q_T/Q correction for unpolarized Drell-Yan process in TMD factorization

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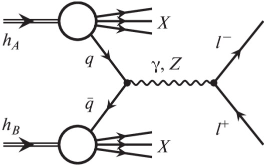
Outline

Introduction to TMD factorization theorem

Drell-Yan reaction

- $h_A(p_1) + h_B(p_2) \to \gamma^*/Z^*(q) + X \to \ell^-(l) + \ell^+(l') + X.$



- Differential cross section and angular coefficients.

$$\frac{d\sigma}{d^4q d\Omega} \sim \sum_n S_n(\theta, \phi)\Sigma_n, \qquad \Sigma_n \sim \mathfrak{L}_n^{\mu\nu} W_{\mu\nu}^{GG'}$$

$$W_{GG'}^{\mu\nu} = \int \frac{d^4y}{(2\pi)^4} e^{-\mathrm{i}qy} \langle p_1, p_2 | J_G^\mu(y) J_{G'}^\nu(0) | p_1, p_2 \rangle$$

- Split original QCD fields into two background and one dynamical component.

$$q = q_{\bar{n}} + q_n + \psi, \qquad A^\mu = A_{\bar{n}}^\mu + A_n^\mu + B^\mu$$

Background fields obey EOMs. Dynamical fields must be integrated out.

Power counting rules

- Λ is a low-energy scale of QCD. TMD factorization is derived in the limit

$$Q^2 \gg \Lambda^2, \qquad Q^2 \gg \boldsymbol{q}_T^2,$$

where

$$Q^2 = q^2, \qquad q^2 = \tau^2 - \boldsymbol{q}_T^2, \qquad \tau^2 = 2q^+q^-$$

- $\bar{n}$-collinear and n-collinear modes ($\{k^+, k^-, k_T\}$).

$$k_{\bar{n}}^\mu \lesssim \{1, \lambda^2, \lambda\}Q, \qquad k_n^\mu \lesssim \{\lambda^2, 1, \lambda\}Q, \qquad \lambda \sim \frac{\Lambda}{Q}$$

Same power counting for gluon fields components.

- "Good" and "bad" spinorial components ($\gamma^+ = n \cdot \gamma$, $\gamma^- = \bar{n} \cdot \gamma$).

$$\xi_{\bar{n}} = \frac{\gamma^-\gamma^+}{2} q_{\bar{n}} \sim \lambda, \qquad \eta_{\bar{n}} = \frac{\gamma^+\gamma^-}{2} q_{\bar{n}} \sim \lambda^2, \quad \eta_{\bar{n}} = -\frac{\gamma^+}{2}\frac{1}{D_+[A_{\bar{n}}]} \not{D}_T[A_{\bar{n}}]\xi_{\bar{n}}$$

TMD factorization at LP

- Leading term of hadronic tensor.

$$\widetilde{W}^{\mu\nu}_{\rm LP} \sim \mathbb{C}_{\rm LP}\left(\frac{\tau^2}{\mu^2}\right)\left\{\mathrm{Tr}\left(\gamma^{\mu}_{G}\overline{\Gamma}^{+}_{m}\gamma^{\nu}_{G'}\overline{\Gamma}^{-}_{n}\right)\Phi^{[\Gamma^{+}_{n}]}_{11}(x_1, b; \mu, \zeta)\overline{\Phi}^{[\Gamma^{-}_{m}]}_{11}(x_2, b; \mu, \bar{\zeta}) + \text{c.c.}\right\}$$

Twist-(1+1) TMD correlator.

$$\Phi^{[\Gamma]}_{11}(x, b) = \int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}e^{-\mathrm{i}x\lambda p^{+}}\langle p, s|\bar{\xi}W^{\dagger}(\lambda n + b)\frac{\Gamma}{2}W\xi(0)|p, s\rangle$$

- Unpolarized case, $\Phi^{[\gamma^{+}]}_{11}(x, b) = f_1(x, b)$.

$$W^{\mu\nu}_{{\rm LP}, f_1 f_1} \sim \mathbb{C}_{\rm LP}\left(\frac{\tau^2}{\mu^2}\right)\int d^2\boldsymbol{k}_{1T}d^2\boldsymbol{k}_{2T}\,\delta^{(2)}(\boldsymbol{q}_T - \boldsymbol{k}_{1T} - \boldsymbol{k}_{2T})$$
$$\boxed{g^{\mu\nu}_{T}}[f_1(x_1, k_{1T}; \mu, \zeta)\bar{f}_1(x_2, k_{2T}; \mu, \bar{\zeta}) + \text{c.c.}]$$

- Gauge invariance is preserved at LP, but broken at NLP.

