Andrea Simonelli

Digging into the deep infrared with TMDs

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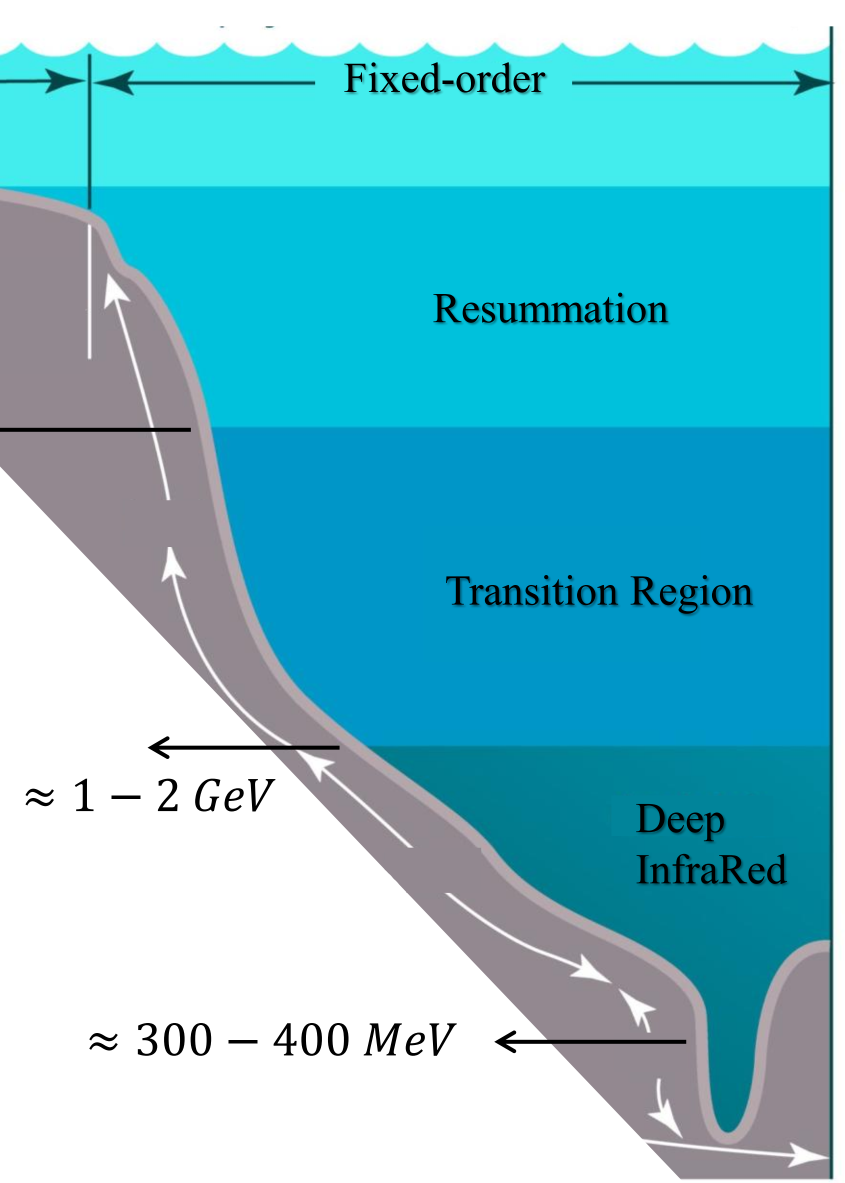
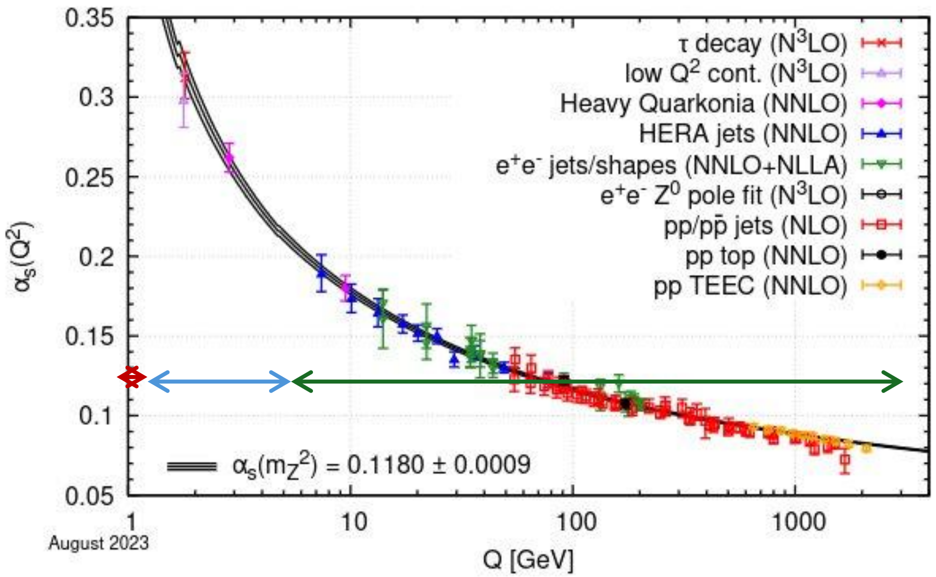
How deep can we probe QCD?

Which observables allow us to probe the **Deep IR**?





Energy scale

Perturbative QCD

Poor convergence of perturbative QCD; non-perturbative effects likely relevant.

Non-perturbative QCD

How deep can we probe QCD?

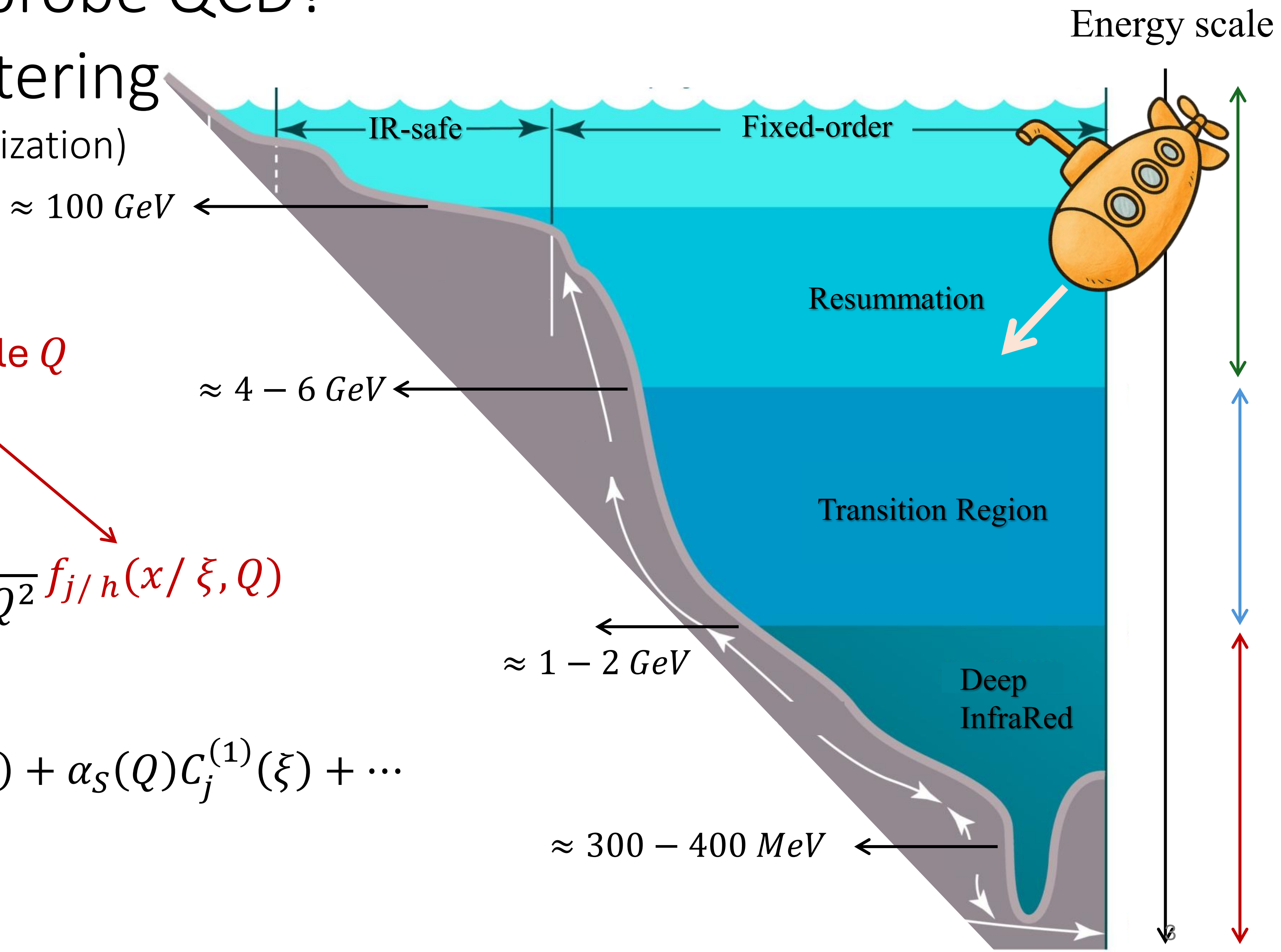
Deep Inelastic Scattering

(and in general, collinear factorization)

Proton (longitudinal)
Structure probed at scale Q

$$\frac{d\sigma}{d\,x\,d\,Q^2} = \sum_j \int_x^1 \frac{d\,\xi}{\xi} \frac{d\,\hat{\sigma}_j}{d\,\xi\,d\,Q^2} f_{j/h}(x/\xi, Q)$$

$$= C_j^{(0)}(\xi) + \alpha_S(Q) C_j^{(1)}(\xi) + \cdots$$



How deep can we probe QCD?

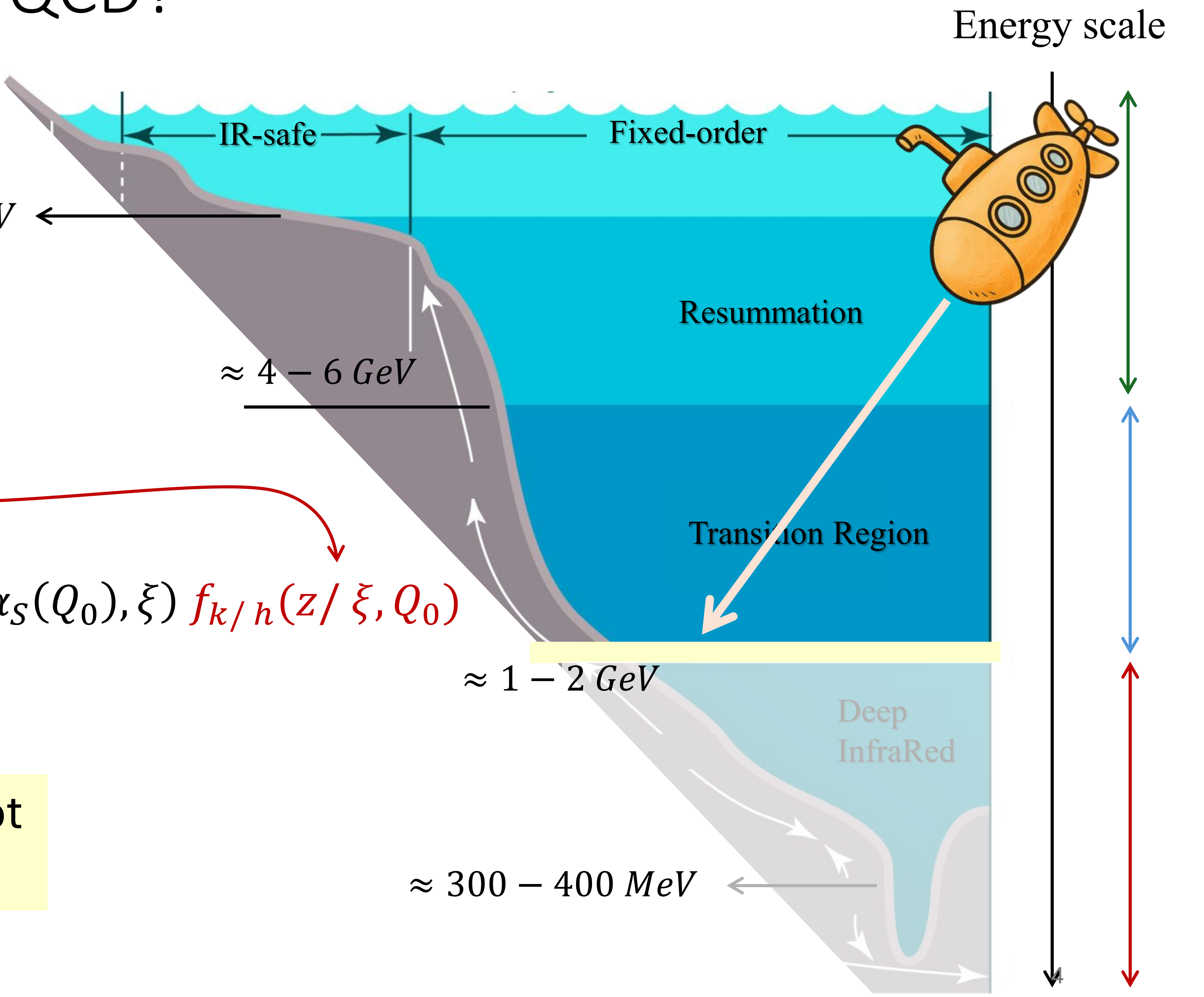
Deep Inelastic Scattering

(and in general, collinear factorization)

Proton (longitudinal)
Structure probed up to scale Q_0

$$f_{j/h}(z,Q) = \sum_k \int_z^1 \frac{d\,\xi}{\xi} E_{j/k}^{DGLAP}(\alpha_S(Q), \alpha_S(Q_0), \xi)\, f_{k/h}(z/\xi, Q_0)$$

As low as Q_0 is large enough to not violate collinear factorization



How deep can we probe QCD?

Thrust distribution

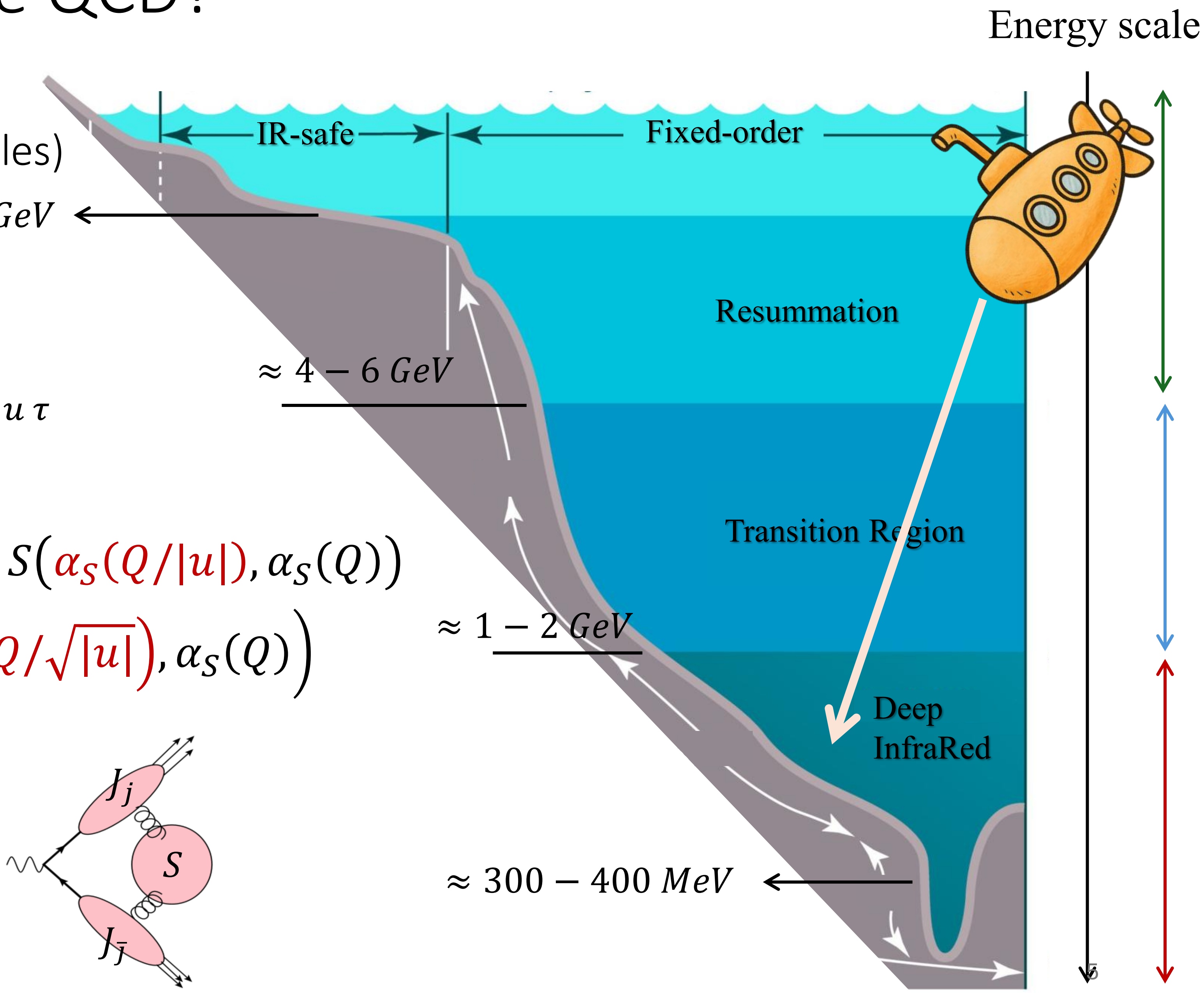
(and in general, event-shape observables)

[Dominated by $|u| \approx 1/\tau$]

$$\frac{d\sigma}{d\tau\, dQ^2} = \sum_j H_{j\bar{j}}\left(\alpha_s(Q)\right) \int \frac{du}{2\pi i} e^{u\tau}$$
$$\times J_j\left(\alpha_s\left(Q/\sqrt{|u|}\right), \alpha_s(Q)\right) S\left(\alpha_s(Q/|u|), \alpha_s(Q)\right)$$
$$\times J_{\bar{j}}\left(\alpha_s\left(Q/\sqrt{|u|}\right), \alpha_s(Q)\right)$$

Strong coupling probed
in the Deep InfraRed

As low as τ goes!



