Nuclear corrections to DVCS

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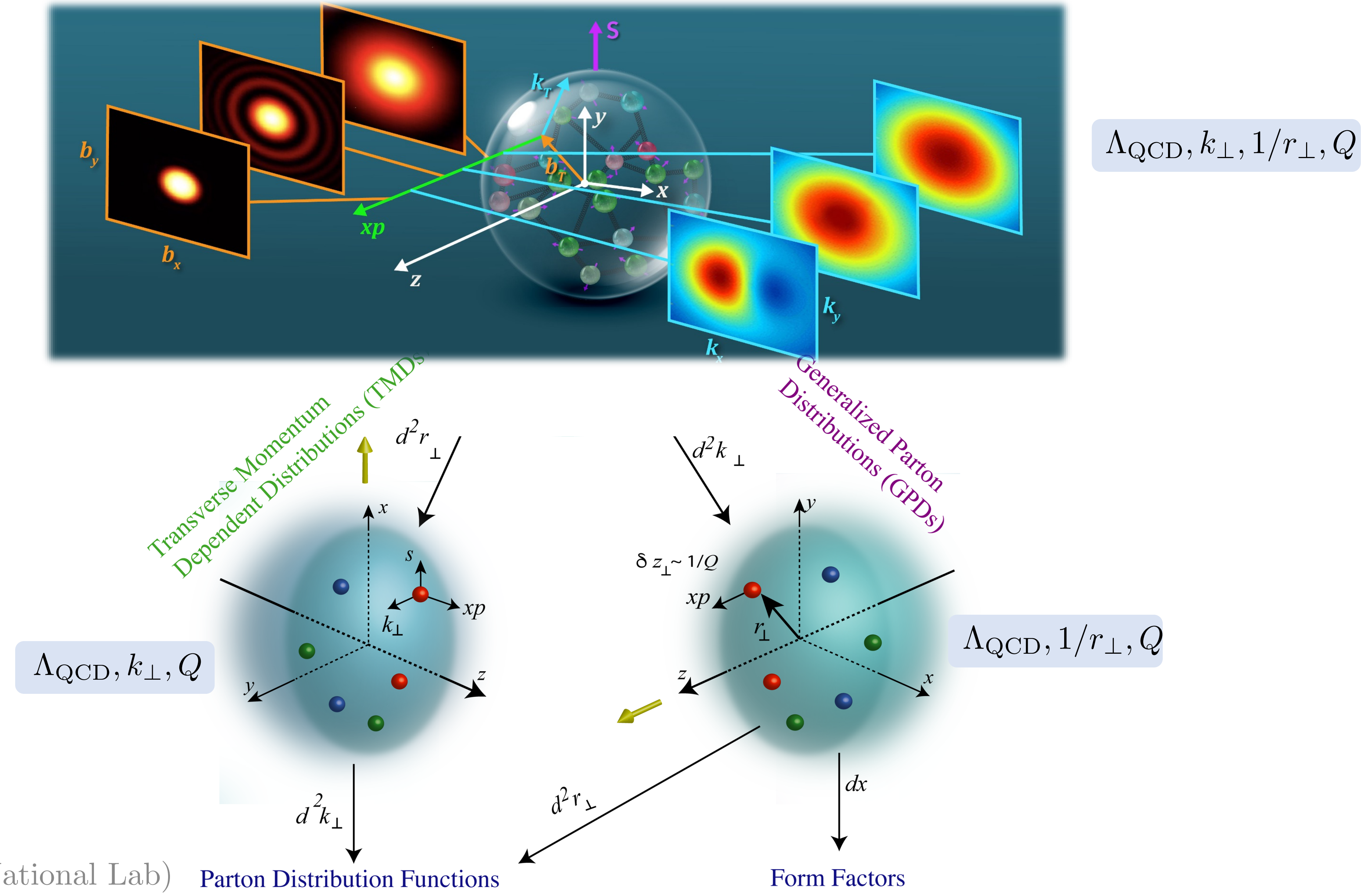
Based on an upcoming paper
Bhattacharya, Ke, Vitev, JT

QCD Evolution 2026

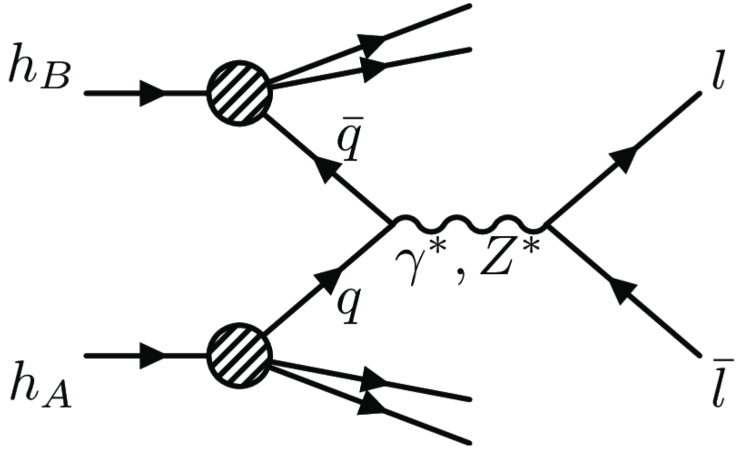
The structure of matter

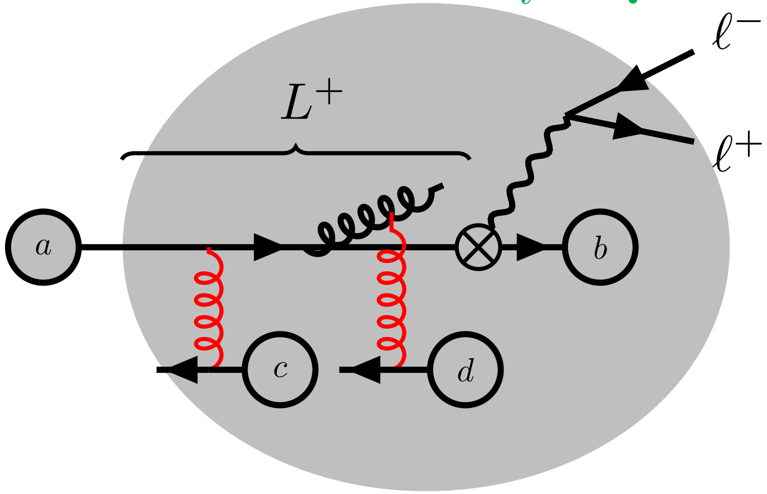
Distributions of partons in hadrons



Power counting nuclear TMD observables

Spectrum of energy and transverse momentum in matter is modified in a non-trivial way in QCD



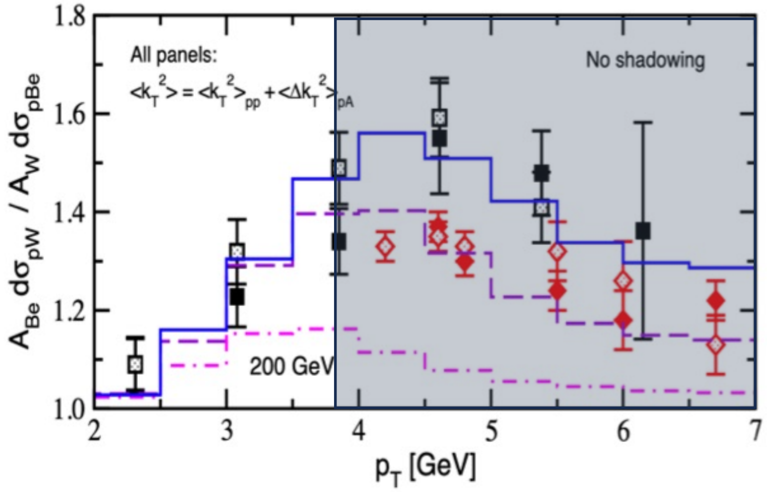


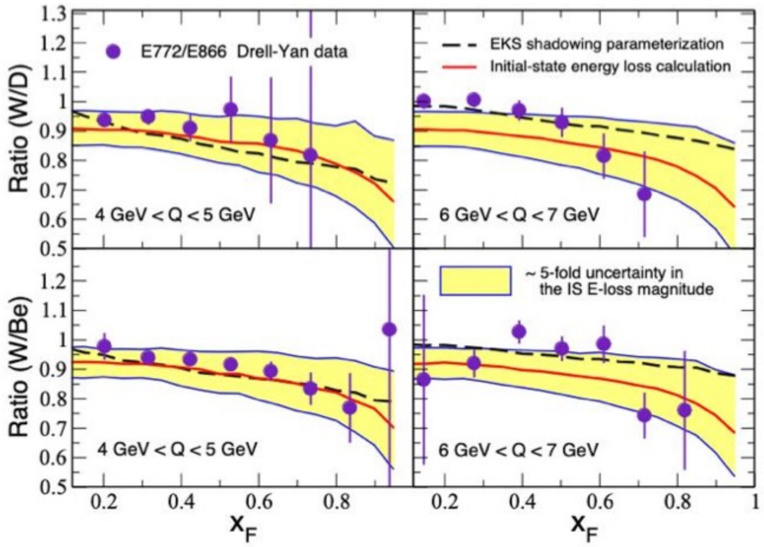
The nuclear medium can affect transverse momentum distributions in three distinct ways

1.) The intrinsic transverse momentum of the quarks in the nucleus differs from that in free nucleons

2.) Interactions between the incoming quark from the proton undergo re-scattering

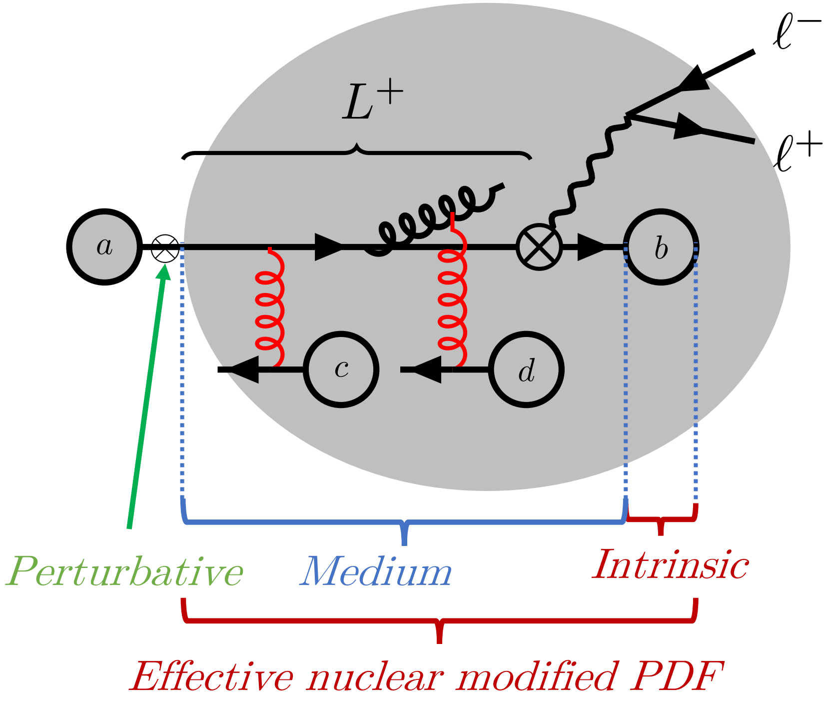
3.) Re-scattering induces energy loss





The structure of matter in the medium: an efficient approximation

nTMDs were originally defined using an approximate scheme by these two



Alrashed, Kang, JT, Xing et al (2021)
Alrashed, Kang, JT, Xing et al (2023)

The nuclear medium can affect transverse momentum distributions in three distinct ways

1.) The intrinsic transverse momentum of the quarks in the nucleus differs from that in free nucleons

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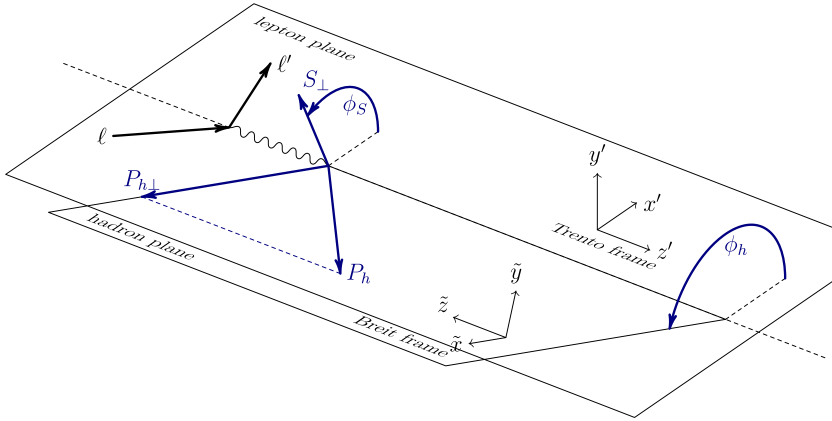
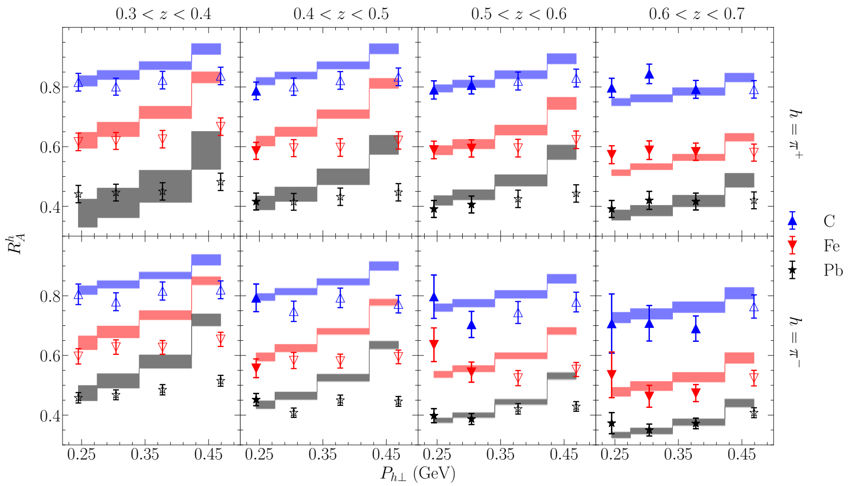
Description of the experimental data

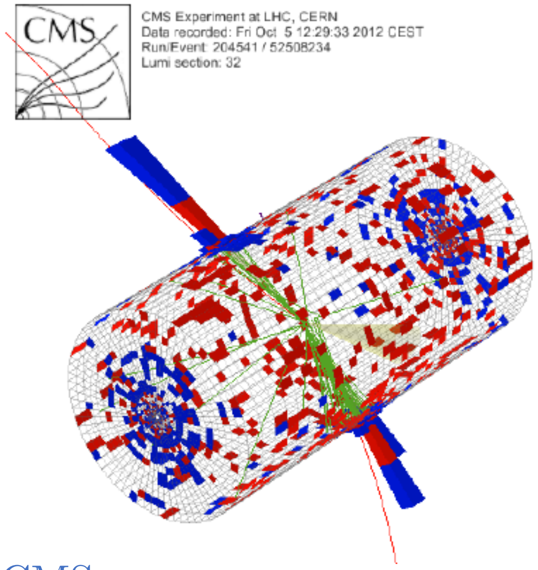
Semi-Inclusive DIS data



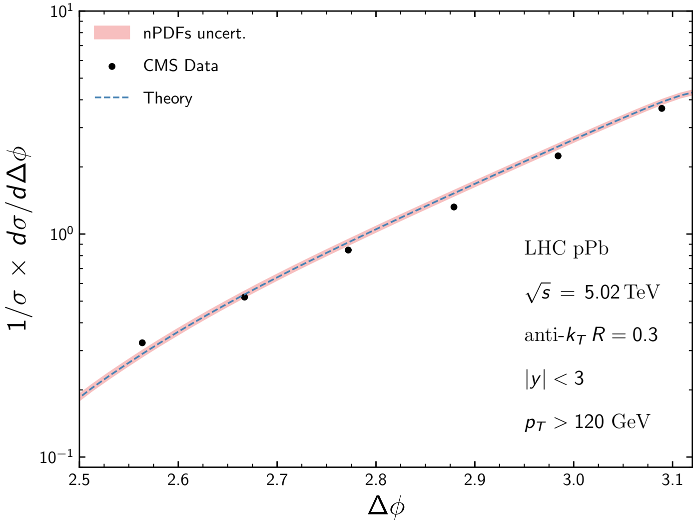
CLAS 12
10.1103/PhysRevC.105.015201



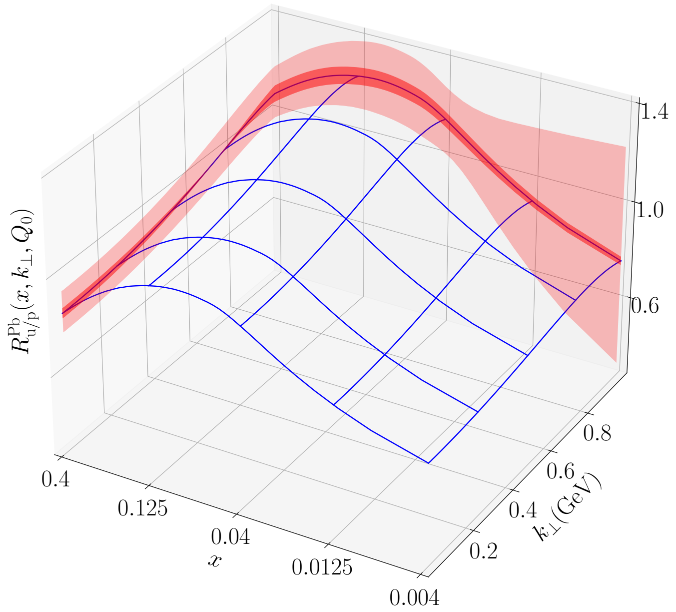
Di-jet measurements at the LHC



CMS
Eur. Phys. J. C 74 (2014) 2951

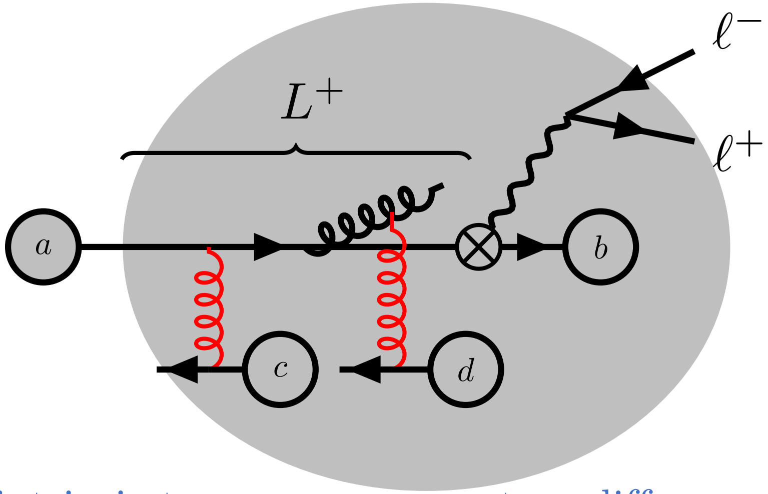


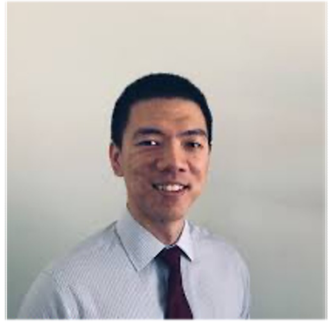
$$R_{u/p}^{\mathrm{Pb}}(x, k_\perp, Q_0) = \frac{f_{u/p}^{\mathrm{Pb}}\left(x, k_\perp, Q_0, Q_0^2\right)}{f_{u/p}\left(x, k_\perp, Q_0, Q_0^2\right)}$$



Power counting nuclear TMD observables

Spectrum of energy and transverse momentum in matter is modified in a non-trivial way in QCD

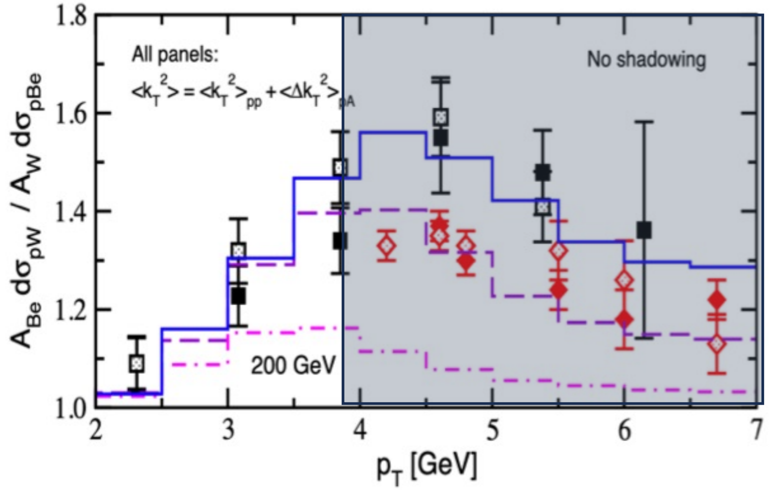




~~1.) The intrinsic transverse momentum difference~~

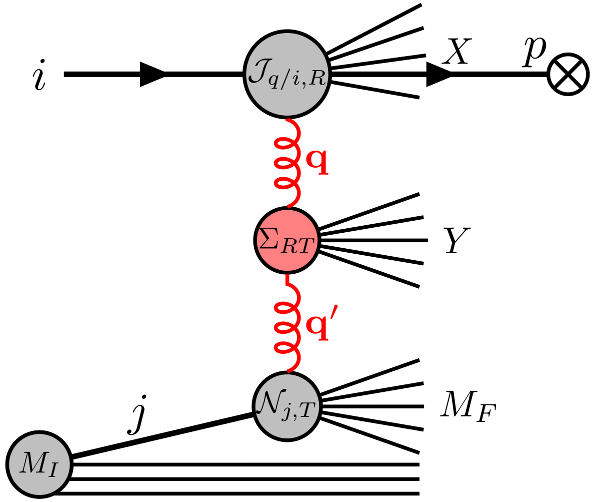
2.) Interactions between the incoming quark from the proton undergo re-scattering

3.) Re-scattering induces energy loss



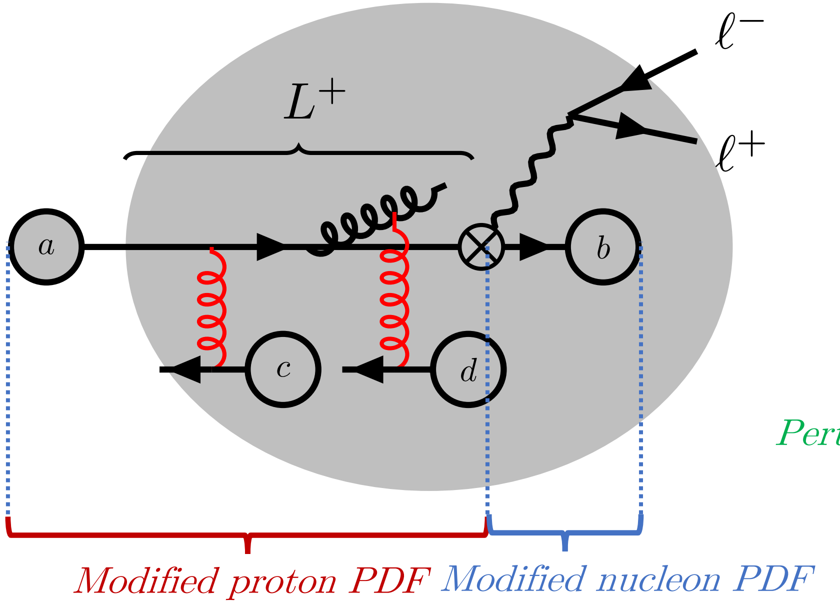
$k_1^\mu \sim Q\left(\lambda^2, 1, \lambda\right)$

$k_2^\mu \sim Q\left(1, \lambda^2, \lambda\right)$

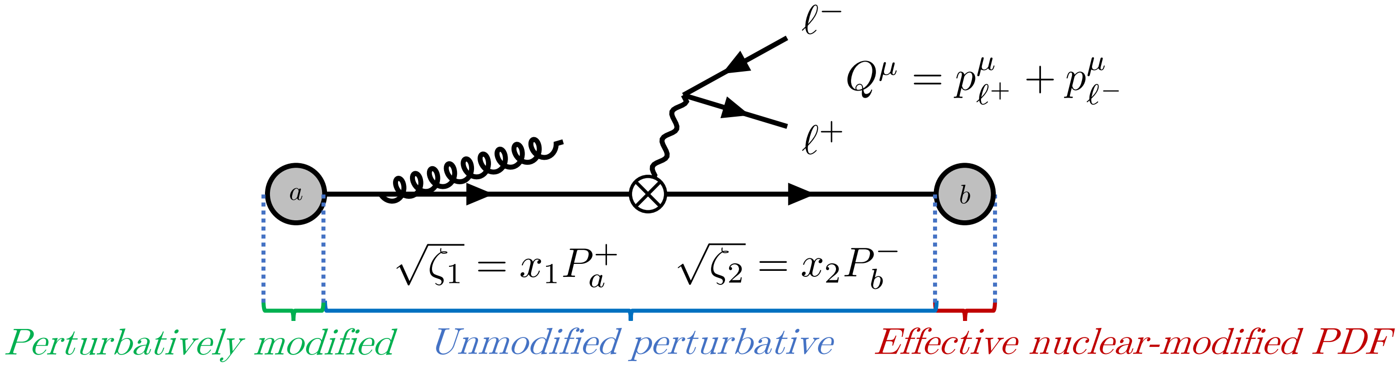


The structure of matter in the medium: modified beam function

Spectrum of energy and transverse momentum in matter is modified in a non-trivial way in QCD

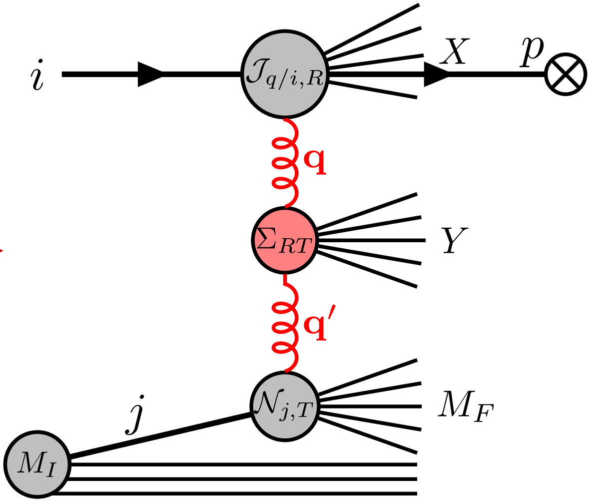


Ke, Vitev, JT (2024)



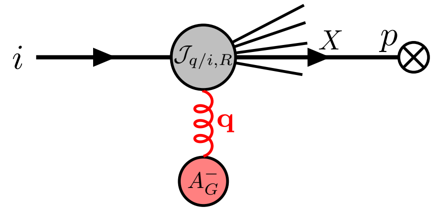
We consider the first-order opacity correction to the incoming beam function

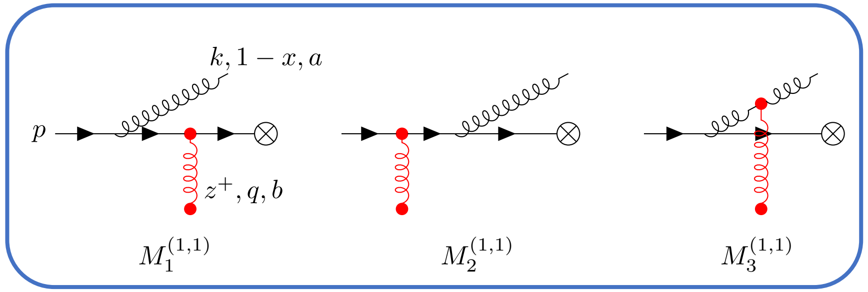
$$\frac{d\sigma_1}{d\mathcal{PS}} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^2 s} H(Q,\mu) \sum_q c_q(Q) \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{P}_T\cdot\mathbf{b}}$$
$$\times \sum_{N\in A} \mathcal{B}_{q/p,1}\left(x_1, b, \mu, \frac{\zeta_1}{\nu^2}; \mu_E, \mathcal{L}_1\right) \mathcal{B}_{\bar{q}/N}\left(x_2, b, \mu, \frac{\zeta_2}{\nu^2}\right) S(b,\mu,\nu)$$



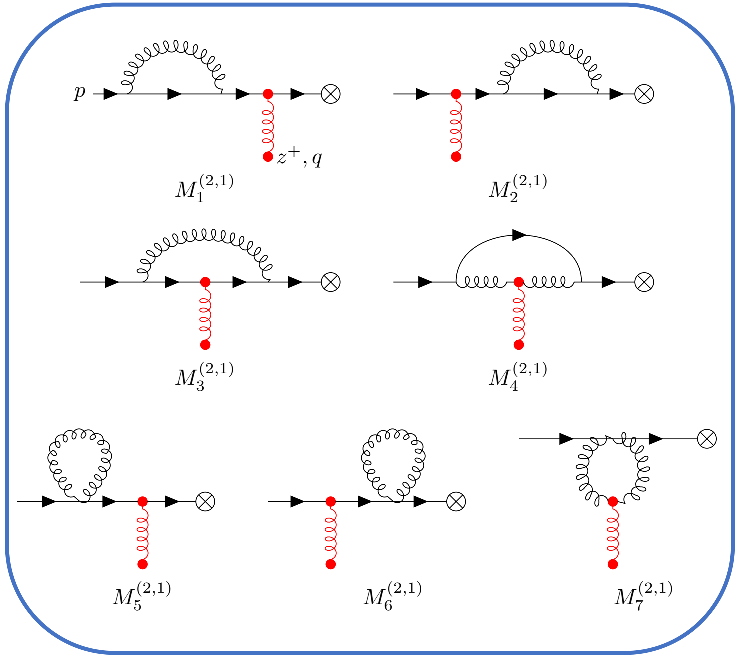
Graphs at one loop for the matching function

One loop graphs can be organized by the type of integration

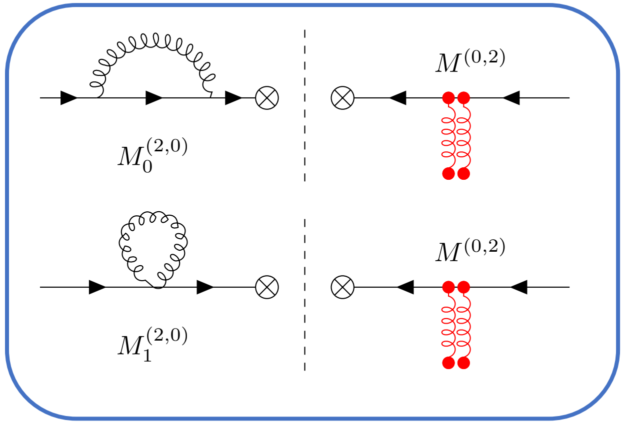




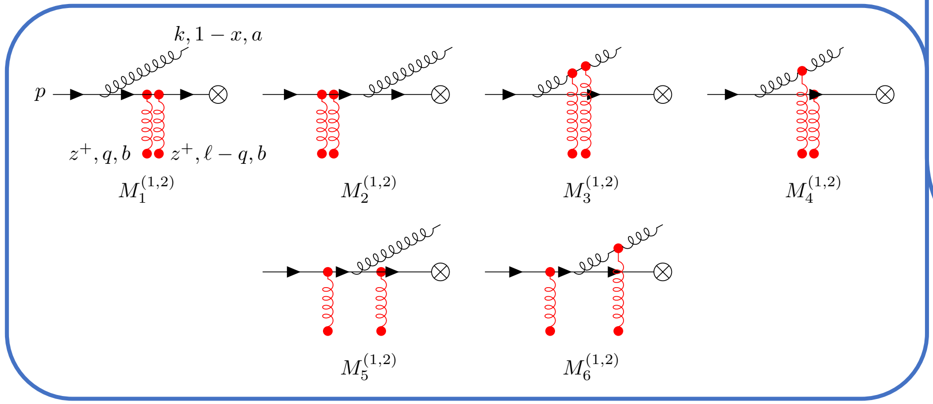
Single Glauber, real emission



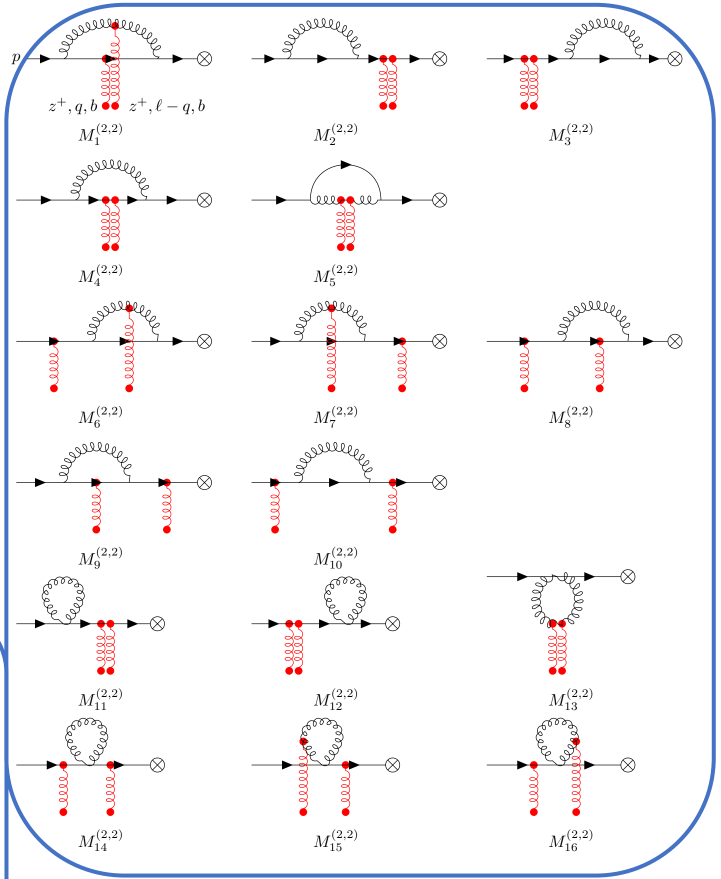
Single Glauber, virtual loop



Wave function



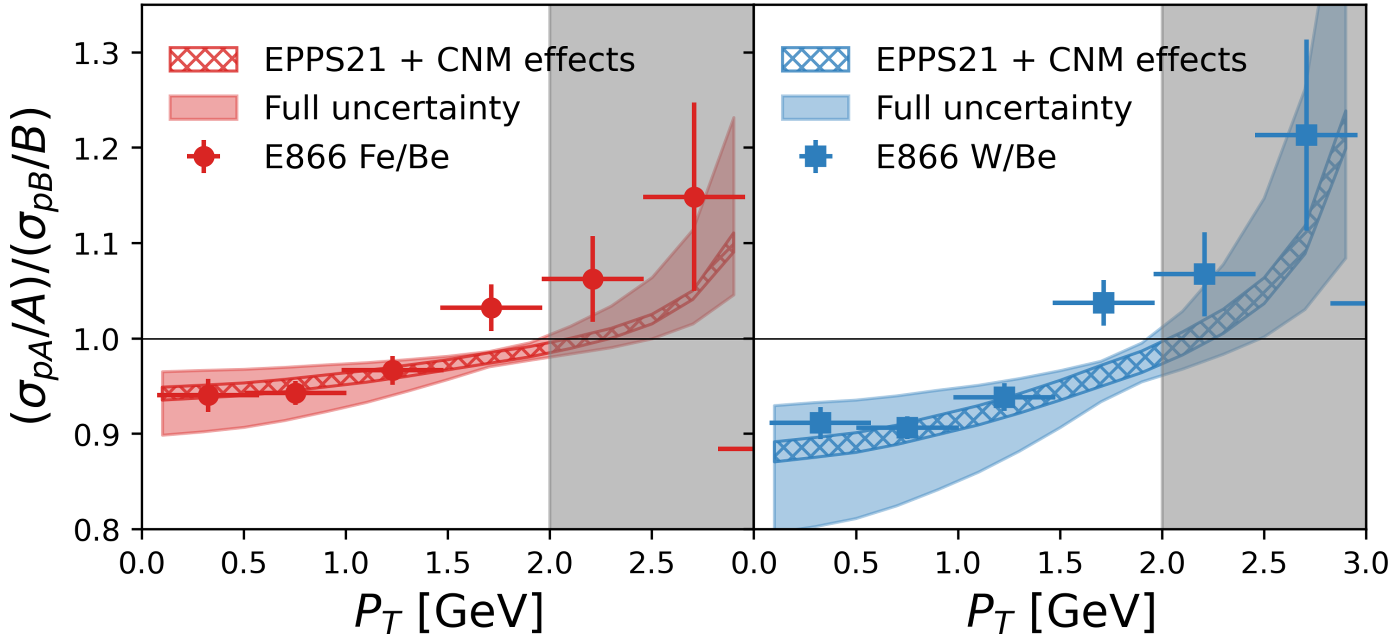
Double Glauber, real emission



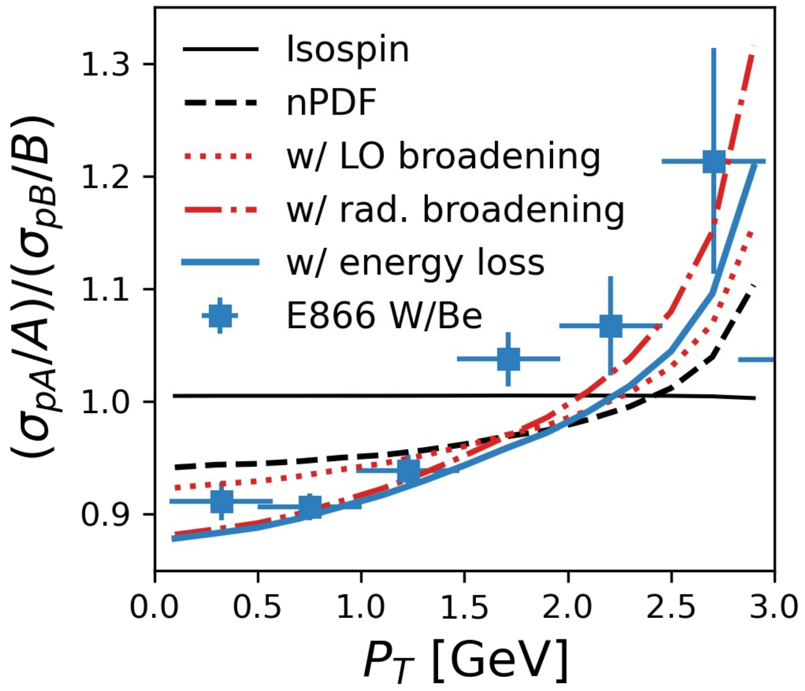
Double Glauber, virtual emission

Example description of data

First principle description of data. Small improvements can be made by adding non-perturbative effects and investigating other perturbative effects



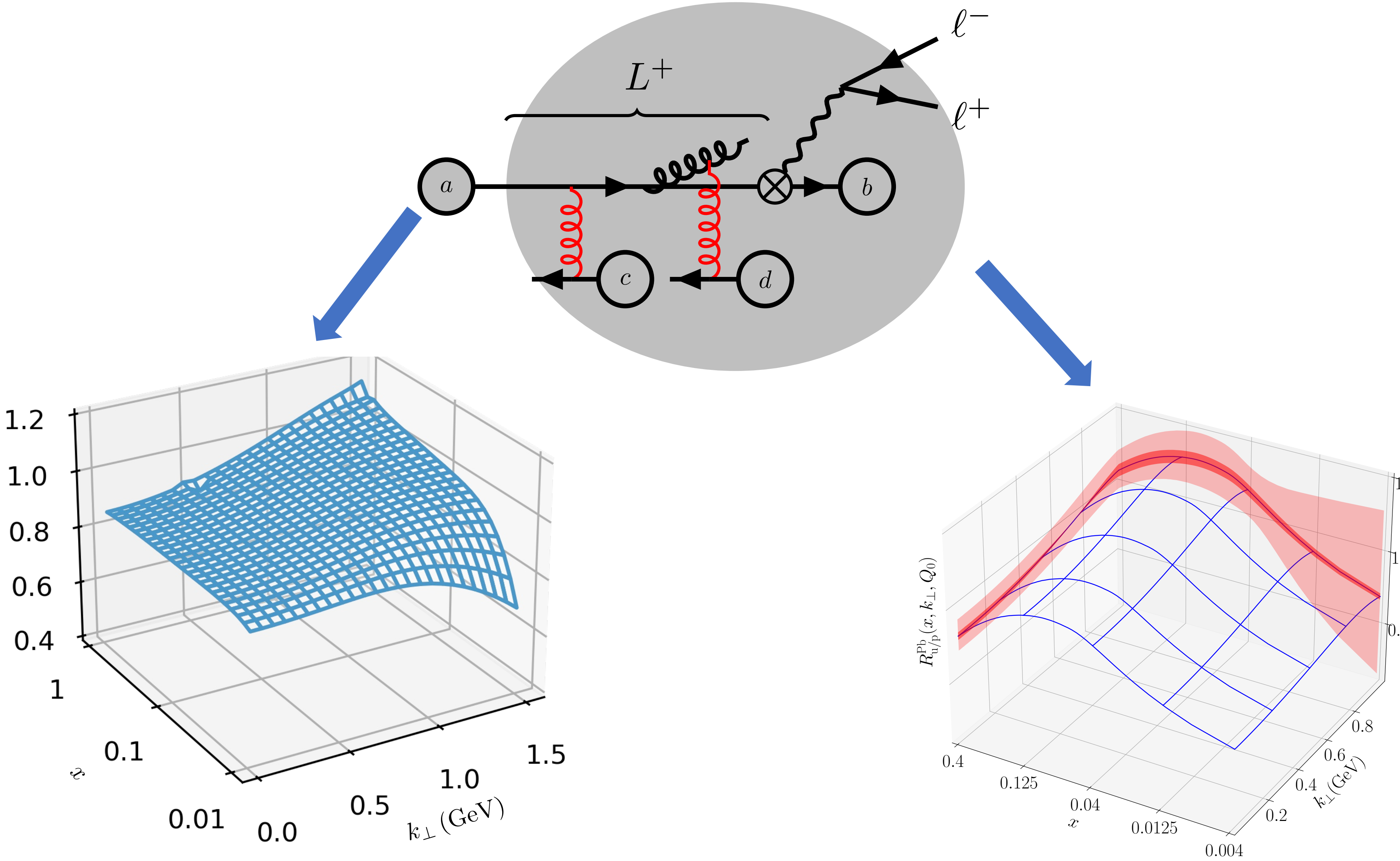
Nuclear size dependence



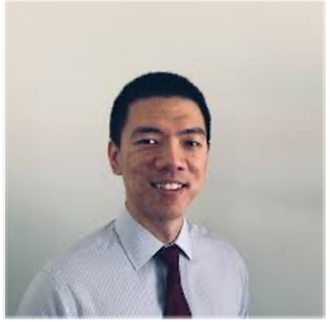
Various effects from RG

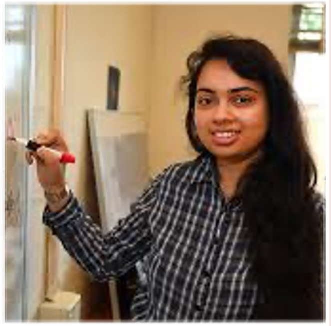
Status of nTMDs

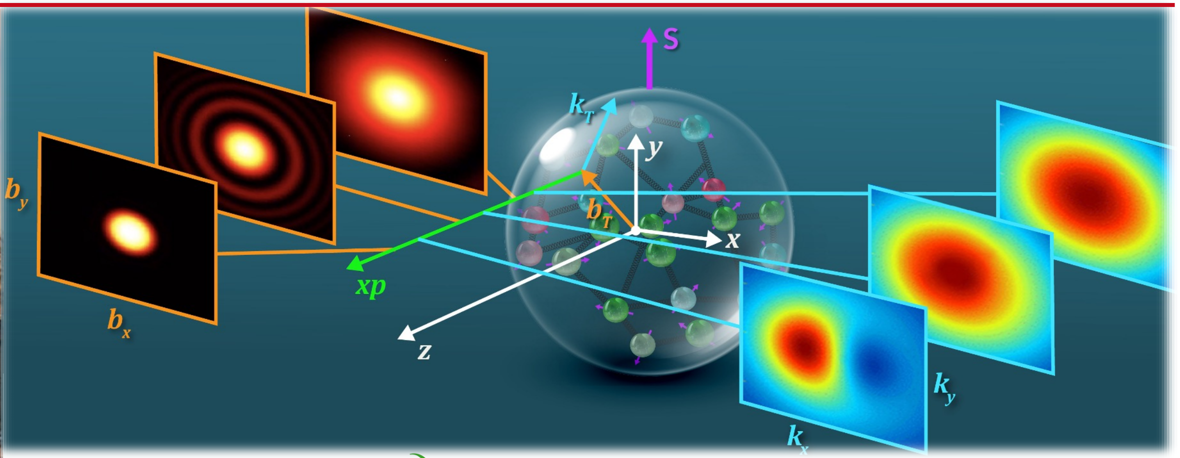
The medium modified beam function and the pheno extracted nuclear modified TMD PDF have been obtained

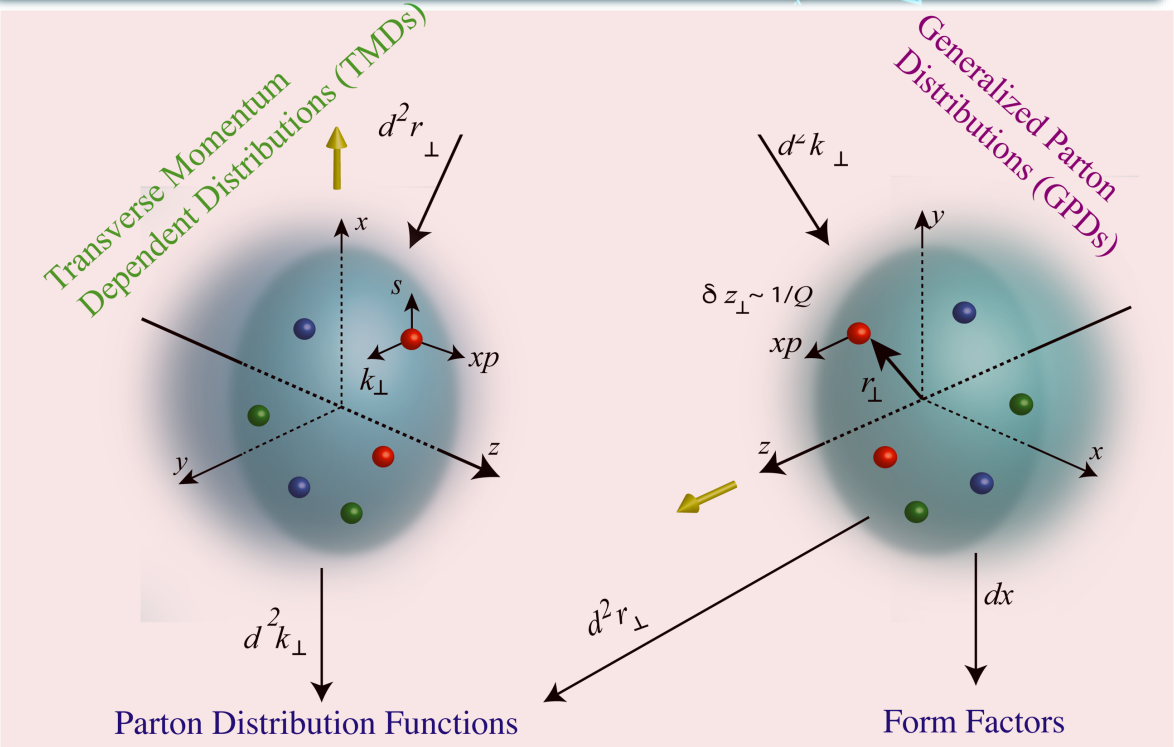


Extending this to GPDs



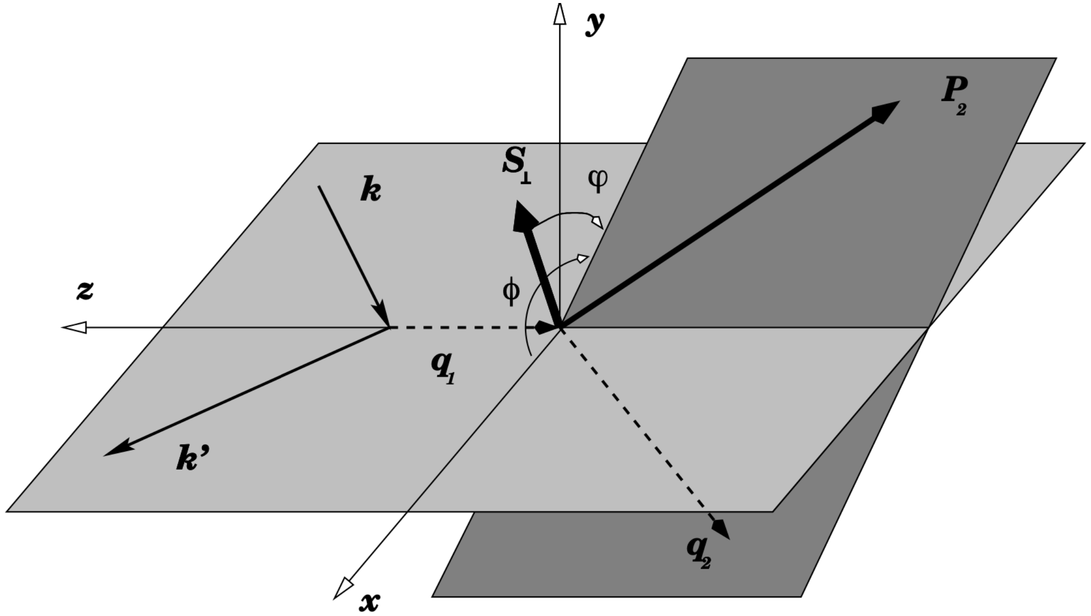




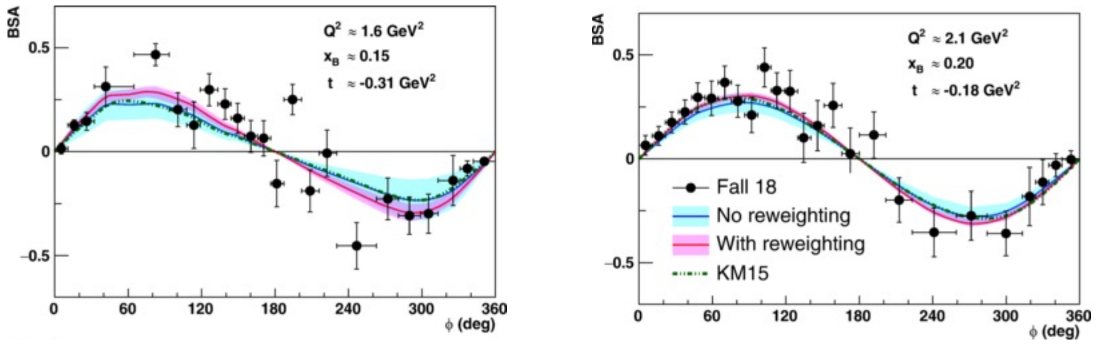


Beam spin asymmetry in DDVCS

Kinematic diagram for DDVCS



$$A_{LU} = \frac{d\sigma^{\rightarrow} - d\sigma^{\leftarrow}}{d\sigma^{\rightarrow} + d\sigma^{\leftarrow}} \propto \mathrm{Im}\left[T^*_{\rm BH} T_{\rm DDVCS}\right].$$



Cross section can be Fourier decomposed

$$|T|^2 = |T_{\rm VCS}|^2 + \mathcal{I} + |T_{\rm BH}|^2,$$

$$|T_{\rm VCS}|^2 \sim \sum_{n=0}^{2}\left[c_n^{\rm VCS}(\varphi_\ell)\cos(n\phi) + s_n^{\rm VCS}(\varphi_\ell)\sin(n\phi)\right],$$

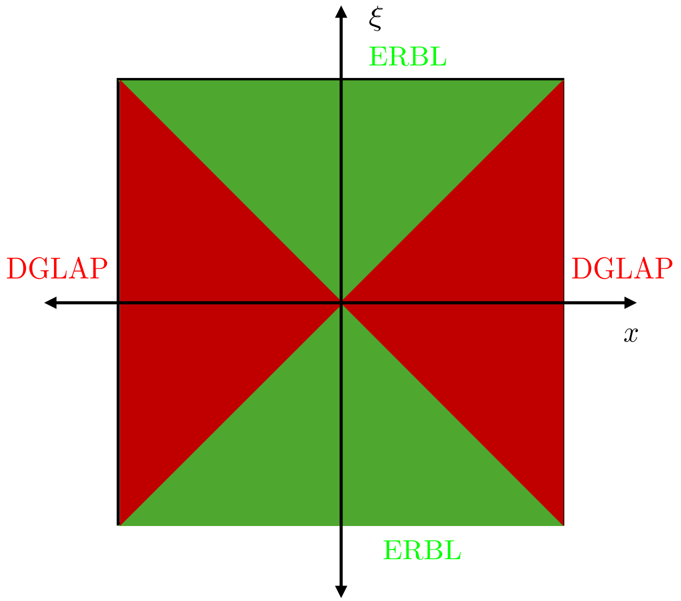
$$\mathcal{I} \sim \sum_{n=0}^{3}\left[c_n^{\rm INT}(\varphi_\ell)\cos(n\phi) + s_n^{\rm INT}(\varphi_\ell)\sin(n\phi)\right],$$

$$c_n^i(\varphi_\ell) = \sum_{m=0}^{2}\left[cc_{nm}^i\cos(m\varphi_\ell) + cs_{nm}^i\sin(m\varphi_\ell)\right],$$

$$s_n^i(\varphi_\ell) = \sum_{m=0}^{2}\left[sc_{nm}^i\cos(m\varphi_\ell) + ss_{nm}^i\sin(m\varphi_\ell)\right].$$

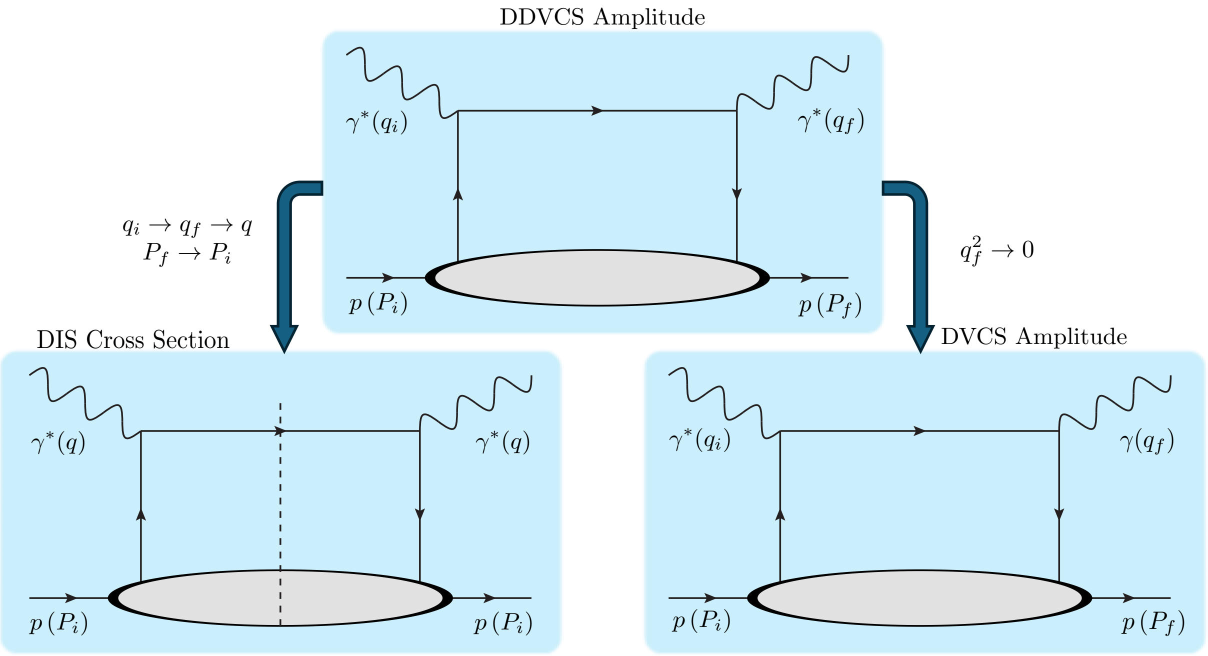
Belitsky, Mueller (2003)

Can access GPDs in off-diagonal and non forward limits

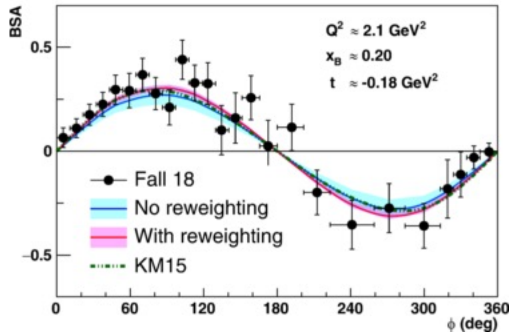


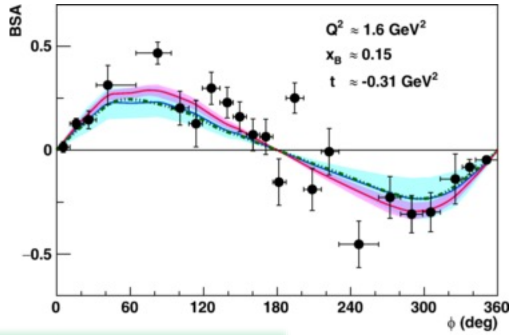
Relationship to DVCS

Hadronic contributions to DDVCS



Data exists for BSA in proton collisions





DVCS limit

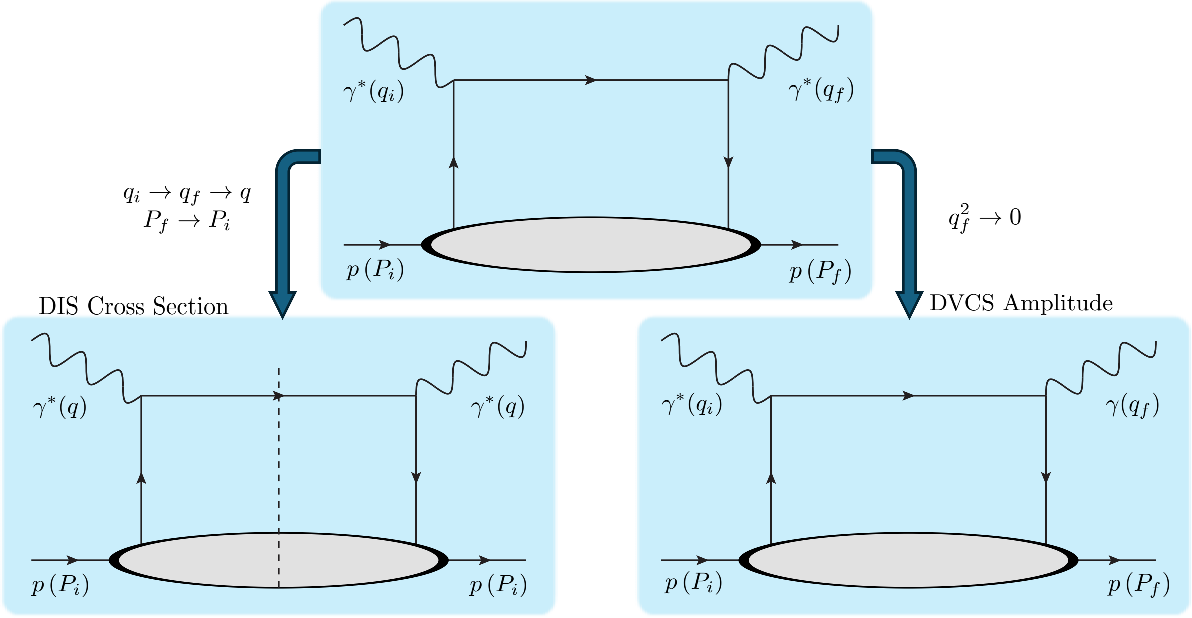
$$\frac{d\sigma}{d\xi\, dt\, dQ^2\, dQ'^2\, d\varphi\, d\Omega_\ell} = \frac{\alpha_{\rm em}^3 x_B y^2}{16\pi^2 Q^4 \sqrt{1+\gamma^2}}$$

$$\times \left[\mathcal{T}^{\mu\alpha}\, \mathcal{L}^{(e)}_{\alpha\beta}\, \frac{\alpha_{\rm em}}{Q'^2} \mathcal{L}^{(\ell)}_{\mu\nu}\, \bar{\mathcal{T}}^{\nu\beta}\right](\xi, t, Q, Q', \varphi, \Omega_\ell) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)$$

$$\left[\mathcal{T}^{\mu\alpha}\, \mathcal{L}^{(e)}_{\alpha\beta}\, \frac{\alpha_{\rm em}}{Q'^2} \mathcal{L}^{(\ell)}_{\mu\nu}\, \bar{\mathcal{T}}^{\nu\beta}\right](\xi, t, Q, Q', \varphi, \Omega_\ell) \to \left[\mathcal{T}^{\alpha}_{\nu}\, \mathcal{L}^{(e)}_{\alpha\beta}\, \bar{\mathcal{T}}^{\nu\beta}\right](\xi, t, Q, Q', \varphi, \Omega_\ell)$$