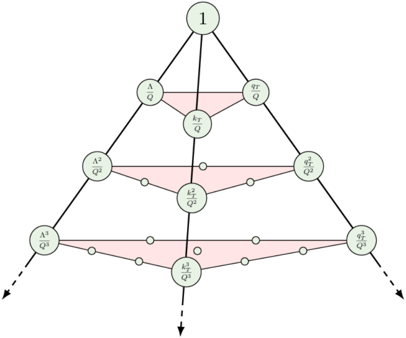
$$q_{\mu}W^{\mu\nu}_{\rm LP} = \mathcal{O}(\lambda^5)$$

- Valid only for $q_T \ll Q \to$ power corrections!

Power corrections in TMD factorization

Power corrections

- Three types of power corrections. Each triangle of the pyramid has the same power counting in λ. [A. Vladimirov, 2307.13054]
  - ⋆ $\Lambda/Q \rightarrow$ higher-twist contributions.
  - ⋆ $k_T/Q \rightarrow$ parton transverse momentum.
  - ⋆ $q_T/Q \rightarrow$ transverse component of gauge boson momentum.



Kinematic Power Corrections

- k_T/Q, k_T^2/Q^2, etc. correspond to transverse derivatives of twist-two TMD correlators. This series can be summed up.

- Unpolarized case.

$$W^{\mu\nu}_{\mathrm{KPC},f_1f_1} \sim \mathbb{C}_{\mathrm{LP}}\left(\frac{Q^2}{\mu^2}\right)\int d\xi_1 d\xi_2 \int d^4k_1 d^4k_2\, \delta^{(4)}(q-k_1-k_2)$$
$$\delta(k_1^+ - \xi_1 p_1^+)\delta(k_2^- - \xi_2 p_2^-)\delta(k_1^2)\delta(k_2^2)$$
$$\boxed{\left((k_1k_2)g^{\mu\nu} - k_1^\mu k_2^\nu - k_2^\mu k_1^\nu\right)}\left[f_1(\xi_1, k_T;\mu, Q^2)\bar{f}_1(\xi_2, k_T;\mu, Q^2) + \text{c.c.}\right]$$

where $\xi_{1,2} = \xi_{1,2}(\boldsymbol{k}_{1T}^2, \boldsymbol{k}_{2T}^2)$.

- Restoration of gauge invariance and frame invariance.

$$q_\mu W^{\mu\nu}_{\mathrm{KPC},f_1f_1} = (k_{1\mu} + k_{2\mu})W^{\mu\nu}_{\mathrm{KPC},f_1f_1} = 0$$

- Numerical improvement as $q_T \to 0$, but also for small values of Q.

Leading TMD approximation

Introduction to leading TMD approximation

- The genuine NLP hadronic tensor contains twist-three TMD correlators.

$$\Phi^{[\Gamma]}_{21,\mu}(\{z\}, b) = g\langle p, s|\bar{\xi}[z_1 n + b, z_2 n + b]F_{\mu+}W^\dagger(z_2 n + b)\frac{\Gamma}{2}W\xi(z_3 n)|p, s\rangle,$$

$$\Phi^{[\Gamma]}_{12,\mu}(\{z\}, b) = g\langle p, s|\bar{\xi}W^\dagger(z_1 n + b)\frac{\Gamma}{2}W(z_2 n)F_{\mu+}[z_2 n, z_3 n]\xi|p, s\rangle$$

- These twist-three TMD correlators behave as $1/b$ in the $b \to 0$ limit. The idea is to make a redefinition of twist-three TMD correlators.

$$\Phi^{[\Gamma]}_{3,\mu} = \widehat{\Phi}^{[\Gamma]}_{3,\mu} + \frac{b^\nu}{b^2}[S \otimes \Phi_2]^{[\Gamma]}_{\mu\nu}, \qquad \lim_{b\to 0}\widehat{\Phi}^{[\Gamma]}_{3,\mu} = \text{finite}$$

The kernel S can be computed perturbatively.

