Quasi Parton Distribution Functions in Covariant Quark Models

Aslı Tandoğan
with Fatma Aslan and Peter Schweitzer

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Outline

1. What is a QPDF?
2. General quark-model framework
3. Numerical Results in Covariant Parton Model

QCD starting point: spacelike correlator and matching

Ji QPDF definition used in this work with a small tweak

$$\tilde{q}(x, \mu, P_z) = \int \frac{dz}{4\pi} e^{-ixP_z z} \langle P|\bar{\psi}(0)\Gamma W(0,z)\psi(z)|P\rangle + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{P_z^2}, \frac{M_N^2}{P_z^2}\right),$$

with $\Gamma = \gamma^z$ in Ji's original construction, and γ^0 also used in later QPDF studies.

We used

$$\tilde{q}(x, \mu, P_z) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{P_z^2}, \frac{M_N^2}{P_z^2}\right).$$

Matching controls perturbative UV differences; the model part of this talk is about finite-P_3 behavior and power corrections.

[Xiandong Ji, Phys. Rev. Lett. 110, 262002 (2013)]

Side note: a QPDF-like object in 1997

Unpolarized and polarized quark distributions in the large–N_c limit

D.I. Diakonov[a], V.Yu. Petrov[b], P.V. Pobylitsa[b,c], M.V. Polyakov[b,c], and C. Weiss[c]

$$P_N = \frac{M_N v}{\sqrt{1-v^2}}, \qquad v \to 1. \tag{2.8}$$

The number of (anti–) quarks can be expressed through the nucleon matrix element of the creation and annihilation operators, a^+, a (for quarks), and b^+, b (for antiquarks). We define the quark and antiquark distribution functions as

$$D_i(x) = \int \frac{d^3k}{(2\pi)^3} 2\pi\delta\left(x - \frac{k^3}{P_N}\right) \langle N_{\mathbf{v}}|a_i^+(\mathbf{k})a_i(\mathbf{k})|N_{\mathbf{v}}\rangle, \tag{2.9}$$

$$\bar{D}_i(x) = \int \frac{d^3k}{(2\pi)^3} 2\pi\delta\left(x - \frac{k^3}{P_N}\right) \langle N_{\mathbf{v}}|b_i^+(\mathbf{k})b_i(\mathbf{k})|N_{\mathbf{v}}\rangle. \tag{2.10}$$

[Diakonov, Petrov, Pobylitsa, Polyakov, Weiss, Phys. Rev. D 56, 4069–4083 (1997)]

A very similar object to QPDF was introduced there for the chiral quark-soliton model.

The purpose was the $P_N \to \infty$ / $v \to 1$ limit, they have not studied $v < 1$

Why do we need model calculations?

- Lattice QCD computes spacelike correlators at finite hadron momentum.
- Present lattice momenta are of order $P_z \sim 2$–$3\,\mathrm{GeV}$, not ∞.
- Matching handles perturbative UV differences; finite-ν power corrections and support effects remain central.

Role of models

Test P_z values for convergence , sum rules, and Γ-dependence in a controlled setting

[Review: K. Cichy and M. Constantinou, Adv.High Energy Phys. 2019 (2019) 3036904 Community report: M. Constantinou et al, Prog.Part.Nucl.Phys. 121 (2021) 103908 more..]

Notation: keep x and x_v separate

PDF

$$f_1^q(x), \qquad x = \frac{k^+}{P^+}$$

Lightlike separation; frame-independent parton variable.