Relationship to DIS

Hadronic contributions to DDVCS



DIS limit

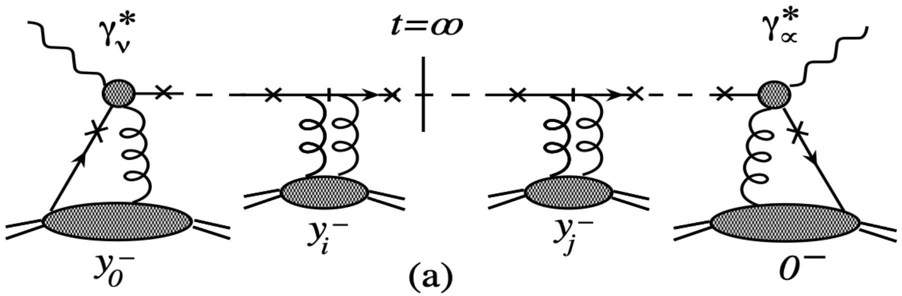
$$\mathcal{T}^{\mu\alpha}(q_i, q_f; P_i, P_f) = \int d^4x\, d^4y\, e^{iq_i\cdot x - iq_f\cdot y}\, \langle p(P_f)|T\{J^\mu(x)J^\alpha(y)\}|p(P_i)\rangle,$$

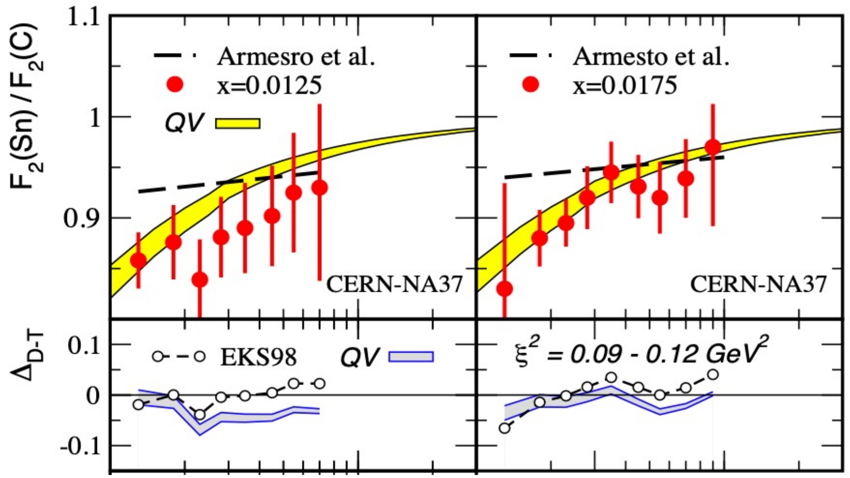
$$W^{\mu\alpha}_{\mathrm{DIS}}(x_B, Q^2) = \frac{1}{\pi}\,\Im\,\mathcal{T}^{\mu\alpha}\Big|_{\substack{q_i=q_f=q\\ P_f=P_i=P}}$$

Shadowing was shown to emerge from multiple scattering

$$F_T^A(x, Q^2) \approx A\, F_T^{(\mathrm{LT})}\left(x + \frac{x\xi^2(A^{1/3}-1)}{Q^2}, Q^2\right) \qquad \xi^2 \sim \frac{1}{r_p^2}$$

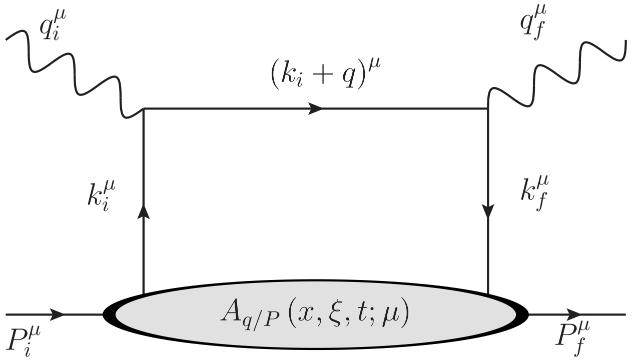
Non-perturbative parameter that controls drag





Power counting in the vacuum

Tree-level diagram for DDVCS



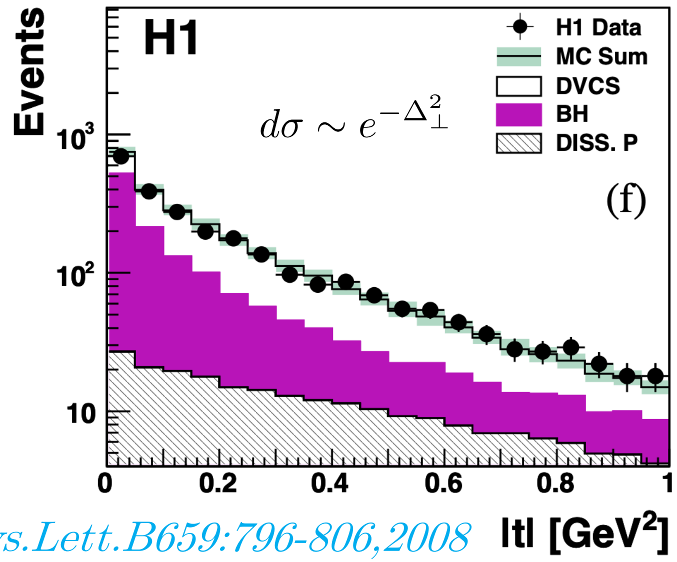
Burkhardt relationship introduces a power counting

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp} H(x, 0, -\mathbf{\Delta}_\perp^2)$$

$$b_\perp \sim R_p \sim \frac{1}{\Lambda_{\rm QCD}} \qquad \Delta_\perp \sim \Lambda_{\rm QCD}$$

$$k_i \sim Q\left(\lambda^2, 1, \lambda\right) \qquad k_f \sim Q\left(\lambda^2, 1, \lambda\right)$$

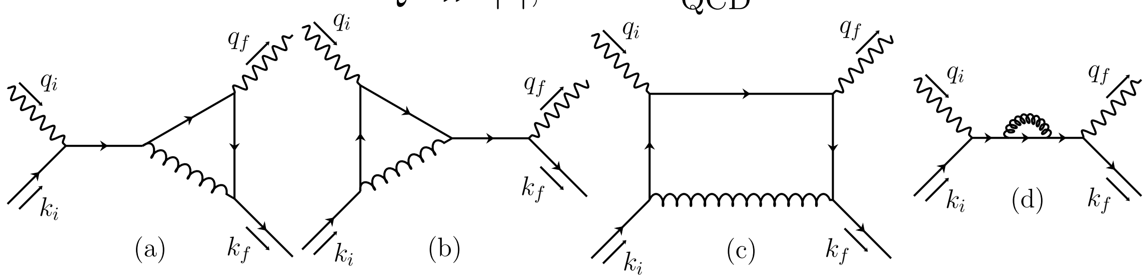
DVCS cross section falls off exponentially in t



H1
Phys.Lett.B659:796-806,2008

DDVCS has the implicit power counting

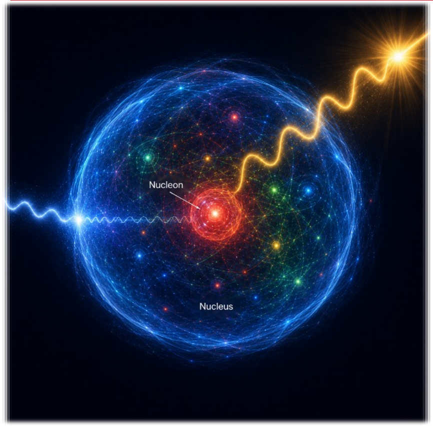
$$Q^2 \gg |t|, m^2 \sim \Lambda_{\rm QCD}^2$$



Infrared physics contained by a single collinear mode

$$\mathcal{T}^{\mu\alpha}_{\rm DVCS}(\xi, t, Q, Q') = i\sum_q e_q^2 \int dx \left[\frac{1}{x-\xi+i\epsilon} - \frac{1}{x+\xi-i\epsilon}\right] H\left(x, \xi, \Delta_\perp\right)$$

DVCS in nuclear matter



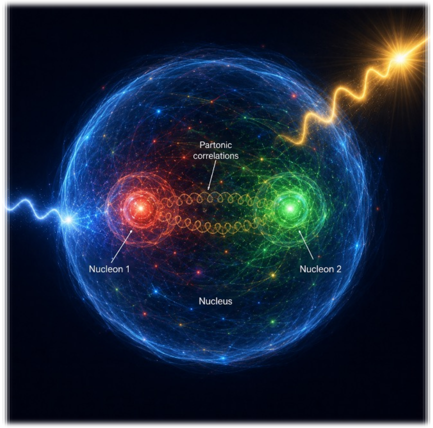
The nuclear states can be decomposed in terms of combinations with different overlap

$$|A(P)\rangle = |A(P)\rangle_{\text{iso}} + |A(P)\rangle_{(ab)} + \cdots + |A(P)\rangle_{(A)}$$

The probe interacts primarily with isolated nucleons embedded within the nuclear medium.

$$|A(P)\rangle_{\text{iso}} = \frac{1}{\sqrt{A!}} \sum_{\lambda_1} \cdots \sum_{\lambda_A} \int \prod_{i=1}^{A} \frac{d^3 p_i}{(2\pi)^3 2E_i} \Psi_A^{\text{iso}}(P; p_1, \lambda_1; \ldots; p_A, \lambda_A)$$
$$\times |N(p_1, \lambda_1) \cdots N(p_A, \lambda_A)\rangle$$

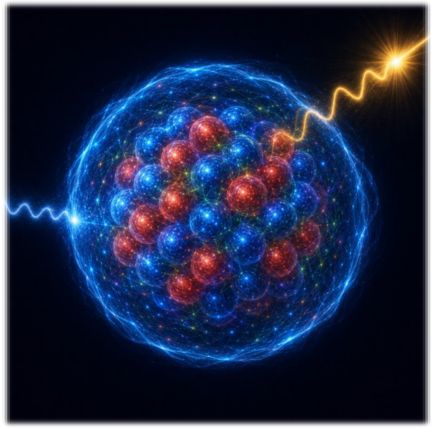
$r_A \sim r_p$



Coherent re-scattering begins to probe correlated nucleon pairs and short-range nuclear structure.

$$|A(P)\rangle_{(ab)} = \frac{1}{\sqrt{A!}} \sum_{\lambda_1} \cdots \sum_{\lambda_A} \int \prod_{i=1}^{A} \frac{d^3 p_i}{(2\pi)^3 2E_i} \Psi_A^{(ab)}(P; p_1, \lambda_1; \ldots; p_A, \lambda_A)$$
$$\times |N(p_1, \lambda_1) \cdots N(p_A, \lambda_A)\rangle$$

$r_A \sim r_p\, A_{eff}^{1/3}$



The scattering process becomes sensitive to coherent many-body dynamics across the entire nucleus.

$$|A(P)\rangle_{(A)} = \frac{1}{\sqrt{A!}} \sum_{\lambda_1} \cdots \sum_{\lambda_A} \int \prod_{i=1}^{A} \frac{d^3 p_i}{(2\pi)^3 2E_i} \Psi_A^{(A)}(P; p_1, \lambda_1; \ldots; p_A, \lambda_A)$$
$$\times |N(p_1, \lambda_1) \cdots N(p_A, \lambda_A)\rangle$$

$r_A \sim r_p\, A^{1/3}$

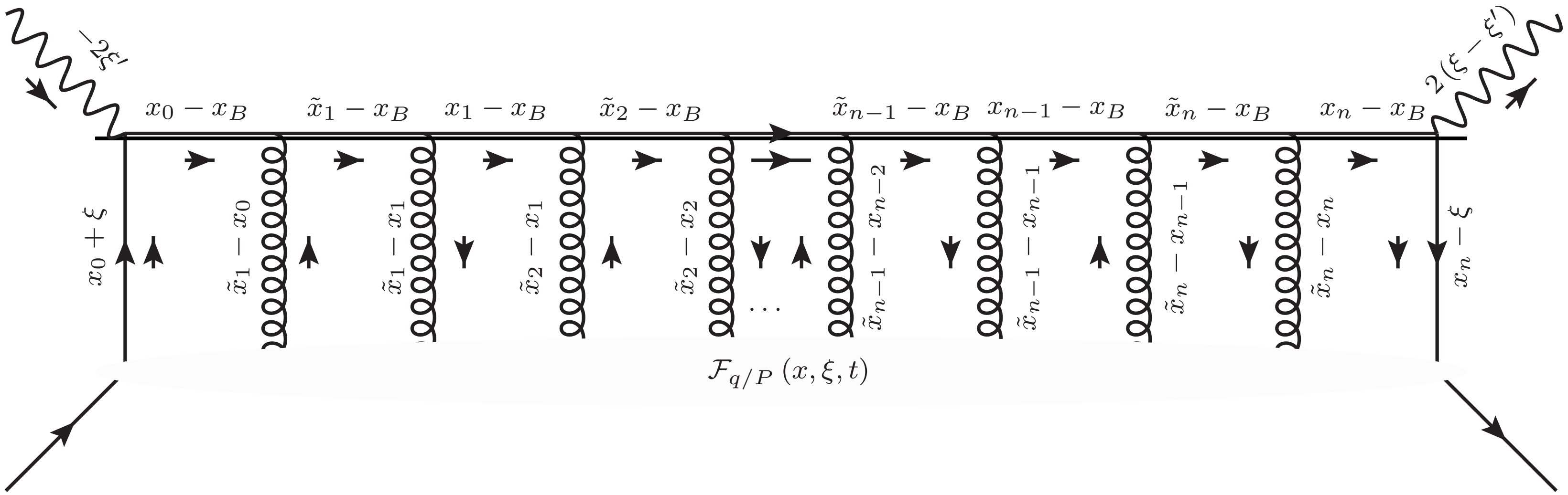
Computation setup

To examine the nuclear power corrections, we follow the methodology of *Qiu-Vitev (2003)*

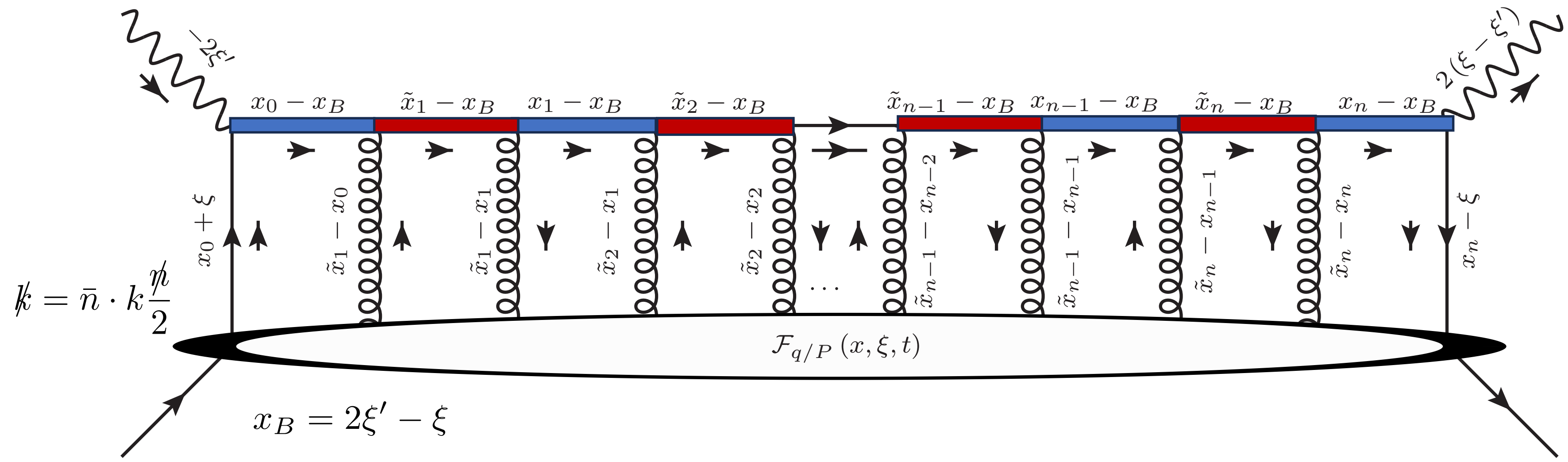
$$\int d^4y\, T\left\{J^\mu(y)\, J^\nu(0)\right\} = \sum_{n=0}^{\infty} \sum_q e_q^2\, \bar{\chi}_c^q(y)\, \gamma^\mu \int d^4y \prod_{i=0}^{n-1} \left\{d^4 y_{n-i}\right\} \Delta(y - y_n)$$
$$\times \prod_{i=0}^{n-1} \left\{-i g_s \not{A}(y_{n-i})\, \Delta(y_{n-i} - y_{n-i-1})\right\} \gamma^\nu \chi_c^q(0)\,,$$

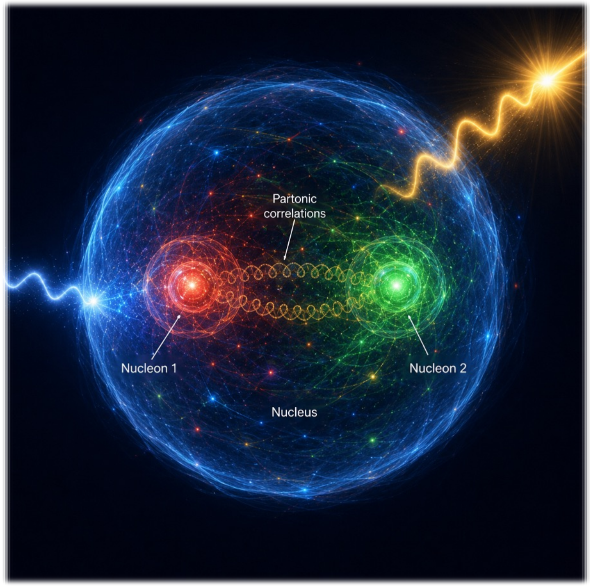
$$\mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{Q^2}\right) \to \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{Q^2} A^{2/3}\right)$$

Example diagram for eight gluons



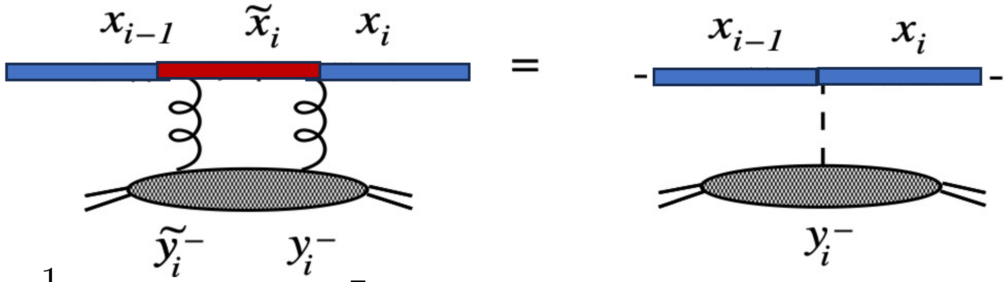
Example diagram for eight gluons





The subsequent interactions give rise to effective Feynman rules

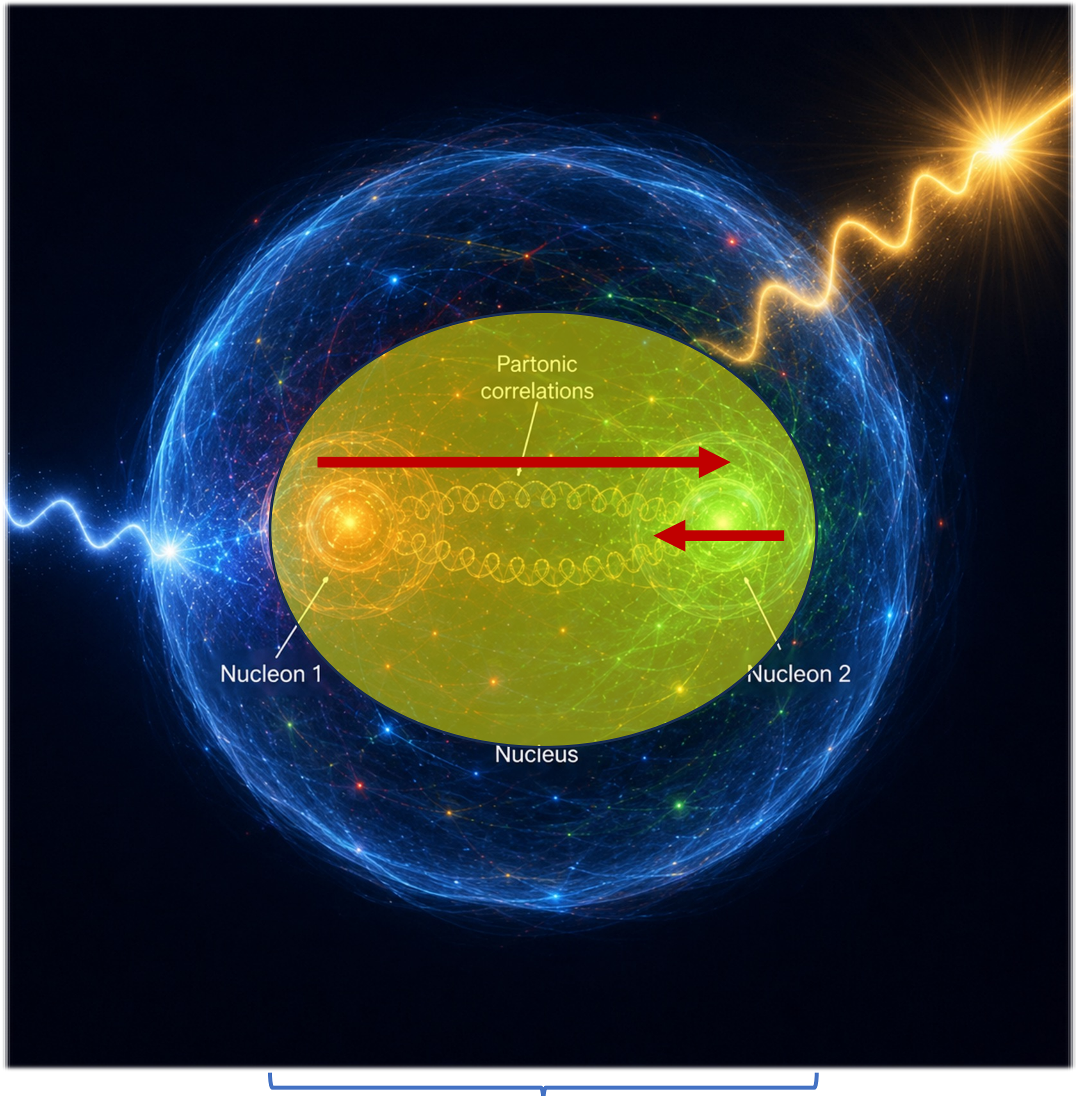
$$x_B \frac{\not{n}}{2} \frac{1}{Q}$$



$$\frac{1}{Q} \frac{\bar{\not{n}}}{2} \frac{1}{x_{i-1} - x_B + i\epsilon} \qquad \frac{1}{Q} \frac{\bar{\not{n}}}{2} \frac{1}{x_i - x_B + i\epsilon}$$

Nuclear shadowing of the GPDs, the imaginary part

Representation of the position-space integration



$$r_A \sim r_0\, A_{eff}^{1/3}$$

Lastly, the integration in position space

$$I_n \equiv \int_0^{L_A} dy_n^- \int_0^{L_A} dy_{n-1}^- \cdots \int_0^{L_A} dy_1^-\, \theta(y_n^- - y_{n-1}^-)\theta(y_{n-1}^- - y_{n-2}^-)\cdots\theta(y_2^- - y_1^-)$$
$$= \int_0^{L_A} dy_n^- \int_0^{y_n^-} dy_{n-1}^- \cdots \int_0^{y_2^-} dy_1^- = \frac{r_p^n A_{eff}^{n/3}}{n!}$$

Lastly, the integration in position space

$$\int \frac{d\lambda_0}{2\pi}\, e^{ix_b\lambda_0}(i\lambda_0)^n \langle p_B|\hat{O}(\lambda_0)|p_B\rangle = \frac{\partial^n}{\partial x_b^n}\left[\int \frac{d\lambda_0}{2\pi}\, e^{ix_b\lambda_0}\langle p_B|\hat{O}(\lambda_0)|p_B\rangle\right]$$
$$= \frac{\partial^n}{\partial x_b^n}\phi_{b/N}(x_b)$$

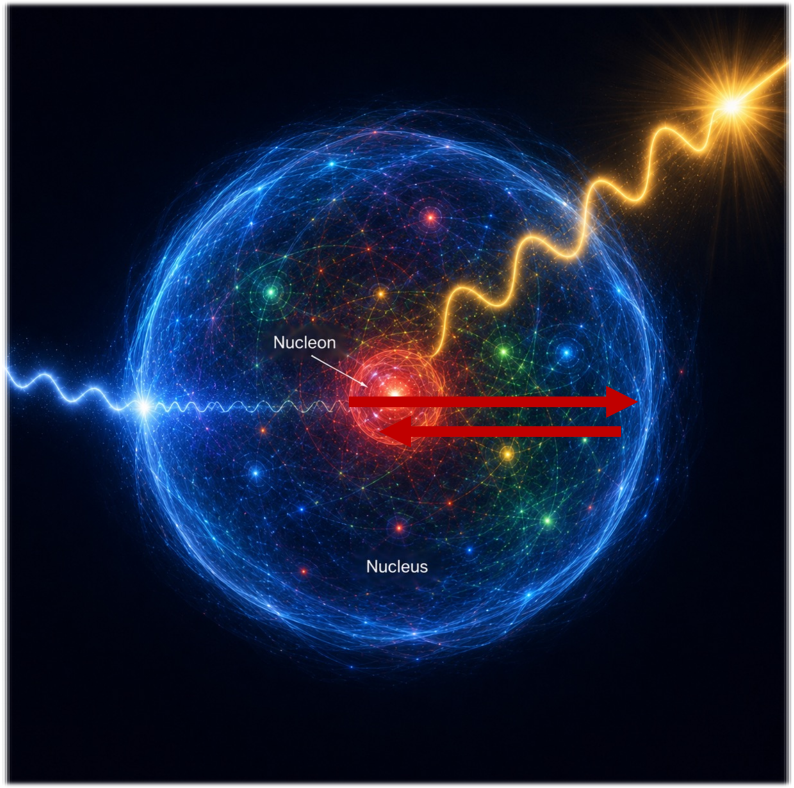
Lastly, the integration in position space

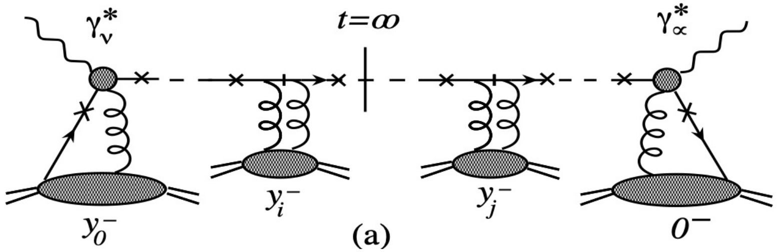
$$H_q(x,\xi,t) \approx \sum_{n=0}^{N} \frac{A}{n!}\left[\frac{\xi^2\left(A^{1/3}-1\right)}{Q^2}\right]^n x^n \frac{d^n H_q(x,\xi,t)}{dx^n}$$

$$H_q(x,\xi,t) \;\rightarrow\; H_q\left(x\left[1+\frac{\xi^2(A^{1/3}-1)}{Q^2}\right],\xi,t\right)$$

Relation to shadowing in DIS

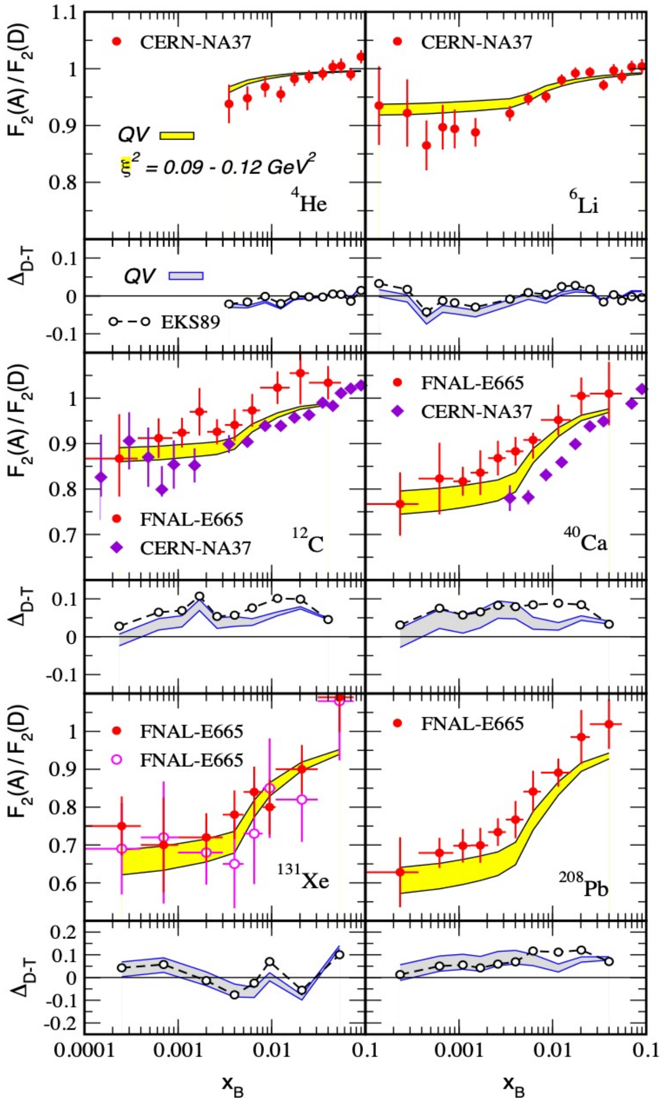
Representation of the space-time integration in DIS





$$\xi^2 \sim \lim_{x\to 0} \frac{x}{2} f_{g/P}(x)$$

Non-perturbative parameter was determined long ago by *Qiu and Vitev (2003)*



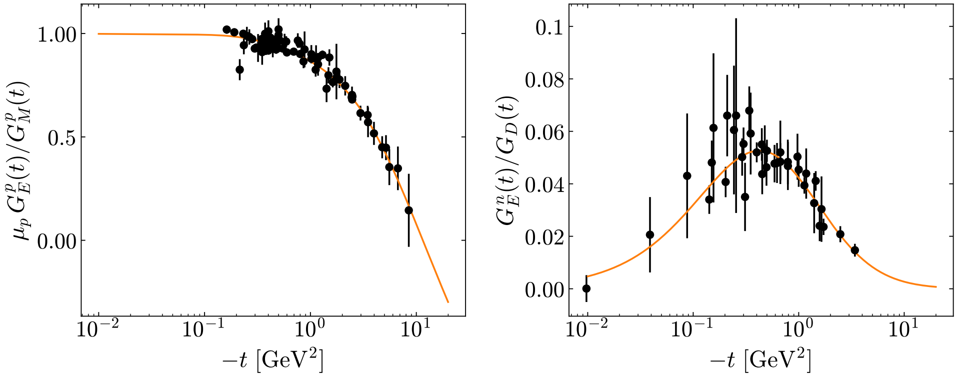
Nuclear modifications to GPDs

Modifications to the GPDs

$$F_A(x,\xi,t) = \frac{F_A^{\rm el}(t)}{F_N^{\rm el}(t)} \left[Z\, F_p(x_{\rm eff},\xi,t) + N\, F_n(x_{\rm eff},\xi,t)\right],$$

$$R_u^A(x,t;Q) = \frac{H_{u/A}(x,x,t;Q)}{H_{u/p}(x,x,t;Q)}$$

Ye, Arrington, Hill, Lee (2017)



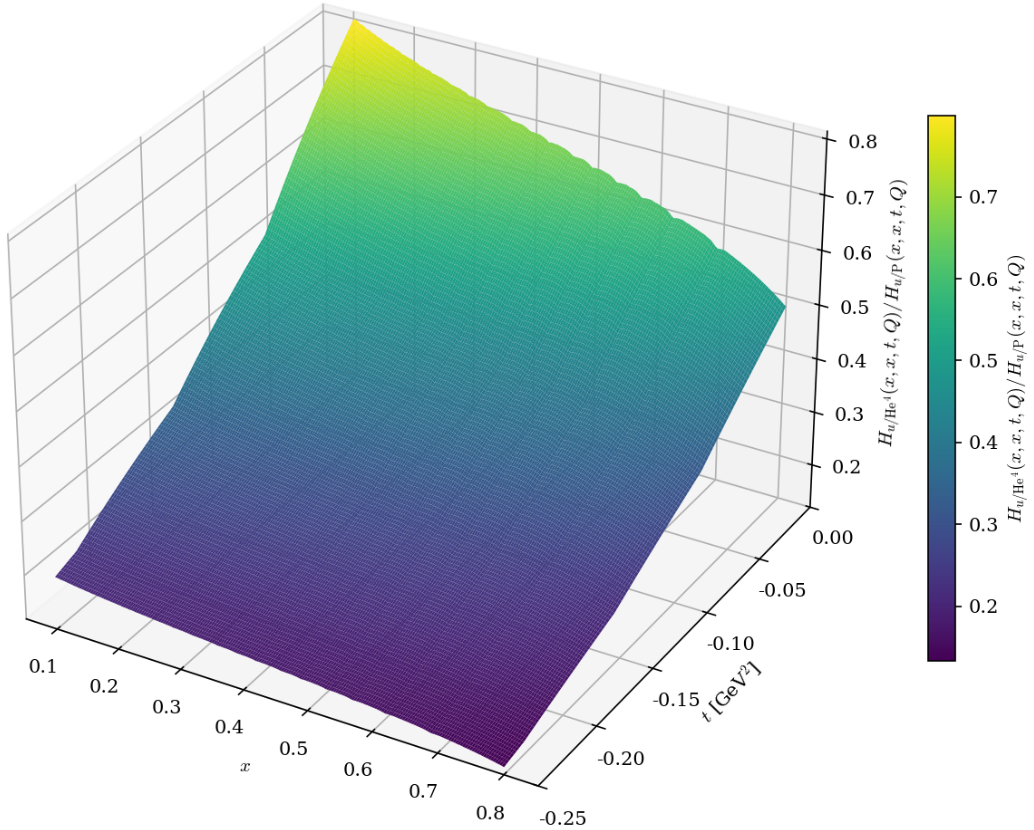
$$F_A^{\rm el}(t) \simeq \exp\left(r_0^2 A^{2/3} t\right),$$

Modifications to the GPDs

$$F(x,\xi,t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\; \delta(x-\beta-\xi\alpha)\, f_F(\beta,\alpha,t)$$

Goloskokov-Kroll 2007

Evolution from Marco



Description of proton DVCS

Full cross section uses formalism of *Belitsky, Mueller, Kirchner 2001*

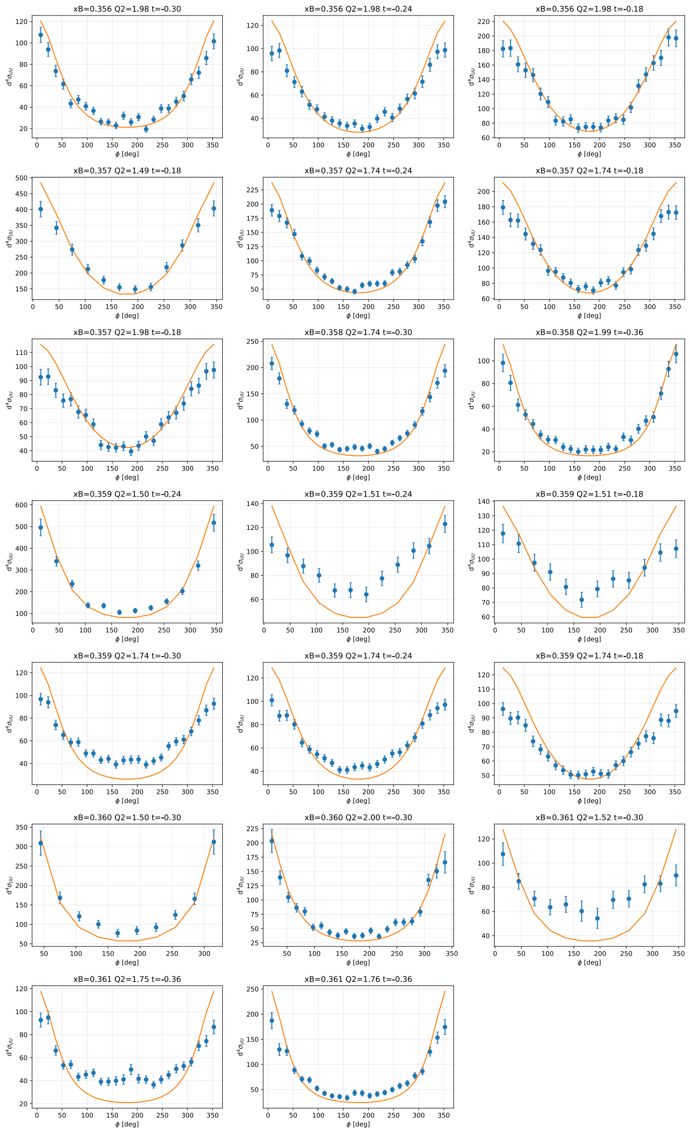
$$\frac{d\sigma}{dx_{\mathrm{B}}dy d|\Delta^2| d\phi d\varphi} = \frac{\alpha^3 x_{\mathrm{B}} y}{16\,\pi^2\,\mathcal{Q}^2\sqrt{1+\epsilon^2}} \left|\frac{\mathcal{T}}{e^3}\right|^2$$

$$|\mathcal{T}_{\mathrm{BH}}|^2 = \frac{e^6}{x_{\mathrm{B}}^2 y^2 (1+\epsilon^2)^2 \Delta^2 \mathcal{P}_1(\phi) P_2(\phi)} \left[c_0^{\mathrm{BH}} + \sum_{n=1}^{2} c_n^{\mathrm{BH}} \cos(n\phi) + s_1^{\mathrm{BH}} \sin\phi \right]$$

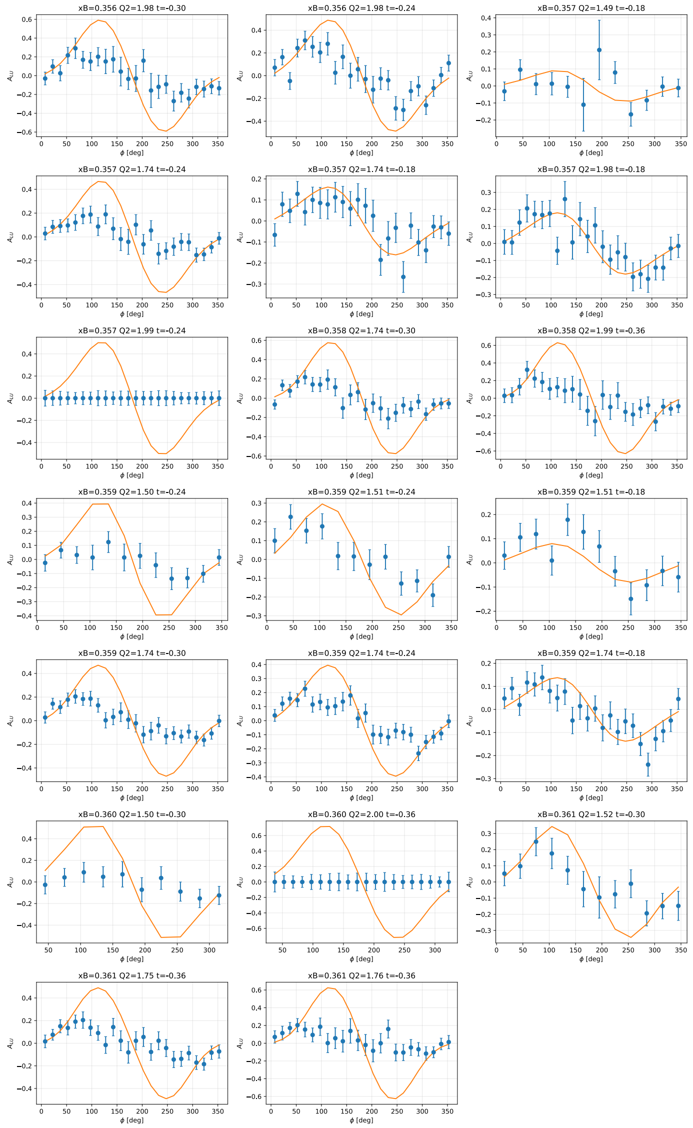
$$|\mathcal{T}_{\mathrm{DVCS}}|^2 = \frac{e^6}{y^2 Q^2} \left[c_0^{\mathrm{DVCS}} + \sum_{n=1}^{2} \left(c_n^{\mathrm{DVCS}} \cos(n\phi) + s_n^{\mathrm{DVCS}} \sin(n\phi) \right) \right]$$

$$\mathcal{I} = \pm \frac{e^6}{x_B y^3 \Delta^2 \mathcal{P}_1(\phi) \mathcal{P}_2(\phi)} \left[c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left(c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right) \right],$$

Unpolarized cross section for CLAS 12 DVCS

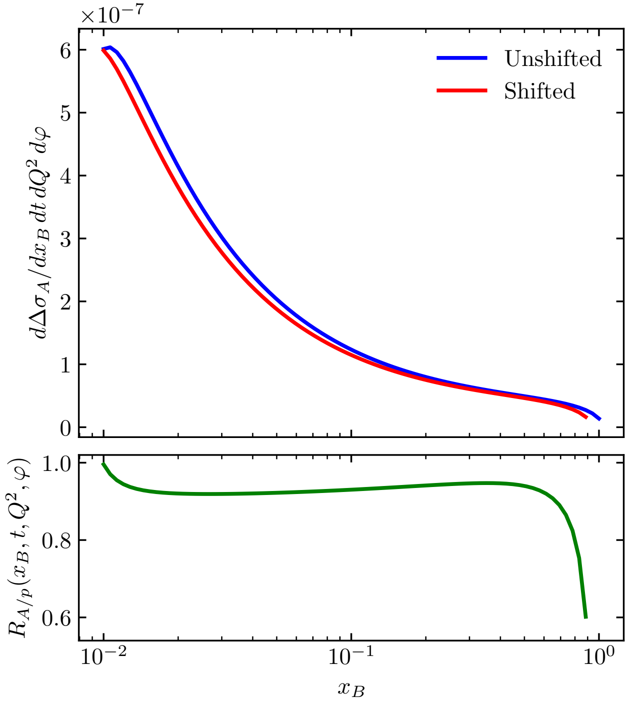


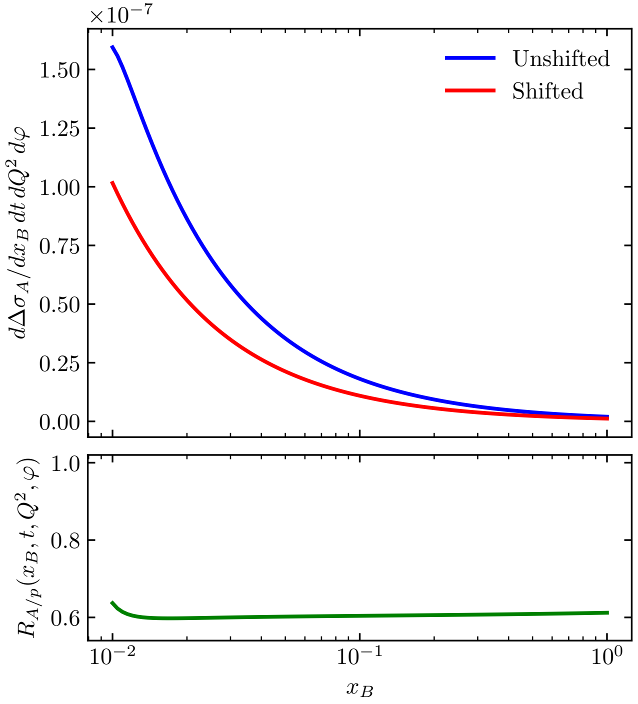
BSA for CLAS 12 DVCS

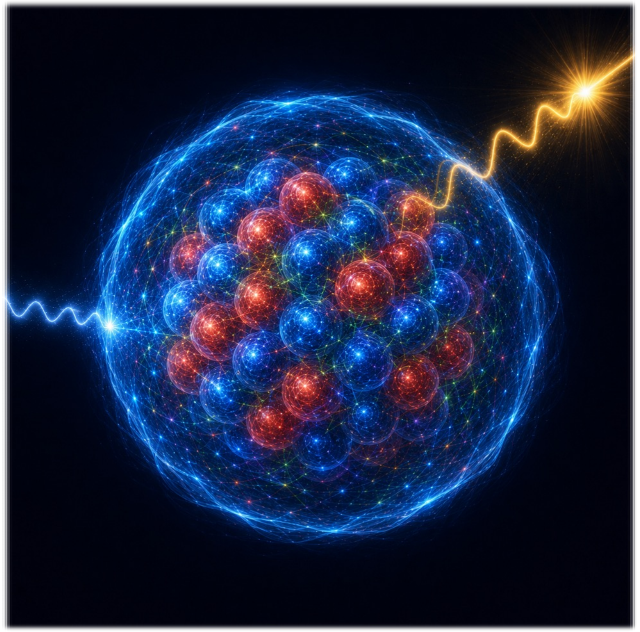


Predictions at Jefferson Lab and the EIC

Preliminary prediction for He4 at JLab and Au197 at the EIC





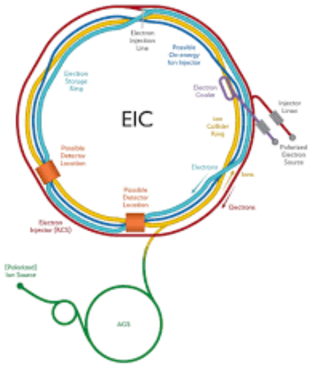


$$F_A(x,\xi,t) = \frac{F_A^{\rm el}(t)}{F_N^{\rm el}(t)}\left[Z\,F_p(x_{\rm eff},\xi,t) + N\,F_n(x_{\rm eff},\xi,t)\right],$$

$$H_q(x,\xi,t) \;\to\; H_q\left(x\left[1+\frac{\xi^2(A^{1/3}-1)}{Q^2}\right],\xi,t\right)$$

Future opportunities

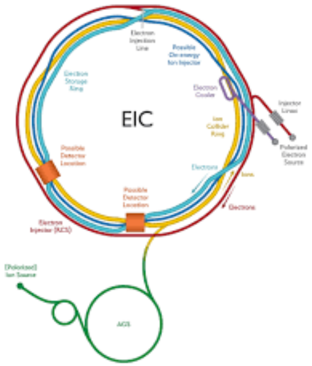
Upcoming measurements at the LHCb, Jlab, and the EIC



Future opportunities

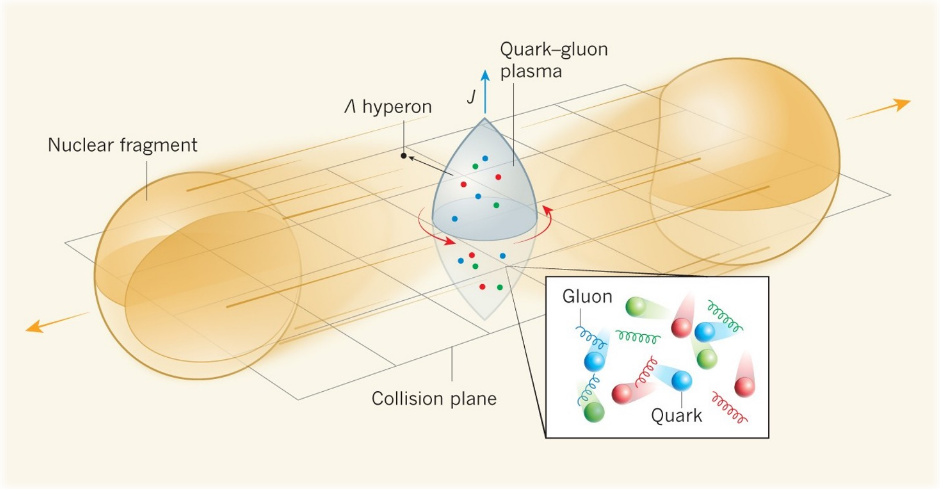
Upcoming measurements at the LHCb, Jlab, and the EIC

Jefferson Lab



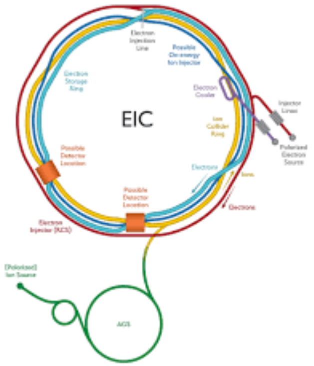
LHCb

Medium modifications to beam function can be used to probe properties of a QGP

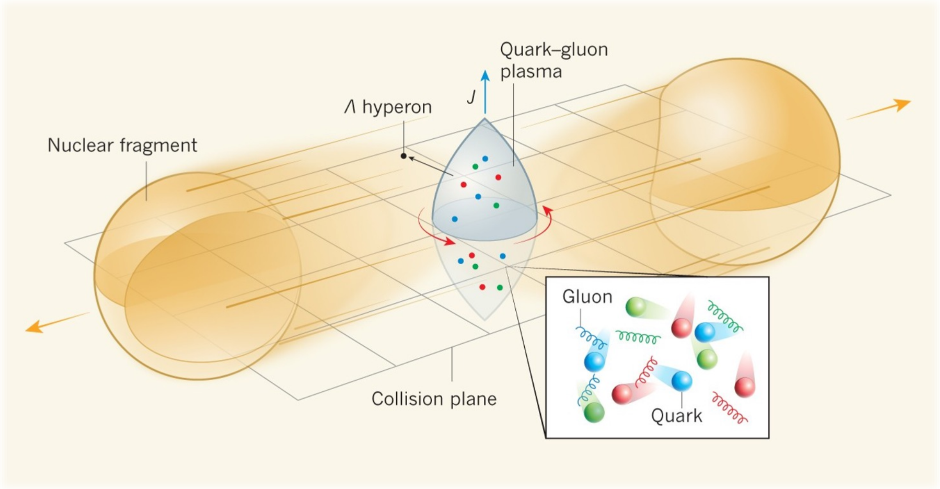


Future opportunities

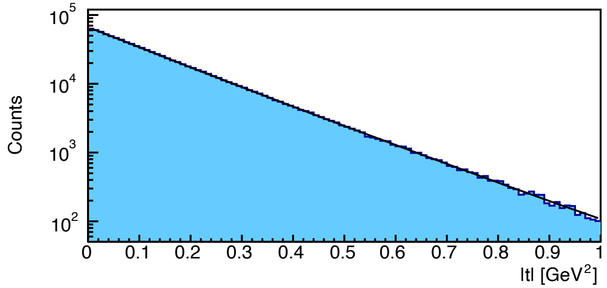
Upcoming measurements at the LHCb, Jlab, and the EIC



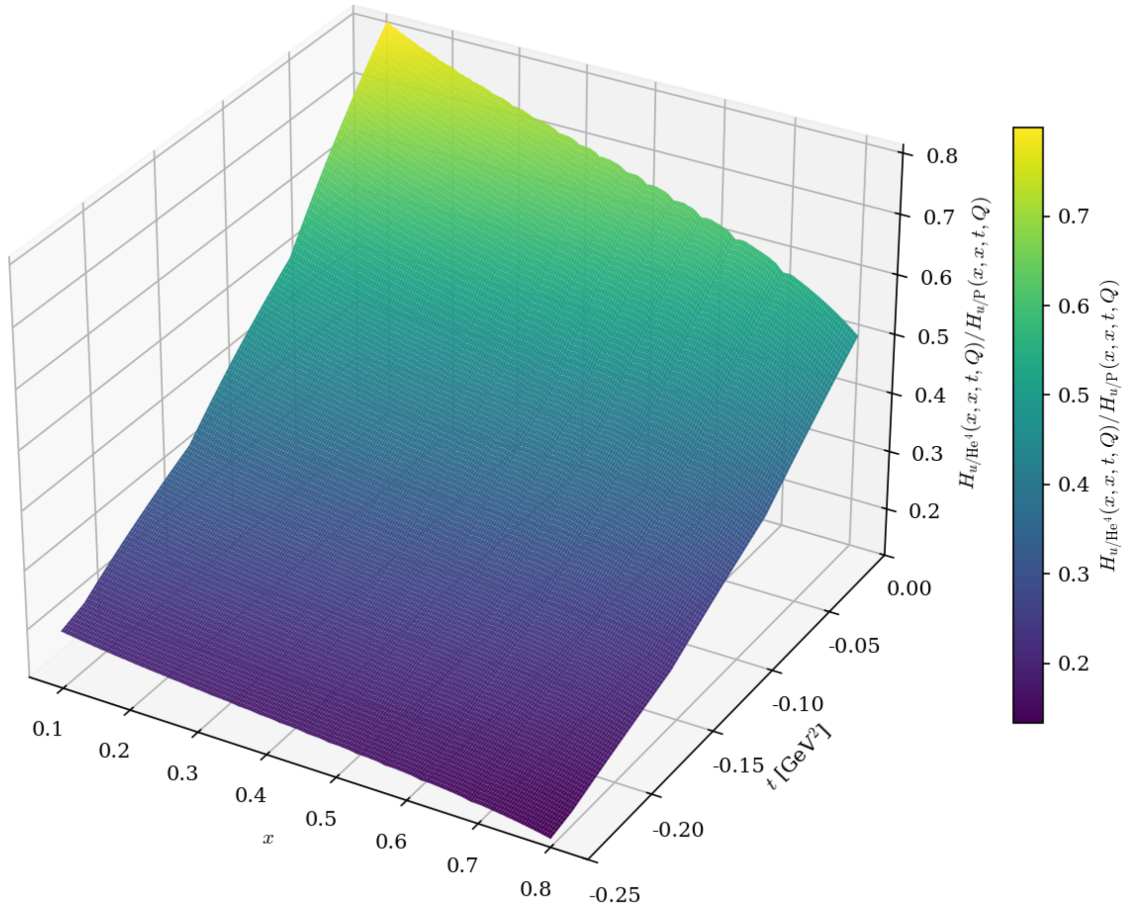
Medium modifications to beam function can be used to probe properties of a QGP



Event generators for EIC and LHCb physics



Questions?