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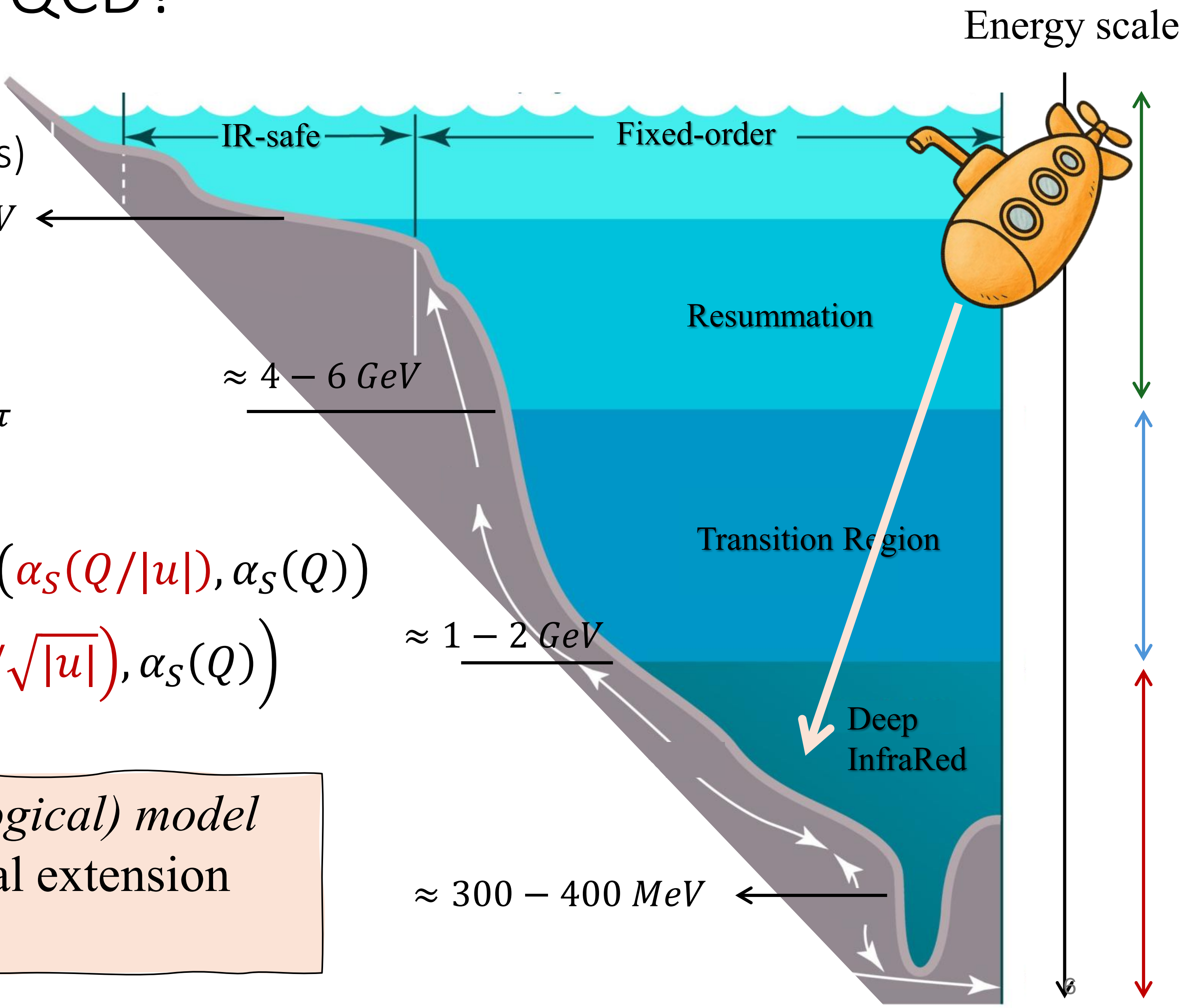


It requires a *Deep IR (phenomenological) model* for the strong coupling for analytical extension below 1-2 GeVs

How deep can we probe QCD?

TMD observables

(DY, SIDIS, e^+e^- annihilation at small q_T)

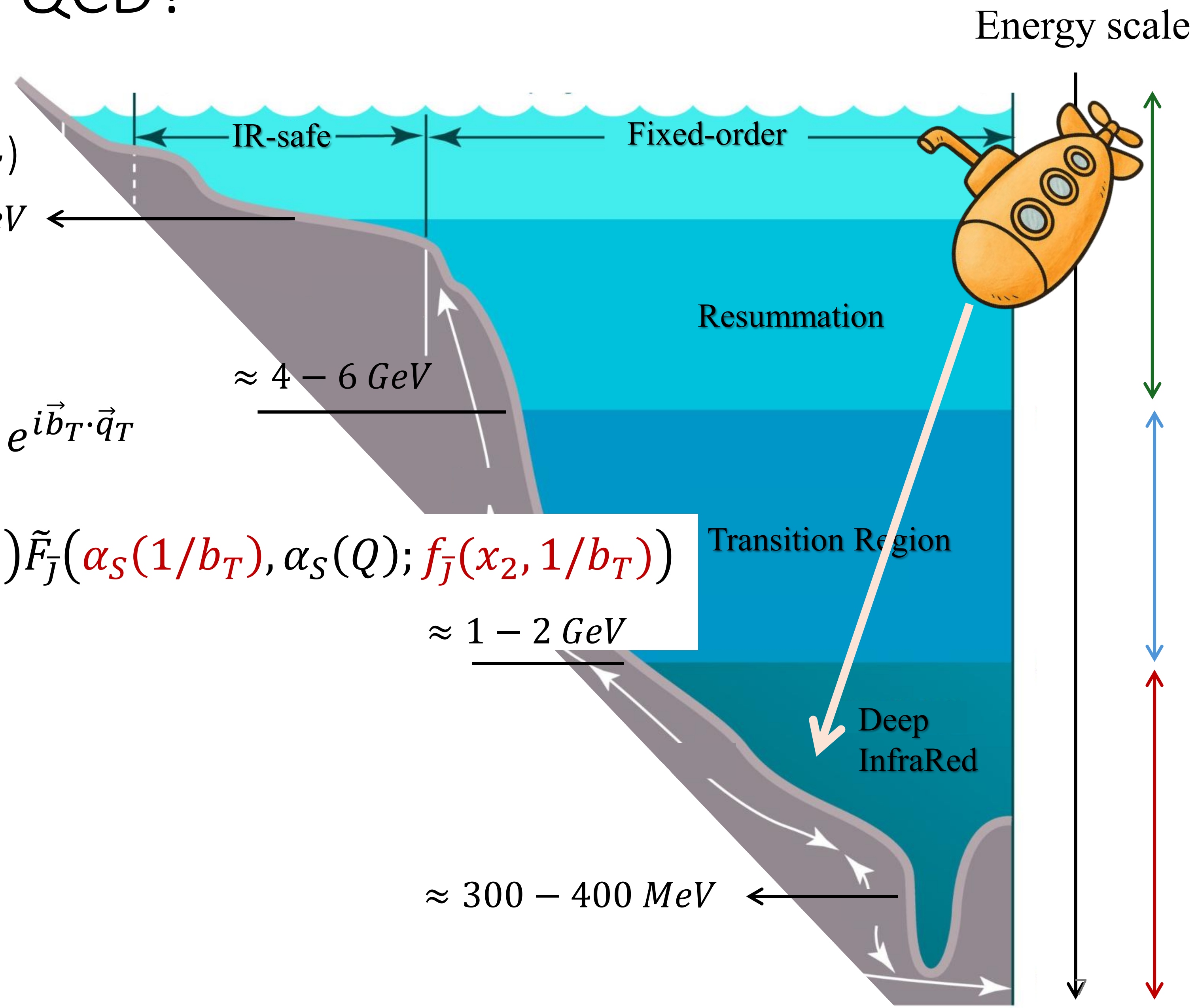
[Dominated by $q_T \approx 1/b_T$]

$$\frac{d\,\sigma}{d\,q_T\,d\,Q^2} = \sum_j H_{j\bar{j}}\left(\alpha_S(Q)\right) \int \frac{d^2\,\vec{b}_T}{(2\,\pi)^2}\, e^{i\vec{b}_T\cdot\vec{q}_T}$$

$$\times\, \tilde{F}_j\left(\alpha_S(1/b_T), \alpha_S(Q); f_j(x_1, 1/b_T)\right) \tilde{F}_{\bar{j}}\left(\alpha_S(1/b_T), \alpha_S(Q); f_{\bar{j}}(x_2, 1/b_T)\right)$$

Strong coupling and PDFs probed in the Deep InfraRed

As low as q_T goes!



How deep can we probe QCD?

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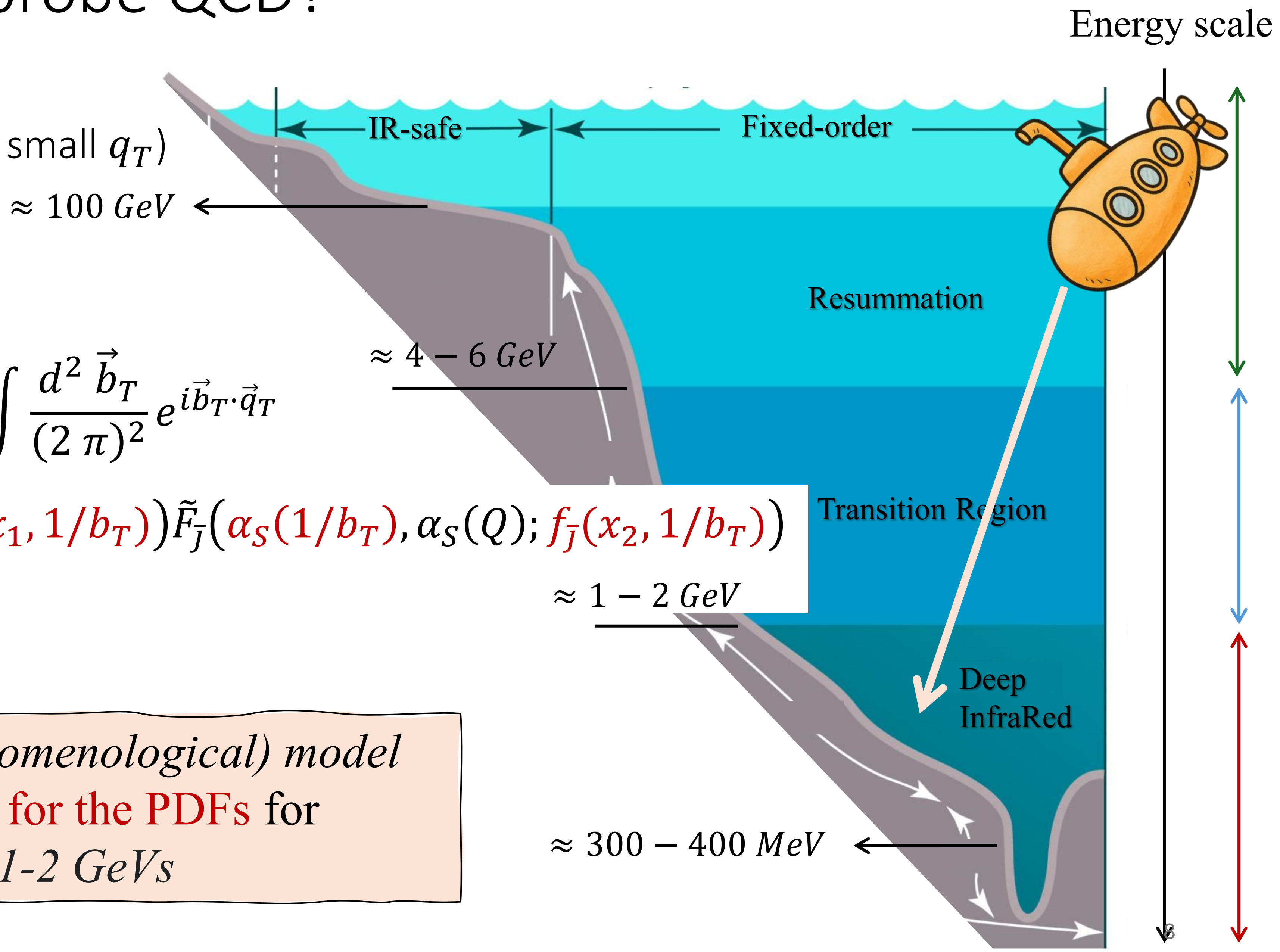


It requires a *Deep IR (phenomenological) model* for the strong coupling and for the PDFs for analytical extension *below 1-2 GeVs*

Transverse Momentum Dependent PDFs

Evolution equations:

- Renormalization group μ
- Light-cone tilt (or rapidity regulator) y_n

$$\begin{cases} \dfrac{\partial \log \tilde{F}_j(x, b_T, \mu, y_n)}{\partial \log \mu} = \gamma_F(a_S(\mu), \log(Q/\mu) \pm y_n) \\ \dfrac{\partial \log \tilde{F}_j(x, b_T, \mu, y_n)}{\partial y_n} = \pm K(a_S(\mu), \log(b_T\, \mu/c_1)) \end{cases}$$

Collins-Soper kernel or
rapidity anomalous dimension (RAD)

$$\frac{dK(a_S(\mu), \log(b_T\, \mu/c_1))}{d\ \log \mu} = -\gamma_K(a_S(\mu))$$

Associated with recoil of soft gluons

Setting the reference scale to the *natural* choice $\mu_b = c_1/b_T$

$$\tilde{F}_j(\mathrm{x}, \mathrm{b_T}, \mu, y_n) = \tilde{F}_j(\mathrm{x}, \mathrm{b_T}, \mu_b, \mu_b^2)\, exp\left\{\log\left(\frac{\mu}{\mu_b}\right) K(a_S(\mu_b), 0) + \int_{\mu_b}^{\mu} \frac{d\,\mu'}{\mu'} \gamma_F(a_S(\mu), \log(\mu/\mu'))\right\}$$

$$\times\, exp\left\{\left[\log\left(\frac{Q}{\mu}\right) \pm y_n\right] K(a_S(\mu), \log(b_T\, \mu/c_1))\right\}$$

How far in b_T can we push perturbative QCD?

$$\tilde{F}_j(\mathrm{x}, \mathrm{b_T}, Q, Q^2)$$

$$= \tilde{F}_j\left(\mathrm{x}, \mathrm{b_T}, \mu_b, \mu_b^2\right) exp\left\{\log\left(\frac{Q}{\mu_b}\right) K(a_S(\mu_b), 0) + \int_{a_S(\mu_b)}^{a_S(Q)} \frac{d\,a}{2\,\beta_{QCD}(a)}\left[\gamma_f(a) - \gamma_k(a)\int_a^{a_S(Q)} \frac{d\,a'}{2\,\beta_{QCD}(a')}\right]\right\}$$

$$a_S(\mu_b)^2\,\gamma_r^{[1]} + \cdots$$

$$f_j(x, \mu_b) + a_S(\mu_b)\int_x^1 \frac{d\,\xi}{\xi} C_{j/k}^{[1]}(\xi) f_k\left(\frac{x}{\xi}, \mu_b\right) + \cdots$$

