$$



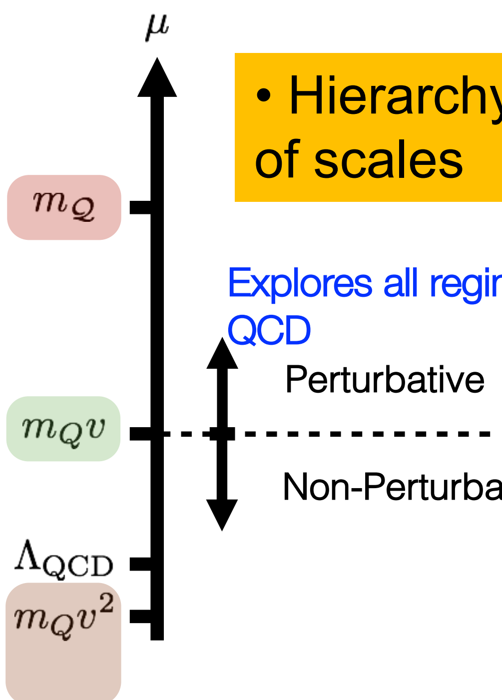
	Q	power counting	DOF in FT	DOF in EFT
Chiral Perturbation Theory (ChPT)	Λ_{QCD}	p/Λ_{QCD}	q, g	K, π
Heavy Quark Effective Theory (HQET)	m_b	Λ_{QCD}/m_b	ψ, A	h_v, A_s
Soft Collinear Effective Theory (SCET)	Q	$p_\perp/Q$	ψ, A	ξ_n, A_n, A_s
Non-Relativistic QCD (NRQCD)	m_Q	p/m_Q	ψ, A	ψ_Q, A_s, A_{us}

Examples of successful EFTs

Production of quarkonia

- Non-Relativistic QCD (NRQCD) - a particular type of effective theory (EFT)

- Hierarchy of scales



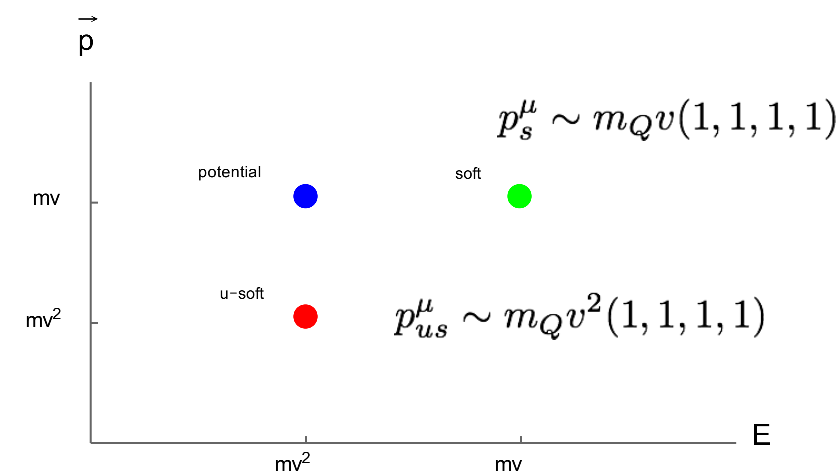
Explores all regimes of QCD

Quarkonium production contains many different scales. Take the J/psi for example

$b\bar{b}: \quad v^2 \sim 0.1$

$c\bar{c}: \quad v^2 \sim 0.3$

- Hard scale : $2m_Q \sim 3$ GeV
- Soft scale : $m_Q v \sim 750$ MeV
- Ultrasoft scale : $m_Q v^2 \sim 450$ MeV



typical momentum of heavy quark: $|\mathbf{p}_Q| \sim m_Q v$

typical kinetic energy of heavy quark: $K_Q \sim m_Q v^2$

NRQCD Lagrangian

Leading order terms

G. Bodwin *et al.* (1995)

S. Cho *et al.* (1996)

Ultra-soft

$$\mathcal{L}_{\text{NRQCD}} = \mathcal{L}_{\text{light}} + \psi^\dagger\left(iD_0 + \frac{\mathbf{D}^2}{2M}\right)\psi + \chi^\dagger\left(iD_0 - \frac{\mathbf{D}^2}{2M}\right)\chi$$

QCD without the heavy flavor

ultra-soft

+ …

NRQCD factorization formula. Short distance cross sections (perturbatively calculable) and long-distance matrix elements (fit to data, scaling relations)

$$d\sigma(a+b\to \mathcal{Q}+X) = \sum_n d\sigma(a+b\to Q\overline{Q}(n)+X)\langle\mathcal{O}_n^{\mathcal{Q}}\rangle$$

- Long-distance matrix elements (LDMEs) are universal non-perturbative parameters in the theory

$$\langle\mathcal{O}^V(n)\rangle = \sum_X \langle 0|\,\chi^\dagger\mathcal{K}\cdot\Gamma\psi\,|V,X\rangle\,\langle V,X|\,\psi\mathcal{K}\cdot\Gamma\chi\,|0\rangle$$

- Describe the hadronization of QQ pairs into the final state quarkonia
- QQ can have different quantum numbers than the final state
- ${}^{2s+1}L_J^{[c]} \to {}^3S_1^{[1]}$ transitions induce further v suppression

pNRQCD and vNRQCD

There are two improved frameworks for dealing with the dynamics below the hard scale. We will come to their discussion shortly

N. Brambilla *et al.* (2000)

pNRQCD

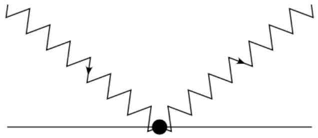
- Soft scale (m_Qv) is integrated out and matched onto a theory with composite quark/anti-quark fields (only ultrasoft d.o.f).

M. Luke *et al.* (2000)

vNRQCD

- Quark and antiquark d.o.f. are explicit. Soft scales and ultrasoft scales are correlated. Soft gluons are important especially if we want to look at low momenta/TMD approach

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{p}\left|p^{\mu}A^{\nu}_{p} - p^{\nu}A^{\mu}_{p}\right|^{2} + \sum_{\mathbf{p}}\psi^{\dagger}_{\mathbf{p}}\left\{iD^{0} - \frac{(\mathbf{p}-i\mathbf{D})^{2}}{2m}\right\}\psi_{\mathbf{p}} \\
& -4\pi\alpha_s \sum_{q,q'\mathbf{p},\mathbf{p}'}\left\{\frac{1}{q^{0}}\psi^{\dagger}_{\mathbf{p}'}\left[A^{0}_{q'},A^{0}_{q}\right]\psi_{\mathbf{p}}\right. \\
& \left. + \frac{g^{\nu 0}(q'-p+p')^{\mu} - g^{\mu 0}(q-p+p')^{\nu} + g^{\mu\nu}(q-q')^{0}}{(\mathbf{p}'-\mathbf{p})^{2}}\psi^{\dagger}_{\mathbf{p}'}\left[A^{\nu}_{q'},A^{\mu}_{q}\right]\psi_{\mathbf{p}}\right\} \\
& \qquad + \psi \leftrightarrow \chi,\ T \leftrightarrow \bar{T} \\
& + \sum_{\mathbf{p},\mathbf{q}}\frac{4\pi\alpha_s}{(\mathbf{p}-\mathbf{q})^{2}}\psi^{\dagger}_{\mathbf{q}}T^{A}\psi_{\mathbf{p}}\chi^{\dagger}_{-\mathbf{q}}\bar{T}^{A}\chi_{-\mathbf{p}} + \ldots
\end{aligned}
$$



- This type of two gluon vertex is important for the self-consistency of the theory

Open questions - LDMEs

- Universality of LDMEs – Different data sets and ranges used, different reactions

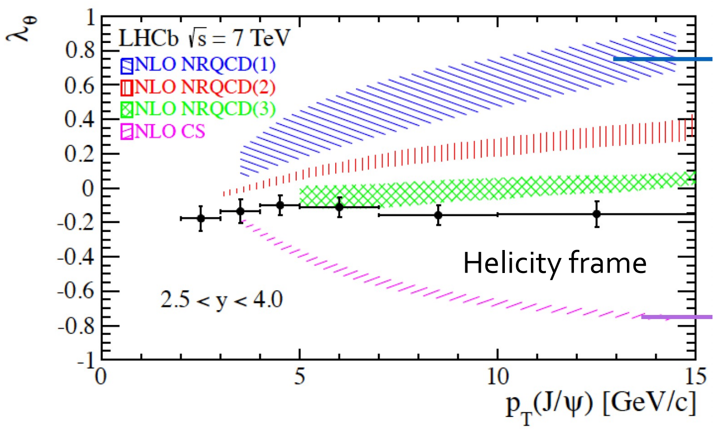
D. Boer et al. (2024)