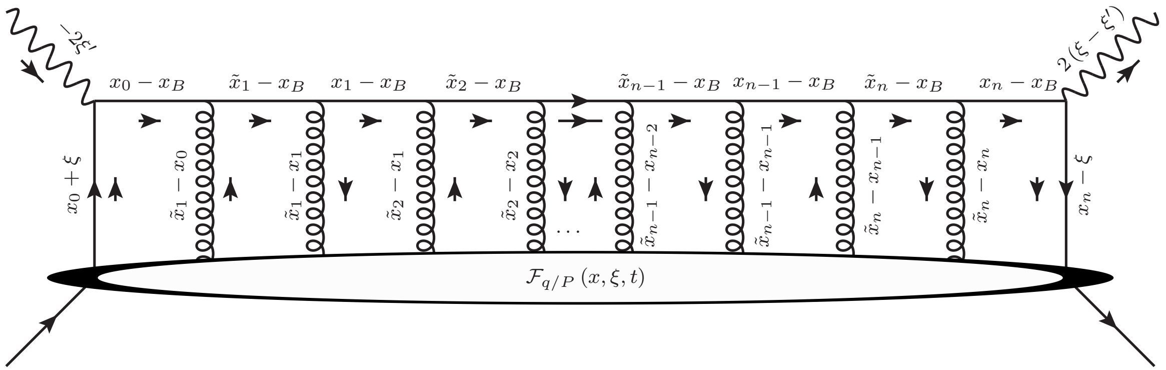


0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
x
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
t [GeV^2]
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$H_{u/\mathrm{He}^4}(x,x,t,Q)/H_{u/\mathrm{P}}(x,x,t,Q)$
0.2
0.3
0.4
0.5
0.6
0.7
$H_{u/\mathrm{He}^4}(x,x,t,Q)/H_{u/\mathrm{P}}(x,x,t,Q)$

Final result

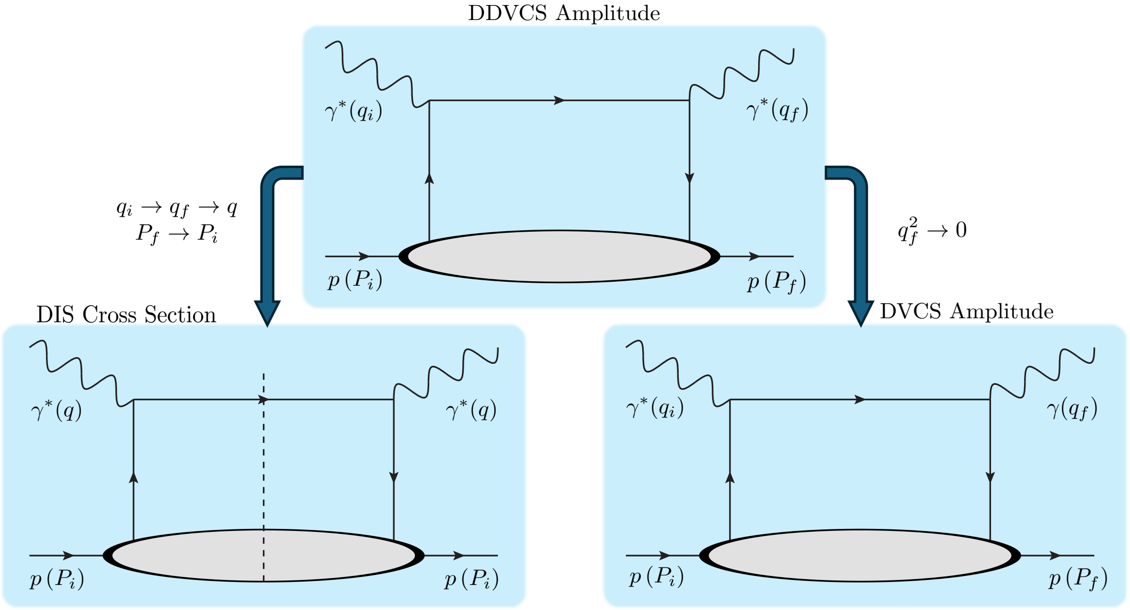
Lastly, the integration in position space

$$\mathcal{M}^{\mu\nu\,\mathrm{F}}_{\mathrm{DVCS}}\left(\xi,\xi',t,Q\right) = -\frac{\bar{n}\cdot P}{4}\left(\frac{1}{32\pi^2 P\cdot q_i}\right)^n \sum_q e_q^2\, g_s^{2n}\int dx_f\, d^4y\, dx_n \frac{d^3k_n}{(2\pi)^3}\frac{e^{iy\cdot(k_f+q_f-k_n)}}{x_n - x_B + i\epsilon}$$
$$\times \prod_{i=0}^{n-1}\left\{dx_{n-i-1}\, d\Phi^{k_g y \tilde{\boldsymbol{k}}}_{n-i}\, d\tilde{\Phi}^{\tilde{y}\tilde{k}_g \boldsymbol{k}}_{n-i}\frac{e^{iy_{n-i}\cdot k_{g\,n-i}}e^{i\tilde{y}_{n-i}\cdot(\tilde{k}_{n-i}-k_{n-i-1}-\tilde{k}_{g\,n-1})}}{[x_{n-i}-x_{n-i-1}-i\epsilon]\,[x_{n-i-1}-x_B+i\epsilon]}\Theta(n\cdot(y_{n-i}-\tilde{y}_{n-i}))\right\}$$
$$\times\,\Psi\left(x_f,\left\{k_{gn},\tilde{k}_{gn}\right\},\xi,t;\mu\right)$$



Lastly, the integration in position space

$$H_q(x,\xi,t) \approx \sum_{n=0}^{N}\frac{A}{n!}\left[\frac{\xi^2\left(A^{1/3}-1\right)}{Q^2}\right]^n x^n\frac{d^nH_q(x,\xi,t)}{dx^n}$$



Lastly, the integration in position space

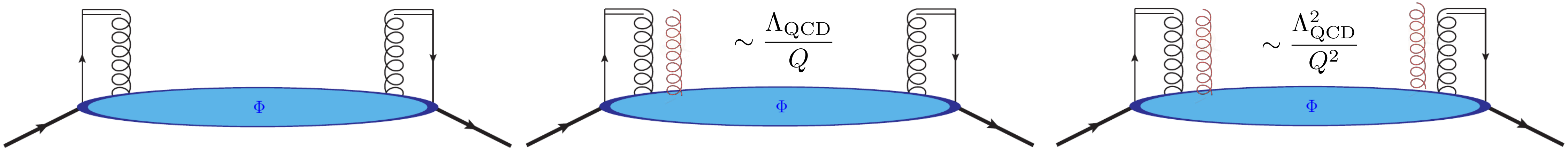
$$H_q(x,\xi,t) \to H_q\left(x_B\left(1+\frac{m^2_{\mathrm{dyn}}}{Q^2}\right),\xi,t\right) \qquad m^2_{\mathrm{dyn}} = \eta^2\left(A^{1/3}-1\right)$$

Power counting nuclear corrections

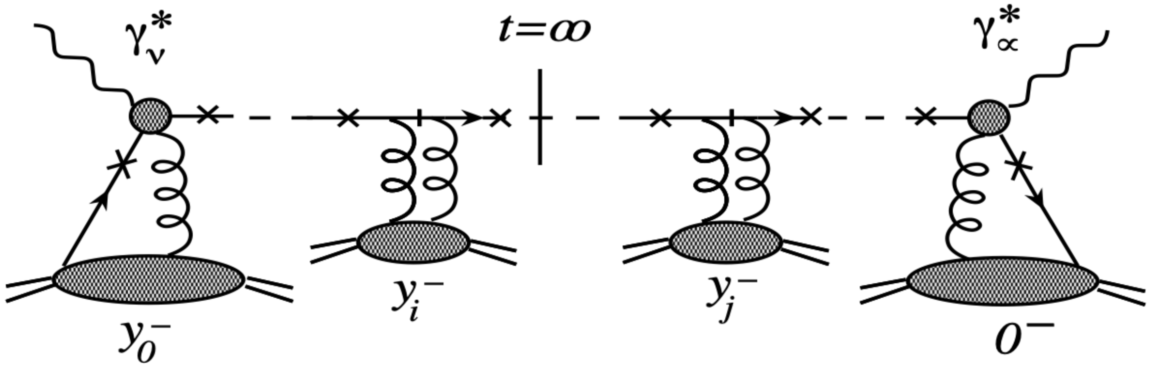
The DIS cross section in the vacuum can be written as

$$\frac{d^2\sigma}{dx\,dy} = \frac{4\pi\alpha_{\rm em}^2}{Q^4}\, S\left[xy^2 F_1(x,Q^2) + \left(1 - y - \frac{xyM^2}{S}\right) F_2(x,Q^2)\right] + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{Q}\right)$$

Dynamical power corrections to cross section associated with additional transverse gluons



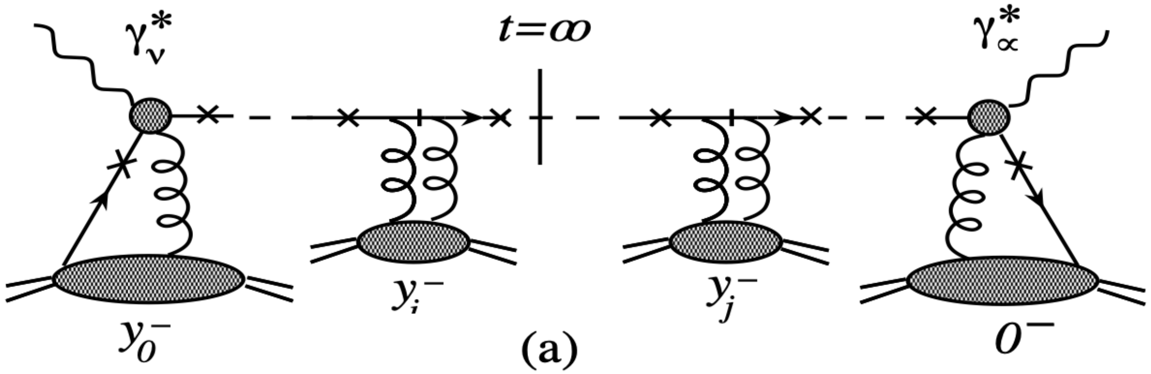
The power corrections are enhanced in nuclear matter *Qiu-Vitev (2003)*



$$\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{Q}\right) \to \mathcal{O}\left(A^{1/3}\frac{\Lambda_{\rm QCD}^2}{Q^2}\right)$$

Resumming power corrections

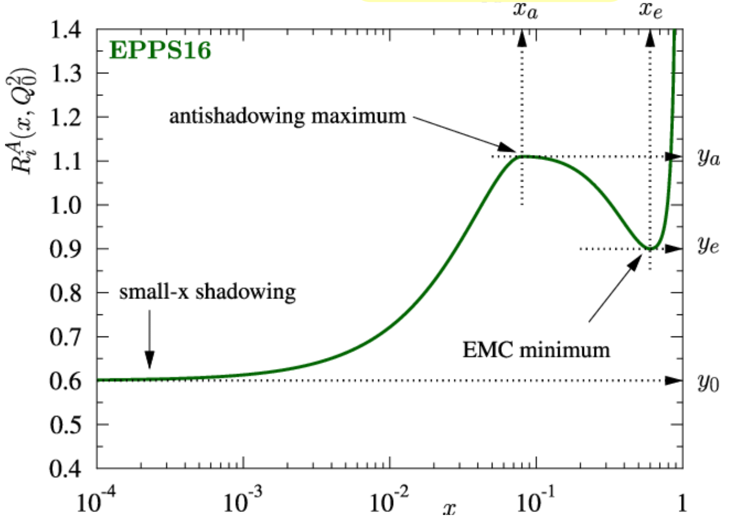
Resummation of nuclear power corrections to all orders *Qiu-Vitev (2003)*

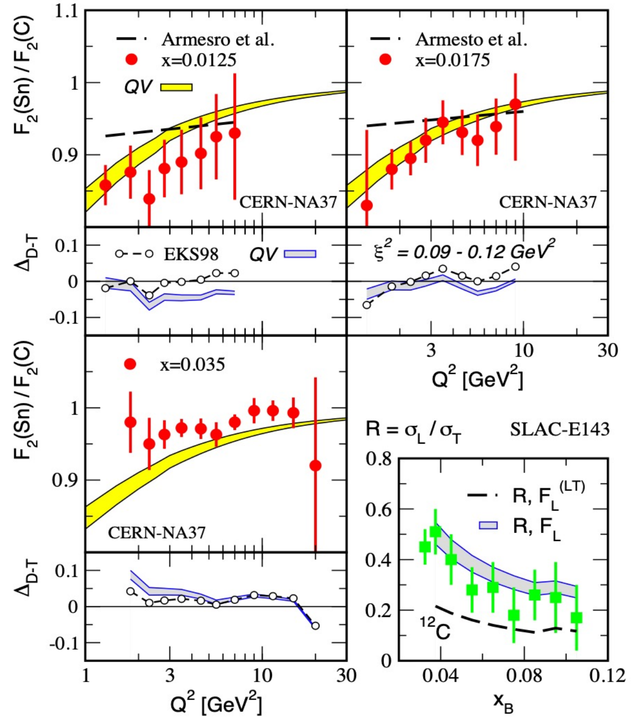


$$A^{1/3}\frac{\Lambda^2_{\rm QCD}}{Q^2} \sim 1$$

Resummation to all orders results in a shift in the PDFs

$$F_T^A(x,Q^2) \approx A\, F_T^{\rm (LT)}\left(x + \frac{x\xi^2(A^{1/3}-1)}{Q^2}, Q^2\right) \qquad \xi^2 \sim \frac{1}{r_p^2}$$

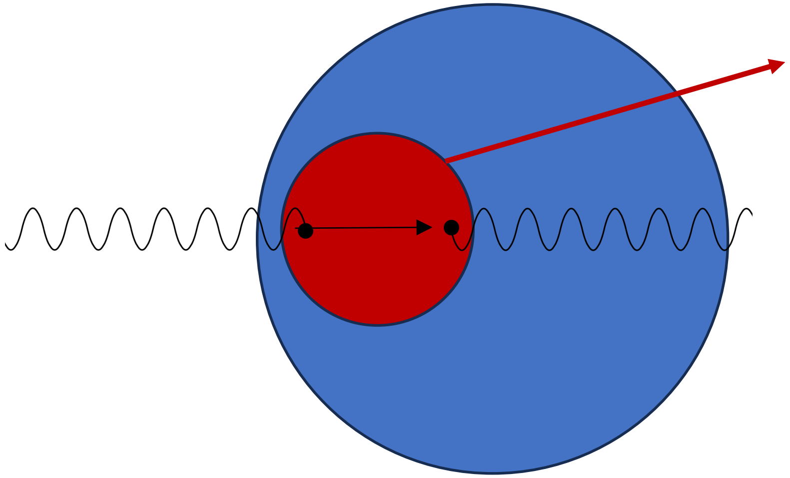




Power counting in nuclear matter

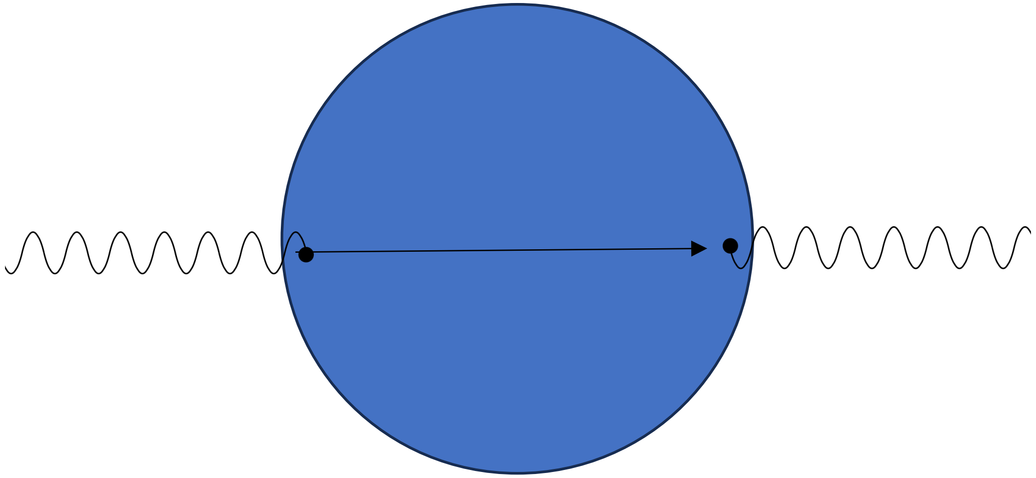
Incoherent DVCS

$$e(\ell_i) + A(P_i) \;\rightarrow\; e(\ell_f) + N(P_f) + (A-1)(P_X) + \gamma^*(q_f)\,,$$



Coherent DVCS

$$e(\ell_i) + A(P_i) \;\rightarrow\; e(\ell_f) + A(P_f) + \gamma^*(q_f)\,,$$

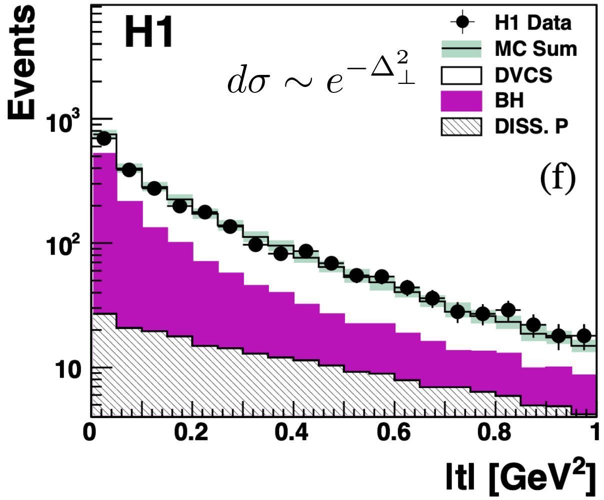


Burkhardt relationship introduces a power counting

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\, e^{-i\mathbf{b}_\perp\cdot\boldsymbol{\Delta}_\perp}\, H(x, 0, -\boldsymbol{\Delta}_\perp^2)$$

$$b_\perp \sim R_{p/A} \sim \frac{c}{\Lambda_{\mathrm{QCD}}}$$

$$\Delta_\perp \sim \frac{\Lambda_{\mathrm{QCD}}}{c}$$

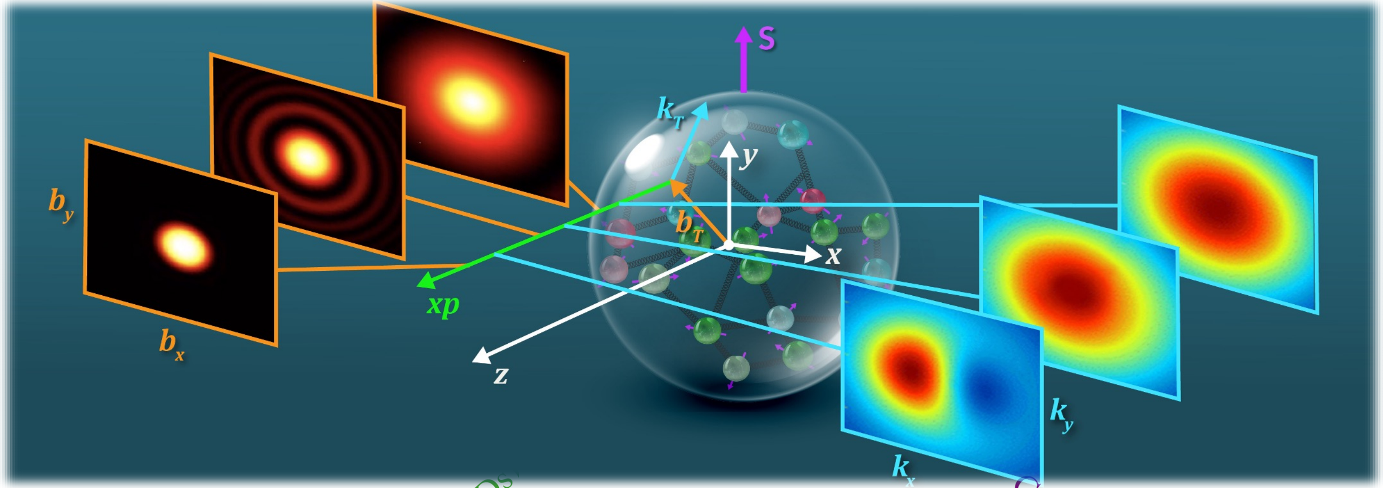


$$b_\perp \sim R_A \sim \frac{A^{1/3}}{\Lambda_{\mathrm{QCD}}}$$

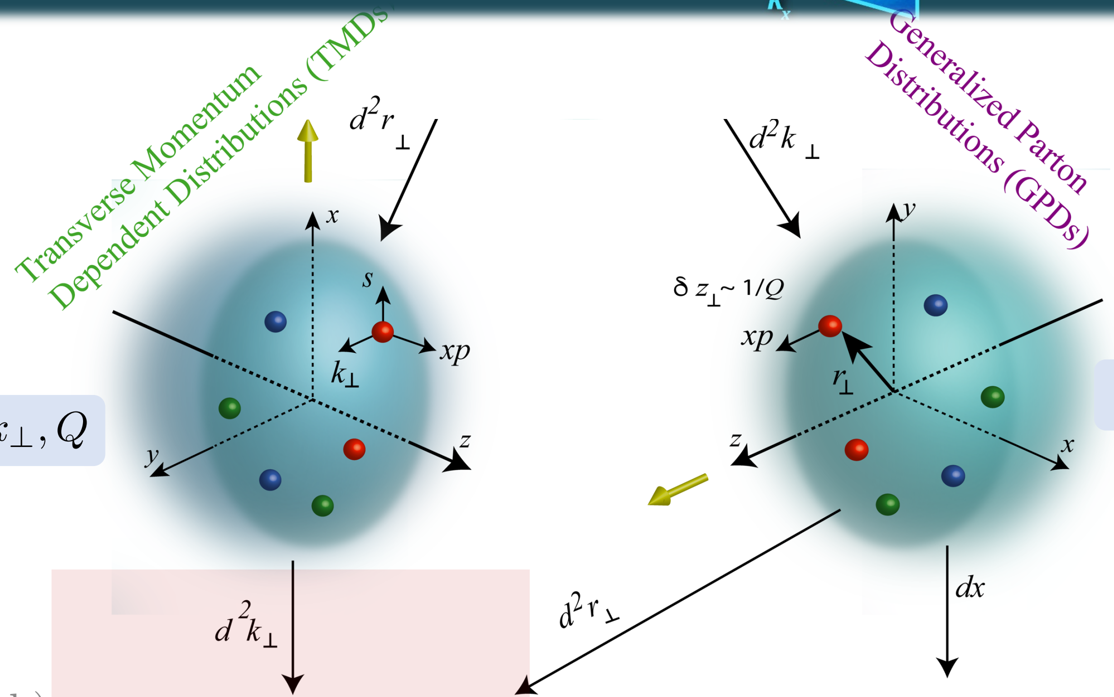
$$\Delta_\perp \sim A^{-1/3}\Lambda_{\mathrm{QCD}}$$

What is known about nuclear matter?

Distributions of partons in hadrons



$\Lambda_{\mathrm{QCD}}, k_\perp, 1/r_\perp, Q$



$\Lambda_{\mathrm{QCD}}, k_\perp, Q$

$\Lambda_{\mathrm{QCD}}, 1/r_\perp, Q$

Parton Distribution Functions

Form Factors

Power counting nuclear corrections

The DIS cross section in the vacuum can be written as

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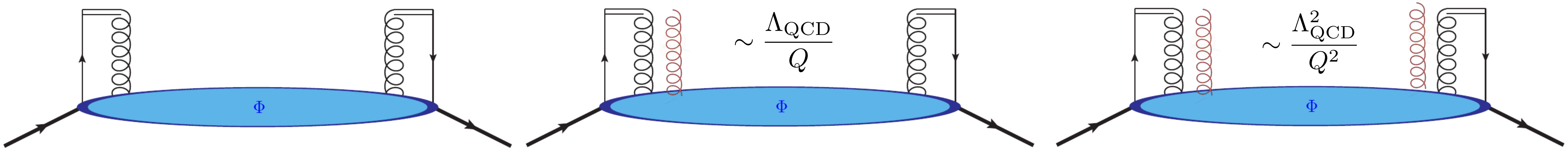
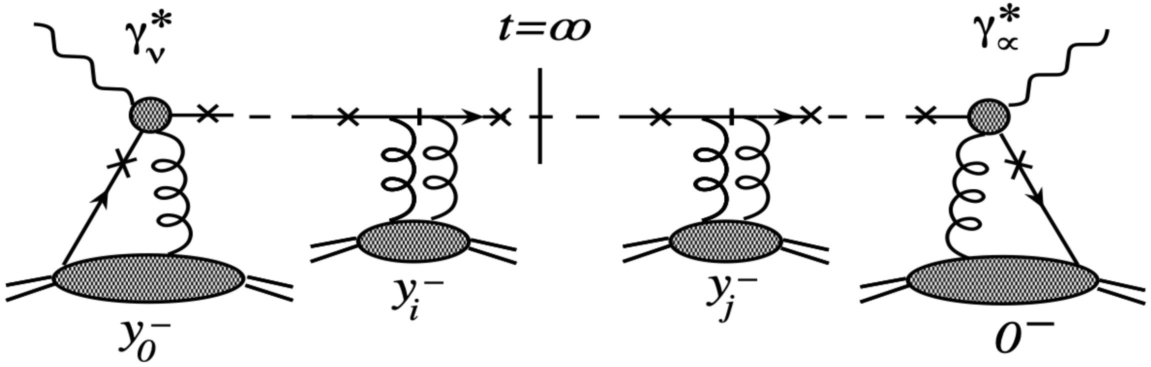
Dynamical power corrections to cross section associated with additional transverse gluons



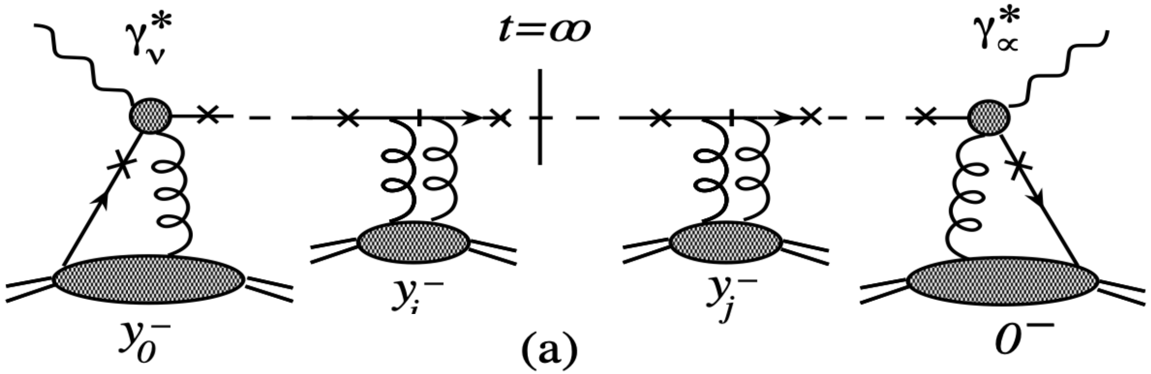
The power corrections are enhanced in nuclear matter *Qiu-Vitev (2003)*



$$\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{Q}\right) \to \mathcal{O}\left(A^{1/3}\frac{\Lambda_{\rm QCD}^2}{Q^2}\right)$$

Resumming power corrections

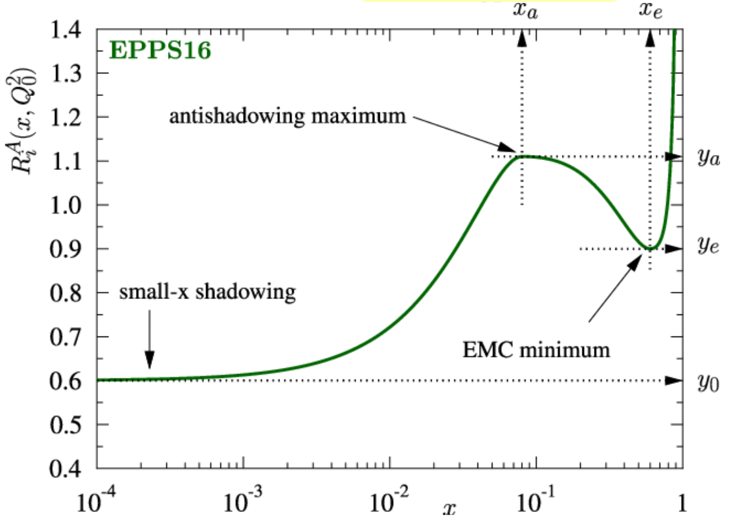
Resummation of nuclear power corrections to all orders *Qiu-Vitev (2003)*

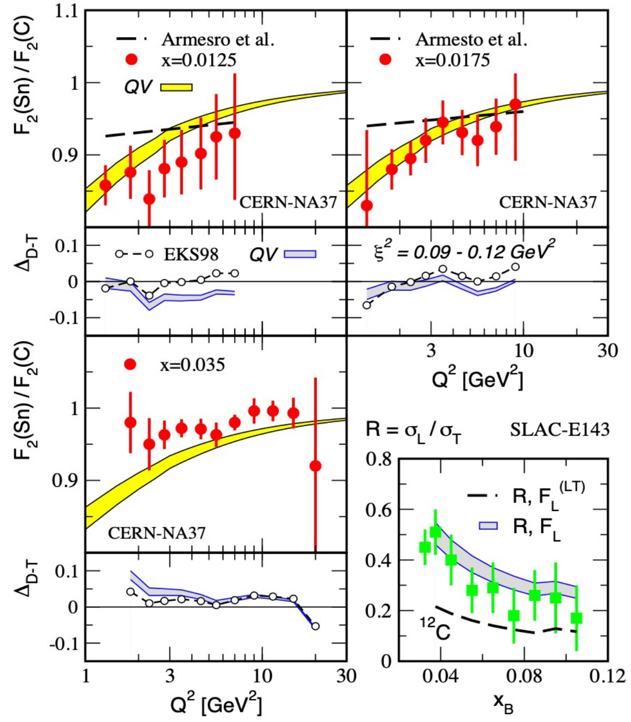


$$A^{1/3}\frac{\Lambda^2_{\rm QCD}}{Q^2} \sim 1$$

Resummation to all orders results in a shift in the PDFs

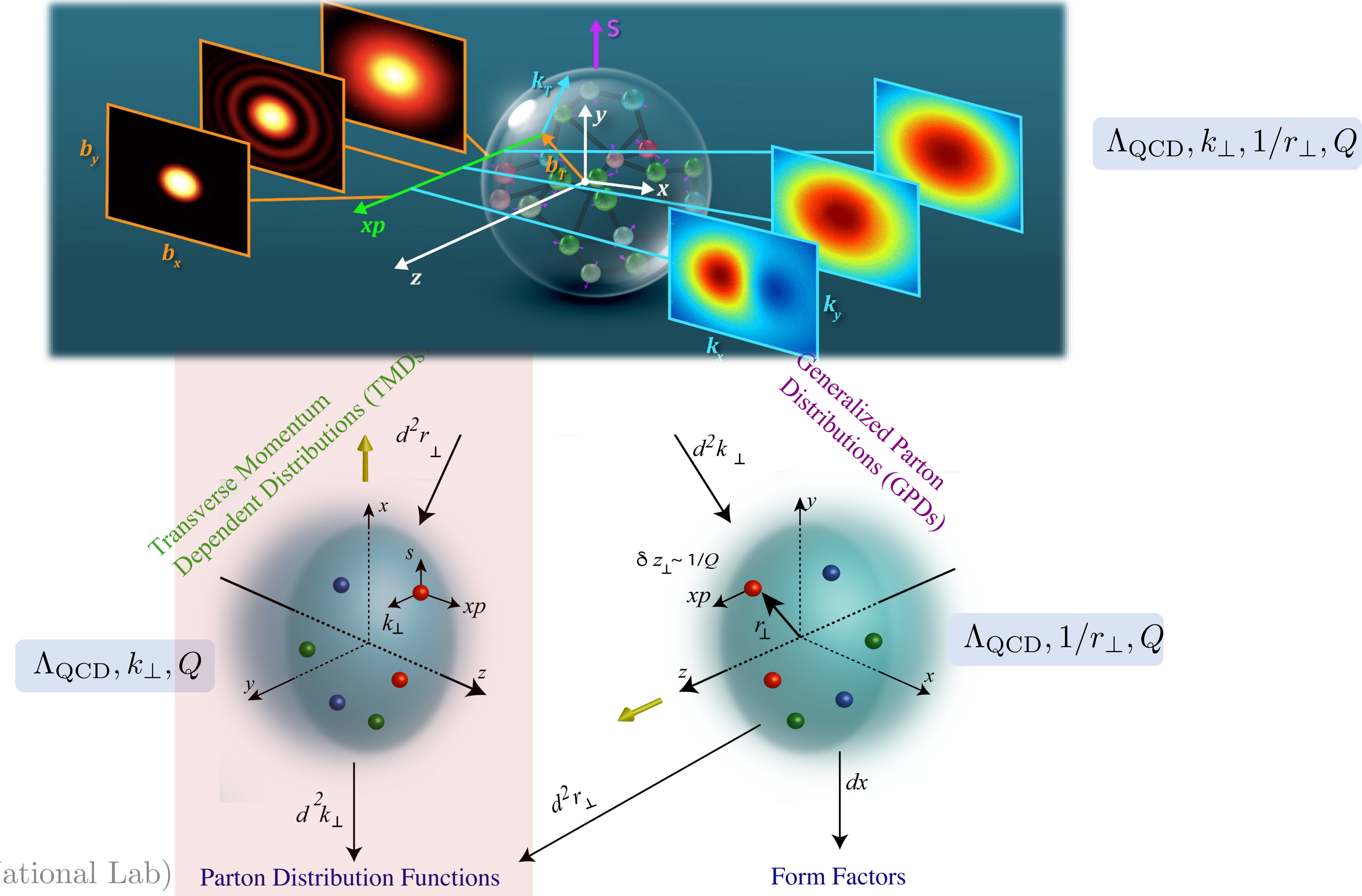
$$F_T^A(x,Q^2) \approx A\,F_T^{(\rm LT)}\left(x + \frac{x\xi^2(A^{1/3}-1)}{Q^2}, Q^2\right) \qquad \xi^2 \sim \frac{1}{r_p^2}$$





Transverse momentum distributions in cold matter

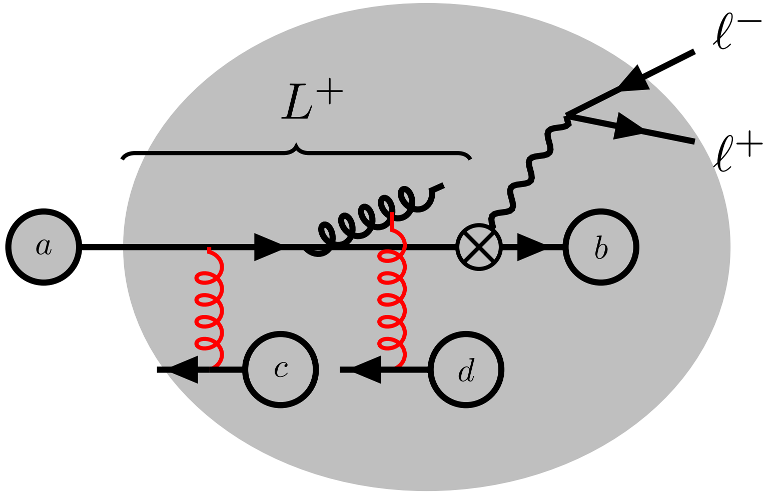
Distributions of partons in hadrons

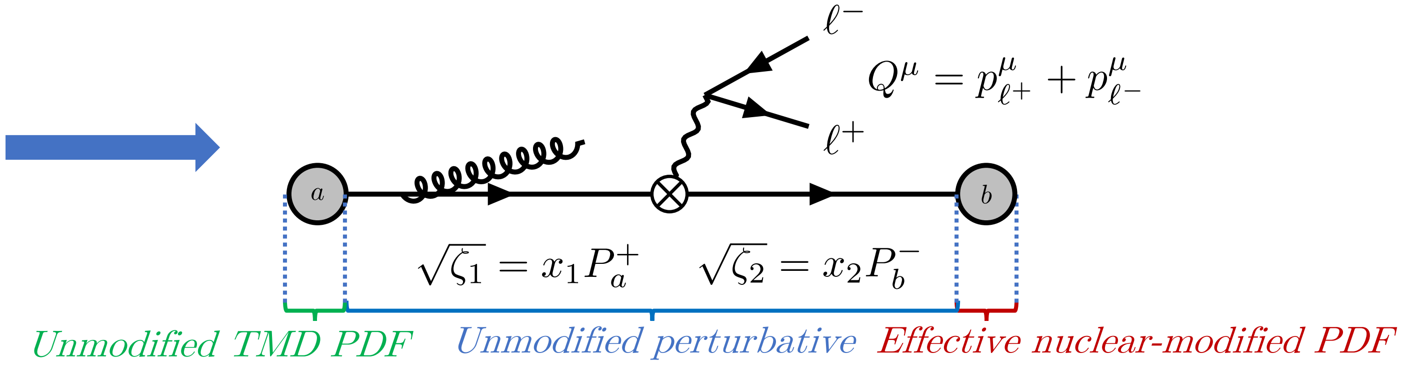


A first approximation

nTMDs were originally defined using an approximate scheme by these two

Alrashed, Anderle, Kang, JT, Xing (2021)



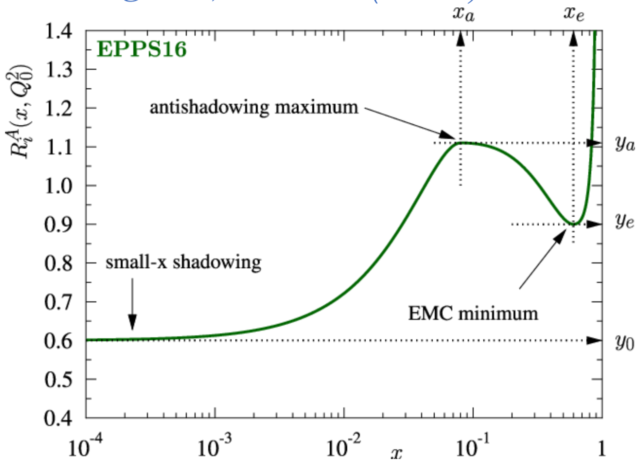


1.) The intrinsic transverse momentum difference

~~2.) Interactions between the incoming quark from the proton undergo re-scattering~~

~~3.) Re-scattering induces energy loss~~

Armesto, Paukkunen, Penín, Salgado, Zurita (2015)



Non-perturbative parameterization is modified to account for nuclear medium effects

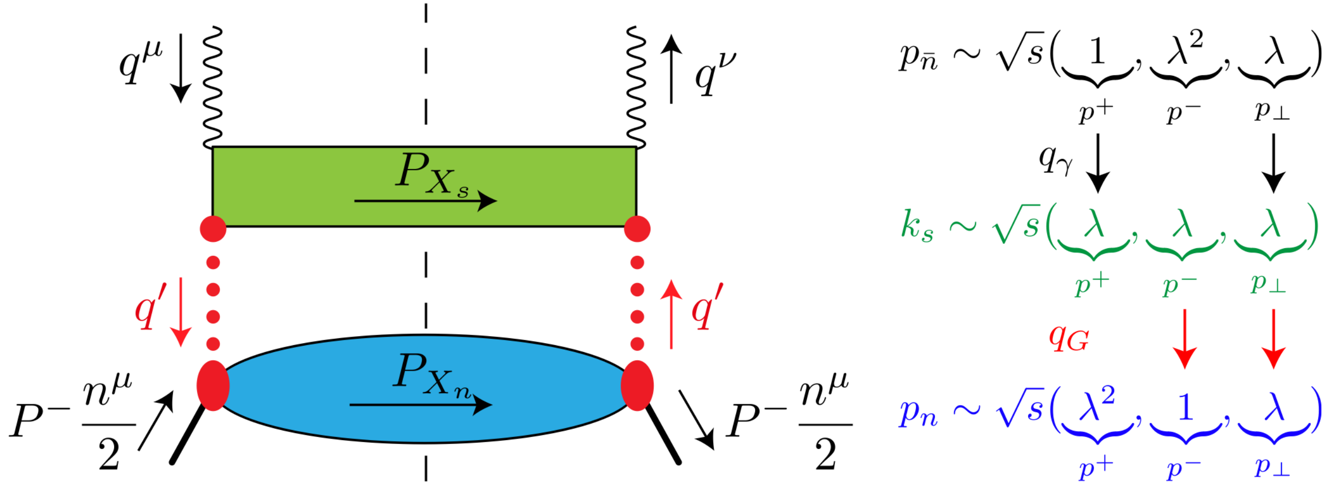
$$f_{1\,q/A}(x,b,\mu,\zeta) = [C \otimes f]_{q/A}(x,b,\mu_i,\zeta_i)\; U(\mu_i,\mu;\zeta)\; Z(b,\zeta_i,\zeta;\mu_i)\; U_{\rm NP}^{f^A}(x,b,\zeta,A)$$

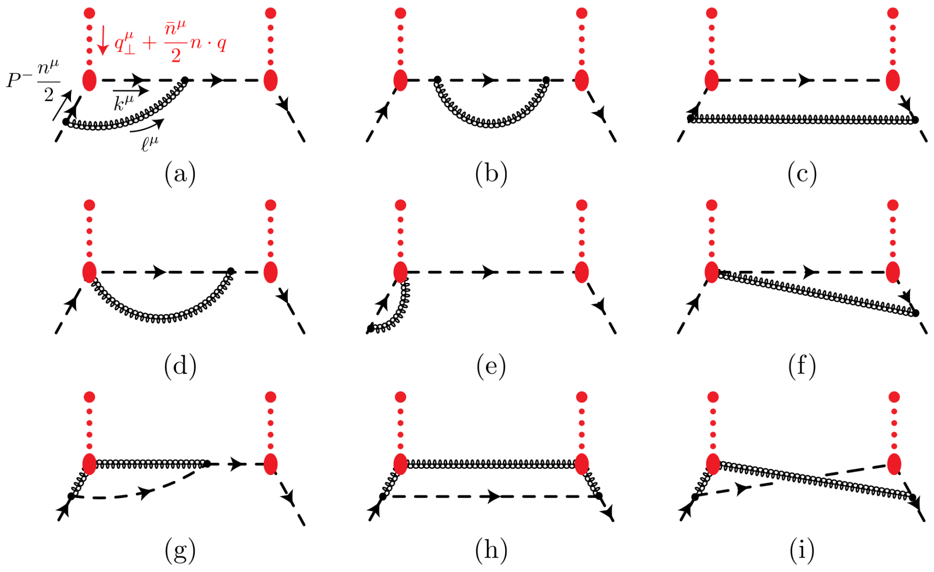
Perturbative *Non-perturbative*

Soft-Collinear Effective Theory with Glaubers

SCET with Glauber gluons has been applied to pp and DIS

$$S_G = \sum_{i,j} \int \mathrm{d}^4x\, \mathcal{O}_{ns}^{ij}(x)\,, \qquad \mathcal{O}_{ns}^{ij}(x) = 8\pi\alpha_s e^{-\mathrm{i}x\cdot\mathcal{P}} \mathcal{O}_n^{iA}(\tilde{x}) \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n A}(\tilde{x})\,,$$





Was shown that rapidity divergences of the collinear function give rise to the BFKL evolution equation

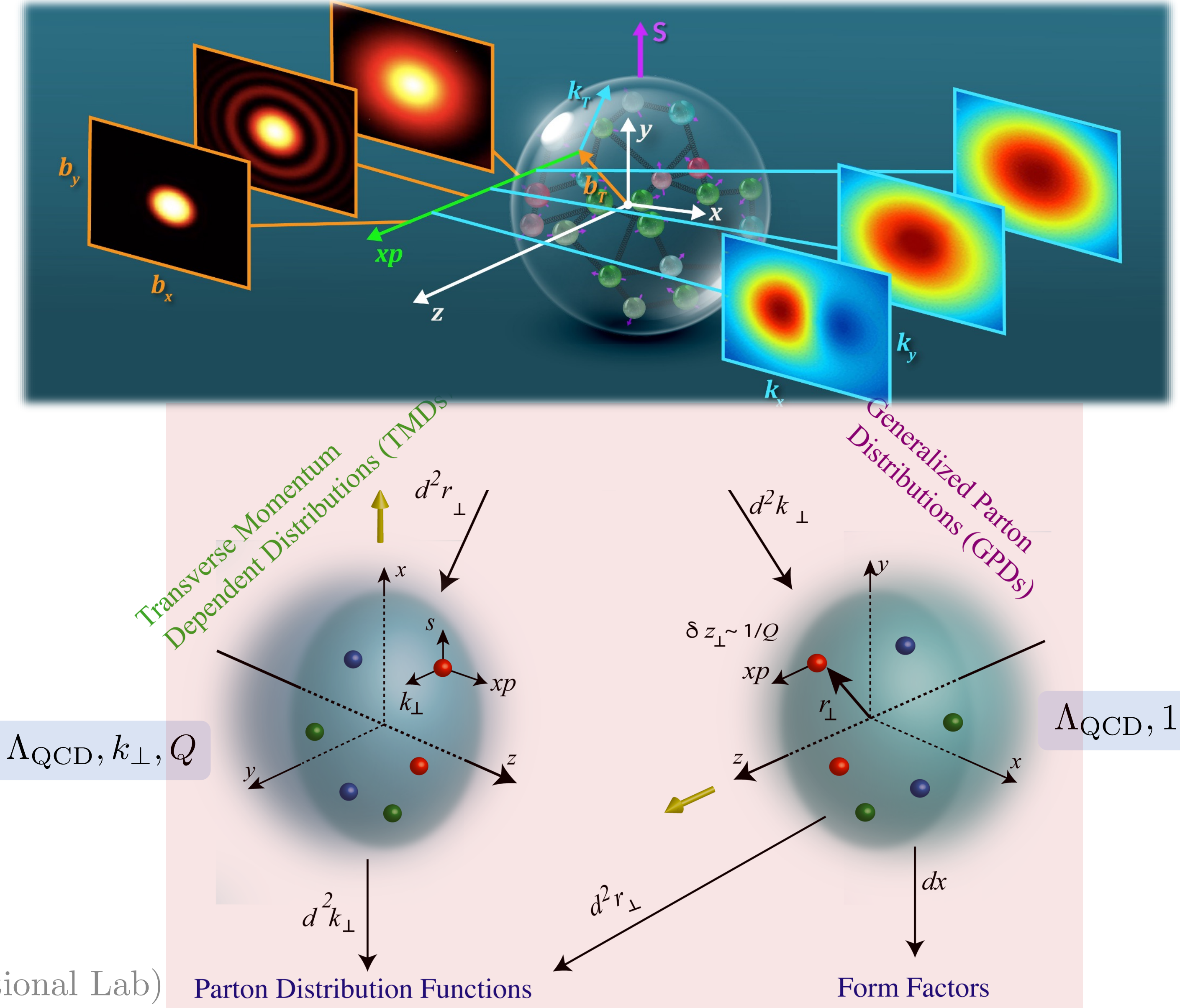
$$\nu \frac{d}{d\nu} C\left(\frac{\nu}{\bar{n}\cdot P}, q_\perp, \epsilon\right) = -C\left(\frac{\nu}{\bar{n}\cdot P}, q_\perp, \epsilon\right)$$

$$-2\frac{\alpha_s C_A}{\pi} \iota^\epsilon \mu^{2\epsilon} \int \frac{d^{2-2\epsilon}k_\perp}{(2\pi)^{1-2\epsilon}} \left\{ \frac{C\left(\frac{\nu}{\bar{n}\cdot P}, \vec{k}_\perp, \epsilon\right)}{(\vec{q}_\perp - \vec{k}_\perp)^2} - \frac{\vec{q}_\perp^{\,2}}{2\vec{k}_\perp^{\,2}(\vec{q}_\perp - \vec{k}_\perp)^2} C\left(\frac{\nu}{\bar{n}\cdot P}, q_\perp, \epsilon\right) \right\}$$

Ovanesyan, Vitev (2011)
Rothstein, Stewart (2016)
Neill, Pathak, Stewart (2022)

What about the other leg of matter?

Distributions of partons in hadrons



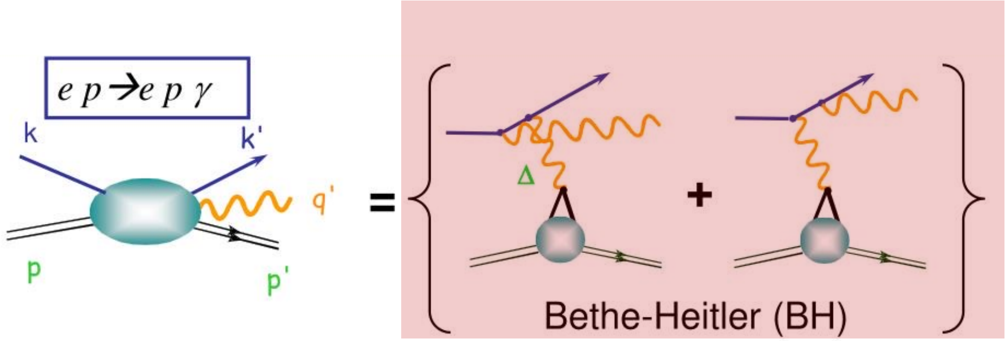
$\Lambda_{\mathrm{QCD}}, k_\perp, 1/r_\perp, Q$

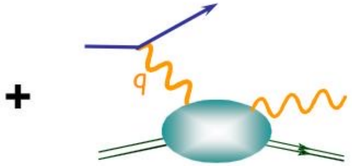
$\Lambda_{\mathrm{QCD}}, k_\perp, Q$

$\Lambda_{\mathrm{QCD}}, 1/r_\perp, Q$

Bethe-Heitler Background

Kinematics of DDVCS





DDVCS

$$\left[\mathcal{T}^{\mu\alpha}\,\mathcal{L}^{(e)}_{\alpha\beta}\,\frac{\alpha_{\text{em}}}{Q'^2}\mathcal{L}^{(\ell)}_{\mu\nu}\,\bar{\mathcal{T}}^{\nu\beta}\right](\xi,t,Q,Q',\varphi,\Omega_\ell) = \frac{\alpha_{\text{em}}}{Q'^2}\mathcal{L}^{(\ell)}_{\mu\nu}\,(Q',\Omega_\ell)\,\Big[$$

$$\mathcal{T}^{\mu\alpha}_{\text{DVCS}}\,(\xi,t,Q,Q')\;\mathcal{L}^{(e)}_{\alpha\beta\,\text{DVCS}}\,(Q,\varphi)\;\bar{\mathcal{T}}^{\nu\beta}_{\text{DVCS}}\,(\xi,t,Q,Q')$$

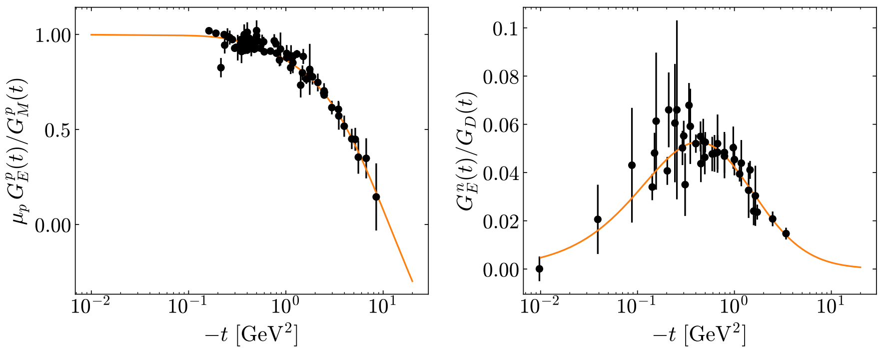
$$+\mathcal{T}^{\mu\alpha}_{\text{BH}}\,(\xi,t,Q)\;\mathcal{L}^{(e)}_{\alpha\beta\,\text{BH}}\,(Q,Q',\varphi)\;\bar{\mathcal{T}}^{\nu\beta}_{\text{BH}}\,(\xi,t,Q)$$

$$+\mathcal{T}^{\mu\alpha}_{\text{DVCS}}\,(\xi,t,Q,Q')\;\mathcal{L}^{(e)}_{\alpha\beta\,\text{I}}\,(Q,Q',\varphi)\;\bar{\mathcal{T}}^{\nu\beta}_{\text{BH}}\,(\xi,t,Q)$$

$$+\mathcal{T}^{\mu\alpha}_{\text{BH}}\,(\xi,t,Q)\;\bar{\mathcal{L}}^{(e)}_{\alpha\beta\,\text{I}}\,(Q,Q',\varphi)\;\bar{\mathcal{T}}^{\nu\beta}_{\text{DVCS}}\,(\xi,t,Q,Q')\Big].$$

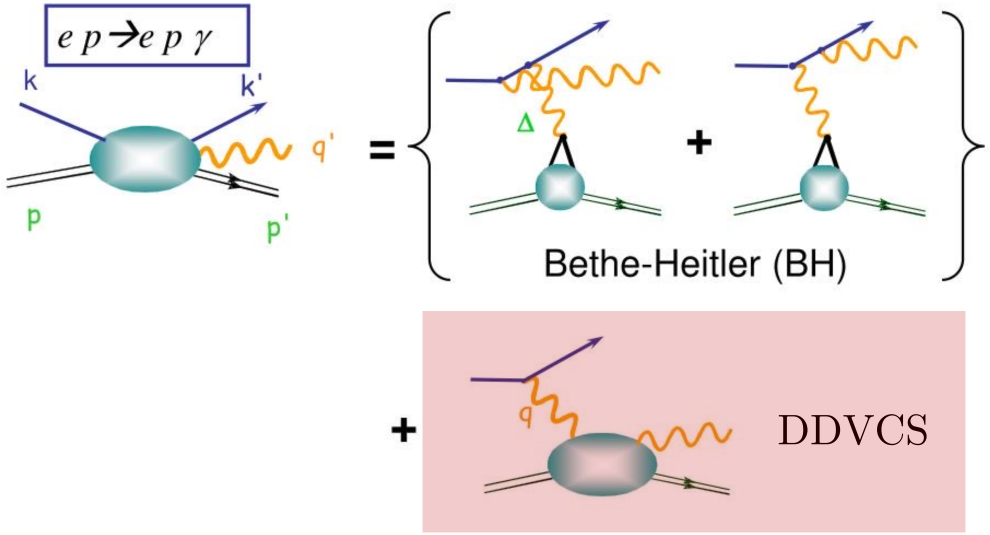
Ye, Arrington, Hill, Lee (2017)

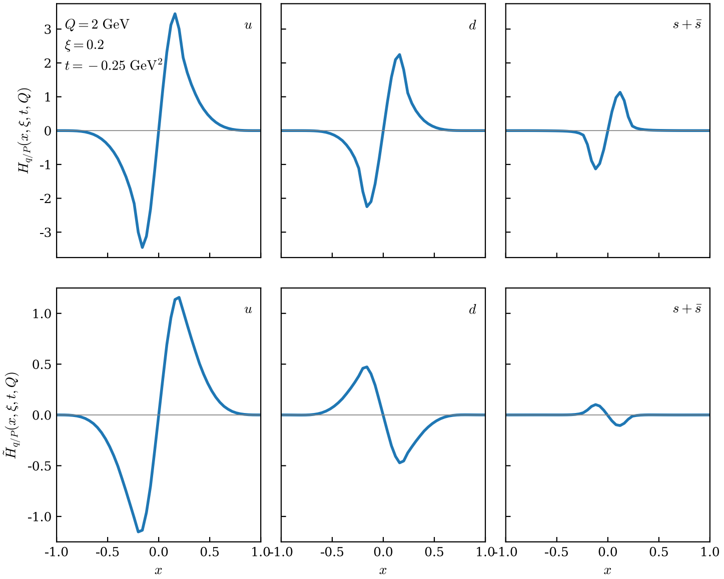
$$\mathcal{T}^{\alpha}_{\text{BH}}\,(\xi,t,Q) = (2\pi)^4\delta^{(4)}(q_i+P_f-P_i)\,\bar{u}(P_f)\left[F_1(t)\gamma^\alpha + F_2(t)\frac{i\sigma^{\alpha\beta}\Delta_\beta}{2M}\right]u(P_i)$$

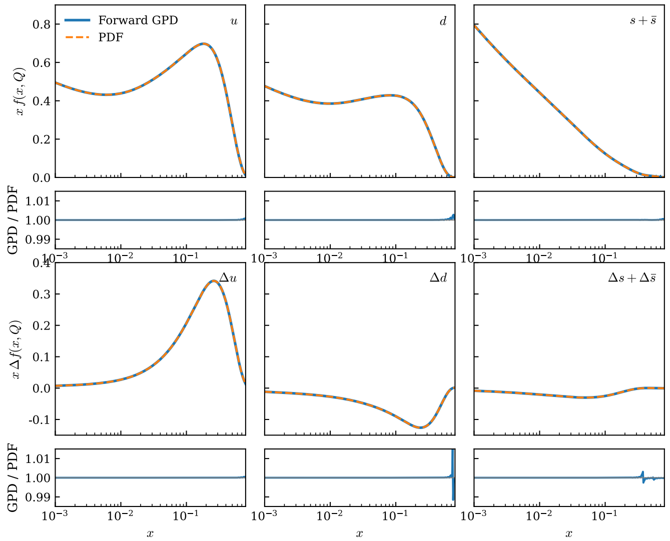


Parameterization of the GPDs

Kinematics of DDVCS







$$\left[\mathcal{T}^{\mu\alpha}\,\mathcal{L}^{(e)}_{\alpha\beta}\,\frac{\alpha_{\rm em}}{Q'^2}\,\mathcal{L}^{(\ell)}_{\mu\nu}\,\bar{\mathcal{T}}^{\nu\beta}\right](\xi,t,Q,Q',\varphi,\Omega_\ell)=\frac{\alpha_{\rm em}}{Q'^2}\,\mathcal{L}^{(\ell)}_{\mu\nu}\,(Q',\Omega_\ell)\,\Big[$$

$$\mathcal{T}^{\mu\alpha}_{\rm DVCS}\,(\xi,t,Q,Q')\;\mathcal{L}^{(e)}_{\alpha\beta\,{\rm DVCS}}\,(Q,\varphi)\;\bar{\mathcal{T}}^{\nu\beta}_{\rm DVCS}\,(\xi,t,Q,Q')$$

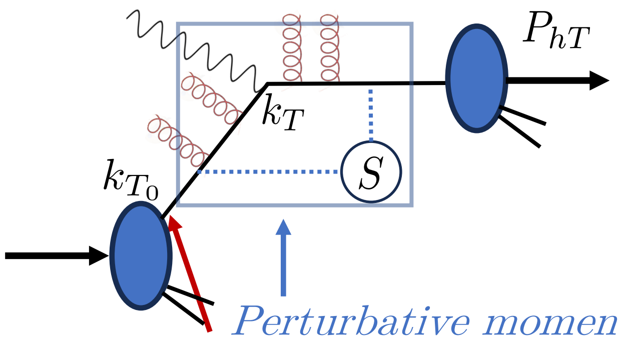
$$+\mathcal{T}^{\mu\alpha}_{\rm BH}\,(\xi,t,Q)\;\mathcal{L}^{(e)}_{\alpha\beta\,{\rm BH}}\,(Q,Q',\varphi)\;\bar{\mathcal{T}}^{\nu\beta}_{\rm BH}\,(\xi,t,Q)$$

$$+\mathcal{T}^{\mu\alpha}_{\rm DVCS}\,(\xi,t,Q,Q')\;\mathcal{L}^{(e)}_{\alpha\beta\,{\rm I}}\,(Q,Q',\varphi)\;\bar{\mathcal{T}}^{\nu\beta}_{\rm BH}\,(\xi,t,Q)$$

$$+\mathcal{T}^{\mu\alpha}_{\rm BH}\,(\xi,t,Q)\;\bar{\mathcal{L}}^{(e)}_{\alpha\beta\,{\rm I}}\,(Q,Q',\varphi)\;\bar{\mathcal{T}}^{\nu\beta}_{\rm DVCS}\,(\xi,t,Q,Q')\Big]\,.$$

Perturbative background

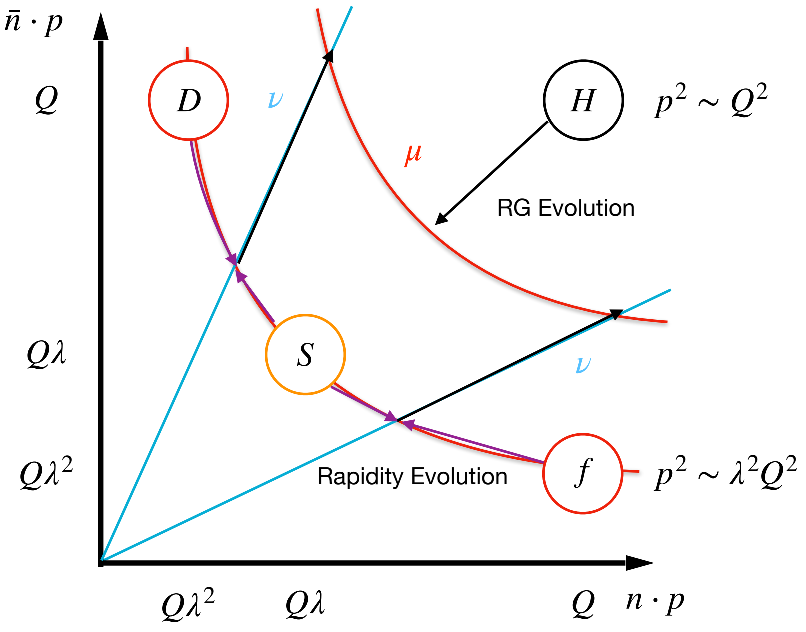
Perturbative Sudakov: accounts for transverse momenta generated from soft and collinear emissions



Perturbative momentum

Intrinsic/non-perturbative momentum

Experiments involve mixture of Perturbative and non-perturbative momentum



Anomalous dimensions

$$\mu \frac{d}{d\mu} \ln F(Q,\mu,\nu) = \gamma^q_{F\,\mu}(Q,\mu,\nu) \qquad \nu \frac{d}{d\nu} \ln G(Q,\mu,\nu) = \gamma^q_{G\,\nu}(Q,\mu,\nu)$$

$$F \in \{H, f, D, S\} \qquad G \in \{f, D, S\}$$

Obtain intrinsic momentum through a fit to data

Available perturbative accuracy

Anomalous dimensions

$$\mu\frac{d}{d\mu}\ln F(Q,\mu,\nu) = \gamma_F^q(Q,\mu,\nu)$$
$$F \in \{H, f, D, S\}$$

$$\mu\frac{d}{d\nu}\ln G(Q,\mu,\nu) = \gamma_G^q(Q,\mu,\nu)$$
$$G \in \{f, D, S\}$$

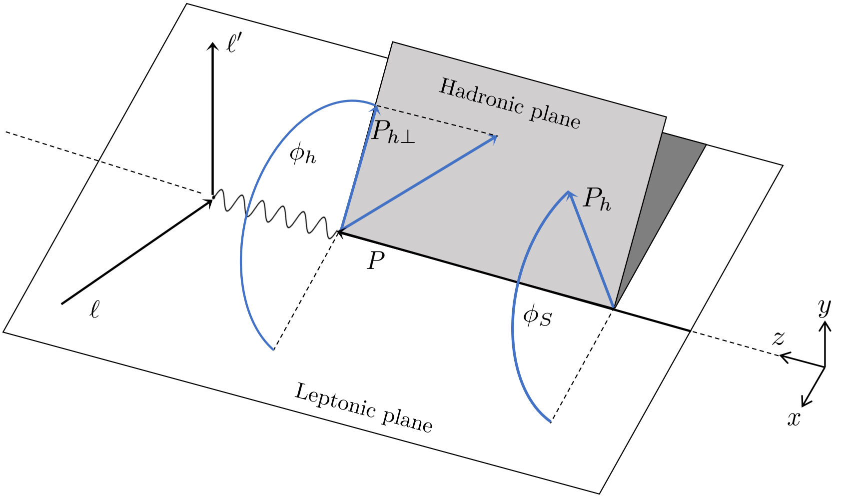
Anomalous dimensions are almost known up to N^{4}LL at this point (no 5-loop cusp)