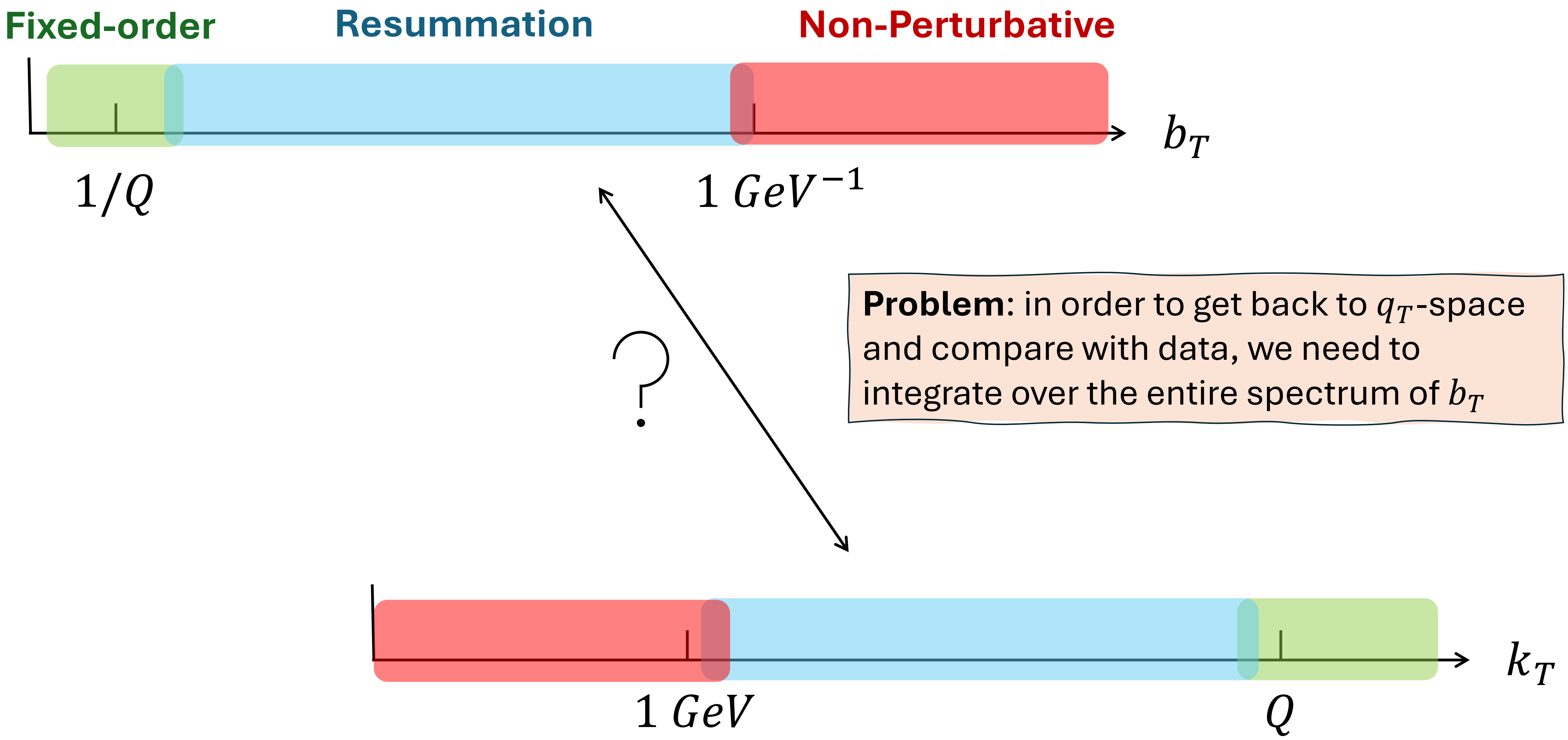
Valid approximations as long as $a_S(\mu_b)$ is perturbative

It can be integrated *analytically*, provided that the integrand can be expanded in powers of a, which in turn is valid as long as $a_S(\mu_b)$ is perturbative

$\mu_b \gtrsim 1\,GeV \longleftrightarrow \mathrm{b_T} \lesssim 1\,GeV^{-1}$

We *know* the TMD up to around $1\,GeV^{-1}$

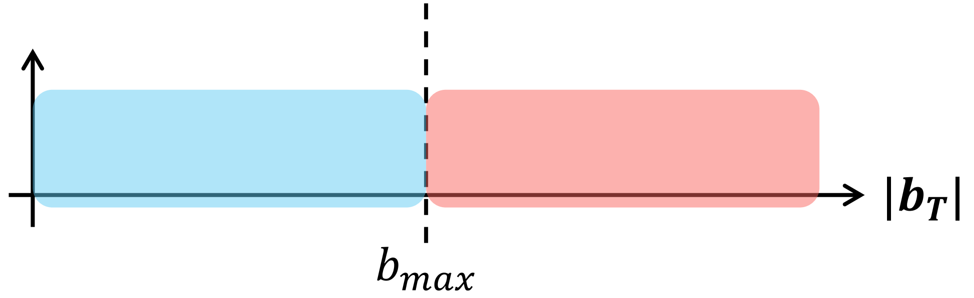
How to get back in transverse momentum space?



Problem: in order to get back to q_T-space and compare with data, we need to integrate over the entire spectrum of b_T

Standard approach

$$F_j(x, |\boldsymbol{b_T}|, \mu, y_n) = F_j(x, |\boldsymbol{b_T}|^*, \mu, y_n) \otimes F_{NP}(x, |\boldsymbol{b_T}|, j) e^{-g_K(|\boldsymbol{b_T}|)\log\left(\frac{Q^2 e^{-y_n}}{\mu}\right)}$$



Completely determined by perturbative QCD

Black boxes that have to be determined by imposing agreement with data

$\approx 1\ GeV^{-1}$

All efforts are dedicated to model and parametrize them

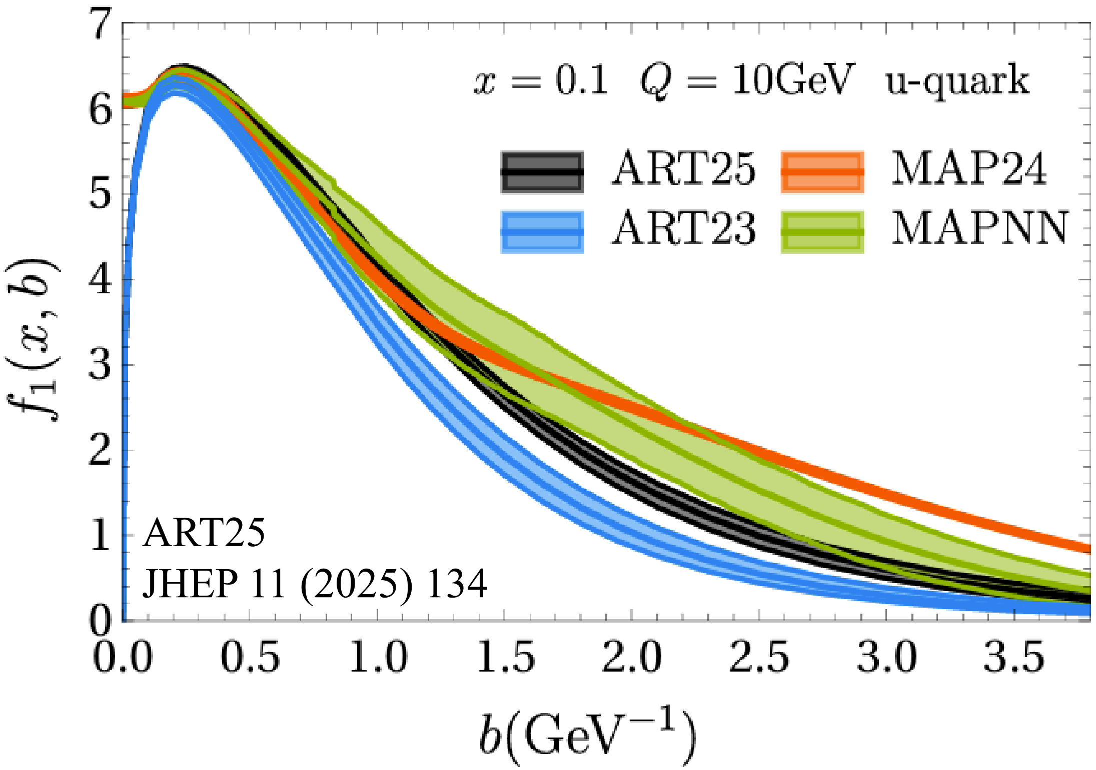
For instance:

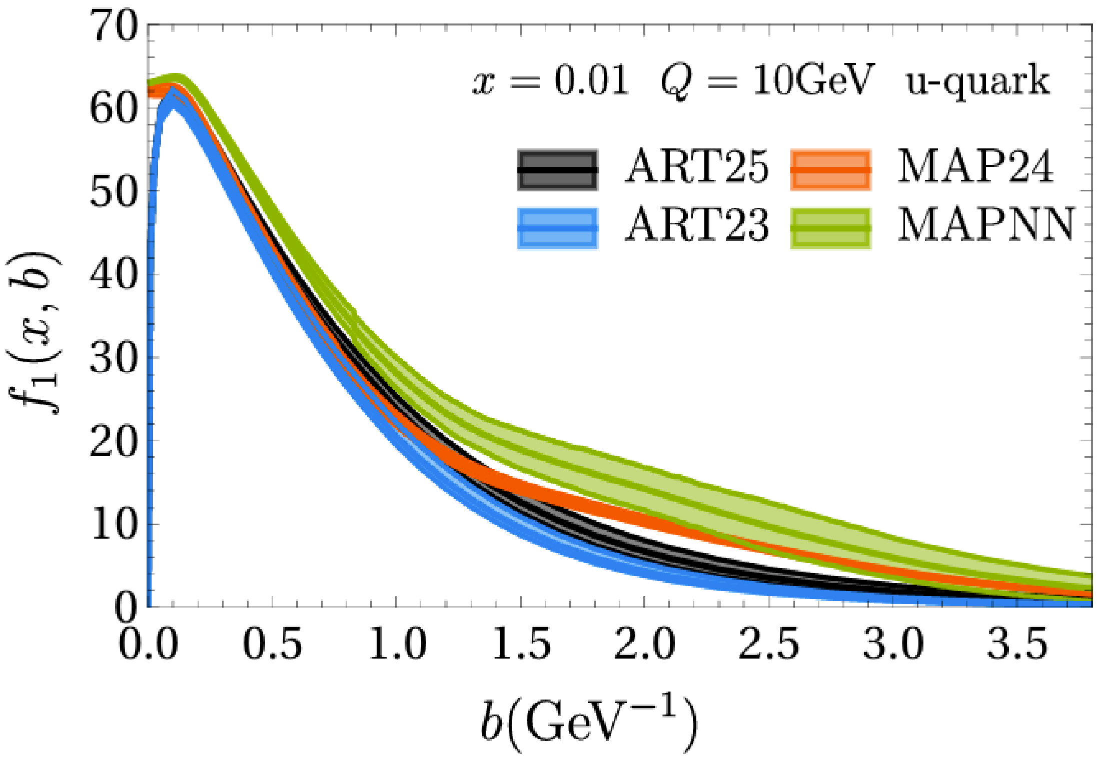
$$|\boldsymbol{b_T}|^* = \frac{|\boldsymbol{b_T}|}{\sqrt{1 + \frac{|\boldsymbol{b_T}|^2}{b_{max}^2}}}$$

Standard hypothesis: the way the transition is implemented (how and where) is *unphysical* and should not affect the predictions.

The b* prescription (functional form and b_{max} value) is fixed, not fitted

Something is not working properly





Impact of prescriptions

The impact of prescriptions in phenomenological extractions of Transverse Momentum Dependent distributions

Matteo Cerutti[1,*] and Andrea Simonelli[2,†] 2603.19088

Data from E288 (130 pts) and E605 (50 pts) experiments:

$4\ GeV \leq Q \leq 13.5\ GeV$ → Energy is low but "still perturbative".

TMD extraction performed at N2LL and with a simple NP ansatz:

$$g_K = \frac{g_2}{2} b_T^2 \qquad g_{j/h} = \frac{g_1}{2} b_T^2$$

We consider different b^*-prescriptions while keeping *the same functional form* for NP models.

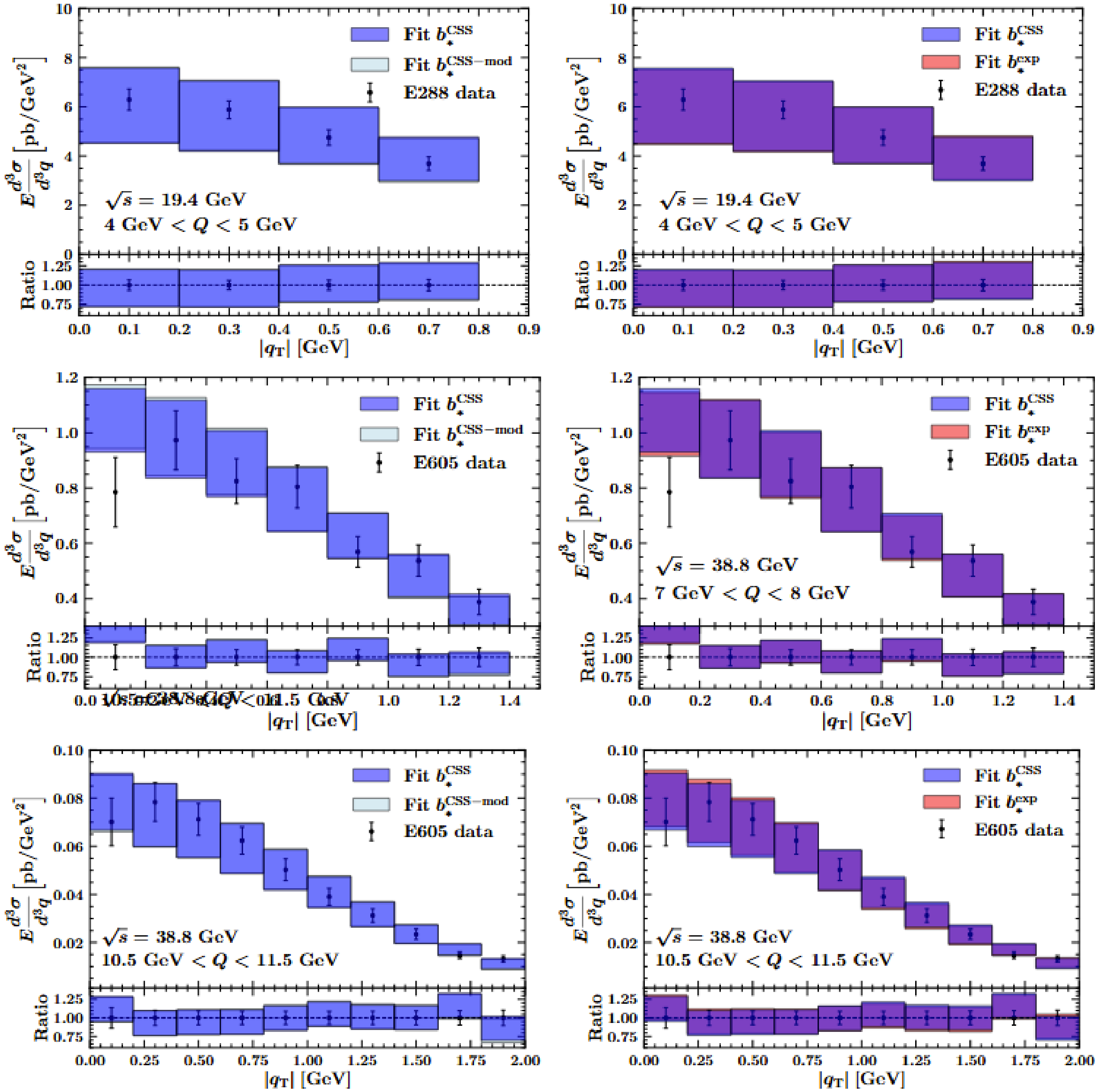
- Different functional form:

$$b_*^{\mathrm{CSS}}(\boldsymbol{b}_T^2) = \sqrt{\frac{\boldsymbol{b}_T^2}{1 + \frac{\boldsymbol{b}_T^2}{b_{\max}^2}}} \qquad b_*^{\exp}(\boldsymbol{b}_T^2) = b_{\max}[1 - \exp(-\boldsymbol{b}_T^4/b_{\max}^4)]^{\frac{1}{4}}$$