Acronym	Reference	J/ψ hadropr.	J/ψ photopr. and e^+e^-	J/ψ polar. in hadropr.	η_c hadropr. ($P_T > 6.5$ GeV)
BK11	Butenschön et al. [104, 105, 106, 107]	✓($P_T > 3$ GeV)	✓	✗	✗
H14	Chao et al. + η_c [114]	✓($P_T > 6.5$ GeV)	✗	✓	✓
Z14	Zhang et al. [115]	✓($P_T > 6.5$ GeV)	✗	✓	✓
G13	Gong et al. [109]	✓($P_T > 7$ GeV)	✗	✓	✗
C12	Chao et al. [108]	✓($P_T > 7$ GeV)	✗	✓	✗
B14	Bodwin et al. [80]	✓($P_T > 10$ GeV)	✗	✓	✗
pNRQCD	Brambilla et al. [110, 116]	✓($P_T > 15$ GeV)	✗	✓	✗✓

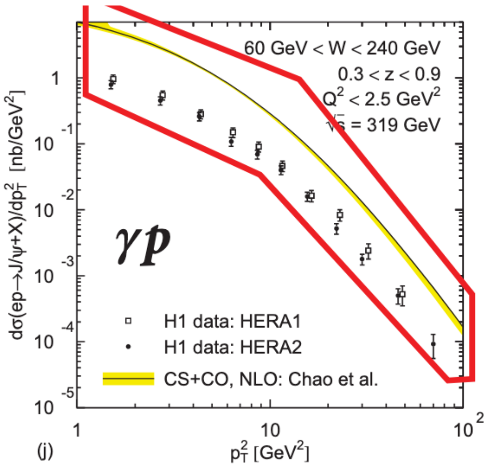
Values

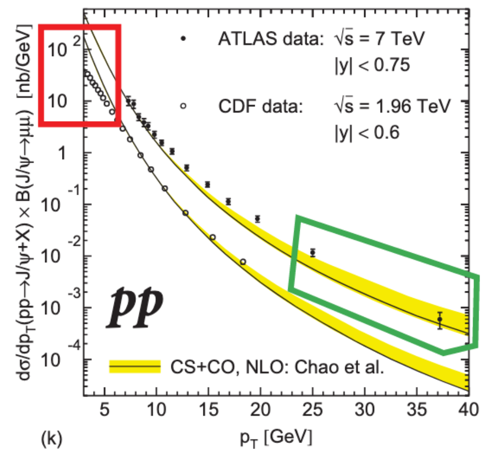
	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]})\rangle$ $\times$ GeV^3	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle$ $\times 10^{-2}$ GeV^3	$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle$ $\times 10^{-2}$ GeV^3	$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle / m_c^2$ $\times 10^{-2}$ GeV^3
B & K	1.32 ± 0.20	0.224 ± 0.59	4.97 ± 0.44	-0.72 ± 0.88
Chao et al.	1.16 ± 0.20	0.30 ± 0.12	8.9 ± 0.98	0.56 ± 0.21
Bodwin et al.	1.32 ± 0.20	1.1 ± 1.0	9.9 ± 2.2	0.49 ± 0.44

Tensions still remain if one attempts a global description



For polarization

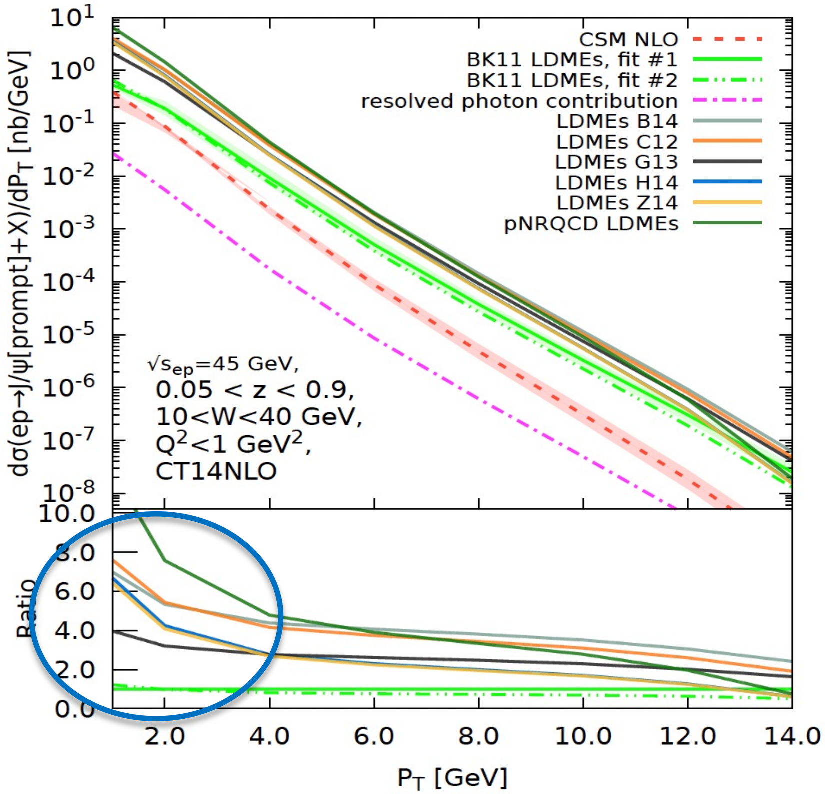


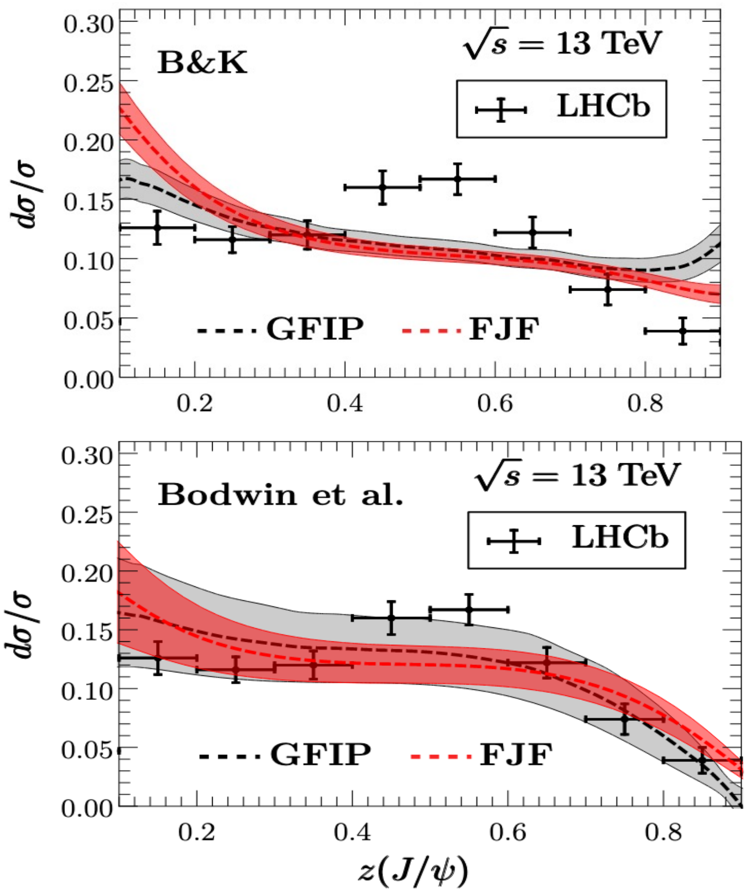


And for cross sections

One approach: Projected EIC impact

- With the current LDMEs, predicted variation of the p_T differential cross section (e.g. J/ψ) can be a factor of 4 to 10. EIC can clearly help constrain the LDMEs much more accurately. Quarkonia in jets are another possibility





D. Boer et al. (2024)

R. Bain al. (2017)

Another approach: pNRQCD analysis of LDMEs

What can theory analysis tell us about the LDMEs? Recently, pNRQCD was applied to quarkonium production matrix elements (LDMEs)

N. Brambilla et al. (2023)

- Color-octet LDMEs in terms of **state-independent** chromo-electric and magnetic correlators, times wave function at the origin

$$\langle O^V(^3S_1^{[8]})\rangle(\mu) = \frac{1}{2N_c m^2}\frac{3|R_V(0)|^2}{4\pi}\mathcal{E}_{10,10}(\mu)$$

$$\langle O^V(^3P_J^{[8]})\rangle = \frac{2J+1}{18N_c}\frac{3|R_V(0)|^2}{4\pi}\mathcal{E}_{00}(\mu)$$

$$\langle O^V(^1S_0^{[8]})\rangle = \frac{1}{6N_c m^2}\frac{3|R_V(0)|^2}{4\pi}c_F^2(m,\mu)\mathcal{B}_{00}(\mu)$$

- Predicts powerful relations between the matrix elements of **different** quarkonium states

$$\langle O^V(^3S_1^{[8]})\rangle(\mu) = \frac{m_{Q'}^2}{m_Q^2}\frac{|R_V(0)|^2}{|R_{V'}(0)|^2}\langle O^{V'}(^3S_1^{[8]})\rangle(\mu)$$