QPDF

$$D^q(x_v, \Gamma, v), \qquad x_v = \frac{k^3}{P^3}$$

Spacelike separation; frame-dependent finite-velocity variable.

$$v = \frac{P_z}{P^0} = \frac{1}{\sqrt{1 + M^2/P_z^2}}, \qquad P^0 = \sqrt{M^2 + P_z^2}.$$

Using v effectively resums target-mass powers $(M^2/P_z^2)^n$ for $P_z > M$.

$$P_z = \frac{Mv}{\sqrt{1-v^2}}, \qquad \lim_{v\to 1} D^q(x_v, \Gamma, v) = f_1^q(x), \quad \Gamma = \gamma^0, \gamma^3.$$

Quark-model operator definitions

$$f_1^q(x) = \int \frac{dz^-}{4\pi}\, e^{-ixP_v^+ z^-} \langle N_v|\bar{\Psi}_q(0)\gamma^+\Psi_q(z)|N_v\rangle\Big|_{z^+=0,\vec{z}_\perp=0},$$

$$D^q(x_v,\Gamma,v) = \int \frac{dz^3}{4\pi}\, e^{-ix_v P_v^3 z^3} \langle N_v|\bar{\Psi}_q(0)\Gamma\Psi_q(z)|N_v\rangle\Big|_{z^\mu=(0,0,0,z^3)}.$$

Antiquark relations

$$f_1^{\bar{q}}(x) = -f_1^q(-x),$$
$$D^{\bar{q}}(x_v,\Gamma,v) = -D^q(-x_v,\Gamma,v).$$

Frame

$$P_v^\mu = \frac{M(1,0,0,v)}{\sqrt{1-v^2}}, \qquad P_v^3 = \frac{Mv}{\sqrt{1-v^2}}.$$

Covariant quark-model assumptions

Quark-model assumptions

No explicit gauge-field degrees of freedom; Lorentz covariance; finite or properly regularized correlators.

- Work with the unintegrated quark correlator $\Phi^q(k, P, S)$.
- Build PDFs and QPDFs from the same covariant object.
- Keep the distinction between model statements and QCD/lattice statements explicit.

Covariant: not tied to the rest frame; boosts and the $v \to 1$ limit can be studied consistently.

PDFs and QPDFs in terms of quark correlator

[Mulders, Tangerman, Nuci Phys. B 461 (1996)]

$$\Phi^q_{ij}(k,P,S) = \int \frac{d^4 z}{(2\pi)^4}\, e^{ik\cdot z} \langle N_v|\bar{\Psi}^q_j(0)\Psi^q_i(z)|N_v\rangle .$$

$$f_1^q(x) = \frac{1}{2P^+}\int d^4 k\, \mathrm{tr}\left[\Phi^q(k,P,S)\gamma^+\right]\delta\left(x - \frac{k^+}{P^+}\right),$$

$$D^q(x_v,\Gamma,v) = \frac{1}{2P^3}\int d^4 k\, \mathrm{tr}[\Phi^q(k,P,S)\Gamma]\,\delta\left(x_v - \frac{k^3}{P^3}\right).$$

In QCD the correlator has Wilson-line structures; in quark models the decomposition reduces from 32 to 12 amplitudes.

PDFs and QPDFs in terms of amplitudes

$$\mathrm{tr}[\Phi^q(k,P,S)\gamma^\mu] = 4\left(P^\mu A_2^q + k^\mu A_3^q\right),$$

$$f_1^q(x) = 2\int d^4k\ (A_2^q + xA_3^q)\,\delta\left(x - \frac{k^+}{P^+}\right),$$

$$D^q(x_v,\gamma^\mu,v) = 2\int d^4k\ \left(\frac{P^\mu}{P^3}A_2^q + \frac{k^\mu}{P^3}A_3^q\right)\delta\left(x_v - \frac{k^3}{P^3}\right), \qquad \mu = 0,3.$$

Still in Quark Models

The $v \to 1$ limit, sum rules, and Γ-dependence follow from the amplitude structure alone, no specific dynamical model needed.

$v \to 1$ limit

$$\lim_{v \to 1} x_v = x, \qquad \lim_{v \to 1} D^q(x_v, \Gamma, v) = f_1^q(x), \qquad \Gamma = \gamma^0, \gamma^3.$$

- The light-cone variable $x = k^+/P^+$ is recovered from the frame-dependent variable $x_v = k^3/P^3$.
- Both $\Gamma = \gamma^0$ and $\Gamma = \gamma^3$ converge to the same unpolarized PDF.
- The finite-v differences are therefore power-correction information, not different PDFs.