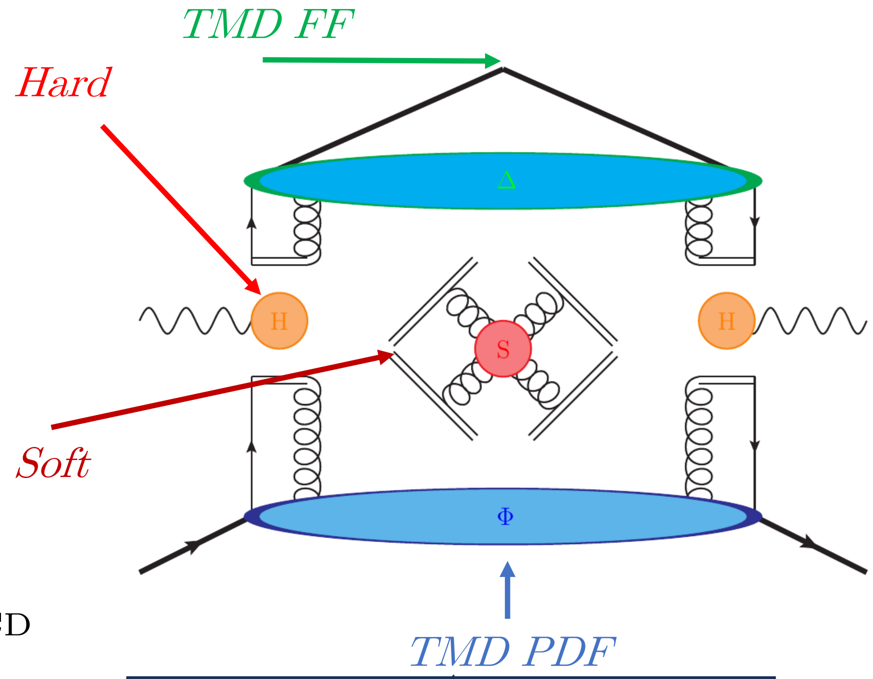
Accuracy	H, $\mathcal{J}$	$\Gamma_{\text{cusp}}(\alpha_s)$	$\gamma_H^q(\alpha_s)$	$\gamma_r^q(\alpha_s)$	$\beta(\alpha_s)$
LL	Tree level	1-loop			1-loop
NLL	Tree level	2-loop	1-loop	1-loop	2-loop
NLL$'$	1-loop	2-loop	1-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	2-loop	3-loop
NNLL$'$	2-loop	3-loop	2-loop	2-loop	3-loop
N^{3}LL	2-loop	4-loop	3-loop	3-loop	4-loop
N^{3}LL$'$	3-loop	4-loop	3-loop	3-loop	4-loop
N^{4}LL	3-loop	5-loop	4-loop	4-loop	5-loop
N^{4}LL$'$	4-loop	5-loop	4-loop	4-loop	5-loop

Lee, Smirnov, and Smirnov (2010)
Gehrmann, Glover, Huber, Ikizlerli, and Studerus (2010)
Ebert, Mistlberger, Vita (2020)
Ebert, Mistlberger, Vita (2020)
Agarwal, von Manteuffel, Panzer, and Schabinger (2021)
Duhr, Mistlberger, Vita (2022)
Moult, Zhu, Zhu (2022)
Herzog, Moch, Ruijl, Ueda, Vermaseren, and Vogt (2019)
Baikov, Chetyrkin, and Kuhn (2017)

Factorization of physics at different scales

Factorization of the cross section in an OPE $\quad Q \gg q_T \gtrsim \Lambda_{\rm QCD}$



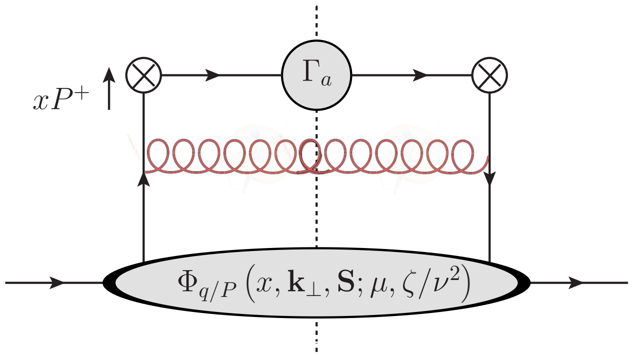


$$d\sigma \sim \sum_i \left|C\left(Q;\mu\right)\right| f \otimes D \otimes S(q_T,\mu)$$

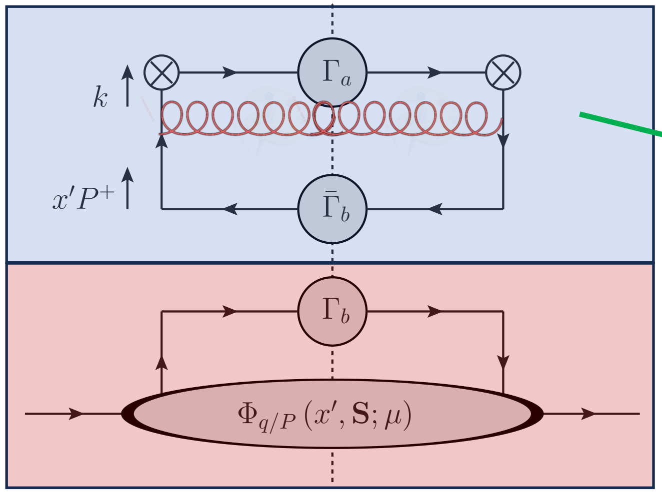
Contains fixed order and large logs

$$\ln\left(\frac{Q}{\mu}\right)$$

Factorization of IR modes $\quad q_T \gtrsim \Lambda_{\rm QCD}$



$=$



Matching coefficient contains fixed order and large logs

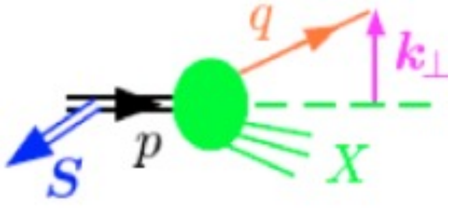
$$\ln\left(\frac{q_T}{\mu}\right)$$

$$f\left(x,q_T,\mu\right) \sim \left[C \otimes f\right]\left(x,q_T,\mu\right)$$

Collinear PDF

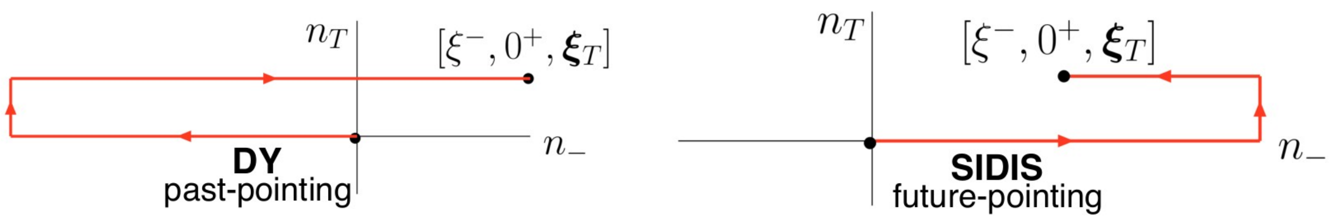
Intro to the Sivers effect

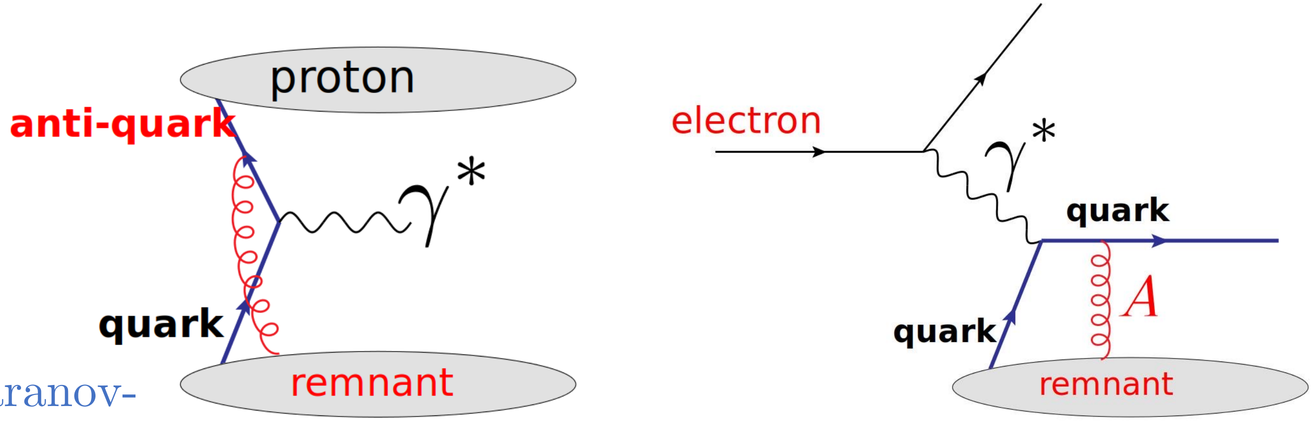
Anomalous dimensions



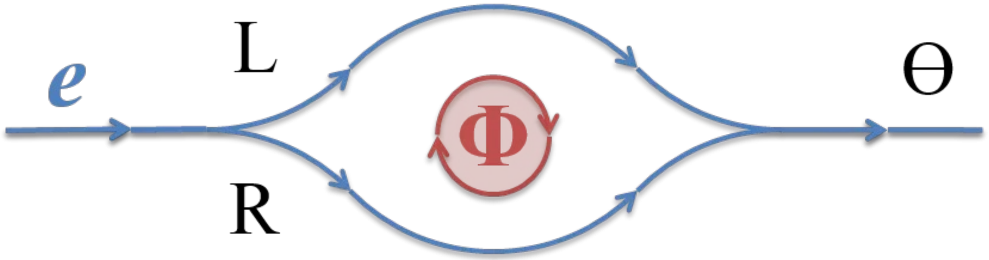
$$\Phi_{q/p}\left(x, \mathbf{k}_\perp, \mathbf{S}, \mu, \zeta\right) = f_{1\,q/p}\left(x, k_\perp, \mu, \zeta\right) - \frac{\epsilon_\perp^{\rho\sigma} k_{\perp\rho} S_{\perp\sigma}}{M} f_{1T\,q/p}^{\perp}\left(x, k_\perp, \mu, \zeta\right)$$

The Sivers function is process dependent





This long-range correlation is similar to the Aharanov-Bohm effect



$$\mathcal{U}^{\bar{n}}\left(0, \xi^-\right) = \mathcal{P}\exp\left[-ig\int_0^{\xi^-} d\bar{n}\cdot x\, n\cdot A(x)\right]$$

$$\psi(\mathbf{r}, t) \to \psi(\mathbf{r}, t)\exp\left(iq\int_{\mathcal{C}} \mathbf{A}\cdot d\mathbf{r}\right)$$

Factorization and resummation

Differential cross section for Semi-Inclusive DIS is given by

$$F_{\rm UU}\left(x,z,\mathbf{P}_{h\perp}\right) = \underbrace{H_{\rm DIS}\left(Q,\mu\right)}_{\text{Hard}} \sum_q e_q^2 \int \frac{bdb}{2\pi} J_0\left(\frac{b\,P_{h\perp}}{z}\right) \underbrace{f_{1\,q/p}\left(x,b,\mu,\zeta_1\right)}_{\text{TMD PDF}} \underbrace{D_{1\,h/q}\left(z,b,\mu,\zeta_2\right)}_{\text{TMD FF}}$$

$$F_{\rm UT}^{\sin\phi_h-\phi_s}\left(x,z,\mathbf{P}_{h\perp}\right) = \underbrace{H_{\rm DIS}\left(Q,\mu\right)}_{\text{Hard}} \sum_q e_q^2 \int \frac{b^2db}{2\pi} J_1\left(\frac{b\,P_{h\perp}}{z}\right) \underbrace{f_{1T\,q/p}^{\perp}\left(x,b,\mu,\zeta_1\right)}_{\text{Sivers function}} \underbrace{D_{1\,h/q}\left(z,b,\mu,\zeta_2\right)}_{\text{TMD FF}}$$

TMDs can be matched onto the collinear distributions

$$f_{1\,q/p}\left(x,b,\mu,\zeta\right) = \left[C\otimes f\right]\left(x,b,\mu_i,\zeta_i\right) U\left(\mu_i,\mu;\zeta\right)\, Z\left(b,\zeta_i,\zeta;\mu_i\right) U_{\rm NP}^{f}\left(x,b,\zeta\right)$$

$$D_{1\,h/q}\left(z,b,\mu,\zeta\right) = \frac{1}{z^2}\left[\hat{C}\otimes D\right]\left(z,b,\mu_i,\zeta_i\right) U\left(\mu_i,\mu;\zeta\right)\, Z\left(b,\zeta_i,\zeta;\mu_i\right) U_{\rm NP}^{D}\left(z,b,\zeta\right)\,.$$

Perturbative: C, $\hat{C}$, $U\left(\mu_i,\mu;\zeta\right)\, Z\left(b,\zeta_i,\zeta;\mu_i\right)$ — *Non-perturbative*: f, D, $U_{\rm NP}$

Large logarithms are are resumed to all orders in the perturbative Sudakov

$$U\left(\mu_i,\mu;\zeta\right) = \exp\left[\int_{\mu_i}^{\mu} \frac{d\mu'}{\mu'}\gamma_\mu\left(\mu',\zeta\right)\right],\; Z\left(b,\mu_i,\mu;\zeta\right) = \left(\frac{\zeta}{\zeta_i}\right)^{\gamma_\zeta\left(b,\mu_i\right)}$$

Matching of the Sivers function

Differential cross section for Semi-Inclusive DIS is given by

$$F_{\mathrm{UT}}^{\sin\phi_h-\phi_s}\left(x,z,\mathbf{P}_{h\perp}\right)=\underbrace{H_{\mathrm{DIS}}\left(Q,\mu\right)}_{\text{Hard}}\sum_q e_q^2\int\frac{b^2 db}{2\pi}J_1\left(\frac{b\,P_{h\perp}}{z}\right)\underbrace{f_{1T\,q/p}^{\perp}\left(x,b,\mu,\zeta_1\right)}_{\text{Sivers function}}\underbrace{D_{1\,h/q}\left(z,b,\mu,\zeta_2\right)}_{\text{TMD FF}}$$

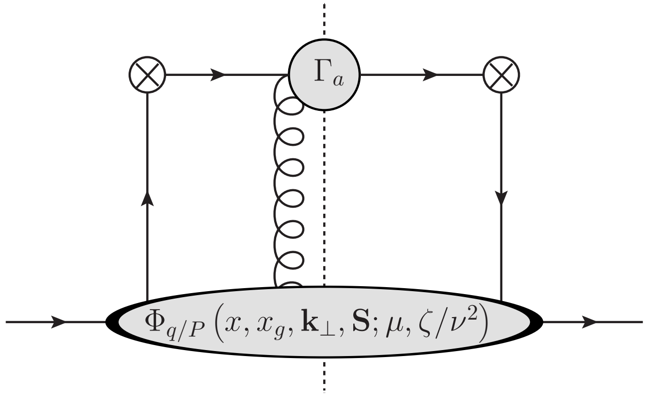
TMDs can be matched onto the collinear distributions

$$f_{1T,q/p}^{\perp}(x,b,\mu,\zeta)=\left[\bar{C}\otimes T_F\right]_{q/p}\left(x,b,\mu_i,\zeta_i\right)\underbrace{U\left(\mu_i,\mu;\zeta\right)\;Z\left(b,\zeta_i,\zeta;\mu_i\right)}_{\text{Same as the unpolarized}}\underbrace{U_{\mathrm{NP}}^{f_{1T}^{\perp}}\left(x,b,\zeta\right)}_{\text{Transverse momentum dependence}}$$

Collinear dependence

Same as the unpolarized

Transverse momentum dependence



$$T_{F\,q/p}(x,x,\mu_0)=\mathcal{N}_q(x)f_{q/p}(x,\mu_0)$$

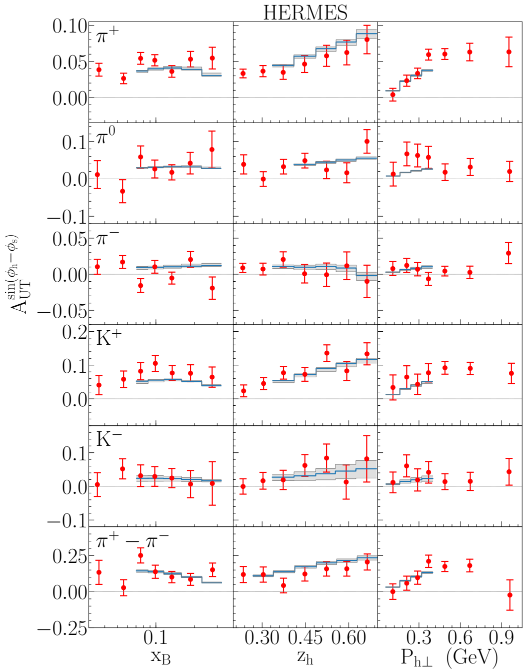
$$\mathcal{N}_q(x)=N_q\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}x^{\alpha_q}(1-x)^{\beta_q}$$

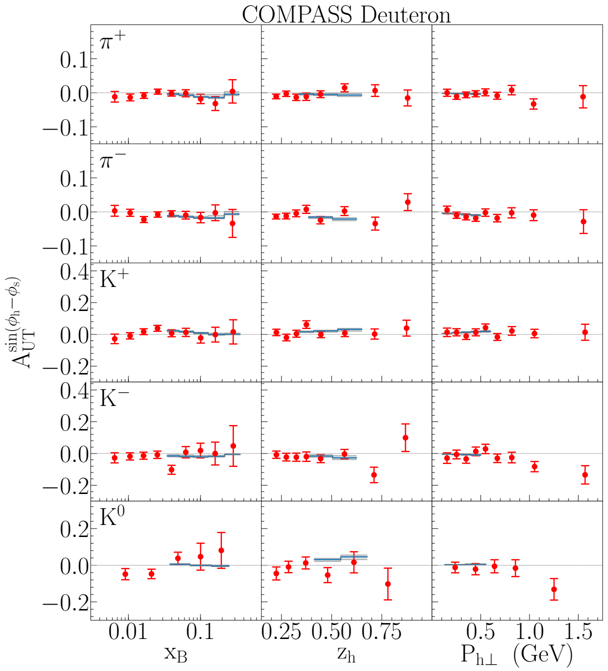
$$U_{\mathrm{NP}}^{f_{1T}^{\perp}}\left(x,b,\zeta\right)=g_1^{f_{1T}^{\perp}}b^2+\frac{g_2}{4}\ln\left(\frac{\zeta}{\zeta_0}\right)\ln\left(\frac{b}{b_*}\right)$$

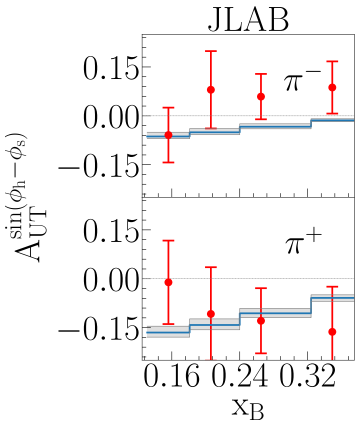
11 params in total

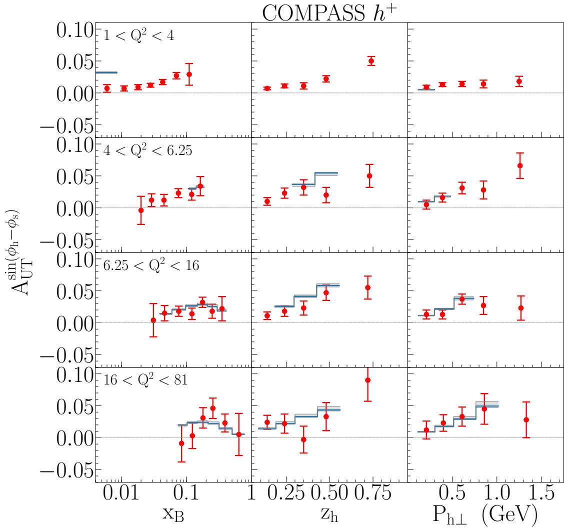
Data description

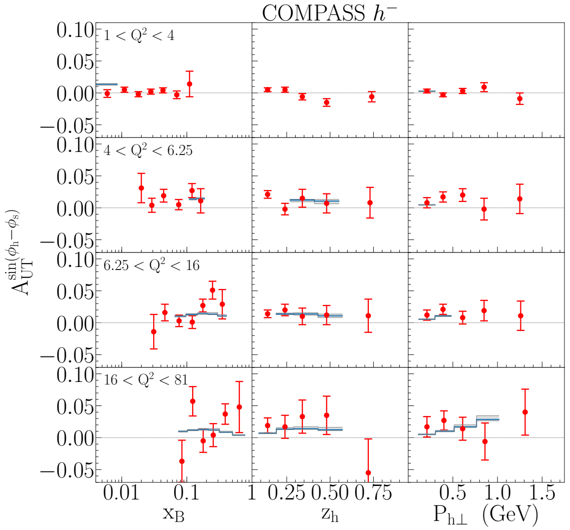
Collab	Ref	Process	Q_{avg}	N_{data}	χ^2/N_{data}
COMPASS	[44]	$ld \to lK^0X$	2.52	7	0.770
		$ld \to lK^-X$	2.80	11	1.325
		$ld \to lK^+X$	1.73	13	0.749
		$ld \to l\pi^-X$	2.50	11	0.719
		$ld \to l\pi^+X$	1.69	12	0.578
	[43]	$lp \to lh^-X$	4.02	31	1.055
		$lp \to lh^+X$	3.93	34	0.898
	[46]	$\pi^-p \to \gamma^*X$	5.34	15	0.658
HERMES	[41]	$lp \to lK^-X$	1.70	14	0.376
		$lp \to lK^+X$	1.73	14	1.339
		$lp \to l\pi^0X$	1.76	13	0.997
		$lp \to l(\pi^+ - \pi^-)X$	1.73	15	1.252
		$lp \to l\pi^-X$	1.67	14	1.498
		$lp \to l\pi^+X$	1.69	14	1.697
JLAB	[45]	$lN \to l\pi^+X$	1.41	4	0.508
		$lN \to l\pi^-X$	1.69	4	1.048
RHIC	[47]	$pp \to W^+X$	M_W	8	2.189
		$pp \to W^-X$	M_W	8	1.684
		$pp \to Z^0X$	M_Z	1	3.270
Total				226	0.989

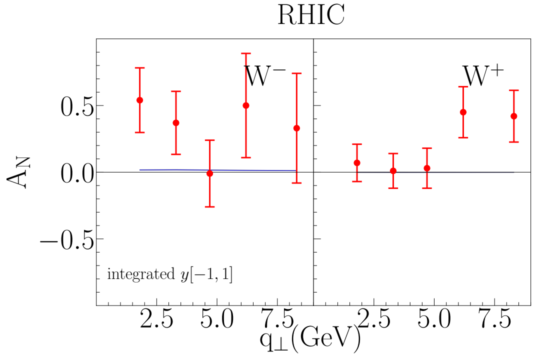






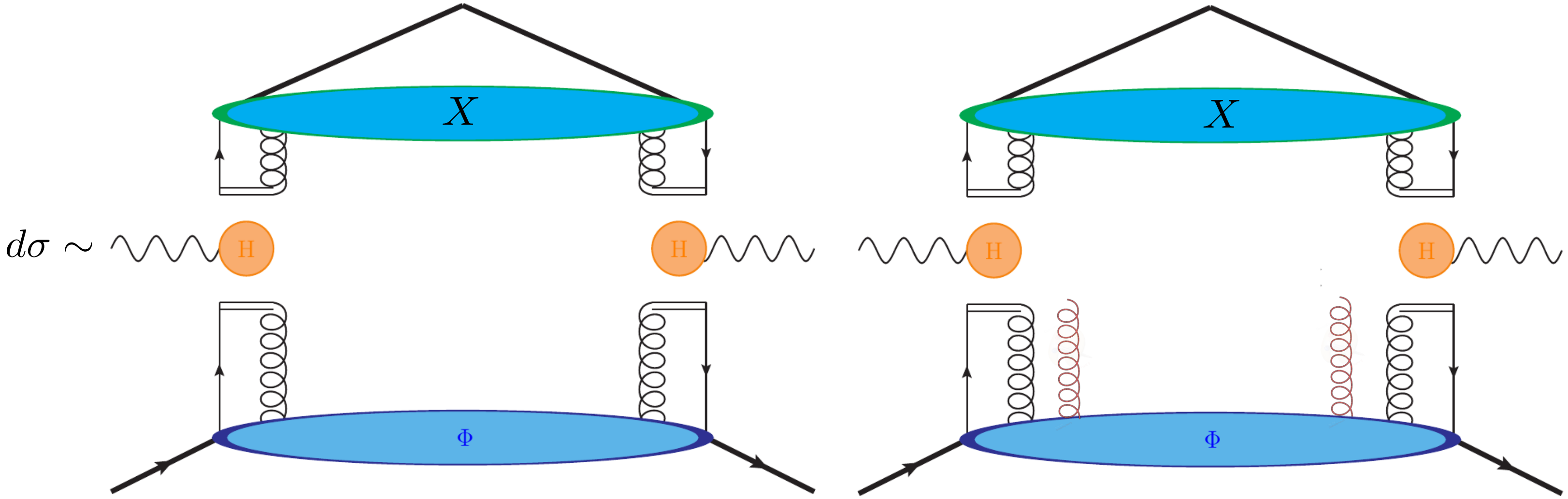






Nuclear modifications to collinear PDFs

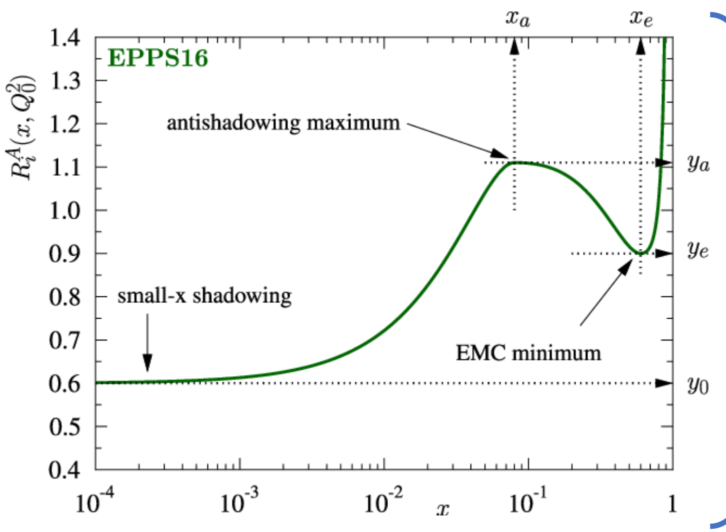
Nuclear medium modification via higher twist



Power suppressed by $\frac{m^2}{Q^2}$

Nuclear enhancement $A^{\#}$

LP TMD factorization cannot address how multiple partons are correlated with one another

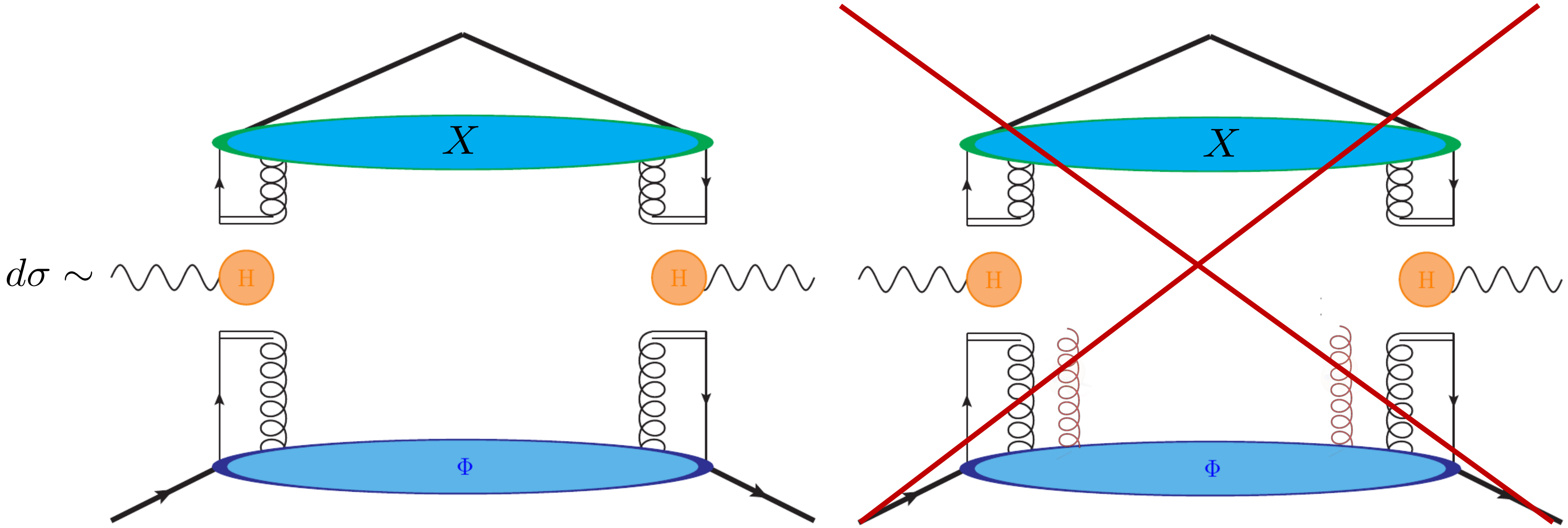


Eskola, Kolhinen, Ruuskanen (1998)

Eskola, Paakkinen, Paukkunen, Salgado (2017)

Method of treating nuclear modifications

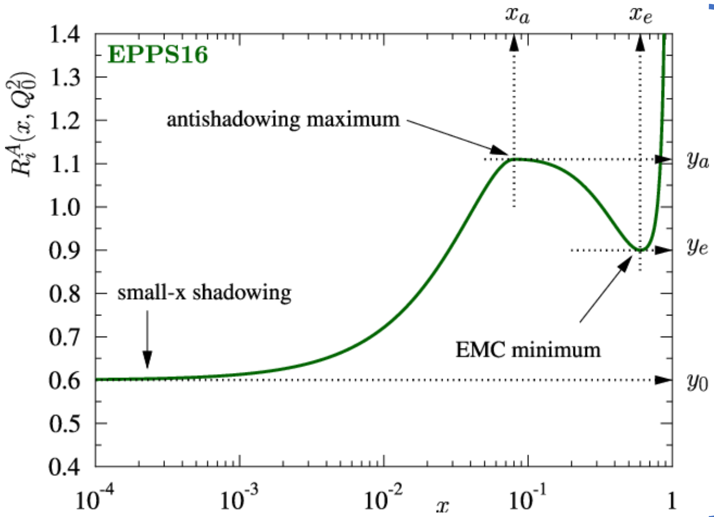
Nuclear medium modification via higher twist



LP TMD factorization cannot address how multiple partons are correlated with one another

$$R_i^A(x,Q) = \frac{f_{i/p}^A(x;Q)}{f_{i/p}(x;Q)}$$

$$R_i^A(x,Q_0^2) = \begin{cases} a_0 + a_1(x-x_a)^2 & x \le x_a \\ b_0 + b_1 x^\alpha + b_2 x^{2\alpha} + b_3 x^{3\alpha} & x_a \le x \le x_e \\ c_0 + (c_1 - c_2 x)(1-x)^{-\beta} & x_e \le x \le 1, \end{cases}$$



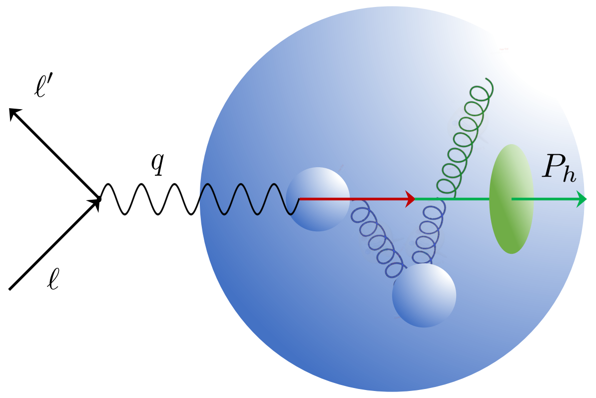
Eskola, Kolhinen, Ruuskanen (1998)

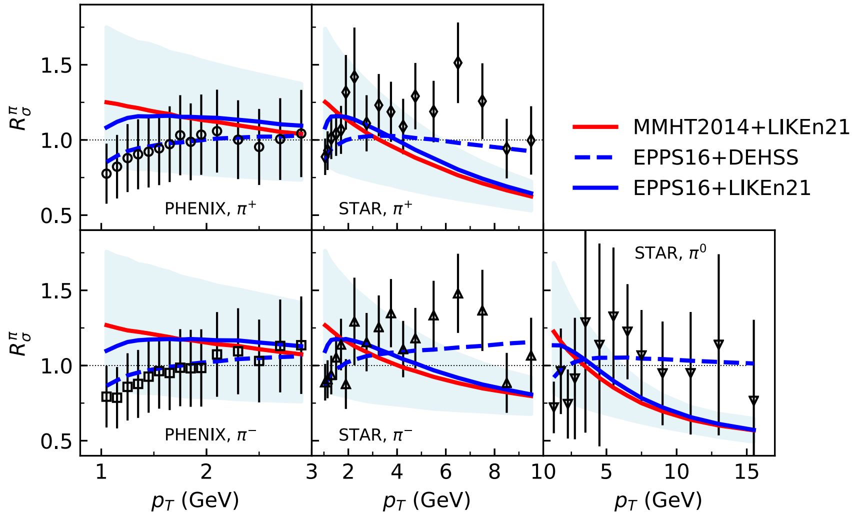
Eskola, Paakkinen, Paukkunen, Salgado (2017)

Nuclear modifications are absorbed into the non-perturbative parameterization.

Effective treatment of medium modifications

Ejected quark undergoes multiple scattering in the nuclear medium, modifies the fragmentation functions





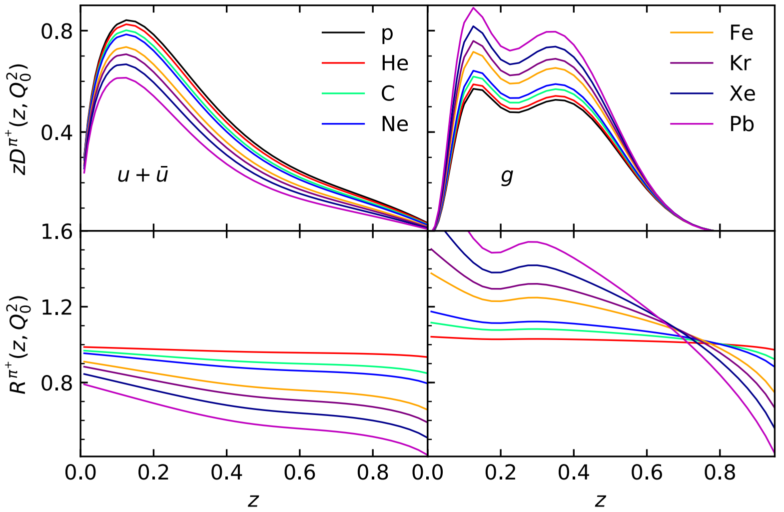
D. de Florian and R. Sassot (2004)
Zurita (2021)

Simultaneous extraction from hadroproduction in p-A collisions from PHENIX and STAR, and Semi-Inclusive DIS (collinear) from HERMES

$$D_i^h(z, Q_0) = \tilde{N}_i z^{\alpha_i}(1-z)^{\beta_i}\left[1 + \gamma_i(1-z)^{\delta_i}\right]$$

$$\tilde{N}_i \;\rightarrow\; \tilde{N}_i\left[1 + N_{i,1}(1 - A^{N_{i,2}})\right]$$

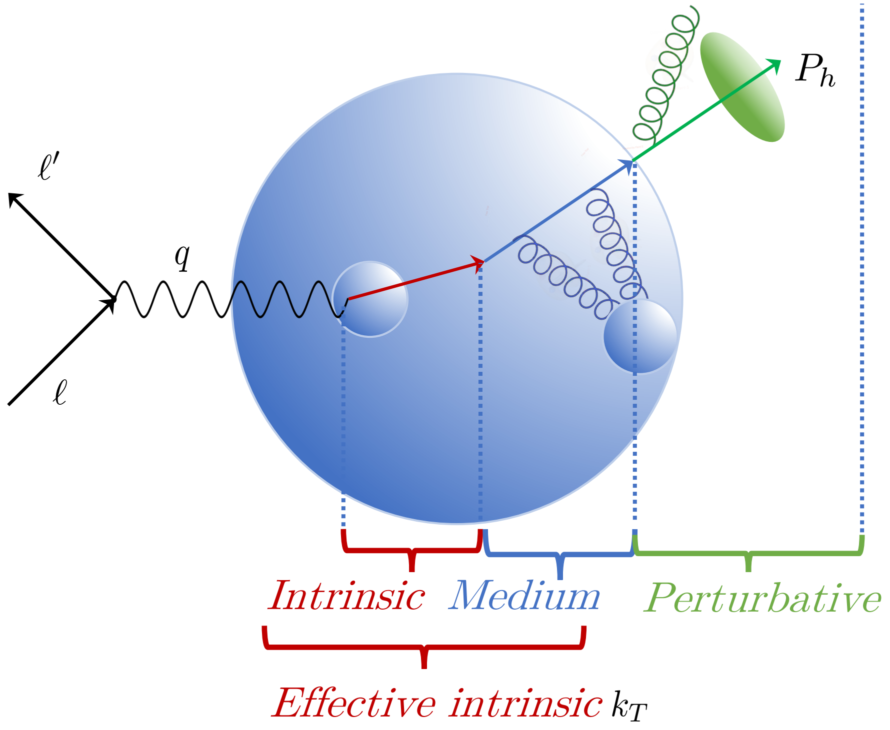
$$c_i \;\rightarrow\; c_i + c_{i,1}(1 - A^{c_{i,2}})$$



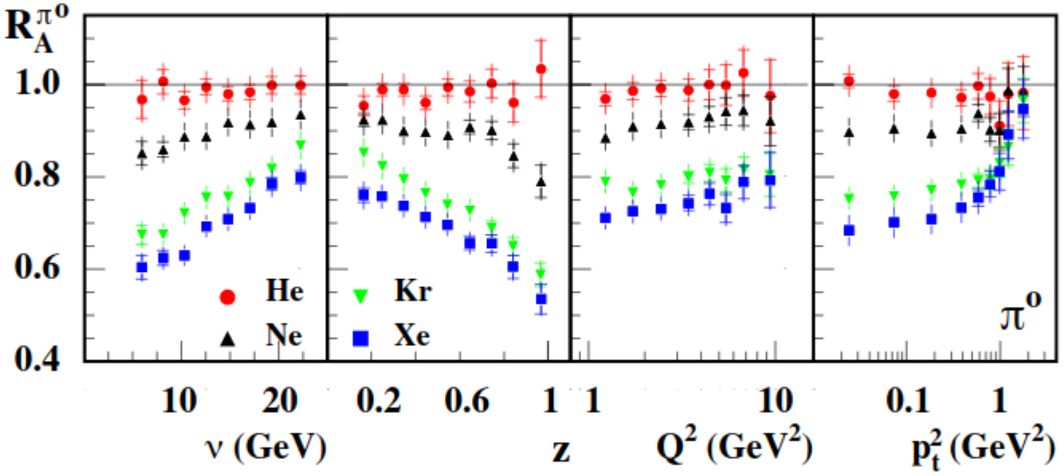
Abelev et al. (STAR) (2010)
Adams et al. (STAR) (2006)
Adare et al. (2013)
Airapetian et al. (HERMES) (2007)

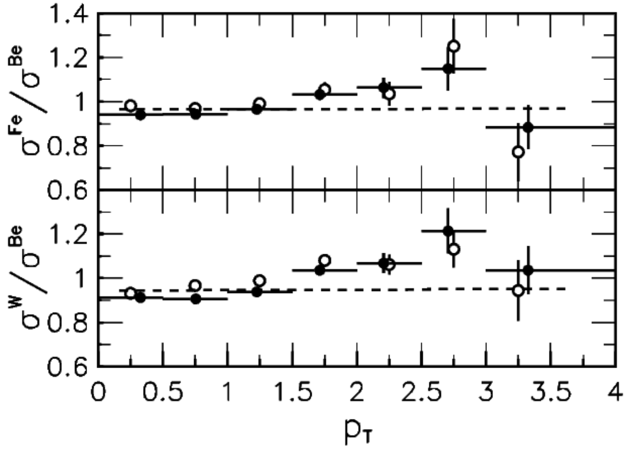
Effective treatment of the transverse momentum broadening

Interaction between partons and interact with the nuclear medium via Glauber exchange



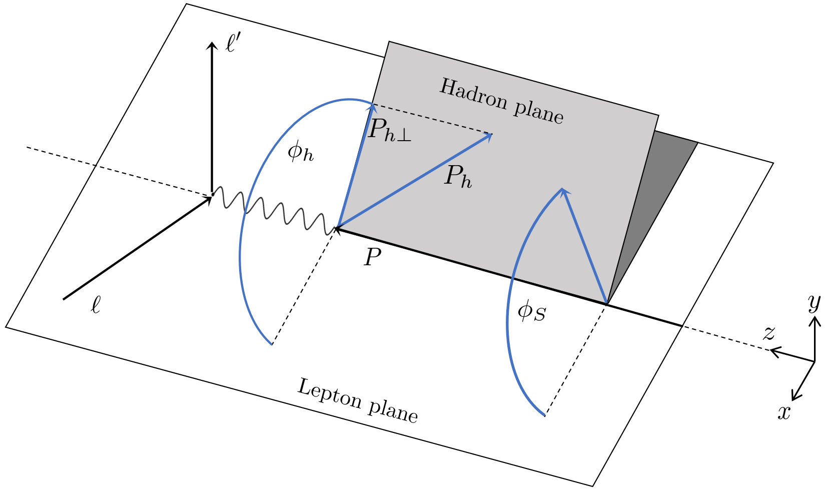
$\sigma_{\text{Heavy}}/\sigma_{\text{Light}}$





Available data

Semi-Inclusive DIS for e-A collisions



Multiplicity ratio

$$R_A^h = \frac{d\sigma_A^h/\mathcal{PS}\, d^2P_{h\perp}}{d\sigma_A/\mathcal{PS}} \frac{d\sigma_D/\mathcal{PS}}{d\sigma_D^h/\mathcal{PS}\, d^2P_{h\perp}}$$

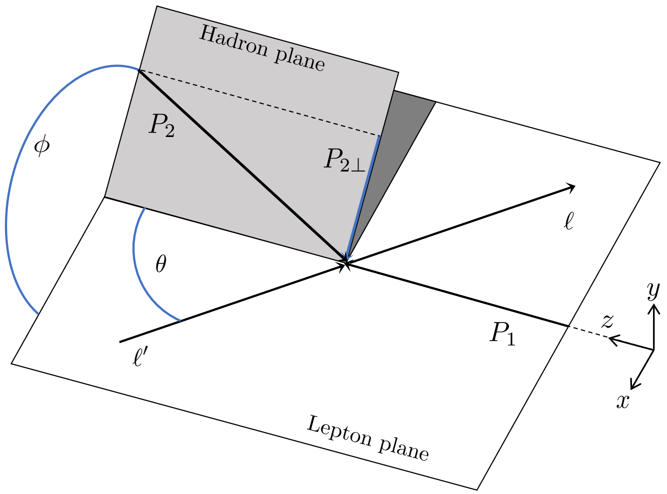
SIDIS cross section $\dfrac{d\sigma_A^h}{\mathcal{PS}\, d^2P_{h\perp}}$

DIS cross section $\dfrac{d\sigma_A}{\mathcal{PS}}$

HERMES ratio for A = He, Ne, Kr, Xe
h = $\pi^+, \pi^-, \pi^0, K^+, K^-, K^0$

Jefferson Lab ratio for A = C, Fe, Pb
h = π^+, π^-

Drell-Yan production in p-A collusions

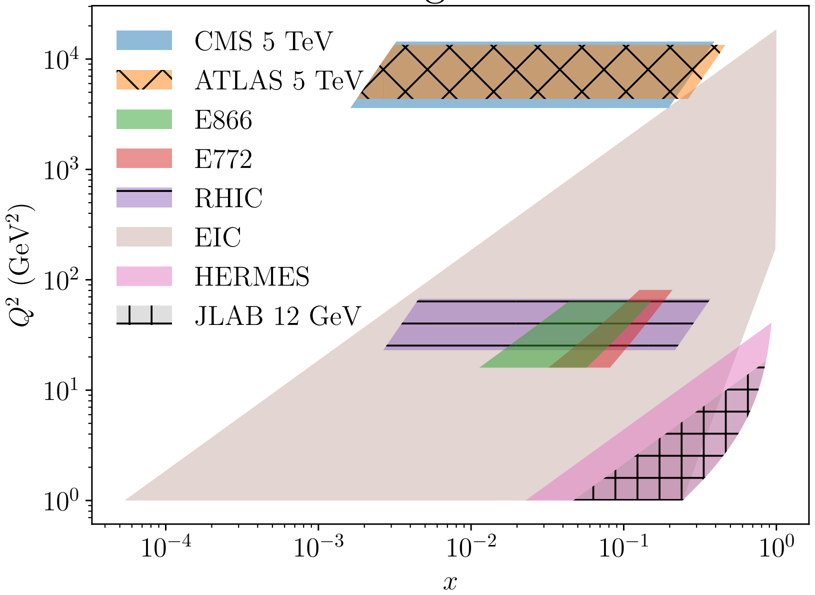


Cross section and cross section ratio for p-A collisions

$$R_{\rm AB} = \frac{d\sigma_A}{d\mathcal{PS}\, d^2q_\perp} \frac{d\mathcal{PS}\, d^2q_\perp}{d\sigma_B}$$

E772: A = C; B = D
E866: A = Fe, W; B = Be
RHIC: A = Au; B = p
ATLAS, CMS: $q_\perp$ distribution p-Pb

Kinematic coverage of the data



Airapetian et al. (HERMES), Nucl. Phys. B 780, 1 (2007)

Dudek et al., Eur. Phys. J. A 48, 187 (2012)

Burkert, in CLAS 12 RICH Detector Workshop (2008)

Alde et al., Phys. Rev. Lett. 64, 2479 (1990)

Vasilev et al. (NuSea), Phys. Rev. Lett. 83, 2304 (1999)

Leung (PHENIX), PoS HardProbes2018, 160 (2018)

Khachatryan et al. (CMS), Phys. Lett. B 759, 36 (2016)

Aad et al. (ATLAS), Phys. Rev. C 92, 044915 (2015)

Available perturbative accuracy

Anomalous dimensions

$$\mu \frac{d}{d\mu} \ln F(Q,\mu,\nu) = \gamma_F^q(Q,\mu,\nu) \qquad \mu \frac{d}{d\nu} \ln G(Q,\mu,\nu) = \gamma_G^q(Q,\mu,\nu)$$

$$F \in \{H, f, D, S\} \qquad G \in \{f, D, S\}$$

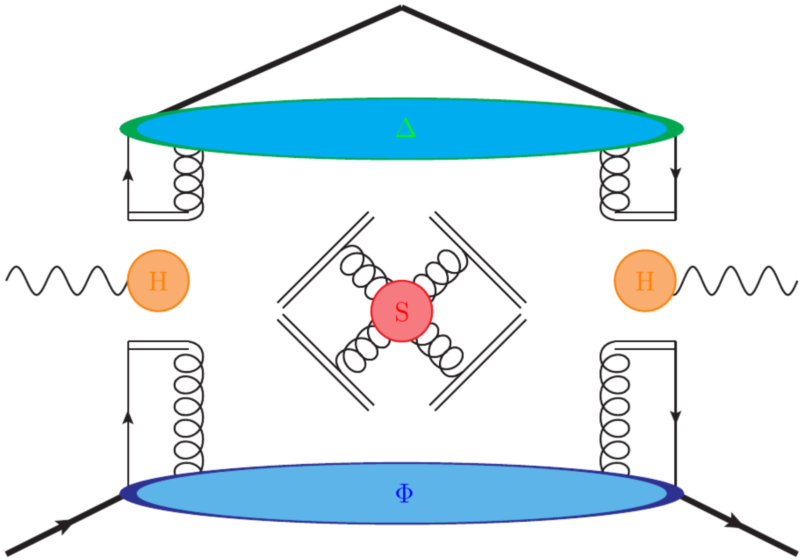
Anomalous dimensions are almost known up to N^{4}LL at this point (no 5-loop cusp)

Accuracy	H, $\mathcal{J}$	$\Gamma_{\rm cusp}(\alpha_s)$	$\gamma_H^q(\alpha_s)$	$\gamma_r^q(\alpha_s)$	$\beta(\alpha_s)$
LL	Tree level	1-loop			1-loop
NLL	Tree level	2-loop	1-loop	1-loop	2-loop
NLL$'$	1-loop	2-loop	1-loop	1-loop	2-loop
NNLL	1-loop	3-loop	2-loop	2-loop	3-loop
NNLL$'$	2-loop	3-loop	2-loop	2-loop	3-loop
N^{3}LL	2-loop	4-loop	3-loop	3-loop	4-loop
N^{3}LL$'$	3-loop	4-loop	3-loop	3-loop	4-loop
N^{4}LL	3-loop	5-loop	4-loop	4-loop	5-loop
N^{4}LL$'$	4-loop	5-loop	4-loop	4-loop	5-loop

Lee, Smirnov, and Smirnov (2010)
Gehrmann, Glover, Huber, Ikizlerli, and Studerus (2010)
Ebert, Mistlberger, Vita (2020)
Ebert, Mistlberger, Vita (2020)
Agarwal, von Manteuffel, Panzer, and Schabinger (2021)
Duhr, Mistlberger, Vita (2022)
Moult, Zhu, Zhu (2022)
Herzog, Moch, Ruijl, Ueda, Vermaseren, and Vogt (2019)
Baikov, Chetyrkin, and Kuhn (2017)

Factorization and resummation in the medium

Differential cross section for Semi-Inclusive DIS is given by



$$\frac{d\sigma}{d\mathcal{PS}\, d^2P_{h\perp}} = \sigma_0 \underbrace{H(Q;\mu)}_{\text{Hard}} \sum_q e_q^2 \int \frac{bdb}{2\pi} J_0\left(\frac{bP_{h\perp}}{z}\right) \underbrace{f^A_{q/N}(x,b;\mu,\zeta_1)}_{\text{nTMD PDF}} \underbrace{D^A_{h/q}(z,b;\mu,\zeta_2)}_{\text{nTMD FF}}$$

TMDs can be matched onto the collinear distributions

$$f_{1\,q/A}(x,b,\mu,\zeta) = [C \otimes f]_{q/A}(x,b,\mu_i,\zeta_i)\, U(\mu_i,\mu;\zeta)\ Z(b,\zeta_i,\zeta;\mu_i)\, U_{\text{NP}}^{f^A}(x,b,\zeta,A)$$

$$D^A_{1\,h/q}(z,b,\mu,\zeta) = \frac{1}{z^2}\left[\hat{C} \otimes D^A\right]_{h/q}(z,b,\mu_i,\zeta_i)\, U(\mu_i,\mu;\zeta)\ Z(b,\zeta_i,\zeta;\mu_i)\, U_{\text{NP}}^{D^A}(z,b,\zeta,A)$$

Perturbative *Non-perturbative*

Large logarithms are are resumed to all orders in the perturbative Sudakov

$$U(\mu_i,\mu;\zeta) = \exp\left[\int_{\mu_i}^{\mu} \frac{d\mu'}{\mu'} \gamma_\mu(\mu',\zeta)\right],\ Z(b,\mu_i,\mu;\zeta) = \left(\frac{\zeta}{\zeta_i}\right)^{\gamma_\zeta(b,\mu_i)}$$

Non-perturbative treatment

Non-perturbative contributions given by

$$f_{1\,q/A}(x,b,\mu,\zeta) = [C \otimes f]_{q/A}(x,b,\mu_i,\zeta_i)\, U(\mu_i,\mu;\zeta)\; Z(b,\zeta_i,\zeta;\mu_i)\, U_{\rm NP}^{f^A}(x,b,\zeta,A)$$

$$D^{A}_{1\,h/q}(z,b,\mu,\zeta) = \frac{1}{z^2}\left[\hat{C} \otimes D^A\right]_{h/q}(z,b,\mu_i,\zeta_i)\, U(\mu_i,\mu;\zeta)\; Z(b,\zeta_i,\zeta;\mu_i)\, U_{\rm NP}^{D^A}(z,b,\zeta,A)$$

EPPS21   In house FF (new), previous analysis used LIKEn

Non-perturbative Sudakov given by

$$U_{\rm NP}^{f^A}(x,b,\zeta) = U_{\rm NP}^{f}(x,b,\zeta)\exp\left\{-g_q^A\left(A^{1/3}-1\right)b^2\left(\frac{\zeta_A}{\zeta}\right)^{\Gamma}\right\}$$

$$U_{\rm NP}^{D^A}(x,b,\zeta) = U_{\rm NP}^{D}(x,b,\zeta)\exp\left\{-g_h^A\left(A^{1/3}-1\right)\frac{b^2}{z^2}\left(\frac{\zeta_A}{\zeta}\right)^{\Gamma}\right\}$$

Parameterization for the medium modified fragmentation

$$D_i^{\pi^+}(z,\mu_0) = \frac{N_i z^{\alpha_i}(1-z)^{\beta_i}[1+\gamma_i(1-z)^{\delta_i}]}{B[2+\alpha_i,\beta_i+1]+\gamma_i B[2+\alpha_i,\beta_i+\delta_i+1]}\,.$$

$$\tilde{N}_i \;\to\; \tilde{N}_i\left[1+N_{i,1}(1-A^{N_{i,2}})\right]$$

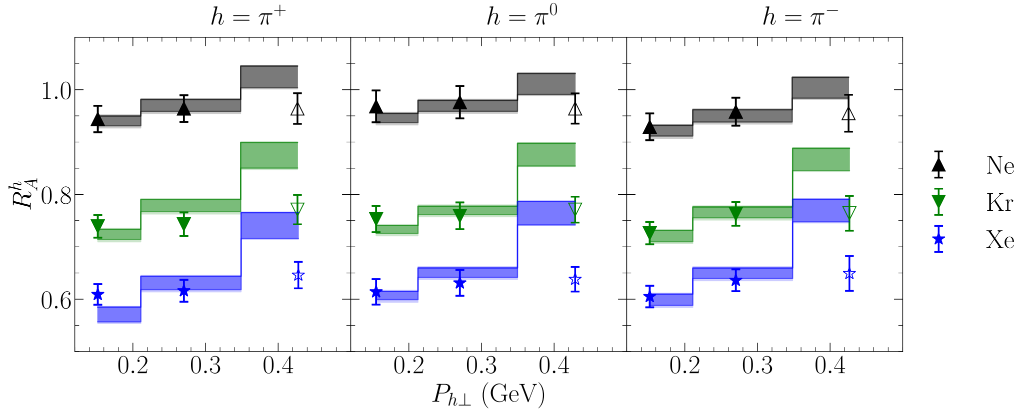
$$c_i \;\to\; c_i + c_{i,1}(1-A^{c_{i,2}})$$

$$\mathbf{p} = \{N_{q1}\,,N_{q2}\,,\gamma_{q1}\,,\gamma_{q2}\,,\delta_{q1}\,,\delta_{q2}\,,g_q^A\,,g_h^A\,,\Gamma\}\;,$$

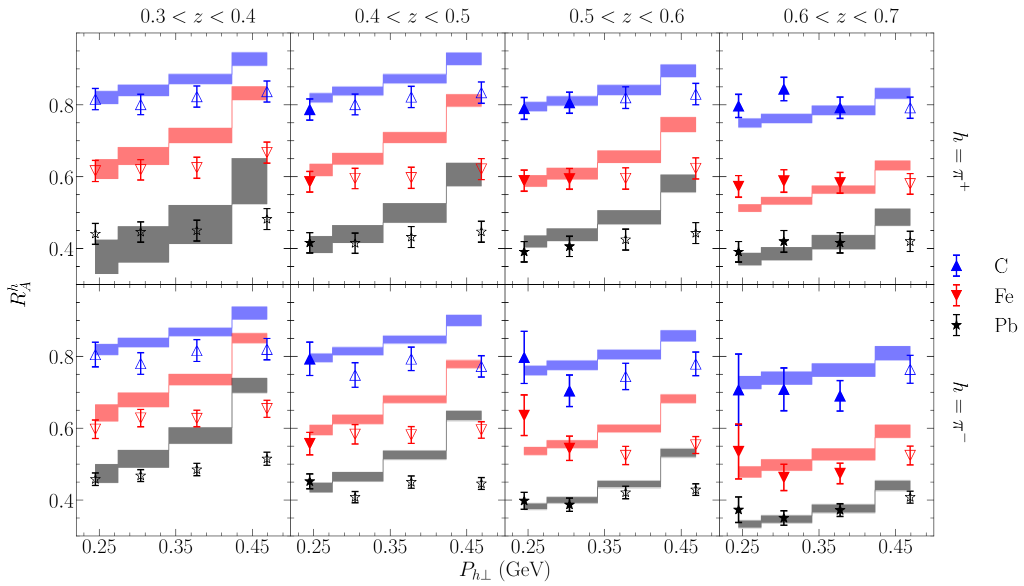
Description of the experimental data

Collaboration	Process	Baseline	Nuclei	N_{data}	χ^2
JLAB [49]	SIDIS(π)	D	C, Fe, Pb	36	41.7
HERMES [40]	SIDIS(π)	D	Ne, Kr, Xe	18	10.2
RHIC [43]	DY	p	Au	4	1.3
E772 [41]	DY	D	C, Fe, W	16	40.2
E866 [42]	DY	Be	Fe, W	28	20.6
CMS [63]	γ^*/Z	N/A	Pb	8	10.4
ATLAS [83]	γ^*/Z	N/A	Pb	7	13.3
Total				117	137.8