- Integrating over the transverse distance yields

$$d\sigma^{\text{gNLP}} = \widehat{d\sigma}^{\text{gNLP}} + \frac{q_T}{Q}d\sigma^{\text{L-TMD}}, \qquad \widehat{d\sigma}^{\text{gNLP}} \sim \frac{\Lambda}{Q}$$

Operator Product Expansion (OPE)

1. Write the twist-three TMD correlators in the path integral formalism and apply background field method.

$$q \to q + \psi, \qquad A_\mu \to A_\mu + B_\mu$$

The QCD action splits as

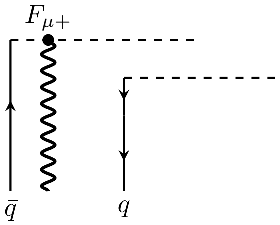
$$S_{\text{QCD}}[q+\psi, A+B] = S_{\text{QCD}}[q, A] + S_{\text{cov.}}[B, \psi] + S_{\text{int}}$$

Last term contains interaction terms between background and dynamical fields.

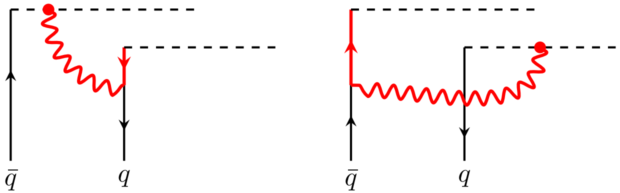
2. Take leading order (LO) term and fix light-cone gauge ($A_+ = 0$). Take terms from S_{int} and contract dynamical fields.

3. After computing the integral, restore gauge invariance.

- Twist-three TMD correlator.

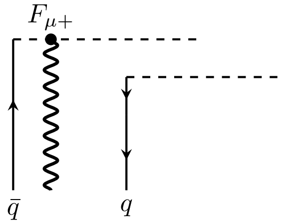


- Projection onto twist-two quark TMD correlator.

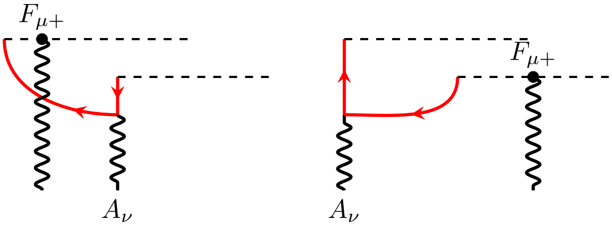


Gluon contribution

- Twist-three TMD correlator.



- Projection onto twist-two gluon TMD correlator.



- Notation: $\theta_{ij} = \theta(x_i, x_j) - \theta(-x_i, -x_j)$. Twist-(1+2) TMD correlator:

$$\Phi_{12,\mu}^{[\Gamma]}\Big|_{\text{quark}} = a_s \frac{C_F}{2} \frac{b^\nu}{b^2} \theta_{12} \mathrm{Tr}\left[\overline{\Gamma}_k^- \left(\gamma_{T\mu}\gamma_{T\nu} - \frac{x_1}{x_3}\gamma_{T\nu}\gamma_{T\mu}\right)\Gamma\right] \Phi_{11}^{[\Gamma_k^+]}(x_3, b),$$

$$\Phi_{12,\mu}^{[\Gamma]}\Big|_{\text{gluon}} = -\frac{a_s}{4} \frac{b_\nu}{b^2} \theta_{13} \mathrm{Tr}\left[\left(\frac{x_1}{x_2}\gamma_T^\nu\gamma_T^\alpha - \frac{x_3}{x_2}\gamma_T^\alpha\gamma_T^\nu\right)\gamma^-\Gamma\right] \Phi_{\mu\alpha}(-x_2, b)$$