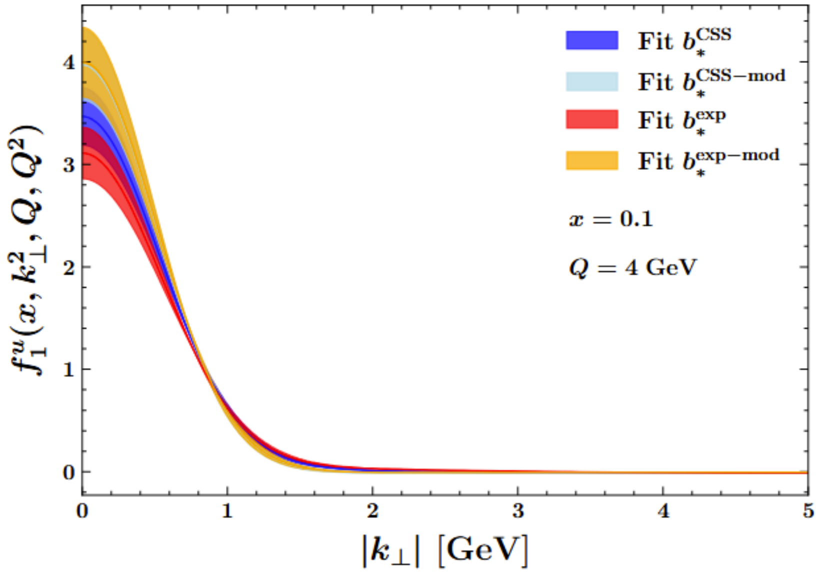
- Different b_{max}:

$$b_{\max} = c_1 \approx 1.12\ \mathrm{GeV}^{-1} \qquad b_{\max} = c_1/2$$

Impact of prescriptions

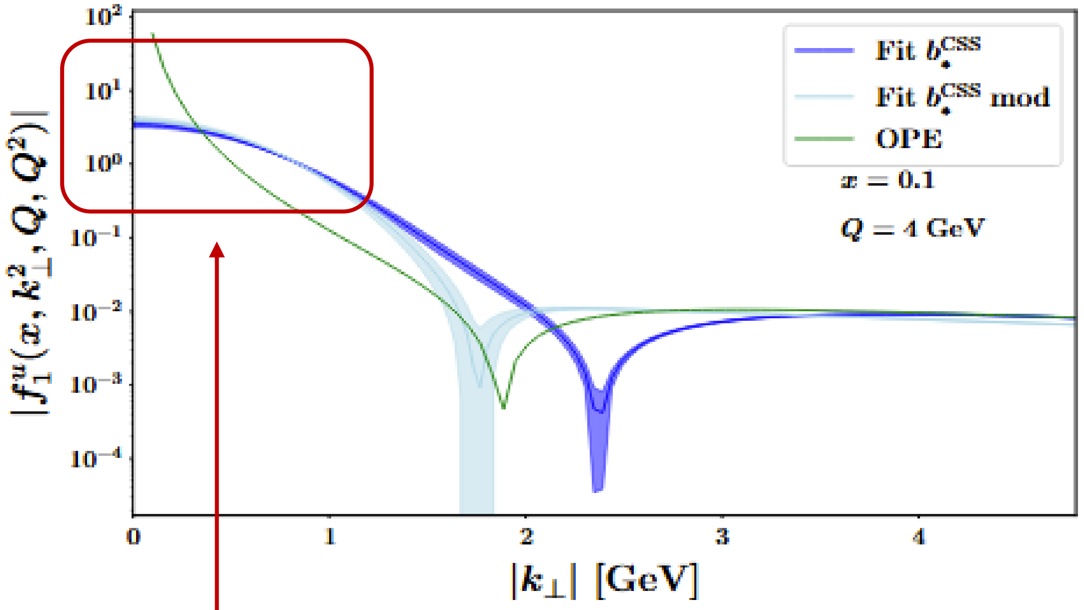


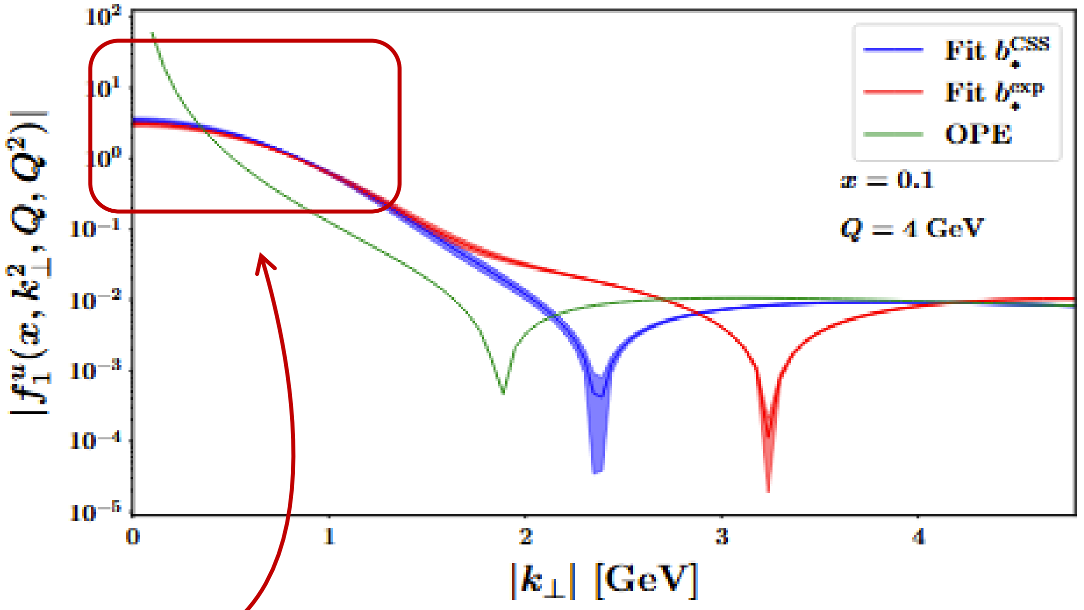
		χ^2/N_{dat}			
Dataset	N_{dat}	b_*^{CSS}	$b_*^{CSS-mod}$	b_*^{exp}	$b_*^{exp-mod}$
E288	130	0.99	0.83	1.00	0.96
E605	50	1.66	1.78	1.40	1.85
Total	180	1.18	1.09	1.11	1.21



It seems that the arbitrariness of our choice just increases the uncertainty bands of the TMDs.

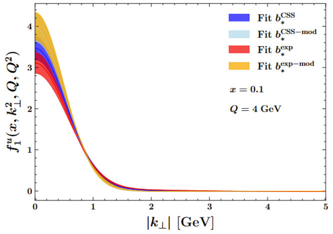
Ambiguity at intermediate k_T



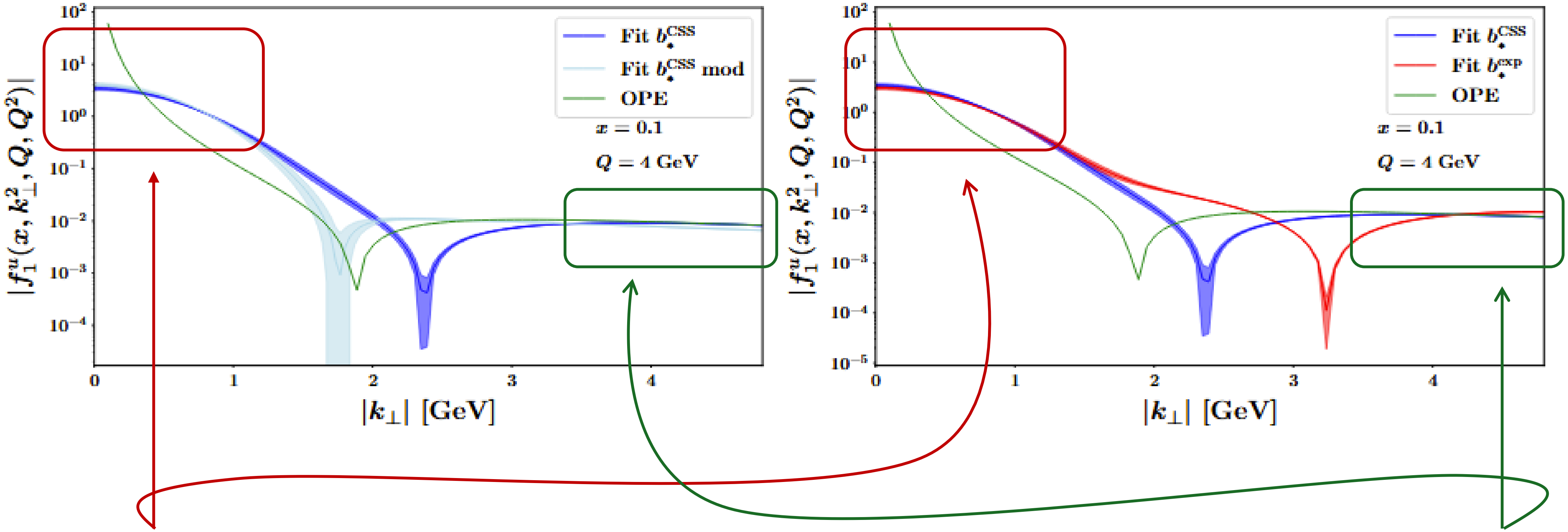


Constrained by data (fit)

See previous plot



Ambiguity at intermediate k_T



Constrained by data (fit)
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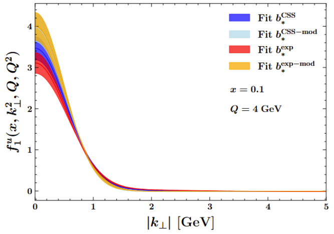
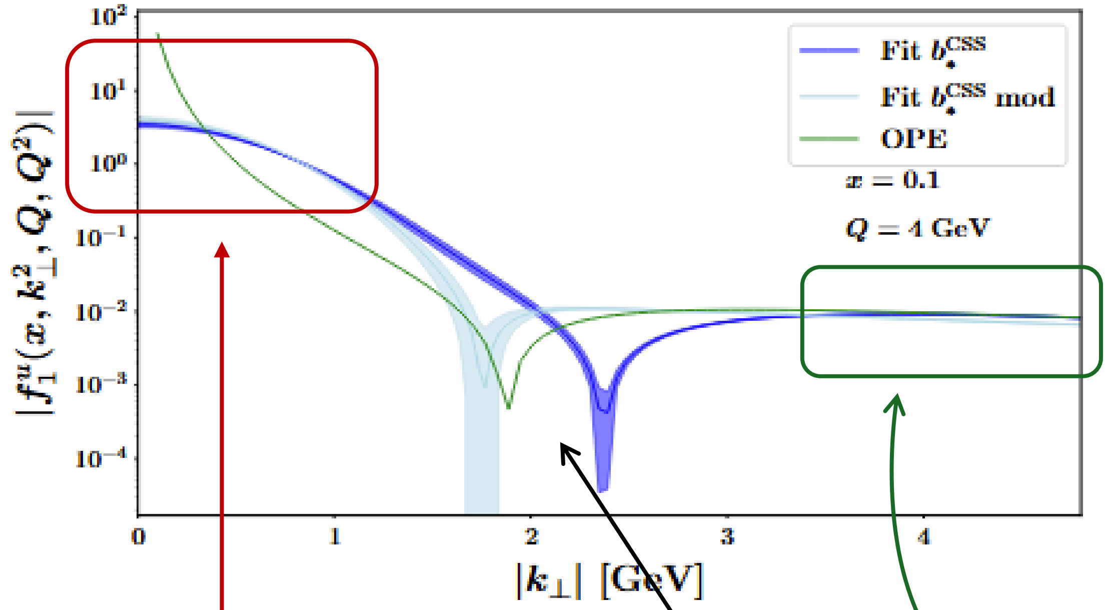
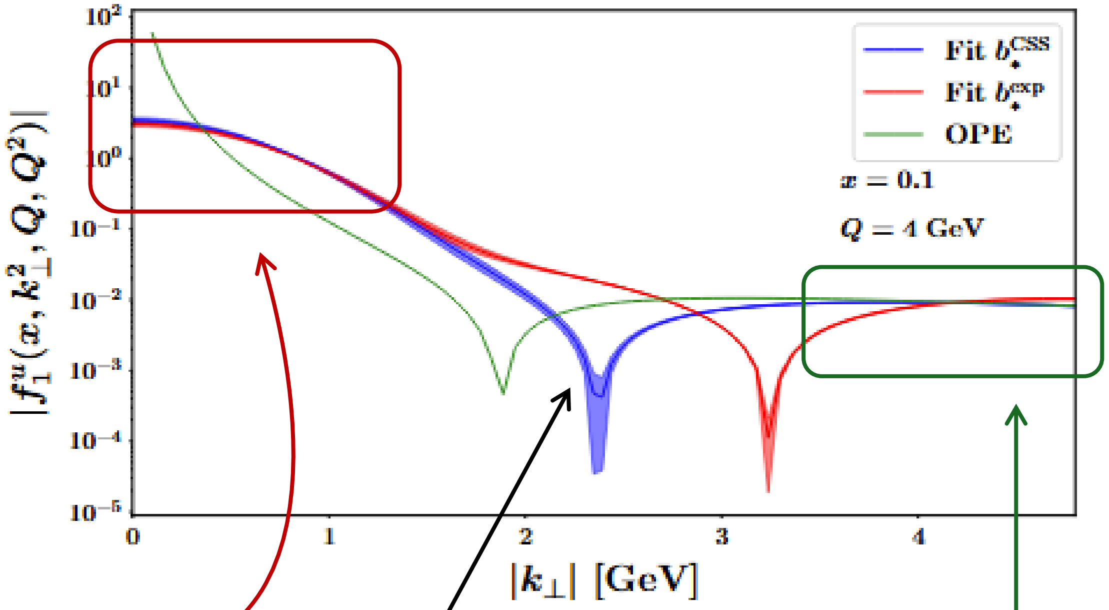


Constrained by fixed-order (OPE)

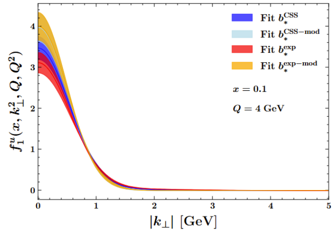
Ambiguity at intermediate k_T





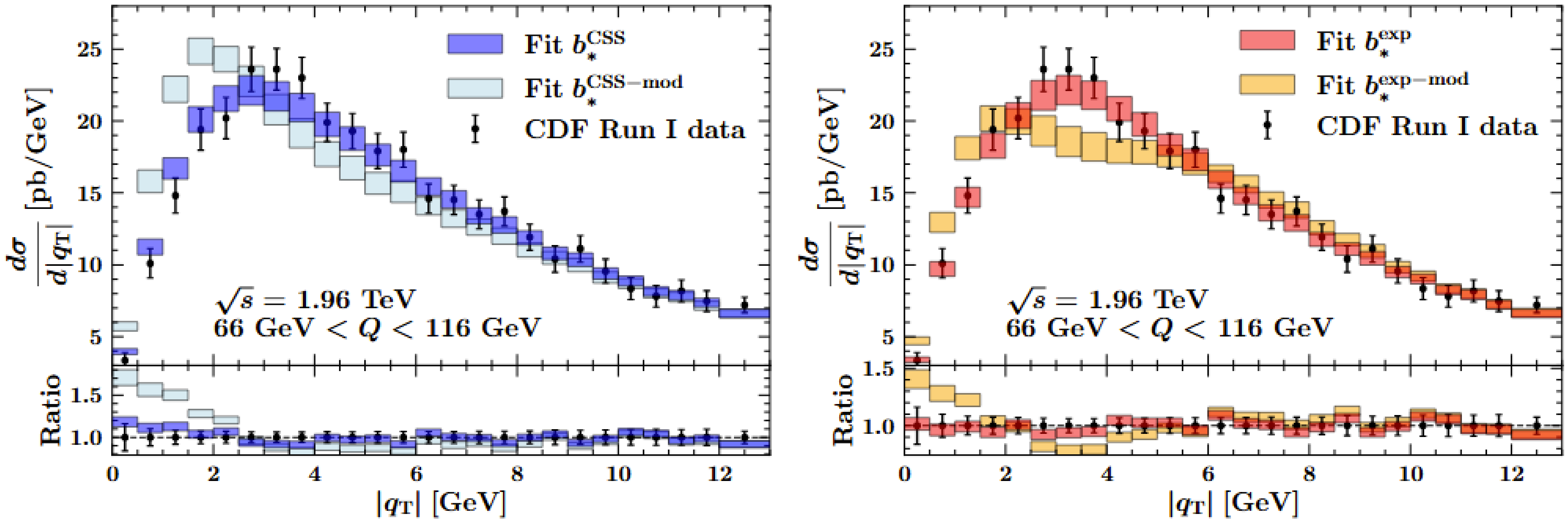
Constrained by data (fit)
See previous plot

Constrained by fixed-order (OPE)



Completely unconstrained in standard approach.
Suspicious: resummation should do the job!