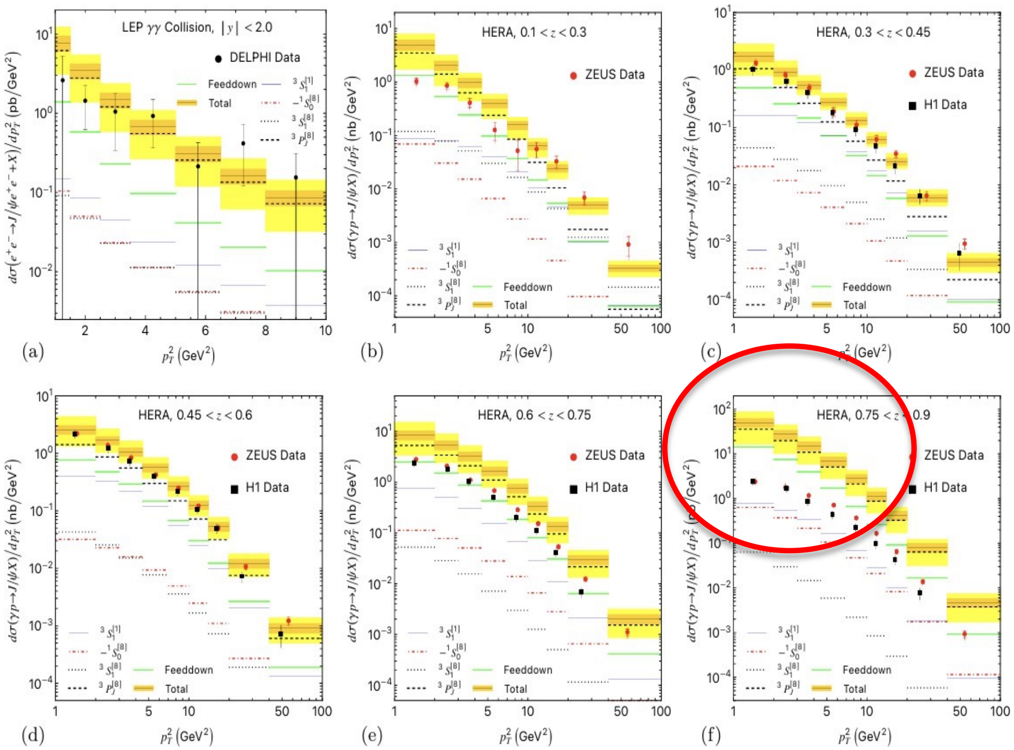
$$\langle O^V(^3P_J^{[8]})\rangle = \frac{|R_V(0)|^2}{|R_{V'}(0)|^2}\langle O^{V'}(^3P_J^{[8]})\rangle$$

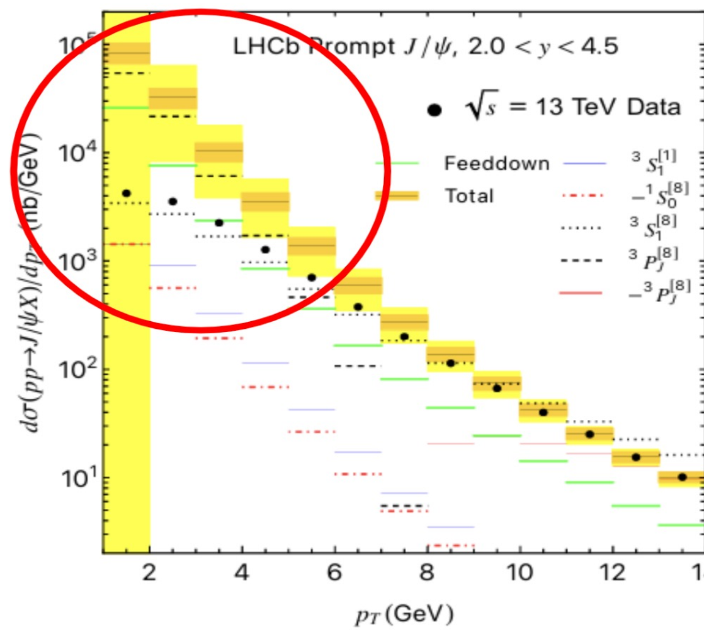
$$\langle O^V(^1S_0^{[8]})\rangle = \frac{m_{Q'}^2}{m_Q^2}\frac{c_F^2(m_Q,\mu)}{c_F^2(m_{Q'},\mu)}\frac{|R_V(0)|^2}{|R_{V'}(0)|^2}\langle O^{V'}(^1S_0^{[8]})\rangle$$

Reduces the number of non-perturbative parameters in S-wave production from 12 to 3

Analysis of LDMEs and pNRQCD relations

What can global analysis tell us about the LDMEs, in particular the scaling relationships?



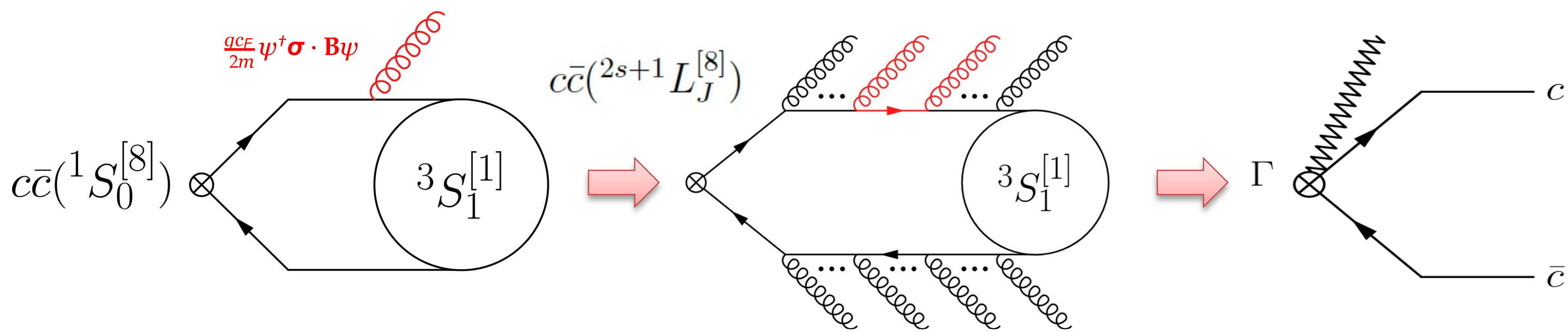


N. Brambilla al. (2025)

In many cases the $^3P_J^{[8]}$ contribution comes out to be too large. Sometimes more than an order of magnitude

Can we address this in vNRQCD?

Recently, color-octet quarkonium production operators were matched onto vNRQCD for the first time. Transitions through soft radiation



Describe the color-octet to color-singlet transition via soft gluon radiation – soft transition functions

- Ultimately what the relations are and factorization in LDMEs will be in vNRQCD depends no the state
- For the color singlet S-wave state there are no complications. Reproduces the classic NRQCD result

M. Copeland et al. (2023)

$$\langle \mathcal{O}^V({}^3S_1^{[1]})\rangle = \frac{3N_c}{2\pi}|R_V^{(0)}(0)|^2$$

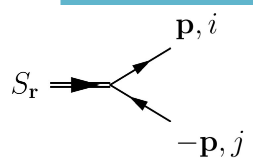
Classic NRQCD

Hubbard-Stratonovich transformation

- Fleming and Mehen showed that a Hubbard-Stratonovich transformation can relate vNRQCD (with quarks) to pNRQCD (without quarks – simply contains singlet and octet operators)

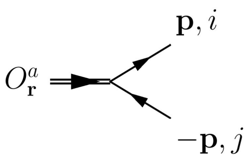
$$\psi^\dagger\psi\psi^\dagger\psi \rightarrow -\frac{1}{4}\sigma^2 + \sigma\psi^\dagger\psi,$$

S. Fleming et al. (2006)



$$-\,ie^{-i\mathbf{p}\,\mathbf{r}}\,\frac{\delta_{ij}}{\sqrt{N_c}}\,V^{(1)}(r)$$

$$V^{(1)}(r) = -C_F\frac{\alpha_s}{r}$$



$$-\,ie^{-i\mathbf{p}\,\mathbf{r}}\,\sqrt{2}\,T^a_{ij}\,V^{(8)}(r)$$

$$V^{(8)}(r) = \left(\frac{C_A}{2} - C_F\right)\frac{\alpha_s}{r}$$

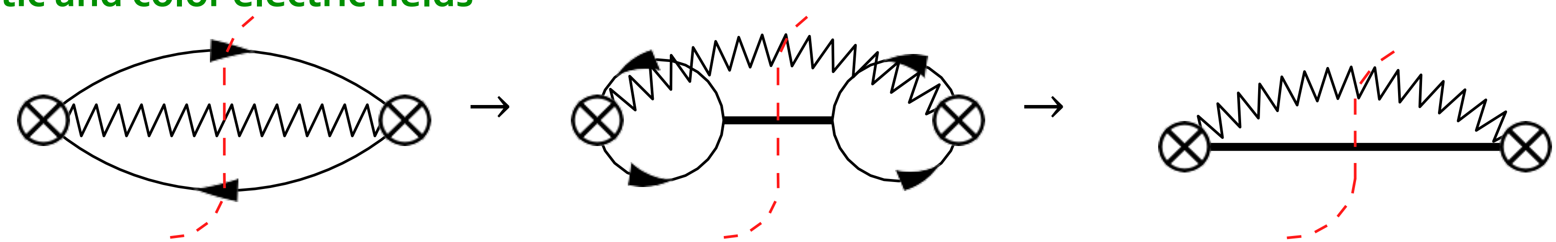
- First, match LDMEs onto the transition operators in vNRQCD. Places the operators in $^3S_1^{[1]}$
- Next, couple to color-single composite fields from the HS transformation.
- Finally, integrate out quark/antiquark loops in the theory.

