Order statistics for multi-jet events

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TÉCNICO LISBOA

Outline

This work focuses on BFKL phenomenology, which remains a very rich field in QCD:

1. High-energy scattering in pQCD.
2. BFKL evolution (Balitsky-Fadin-Kuraev-Lipatov).
3. Order statistics for multi-jet events.

Resummation at High Energies

At very high center–of–mass energies, perturbative QCD calculations receive large logarithmic contributions of the form

$$\left(\alpha_s \ln \frac{s}{s_0}\right)^n .$$

Even if the strong coupling α_s is small, the logarithms can become large enough that

$$\alpha_s \ln \frac{s}{s_0} \sim \mathcal{O}(1),$$

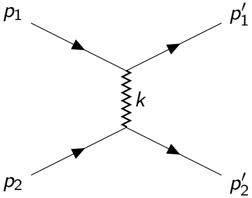
making fixed–order perturbation theory unreliable. To recover predictive power, one must resum these logarithmically enhanced contributions to all orders in perturbation theory.

Physical picture

BFKL resummation $\Longrightarrow$ systematic summation of high–energy logarithms.

The Reggeized Gluon

BFKL resummation is connected with the old Regge–Gribov Theory, and relies on the assumption that the gluon *reggeizes*.

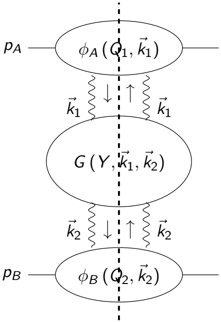


The reggeized gluon is a new degree of freedom arising in the high energy limit, with a "dressed" propagator:

$$D_{\mu\nu} = -\frac{i\eta_{\mu\nu}}{k^2} \quad \rightarrow \quad D^{\mathcal{R}}_{\mu\nu} = -\frac{i\eta_{\mu\nu}}{k^2}\left(\frac{s}{\mathbf{k}^2}\right)^{\epsilon(\mathbf{k})},$$

where $\mathbf{k}$ is the hard scale of the process, and $\epsilon(\mathbf{k})$ is called *gluon Regge trajectory*.

BFKL resummation



Due to the reggeization of the gluon, in BFKL formalism the partonic hard cross-section can be factorised as

$$\sigma_p(Q_1, Q_2, Y) = \int \mathrm{d}^2\vec{k}_1\,\mathrm{d}^2\vec{k}_2\,\phi_A(Q_1,\vec{k}_1)\,\phi_B(Q_2,\vec{k}_2)\,G(\vec{k}_1,\vec{k}_2,Y),$$

where impact factors are process dependent, and the gluon Green's function is universal.

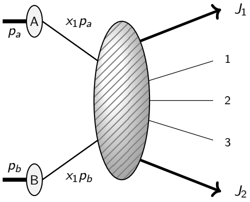
Phenomenological challenges

Due to the resummation, BFKL dynamics should describe the high-energy behaviour of scattering processes, and it should exhibit different predictions compared to the non-resummed dynamics.

However, detecting a clear BFKL signature is not an easy task:

- Nonetheless different classes of observables are well described by BFKL dynamics (inclusive DIS, jet azimuthal decorrelation), good fits are also obtained using non-BFKL approaches (DGLAP-based approaches).
- Several observables have been proposed in the literature in order to disentangle the BFKL dynamics (Mueller-Navelet jets, Mueller-Tang jets).
- It's important to study differential distributions, or in general more exclusive observables.

Order statistics for multi-jet events



We consider the following events:

- Proton-proton scattering at 13 TeV.
- Jet activity between the rapidity boundary jets.
- Rapidity window (-4.7, 4.7).
- $p_{T_{jet}} > 20$ GeV.
- Anti-k_T algorithm with R $= 0.4$.

PYTHIA8 vs HERWIG7 vs BFKLex

Ordering jets by rapidity is connected with order statistics: the idea is that the rapidity distribution of ordered jets reflects the parent dynamics. Different underlying dynamics lead to different ordered rapidity distributions.

We investigate differences between collinear and BFKL dynamics by comparing collinear Monte Carlo event generators with BFKLex.

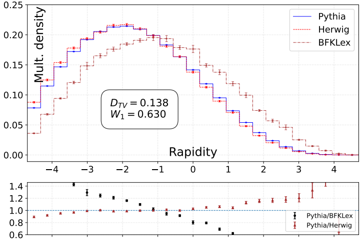
BFKLex is a Monte Carlo generator that implements the iterative solution of the BFKL equation at LLA and NLLA.

- PYTHIA8 and HERWIG7: leading-order matrix elements, collinear parton-shower based dynamics, MPI, hadronization.
- BFKLex: high energy logarithmic resummation. We consider only the leading logarithmic approximation (LLA).

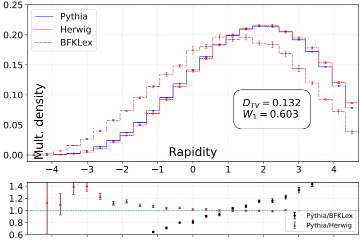
Observables

We investigate the following observables:

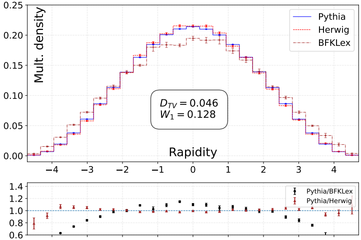
1. The rapidity distribution of each jet in three-jet events.
2. Rapidity distribution of the most backward and most forward jets, for all multiplicity events.
3. Same as above, but for the second most backward and second most forward jets.