Postdicting high-energy data



		χ^2/N_{dat}			
Dataset	N_{dat}	b_*^{CSS}	$b_*^{\mathrm{CSS-mod}}$	b_*^{exp}	$b_*^{\mathrm{exp-mod}}$
CDF I	25	0.83	6.16	0.57	2.87

Postdicting high-energy data

Global fits *mitigate* but *not solve* the issue

		χ^2/N_{dat}			
Dataset	N_{dat}	b_*^{CSS}	$b_*^{\text{CSS-mod}}$	b_*^{exp}	$b_*^{\text{exp-mod}}$
CDF I	25	0.83	6.16	0.57	2.87

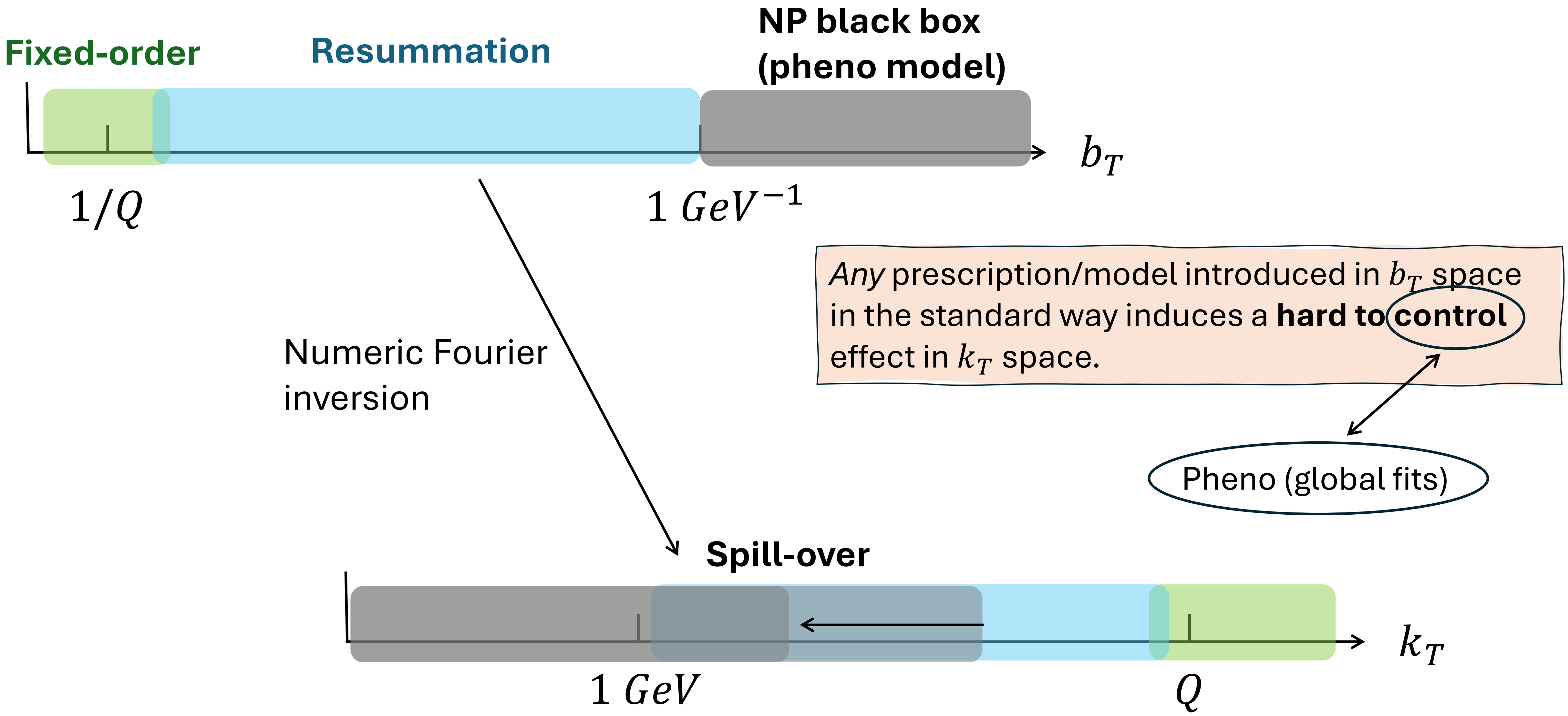
		χ^2/N_{dat}			
Dataset	N_{dat}	b_*^{CSS}	$b_*^{\text{CSS-mod}}$	b_*^{exp}	$b_*^{\text{exp-mod}}$
E288	130	0.98	0.89	1.01	0.96
E605	50	1.72	2.12	1.38	1.96
CDF I	25	0.74	3.38	0.56	2.19
Total	205	1.13	1.49	1.04	1.36

The standard approach *requires* **global fits**:

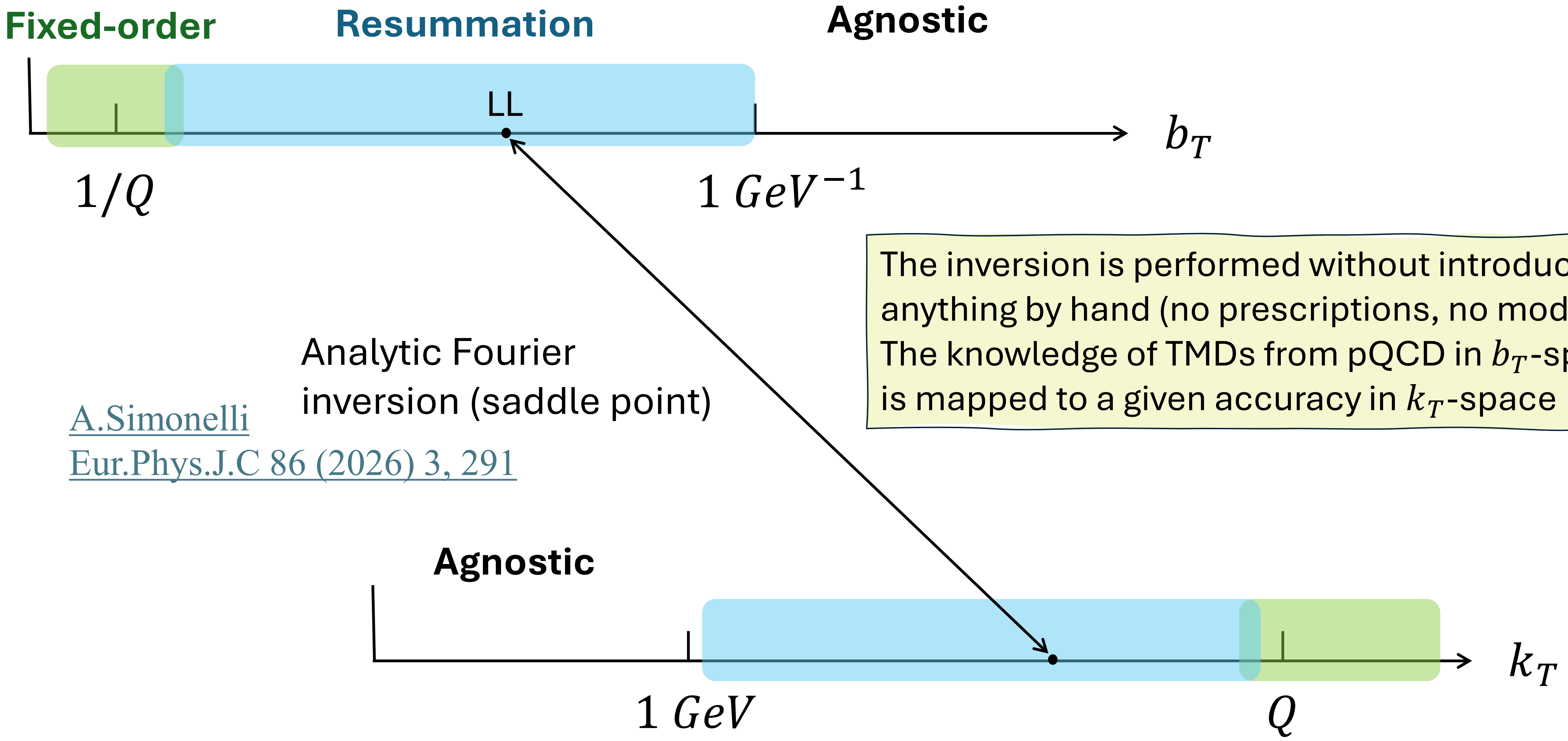
- The low-energy data mainly constraint the NP region at very low k_T
- The high-energy data provide information about the intermediate region, helping to select the "proper prescription" at a given NP model.

Note that the global fit selects **two** “good” prescriptions.

The standard way to get back to k_T



A cleaner alternative



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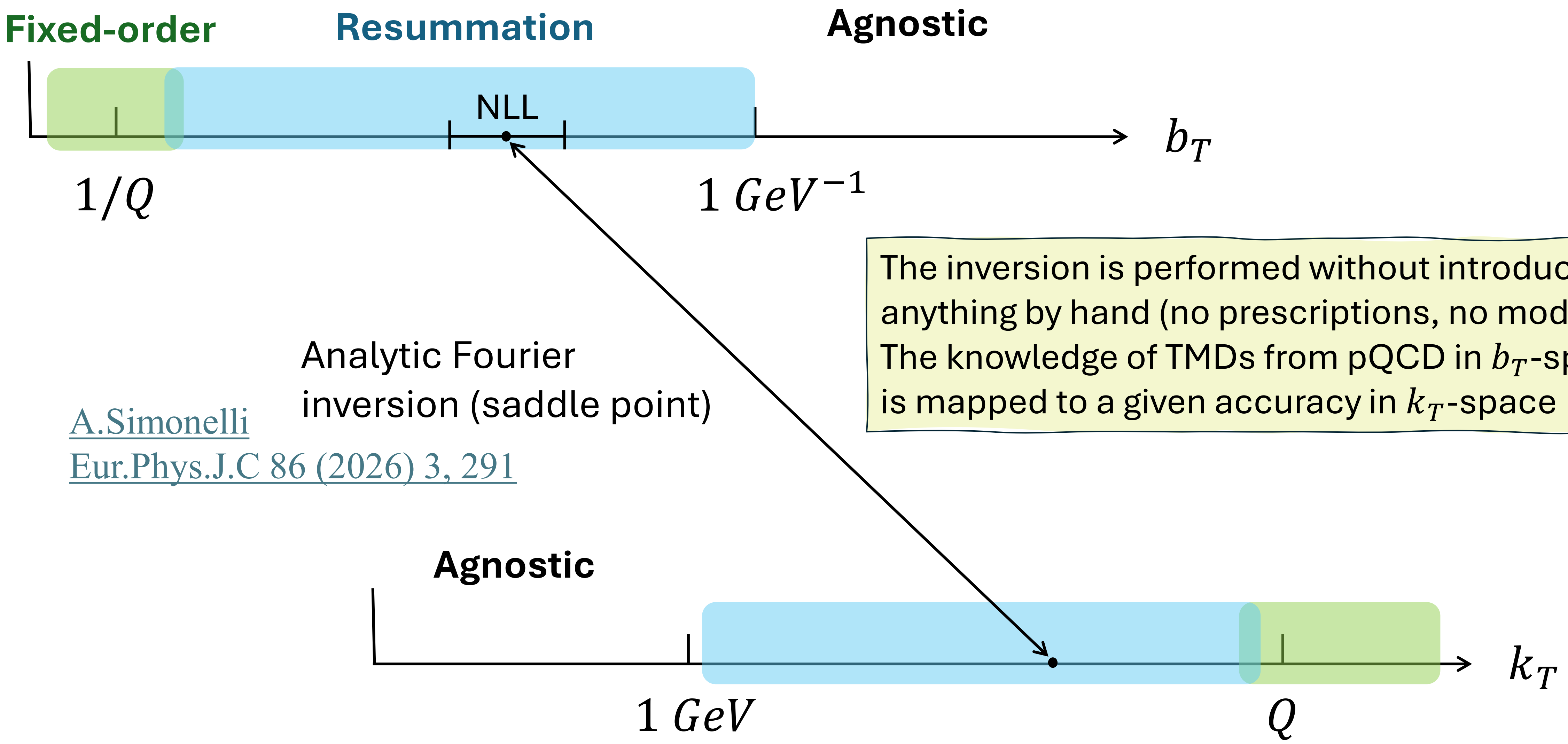
The inversion is performed without introducing anything by hand (no prescriptions, no models). The knowledge of TMDs from pQCD in b_T-space is mapped to a given accuracy in k_T-space

A cleaner alternative



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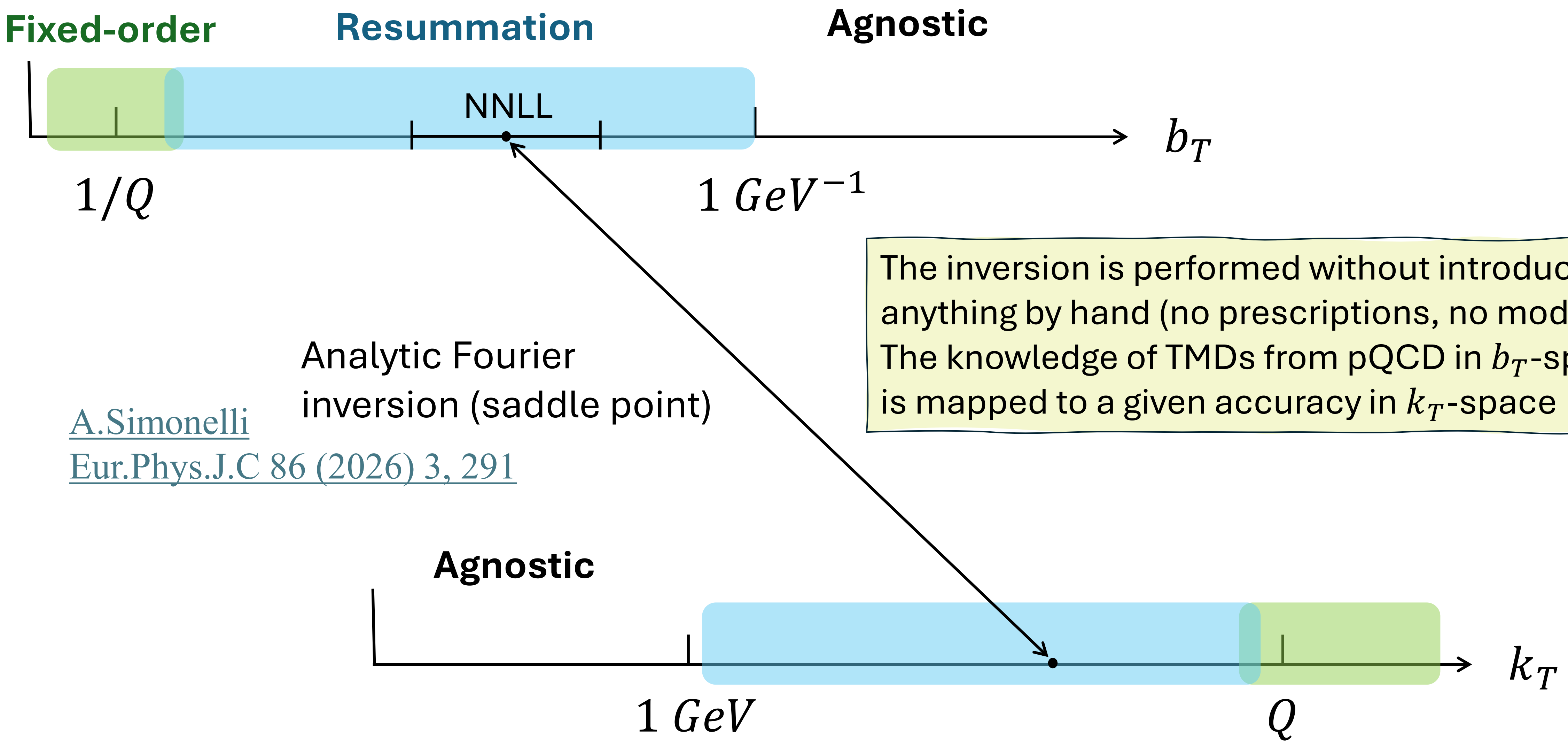