- Technical note: special rapidity divergence must be subtracted. Substitute the parametrization for the TMD-twist-(1+1) TMD correlators.

$$\Phi_{11}^{[\gamma^+]}(x, b) = f_1(x, b) + \ldots,$$
$$\Phi_{11}^{[\gamma^+\gamma^5]}(x, b) = \ \ldots,$$
$$\Phi_{11}^{[\mathrm{i}\sigma^{\alpha+}\gamma^5]}(x, b) = \mathrm{i}\epsilon_T^{\alpha\mu} b_\mu M h_1^\perp(x, b) + \ldots$$

- Find the corresponding expressions for $\Gamma \in \{\gamma^+, \gamma^+\gamma^5, \mathrm{i}\sigma^{\alpha+}\gamma^5\}$.

- Compare with the parametrization for the twist-(1+2) TMD correlator.

$$\begin{aligned}
\boldsymbol{\Phi}^{[\gamma^+]}_{12,\mu}(x_{1,2,3}, b) &= \mathrm{i} b^\mu M^2 \boldsymbol{f}^{\perp}_{12}(x_{1,2,3}, b), \\
\boldsymbol{\Phi}^{[\gamma^+\gamma^5]}_{12,\mu}(x_{1,2,3}, b) &= -\mathrm{i}\epsilon_T^{\mu\nu} b_\nu M^2 \boldsymbol{g}^{\perp}_{12}(x_{1,2,3}, b) + \ldots, \\
\boldsymbol{\Phi}^{[\mathrm{i}\sigma^{\alpha+}\gamma^5]}_{12,\mu}(x_{1,2,3}, b) &= \epsilon_T^{\mu\alpha} M \boldsymbol{h}_{12}(x_{1,2,3}, b) + \ldots \\
&\quad + (b^\mu \epsilon_T^{\alpha\beta} + b^\alpha \epsilon_T^{\mu\beta}) b_\beta M^3 \boldsymbol{h}^{\perp}_{12}(x_{1,2,3}, b) + \ldots
\end{aligned}$$

- After some combinations of distributions, we get twist-three TMDPDFs in terms of twist-two ones.

$$\boldsymbol{f}^{\perp}_{2}\Big|_{\text{L-TMD}} = \frac{a_s}{b^2 M^2}\left[4C_F \theta_{23}(f_1(-x_1, b) - f_1(x_3, b)) + \frac{x_1}{x_2}\theta_{13} f_g(-x_2, b)\right],$$

$$\bar{\boldsymbol{f}}^{\perp}_{2}\Big|_{\text{L-TMD}} = \frac{-a_s}{b^2 M^2}\left[4C_F \theta_{23}(\bar{f}_1(-x_1, b) - \bar{f}_1(x_3, b)) + \frac{x_1}{x_2}\theta_{13} f_g(-x_2, b)\right]$$

Final result

- gNLP coefficients in the L-TMD approximation.

$$\Sigma_1^{\text{L-TMD}} = \frac{8\pi\alpha_{\text{em}}^2}{3N_c s Q^2} \sum_{f,G,G'} Q^4 \Delta_G^* \Delta_{G'} \boxed{\frac{-2|\boldsymbol{q}_T|}{Q}} z_{+\ell}^{GG'} z_{+f}^{GG'} \mathcal{J}_+^{\text{L-TMD}},$$

$$\Sigma_3^{\text{L-TMD}} = \frac{8\pi\alpha_{\text{em}}^2}{3N_c s Q^2} \sum_{f,G,G'} Q^4 \Delta_G^* \Delta_{G'} \boxed{\frac{-4|\boldsymbol{q}_T|}{\tau}} z_{-\ell}^{GG'} z_{-f}^{GG'} \mathcal{J}_-^{\text{L-TMD}}$$

- Convolutions finite at $q_T \to 0$. First TMDPDF evaluated at (x_1, b). Second one evaluated at (x_2, b).