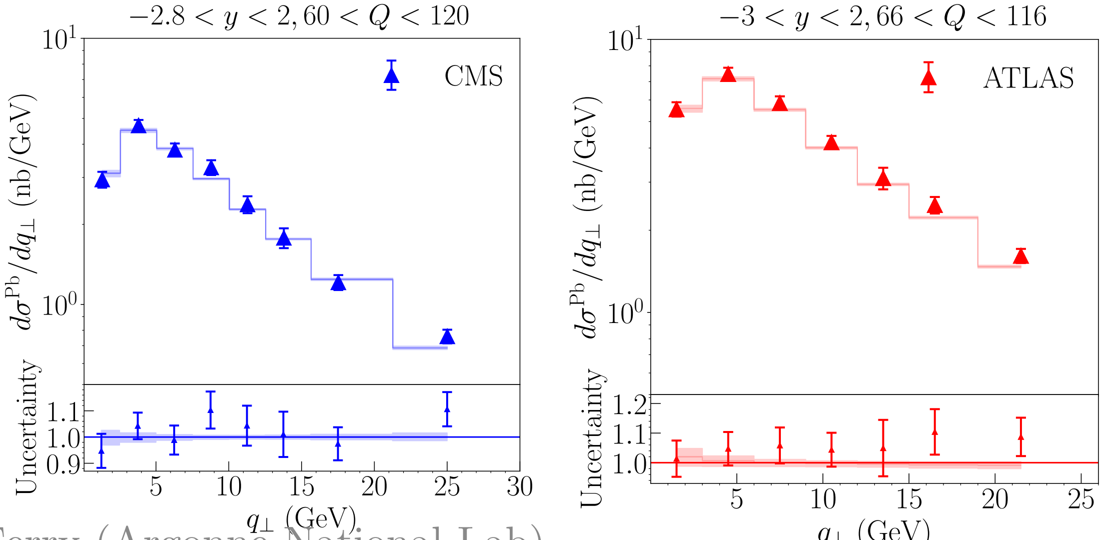
HERMES



JLab



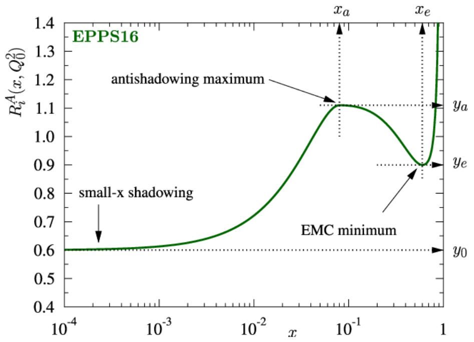
LHC



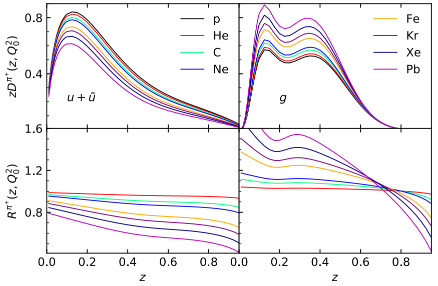
Three-dimensional images

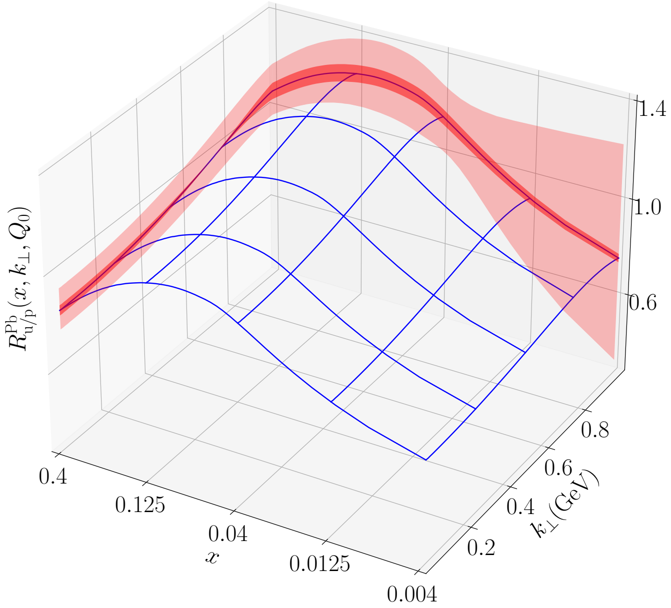
Ratios defined for nPDF and nFF

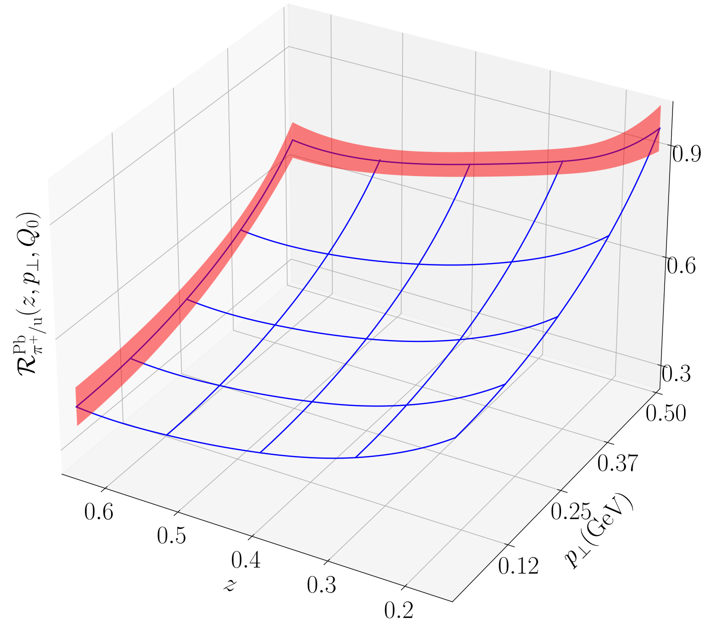
$$R_i^A\left(x, Q_0^2\right) = \frac{f_{i/p}^A\left(x, Q_0^2\right)}{f_{i/p}\left(x, Q_0^2\right)}$$



$$R_i^A\left(z, Q_0^2\right) = \frac{D_{h/i}^A\left(z, Q_0^2\right)}{D_{h/i}\left(z, Q_0^2\right)}$$







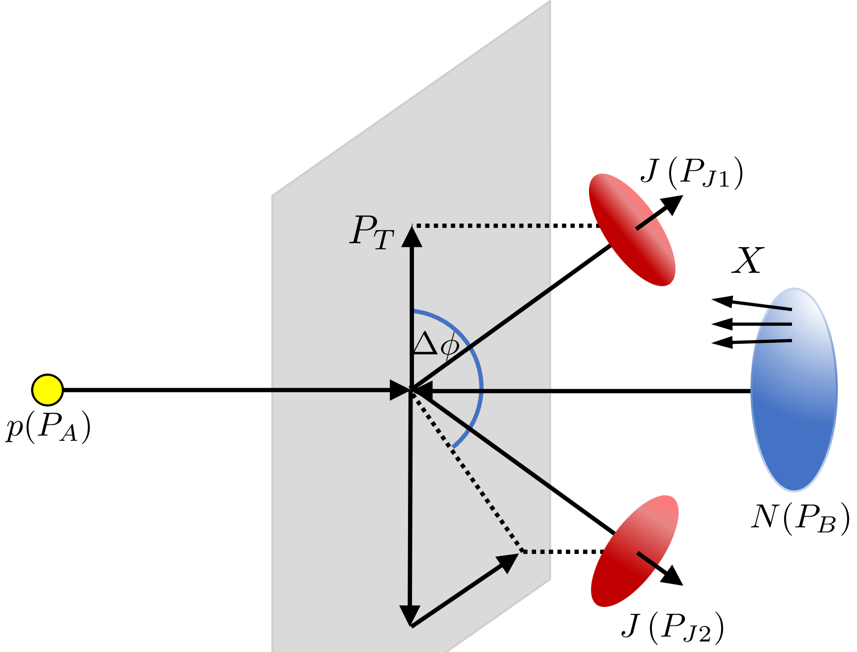
$$R_{u/p}^{Pb}\left(x, k_\perp, Q_0\right) = \frac{f_{u/p}^{Pb}\left(x, k_\perp, Q_0, Q_0^2\right)}{f_{u/p}\left(x, k_\perp, Q_0, Q_0, Q_0^2\right)}$$

$$\mathcal{R}_{\pi^+/u}^{Pb}\left(z, p_\perp, Q_0\right) = \frac{D_{\pi^+/u}^{Pb}\left(z, p_\perp, Q_0, Q_0^2\right)}{D_{\pi^+/u}\left(z, p_\perp, Q_0, Q_0^2\right)}$$

Di-jet decorrelations in pp

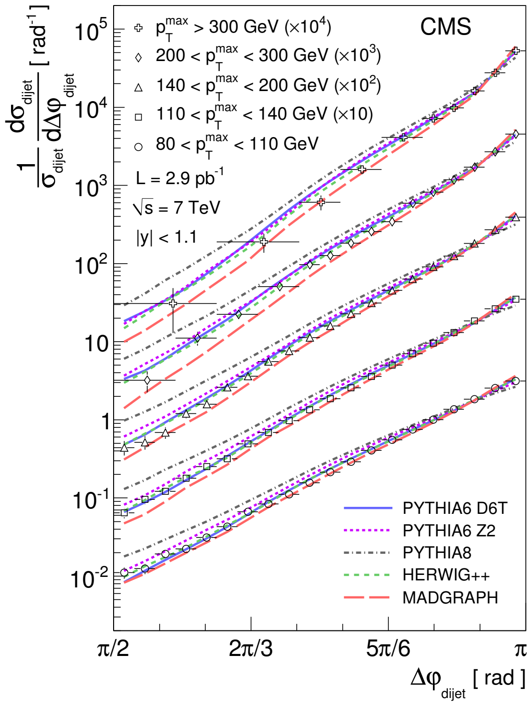
Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs

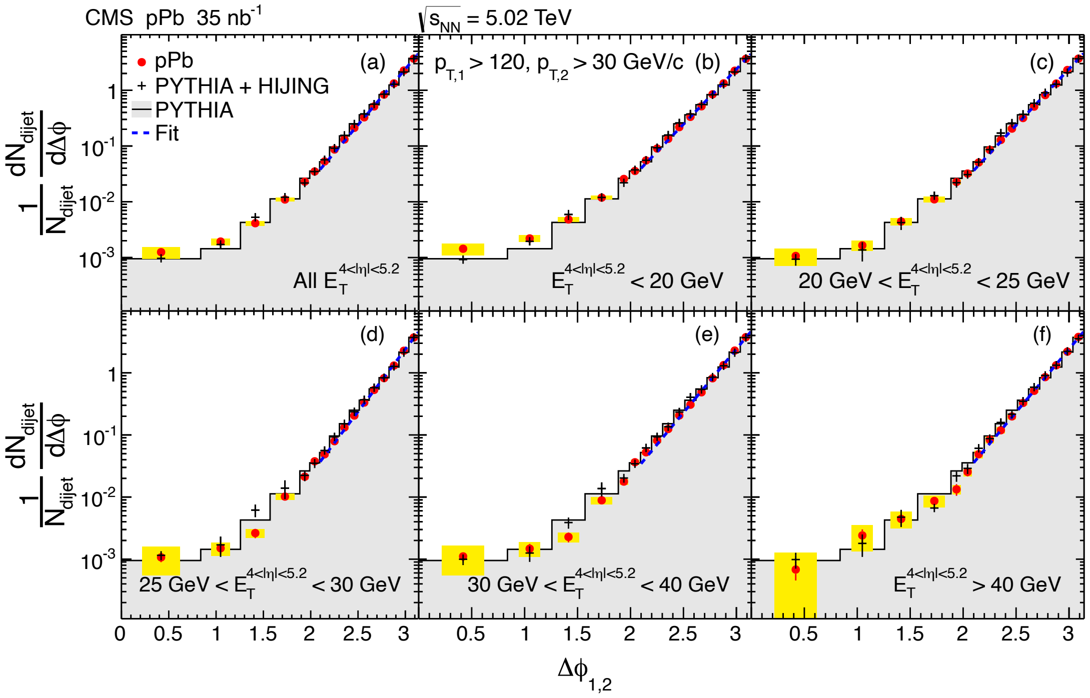
Back-to-back region is sensitive to the 1+1 dimensional TMDs



CMS Measurements in pp and pA collisions

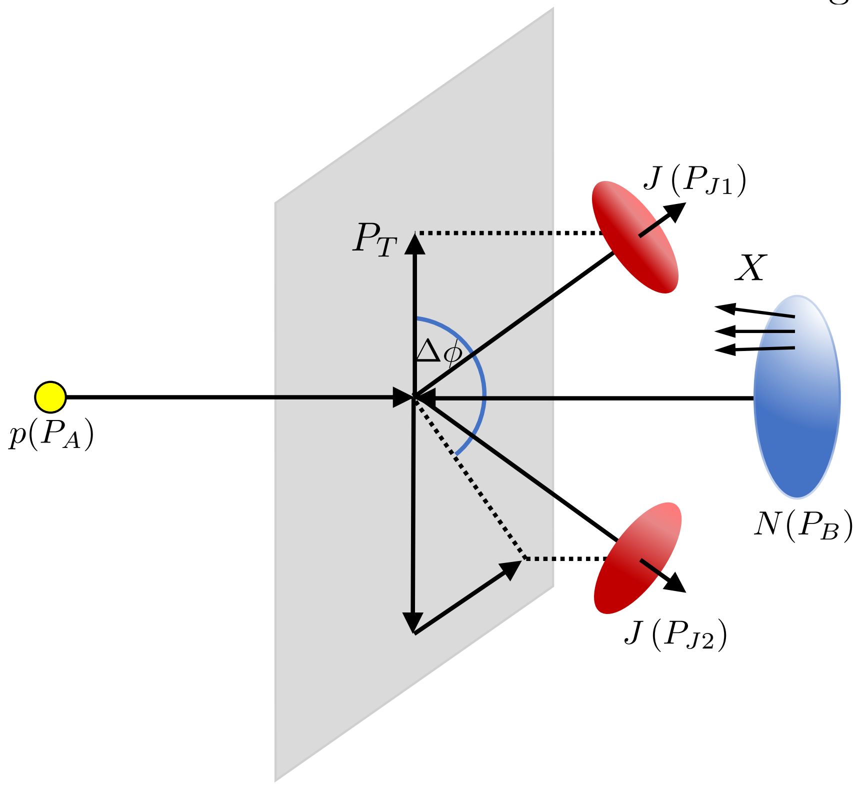
Phys.Rev.Lett.106:122003,2011
Eur. Phys. J. C 74 (2014) 2951
Phys. Rev. Lett. 121, 062002 (2018)

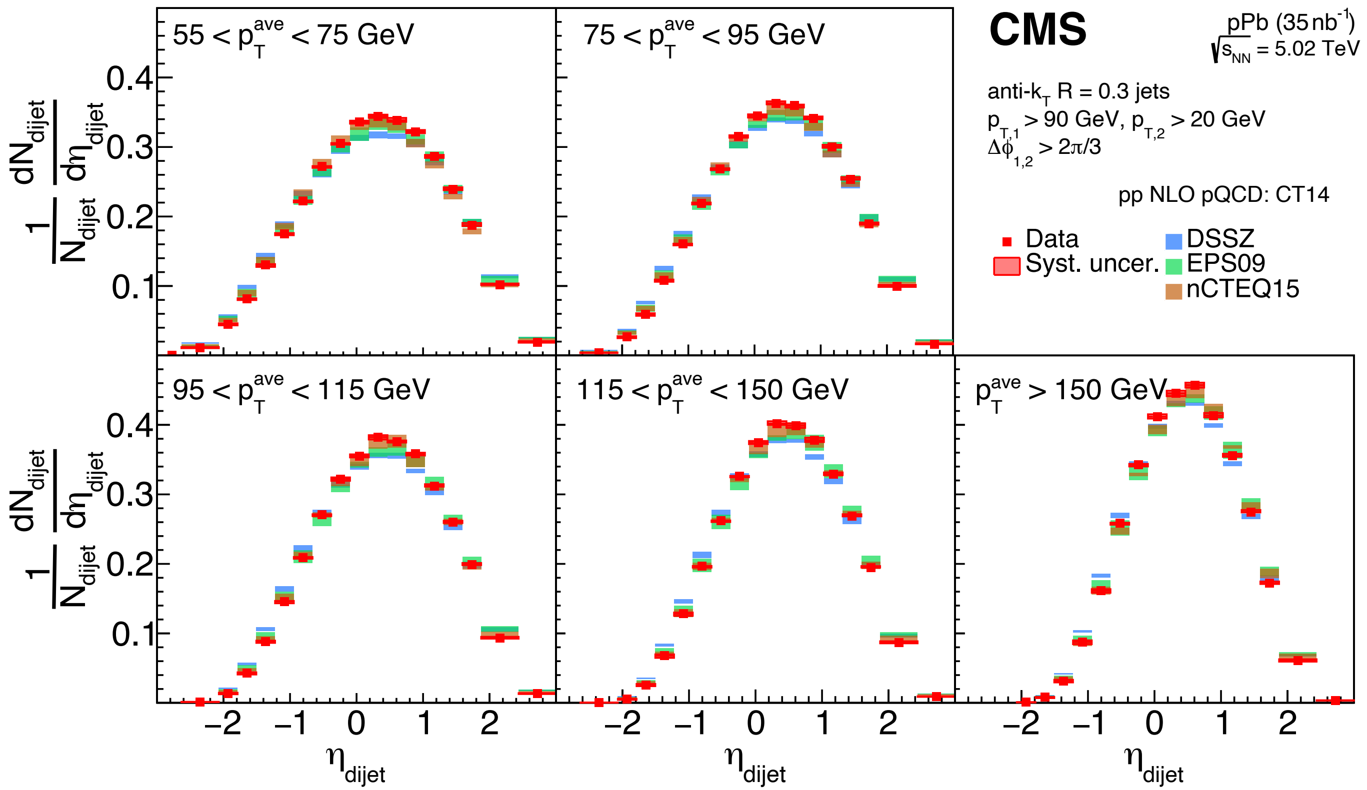




Di-jet decorrelations in pp continued

Additional measurements of the integrated azimuthal angle decorrelation





Integration in region $\Delta\phi > 2\pi/3$ performed using a collinear approximation. However, there are issues with this approach as $\Delta\phi \to \pi$ due to large logarithms.

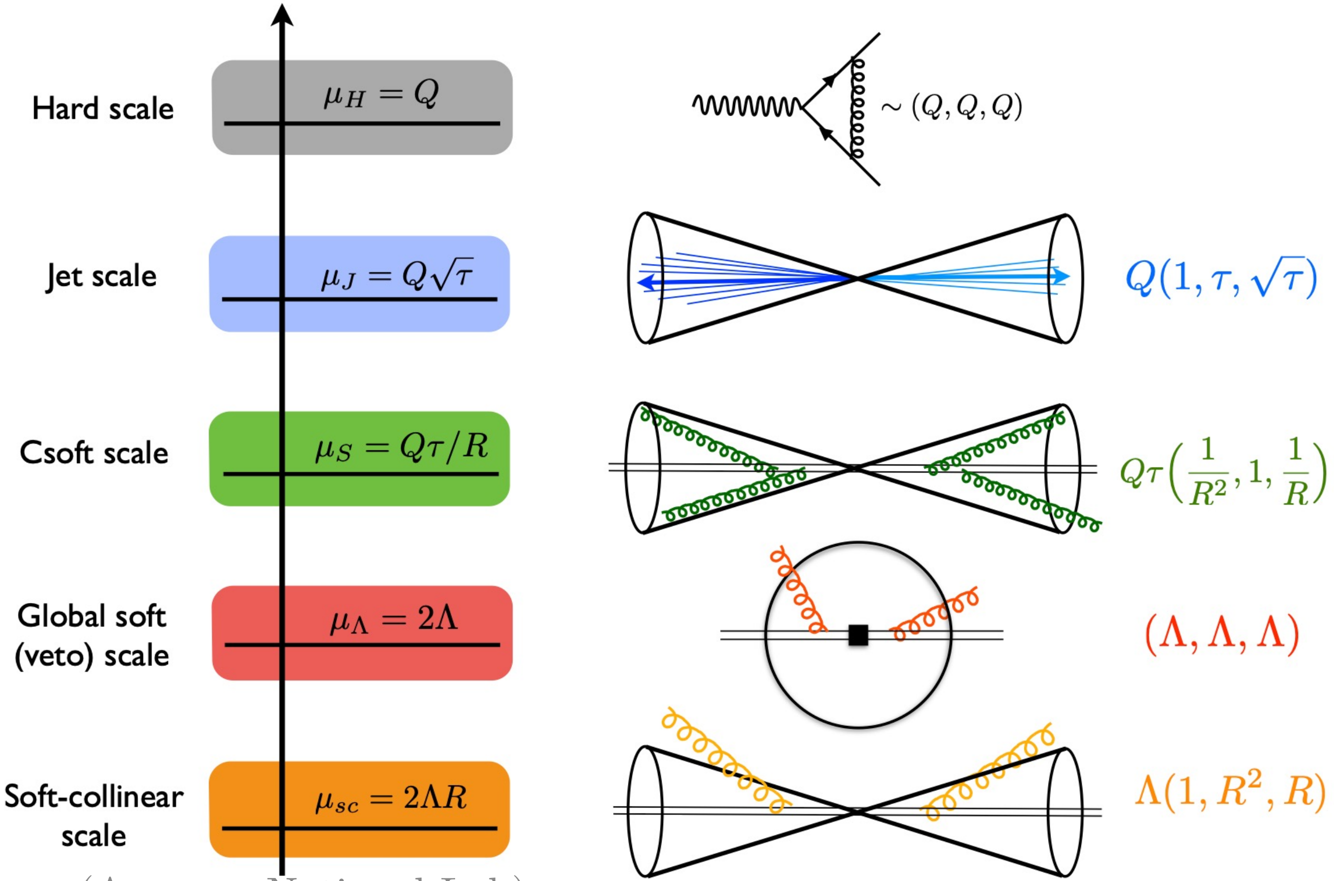
QCD modes in SCET

SCET is an EFT which captures soft and collinear emissions along the directions

$$\mathcal{L}_{\mathrm{QCD}} = \sum_q \bar{\psi}\, i\not{D}\, \psi - \frac{1}{4} G^A_{\mu\nu} G^{A\,\mu\nu} + \mathcal{L}_{\mathrm{gauge-fix}} + \mathcal{L}_{\mathrm{ghost}}$$

$$\psi \to \psi_s + \psi_c \qquad A^\mu \to A^\mu_s + A^\mu_c$$

$$\mathcal{L}_{\mathrm{SCET}} = \bar{\psi}_s\, i\not{D}_s\, \psi_s - \frac{1}{4} G^A_{\mu\nu\,s} G^{A\,\mu\nu}_s + \bar{\xi}\frac{\not{\bar{n}}}{2}\left[in\cdot D + i\not{D}_{c\perp} \frac{1}{i\bar{n}\cdot D_c} i\not{D}_{c\perp}\right]\xi - \frac{1}{4} G^A_{\mu\nu\,c} G^{A\,\mu\nu}_c$$



Bauer, Fleming, Luke 2000

Bauer, Fleming, Pirjol, Stewart 2001

Bauer, Stewart 2001

Bauer, Pirjol, Stewart 2002

Beneke, Chapovsky, Diehl, Feldmann 2002

Beneke, Feldmann 2003

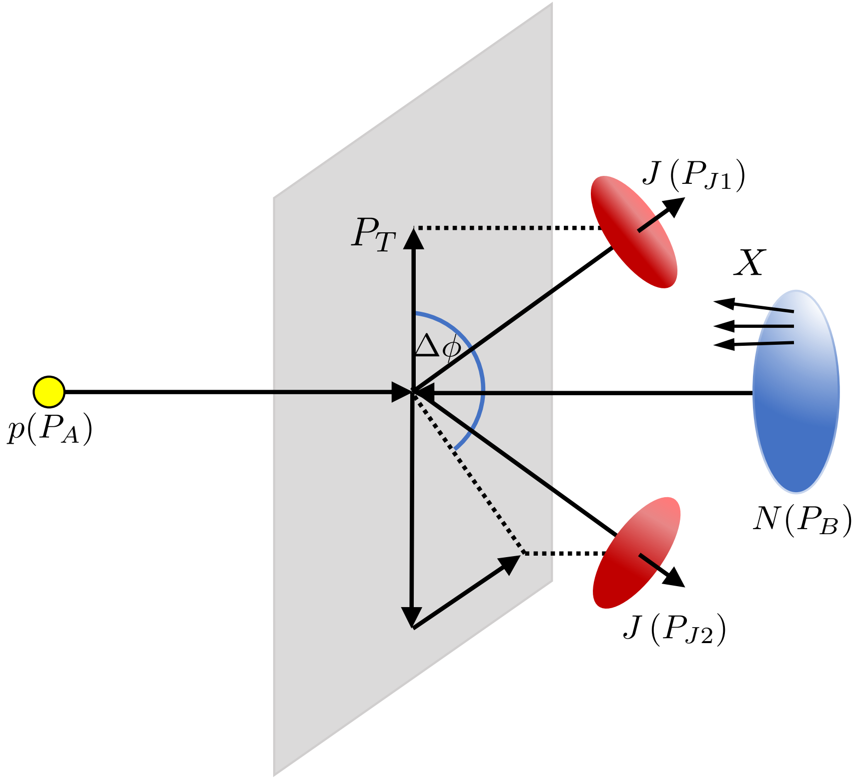
Hill, Neubert 2003

Echevarria, Idilbi, Scimemi 2011

Chien, Hornig, Lee 2015

Factorization in SCET

Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs

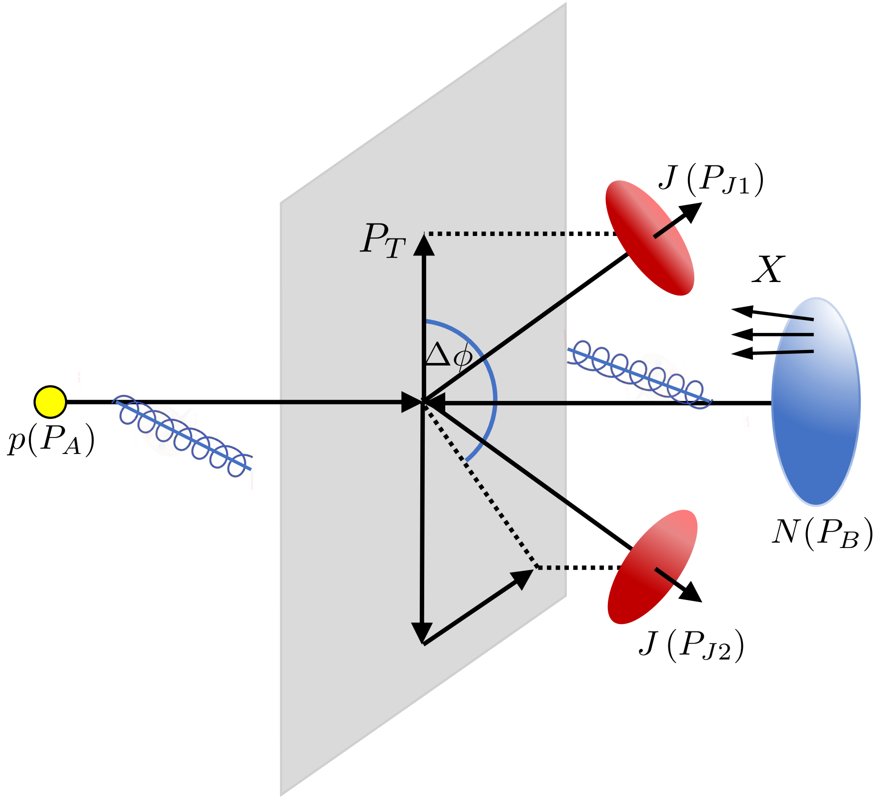


Factorization and resummation derived in a SCET framework

$$\textbf{hard}: \; p_h^\mu \sim p_T(1,1,1)$$

Factorization in SCET

Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs

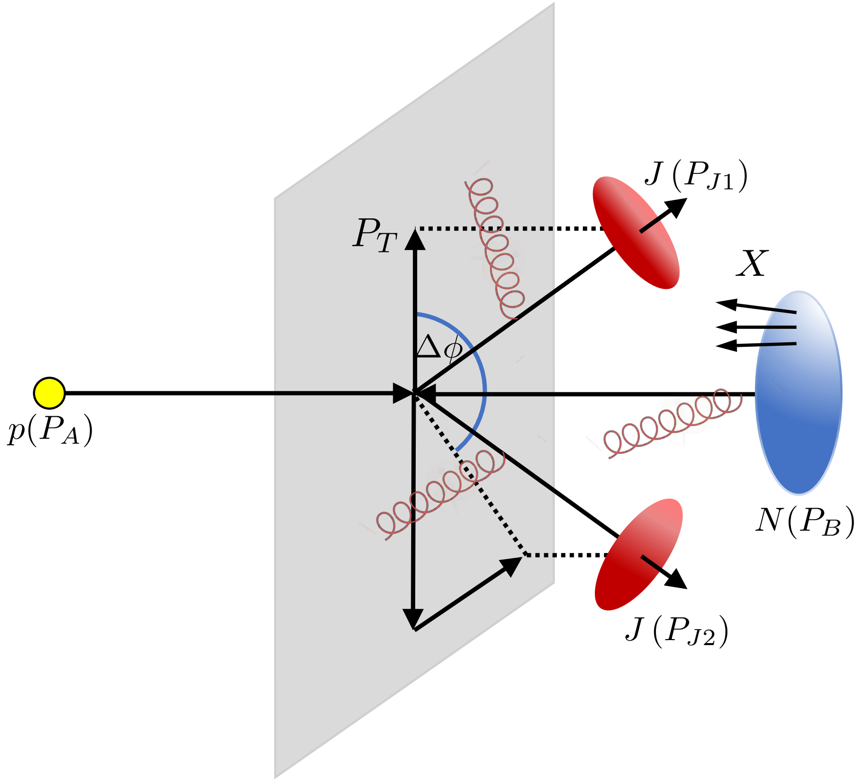


Factorization and resummation derived in a SCET framework

$$\textbf{hard}: \quad p_h^\mu \sim p_T(1,1,1)$$
$$n_{a,b}\textbf{-collinear}: \quad p_{c_i}^\mu \sim p_T\,(\delta\phi^2, 1, \delta\phi)_{n_i\bar{n}_i},$$

Factorization in SCET

Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs

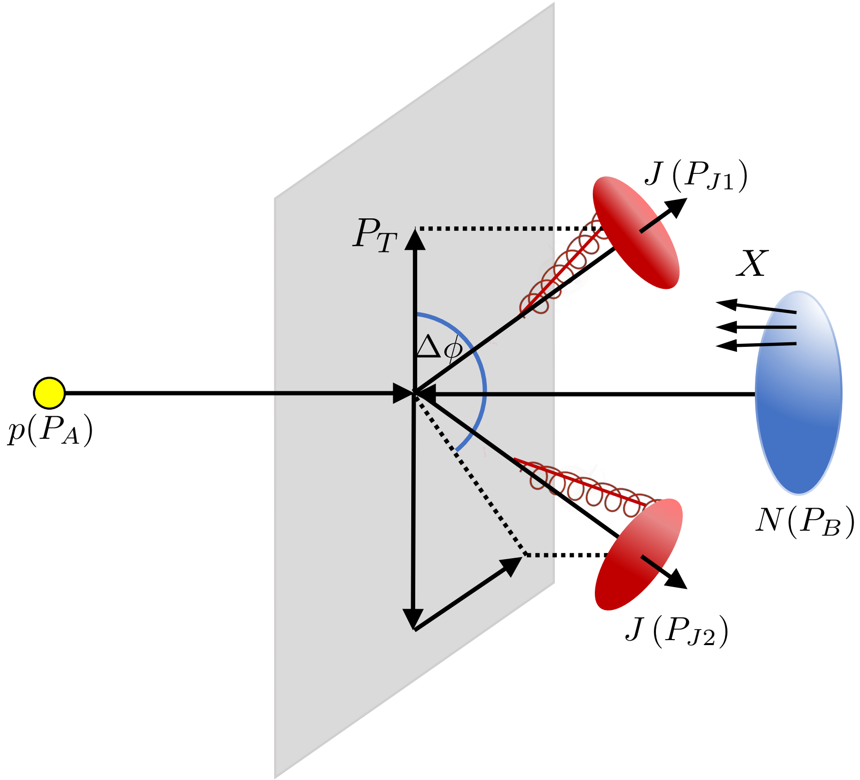


Factorization and resummation derived in a SCET framework

$$\begin{aligned} \textbf{hard}: &\quad p_h^\mu \sim p_T(1,1,1) \\ n_{a,b}\textbf{-collinear}: &\quad p_{c_i}^\mu \sim p_T\,(\delta\phi^2, 1, \delta\phi)_{n_i\bar{n}_i}, \\ \textbf{soft}: &\quad p_s^\mu \sim p_T\,(\delta\phi, \delta\phi, \delta\phi), \end{aligned}$$

Factorization in SCET

Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs

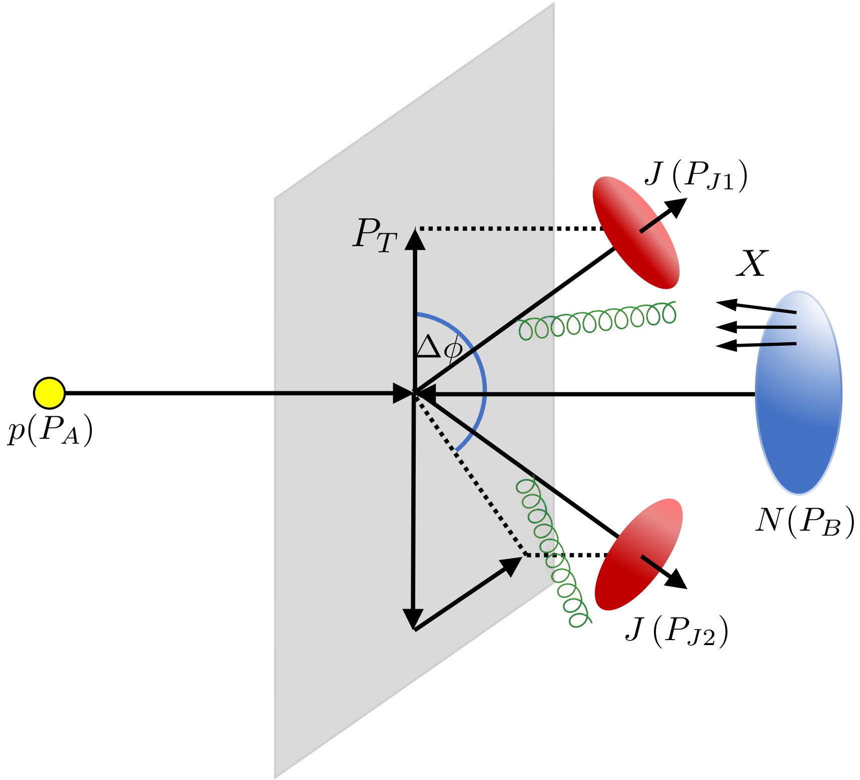


Factorization and resummation derived in a SCET framework

$$\begin{aligned} \textbf{hard}: &\quad p_h^\mu \sim p_T(1,1,1) \\ n_{a,b}\textbf{-collinear}: &\quad p_{c_i}^\mu \sim p_T\,(\delta\phi^2, 1, \delta\phi)_{n_i\bar{n}_i}, \\ \textbf{soft}: &\quad p_s^\mu \sim p_T\,(\delta\phi, \delta\phi, \delta\phi), \\ n_{c,d}\textbf{-jet}: &\quad p_{c_i}^\mu \sim p_T\,(R^2, 1, R)_{n_i\bar{n}_i}, \end{aligned}$$

Factorization in SCET

Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs

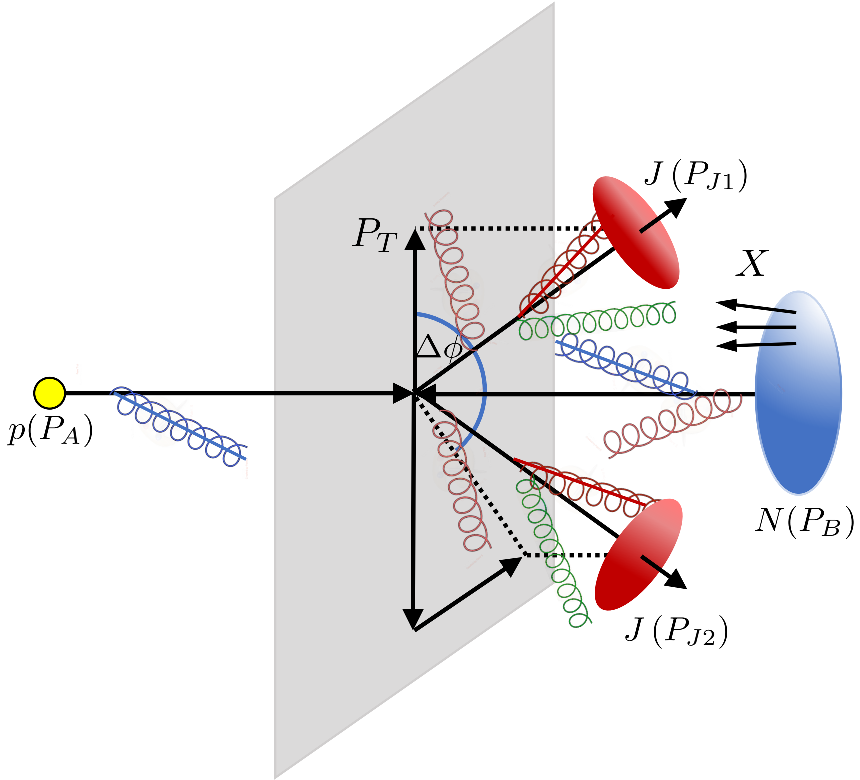


Factorization and resummation derived in a SCET framework

$$\begin{aligned}
\textbf{hard}: &\quad p_h^\mu \sim p_T(1,1,1) \\
n_{a,b}\textbf{-collinear}: &\quad p_{c_i}^\mu \sim p_T\,(\delta\phi^2, 1, \delta\phi)_{n_i\bar{n}_i}, \\
\textbf{soft}: &\quad p_s^\mu \sim p_T\,(\delta\phi, \delta\phi, \delta\phi), \\
n_{c,d}\textbf{-jet}: &\quad p_{c_i}^\mu \sim p_T\,(R^2, 1, R)_{n_i\bar{n}_i}, \\
n_{c,d}\textbf{-collinear-soft}: &\quad p_{cs_i}^\mu \sim \frac{p_T\,\delta\phi}{R}(R^2, 1, R)_{n_i\bar{n}_i},
\end{aligned}$$

Factorization in SCET

Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs



Factorization and resummation derived in a SCET framework

$$
\begin{aligned}
\textbf{hard}: &\quad p_h^\mu \sim p_T(1,1,1) \\
n_{a,b}\textbf{-collinear}: &\quad p_{c_i}^\mu \sim p_T\,(\delta\phi^2, 1, \delta\phi)_{n_i\bar{n}_i}, \\
\textbf{soft}: &\quad p_s^\mu \sim p_T\,(\delta\phi, \delta\phi, \delta\phi), \\
n_{c,d}\textbf{-jet}: &\quad p_{c_i}^\mu \sim p_T\,(R^2, 1, R)_{n_i\bar{n}_i}, \\
n_{c,d}\textbf{-collinear-soft}: &\quad p_{cs_i}^\mu \sim \frac{p_T\,\delta\phi}{R}(R^2, 1, R)_{n_i\bar{n}_i},
\end{aligned}
$$

Factorization and resummation at NLL:

$$
\begin{aligned}
\frac{\mathrm{d}^4\sigma_{pp}}{\mathrm{d}y_c\,\mathrm{d}y_d\,\mathrm{d}p_T^2\,\mathrm{d}\delta\phi} &= \sum_{abcd} \frac{p_T}{16\pi\hat{s}^2}\frac{1}{1+\delta_{cd}}\int_0^\infty \frac{2\mathrm{d}b}{\pi}\cos(bp_T\delta\phi)x_a\tilde{f}_{a/p}(x_a,\mu_{b_*})x_b\tilde{f}_{b/p}(x_b,\mu_{b_*}) \\
&\times \exp\left\{-\int_{\mu_{b_*}}^{\mu_h}\frac{\mathrm{d}\mu}{\mu}\left[\gamma_{\mathrm{cusp}}\,(\alpha_s)\,C_H\ln\frac{\hat{s}}{\mu^2}+2\gamma_H\,(\alpha_s)\right]\right\} \\
&\times \sum_{KK'}\exp\left[-\int_{\mu_{b_*}}^{\mu_h}\frac{\mathrm{d}\mu}{\mu}\gamma_{\mathrm{cusp}}\,(\alpha_s)\,(\lambda_K+\lambda_{K'}^*)\right]H_{KK'}\,(\hat{s},\hat{t},\mu_h)\,W_{K'K}\,(b_*,\mu_{b_*}) \\
&\times \exp\left[-\int_{\mu_{b_*}}^{\mu_j}\frac{\mathrm{d}\mu}{\mu}\Gamma^{J_c}\,(\alpha_s)-\int_{\mu_{b_*}}^{\mu_j}\frac{\mathrm{d}\mu}{\mu}\Gamma^{J_d}\,(\alpha_s)\right]U_{\mathrm{NG}}^c\,(\mu_{b_*},\mu_j)\,U_{\mathrm{NG}}^d\,(\mu_{b_*},\mu_j) \\
&\times \exp\left[-S_{\mathrm{NP}}^a(b,Q_0,\sqrt{\hat{s}})-S_{\mathrm{NP}}^b(b,Q_0,\sqrt{\hat{s}})\right].
\end{aligned}
$$

Do we observe factorization breaking effects?

Glauber mode note treated in our paper

$$\begin{aligned} \textbf{hard}: &\ p_h^\mu \sim p_T(1,1,1) \\ n_{a,b}\textbf{-collinear}: &\ p_{c_i}^\mu \sim p_T\,(\delta\phi^2, 1, \delta\phi)_{n_i\bar{n}_i}, \\ \textbf{soft}: &\ p_s^\mu \sim p_T\,(\delta\phi, \delta\phi, \delta\phi), \\ n_{c,d}\textbf{-jet}: &\ p_{c_i}^\mu \sim p_T\,(R^2, 1, R)_{n_i\bar{n}_i}, \\ n_{c,d}\textbf{-collinear-soft}: &\ p_{cs_i}^\mu \sim \frac{p_T\,\delta\phi}{R}(R^2, 1, R)_{n_i\bar{n}_i}, \\ n_G\textbf{-Glauber}: &\ p_G^\mu \sim p_T(\delta\phi^2, \delta\phi^2, \delta\phi)_{n_i\bar{n}_i} \end{aligned}$$

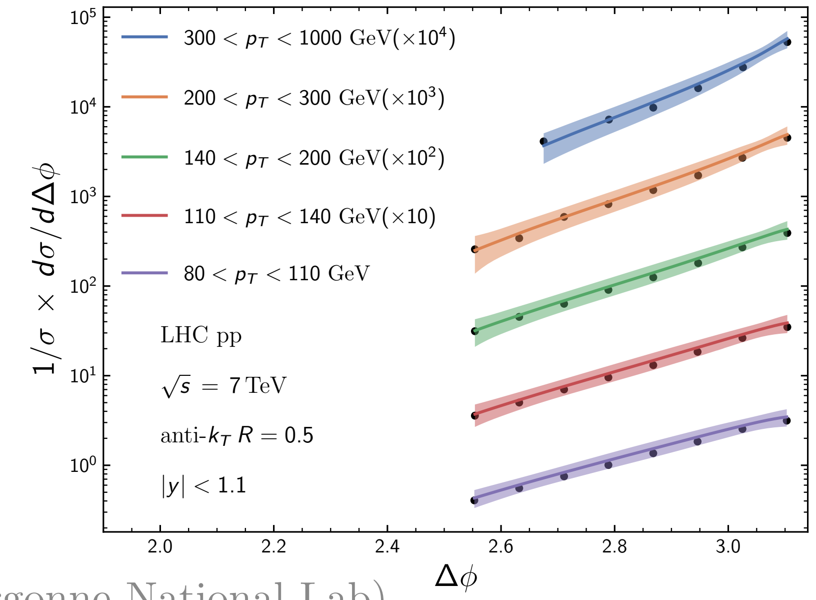
Factorization breaking effects in dijet production studied in

Collins, Qiu Phys.Rev.D75:114014,2007
Collins (2007)

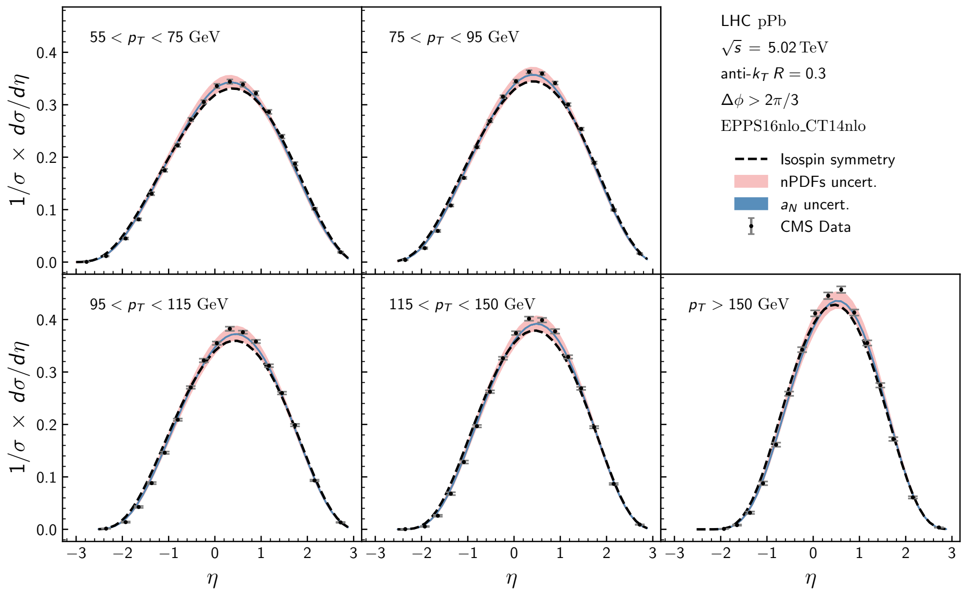
The experimental data is well-described by the experimental data in the back-to-back region, within the error bars

$$\frac{d^4\sigma_{pp}}{dy_c\, dy_d\, dp_T^2\, d\delta\phi}$$

Phys.Rev.Lett.106:122003,2011



Phys. Rev. Lett. 121, 062002 (2018)



Nuclear modifications to this process

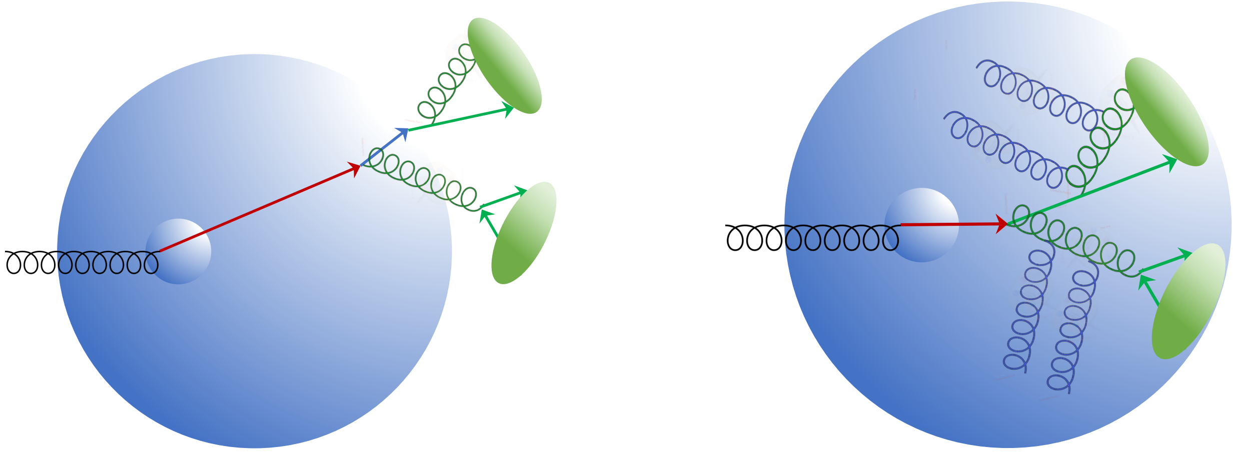
Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs

nTMDs can be matched onto the collinear distributions

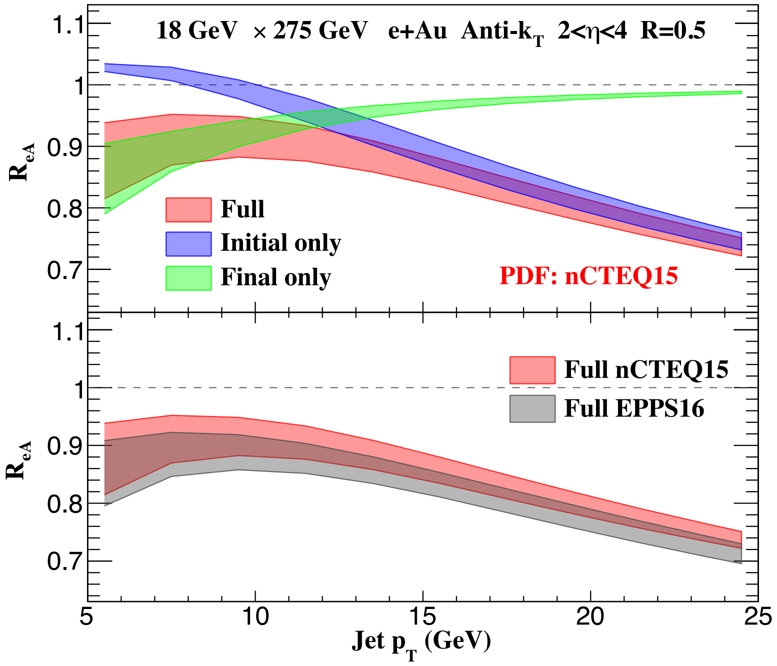
$$f_{q/N}^{A}(b,x;\mu,\zeta_1) = [C \otimes f](x;\mu_i)\, \exp\left[-S_{\text{pert}}(b;\mu_i,\mu,\zeta_i,\zeta_1) - S_{\text{NP}}^{f\,A}(b;Q_0,\mu,\zeta_i,\zeta)\right]$$

Perturbative *Non-perturbative: Contains all medium contributions*

We ignore all final-state interactions between the jets and the medium.
High energy jets are not expected to be affected by the medium

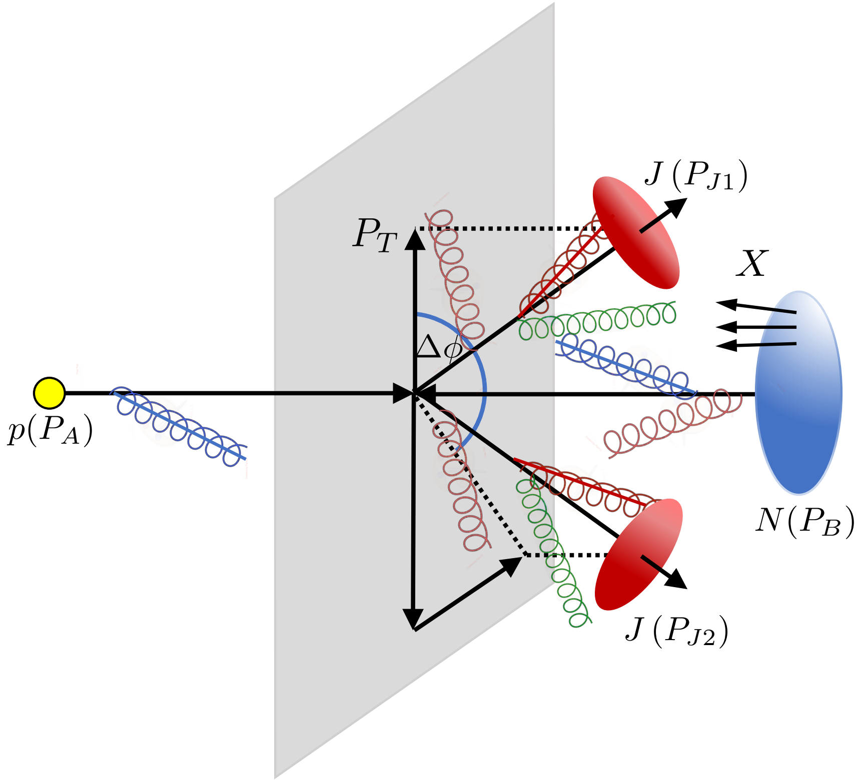


Li, Vitev (2021)



Factorization in pA

Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs



Factorization and resummation derived in a SCET framework

$$\begin{aligned}
\textbf{hard}: &\quad p_h^\mu \sim p_T(1,1,1)\\
n_{a,b}\textbf{-collinear}: &\quad p_{c_i}^\mu \sim p_T\,(\delta\phi^2, 1, \delta\phi)_{n_i\bar{n}_i},\\
\textbf{soft}: &\quad p_s^\mu \sim p_T\,(\delta\phi, \delta\phi, \delta\phi),\\
n_{c,d}\textbf{-jet}: &\quad p_{c_i}^\mu \sim p_T\,(R^2, 1, R)_{n_i\bar{n}_i},\\
n_{c,d}\textbf{-collinear-soft}: &\quad p_{cs_i}^\mu \sim \frac{p_T\,\delta\phi}{R}(R^2, 1, R)_{n_i\bar{n}_i},
\end{aligned}$$

Factorization and resummation:

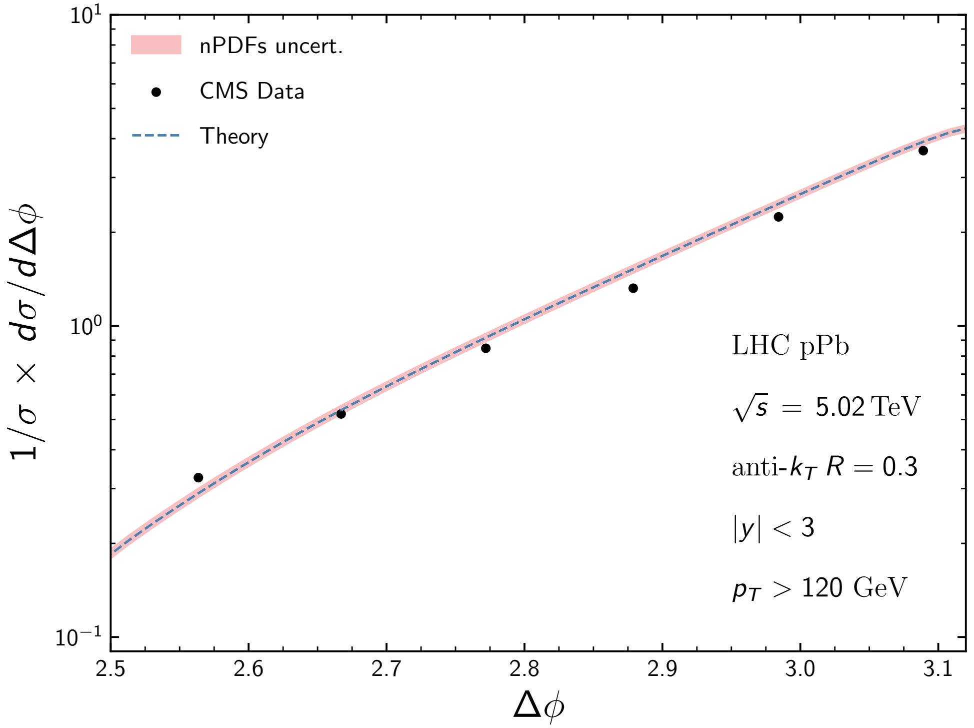
$$\begin{aligned}
\frac{\mathrm{d}^4\sigma_{pA}}{\mathrm{d}y_c\,\mathrm{d}y_d\,\mathrm{d}p_T^2\,\mathrm{d}\delta\phi} &= \sum_{abcd}\frac{p_T}{16\pi\hat{s}^2}\frac{1}{1+\delta_{cd}}\int_0^\infty \frac{2\mathrm{d}b}{\pi}\cos(bp_T\delta\phi)x_a\tilde{f}_{a/p}(x_a,\mu_{b_*})x_b\tilde{f}_{b/A}(x_b,\mu_{b_*})\\
&\times \exp\left\{-\int_{\mu_{b_*}}^{\mu_h}\frac{\mathrm{d}\mu}{\mu}\left[\gamma_{\text{cusp}}\left(\alpha_s\right)C_H\ln\frac{\hat{s}}{\mu^2}+2\gamma_H\left(\alpha_s\right)\right]\right\}\\
&\times \sum_{KK'}\exp\left[-\int_{\mu_{b_*}}^{\mu_h}\frac{\mathrm{d}\mu}{\mu}\gamma_{\text{cusp}}\left(\alpha_s\right)\left(\lambda_K+\lambda_{K'}^*\right)\right]H_{KK'}\left(\hat{s},\hat{t},\mu_h\right)W_{K'K}\left(b_*,\mu_{b_*}\right)\\
&\times \exp\left[-\int_{\mu_{b_*}}^{\mu_j}\frac{\mathrm{d}\mu}{\mu}\Gamma^{J_c}\left(\alpha_s\right)-\int_{\mu_{b_*}}^{\mu_j}\frac{\mathrm{d}\mu}{\mu}\Gamma^{J_d}\left(\alpha_s\right)\right]U_{\text{NG}}^c\left(\mu_{b_*},\mu_j\right)U_{\text{NG}}^d\left(\mu_{b_*},\mu_j\right)\\
&\times \exp\left[-S_{\text{NP}}^a(b,Q_0,\sqrt{\hat{s}})-S_{\text{NP}}^{b,A}(b,Q_0,\sqrt{\hat{s}})\right]
\end{aligned}$$

Description of pA data

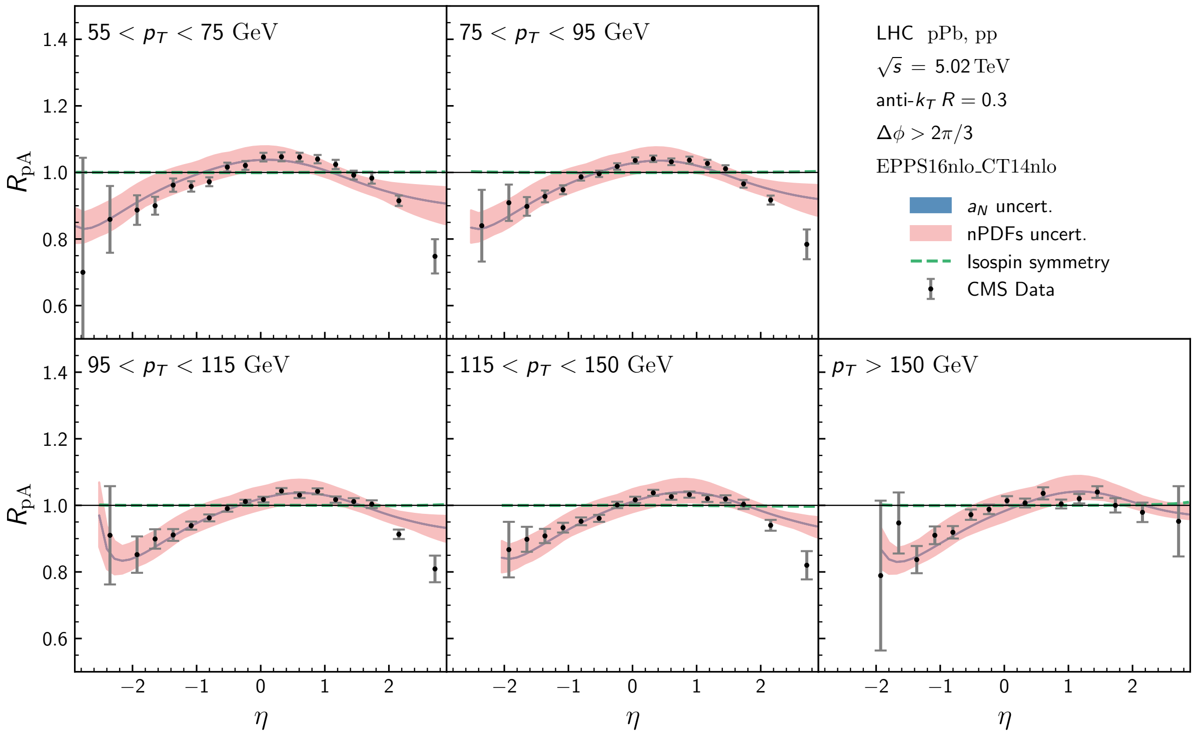
Strong consistency with the CMS measurements of the azimuthal angle decorrelation in pA and the ratio of the integrated azimuthal angle decorrelation.

$$\frac{\mathrm{d}^4\sigma_{pA}}{\mathrm{d}y_c\,\mathrm{d}y_d\,\mathrm{d}p_T^2\,\mathrm{d}\delta\phi}$$

$$R_{\mathrm{pA}} = \frac{1}{A}\frac{\mathrm{d}^4\sigma_{pA}}{\mathrm{d}y_c\,\mathrm{d}y_d\,\mathrm{d}p_T^2\,\mathrm{d}\Delta\phi}\Bigg/\frac{\mathrm{d}^4\sigma_{pp}}{\mathrm{d}y_c\,\mathrm{d}y_d\,\mathrm{d}p_T^2\,\mathrm{d}\Delta\phi}$$



Eur. Phys. J. C 74 (2014) 2951

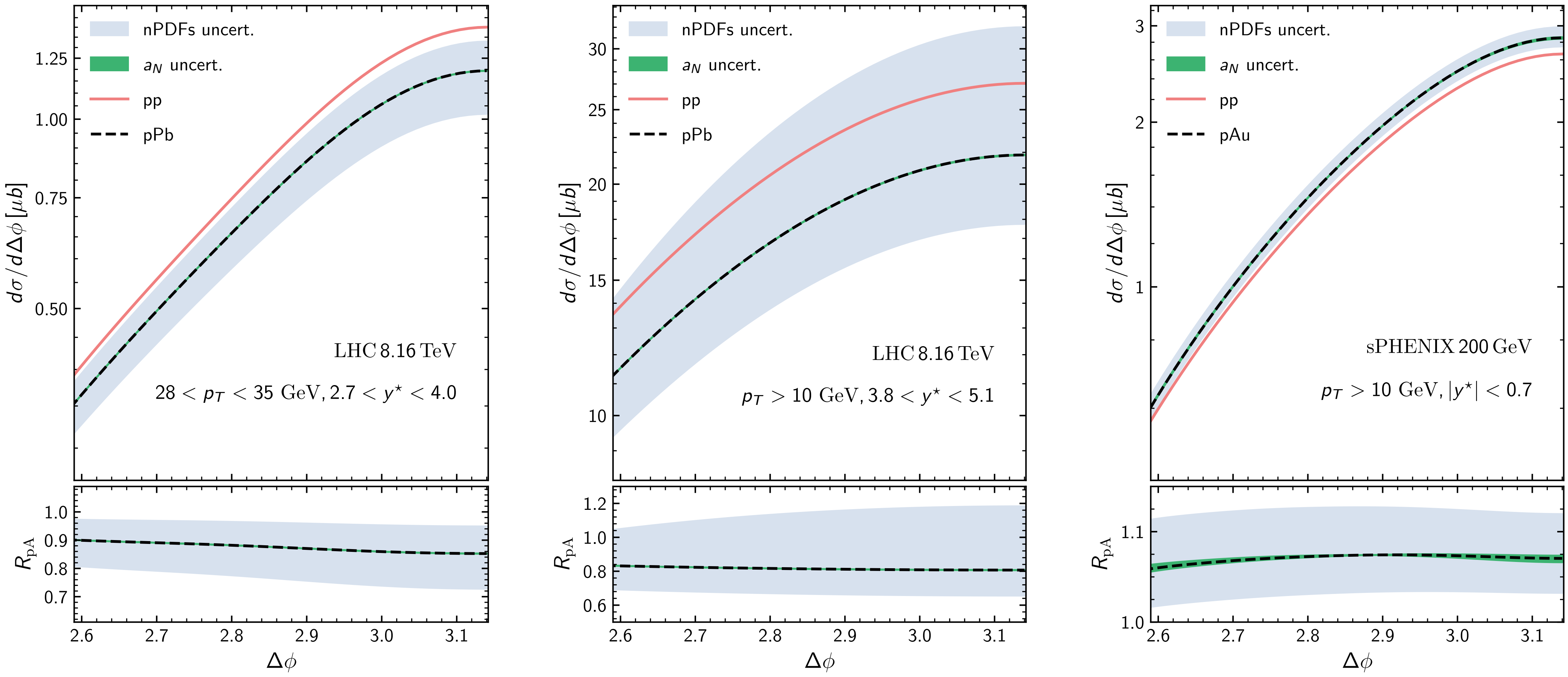


Phys. Rev. Lett. 121, 062002 (2018)

Red band is the uncertainty from the EPPS sets, small blue band from the uncertainty of the nTMDs and is very small for high pT jet production.