$$\langle \mathcal{O}^V({}^1S_0^{[8]})\rangle = \frac{1}{3N_cM^2}\,\langle \mathcal{O}_{\boldsymbol{\mathcal{B}}}\rangle\,\frac{3|R_V(0)|^2}{2\pi} + \mathcal{O}(\mathrm{v}^2)$$ **recover pNRQCD**

$$\langle \mathcal{O}^V({}^3S_1^{[8]})\rangle = \frac{1}{N_cM^2}\,\langle \mathcal{O}_{\boldsymbol{\mathcal{E}}\boldsymbol{\mathcal{E}}}\rangle\,\frac{3|R_V(0)|^2}{2\pi}$$ **recover pNRQCD**

Expressed in terms of exp. values of color magnetic and color electric fields

$$\langle \mathcal{O}_{\boldsymbol{\mathcal{B}}}\rangle = \langle 0|\left(\frac{1}{v\cdot\mathcal{P}}\,g\boldsymbol{\mathcal{B}}^a_s\right)^2|0\rangle \qquad \langle \mathcal{O}_{\boldsymbol{\mathcal{E}}\boldsymbol{\mathcal{E}}}\rangle = \langle 0|\left[\frac{1}{2}\frac{1}{v\cdot\mathcal{P}}\left(d^{abc}g^2\boldsymbol{\mathcal{E}}^b\cdot\boldsymbol{\mathcal{E}}^c\right)\right]^2|0\rangle$$



The P-wave states

- For P-wave states we find different results

M. Copeland et al. (2026)

LDME	pNRQCD	vNRQCD
$\langle \mathcal{O}^V(^3S_1^{[8]})\rangle$	$\frac{1}{N_c M^2}\frac{3\|R_V(0)\|^2}{2\pi}\mathcal{E}_{10,10}$	$\frac{1}{N_c M^2}\frac{3\|R_V(0)\|^2}{2\pi}\langle \mathcal{O}_{\mathcal{E}\mathcal{E}}\rangle$
$\langle \mathcal{O}^V(^1S_0^{[8]})\rangle$	$\frac{1}{3N_c M^2}\frac{3\|R_V(0)\|^2}{2\pi}c_F^2\mathcal{B}_{00}$	$\frac{1}{3N_c M^2}\frac{3\|R_V(0)\|^2}{2\pi}\langle \mathcal{O}_{\mathcal{B}}\rangle$
$\langle \mathcal{O}^V(^3P_J^{[8]})\rangle$	$\frac{2J+1}{18N_c}\frac{3\|R_V(0)\|^2}{4\pi}\mathcal{E}_{00}$	$\frac{(m_Q\langle v^2\rangle_V)^2}{108N_c}(2J+1)\frac{3\left\|R_V(0)\right\|^2}{2\pi}\langle \mathcal{O}_{\mathcal{E}}\rangle$

- Depend on the derivative of the wavefunction at the origin

$$\overline{\nabla^2 R_V^{(0)}}(0) \approx m_Q^2\,\langle v^2\rangle_V\, R_V(0) + \mathcal{O}(v^2).$$

- Can be related to the binding energy

$$m_Q\,\langle v^2\rangle_V = (M_V - 2m_Q)$$

Gremm-Kapustin relationship

- Modification of scaling relationship in vNRQCD vs pNRQCD of P waves
- Explicit velocity suppression term of the operator
- Together with numerical prefactors these go in the right direction of suppressing the $^3P_J^{[8]}$ LDME

$$\langle \mathcal{O}^V(^3P_J^{[8]})\rangle = \left(\frac{m_Q\,\langle v^2\rangle_V}{m_{Q'}\,\langle v^2\rangle_{V'}}\right)^2 \frac{\left|R_V(0)\right|^2}{\left|R_{V'}(0)\right|^2}\langle \mathcal{O}^{V'}(^3P_J^{[8]})\rangle$$

Explicit v suppression

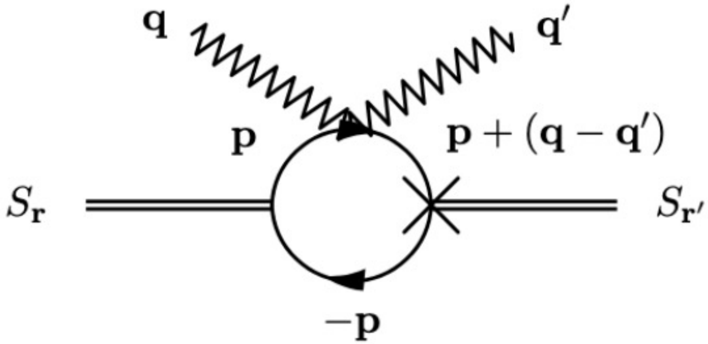
	$\langle \mathcal{O}^V(^3S_1^{[1]})\rangle$	$\langle \mathcal{O}^V(^3S_1^{[8]})\rangle$	$\langle \mathcal{O}^V(^1S_0^{[8]})\rangle$	$\langle \mathcal{O}^V(^3P_J^{[8]})\rangle/m_Q^2$
v-scaling	v^3	v^7	v^6	v^8

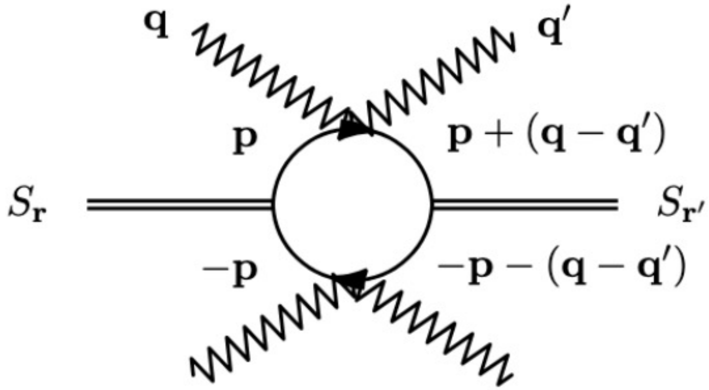
Violation of LDME factorization in the general case

Soft gluons can spoil refactorization of LDMEs

M. Copeland et al. (2026)

- Recall the two-gluon vertex. A single insertion will put the state off-shell - not considered
- One can have two insertions, however. And this ends up not being velocity suppressed





- The original hope was to resum all of the double soft gluon insertions as a Wilson line, do a field redefinition
- Unfortunately it doesn't work. It is not a unitary operator

$$\alpha_s(mv)\frac{1}{p^0+\bar{q}_{us}-\frac{(\mathbf{p}+\bar{\mathbf{q}})^2}{2m}}\frac{i}{2}f^{abc}T^cU^{(0)}_{\mu\nu}(\mathbf{q},\mathbf{q}')\epsilon^{\mu,a}(\mathbf{q})\epsilon^{\nu,b}(\mathbf{q}')\sqrt{2m}\xi$$

$$\to \alpha_s(mv)\sum_{\mathbf{p},\mathbf{q},\mathbf{q}'}\frac{i}{2}f^{abc}T^cU^{(0)}_{\mu\nu}(\mathbf{q},\mathbf{q}')A^{\mu,a}_{\mathbf{q}}A^{\nu,b}_{\mathbf{q}'}\frac{1}{i\overleftrightarrow{\partial}_0-\frac{\overleftrightarrow{\mathcal{P}}^2}{2m}}\psi_{\mathbf{p}}$$

$$W_{AA}=\sum_{\text{perm.}}\exp\left[\sum_{\mathbf{q},\mathbf{q}'}\frac{i\alpha_s(mv)}{2}f^{abc}T^cU^{(0)}_{\mu\nu}(\mathbf{q},\mathbf{q}')A^{\mu,a}_{\mathbf{q}}A^{\nu,b}_{\mathbf{q}'}\frac{1}{i\overleftrightarrow{\partial}_0-\frac{(\mathcal{P}-\mathcal{P}^\dagger)^2}{2m}}\right]$$

- Reason – NR expansion of the propagator

Specific case of production

Look at the case when the heavy quark-antiquark pair is produces at the same point. A way out of the predicament