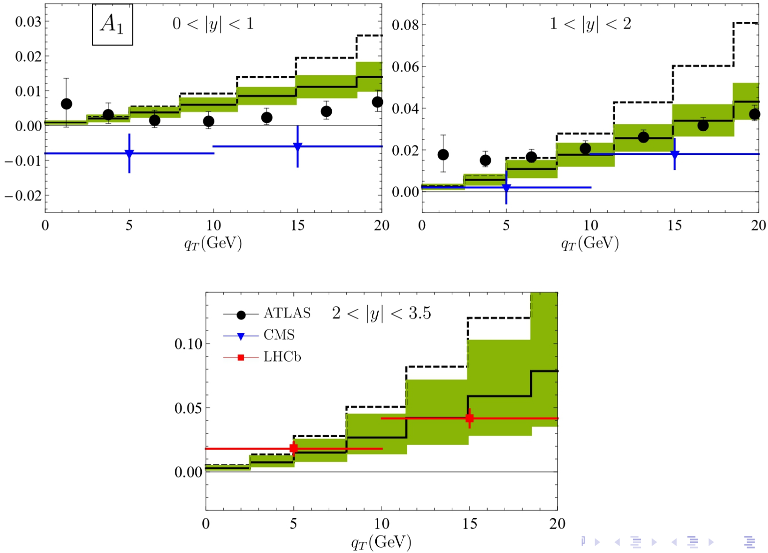
$$\mathcal{J}_\pm^{\text{L-TMD}} = \frac{1}{2}\int_0^\infty \frac{b\,db}{2\pi}(J_0(b|\boldsymbol{q}_T|) + J_2(b|\boldsymbol{q}_T|))\left(\frac{\tau^2}{\zeta_\mu}\right)^{-2\mathcal{D}(b,\mu)} \Big\{ s_{qq}\otimes f_1\bar{f}_1 \pm s_{qq}\otimes \bar{f}_1 f_1 - f_1 s_{qq}\otimes \bar{f}_1 \mp \bar{f}_1 s_{qq}\otimes f_1 + s_{qg}\otimes f_g(f_1 \pm \bar{f}_1) - (f_1 \pm \bar{f}_1) s_{qg}\otimes f_g \Big\}$$

- Quark and gluon coefficient functions (at a_s order):

$$s_{qq}(x) = \frac{4a_s C_F}{(1-x)_+}, \quad s_{qg}(x) = a_s(1-x), \quad s\otimes f(x) = \int_x^1 \frac{dy}{y} s(y) f\left(\frac{x}{y}\right)$$

Theoretical prediction vs. experimental data

- Coefficient $A_1 = \Sigma_1/\Sigma_U$. $Q \in [80, 100]$ GeV; ATLAS, CMS and LHCb; $\sqrt{s} = 8$ TeV run. [A. Arroyo-Castro, I. Scimemi, A. Vladimirov, 2503.24336].



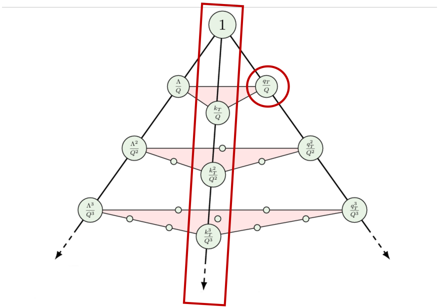
Final remarks

Final remarks (I)

- **Conclusions**.

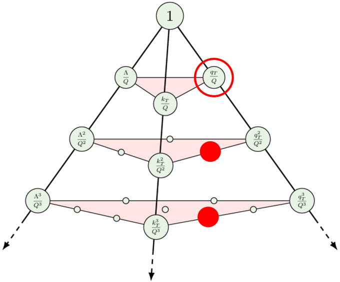
q_T/Q correction is "hidden" in twist-three terms

Leading TMD approximation is in complete agreement with data



- **What are we working on?**

Compute $k_T q_T / Q^2$, $k_T^2 q_T / Q^3$, $k_T^3 q_T / Q^4$, etc.



- **Long-term goals?** Compute q_T^2/Q^2, q_T^3/Q^3, etc.

Thank you for your attention!