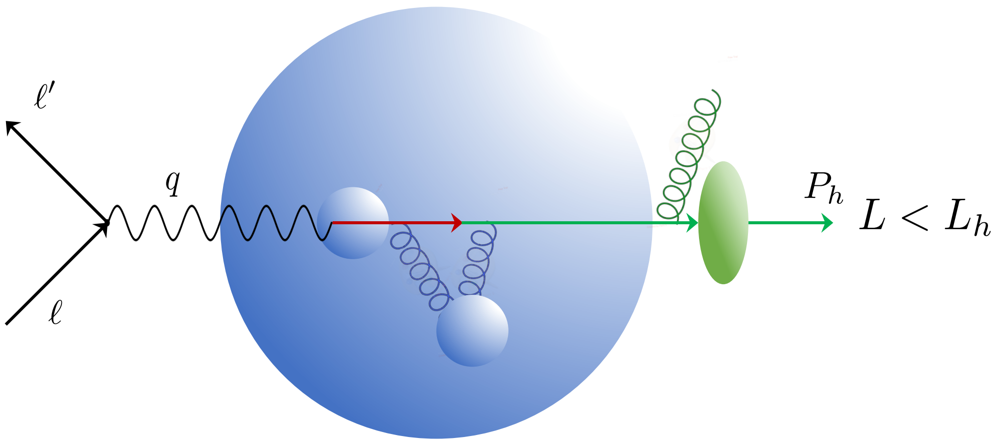
Predictions at ATLAS, ALICE, and sPHENIX

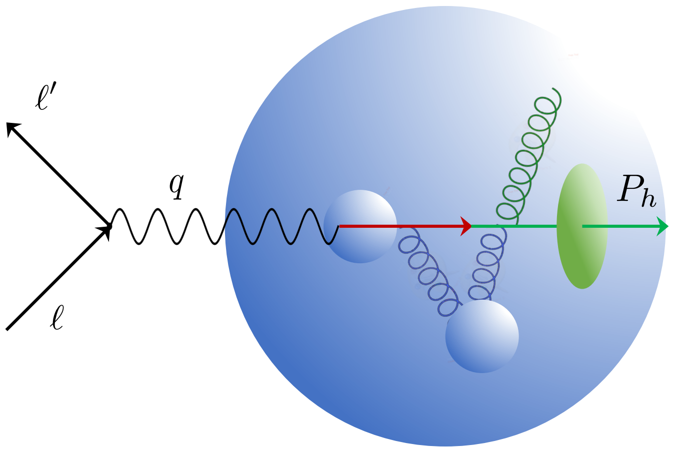
At the LHC, the collinear uncertainties are more dominant due to the large perturbative transverse momenta that are generated. Uncertainty band of the broadening becomes larger at lower center of mass energies



Fragmentation in the medium from first principles

Medium introduces three length scales to the problem $L \sim A^{1/3}/\Lambda_{\mathrm{QCD}}$ $L_h \sim \nu/m_h^2$ λ





$L > L_h$

Thin medium: NP input only from medium properties

Large medium: requires additional NP input from hadronization
also require input from hadronic collisions

Number of collisions goes like $\chi = L/\lambda$

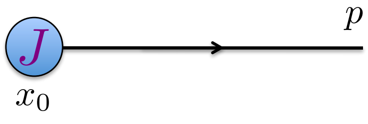
Dilute limit: Opacity expansion or higher twist $\chi \lesssim 1$

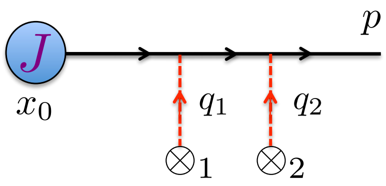
Gyulassy-Levai-Vitev (2000) Guo, Wang (2000)

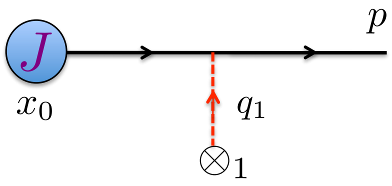
Dense limit: Opacity expansion or higher twist $\chi \gg 1$

Baier, Dokshitzer, Mueller, Peigné, Schiff (1996) Zakharov (1997)

Can be treated in Glauber SCET Ovanesyan and Vitev (2011) $p_G^\mu \sim Q\left(\lambda^2, \lambda^2, \lambda\right)$

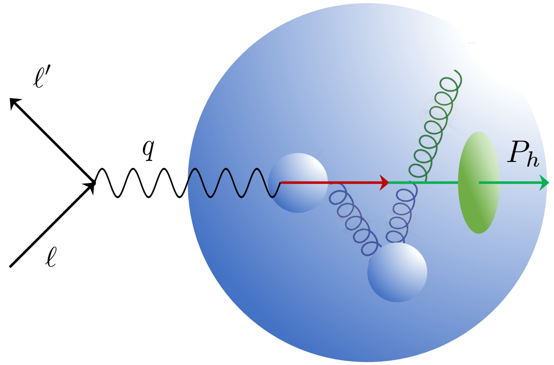
$A_1^{(0)q} =$ 

$A_1^{(2)q} =$ 

$A_1^{(1)q} =$ 

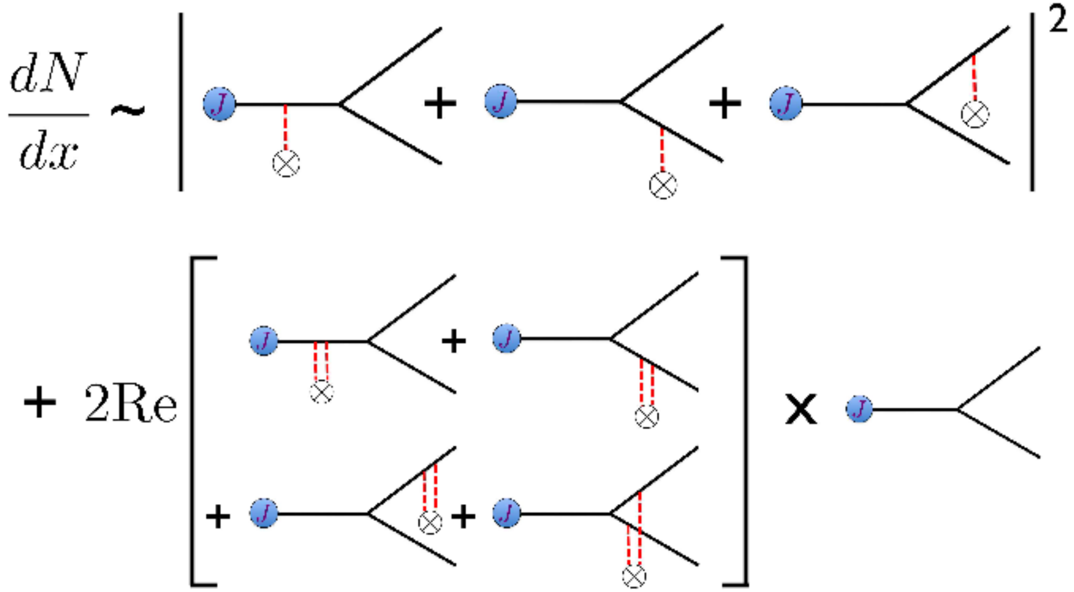
Medium modified DGLAP evolution

Previous work has been done in QCD and SCET to derive medium modified evolution equations



$$\frac{\partial \tilde{D}_{h/j}(z;\mu)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu)}{2\pi} \sum_i \left[\tilde{P}_{ij} \otimes \tilde{D}_{h/i}\right](z;\mu)$$

$$\tilde{P}_{ij}(z;\mu) = \tilde{P}_{ij}(z) + \Delta\tilde{P}_{ij}(z;\mu)$$

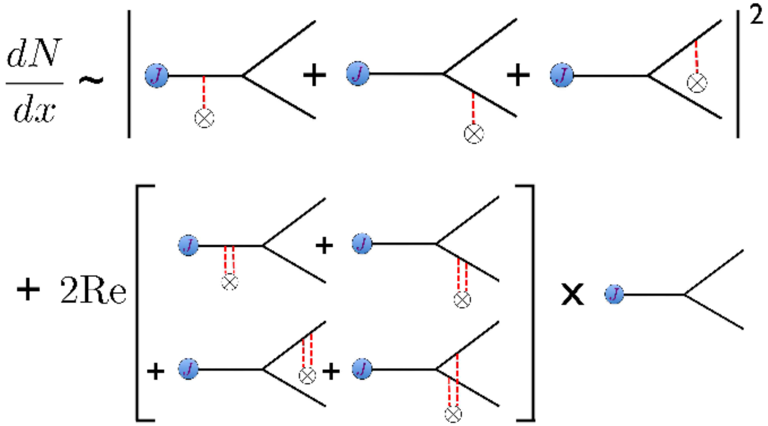


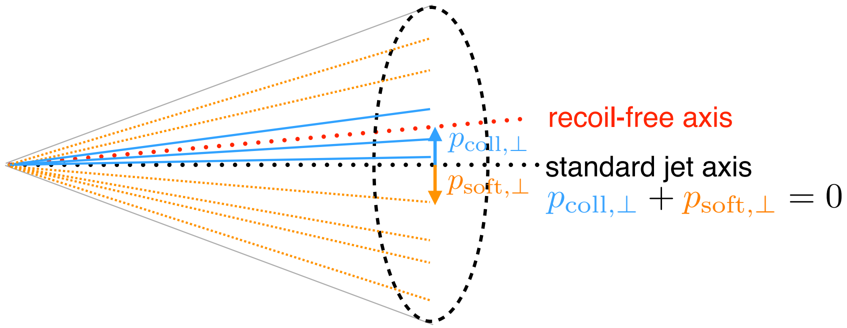
Ovanesyan, Vitev (2012)

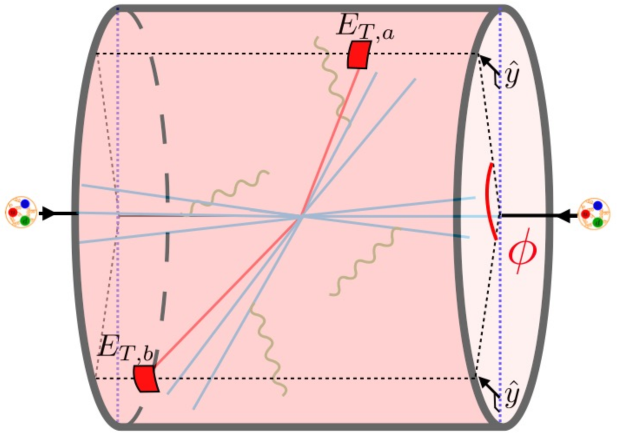
Medium modification can be implemented into the fit, but introduces additional scales. Future work in this community will involve including the medium modified DGLAP into the fit, as well as calculating the medium modifications to the RG and Collins-evolution of the TMDs.

Future work

Graphs for medium modified evolution can be applied to final-state functions for TMD measurements (exclusive jet functions, EECs)





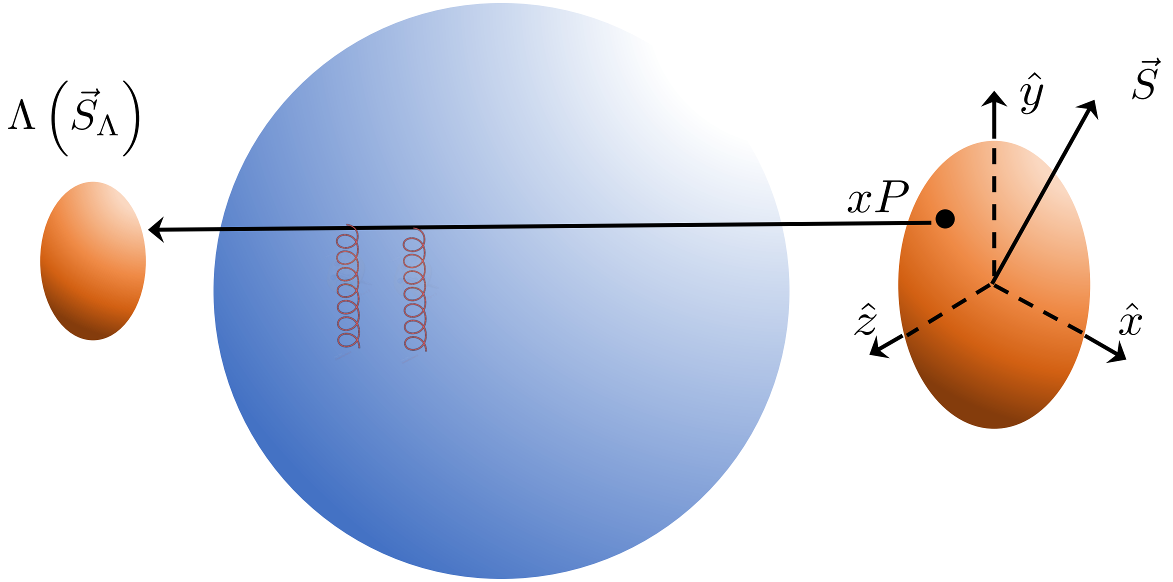


Medium modified DGLAP can be applied in a global analysis

$$\frac{\partial \tilde{D}_{h/j}(z;\mu)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu)}{2\pi} \sum_i \left[\tilde{P}_{ij} \otimes \tilde{D}_{h/i}\right](z;\mu)$$

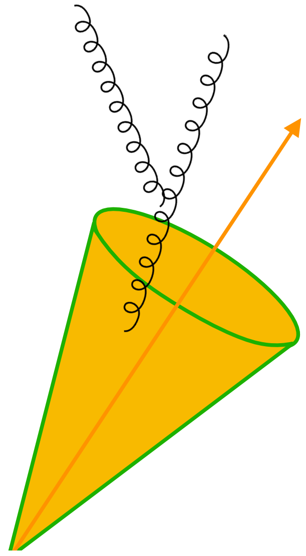
$$\tilde{P}_{ij}(z;\mu) = \tilde{P}_{ij}(z) + \Delta\tilde{P}_{ij}(z;\mu)$$

Medium modified spin physics as a new probe of medium properties



Non-global logs

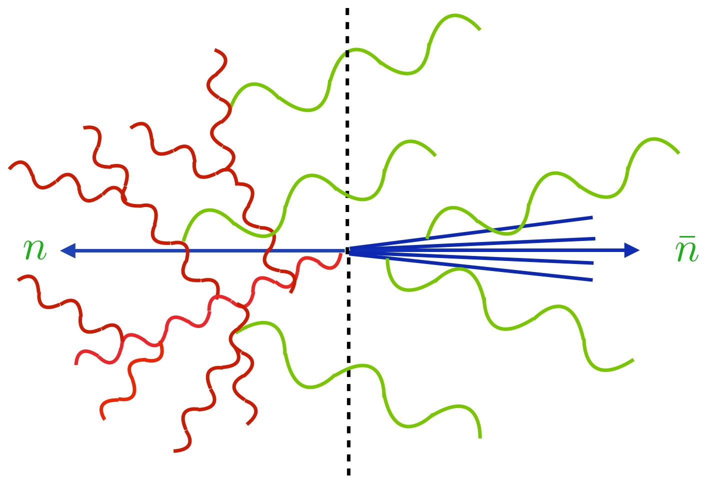
Standard jets complicate resummation



$$-\frac{C_A C_a}{2}\left(\frac{\alpha_s}{\pi}\right)^2\frac{\pi^2}{24}\ln^2\left(\frac{P_\perp^2}{\mu_b^2}\right)$$

Dasgupta, Salam (2001)   Larkoski, Moult, Neill (2016)

$$m_H \gg m_L$$



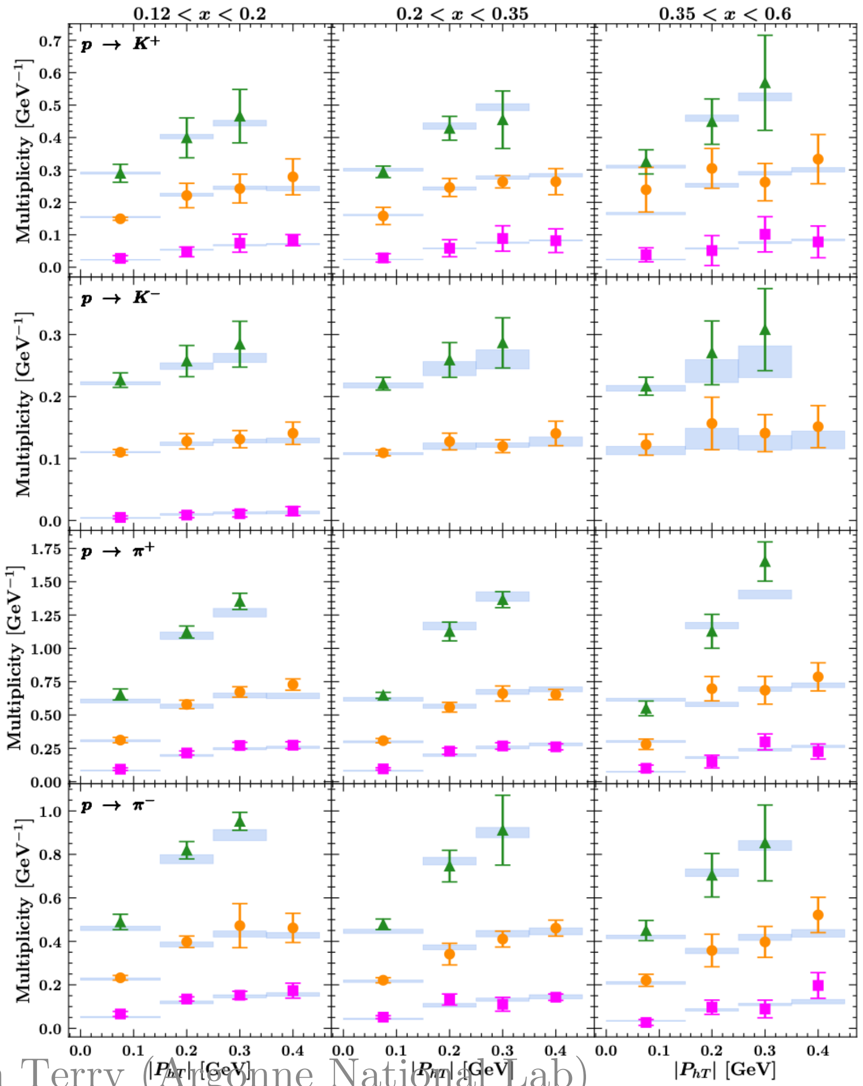
NGLs for jet at NLL is the same as jet mass in e+ e-   Becher, Neubert, Rothen and Shao (2016)

$$U_{\rm NG}^k(\mu_{cs},\mu_j) = \exp\left[-C_A C_k \frac{\pi^2}{3} u^2 \frac{1+(au)^2}{1+(bu)^c}\right],$$

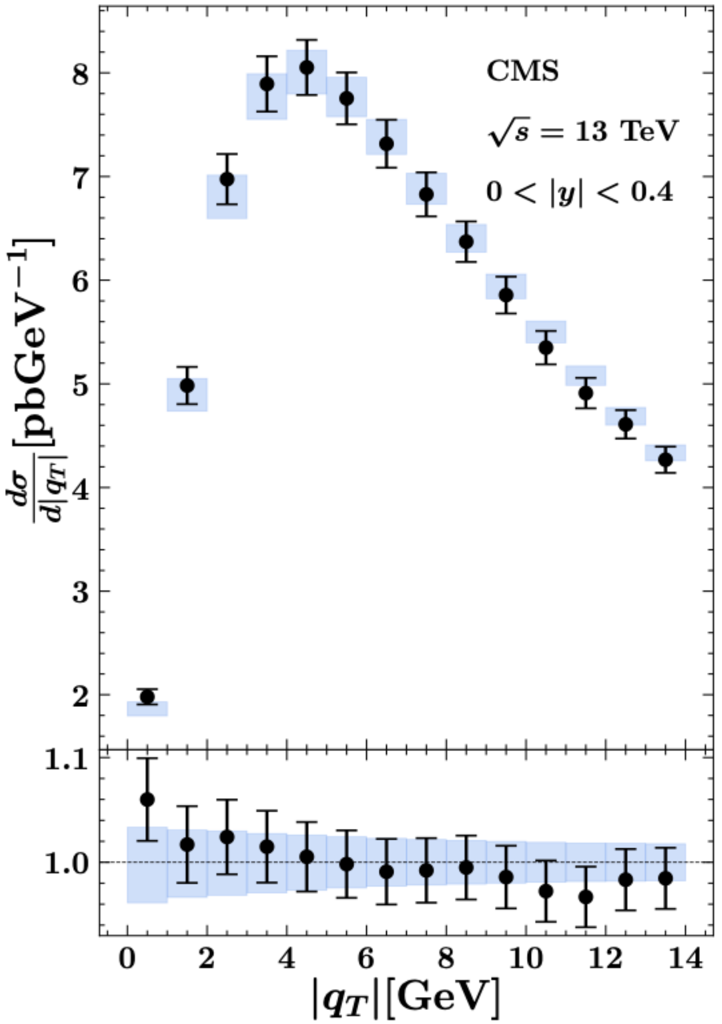
Map Collaboration

Extraction at NNLO+N^{3}LL, SIDIS+DY

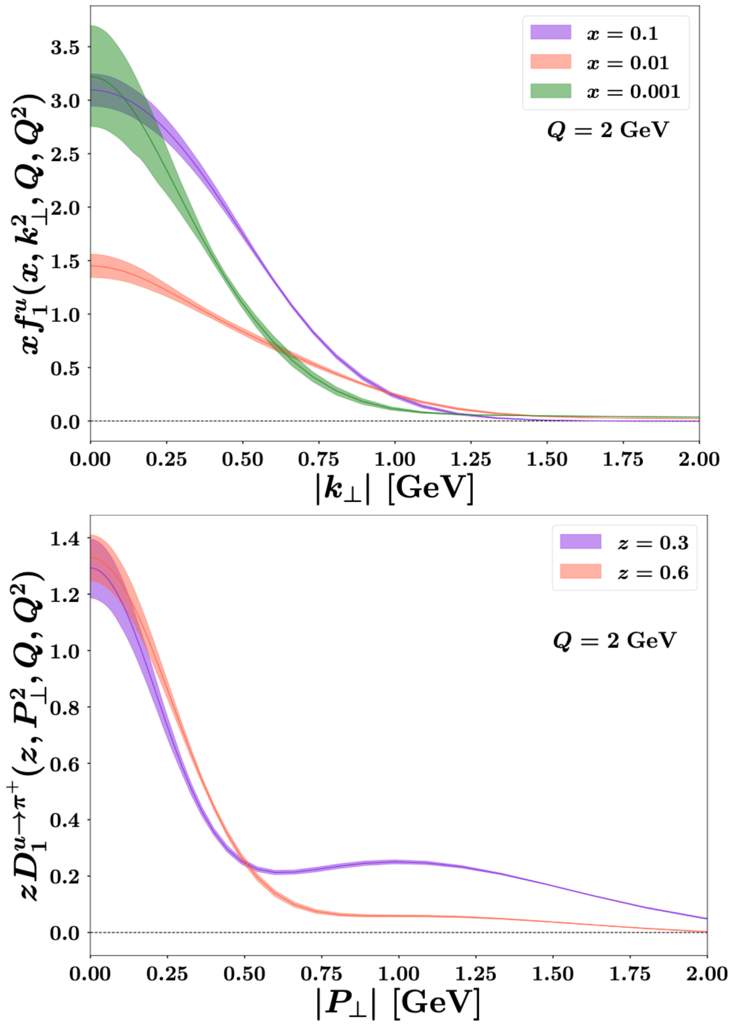
Hermes multiplicity data



CMS $q_\perp$ distribution



TMDs

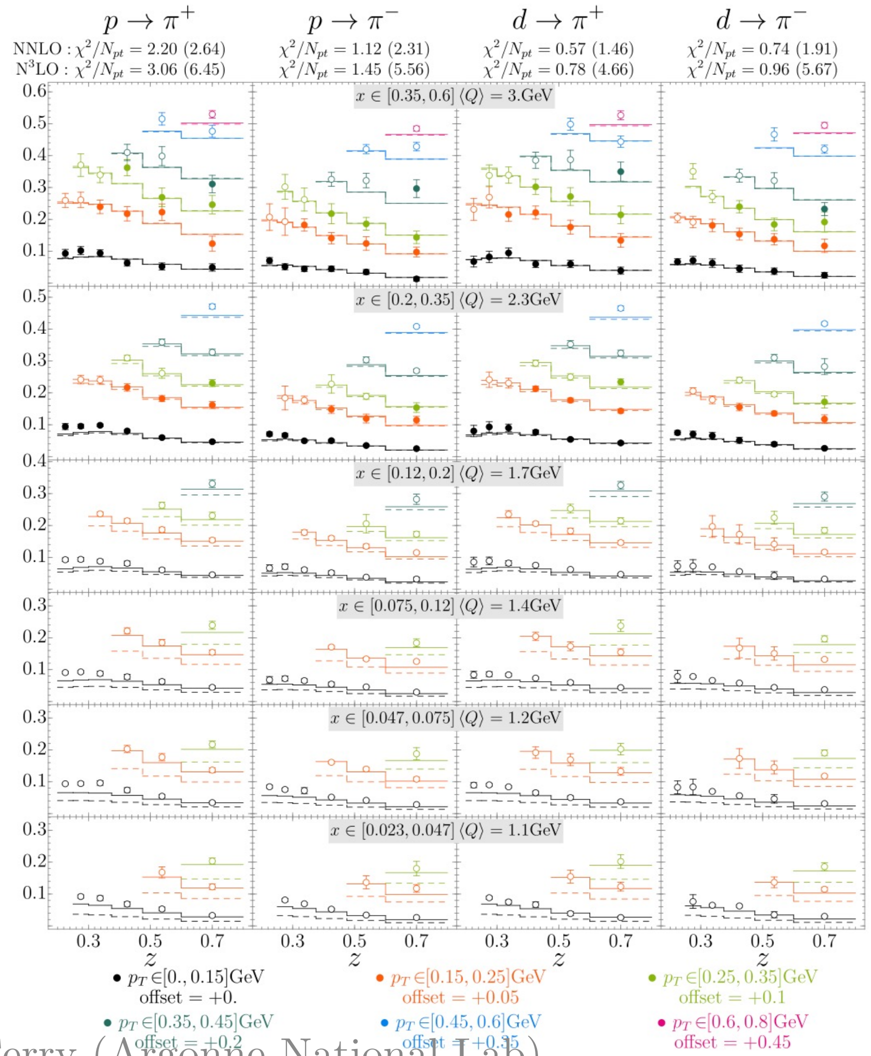


Artemides

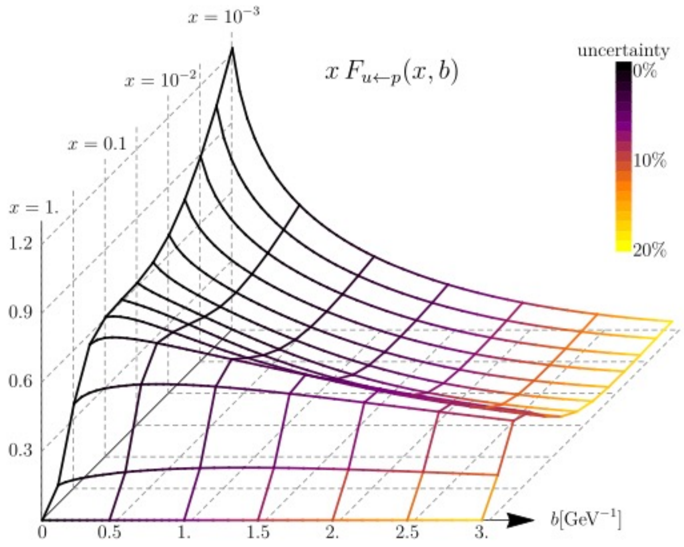
Extraction obtained at NNLO+N^{3}LL, SIDIS+DY

Hermes Multiplicity data

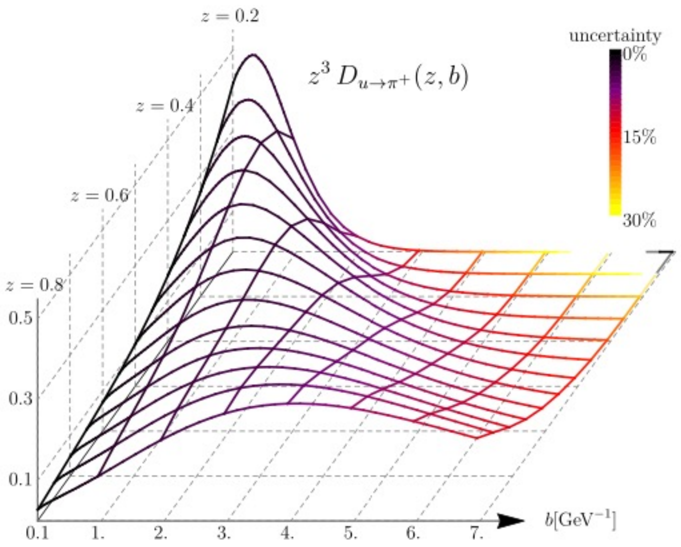
$$\Phi_n = 1 - \frac{\sin\left(\frac{\mathbf{Q}_n^2 L^+}{2x(1-x)p_1^+}\right)}{\frac{\mathbf{Q}_n^2 L^+}{2x(1-x)p_1^+}}$$



TMD PDF



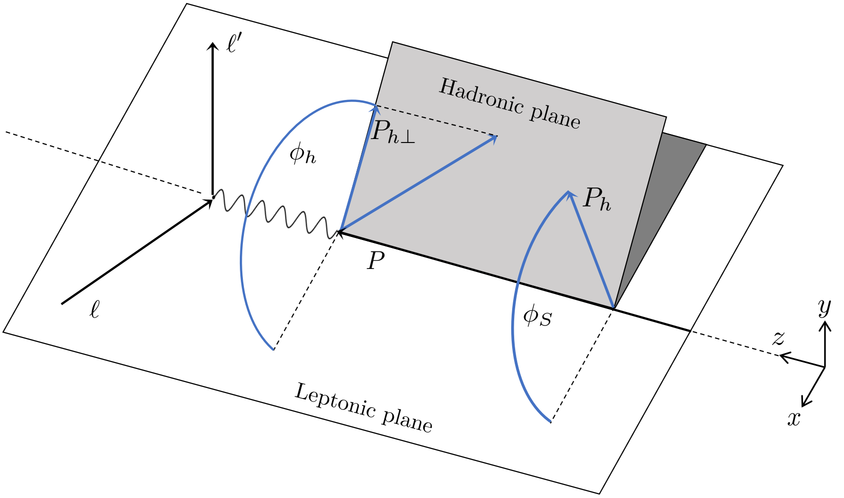
TMD FF

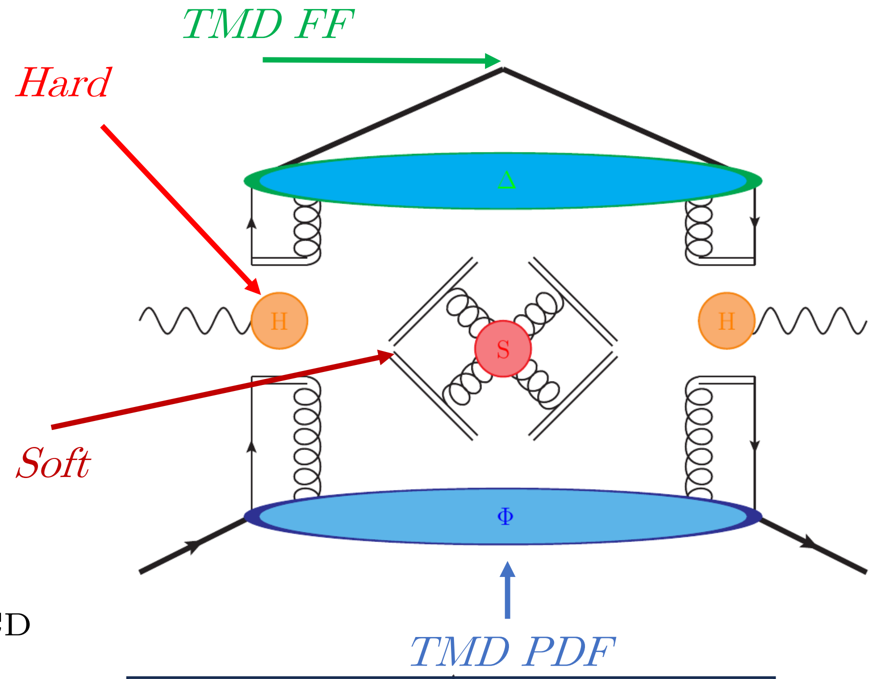


$b \sim 1/q_T$

Factorization of physics at different scales

Factorization of the cross section in an OPE $Q \gg q_T \gtrsim \Lambda_{\rm QCD}$



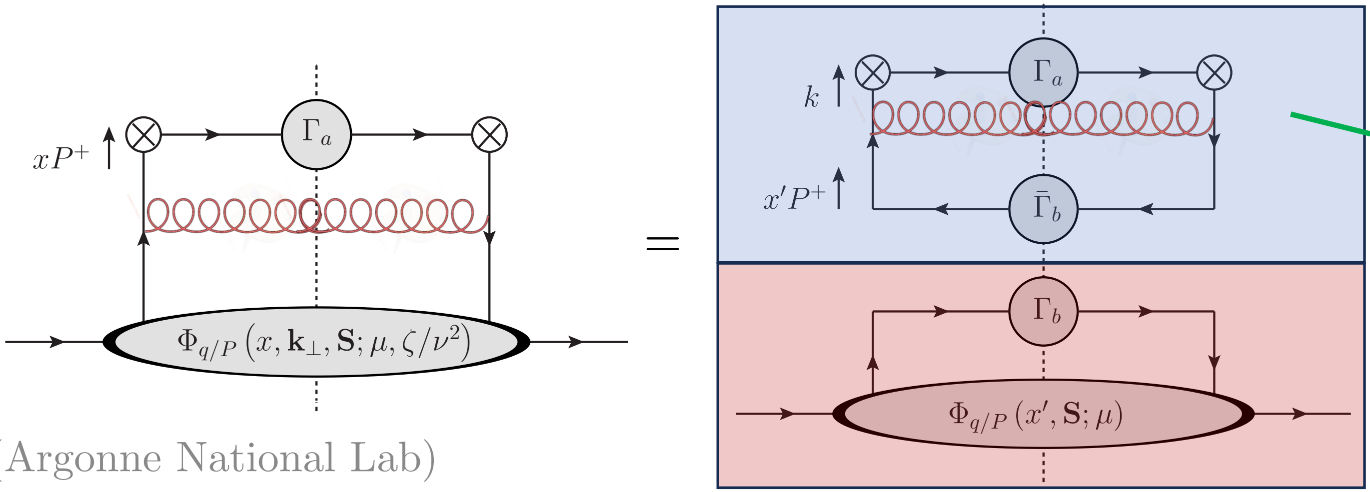


$$d\sigma \sim \sum_i \left|C\left(Q;\mu\right)\right| f \otimes D \otimes S(q_T,\mu)$$

Contains fixed order and large logs

$$\ln\left(\frac{Q}{\mu}\right)$$

Factorization of IR modes $q_T \gtrsim \Lambda_{\rm QCD}$



Matching coefficient contains fixed order and large logs

$$\ln\left(\frac{q_T}{\mu}\right)$$

$$f(x,q_T,\mu) \sim \left[C \otimes f\right](x,q_T,\mu)$$

Collinear PDF

Soft-Collinear Effective Theory

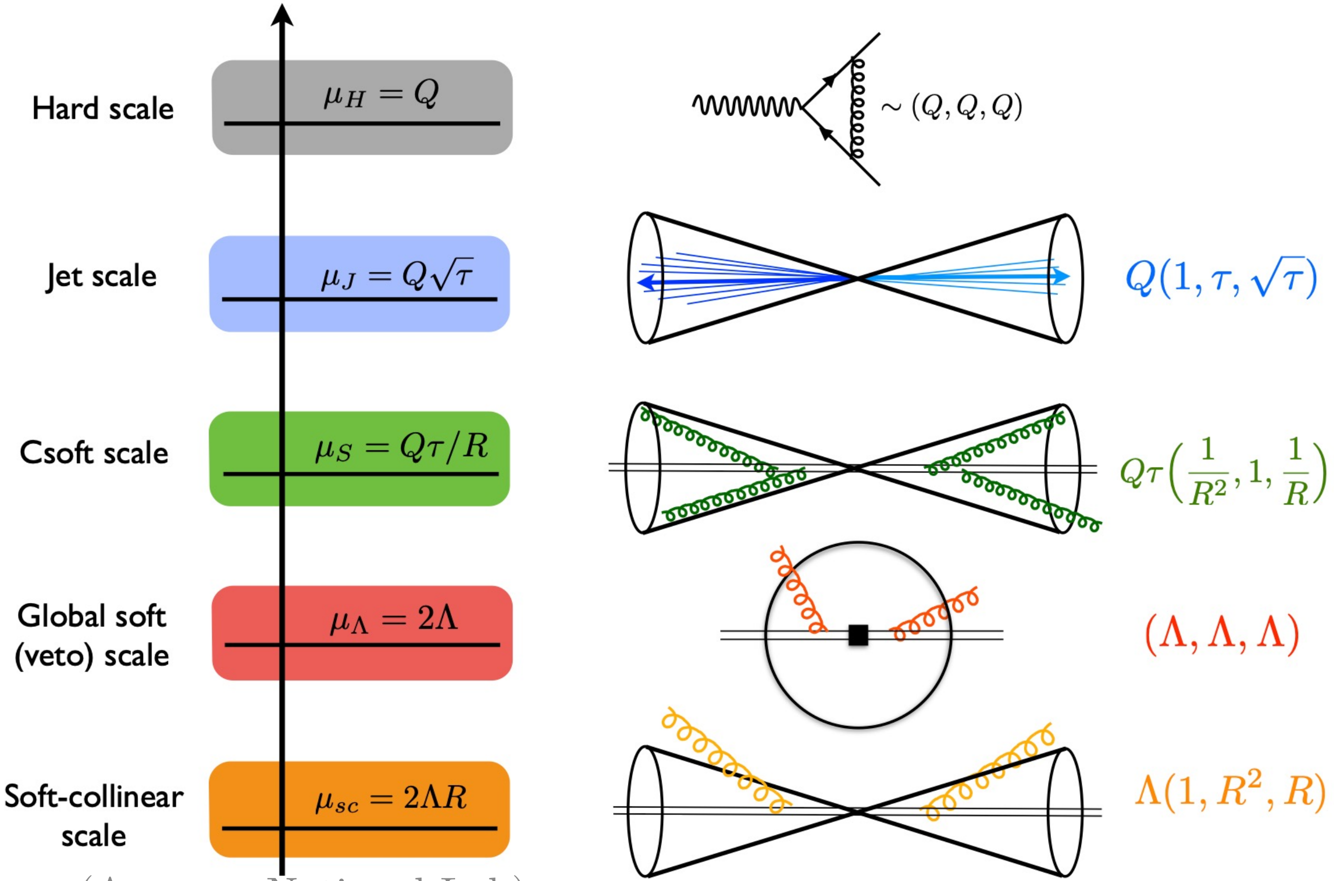
SCET is an EFT which captures soft and collinear emissions along the directions

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{\psi}\, i\not{D}\, \psi - \frac{1}{4} G^A_{\mu\nu} G^{A\,\mu\nu} + \mathcal{L}_{\text{gauge-fix}} + \mathcal{L}_{\text{ghost}}$$

$$\psi \to \psi_s + \psi_c \qquad A^\mu \to A^\mu_s + A^\mu_c$$

$$\longrightarrow$$

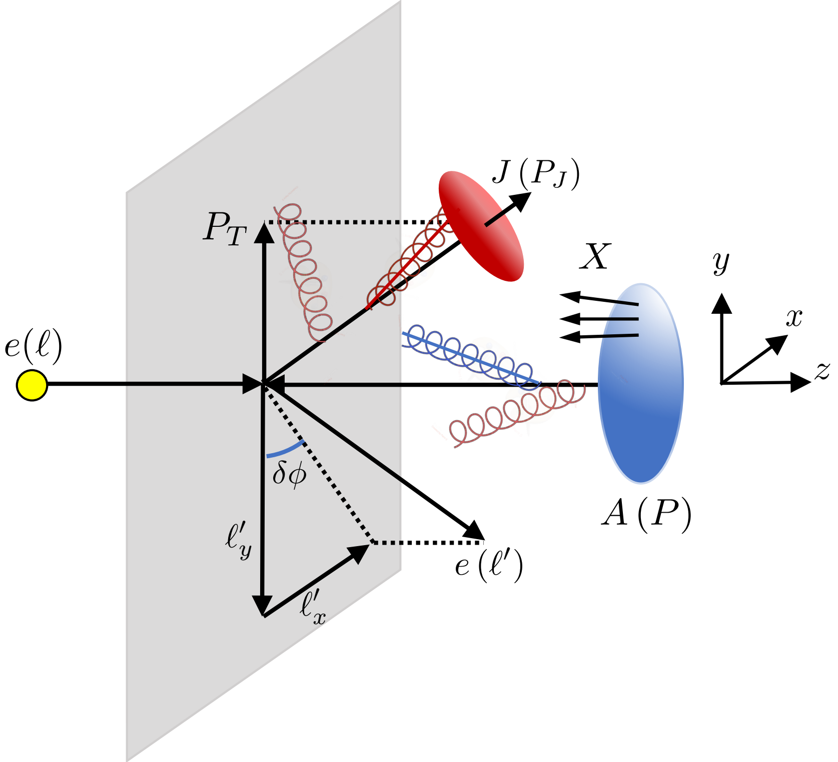
$$\mathcal{L}_{\text{SCET}} = \bar{\psi}_s\, i\not{D}_s\, \psi_s - \frac{1}{4} G^A_{\mu\nu\,s} G^{A\,\mu\nu}_s + \bar{\xi}\frac{\not{\bar{n}}}{2}\left[in\cdot D + i\not{D}_{c\perp}\frac{1}{i\bar{n}\cdot D_c} i\not{D}_{c\perp}\right]\xi - \frac{1}{4} G^A_{\mu\nu\,c} G^{A\,\mu\nu}_c$$



Bauer, Fleming, Luke 2000
Bauer, Fleming, Pirjol, Stewart 2001
Bauer, Stewart 2001
Bauer, Pirjol, Stewart 2002
Beneke, Chapovsky, Diehl, Feldmann 2002
Beneke, Feldmann 2003
Hill, Neubert 2003
Echevarria, Idilbi, Scimemi 2011

Back-to-back lepton-jet production

Process proposed by: *Liu, Ringer, Vogelsang, Yuan (2019)*



Less sensitive to non-perturbative physics

Lepton-jet transverse momentum imbalance

$$\vec{q}_T = \vec{P}_{JT} + \vec{\ell}_T$$

TMD region

$$\frac{|\vec{q}_T|}{\left|\vec{P}_{JT}\right|} \ll 1$$

Better than SIDIS in that there is no sensitivity to FFs
Worse due to the perturbative accuracy

Novel factorization using recoil-free jets

Hard: $P_{JT}\,(1,1,1)$

Collinear: $P_{JT}\left(\lambda^2,1,\lambda\right)$

Jet: $P_{JT}\left(1,\lambda^2,\lambda\right)$

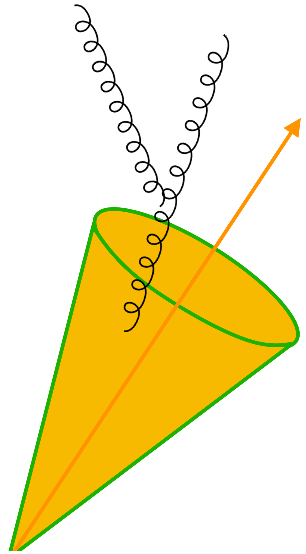
Global soft: $P_{JT}\,(\lambda,\lambda,\lambda)$

$$\frac{d\sigma_p}{d^2\ell'_T\,dy\,d\delta\phi} = \frac{\sigma_0\,\ell'_T}{1-y}\,H\,(Q,\mu_H)\int\frac{db}{2\pi}\cos\left(b\ell'_T\delta\phi\right)\sum_q e_q^2\, f_{q/p}\left(x_B,b,\mu_H,\zeta_{\mathcal{B}}\right)\,\mathcal{J}_q\left(b,\mu_H,\zeta_{\mathcal{J}}\right)$$

$$\frac{d\sigma_A}{d^2\ell'_T\,dy\,d\delta\phi} = \frac{\sigma_0\,\ell'_T}{1-y}\,H\,(Q,\mu_H)\int\frac{db}{2\pi}\cos\left(b\ell'_T\delta\phi\right)\sum_q e_q^2\, f_{q/A}\left(x_B,b,\mu_H,\zeta_{\mathcal{B}}\right)\,\mathcal{J}_q^A\left(b,\mu_H,\zeta_{\mathcal{J}}\right)$$

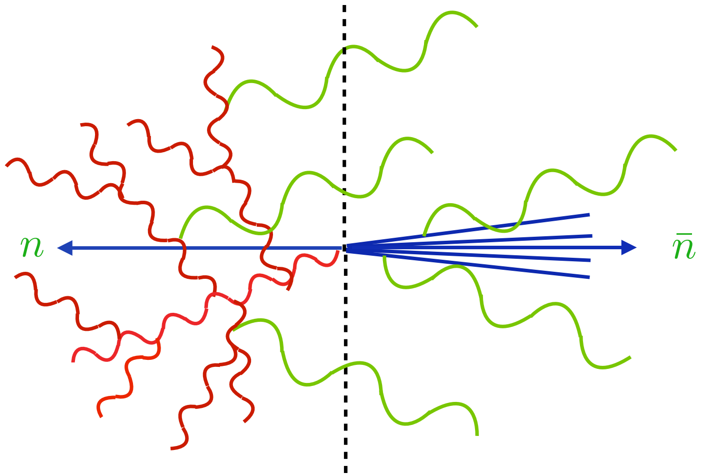
Non-global logs

Standard jets complicate resummation *Dasgupta, Salam (2001)* Larkoski, Moult, Neill (2016)



$$-\frac{C_A C_a}{2}\left(\frac{\alpha_s}{\pi}\right)^2\frac{\pi^2}{24}\ln^2\left(\frac{P_\perp^2}{\mu_b^2}\right)$$

$$m_H \gg m_L$$



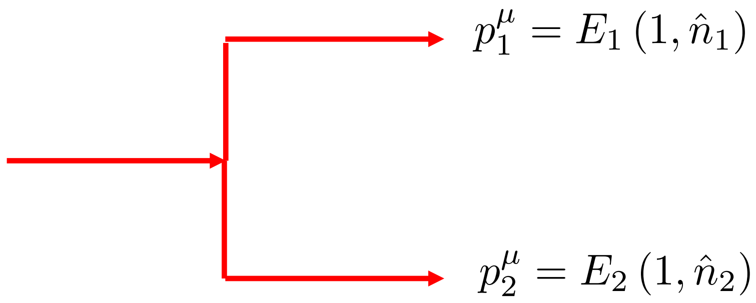
NGLs for jet at NLL is the same as jet mass in e+ e- *Becher, Neubert, Rothen and Shao (2016)*

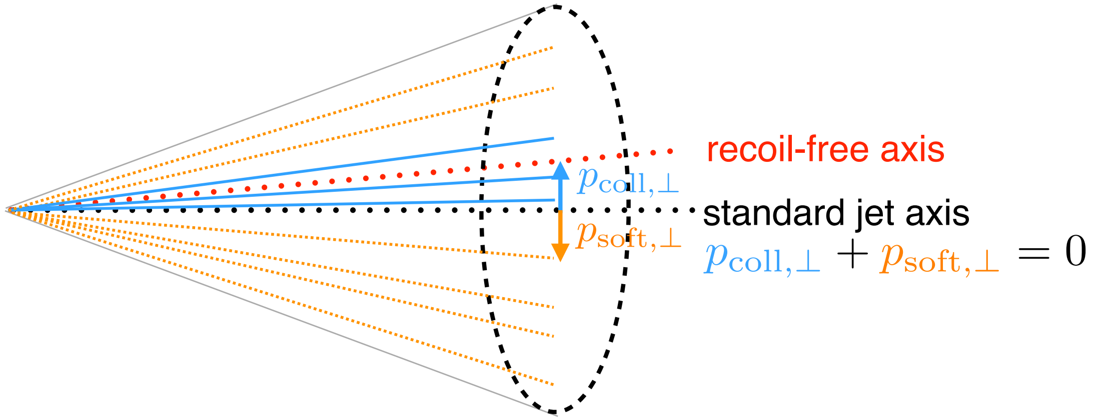
$$U_{\rm NG}^k(\mu_{cs},\mu_j) = \exp\left[-C_A C_k \frac{\pi^2}{3} u^2 \frac{1+(au)^2}{1+(bu)^c}\right],$$

Winner take all jet axis

Direction of recoil-free jet is insensitive all soft emissions, jet points in direction of most energetic hadron. Thus no NGLS

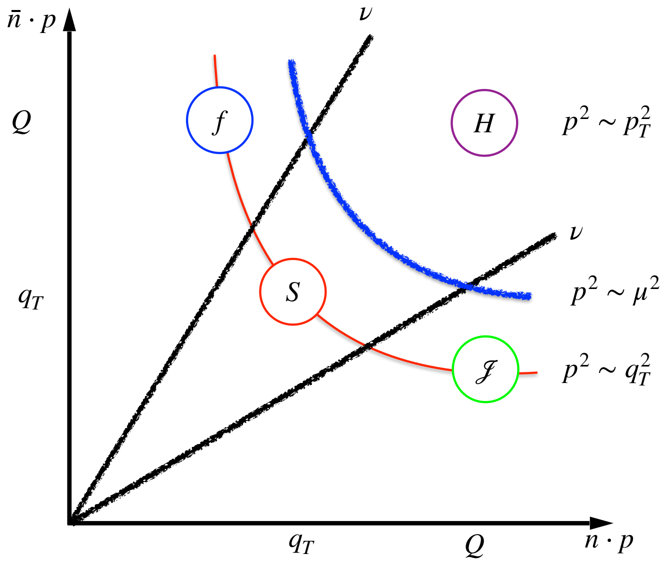
Larkoski, Neill, and Thaler (2014)





$$p_0^\mu = (E_1 + E_2)\left[\hat{p}_1^\mu \Theta\,(E_1 - E_2) + \hat{p}_2^\mu \Theta\,(E_2 - E_1)\right]$$

Direction of jet and total jet momentum have a transverse momentum relative to one another, contains rapidity divergence



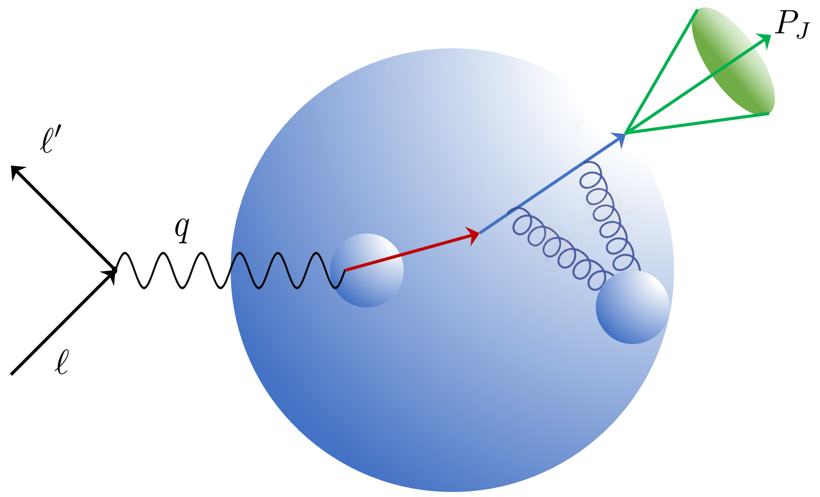
$$\mu \frac{d}{d\mu} \ln \mathcal{J}(Q,\mu,\nu) = \gamma^q_{\mathcal{J}\,\mu}(Q,\mu,\nu) \qquad \gamma^q_{\mathcal{J}\,\mu}(Q,\mu,\nu) = \gamma^q_{D\,\mu}(Q,\mu,\nu)$$

$$\nu \frac{d}{d\nu} \ln \mathcal{J}(Q,\mu,\nu) = \gamma^q_{\mathcal{J}\,\nu}(Q,\mu,\nu) \qquad \gamma^q_{\mathcal{J}\,\nu}(Q,\mu,\nu) = \gamma^q_{D\,\nu}(Q,\mu,\nu)$$

$$\mathcal{J}_q\left(b, \mu, \zeta_{\mathcal{J}}/\nu^2\right) = 1 + \frac{\alpha_s C_F}{4\pi}\left[3L + 2L\ln\left(\frac{\nu^2}{\zeta_{\mathcal{J}}}\right) + 7 - \frac{2\pi^2}{3} - 6\ln 2\right] + \mathcal{O}\left(\alpha_s^2\right)$$

Treatment of medium modifications to the jet

We want to take the jets to be energetic so that $L/L_h \sim \frac{A^{1/3}\Lambda_{\rm QCD}}{\nu} \ll 1$



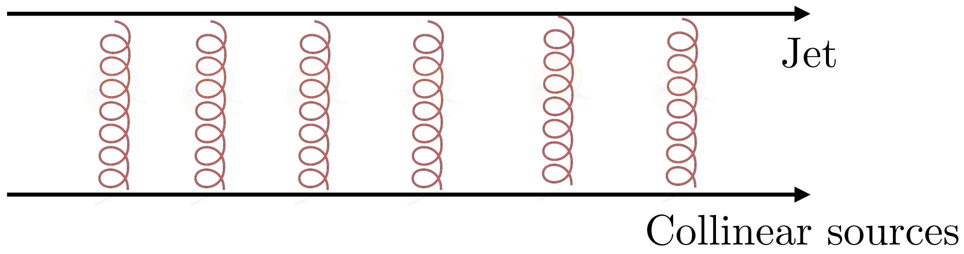
Single gluon exchange

$$\frac{d\sigma}{d^2q_T} = \frac{\alpha_s C_F}{\pi}\frac{1}{\left(q_T^2+m^2\right)^2}$$

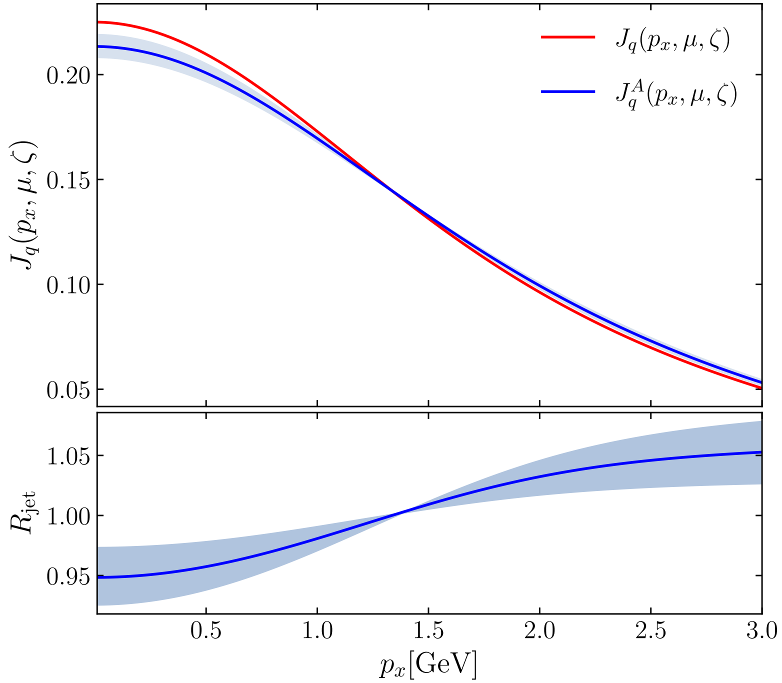
Infinite number of gluon exchanges

$$\frac{d\sigma_{n\to\infty}}{db} = \exp\left(\frac{\rho_G L}{m^2}\alpha_s C_F\left(mbK_1\left(mb\right)-1\right)\right)$$