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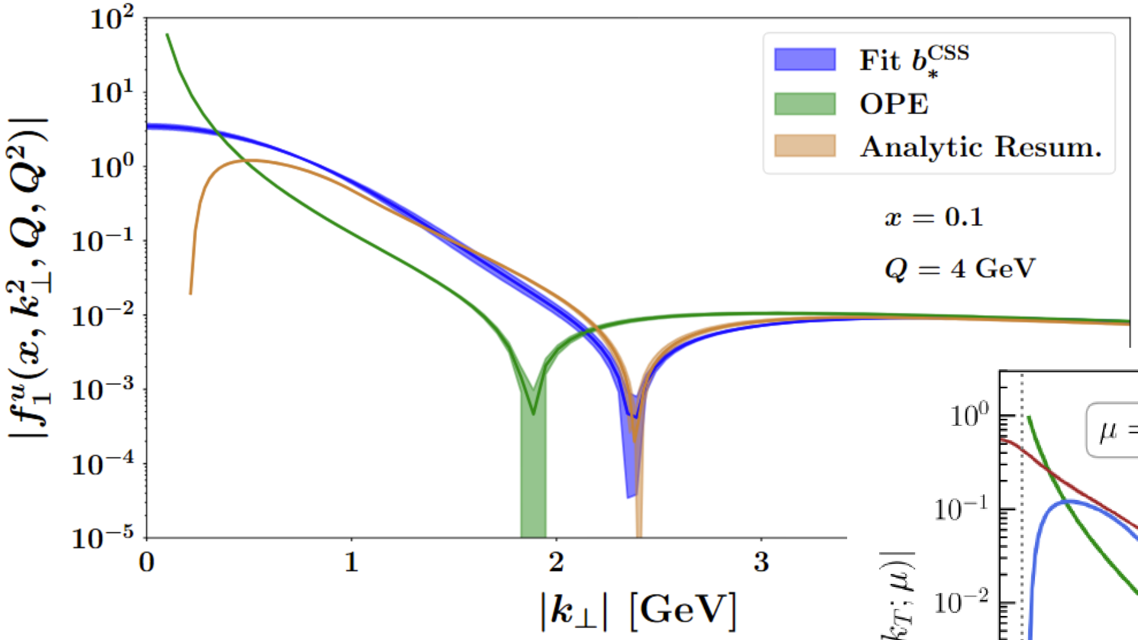
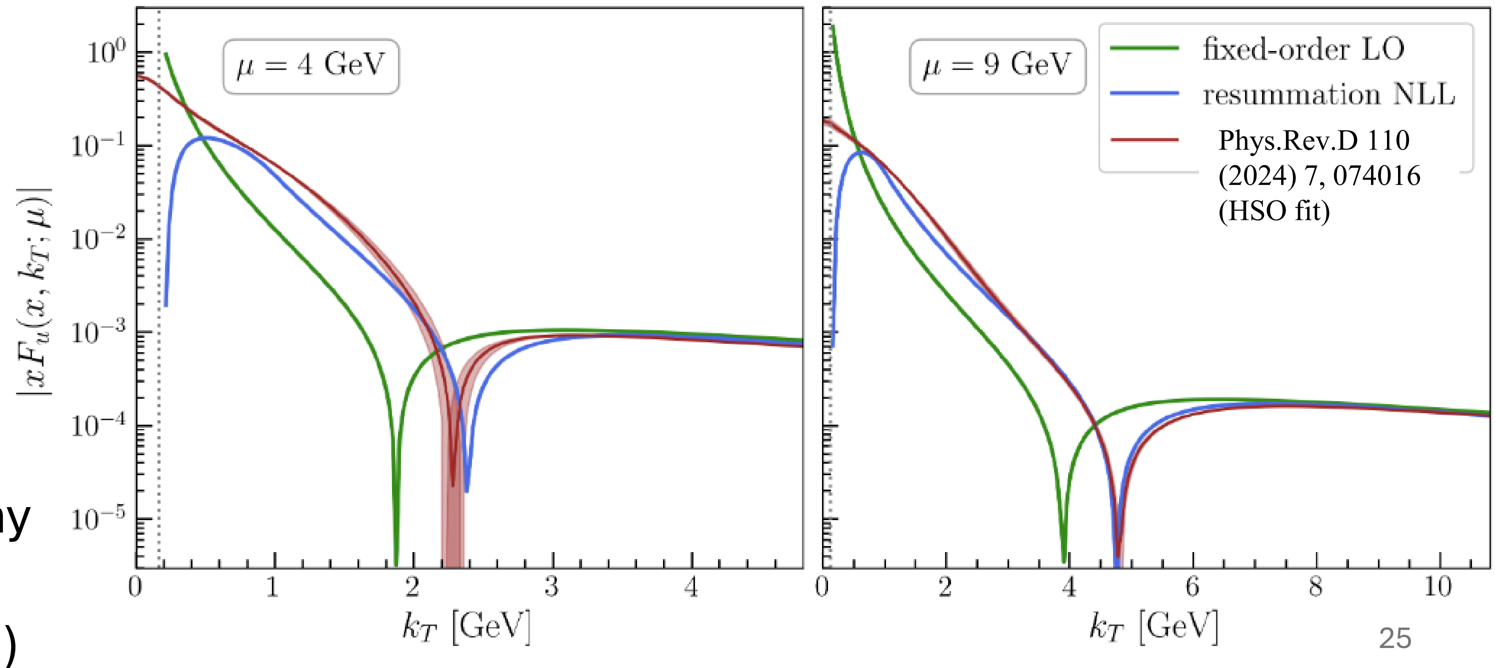
Analytic resummation in k_T space

This expression reproduces one of the best curve of the DY stress test in the intermediate region

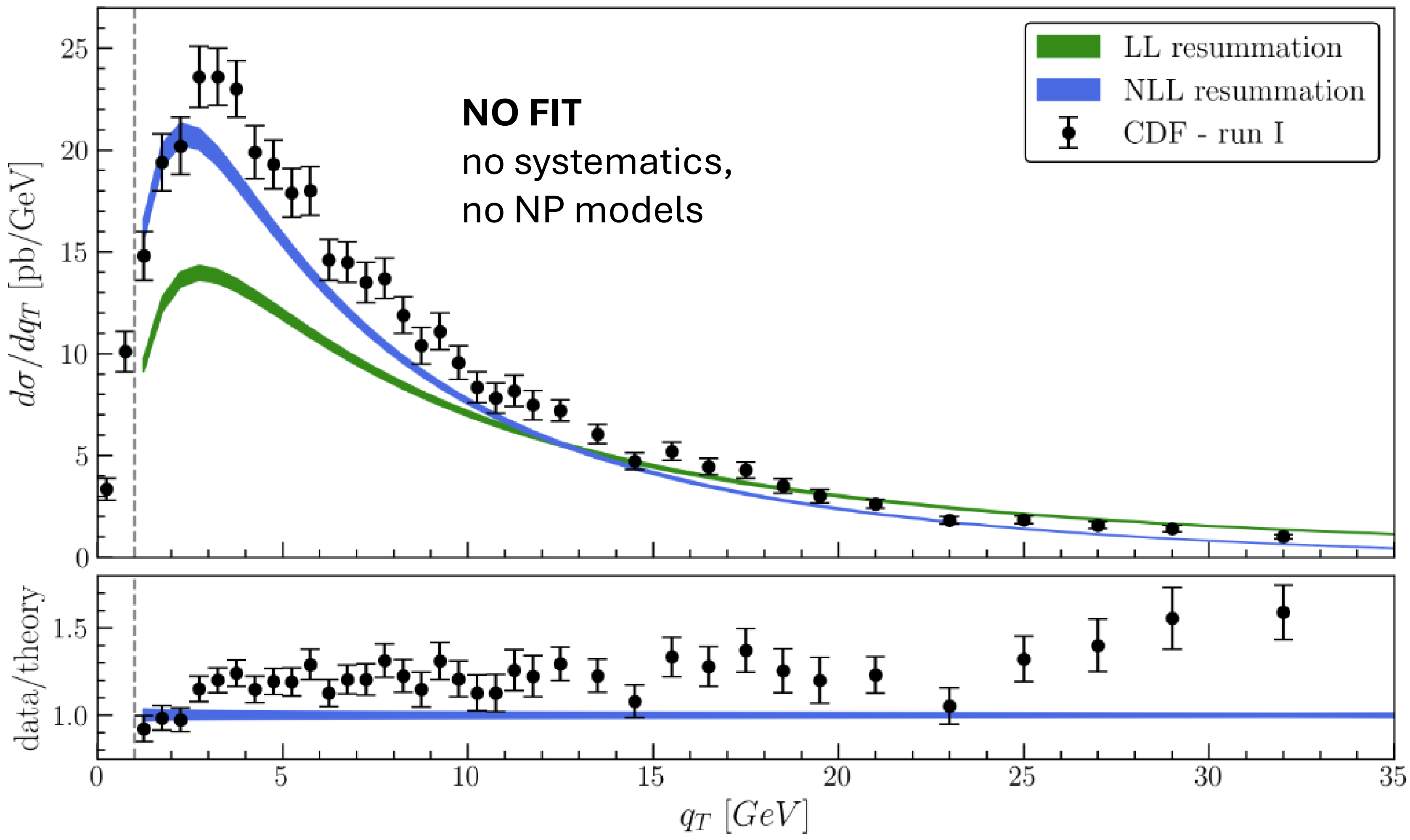


As well as the result from an actual fit



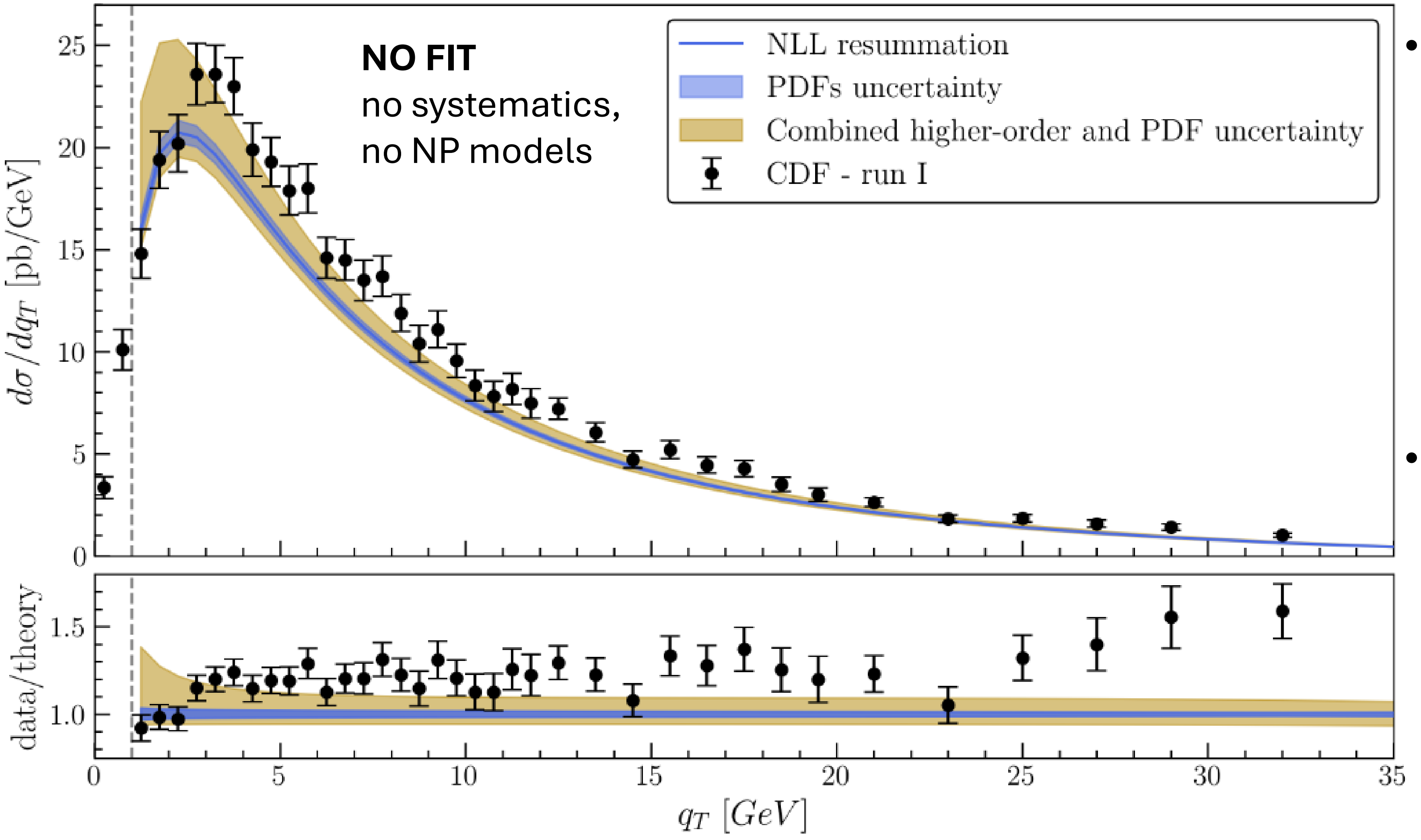
With the significative advantage of being completely perturbative and free from any contamination due to prescriptions and NP models (look at the size of the bands!)

Analytic resummation



- Clear improvement from LL to NLL precision
- Below $q_T \approx$ 1 GeV the full-perturbative cross-section is no longer applicable as it stands.
- Above $q_T \approx$ 25 GeV the agreement with data deteriorates. This indicates the region *where* the matching to fixed-order collinear factorization becomes relevant.

Analytic resummation extra bonuses



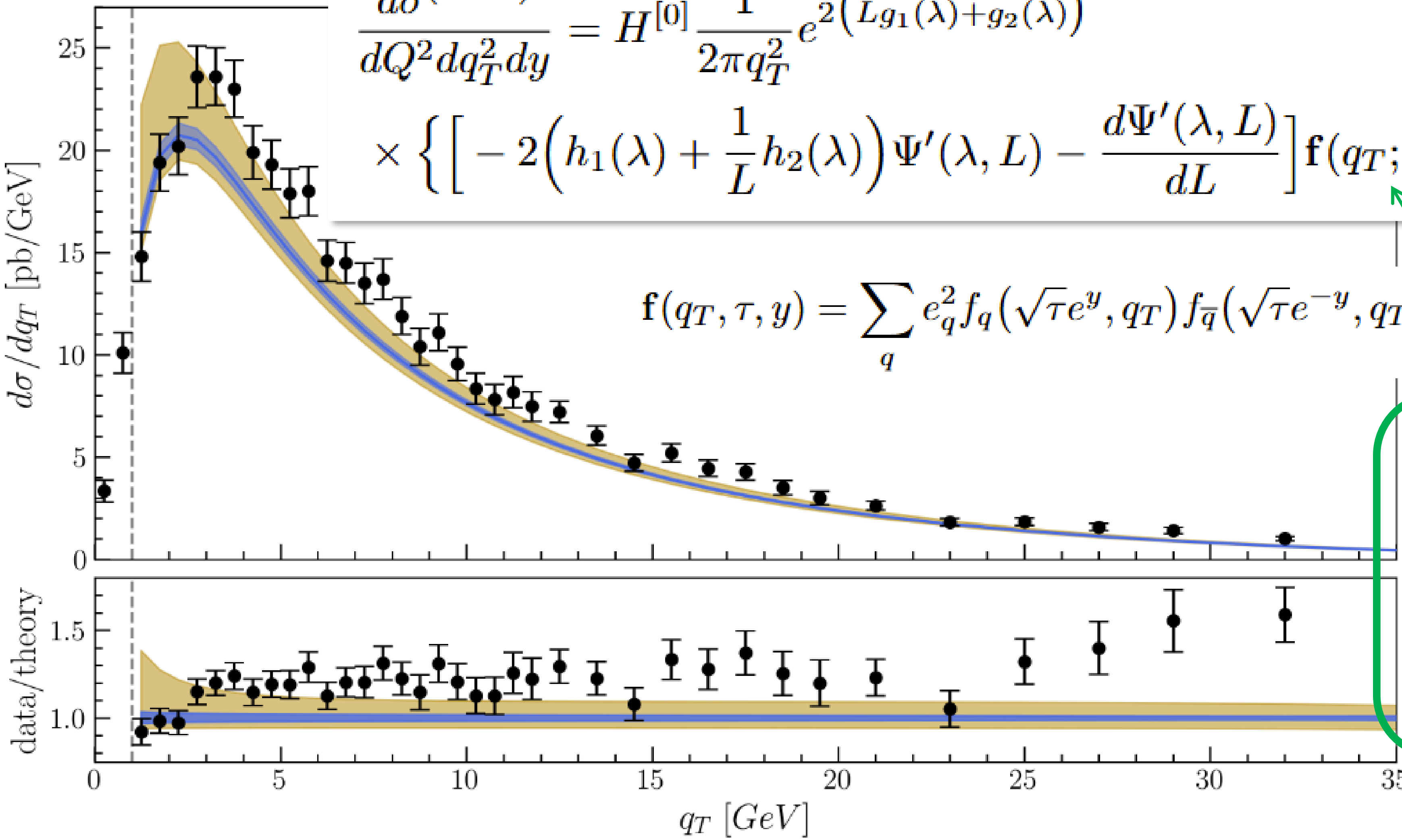
- Simple estimate of missing higher-order corrections based on scale variation. This is a new result, and it would be difficult to obtain within the standard approach.
- All data above $q_T \approx Q_0$ can be used to extract the usual PDFs, which are thus elevated from simple inputs to active ingredients of the phenomenology