Jet 1

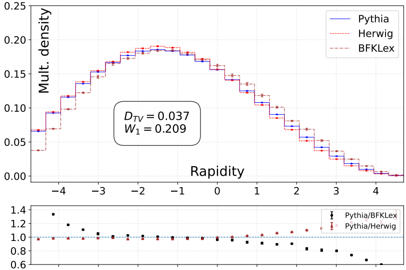


Jet 3

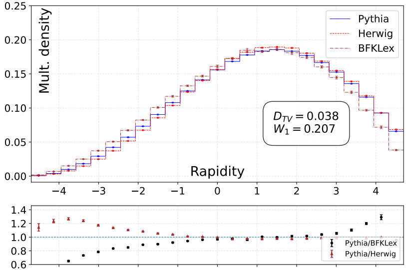


Jet 2

MB and MF rapidity distributions

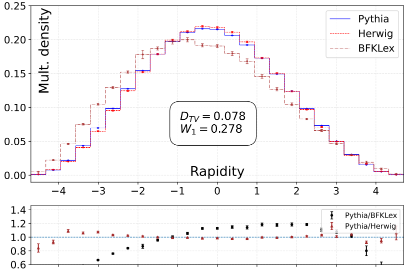


MB

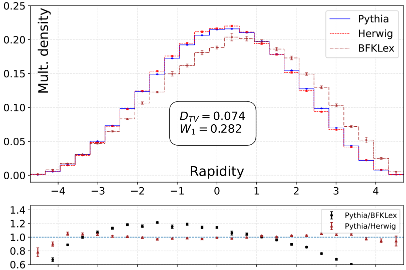


MF

SMB and SMF rapidity distributions



SMB



SMF

D_{TV} and W_1 distances

To quantify the separation of two unit-normalized rank histograms we use two complementary shape distances. Defining $q_i = h_i \Delta y_i$ and $p_i = g_i \Delta y_i$,

- The total-variation distance measures the fraction of probability weight that must be reassigned among bins,

$$D_{TV} = \frac{1}{2} \sum_{i=1}^{N_{bin}} |p_i - q_i|.$$

- The Wasserstein distance is sensitive to horizontal displacements of probability weight,

$$W_1 = \sum_{i=1}^{N_{bin}-1} \left| \sum_{j=1}^{i} (p_j - q_j) \right| \Delta y_{i,i+1}.$$

The quantities D_{TV} and W_1 are used here as shape-distance measures between normalized predictions, not as goodness-of-fit tests or exclusion criteria.

Experimental cuts

Then, we conducted another analysis, with new experimental cuts:

- $p_{T_{jet}} > 60$ GeV.
- HT2 $= 250$ GeV, where HT2 is the p_T sum of the two leading jets in transverse momentum.

Cuts are experimentally motivated by the following ATLAS analysis:

Measurements of jet cross-section ratios in 13 TeV proton–proton collisions with ATLAS

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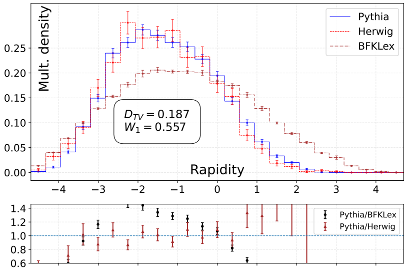
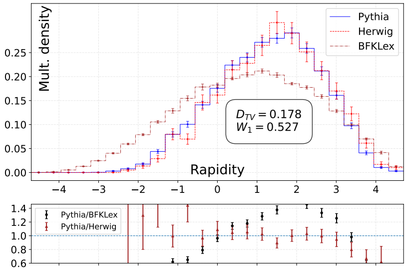
Three-jet events rapidity distributions

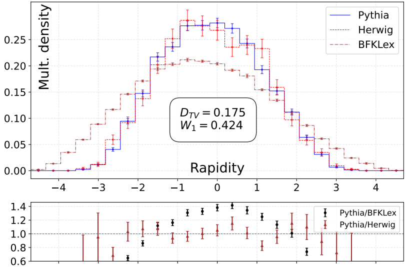


Jet 1

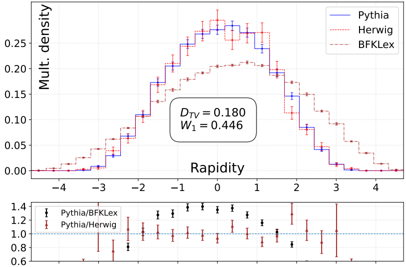


Jet 3

SMB and SMF rapidity distributions



SMB



SMF

Conclusion

In conclusion, the message of the talk is:

1. Rapidity-ordered jet distributions in multi-jet events are analysed in order to obtain a fingerprint of BFKL dynamics.
2. Significant differences are observed by focusing on the differences between Monte Carlo event generators BFKLex, PYTHIA8, and HERWIG7.
3. This study potentially opens the door to the investigation of new complementary exclusive observables. We will soon compare 2D rapidity distributions and rapidity correlations among ordered jets.
4. The ultimate goal is obviously to compare the Monte Carlo predictions to experimental data. We are in touch with CMS collaborators, and we aim to do it soon.