We consider an infinite chain of Glauber gluons



Modified jet function under this approximation

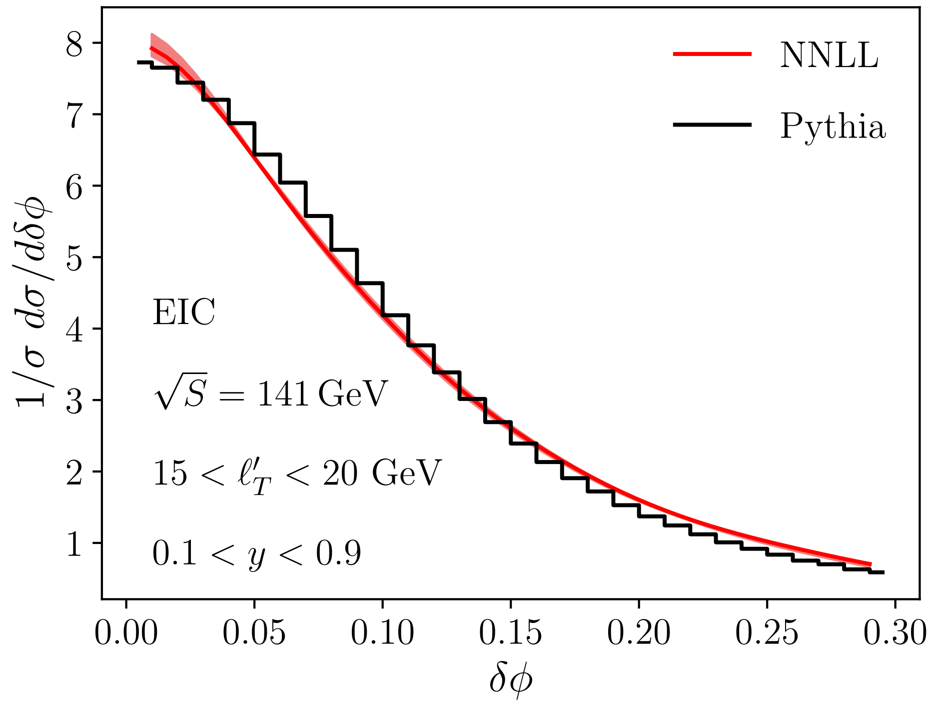


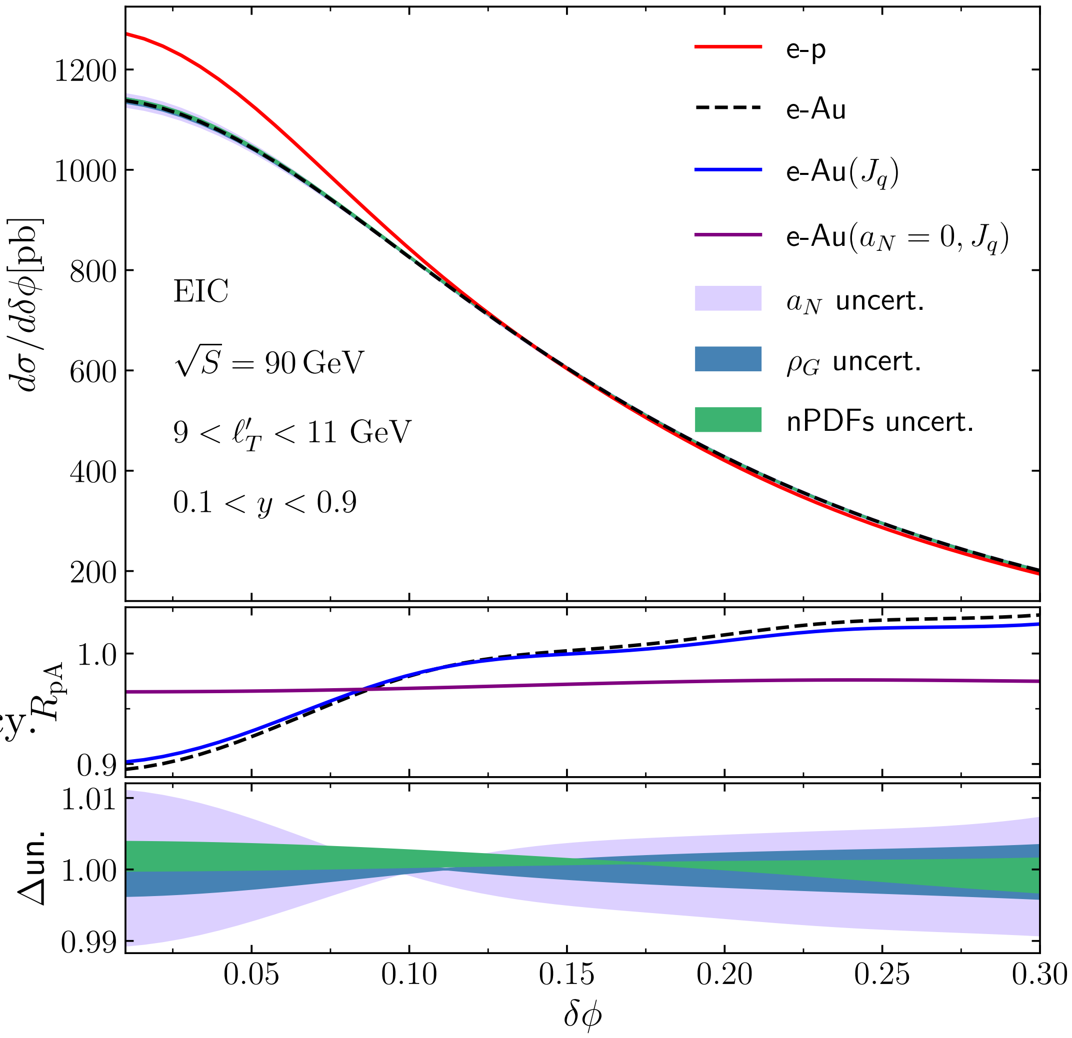
Modification to the jet

$$\mathcal{J}_q^A\left(b,\mu,\nu\right) = \frac{d\sigma_{n\to\infty}}{db}\mathcal{J}_q\left(b,\mu,\nu\right)$$

Predictions at the EIC

Comparison of our results with Pythia at NNLL

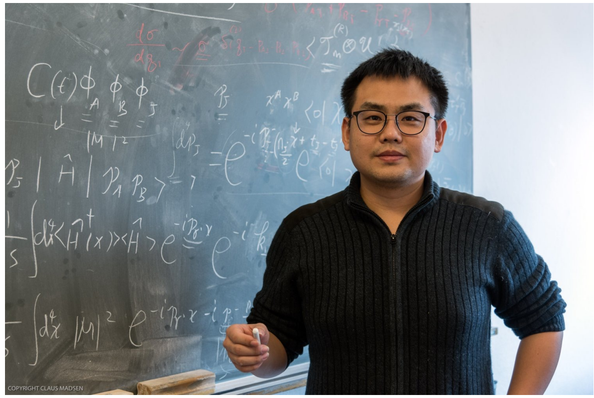


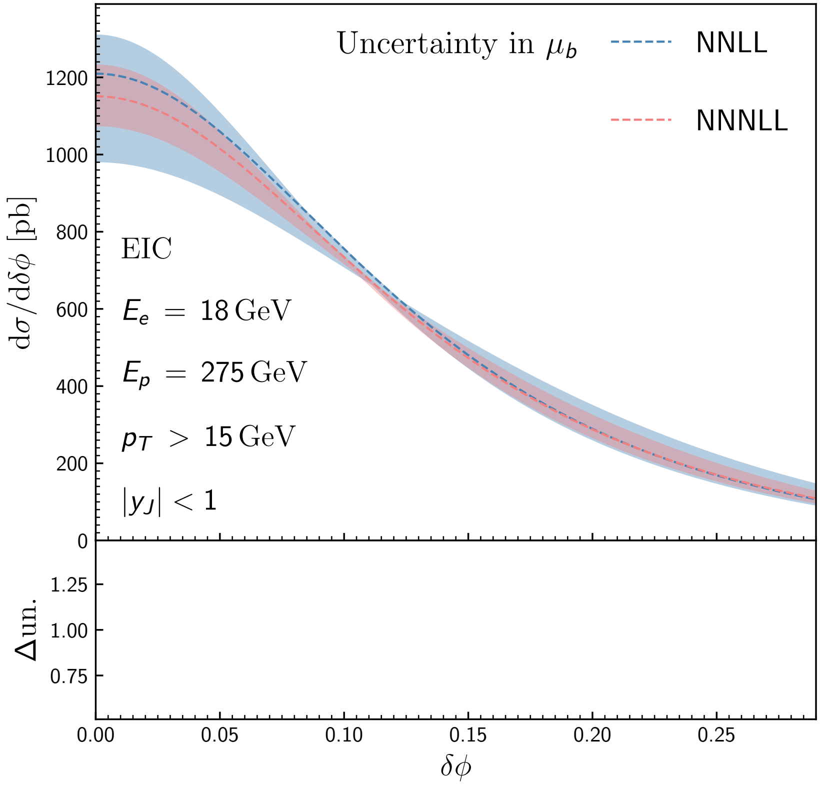


Perturbative ingredients are known to have N3LL accuracy.
Only missing the 3-loop jet function and the 5-loop cusp anomalous dimension to reach N4LL

Higher order results

Results have been taken one step higher by *Fang, Gao, Li, Shao (2024)*





$$J_q(b,\mu,\zeta) = 1 + \frac{\alpha_s C_F}{4\pi}\left[-L_b^2 + L_b(3+2L_\zeta) - \frac{5\pi^2}{6} + 7 - 6\ln 2\right]$$
$$+ \left(\frac{\alpha_s}{4\pi}\right)^2 \left\{ C_F^2\left[\frac{L_b^4}{2} - L_b^3(3+2L_\zeta) + L_b^2\left(2L_\zeta^2 + 6L_\zeta - \frac{5}{2} + 6\ln 2 + \frac{5\pi^2}{6}\right)\right.\right.$$
$$\left. + L_b\left(L_\zeta\left(14 - 12\ln 2 - \frac{5\pi^2}{3}\right) + \frac{45}{2} - 18\ln 2 - \frac{9\pi^2}{2} + 24\zeta_3\right)\right]$$
$$+ C_F C_A\left[-\frac{22}{9}L_b^3 + L_b^2\left(\frac{11}{3}L_\zeta - \frac{35}{18} + \frac{\pi^2}{3}\right) + L_\zeta\left(\frac{404}{27} - 14\zeta_3\right)\right.$$
$$\left. + L_b\left(L_\zeta\left(\frac{134}{9} - \frac{2\pi^2}{3}\right) + \frac{57}{2} - 22\ln 2 - \frac{11\pi^2}{9} - 12\zeta_3\right)\right]$$
$$+ C_F T_F n_f\left[\frac{8}{9}L_b^3 + L_b^2\left(\frac{2}{9} - \frac{4}{3}L_\zeta\right) + L_b\left(-\frac{40}{9}L_\zeta - 10 + 8\ln 2 + \frac{4\pi^2}{9}\right)\right.$$
$$\left.\left. - \frac{112}{27}L_\zeta\right] + j_2\right\},$$

Constant that was obtained numerically

Power counting

Power counting the observable requires examining several scales

$$\tau_f = \frac{1}{p^-} = \frac{x(1-x)p^+}{\mathbf{p}^2} \qquad e^{iL^+/\tau_f}$$

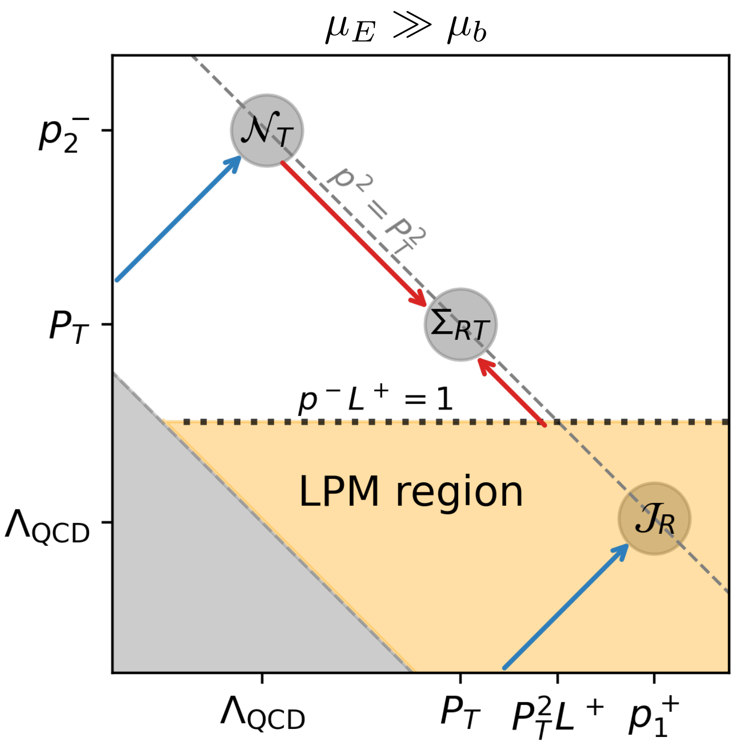
$\tau_f \ll L^+$
LPM phase rapidly oscillates

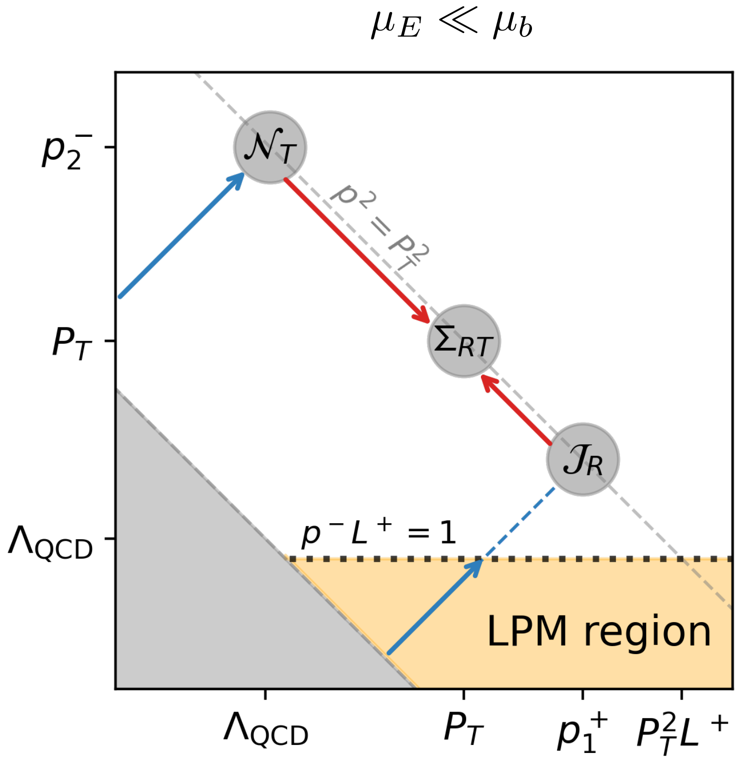
$\tau_f \gg L^+$
LPM phase contributes

Power counting the observable requires examining several scales. Collinear logs are different in these cases

$$\Lambda_{\rm QCD}^2 \ll \mu_E^2 \ll Q^2$$

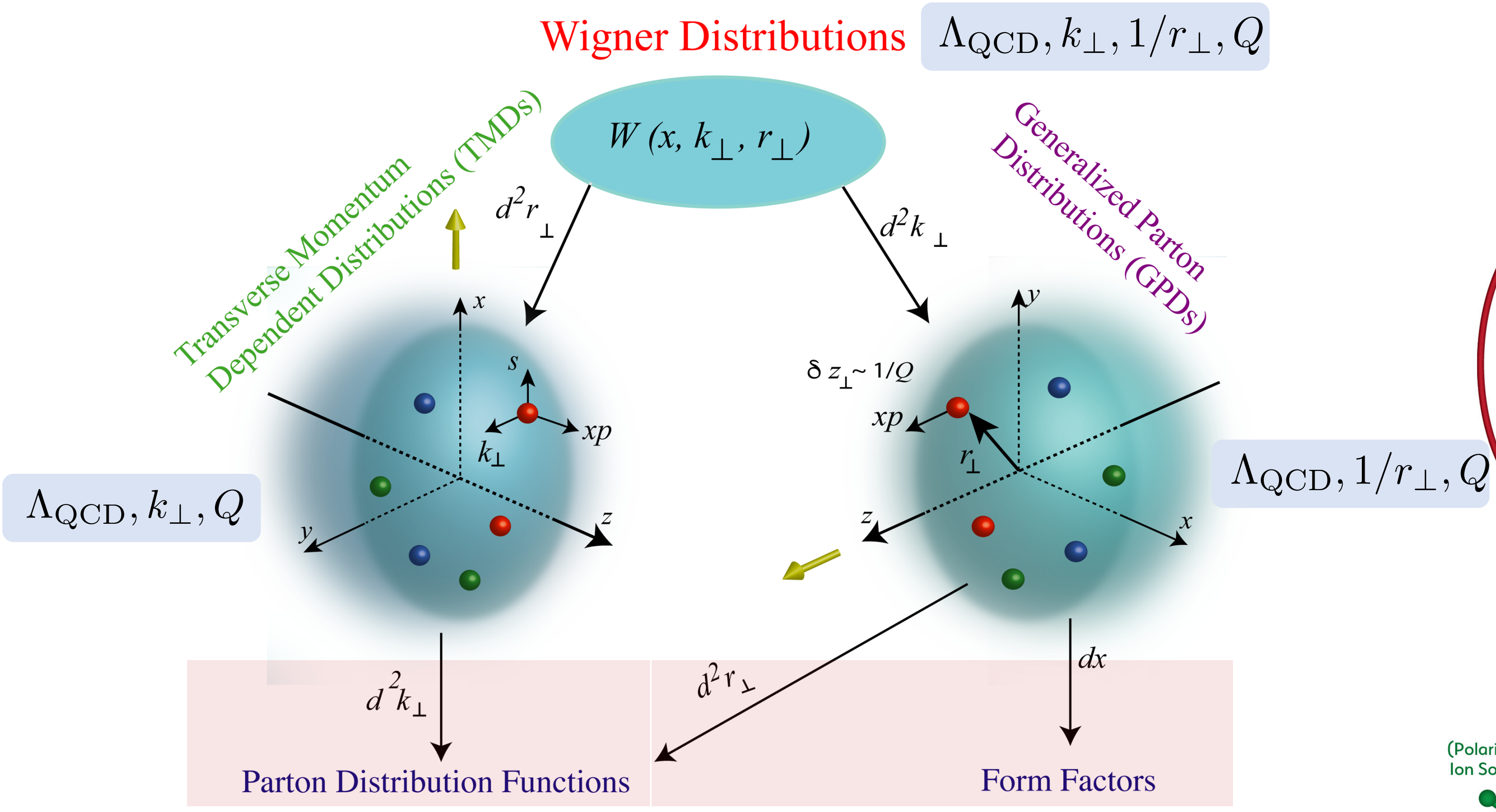
$$\mu_E^2 = p_1^+/L^+$$





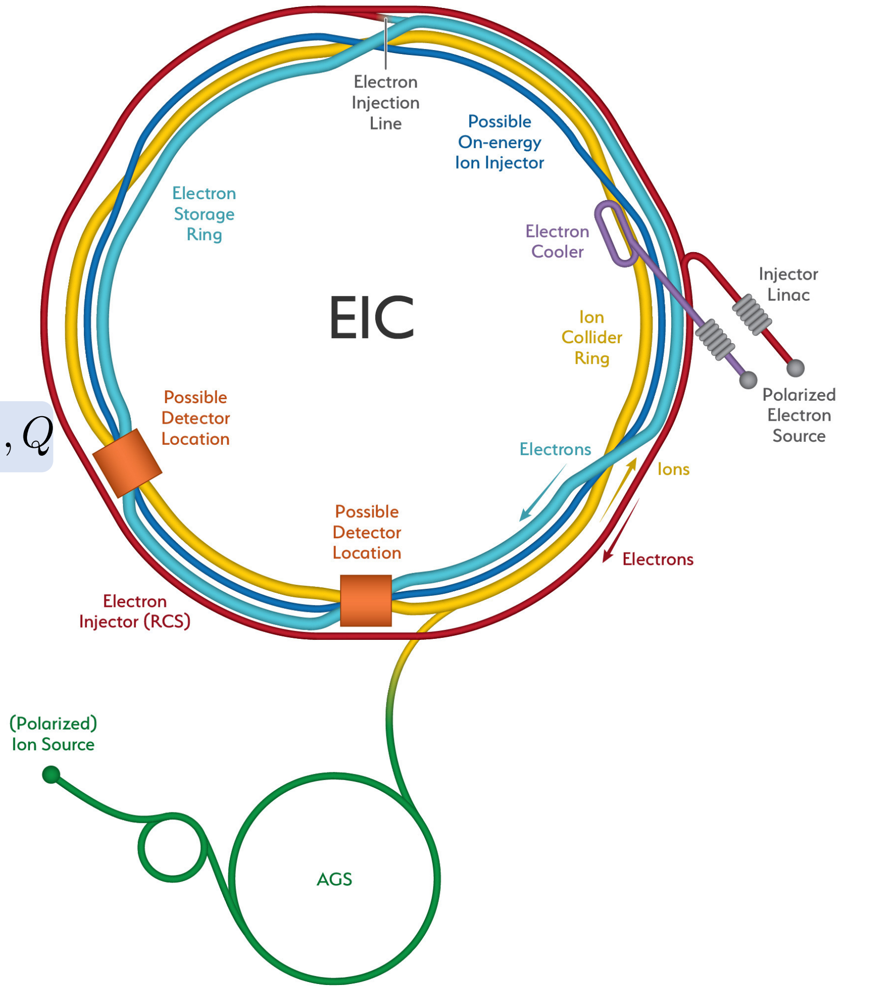
The structure of matter: Nuclear matter

Distributions of partons in hadrons



The reach of EFTs

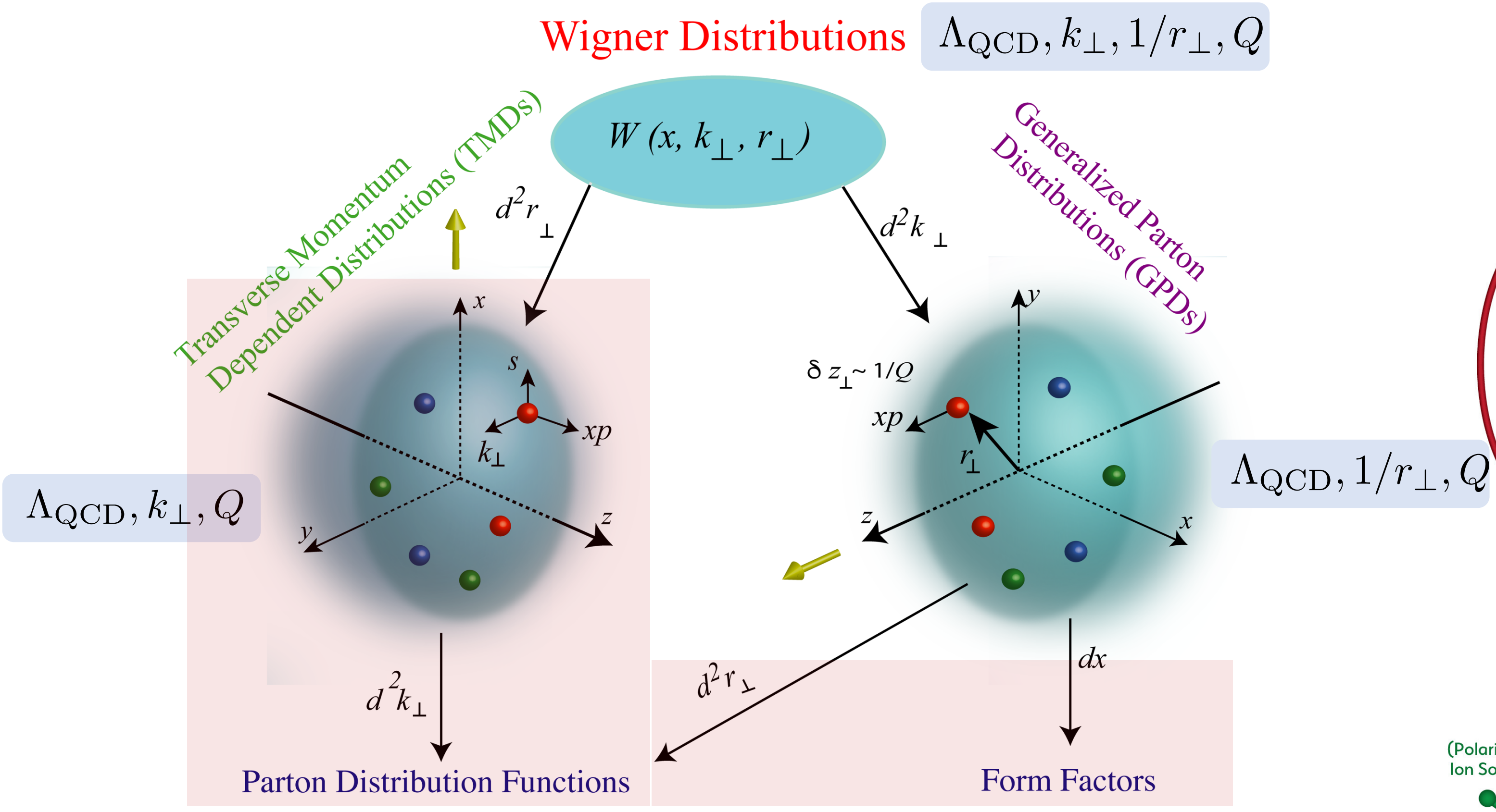
Highly differential distributions require high luminosity



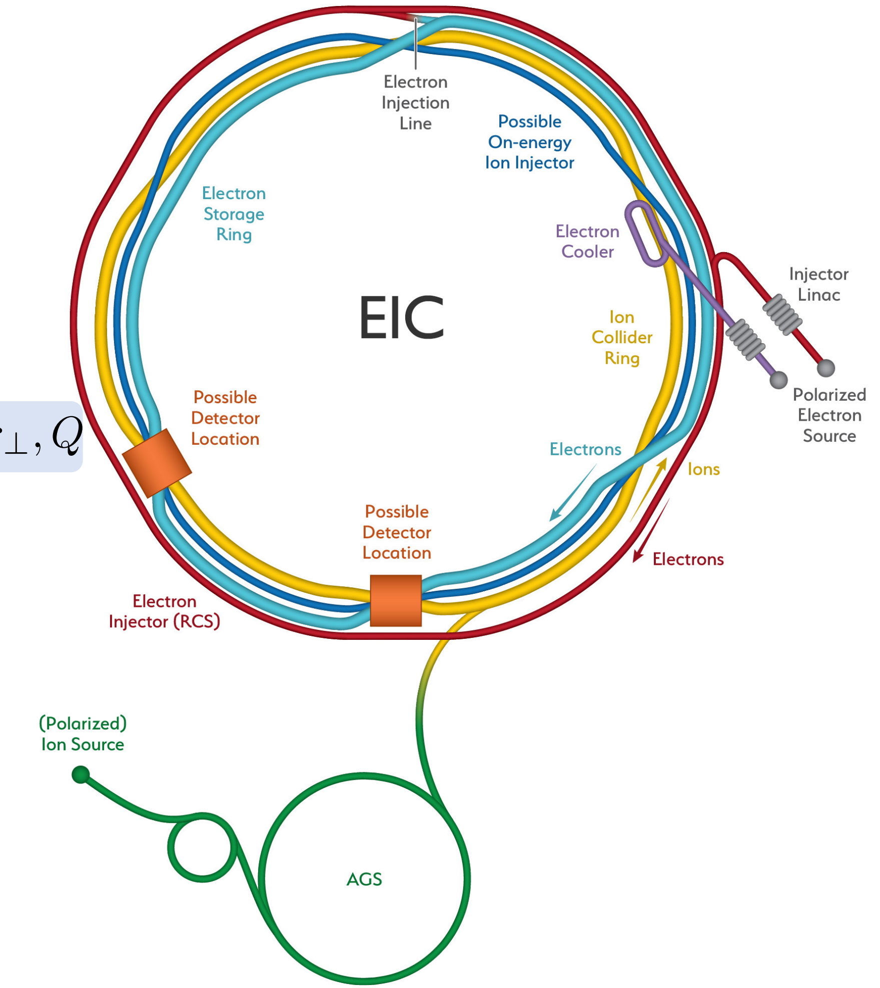
The structure of matter in the vacuum

Distributions of partons in hadrons

Highly differential distributions require high luminosity



The reach of EFTs



Medium induced collinear divergences

The one-loop computation yields a medium-induced collinear divergence

$$\sum_{T,j} x f_{q/p}(x) \otimes \mathcal{J}^{(1),\mathrm{coll}}_{q/q,F} \otimes_\perp \Sigma^{(0)}_{FT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} \otimes f_{j/N} \rho_0^- L^+$$

$$= \frac{\alpha_s^2(\mu^2)\rho_G^- L^+}{8\mu_E^2} B(w) \left(\frac{1}{2\epsilon} + \ln \frac{\mu^2}{\gamma(w)\mu_E^2} \right) 2C_F \left(\frac{2C_A + C_F}{x} - 2C_A \frac{d}{dx} \right) \left[x f_{q/a}(x) \right]$$

Medium-induced collinear divergence

Collinear divergences give rise to a medium modified DGLAP evolution equation

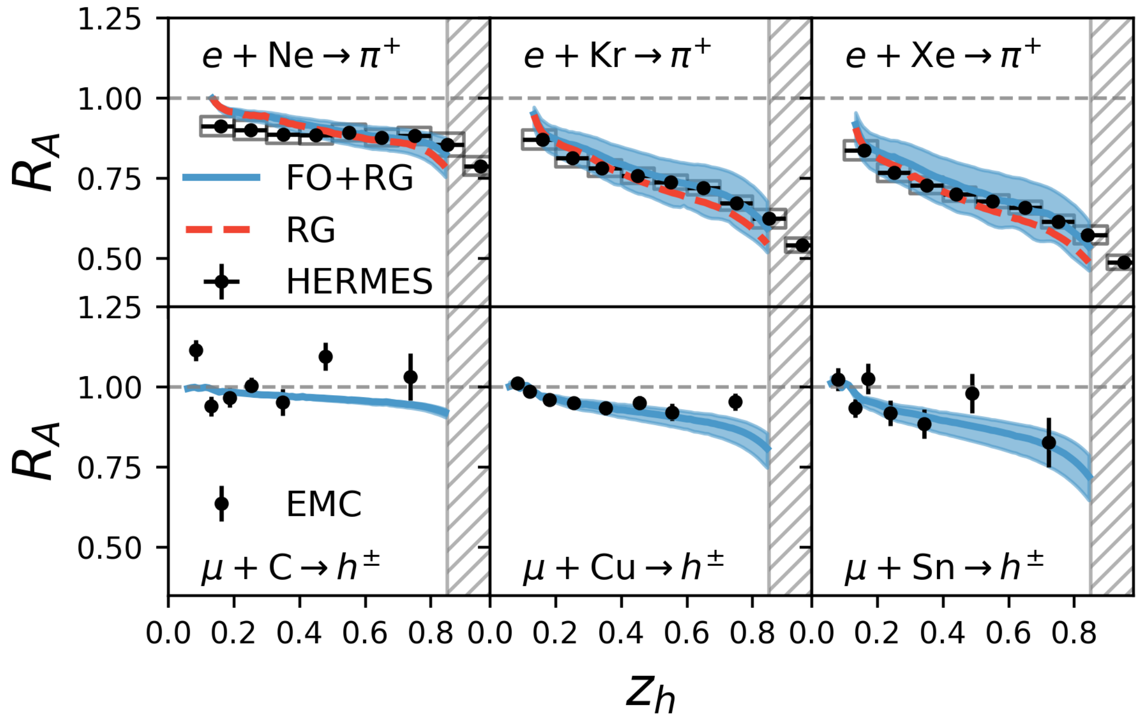
Flavor non-singlet $(q - \bar{q})$

Flavor singlet $(q + \bar{q}, g)$

$$\frac{\partial F_{q-\bar{q}}}{\partial t_\mu} = \left(4C_F C_A \frac{\partial}{\partial x} - \frac{4C_F C_A + 2C_F^2}{x} \right) F_{q-\bar{q}}$$

$$\frac{\partial F_{q+\bar{q}}}{\partial t_\mu} = \left(4C_F C_A \frac{\partial}{\partial x} - \frac{4C_F C_A + 2C_F^2}{x} \right) F_{q+\bar{q}} + C_F \frac{F_g}{x}$$

$$\frac{\partial F_g}{\partial t_\mu} = \left(4C_A^2 \frac{\partial}{\partial x} - \frac{2N_f C_F}{x} \right) F_g + 2C_F^2 \sum_q \frac{F_{q+\bar{q}}}{x}$$

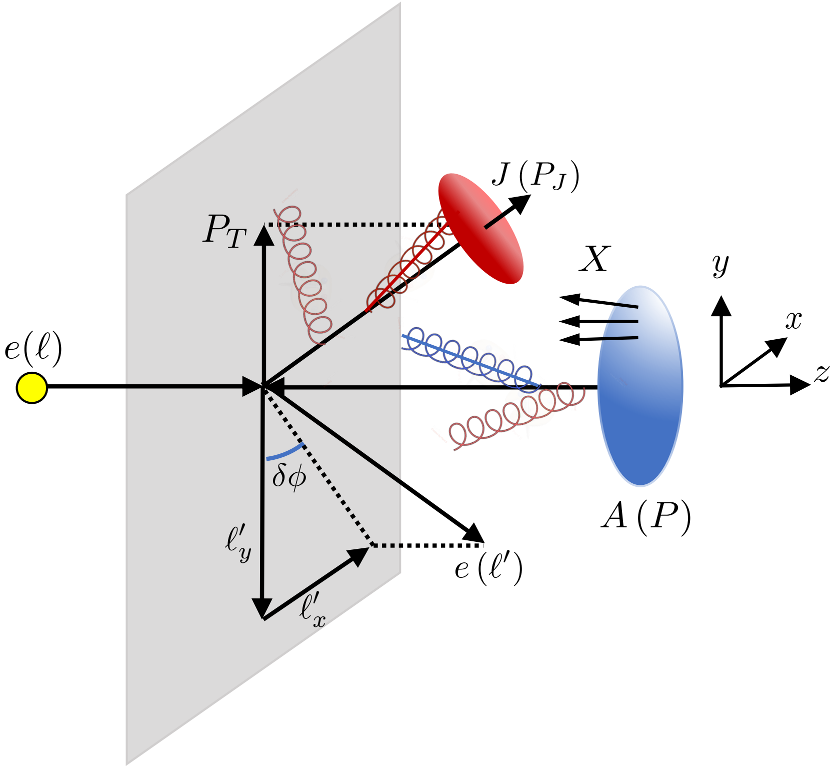


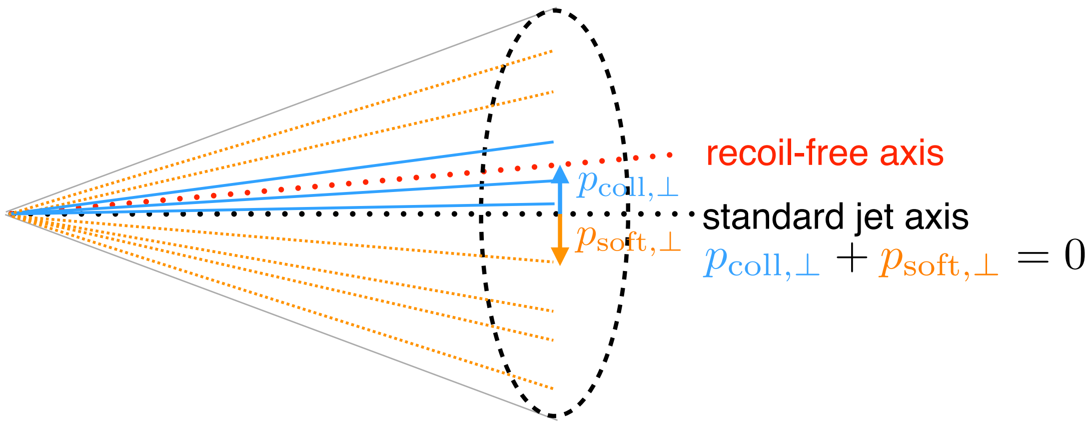
Motivation

Past work

Back-to-back lepton-jet production

Process proposed by: *Liu, Ringer, Vogelsang, Yuan (2019)*





Bertolini, Chan, and Thaler (2013)
Larkoski, Neill, and Thaler (2014)

Factorization using recoil-free jets: *Fang, Ke, Shao, JT (2023)*

Hard: $P_{JT}\,(1,1,1)$

Collinear: $P_{JT}\,(\lambda^2,1,\lambda)$

Jet: $P_{JT}\,(1,\lambda^2,\lambda)$

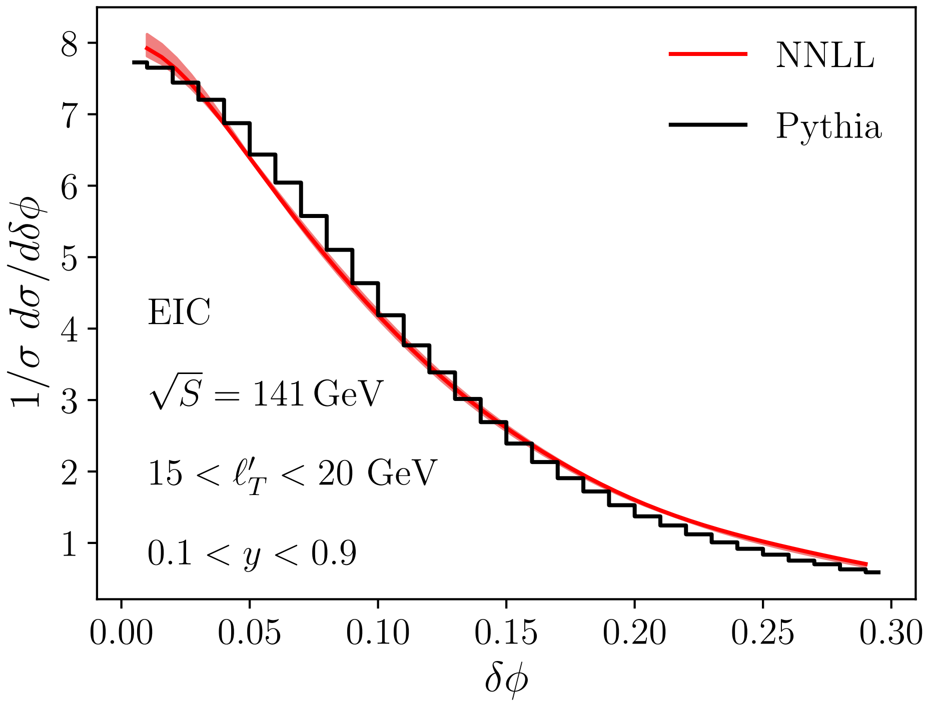
Global soft: $P_{JT}\,(\lambda,\lambda,\lambda)$

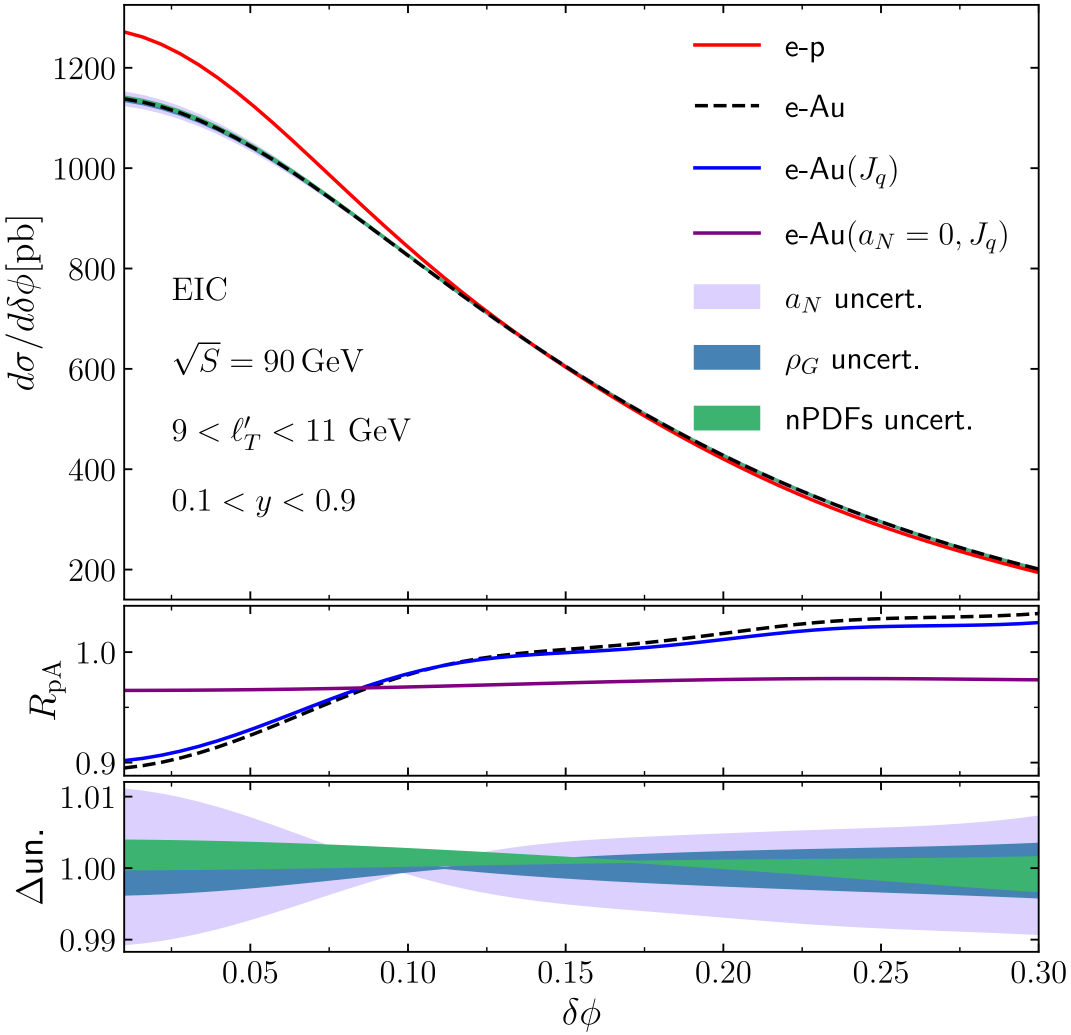
$$\frac{d\sigma_p}{d^2\ell'_T\,dy\,d\delta\phi} = \frac{\sigma_0\,\ell'_T}{1-y}\,H\,(Q,\mu_H)\int\frac{db}{2\pi}\cos\,(b\ell'_T\delta\phi)\sum_q e_q^2\,f_{q/p}\,(x_B,b,\mu_H,\zeta_{\mathcal{B}})\;\mathcal{J}_q\,(b,\mu_H,\zeta_{\mathcal{J}})$$

$$\frac{d\sigma_A}{d^2\ell'_T\,dy\,d\delta\phi} = \frac{\sigma_0\,\ell'_T}{1-y}\,H\,(Q,\mu_H)\int\frac{db}{2\pi}\cos\,(b\ell'_T\delta\phi)\sum_q e_q^2\,f_{q/A}\,(x_B,b,\mu_H,\zeta_{\mathcal{B}})\;\mathcal{J}_q^A\,(b,\mu_H,\zeta_{\mathcal{J}})$$

Predictions at the EIC

Comparison of our results with Pythia at NNLL





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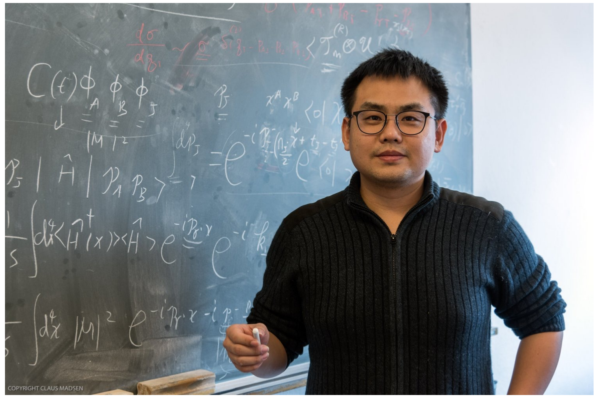
Higher order results

Results have been taken one step higher by: *Fang, Gao, Li, Shao (2024)*



$$J_q(b,\mu,\zeta) = 1 + \frac{\alpha_s C_F}{4\pi}\left[-L_b^2 + L_b(3+2L_\zeta) - \frac{5\pi^2}{6} + 7 - 6\ln 2\right]$$
$$+\left(\frac{\alpha_s}{4\pi}\right)^2\Bigg\{C_F^2\Bigg[\frac{L_b^4}{2} - L_b^3(3+2L_\zeta) + L_b^2\left(2L_\zeta^2 + 6L_\zeta - \frac{5}{2} + 6\ln 2 + \frac{5\pi^2}{6}\right)$$
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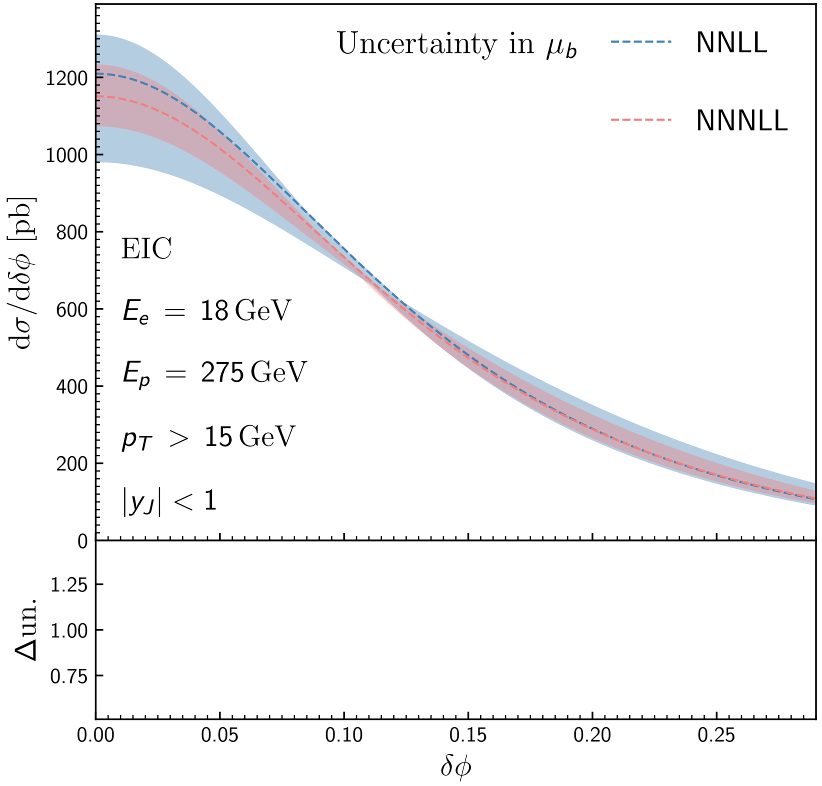
$$+ C_F C_A\Bigg[-\frac{22}{9}L_b^3 + L_b^2\left(\frac{11}{3}L_\zeta - \frac{35}{18} + \frac{\pi^2}{3}\right) + L_\zeta\left(\frac{404}{27} - 14\zeta_3\right)$$
$$+ L_b\left(L_\zeta\left(\frac{134}{9} - \frac{2\pi^2}{3}\right) + \frac{57}{2} - 22\ln 2 - \frac{11\pi^2}{9} - 12\zeta_3\right)\Bigg]$$
$$+ C_F T_F n_f\Bigg[\frac{8}{9}L_b^3 + L_b^2\left(\frac{2}{9} - \frac{4}{3}L_\zeta\right) + L_b\left(-\frac{40}{9}L_\zeta - 10 + 8\ln 2 + \frac{4\pi^2}{9}\right)$$
$$-\frac{112}{27}L_\zeta\Bigg] + j_2\Bigg\},$$

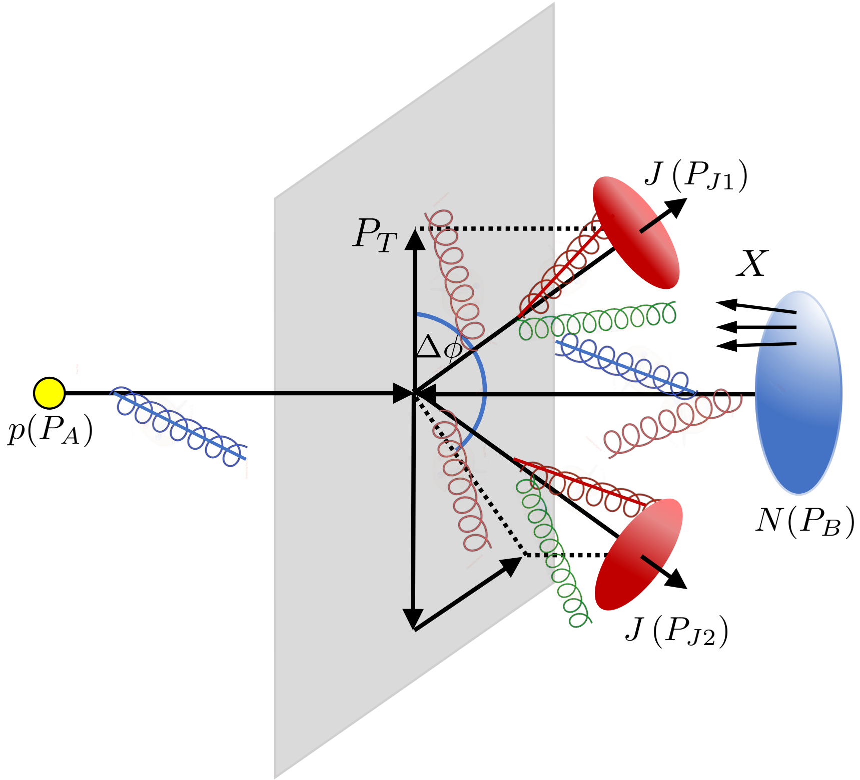
Constant that was obtained numerically



Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi (2019)

Factorization in pp and pA

Azimuthal angle decorrelations of di-jets measured at the CMS, are sensitive to nTMDs



Factorization and resummation derived in a SCET framework

$$\begin{aligned}
\textbf{hard}: &\quad p_h^\mu \sim p_T(1,1,1)\\
n_{a,b}\textbf{-collinear}: &\quad p_{c_i}^\mu \sim p_T\,(\delta\phi^2,1,\delta\phi)_{n_i\bar n_i},\\
\textbf{soft}: &\quad p_s^\mu \sim p_T\,(\delta\phi,\delta\phi,\delta\phi),\\
n_{c,d}\textbf{-jet}: &\quad p_{c_i}^\mu \sim p_T\,(R^2,1,R)_{n_i\bar n_i},\\
n_{c,d}\textbf{-collinear-soft}: &\quad p_{cs_i}^\mu \sim \frac{p_T\,\delta\phi}{R}(R^2,1,R)_{n_i\bar n_i},
\end{aligned}$$

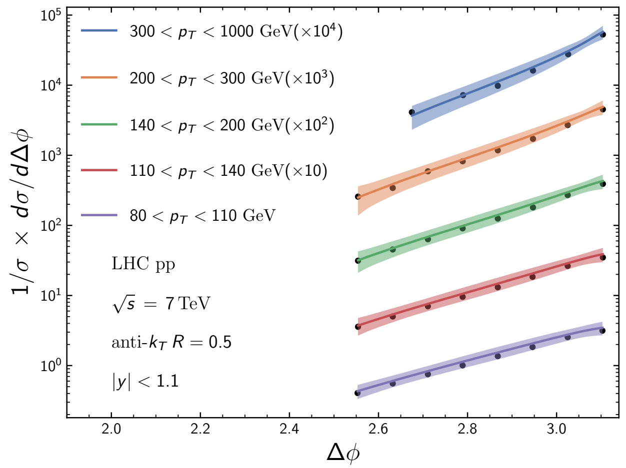
$$\begin{aligned}
\frac{\mathrm{d}^4\sigma_{pA}}{\mathrm{d}y_c\,\mathrm{d}y_d\,\mathrm{d}p_T^2\,\mathrm{d}\delta\phi} &= \sum_{abcd}\frac{p_T}{16\pi\hat s^2}\frac{1}{1+\delta_{cd}}\int_0^\infty\frac{2\mathrm{d}b}{\pi}\cos(bp_T\delta\phi)x_a\tilde f_{a/p}(x_a,\mu_{b_*})x_b\tilde f_{b/A}(x_b,\mu_{b_*})\\
&\times\exp\left\{-\int_{\mu_{b_*}}^{\mu_h}\frac{\mathrm{d}\mu}{\mu}\left[\gamma_{\rm cusp}\left(\alpha_s\right)C_H\ln\frac{\hat s}{\mu^2}+2\gamma_H\left(\alpha_s\right)\right]\right\}\\
&\times\sum_{KK'}\exp\left[-\int_{\mu_{b_*}}^{\mu_h}\frac{\mathrm{d}\mu}{\mu}\gamma_{\rm cusp}\left(\alpha_s\right)\left(\lambda_K+\lambda_{K'}^*\right)\right]H_{KK'}\left(\hat s,\hat t,\mu_h\right)W_{K'K}\left(b_*,\mu_{b_*}\right)\\
&\times\exp\left[-\int_{\mu_{b_*}}^{\mu_j}\frac{\mathrm{d}\mu}{\mu}\Gamma^{J_c}\left(\alpha_s\right)-\int_{\mu_{b_*}}^{\mu_j}\frac{\mathrm{d}\mu}{\mu}\Gamma^{J_d}\left(\alpha_s\right)\right]U_{\rm NG}^c\left(\mu_{b_*},\mu_j\right)U_{\rm NG}^d\left(\mu_{b_*},\mu_j\right)\\
&\times\exp\left[-S_{\rm NP}^a(b,Q_0,\sqrt{\hat s})-S_{\rm NP}^{b,A}(b,Q_0,\sqrt{\hat s})\right]
\end{aligned}$$

Description of pp and pA data

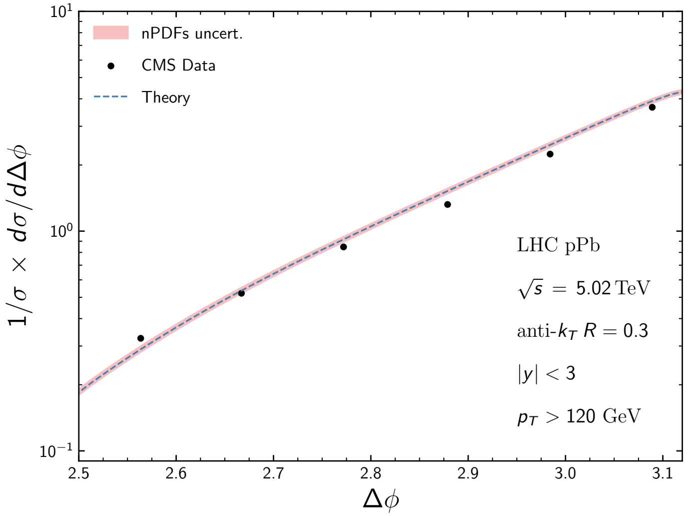
Strong consistency with the CMS measurements of the azimuthal angle decorrelation in pA and the ratio of the integrated azimuthal angle decorrelation.