- Reason : proportional to $\sim 1/V(r)$, but $r \to 0$ in production

Use the equations of motion for composite fields

Integrate out quark loops

$$\left(E_{Q\bar{Q}} + \frac{\nabla_{\mathbf{r}}^2}{m}\right) S_{\mathbf{r}} = V^{(1)}(r) S_{\mathbf{r}}$$

$$\lim_{a\to 0} \frac{\alpha_s(mv)^2}{4\sqrt{N_c}} \sum_{X_s, X_{us}} \langle 0| \sigma^i i \int d^3\mathbf{r} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \Bigg[\frac{1}{V^{(1)}(r)} \frac{1}{V(r) - \bar{q}^0_{us} - \frac{(-i\nabla - \mathbf{q})^2}{2m}}$$
$$\times \frac{1}{\bar{q}^0_{us} - \frac{\nabla^2}{2m} + \frac{(-i\nabla - \bar{\mathbf{q}})^2}{2m}} + \frac{1}{V(r) - \frac{-2i\nabla\cdot\bar{\mathbf{q}} + \bar{\mathbf{q}}^2}{m}} \frac{1}{\bar{q}^0_{us} - \frac{(-i\nabla - \bar{\mathbf{q}})^2}{2m} + V(r)} \frac{1}{-\bar{q}^0_{us} + \frac{(-i\nabla - \bar{\mathbf{q}})^2}{2m} + \frac{\nabla^2}{2m}} \Bigg]$$
$$\times \left[-i f^{abc} T^c \frac{2q^0}{\bar{\mathbf{q}}^2} \boldsymbol{\mathcal{E}}^a_{\mathbf{q}} \cdot \boldsymbol{\mathcal{E}}^b_{\mathbf{q}'} \right] \left[-i f^{a'b'c'} \bar{T}^{c_1} \frac{2q^0}{\bar{\mathbf{q}}^2} \boldsymbol{\mathcal{E}}^{a'}_{-\mathbf{q}} \cdot \boldsymbol{\mathcal{E}}^{b'}_{-\mathbf{q}'} \right]$$
$$\times V^{(1)}(r) \delta^{(3)}(\mathbf{r} - \mathbf{a}) e^{i\bar{\mathbf{q}}\cdot\mathbf{r}} S^\dagger_{\mathbf{r}} \Big] |V, X_s, X_{us}\rangle \,.$$

Simply more singular potentials in the denominator

- It is a practical solution, but not nearly as elegant as the emergence of a Wilson line and field redefinition would have been
- It also does not work at non-zero separation (r). Pointing to the fact that the soft gluons do not factorize in general

TMD formalism for quarkonium production

Fragmentation mechanism and TMD formalism at small and intermediate p_T – require very different factorization

Non-perturbative physics captured in **shape functions** (state transitions implicit)

$$\frac{d\sigma}{dy\, d^2q_\perp} = \frac{4M^4\, H(M^2,\mu^2)}{2sM^2(N_c^2-1)}\, \Gamma^*_{\rho\sigma}\Gamma_{\mu\nu}(2\pi) \int d^2\boldsymbol{k}_{n\perp} d^2\boldsymbol{k}_{\bar{n}\perp} d^2\boldsymbol{k}_{s\perp}\, \delta^{(2)}\left(\boldsymbol{q}_\perp - \boldsymbol{k}_{n\perp} - \boldsymbol{k}_{\bar{n}\perp} - \boldsymbol{k}_{s\perp}\right)$$
$$\times\, G^{\sigma\nu}_{g/A}(x_A,\boldsymbol{k}_{n\perp},S_A;\zeta_A,\mu)\, G^{\rho\mu}_{g/B}(x_B,\boldsymbol{k}_{\bar{n}\perp},S_B;\zeta_B,\mu)\, S_{\eta_Q}\left[{}^1S_0^{[1]}\right](\boldsymbol{k}_{s\perp};\mu)\,,$$

$$\tilde{S}_{\eta_Q}\left[{}^1S_0^{[1]}\right](\xi_T;\mu) = \frac{\tilde{S}^{(0)}_{\eta_Q}\left[{}^1S_0^{[1]}\right](\xi_T;\mu)}{\tilde{S}(\xi_T;\mu)}$$

M. Echevarria (2019)

The next step in the development is **soft transition function** (state transitions explicit). **Manifest power and velocity counting**

S. Fleming *et al.* (2019)

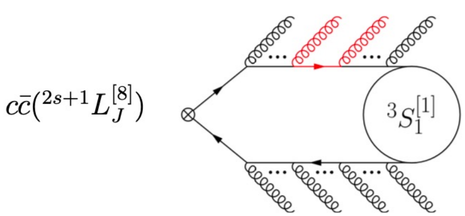
- New NRQCD factorization formula in terms of TMDShFs and TMDSTFs

$$\frac{d\sigma_{e+p\to J/\psi+X}}{dx dz d\mathbf{P}_{T,\psi}} = \sum_n \int d^2\mathbf{k}_T d^2\mathbf{q}_T\, d\hat{\sigma}_{\gamma^*+p\to c\bar{c}(n)+X}(x,z,\mathbf{k}_T)\Big[S_{n\to J/\psi}(\mathbf{q}_T)$$
$$+\, T_{n\to J/\psi}(\mathbf{q}_T)\Big]\delta^{(2)}(\mathbf{P}_{\psi,T} - \mathbf{q}_T - \mathbf{k}_T)\,.$$

M. Copeland *et al.* (2025)

where

$$d\hat{\sigma}_{\gamma^*+p\to c\bar{c}(n)+X}(x,z,\mathbf{k}_T) = \sum_n H^n_{\alpha\alpha'}\, f^{\alpha\alpha'}_{g/p}(x,\mathbf{k}_T)\,\delta(1-z) + \mathcal{O}(\alpha_s)$$



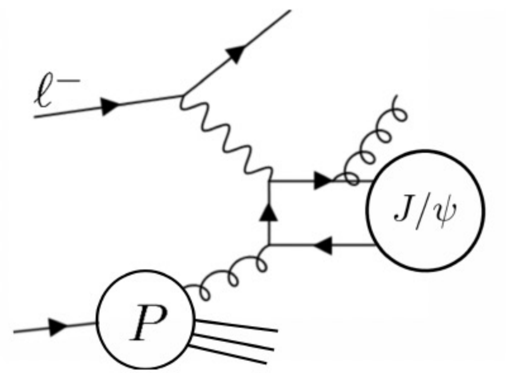
Ongoing work to understand the connection and the TMD framework

S. Romera *et al.* (2026-)

Application of refactorization to TMDSTFs

M.Copeland *et al.* (2026)

- TMDSTF leading in the velocity power counting. Apply the matrix element refactorization techniques



- At the lowest order there is one TMDSTF $T^V_{{}^1S_0^{[8]}}(\mathbf{b}_T)$
- Depends on the n-direction radiation from the initial proton (soft Wilson lines). So it is not universal

$$T_{{}^1S_0^{[8]}\to J/\psi}(\mathbf{q}_T) = \frac{1}{(N_c^2-1)\sqrt{S(\mathbf{b}_T)}} \sum\!\!\!\!\!\!\int_{X_s} \int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{-i\mathbf{b}_T\cdot\mathbf{q}_T}$$
$$\times \mathrm{Tr}_c \langle 0| \left[\mathcal{S}_n^{\dagger ae} \chi^\dagger_{\mathbf{p}\bar{\mathbf{Q}}} \left[\frac{c_F}{2MN_c} \frac{1}{v\cdot\mathcal{P}} \boldsymbol{\sigma}\cdot g\mathbf{B}_s^a \right] \psi_{\mathbf{p}\mathbf{Q}} \right](\mathbf{b}_T) |J/\psi, X_s\rangle$$
$$\times \langle J/\psi, X_s| \left[\mathcal{S}_n^{a'e} \psi^\dagger_{\mathbf{p}\mathbf{Q}} \left[\frac{c_F}{2MN_c} \frac{1}{v\cdot\mathcal{P}} \boldsymbol{\sigma}\cdot g\mathbf{B}_s^{a'} \right] \psi_{\mathbf{p}\mathbf{Q}} \right](0) |0\rangle$$