Analytic resummation extra bonuses

$$\frac{d\sigma^{(NLL)}}{dQ^2 dq_T^2 dy} = H^{[0]} \frac{1}{2\pi q_T^2} e^{2\left(L g_1(\lambda)+g_2(\lambda)\right)}$$

$$\times \left\{ \left[-2\left(h_1(\lambda) + \frac{1}{L} h_2(\lambda)\right) \Psi'(\lambda, L) - \frac{d\Psi'(\lambda, L)}{dL} \right] \mathbf{f}(q_T; \tau, y) + \Psi'(\lambda, L) \frac{d\mathbf{f}(q_T; \tau, y)}{d\log q_T} \right\}$$

$$\mathbf{f}(q_T, \tau, y) = \sum_q e_q^2 f_q(\sqrt{\tau} e^{y}, q_T) f_{\bar{q}}(\sqrt{\tau} e^{-y}, q_T)$$



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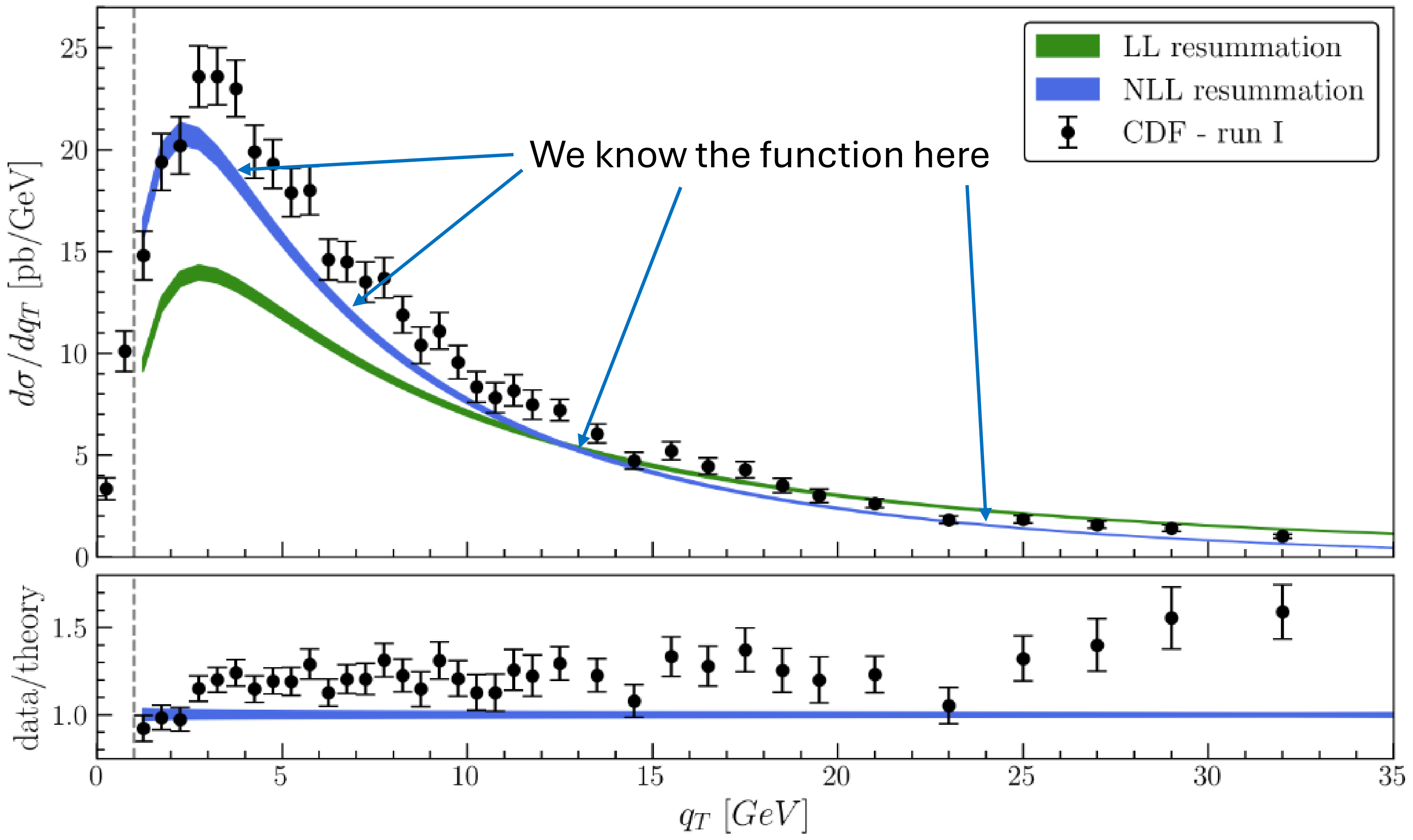
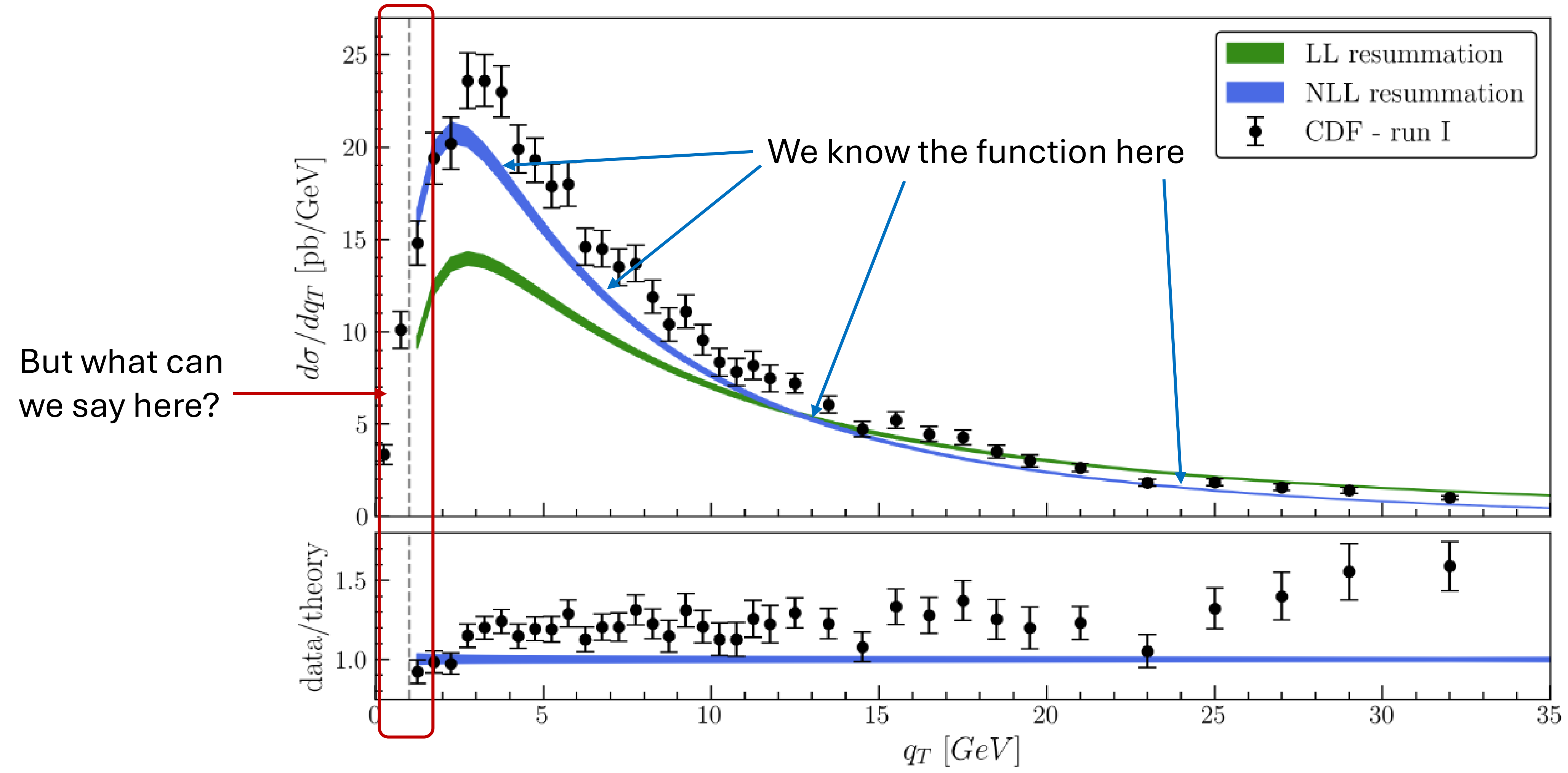
Analytic continuation into the Deep Infrared

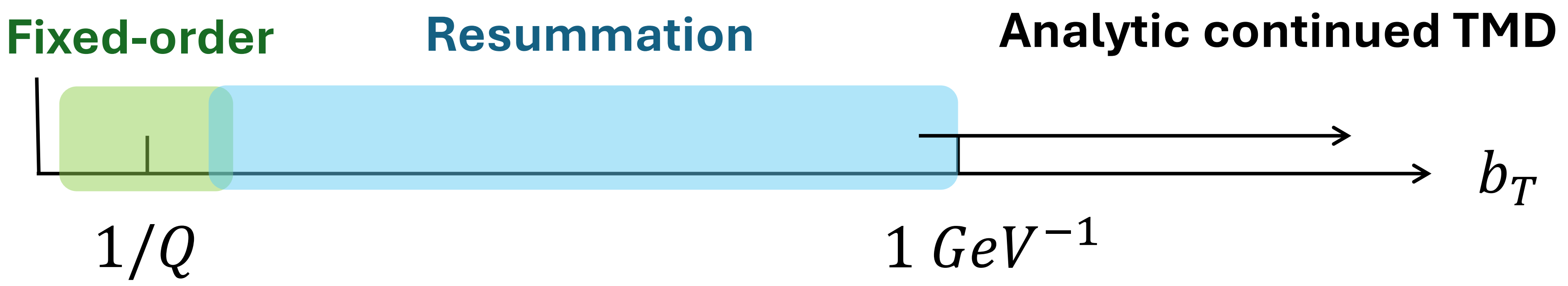


Analytic continuation into the Deep Infrared



The region of q_T below Q_0 becomes larger and larger as the energy decreases, making its proper description essential.

Analytic continuation into the Deep Infrared



We need **two** Deep IR models:

- One for the QCD Beta function (e.g. α_S in the Deep IR)

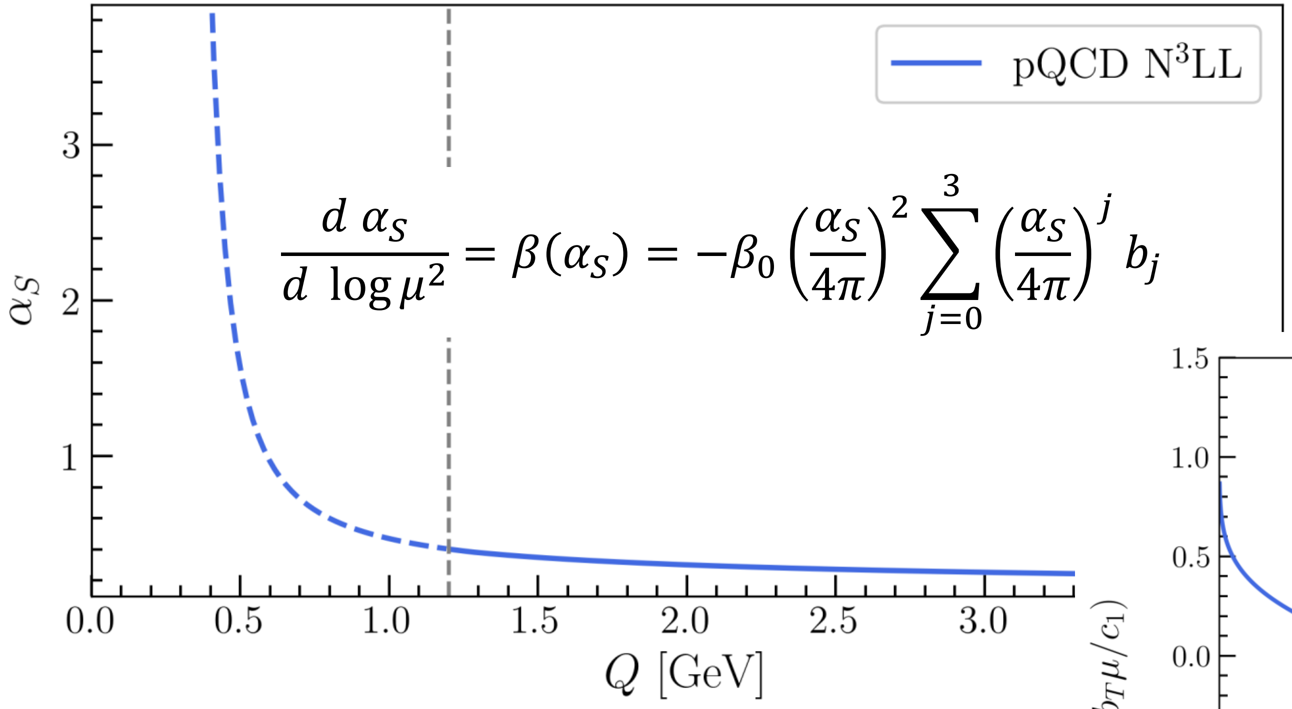
$$\frac{d\ \alpha_S(\mu)}{d\ \log\mu^2} = \beta_{QCD}^{pQCD}(\mu) \quad \rightarrow \quad \beta_{QCD}^{Deep\ IR}(\mu, Q_0, pars) \quad \text{for } Q \leq Q_0$$

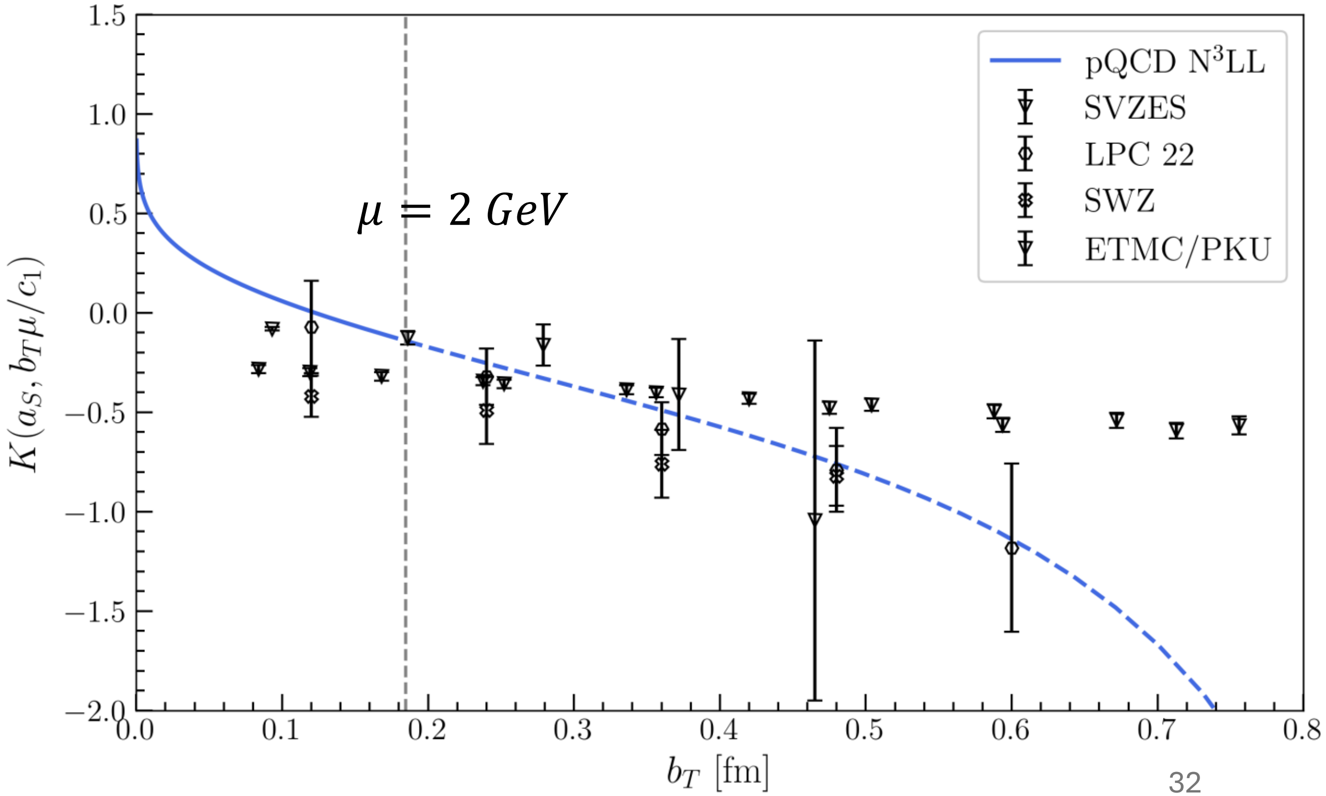
- One for DGLAP evolution (e.g. PDFs in the Deep IR)

$$\frac{d\ f_j(x,\mu)}{d\ \log\mu^2} = P_{j/k}^{pQCD}(\mu) \otimes f_k(x,\mu) \quad \rightarrow \quad P_{\frac{j}{k}}^{Deep\ IR}(\mu, Q_0, pars) \otimes f_k(x,\mu) \quad \text{for } Q \leq Q_0$$