Back up

Composite background field method

- Path integral. (Anti-)causal sectors.
- We split fields into background (which obey EOMs) and dynamical components:

$$q^{(\pm)} = q_{\bar{n}}^{(\pm)} + q_n^{(\pm)} + \psi^{(\pm)},$$
$$A_\mu^{(\pm)} = A_{\bar{n},\mu}^{(\pm)} + A_{n,\mu}^{(\pm)} + B_\mu^{(\pm)}.$$

- Background field gauge:

$$D_\mu[A_{\bar{n}}^{(\pm)} + A_n^{(\pm)}]B^{(\pm)\mu} = 0.$$

- Hadronic tensor is matrix element of effective operator:

$$\mathcal{J}_{GG'}^{\mu\nu}(y) \sim J_G^{\mu(-)}[q](y) J_{G'}^{\nu(+)}[q](0) e^{iS_{\text{int}}^{(+)} - iS_{\text{int}}^{(-)}}\Big|_{q=\psi+q_{\bar{n}}+q_n}.$$

- Light-cone gauge:

$$A_{\bar{n}}^{+(\pm)} = 0, \quad A_n^{-(\pm)} = 0.$$

gNLP angular coefficients

$$\frac{Q}{2M}\Pi_1^{\text{gNLP}} = z_{+q}^{GG'}\mathcal{J}_1^R[\{f_1\mathbf{f}_2^\perp + \mathbf{f}_2^\perp f_1\}_A] - 2r_{+q}^{GG'}\mathcal{J}_1^R[\{h_1^\perp\mathbf{h}_\ominus + \mathbf{h}_\ominus h_1^\perp\}_A]$$
$$+z_{+q}^{GG'}\mathcal{J}_1^I[\{f_1\mathbf{g}_2^\perp + \mathbf{g}_2^\perp f_1\}_A] - 2r_{+q}^{GG'}\mathcal{J}_1^I[\{h_1^\perp\mathbf{h}_\oplus + \mathbf{h}_\oplus h_1^\perp\}_A],$$

$$\frac{\mathcal{T}}{4M}\Pi_3^{\text{gNLP}} = z_{-q}^{GG'}\mathcal{J}_1^R[\{f_1\mathbf{f}_2^\perp + \mathbf{f}_2^\perp f_1\}_S] + 2ir_{-q}^{GG'}\mathcal{J}_1^R[\{h_1^\perp\mathbf{h}_\oplus + \mathbf{h}_\oplus h_1^\perp\}_S]$$
$$+z_{-q}^{GG'}\mathcal{J}_1^I[\{f_1\mathbf{g}_2^\perp + \mathbf{g}_2^\perp f_1\}_S] - 2ir_{-q}^{GG'}\mathcal{J}_1^I[\{h_1^\perp\mathbf{h}_\ominus + \mathbf{h}_\ominus h_1^\perp\}_S],$$

$$\frac{\mathcal{T}}{2M}\Pi_6^{\text{gNLP}} = z_{-q}^{GG'}\mathcal{J}_1^R[\{f_1\mathbf{g}_2^\perp + \mathbf{g}_2^\perp f_1\}_S] - 2ir_{-q}^{GG'}\mathcal{J}_1^R[\{h_1^\perp\mathbf{h}_\ominus + \mathbf{h}_\ominus h_1^\perp\}_S]$$
$$-z_{-q}^{GG'}\mathcal{J}_1^I[\{f_1\mathbf{f}_2^\perp + \mathbf{f}_2^\perp f_1\}_S] - 2ir_{-q}^{GG'}\mathcal{J}_1^I[\{h_1^\perp\mathbf{h}_\oplus + \mathbf{h}_\oplus h_1^\perp\}_S],$$

$$\frac{Q}{4M}\Pi_7^{\text{gNLP}} = z_{+q}^{GG'}\mathcal{J}_1^R[\{f_1\mathbf{g}_2^\perp + \mathbf{g}_2^\perp f_1\}_A] - 2r_{+q}^{GG'}\mathcal{J}_1^R[\{h_1^\perp\mathbf{h}_\oplus + \mathbf{h}_\oplus h_1^\perp\}_A]$$
$$-z_{+q}^{GG'}\mathcal{J}_1^I[\{f_1\mathbf{f}_2^\perp + \mathbf{f}_2^\perp f_1\}_A] + 2r_{+q}^{GG'}\mathcal{J}_1^I[\{h_1^\perp\mathbf{h}_\ominus + \mathbf{h}_\ominus h_1^\perp\}_A]$$