Important for example, as it can access the gluon TMDPDF

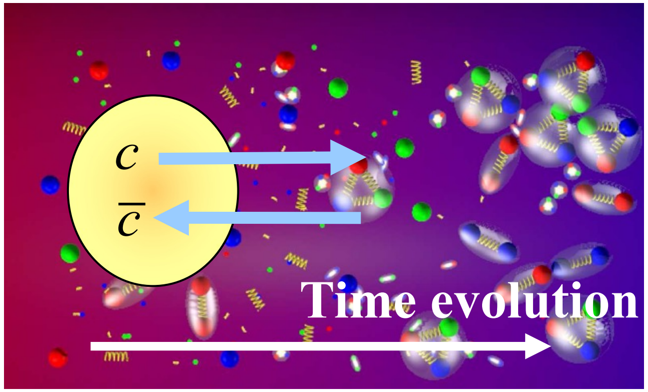
- Using similar techniques and mutipole expanding in b we can refactorize the TMDSTF

$$T^V_{{}^1S_0^{[8]}}(\mathbf{b}_T) = \frac{1}{3N_cM^2} \langle \mathcal{O}_{n\mathcal{B}}(\mathbf{b}_T)\rangle \frac{3|R_V(0)|^2}{2\pi} \qquad \langle \mathcal{O}_{n\mathcal{B}}(\mathbf{b}_T)\rangle = \frac{1}{(N_c^2-1)\sqrt{S(\mathbf{b}_T)}} \mathrm{Tr}_c \langle 0| \left[\mathcal{S}_n^{\dagger ae} \frac{1}{v\cdot\mathcal{P}} g\boldsymbol{\sigma}\cdot\boldsymbol{\mathcal{B}}_s^a \right]^i (\mathbf{b}_T) \left[\mathcal{S}_n^{a'e} \frac{1}{v\cdot\mathcal{P}} g\boldsymbol{\sigma}\cdot\boldsymbol{\mathcal{B}}_s^a \right]^j (0) |0\rangle$$

- We find relations between the TMDSTFs of different S-wave quarkonium states
- Significantly increases the predictive power of the framework. - J/ψ, Υ, $\psi(2S)$, etc. TMD cross sections are related

$$T^V_{{}^1S_0^{[8]}}(\mathbf{b}_T) = \frac{m^2_{Q'}}{m^2_Q} \frac{|R_V(0)|^2}{|R_{V'}(0)|^2} T^{V'}_{{}^1S_0^{[8]}}(\mathbf{b}_T)$$

NRQCD in the nuclear medium



- At the Lagrangian level

Y. Makris et al. (2019)

$$\mathcal{L}_{\mathrm{NRQCD}_G} = \mathcal{L}_{\mathrm{NRQCD}} + \mathcal{L}_{Q-G/C}(\psi, A^{\mu,a}_{G/C}) + \mathcal{L}_{g-G/C}(A^{\mu,b}_{s}, A^{\mu,a}_{G/C}) \quad + \psi \longleftrightarrow \chi$$

- Glauber gluons - transverse to the direction of propagation contribution
- Coulomb gluons - isotropic momentum distribution

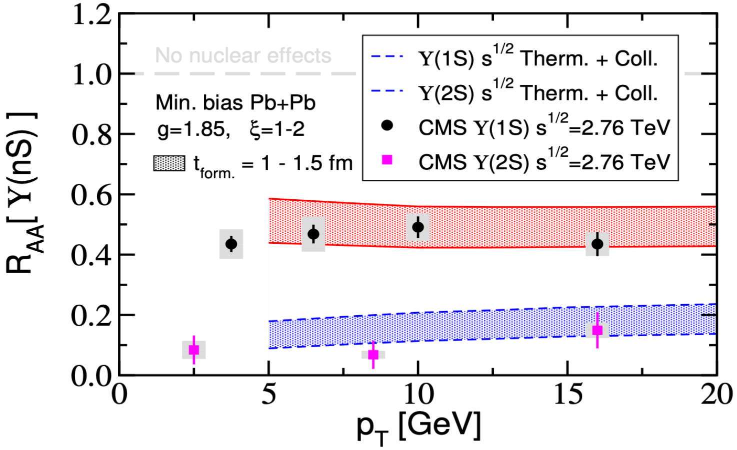
Results: depend on the type of the source of scattering in the medium at higher orders only

$$\mathcal{L}^{(0)}_{Q-G/C}(\psi, A^{\mu,a}_{G/C}) = \sum_{\mathbf{p},q_T} \psi^{\dagger}_{\mathbf{p}+\mathbf{q}_T}\Big(-gA^{0}_{G/C}\Big)\psi_{\mathbf{p}} \quad (collinear/static/soft).$$

$$\mathcal{L}^{(1)}_{Q-G}(\psi, A^{\mu,a}_{G}) = g\sum_{\mathbf{p},q_T} \psi^{\dagger}_{\mathbf{p}+\mathbf{q}_T}\Big(\frac{2A^{\mathbf{n}}_{G}(\mathbf{n}\cdot\boldsymbol{\mathcal{P}}) - i\Big[(\boldsymbol{\mathcal{P}}_{\perp}\times\mathbf{n})A^{\mathbf{n}}_{G}\Big]\cdot\boldsymbol{\sigma}}{2m}\Big)\psi_{\mathbf{p}} \quad (collinear)$$

$$\mathcal{L}^{(1)}_{Q-C}(\psi, A^{\mu,a}_{C}) = 0 \quad (static)$$

$$\mathcal{L}^{(1)}_{Q-C}(\psi, A^{\mu,a}_{C}) = g\sum_{\mathbf{p},q_T} \psi^{\dagger}_{\mathbf{p}+\mathbf{q}_T}\Big(\frac{2\mathbf{A}_{C}\cdot\boldsymbol{\mathcal{P}} + [\boldsymbol{\mathcal{P}}\cdot\mathbf{A}_{C}] - i\Big[\boldsymbol{\mathcal{P}}\times\mathbf{A}_{C}\Big]\cdot\boldsymbol{\sigma}}{2m}\Big)\psi_{\mathbf{p}} \quad (soft)$$



Phenomenology at high transverse momentum, extend to low transverse momentum

Other approaches

Y. Akamatsu et al. (2015)

N. Brambilla et al. (2022)

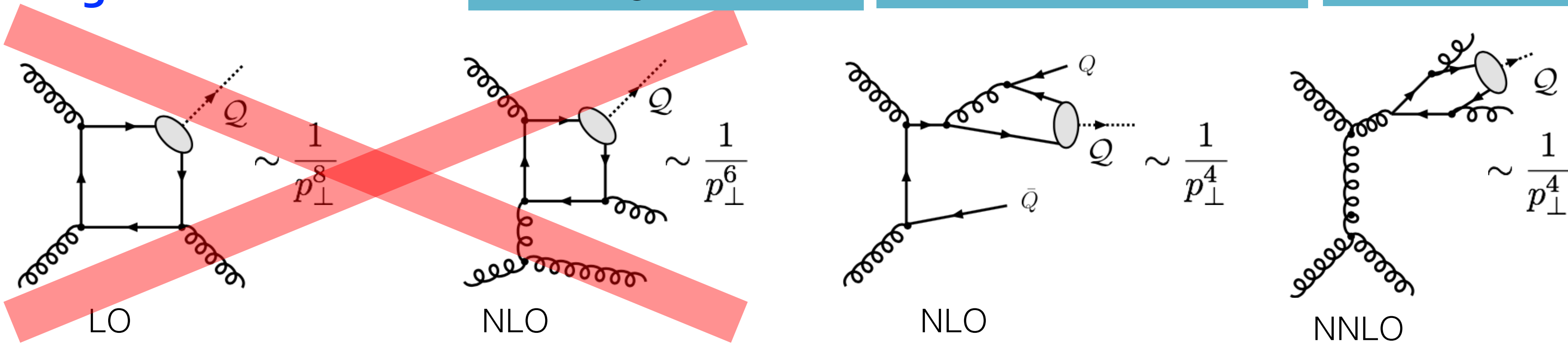
Conclusions

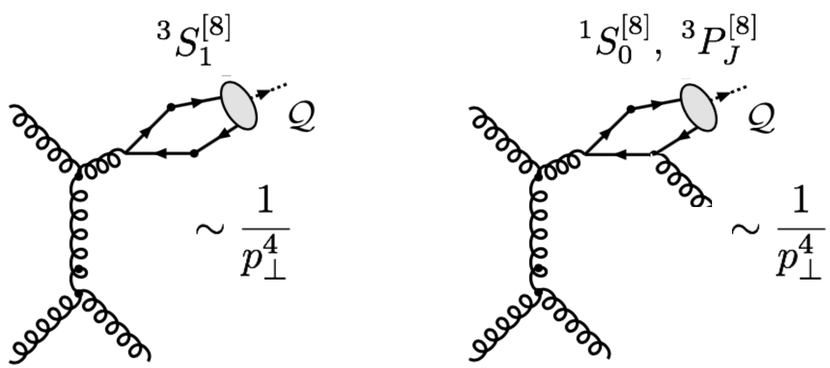
- Quick overview of quarkonium production in NRQCD, including successes and challenges
- Clarified the connection between pNRQCD and vNRQCD. Derived new factorization for quarkonium production matrix elements in vNRQCD using Hubbard-Stratonovich transformation and novel field redefinitions
- Verified some pNRQCD for but derived new relations for P-wave states. Expectations to reduce tensions with experimental data at low p_T
- Applied the formalism to TMD factorization and TMDSFs. Demonstrated state-independence property for TMDSTFs in SIDIS, significantly improving predictive power for quarkonium production in the TMD framework
- In the future better understand the relation between TNDSFs and TMDSTFs
- Extend the formalism to reactions with nuclei/nuclear matter effects, especially at low p_T

Energy loss and quarkonia?