Phys.Rev.Lett.106:122003,2011

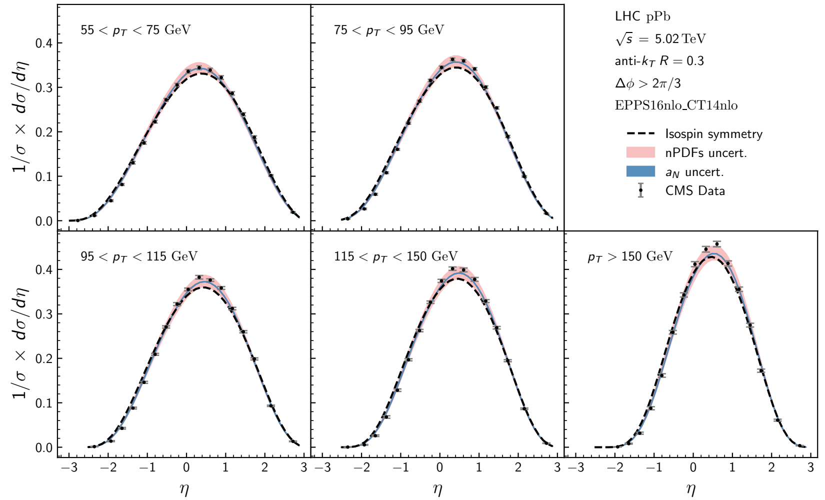
$$\frac{d^4\sigma_{pp}}{dy_c\,dy_d\,dp_T^2\,d\delta\phi}$$



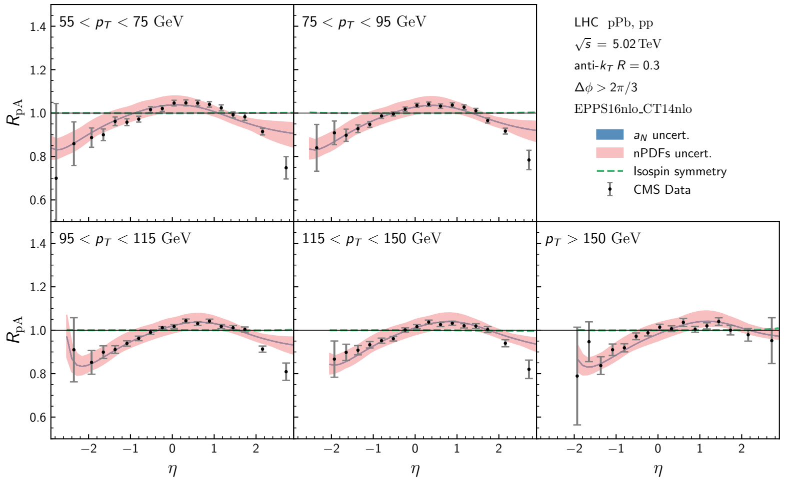
$$\frac{d^4\sigma_{pA}}{dy_c\,dy_d\,dp_T^2\,d\delta\phi}$$



Phys. Rev. Lett. 121, 062002 (2018)



$$R_{pA} = \frac{1}{A}\frac{d^4\sigma_{pA}}{dy_c\,dy_d\,dp_T^2\,d\Delta\phi} \Big/ \frac{d^4\sigma_{pp}}{dy_c\,dy_d\,dp_T^2\,d\Delta\phi}$$



Eur. Phys. J. C 74 (2014) 2951

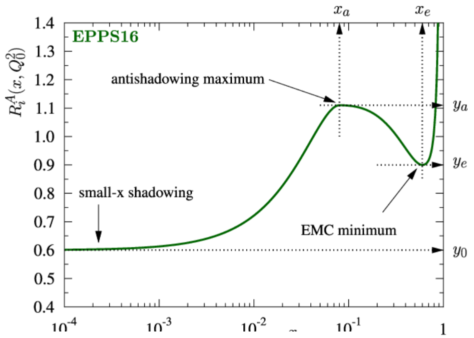
Phys. Rev. Lett. 121, 062002 (2018)

Towards first principles

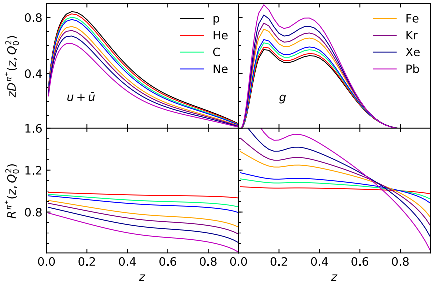
Three-dimensional images

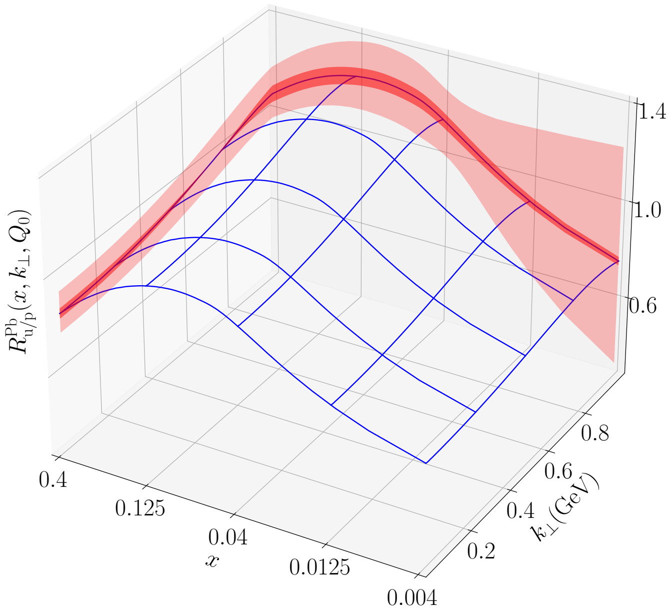
Ratios defined for nPDF and nFF *Alrashed, JT et al: Phys. Rev. Lett. 129 (2022)*

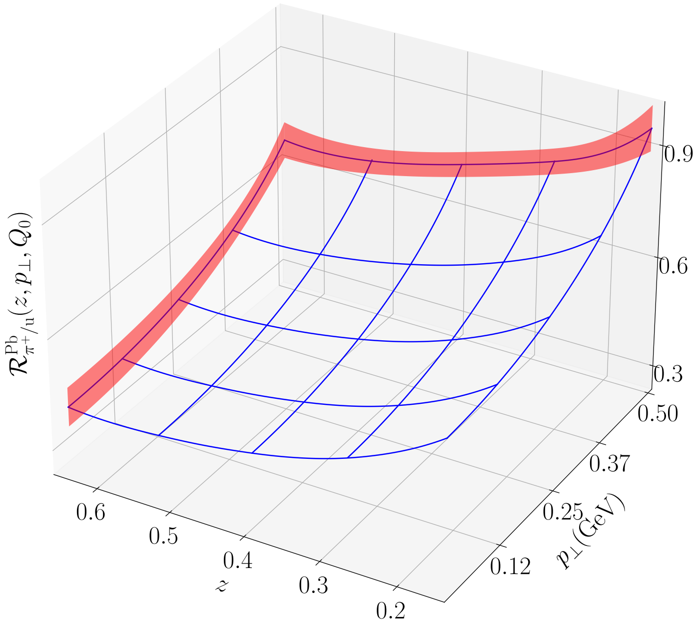
$$R_i^A\left(x, Q_0^2\right) = \frac{f_{i/p}^A\left(x, Q_0^2\right)}{f_{i/p}\left(x, Q_0^2\right)}$$



$$R_i^A\left(z, Q_0^2\right) = \frac{D_{h/i}^A\left(z, Q_0^2\right)}{D_{h/i}\left(z, Q_0^2\right)}$$





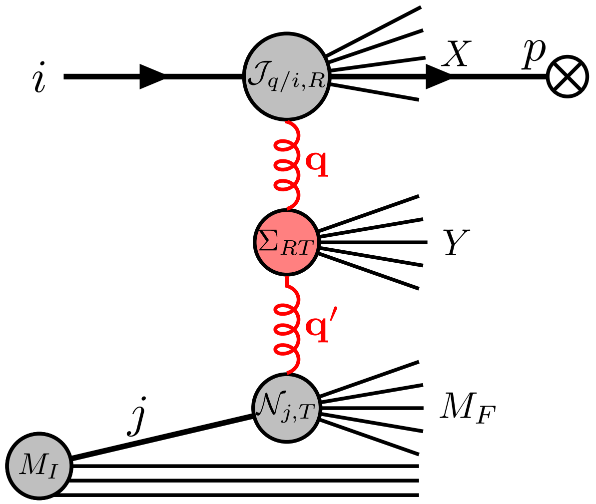


$$R_{u/p}^{\rm Pb}(x, k_\perp, Q_0) = \frac{f_{u/p}^{\rm Pb}\left(x, k_\perp, Q_0, Q_0^2\right)}{f_{u/p}\left(x, k_\perp, Q_0, Q_0^2\right)}$$

$$\mathcal{R}_{\pi^+/u}^{\rm Pb}\left(z, p_\perp, Q_0\right) = \frac{D_{\pi^+/u}^{\rm Pb}\left(z, p_\perp, Q_0, Q_0^2\right)}{D_{\pi^+/u}\left(z, p_\perp, Q_0, Q_0^2\right)}$$

Overview of graphs at one loop

In this paper, we study the correlations between the incoming proton and the medium



$$\mathcal{B}_{q/p,1} = \sum_{i=q,g} \sum_{j=q,\bar{q},g} \sigma_{q/i,j} \otimes f_{i/p} \otimes f_{j/N} \cdot \rho_0^- L^+,$$

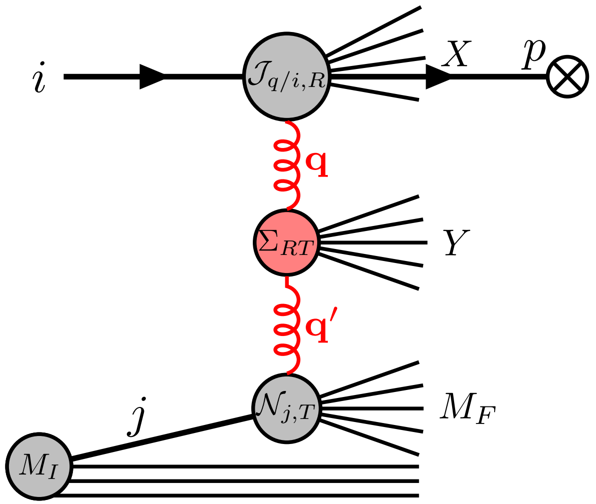
$$\sigma^{(0)}_{q/q,j} + \sigma^{(1)}_{q/q,j} = \left(\mathcal{J}^{(0)}_{q/q,F} + \mathcal{J}^{(1),\mathrm{rap}}_{q/q,F}\right) \otimes_\perp \Sigma^{(0)}_{FT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \\ + \mathcal{J}^{(1),\mathrm{coll}}_{q/q,F} \otimes_\perp \Sigma^{(0)}_{FT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \mathcal{J}^{(1),\mathrm{coll}}_{q/q,A} \otimes_\perp \Sigma^{(0)}_{AT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} \\ + \mathcal{J}^{(1),\mathrm{rap}}_{q/q,A} \otimes_\perp \Sigma^{(0)}_{AT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \mathcal{J}^{(0)}_{q/q,F} \otimes_\perp \Sigma^{(1)}_{FT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \mathcal{J}^{(0)}_{q/q,F} \otimes_\perp \Sigma^{(0)}_{FT} \otimes_\perp \mathcal{N}^{(1)}_{j,T} \\ + \Delta\sigma^{\mathrm{NLO}}_{q/q,j}\,,$$

Here we use the short-hand

$$\left[\mathcal{J}_{q/i,R} \otimes_\perp \Sigma_{RT} \otimes_\perp \mathcal{N}_{j,T}\right](x_1, \mathbf{p}, \mu) = \int \frac{d^{2-2\epsilon}\mathbf{q}}{(2\pi)^{2-2\epsilon}} \frac{d^{2-2\epsilon}\mathbf{q}'}{(2\pi)^{2-2\epsilon}} \left[\mathcal{J}_{q/i,R}(x_1, \mathbf{p}, \mathbf{q}, \mu, \nu) \frac{g_s^2}{\mathbf{q}^2}\right] \\ \times \left[\left(\frac{g_s^2}{\mathbf{q}^2}\right)^{-1} \Sigma_{RT}\left(\mathbf{q}, \mathbf{q}', \nu, \nu'\right) \left(\frac{g_s^2}{\mathbf{q}'^2}\right)^{-1}\right] \times \left[\frac{g_s^2}{\mathbf{q}'^2} \mathcal{N}_{j,T}\left(\mathbf{q}', \nu'\right)\right]$$

Overview of graphs at one loop

In this paper, we study the correlations between the incoming proton and the medium

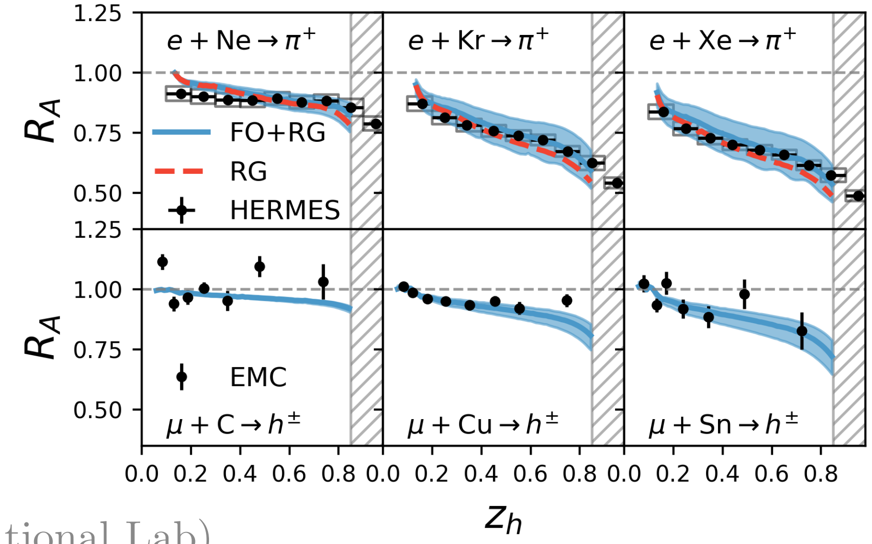


$$\mathcal{B}_{q/p,1} = \sum_{i=q,g} \sum_{j=q,\bar{q},g} \sigma_{q/i,j} \otimes f_{i/p} \otimes f_{j/N} \cdot \rho_0^- L^+ ,$$

$$\begin{aligned}
\sigma^{(0)}_{q/q,j} + \sigma^{(1)}_{q/q,j} = & \left(\mathcal{J}^{(0)}_{q/q,F} + \mathcal{J}^{(1),\mathrm{rap}}_{q/q,F} \right) \otimes_\perp \Sigma^{(0)}_{FT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \\
& + \mathcal{J}^{(1),\mathrm{coll}}_{q/q,F} \otimes_\perp \Sigma^{(0)}_{FT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \mathcal{J}^{(1),\mathrm{coll}}_{q/q,A} \otimes_\perp \Sigma^{(0)}_{AT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} \\
& + \mathcal{J}^{(1),\mathrm{rap}}_{q/q,A} \otimes_\perp \Sigma^{(0)}_{AT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \mathcal{J}^{(0)}_{q/q,F} \otimes_\perp \Sigma^{(1)}_{FT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \mathcal{J}^{(0)}_{q/q,F} \otimes_\perp \Sigma^{(0)}_{FT} \otimes_\perp \mathcal{N}^{(1)}_{j,T} \\
& + \Delta\sigma^{\mathrm{NLO}}_{q/q,j} ,
\end{aligned}$$

Collinear divergent terms

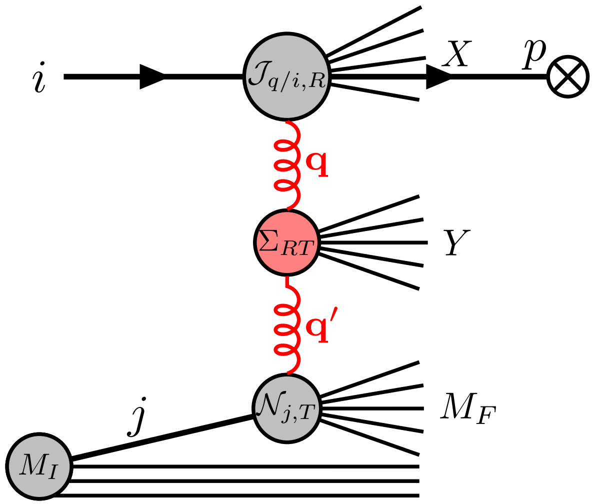
Collinear divergent terms have been considered in Semi-Inclusive DIS



Ke, Vitev (2023)

Overview of graphs at one loop

In this paper, we study the correlations between the incoming proton and the medium



$$\mathcal{B}_{q/p,1} = \sum_{i=q,g} \sum_{j=q,\bar{q},g} \sigma_{q/i,j} \otimes f_{i/p} \otimes f_{j/N} \cdot \rho_0^- L^+,$$

$$\sigma^{(0)}_{q/q,j} + \sigma^{(1)}_{q/q,j} = \left(\mathcal{J}^{(0)}_{q/q,F} + \mathcal{J}^{(1),\text{rap}}_{q/q,F}\right) \otimes_\perp \Sigma^{(0)}_{FT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} +$$
$$+ \mathcal{J}^{(1),\text{coll}}_{q/q,F} \otimes_\perp \Sigma^{(0)}_{FT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \mathcal{J}^{(1),\text{coll}}_{q/q,A} \otimes_\perp \Sigma^{(0)}_{AT} \otimes_\perp \mathcal{N}^{(0)}_{j,T}$$
$$+ \mathcal{J}^{(1),\text{rap}}_{q/q,A} \otimes_\perp \Sigma^{(0)}_{AT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \mathcal{J}^{(0)}_{q/q,F} \otimes_\perp \Sigma^{(1)}_{FT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \mathcal{J}^{(0)}_{q/q,F} \otimes_\perp \Sigma^{(0)}_{FT} \otimes_\perp \mathcal{N}^{(1)}_{j,T}$$
$$+ \Delta\sigma^{\text{NLO}}_{q/q,j},$$

Collinear divergent terms

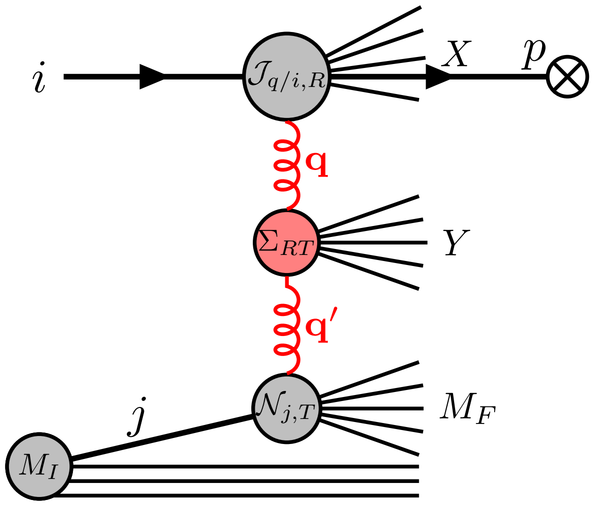
Collinear rapidity divergent terms

Here we use the short-hand

$$\left[\mathcal{J}_{q/i,R} \otimes_\perp \Sigma_{RT} \otimes_\perp \mathcal{N}_{j,T}\right](x_1, \mathbf{p}, \mu) = \int \frac{d^{2-2\epsilon}\mathbf{q}}{(2\pi)^{2-2\epsilon}} \frac{d^{2-2\epsilon}\mathbf{q}'}{(2\pi)^{2-2\epsilon}} \left[\mathcal{J}_{q/i,R}(x_1, \mathbf{p}, \mathbf{q}, \mu, \nu) \frac{g_s^2}{\mathbf{q}^2}\right]$$
$$\times \left[\left(\frac{g_s^2}{\mathbf{q}^2}\right)^{-1} \Sigma_{RT}(\mathbf{q}, \mathbf{q}', \nu, \nu') \left(\frac{g_s^2}{\mathbf{q}'^2}\right)^{-1}\right] \times \left[\frac{g_s^2}{\mathbf{q}'^2} \mathcal{N}_{j,T}(\mathbf{q}', \nu')\right]$$

Overview of graphs at one loop

In this paper, we study the correlations between the incoming proton and the medium



$$\mathcal{B}_{q/p,1} = \sum_{i=q,g} \sum_{j=q,\bar{q},g} \sigma_{q/i,j} \otimes f_{i/p} \otimes f_{j/N} \cdot \rho_0^- L^+,$$

$$\begin{aligned}\sigma^{(0)}_{q/q,j} + \sigma^{(1)}_{q/q,j} = & \left(\mathcal{J}^{(0)}_{q/q,F} + \mathcal{J}^{(1),\text{rap}}_{q/q,F}\right) \otimes_\perp \Sigma^{(0)}_{FT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \\ & + \mathcal{J}^{(1),\text{coll}}_{q/q,F} \otimes_\perp \Sigma^{(0)}_{FT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \mathcal{J}^{(1),\text{coll}}_{q/q,A} \otimes_\perp \Sigma^{(0)}_{AT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} \\ & + \mathcal{J}^{(1),\text{rap}}_{q/q,A} \otimes_\perp \Sigma^{(0)}_{AT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \mathcal{J}^{(0)}_{q/q,F} \otimes_\perp \Sigma^{(1)}_{FT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \mathcal{J}^{(0)}_{q/q,F} \otimes_\perp \Sigma^{(0)}_{FT} \otimes_\perp \mathcal{N}^{(1)}_{j,T} \\ & + \Delta\sigma^{\text{NLO}}_{q/q,j}\,,\end{aligned}$$

Collinear divergent terms

Collinear rapidity divergent terms

Soft and anti-collinear rapidity divergent terms

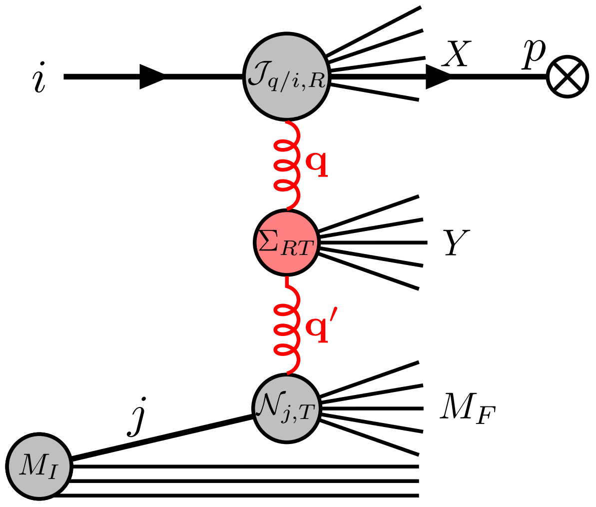
Here we use the short-hand

$$\left[\mathcal{J}_{q/i,R} \otimes_\perp \Sigma_{RT} \otimes_\perp \mathcal{N}_{j,T}\right](x_1, \mathbf{p}, \mu) = \int \frac{d^{2-2\epsilon}\mathbf{q}}{(2\pi)^{2-2\epsilon}} \frac{d^{2-2\epsilon}\mathbf{q}'}{(2\pi)^{2-2\epsilon}} \left[\mathcal{J}_{q/i,R}(x_1, \mathbf{p}, \mathbf{q}, \mu, \nu) \frac{g_s^2}{\mathbf{q}^2}\right]$$

$$\times \left[\left(\frac{g_s^2}{\mathbf{q}^2}\right)^{-1} \Sigma_{RT}\left(\mathbf{q}, \mathbf{q}', \nu, \nu'\right) \left(\frac{g_s^2}{\mathbf{q}'^2}\right)^{-1}\right] \times \left[\frac{g_s^2}{\mathbf{q}'^2} \mathcal{N}_{j,T}\left(\mathbf{q}', \nu'\right)\right]$$

Overview of graphs at one loop

In this paper, we study the correlations between the incoming proton and the medium



$$\mathcal{B}_{q/p,1} = \sum_{i=q,g} \sum_{j=q,\bar{q},g} \sigma_{q/i,j} \otimes f_{i/p} \otimes f_{j/N} \cdot \rho_0^- L^+,$$

$$\sigma^{(0)}_{q/q,j} + \sigma^{(1)}_{q/q,j} = \left(\mathcal{J}^{(0)}_{q/q,F} + \mathcal{J}^{(1),\mathrm{rap}}_{q/q,F}\right) \otimes_\perp \Sigma^{(0)}_{FT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \\ + \mathcal{J}^{(1),\mathrm{coll}}_{q/q,F} \otimes_\perp \Sigma^{(0)}_{FT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \mathcal{J}^{(1),\mathrm{coll}}_{q/q,A} \otimes_\perp \Sigma^{(0)}_{AT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} \\ + \mathcal{J}^{(1),\mathrm{rap}}_{q/q,A} \otimes_\perp \Sigma^{(0)}_{AT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \mathcal{J}^{(0)}_{q/q,F} \otimes_\perp \Sigma^{(1)}_{FT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \mathcal{J}^{(0)}_{q/q,F} \otimes_\perp \Sigma^{(0)}_{FT} \otimes_\perp \mathcal{N}^{(1)}_{j,T} \\ + \Delta\sigma^{\mathrm{NLO}}_{q/q,j},$$

Collinear divergent terms

Collinear rapidity divergent terms

Soft and anti-collinear rapidity divergent terms

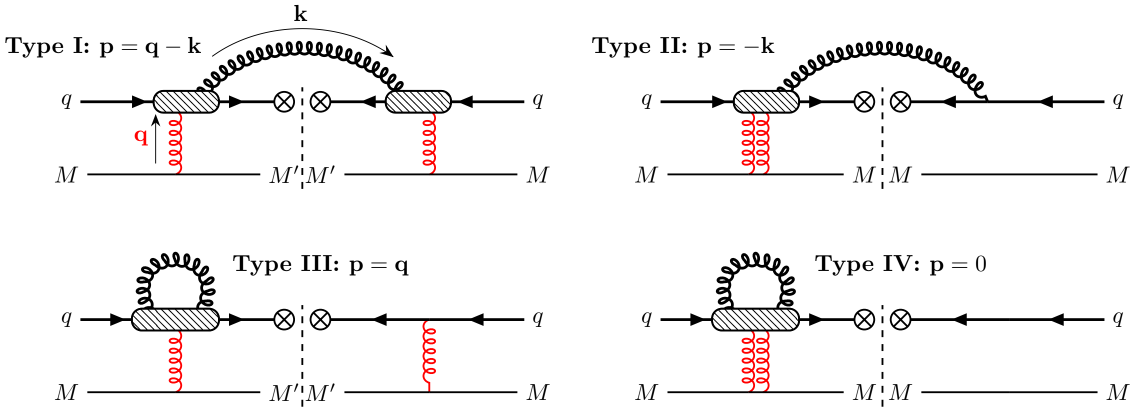
Finite terms

Here we use the short-hand

$$\left[\mathcal{J}_{q/i,R} \otimes_\perp \Sigma_{RT} \otimes_\perp \mathcal{N}_{j,T}\right](x_1, \mathbf{p}, \mu) = \int \frac{d^{2-2\epsilon}\mathbf{q}}{(2\pi)^{2-2\epsilon}} \frac{d^{2-2\epsilon}\mathbf{q}'}{(2\pi)^{2-2\epsilon}} \left[\mathcal{J}_{q/i,R}(x_1, \mathbf{p}, \mathbf{q}, \mu, \nu) \frac{g_s^2}{\mathbf{q}^2}\right] \\ \times \left[\left(\frac{g_s^2}{\mathbf{q}^2}\right)^{-1} \Sigma_{RT}(\mathbf{q}, \mathbf{q}', \nu, \nu') \left(\frac{g_s^2}{\mathbf{q}'^2}\right)^{-1}\right] \times \left[\frac{g_s^2}{\mathbf{q}'^2} \mathcal{N}_{j,T}(\mathbf{q}', \nu')\right]$$

Collinear matching function at one loop

The finite contributions to the collinear function can be organized based on the type of emission



We use the short-hand

Type K	$\mathcal{I}_{K,F}(x,\mathbf{k},\mathbf{q})$	$\mathcal{I}_{K,A}(x,\mathbf{k},\mathbf{q})$
I	$\frac{1}{\mathbf{Q}_1^2} + 2\frac{\mathbf{Q}_2}{\mathbf{Q}_2^2}\cdot\left(\frac{\mathbf{Q}_2}{\mathbf{Q}_2^2} - \frac{\mathbf{Q}_1}{\mathbf{Q}_1^2}\right)\phi_2$	$\frac{1}{\mathbf{Q}_3^2} - \frac{\mathbf{Q}_1}{\mathbf{Q}_1^2}\cdot\frac{\mathbf{Q}_3}{\mathbf{Q}_3^2} + \frac{\mathbf{Q}_2}{\mathbf{Q}_2^2}\cdot\left(\frac{\mathbf{Q}_1}{\mathbf{Q}_1^2} - \frac{\mathbf{Q}_3}{\mathbf{Q}_3^2}\right)\phi_2$
II	$-\frac{1}{\mathbf{Q}_1^2}$	$\frac{\mathbf{Q}_1}{\mathbf{Q}_1^2}\cdot\left(\frac{\mathbf{Q}_1}{\mathbf{Q}_1^2} - \frac{\mathbf{Q}_3}{\mathbf{Q}_3^2}\right)(\phi_1 - 1)$
III	$-2\frac{\mathbf{Q}_2}{\mathbf{Q}_2^2}\cdot\left(\frac{\mathbf{Q}_2}{\mathbf{Q}_2^2} - \frac{\mathbf{Q}_1}{\mathbf{Q}_1^2}\right)\phi_2$	$-\frac{\mathbf{Q}_1\cdot\mathbf{Q}_2}{\mathbf{Q}_1^2\mathbf{Q}_2^2}\phi_2 + \frac{\mathbf{Q}_1}{\mathbf{Q}_1^2}\cdot\frac{\mathbf{Q}_4}{\mathbf{Q}_4^2}\phi_4$
IV	0	$-\frac{1}{\mathbf{Q}_1^2}\phi_1 + \frac{\mathbf{Q}_1}{\mathbf{Q}_1^2}\cdot\frac{\mathbf{Q}_3}{\mathbf{Q}_3^2}\phi_3$

The finite contributions to the collinear function can be organized based on the type of emission

$$\mathcal{J}^{(1)}_{q/q,R}(x,\mathbf{p},\mathbf{q}) = \frac{g_s^2 C_F}{2\pi} p_{qq,\epsilon}(x) \int d^{2-2\epsilon}\mathbf{k}\left[\delta^{(2-2\epsilon)}(\mathbf{p}-\mathbf{q}+\mathbf{k})\mathcal{I}_{\mathrm{I,R}} + \delta^{(2-2\epsilon)}(\mathbf{p}+\mathbf{k})\mathcal{I}_{\mathrm{II,R}}\right]$$
$$+ \frac{g_s^2 C_F}{2\pi}\delta(1-x)\int_0^1 dx' p_{qq,\epsilon}(x')\int d^{2-2\epsilon}\mathbf{k}\left[\delta^{(2-2\epsilon)}(\mathbf{p}-\mathbf{q})\mathcal{I}_{\mathrm{III,R}} + \delta^{(2-2\epsilon)}(\mathbf{p})\mathcal{I}_{\mathrm{IV,R}}\right]$$

$$\mathbf{Q}_1 = x\mathbf{k} - (1-x)(\mathbf{p}_0 - \mathbf{k})$$
$$\mathbf{Q}_2 = x\mathbf{k} - (1-x)(\mathbf{p}_0 - \mathbf{k} + \mathbf{q})$$
$$\mathbf{Q}_3 = x(\mathbf{k}-\mathbf{q}) - (1-x)(\mathbf{p}_0 - \mathbf{k} + \mathbf{q})$$
$$\mathbf{Q}_4 = x(\mathbf{k}+\mathbf{q}) - (1-x)(\mathbf{p}_0 - \mathbf{k})$$

Generate collinear divergences

Generate rapidity divergences (focus of this talk)

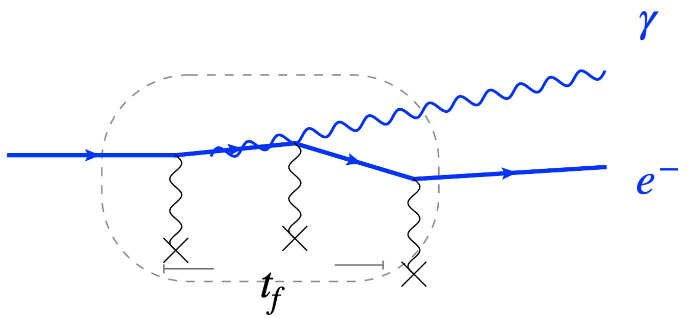
Coherence in the scattering

There are two competing time-scales in the process

$\tau_f \sim \frac{1}{p^-}$ — *Time-scale for emission*

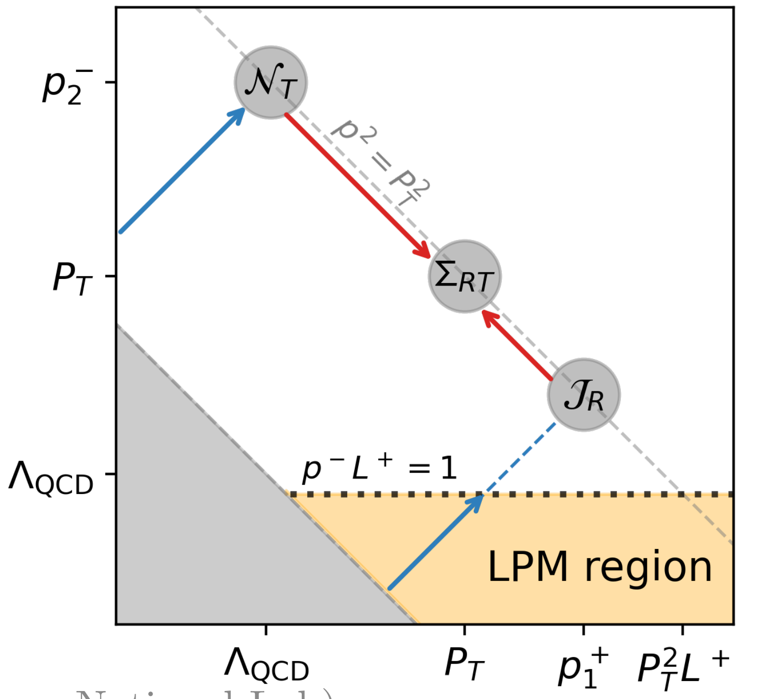
L^+ — *Time-scale for traversing the medium*

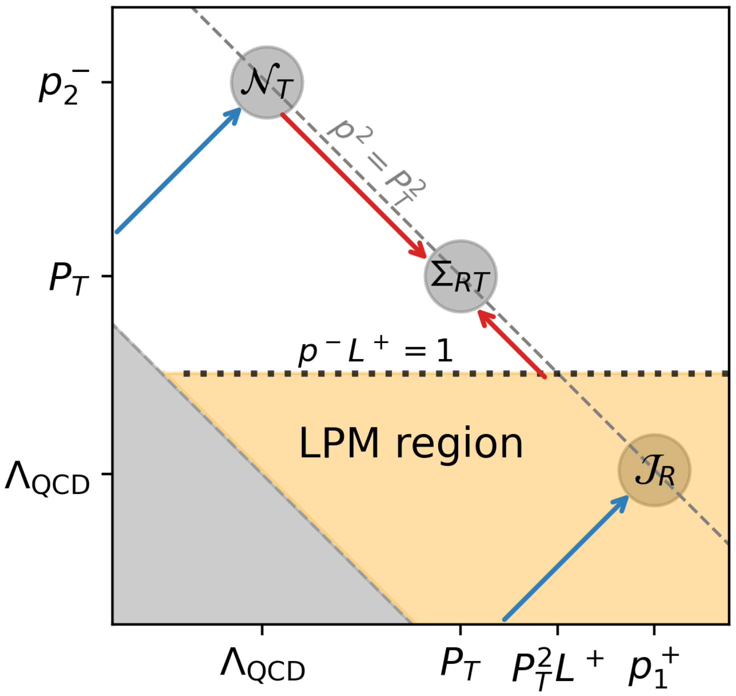
e^{iL^+/τ_f} — *LPM phase*



There are two regions with distinct power countings

$\tau_f \ll L^+$	$\tau_f \gg L^+$
Emissions break coherence (GB region)	Coherence is unbroken (LPM region)





Rapidity divergence in the collinear function

The one loop contribution to the collinear function takes on a simple form

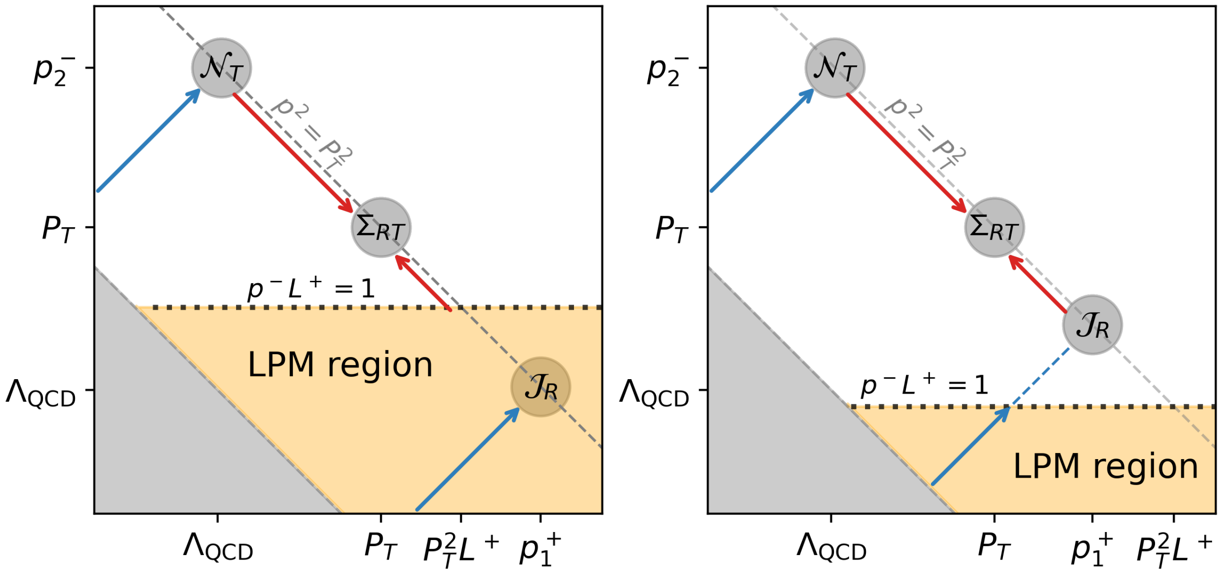
$$\mathcal{J}^{(0)}_{q/q,F} \otimes_\perp \Sigma^{(0)}_{FT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} + \mathcal{J}^{(1),\mathrm{rap}}_{q/q,A} \otimes_\perp \Sigma^{(0)}_{AT} \otimes_\perp \mathcal{N}^{(0)}_{j,T} = \int \frac{d^{2-2\epsilon}\mathbf{p}}{(2\pi)^{-2\epsilon}} e^{-i\mathbf{p}\cdot\mathbf{b}} \int \frac{d^{2-2\epsilon}\mathbf{q}}{(2\pi)^{2-2\epsilon}} \int \frac{d^{2-2\epsilon}\mathbf{q}'}{(2\pi)^{2-2\epsilon}}$$

$$\eta(x) = \left(\frac{(1-x)p_1^+}{\nu}\right)^{-\tau}$$

$$\times \left[1 + \left(-\frac{1}{\tau} + \mathcal{L}_n\right)\hat{\mathcal{C}}\right]\left[\frac{\mathcal{J}^{(0)}_{q/q,F}}{\mathbf{q}^2}\right]\mathbf{q}^2\mathbf{q}'^2\Sigma^{(0)}_{FT}\frac{\mathcal{N}^{(0)}_{j,T}}{\mathbf{q}'^2}$$

$$\hat{\mathcal{C}}[v(\mathbf{q}^2)] = \frac{g_s^2 C_A}{\pi}\int \frac{d^{2-2\epsilon}\mathbf{k}}{(2\pi)^{2-2\epsilon}}\left[\frac{1}{(\mathbf{q}-\mathbf{k})^2}v(\mathbf{k}^2) - \frac{\mathbf{q}^2}{2\mathbf{k}^2(\mathbf{q}-\mathbf{k})^2}v(\mathbf{q}^2)\right]$$

The one loop contribution to the collinear function takes on a simple form

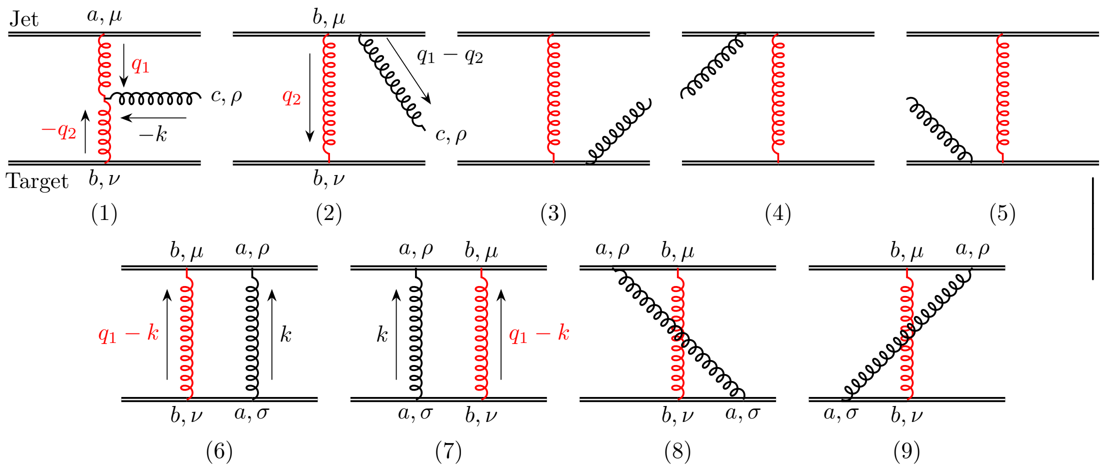


Different natural rapidity scale enters into the different regions

$$\mathcal{L}_n \sim \ln \frac{\min\{2L^+\mu_b^2, x_1 P_a^+\}}{\nu}$$

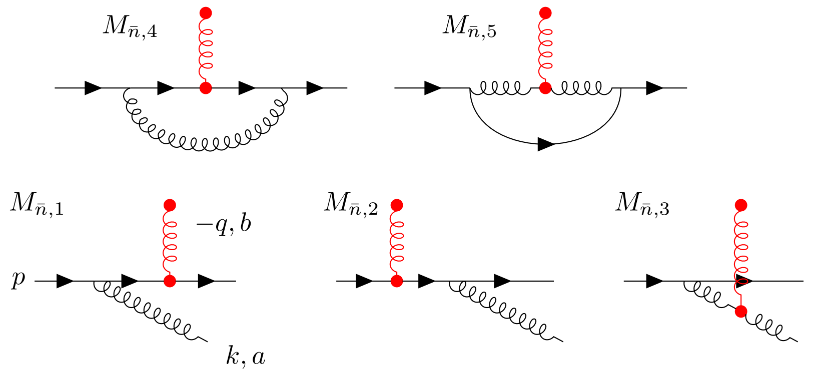
Soft and anti-collinear contribution

Graphs from the soft sector $l^\mu \sim Q\,(\lambda, \lambda, \lambda)$



$$\left|\frac{l_z}{\nu}\right|^{-\tau/2}$$

Graphs from the anti-collinear sector



Double Glauber exchange graphs are scaleless

Soft and collinear functions at one loop

$$\mathcal{J}^{(0)}_{q/q,F} \otimes_\perp \Sigma^{(1)}_{FT} \otimes_\perp \mathcal{N}^{(0)}_{j,T}$$

$$= \int \frac{d^{2-2\epsilon}\mathbf{p}}{(2\pi)^{-2\epsilon}} e^{-i\mathbf{p}\cdot\mathbf{b}} \int \frac{d^{2-2\epsilon}\mathbf{q}}{(2\pi)^{2-2\epsilon}} \int \frac{d^{2-2\epsilon}\mathbf{q}'}{(2\pi)^{2-2\epsilon}} \frac{\mathcal{J}^{(0)}_{q/q,F}}{\mathbf{q}^2} \left(\frac{2}{\tau} + \mathcal{L}_s\right) \hat{\mathcal{C}}\left[\mathbf{q}^2 \mathbf{q}'^2 \Sigma^{(0)}_{FT}\right] \frac{\mathcal{N}^{(0)}_{j,T}}{\mathbf{q}'^2} \qquad \mathcal{L}_s \sim \ln\left(\frac{\mu_b^2}{\nu^2}\right)$$

$$\mathcal{J}^{(0)}_{q/q,F} \otimes_\perp \Sigma^{(0)}_{FT} \otimes_\perp \mathcal{N}^{(1)}_{j,T}$$

$$= \int \frac{d^{2-2\epsilon}\mathbf{p}}{(2\pi)^{-2\epsilon}} e^{-i\mathbf{p}\cdot\mathbf{b}} \int \frac{d^{2-2\epsilon}\mathbf{q}}{(2\pi)^{2-2\epsilon}} \int \frac{d^{2-2\epsilon}\mathbf{q}'}{(2\pi)^{2-2\epsilon}} \frac{\mathcal{J}^{(0)}_{q/q,F}}{\mathbf{q}^2} \mathbf{q}^2 \mathbf{q}'^2 \Sigma^{(0)}_{FT} \left(-\frac{1}{\tau} + \mathcal{L}_{\bar{n}}\right) \hat{\mathcal{C}}\left[\frac{\mathcal{N}^{(0)}_{j,T}}{\mathbf{q}'^2}\right]$$

The BFKL evolution equations

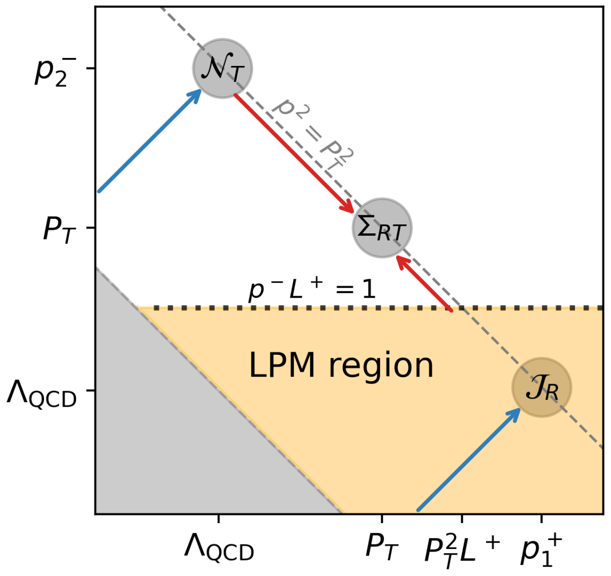
Each sector obeys a BFKL evolution equation

$$\frac{g_s^2}{\mathbf{q}^2}\frac{\partial \mathcal{J}_{q/i,R}(x,\mathbf{p},\mathbf{q};\nu)}{\partial \ln \nu} = -\hat{\mathcal{C}}\left[\frac{g_s^2}{\mathbf{q}^2}\mathcal{J}_{q/i,R}(x,\mathbf{p},\mathbf{q};\nu)\right],$$

$$\frac{g_s^2}{\mathbf{q}'^2}\frac{\partial \mathcal{N}_{j,T}(\mathbf{q}';\nu')}{\partial \ln \nu'} = -\hat{\mathcal{C}}\left[\frac{g_s^2}{\mathbf{q}'^2}\mathcal{N}_{j,T}(\mathbf{q}';\nu')\right],$$

$$\left(\frac{g_s^2}{\mathbf{q}^2}\right)^{-1}\left(\frac{g_s^2}{\mathbf{q}'^2}\right)^{-1}\frac{\partial \Sigma_{RT}(\mathbf{q},\mathbf{q}';\nu,\nu')}{\partial \ln \nu} = \hat{\mathcal{C}}\left[\left(\frac{g_s^2}{\mathbf{q}^2}\right)^{-1}\left(\frac{g_s^2}{\mathbf{q}'^2}\right)^{-1}\Sigma_{RT}(\mathbf{q},\mathbf{q}';\nu,\nu')\right].$$

$$\frac{\partial}{\partial \ln \nu}\left[\mathcal{J}_{q/i,R}\otimes_\perp \Sigma_{RT}\otimes_\perp \mathcal{N}_{j,T}\right](x_1,\mathbf{p},\mu) = 0$$

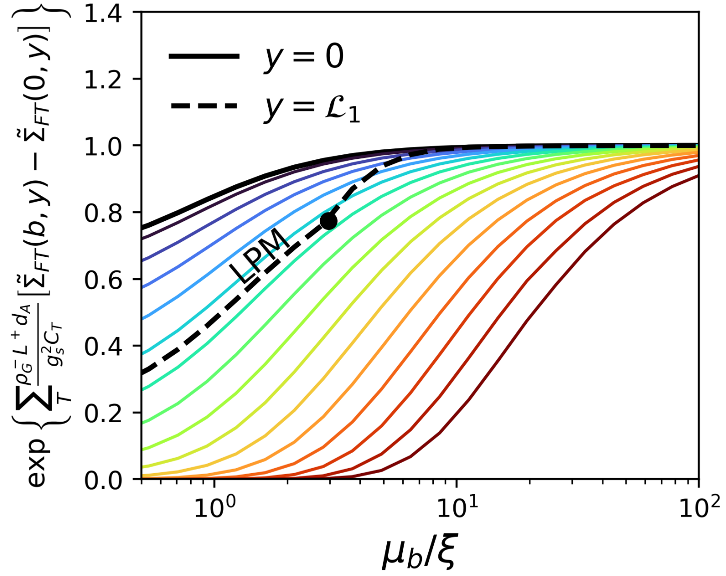


Each sector obeys a BFKL evolution equation

$$\mathcal{B}^{(0)}_{q/i,0} + \chi \mathcal{B}^{(0)}_{q/i,1} + \cdots$$

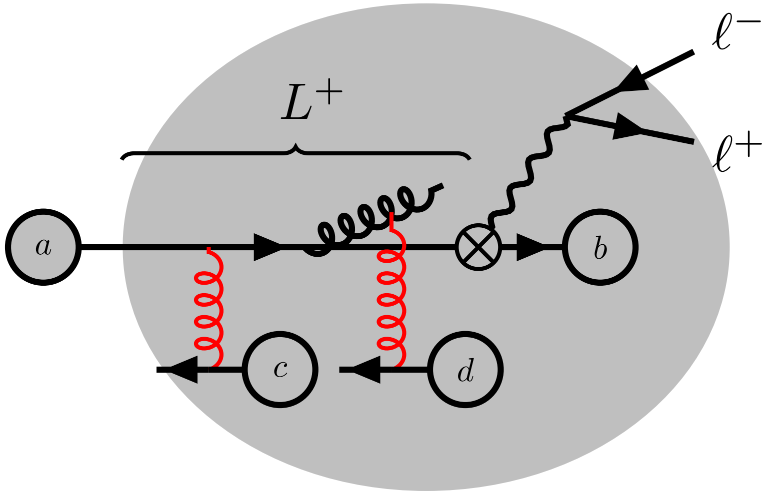
$$= \delta(1-x_1)\exp\left\{\sum_j \rho_0^- L^+ \int dx_t f_{j/N}(x_t)\mathcal{N}^{(0)}_{j,T}\left[\tilde{\Sigma}^{(0)}_{FT}(b) - \tilde{\Sigma}^{(0)}_{FT}(0)\right]\right\}$$

$$\tilde{\Sigma}_{RT}(b,y) = \int d^2\mathbf{b}'\tilde{v}_R(\mathbf{b}-\mathbf{b}',0)\tilde{v}_T(\mathbf{b}',y)$$



The Landau–Pomeranchuk–Migdal effect

Spectrum of energy and transverse momentum in matter is modified in a non-trivial way in QCD



Cross section can be written as an expansion in the opacity

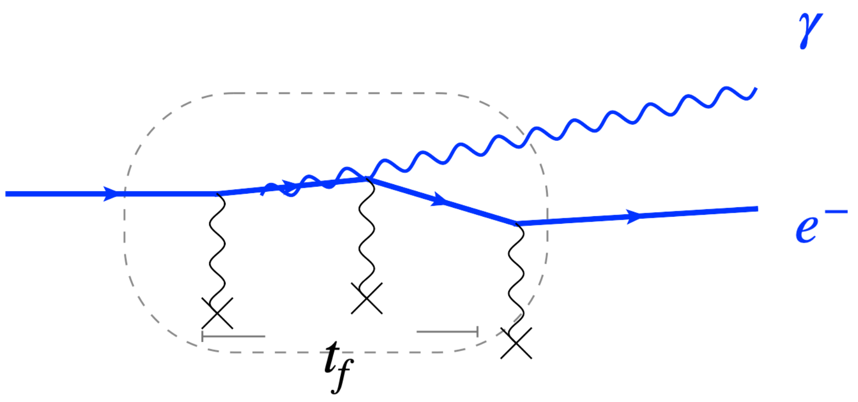
$$\frac{d\sigma}{d\mathcal{PS}} = \sum_{n=0}^{\infty} \frac{1}{n!}\chi^n \frac{d\sigma_n}{d\mathcal{PS}}$$

χ^n is the average number of active partons that contribute

$\chi^n \ll 1$

Spectrum of energy and transverse momentum in matter is modified in a non-trivial way in QCD

Landau et al (1953), Migdal (1956)



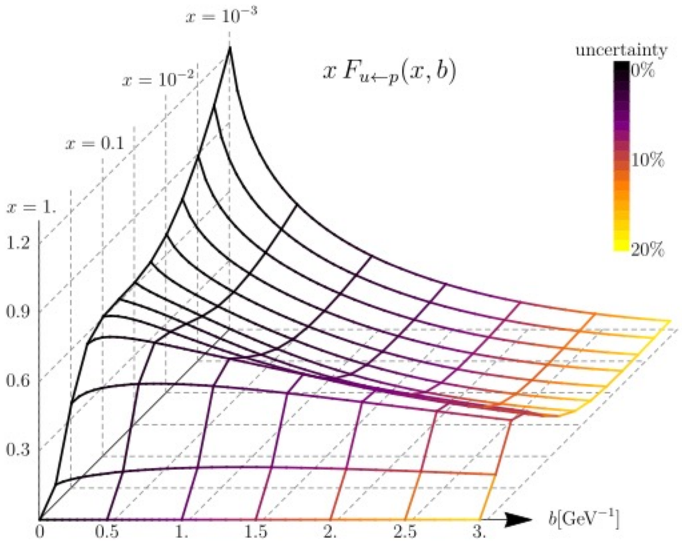
Images of proton structure

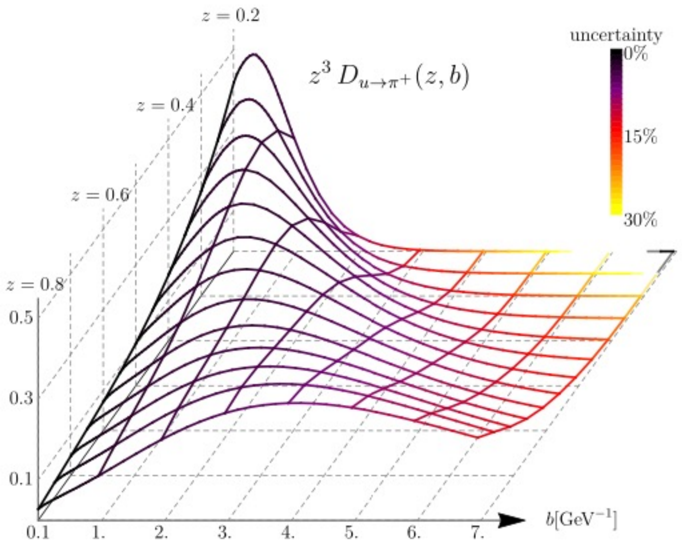
Global extraction from data requires evolution equations for TMDs

$$\frac{d}{d\ln\mu}\ln F(w,b,\mu,\zeta)=\gamma_\mu\left(\mu,\zeta\right)\ ,\qquad \frac{d}{d\ln\zeta}\ln F(w,b,\mu,\zeta)=\gamma_\zeta\left(b,\mu\right)\ ,$$

Approximations to N^4LL TMDs have been extracted in *Moos, Scimemi, Vladimirov, Zurita (2023)*

True N^4LL requires full 5 loop cusp anomalous dimension and evolution of PDF at N^3LO

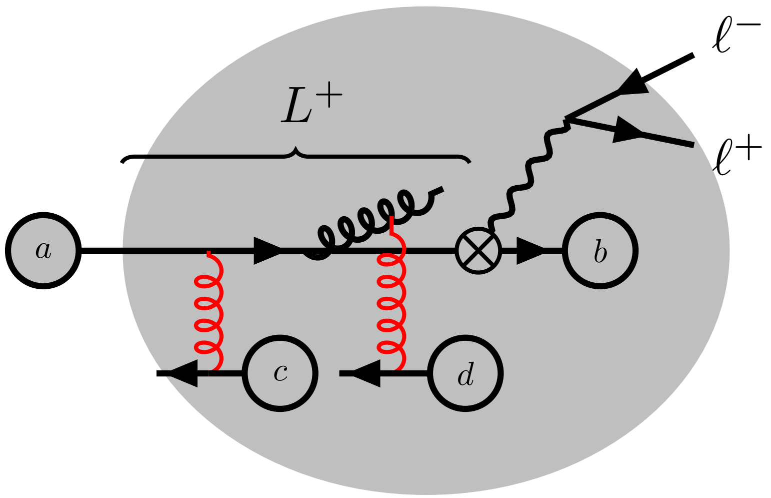




Gutierrez-Reyes, Scimemi, Vladimirov (2020)

The structure of matter in the medium

Spectrum of energy and transverse momentum in matter is modified in a non-trivial way in QCD



Cross section can be written as an expansion in the opacity

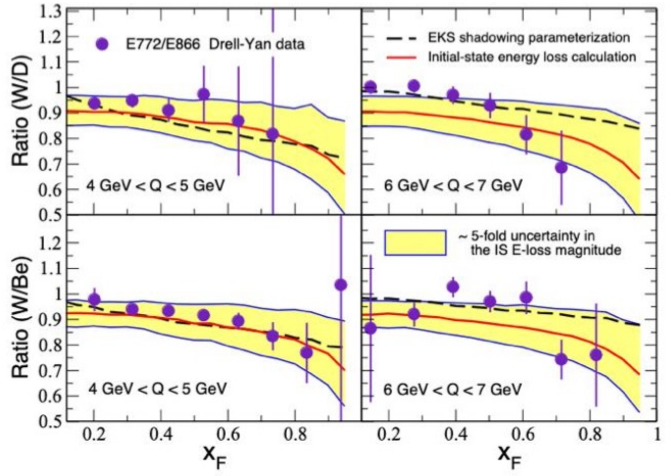
$$\frac{d\sigma}{d\mathcal{PS}} = \sum_{n=0}^{\infty} \frac{1}{n!}\chi^n \frac{d\sigma_n}{d\mathcal{PS}}$$

χ^n is the average number of active partons that contribute

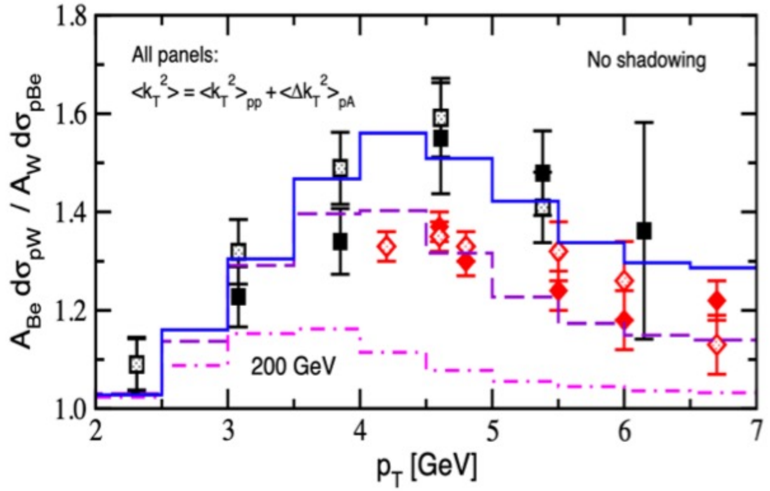
$$\chi^n \ll 1$$

Gyulassy-Levai-Vitev (2000) Guo, Wang (2000)

Stimulated emissions in Drell-Yan result in energy loss



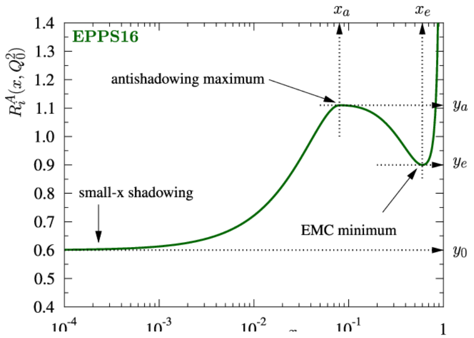
Stimulated emissions also generate additional transverse momentum



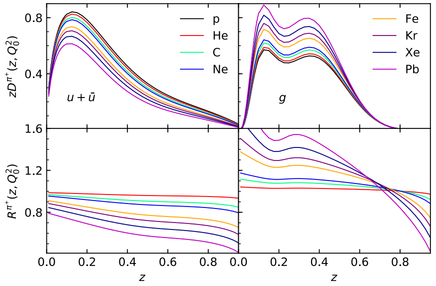
Three-dimensional images

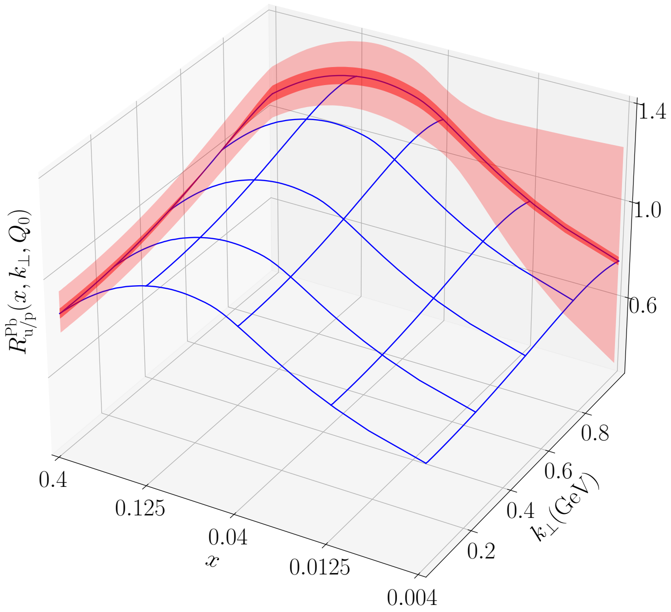
Ratios defined for nPDF and nFF *Alrashed, JT et al: Phys. Rev. Lett. 129 (2022)*

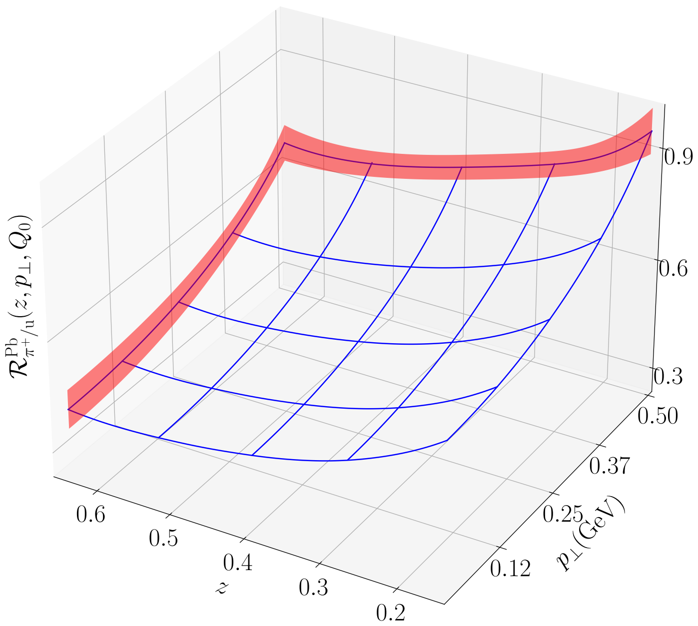
$$R_i^A\left(x, Q_0^2\right) = \frac{f_{i/p}^A\left(x, Q_0^2\right)}{f_{i/p}\left(x, Q_0^2\right)}$$



$$R_i^A\left(z, Q_0^2\right) = \frac{D_{h/i}^A\left(z, Q_0^2\right)}{D_{h/i}\left(z, Q_0^2\right)}$$







$$R_{u/p}^{\text{Pb}}(x, k_\perp, Q_0) = \frac{f_{u/p}^{\text{Pb}}\left(x, k_\perp, Q_0, Q_0^2\right)}{f_{u/p}\left(x, k_\perp, Q_0, Q_0^2\right)}$$

$$\mathcal{R}_{\pi^+/u}^{\text{Pb}}\left(z, p_\perp, Q_0\right) = \frac{D_{\pi^+/u}^{\text{Pb}}\left(z, p_\perp, Q_0, Q_0^2\right)}{D_{\pi^+/u}\left(z, p_\perp, Q_0, Q_0^2\right)}$$

The structure of matter: Nuclear matter