Theoretical approach

- New twist-3 TMD correlators:

$$\widehat{\Phi}_{\bullet,\mu}(x_{1,2,3}, b; \mu) = \Phi_{\bullet,\mu}(x_{1,2,3}, b; \mu) - \frac{b^\nu}{b^2}[S_\bullet \otimes \Phi_{11}]_{\mu\nu}(x_{1,2,3}, b; \mu)$$

such that $\lim_{b\to 0} \widehat{\Phi}^{[\Gamma]}_{\bullet,\mu}(x_{1,2,3}, \mu) = \text{finite}$.

- Operator Product Expansion (OPE) for the twist-2 TMD correlator:

$$\Phi_{11}(x, b) = C_{2/f}(\mathbf{L}_b; \mu_{\text{OPE}}) \otimes f_{\text{coll}}(x; \mu_{\text{OPE}}) + \mathcal{O}(b^2), \quad \mathbf{L}_b = \ln(\mu_{\text{OPE}}^2 b^2)$$

This relation can be inverted perturbatively (set $\mu_{\text{OPE}} = \mu$):

$$f_{\text{coll}}(x; \mu) = C^{-1}_{2/f}(\mathbf{L}_b; \mu) \otimes \Phi_{11}(x, b) + \mathcal{O}(b^2)$$

- Twist-3 OPE:

$$\Phi^\mu_\bullet(x_{1,2,3}, b; \mu) = \frac{b_\nu}{b^2} C^{\mu\nu}_{3/f}(\mathbf{L}_b; \mu) \otimes f_{\text{coll}}(x; \mu) + \mathcal{O}(b^0)$$

- Altogether:

$$[S_\bullet \otimes \Phi_{11}]^{\mu\nu}(x_{1,2,3}, b; \mu) = C^{\mu\nu}_{3/f}(\mathbf{L}_b; \mu) \otimes C^{-1}_{2/f}(\mathbf{L}_b; \mu) \otimes \Phi_{11}(x, b)$$

Matching of twist-three TMD distributions

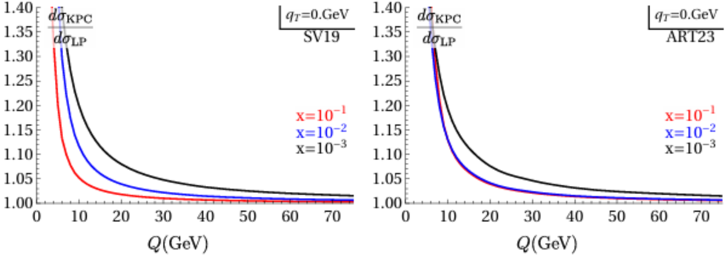
$$f_2^{\perp} = \frac{1}{b^2 M^2}\left[4 a_s C_F \theta_{23} \left(f_1(-x_1, b) - f_1(x_3, b)\right) + a_s \theta_{13} \frac{x_1}{x_2} f_g(-x_2, b)\right],$$

$$\mathbf{g}_2^{\perp} = 0,$$

$$\bar{\mathbf{f}}_2^{\perp} = \frac{-1}{b^2 M^2}\left[4 a_s C_F \theta_{23} \left(\bar{f}_1(-x_1, b) - \bar{f}_1(x_3, b)\right) + a_s \theta_{13} \frac{x_1}{x_2} f_g(-x_2, b)\right],$$

$$\bar{\mathbf{g}}_2^{\perp} = 0$$

- Ratio between KPCs and LP Drell-Yan cross section at $q_T = 0$ [A. Vladimirov, 2307.13054].



- Fraction momentum.

$$\xi_1 = \frac{x_1}{2}\left(1 + \frac{\boldsymbol{k}_{1T}^2}{\tau^2} - \frac{\boldsymbol{k}_{2T}^2}{\tau^2} + \frac{\sqrt{\lambda(\boldsymbol{k}_{1T}^2, \boldsymbol{k}_{2T}^2, \tau^2)}}{\tau^2}\right)$$