Singlet contribution

S. Fleming *et al.* (2012) | M. Baumgart *et al.* (2014) | Y. Ma *et al.* (2014)

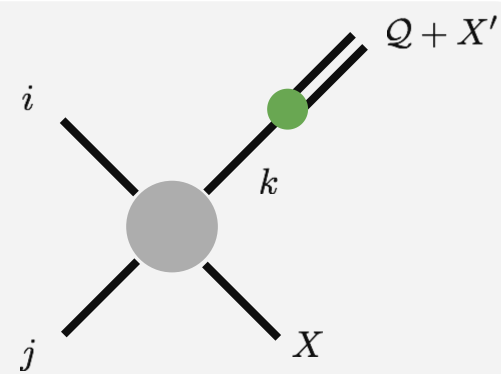




LO

NLO

(single) Parton fragmentation process



Octet contribution

Only a subset of contributions survive, now interpretable as parton fragmentation in quarkonia

LP example and applicability

$$\frac{d\sigma_h}{dp_\perp}(p_\perp) = \sum_i \int_z^1 \frac{dx}{x} \frac{d\sigma_i}{dp_\perp}\left(\frac{p_\perp}{x}, \mu\right) D_{i/h}(x,\mu) + \mathcal{O}\left(\frac{m_h^2}{p_\perp^2}\right)$$

$p_T \gg m_Q$

$$\ln\left(\frac{\mu}{p_T}\right) - \ln\left(\frac{\mu}{2m_Q}\right) \quad d_{i/n}(x,\mu)\langle \mathcal{O}_n^h \rangle$$

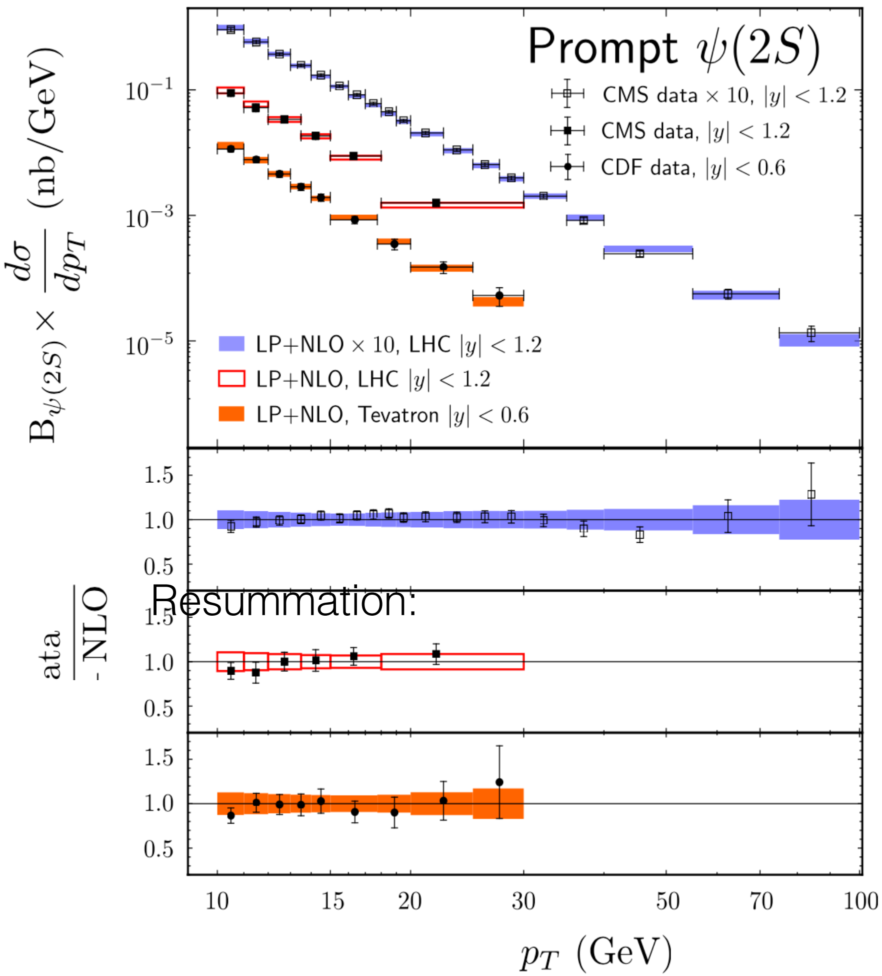
DGLAP Evolution

$$\mu \frac{d}{d\mu} D_{i/h}(z,\mu) = \sum_i \int_z^1 \frac{dx}{x} P_{ij}(x) D_{i/h}\left(\frac{z}{x}, \mu\right)$$

Resummation of $\ln(p_T/m_h)$

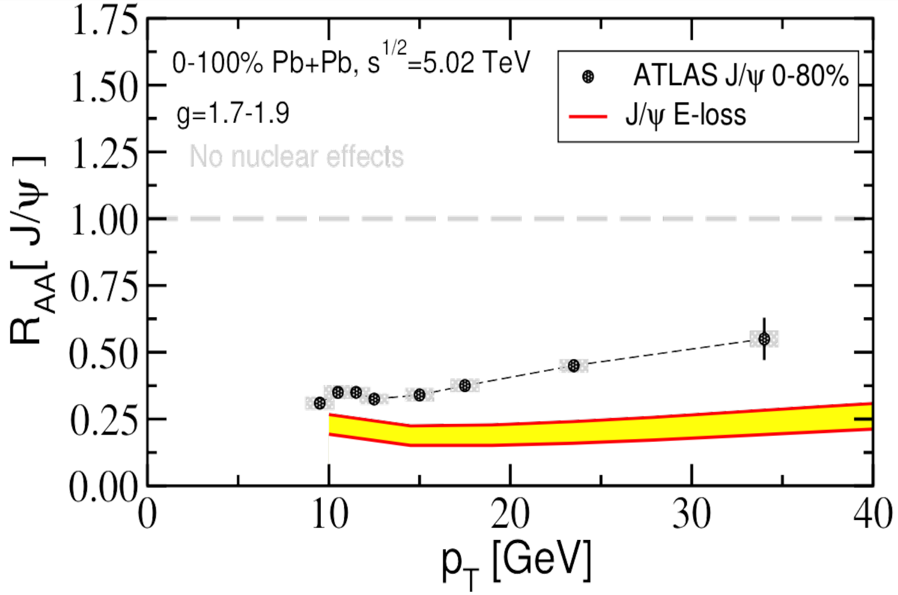
Contributions we take

	χ_{cJ}	$J/\psi(1S)/\psi(2S)$		
Initiating parton / Mechanism	$^3P_J^{[1]}$	$^3S_1^{[8]}$	$^3P_J^{[8]}/^1S_0^{[8]}$	$^3S_1^{[1]}$
g	α_s^2	α_s	α_s^2	α_s^3
Q	α_s^2	α_s^2	α_s^3	α_s^2
q	α_s^3	α_s^2	α_s^3	α_s^4



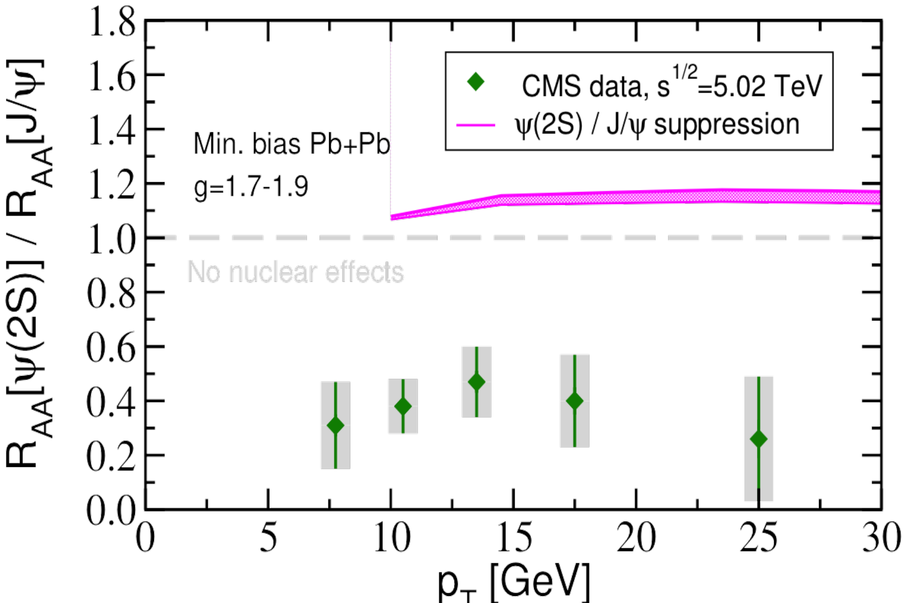
G. Bodwin *et al. (2016)*

E-loss and suppression and the ratio ψ(2S) / J/ψ



- In the double suppression ratio $R_{AA}(\psi(2S))/R_{AA}(J/\psi)$ the discrepancy is not simply in magnitude. There is a discrepancy in the sign of prediction

Makris et all. (2019)



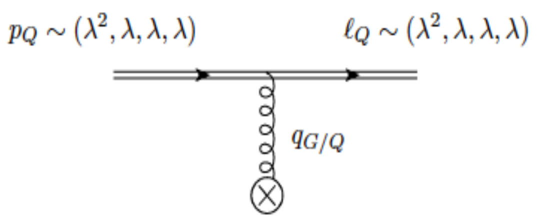
- Suppression of J/ψ overestimated by factor of 2 to 3.