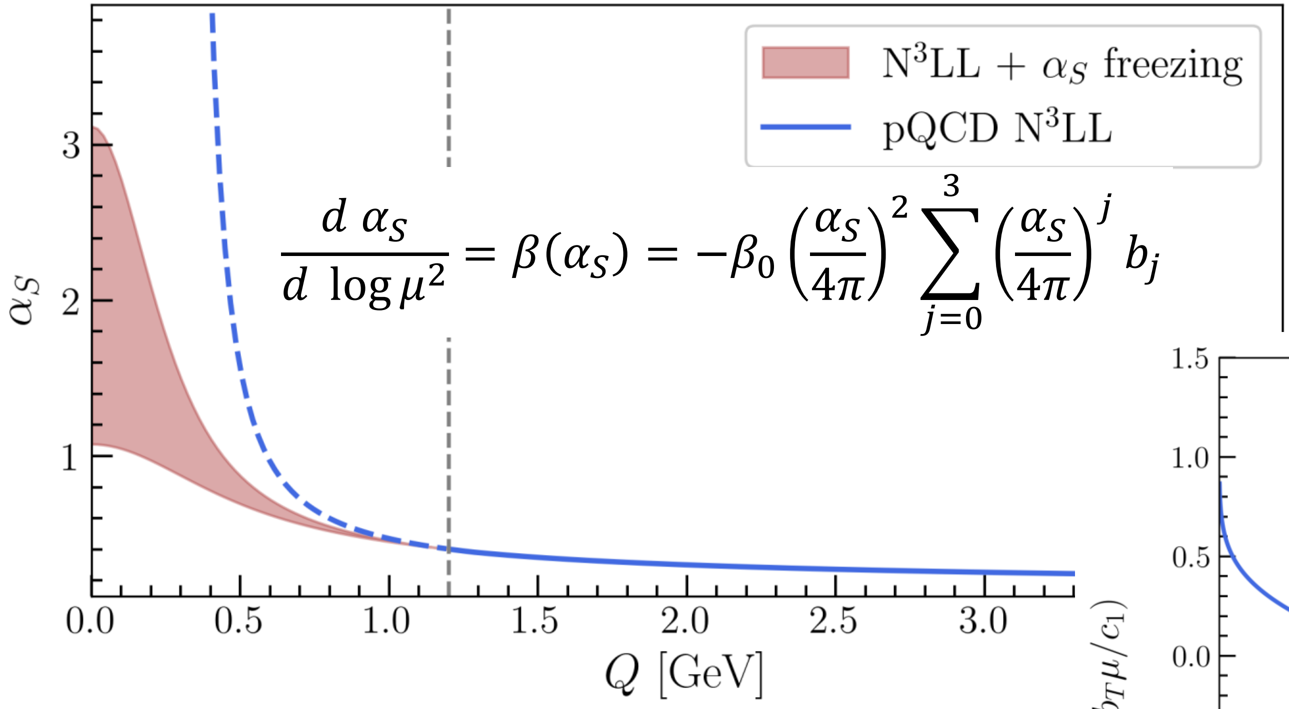
For instance: effect on CS-kernel

$$\frac{d\,\alpha_S}{d\ \log \mu^2} = \beta(\alpha_S) = -\beta_0 \left(\frac{\alpha_S}{4\pi}\right)^2 \sum_{j=0}^{3} \left(\frac{\alpha_S}{4\pi}\right)^j b_j$$





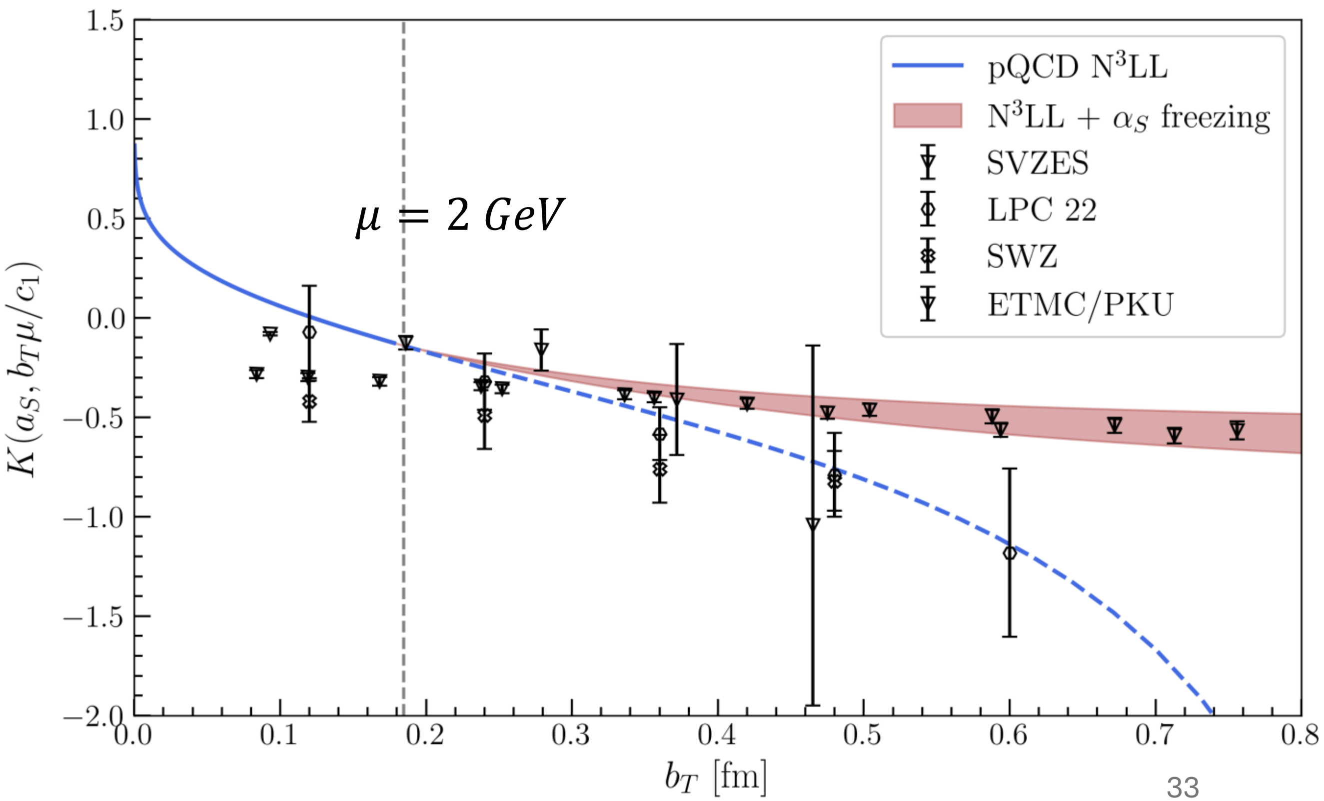
For instance: effect on CS-kernel



$$\frac{d\,\alpha_S}{d\,\log\mu^2} = \beta(\alpha_S) = -\beta_0\left(\frac{\alpha_S}{4\pi}\right)^2\sum_{j=0}^{3}\left(\frac{\alpha_S}{4\pi}\right)^j b_j$$

Here we assumed the deep IR model:

- $Q_0 = 1.2\ GeV$
- $450\ MeV \leq m_g \leq 650\ MeV$



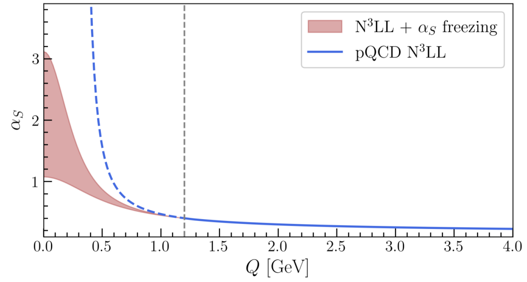
The freezing of alpha strong implies the large b_T behavior:

$$K(\alpha_S, b_T\mu/c_1) = K_0^\infty + \frac{1}{b_T^2}K_1^\infty + \cdots$$

InfraRed PDFs as non-perturbative models in TMDs

- **Consistency with DGLAP structure**

Strong coupling analytically continued into the Deep IR



$$f_j^{IR}(x,\mu) = \int_x^1 \frac{dz}{z} E_{jk}^{IR}(z, \alpha_S^{IR}, \alpha_0) f_k^{input}(x/z, Q_0)$$

DGLAP evolutor analytically continued into the Deep IR

Input model at scale Q_0 (as usual)

$$\frac{d\, E_{jk}^{IR}(z, \alpha_S^{IR}, \alpha_0)}{d\, \log\mu^2} = \int_z^1 \frac{d\,\xi}{\xi} P_{jl}^{IR}(\xi, \alpha_S^{IR}) E_{lk}^{IR}(x/\xi, \alpha_S^{IR}, \alpha_0)$$

DGLAP kernels analytically continued into the Deep IR.

Instead of fitting generic unknown non-perturbative factors, let's parametrize fundamental QCD evolution at low scales

InfraRed PDFs as non-perturbative models in TMDs

- Consistency with DGLAP structure

$$f_j^{IR}(x,\mu) = \int_x^1 \frac{d\,z}{z} E_{jk}^{IR}(z, \alpha_S^{IR}, \alpha_0) f_k^{input}(x/z, Q_0)$$

Compare against standard approach: $F_j(x, b_T, \mu) = f_j^{pQCD}(x, \mu_b^*) F_j^{NP}(x, b_T) exp\{S(\mu_b \to \mu)\}$

Effectively an (implicit) model for PDFs in the Deep IR, but **not compatible** with DGLAP structure (product instead of convolution)

$$f_j^{IR}(x,\mu) = f_j^{pQCD}(x, \mu^*) F_j^{NP}(x, c_1/\mu)$$

Obtained from (usual) perturbative DGLAP evolution

Instead of fitting F_j^{NP} let's focus on finding the appropriate model for the DGLAP splitting kernels P_{jk} at low scales

Automatic x and flavor dependence in the NP model

InfraRed PDFs as non-perturbative models in TMDs

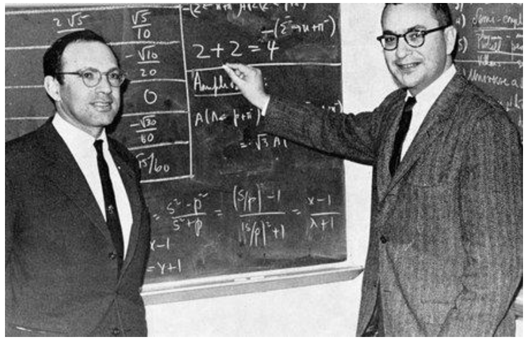
- **Consistency with inverse Fourier transform** $\left[F_j(x, b_T, \mu) \propto f_j^{IR}(x, 1/b_T\,), b_T \to \infty \right]$

$f_j^{IR}(x,\mu) \longrightarrow 0$ when the scale goes to zero, i.e. $\mu \to 0$

What should PDFs do at extremely low energies?

Any hint from NRQCD?

Enhance valence quarks and suppress gluons and sea? Perhaps recovering static quark model?



Is it correct to postulate that *they vanish* at zero energy?

*Note that the sole freezing of α_S in the deep IR is not sufficient to fulfil this constrain

InfraRed PDFs as non-perturbative models in TMDs

- Consistency with Valence Sum Rule

$$\int_0^1 dx\, E_V^{IR}(x, \alpha_S^{IR}, \alpha_0) = 1$$

Evolutor for **valence** PDF analytically continued into the Deep IR

For instance:

$$E_V^{IR}\left(x, \alpha_S^{IR}, \alpha_0\right) = \delta(1-x) + [V_{NP}(x)]_+$$

With the NP-model V_{NP} goes as $\sim 1/(1-x)$ at large x

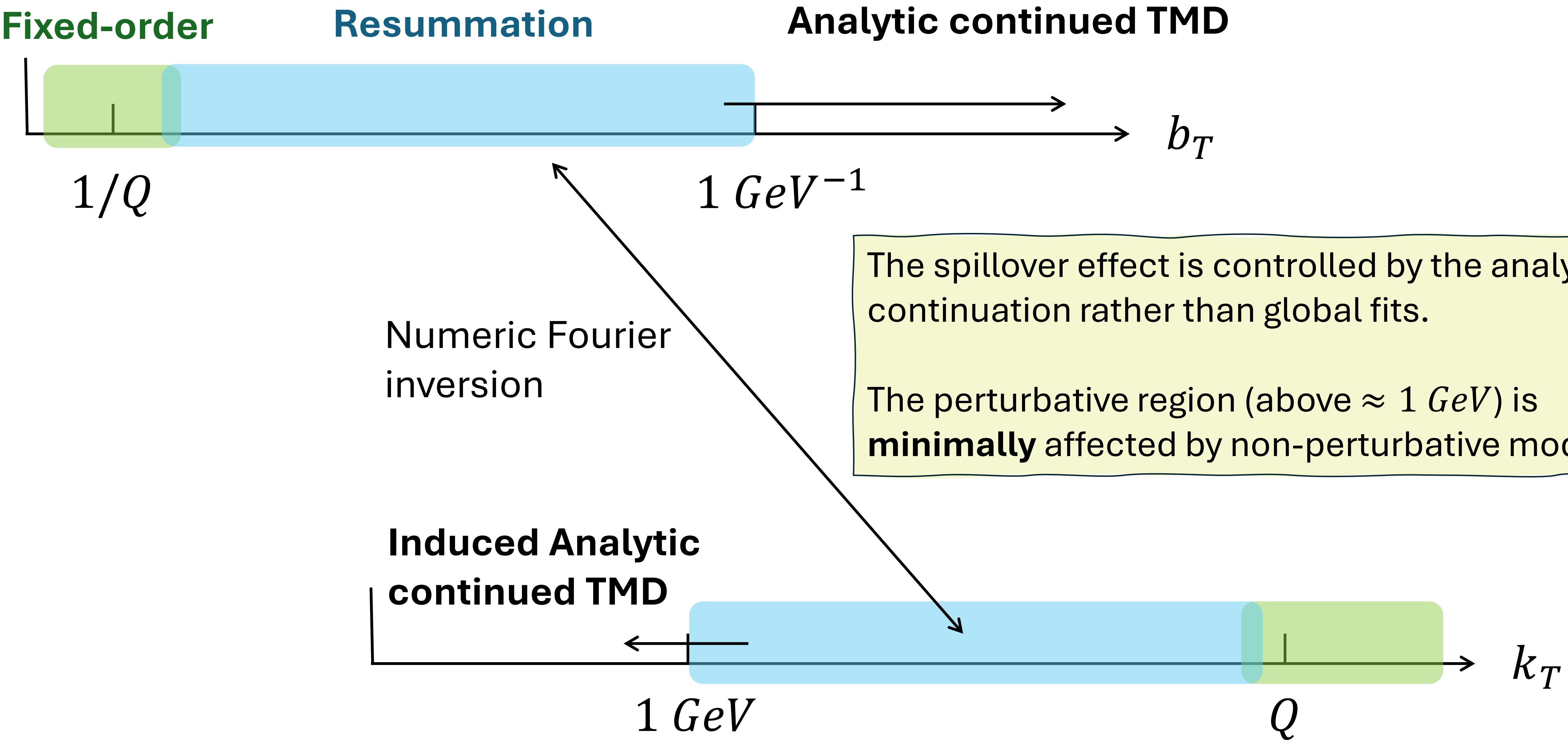
- Consistency with Momentum Sum Rule

$$\int_0^1 dx\, x\left(E_{qq}^{IR}\left(x, \alpha_S^{IR}, \alpha_0\right) + \boldsymbol{E_{gq}^{IR}}(x, \alpha_S^{IR}, \alpha_0)\right) = 1$$

$$\int_0^1 dx\, x\left(E_{qg}^{IR}\left(x, \alpha_S^{IR}, \alpha_0\right) + \boldsymbol{E_{gg}^{IR}}(x, \alpha_S^{IR}, \alpha_0)\right) = 1$$

Gluon and Quark TMD PDFs are *related* through the **singlet** sector of Deep IR DGLAP evolution

Analytic continuation into the Deep Infrared



The spillover effect is controlled by the analytic continuation rather than global fits.

The perturbative region (above $\approx 1\ GeV$) is **minimally** affected by non-perturbative modelling.

How deep can we probe QCD with TMDs?

*TMD phenomenology can be used to probe the Deep IR regime of **parton distributions** and of the **strong coupling**, opening a window onto the inner structure of QCD in a domain that is typically inaccessible.*

*In this perspective, the structure of the hadrons is an **emergent property** of QCD.*

Thank you!