The energy loss based quarkonium suppression in the p_T range measured by ATLAS and CMS (up to 40 GeV) is strongly challenged

Allowed interactions in the medium

- At the level of the Lagrangian

$$\mathcal{L}_{\mathrm{NRQCD}_G} = \mathcal{L}_{\mathrm{NRQCD}} + \mathcal{L}_{Q-G/C}(\psi, A^{\mu,a}_{G/C}) + \mathcal{L}_{g-G/C}(A^{\mu,b}_{s}, A^{\mu,a}_{G/C}) + \psi \longleftrightarrow \chi$$



Possible scaling for the virtual gluons interacting with the heavy quarks

	0 1 2 3	+ − ⊥
(1)	$q_G \sim (\lambda^2, \lambda^1, \lambda^1, \lambda^2)$	$\sim (\lambda^2, \lambda^2, \boldsymbol{\lambda}_\perp)_n$
(2)	$q_C \sim (\lambda^2, \lambda^1, \lambda^1, \lambda^1)$	$\sim (\lambda^1, \lambda^1, \boldsymbol{\lambda}_\perp)_n$

- Energy component must always be suppressed
- Glauber gluons - transverse to the direction of propagation contribution
- Coulomb gluons - isotropic momentum distribution

- Calculated the leading power and next to leading power contributions 3 different ways

Background field method	**Perform a shift in the gluon field in the NRQCD Lagrangian then perform the power-counting**
Hybrid method	**From the full QCD diagrams for single effective Glauber/Coulomb gluon perform the corresponding power-counting, read the Feynman rules**
Matching method	**Full QCD diagrams describing the forward scattering of incoming heavy quark and a light quark or a gluon. We also derive the tree level expressions of the effective fields in terms of the QCD ingredients**

Example of the background field method

- Perform the label momentum representation and field substitution (u.s. -> u.s. + Glauber)

$$\psi(x) \to \sum_{\mathbf{p}} \psi_{\mathbf{p}}(x) \,,$$

$$iD_\mu \to \mathcal{P}_\mu + i\partial_\mu - g(A_U^\mu + A_{G/C}^\mu)$$

$$iD_t = \underbrace{i\partial_t - gA_U^0 - gA_G^0}_{\sim \lambda^2} \,,$$

$$i\mathbf{D} = \underbrace{\boldsymbol{\mathcal{P}}}_{\sim \lambda} - (\underbrace{i\boldsymbol{\partial} + g\mathbf{A}_U + g\mathbf{n}A_G^{\mathbf{n}}}_{\sim \lambda^2}) + \mathcal{O}(\lambda^3) \,,$$

$$\mathbf{E} = \partial_t(\mathbf{A}_U + \mathbf{A}_G) + (\boldsymbol{\partial} + i\boldsymbol{\mathcal{P}})(A_U^0 + A_G^0) + gT^c f^{cba}(A_U^0 + A_G^0)^b(\mathbf{A}_U + \mathbf{A}_G)^a$$
$$= \underbrace{i\boldsymbol{\mathcal{P}}_\perp A_G^0}_{\sim \lambda^3} + \mathcal{O}(\lambda^4) \,,$$

$$\mathbf{B} = -(\boldsymbol{\partial} + i\boldsymbol{\mathcal{P}}) \times (\mathbf{A}_U + \mathbf{A}_G) + \frac{g}{2}T^c f^{cba}(\mathbf{A}_U + \mathbf{A}_G)^b(\mathbf{A}_U + \mathbf{A}_G)^a$$
$$= -\underbrace{(i\boldsymbol{\mathcal{P}}_\perp \times \mathbf{n})\, A_G^{\mathbf{n}}}_{\sim \lambda^3} + \mathcal{O}(\lambda^4) \,.$$

Example for a collinear source (note results depend on the type of source)

Substitute, expand and collect terms up to order λ^3

- Results: depend on the type of the source of scattering in the medium

Leading medium corrections

$$\mathcal{L}^{(0)}_{Q-G/C}(\psi, A^{\mu,a}_{G/C}) = \sum_{\mathbf{p},\mathbf{q}_T} \psi^\dagger_{\mathbf{p}+\mathbf{q}_T}\Big(-gA^0_{G/C}\Big)\psi_{\mathbf{p}} \quad (collinear/static/soft).$$

Sub-leading medium corrections

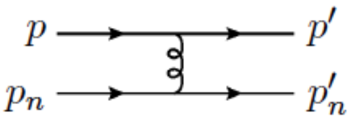
$$\mathcal{L}^{(1)}_{Q-G}(\psi, A^{\mu,a}_G) = g\sum_{\mathbf{p},\mathbf{q}_T} \psi^\dagger_{\mathbf{p}+\mathbf{q}_T}\Big(\frac{2A^{\mathbf{n}}_G(\mathbf{n}\cdot\boldsymbol{\mathcal{P}}) - i\Big[(\boldsymbol{\mathcal{P}}_\perp \times \mathbf{n})A^{\mathbf{n}}_G\Big]\cdot\boldsymbol{\sigma}}{2m}\Big)\psi_{\mathbf{p}} \quad (collinear)$$

$$\mathcal{L}^{(1)}_{Q-C}(\psi, A^{\mu,a}_C) = 0 \quad (static)$$

$$\mathcal{L}^{(1)}_{Q-C}(\psi, A^{\mu,a}_C) = g\sum_{\mathbf{p},\mathbf{q}_T} \psi^\dagger_{\mathbf{p}+\mathbf{q}_T}\Big(\frac{2\mathbf{A}_C\cdot\boldsymbol{\mathcal{P}} + [\boldsymbol{\mathcal{P}}\cdot\mathbf{A}_C] - i\Big[\boldsymbol{\mathcal{P}}\times\mathbf{A}_C\Big]\cdot\boldsymbol{\sigma}}{2m}\Big)\psi_{\mathbf{p}} \quad (soft)$$

The QCD forward scattering diagram expansion

- Looking at t-channel scattering we can also extract the form of the Glauber/Coulomb fields in terms of QCD ingredients (and recover Lagrangian)

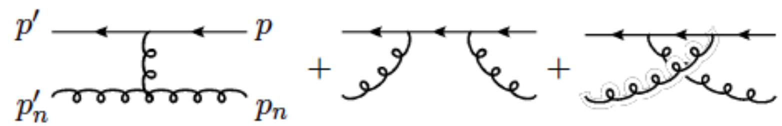
$t_{coll.} =$ 

Glauber field for collinear source

$$A_G^{\mu,a} = \frac{n^\mu}{\mathbf{q}_T^2} \sum_\ell \bar{\xi}_{n,\ell-\mathbf{q}_T} \frac{\bar{\not{n}}}{2} (gT^a) \xi_{n,\ell}$$

Coulomb field for soft source

$$A_C^{\mu,a} \equiv \frac{1}{\mathbf{q}^2} \sum_\ell \bar{\phi}_{\ell-\mathbf{q}} \gamma^\mu (gT^A) \phi_\ell$$

$t_{g-coll.} =$ 

$$= t^{(0)}_{g-coll.} + t^{(1)}_{g-coll.} + \mathcal{O}(\lambda^2) \, .$$

Glauber field for collinear source

$$A_G^{\mu,a} = \frac{i}{2} g f^{abc} \frac{n^\mu}{\mathbf{q}_T^2} \sum_\ell \left[\bar{n} \cdot \mathcal{P} \left(\mathbf{B}^{b(0)}_{n\perp,\ell-\mathbf{q}_T} \cdot \mathbf{B}^{c(0)}_{n\perp,\ell} \right) \right]$$

Coulomb field for soft source

Y. Makris et al. (2019)

$$A_C^{\mu,a} = f^{abc} \frac{ig}{2\,\mathbf{q}^2} \sum_\ell \left\{ \left[\mathcal{P}^\mu \left(\mathbf{B}^{b(0)}_{s,\ell-\mathbf{q}} \cdot \mathbf{B}^{c(0)}_{s,\ell} \right) \right] - 2 (\mathbf{B}^{c(0)}_{s,\ell} \cdot \left[\boldsymbol{\mathcal{P}}) B^{\mu,b(0)}_{s,\ell-\mathbf{q}} \right] - 2 (\mathbf{B}^{b(0)}_{s,\ell-\mathbf{q}} \cdot \left[\boldsymbol{\mathcal{P}}) B^{\mu,c(0)}_{s,\ell} \right] \right\}$$

- Note that for the gluon the last 2 diagrams are necessary for gauge invariance but the first diagram the leading forward scattering contribution
- In the medium the momentum exchange can get dressed ~ Debye screening