Flavor number sum rule

PDF

$$\int_{-1}^{1} dx\, f_1^q(x) = 2\int d^4k\left(A_2^q + \frac{k^0}{M}A_3^q\right) = N^q.$$

QPDF $\Gamma = \gamma^0$

$$\int_{-\infty}^{\infty} dx_v\, D^q(x_v, \gamma^0, v) = \frac{1}{v}\int_{-1}^{1} dx\, f_1^q(x)$$
$$= \frac{N^q}{v}.$$

QPDF $\Gamma = \gamma^3$

$$\int_{-\infty}^{\infty} dx_v\, D^q(x_v, \gamma^3, v) = \int_{-1}^{1} dx\, f_1^q(x)$$
$$= N^q.$$

Agrees with Bhattacharya, Cocuzza, Metz, Phys.Rev.D 102 (2020) 5, 054021

Momentum sum rule

$$\int_{-1}^{1} dx\, x f_1^q(x) = A^q(0).$$

$\Gamma = \gamma^0$

$$\int_{-\infty}^{\infty} dx_v\, x_v D^q(x_v, \gamma^0, v) = \frac{1}{v}\int_{-1}^{1} dx\, x f_1^q(x) = \frac{A^q(0)}{v}.$$

$\Gamma = \gamma^3$

$$\int_{-\infty}^{\infty} dx_v\, x_v D^q(x_v, \gamma^3, v) = A^q(0) - \frac{1-v^2}{v^2}\bar{c}^q(0).$$

Notice $\Gamma = \gamma^3$: the second moment gives access to $\bar{c}^q(0)$, the EMT form factor for non-conservation of individual quark/gluon EMT contributions.

The γ^3 momentum sum rule exposes $\bar{c}^q(0)$

$$\langle N|\hat{T}_q^{\mu\nu}(0)|N\rangle = \bar{u}_N\Big[A^q(t)\tfrac{\bar{P}^\mu\bar{P}^\nu}{M} + J^q(t)\tfrac{i(\bar{P}^\mu\sigma^{\nu\rho}+\bar{P}^\nu\sigma^{\mu\rho})\Delta_\rho}{2M} + D^q(t)\tfrac{\Delta^\mu\Delta^\nu-g^{\mu\nu}\Delta^2}{4M} + \bar{c}^q(t)Mg^{\mu\nu}\Big]u_N.$$

$$\int dx_v\, x_v D^q(x_v,\gamma^3,v) = A^q(0) - \frac{1-v^2}{v^2}\bar{c}^q(0), \qquad \sum_a \bar{c}^a(t) = 0.$$

- $A^q(0)$ is the usual quark contribution to the momentum sum rule
- $\bar{c}^q(0)$ enters only through the $\Gamma = \gamma^3$ QPDF momentum sum rule
- This statement is general in the quark-model framework

Which Γ is preferred? Coordinate-space view

$C^\mu(P,z) \equiv \langle N|\bar{\Psi}^q(0)\gamma^\mu\Psi^q(z)|N\rangle$: the nucleon matrix element of the quark bilinear, decomposed by Lorentz invariance (Ioffe time $\nu = -P\cdot z$):

$$C^\mu(P,z) = 2P^\mu M^q(\nu,z^2) + 2z^\mu M^2 J^q(\nu,z^2), \qquad \nu = -P\cdot z.$$

$$vD^q(x_v,\gamma^0,v) = \int \frac{d\nu}{2\pi}\, e^{-i\nu x_v} M^q(\nu,-z_3^2),$$

$$D^q(x_v,\gamma^3,v) = \int \frac{d\nu}{2\pi}\, e^{-i\nu x_v}\left[M^q(\nu,-z_3^2) + \frac{M^2}{P_3^2}\nu J^q(\nu,-z_3^2)\right].$$

Coordinate-space conclusion

$\Gamma = \gamma^0$ looks cleaner: the power-suppressed J^q term is absent. $\Gamma = \gamma^3$ carries an explicit M^2/P_3^2 correction.

Which Γ is preferred? Momentum-space view

$$D^q(x_v, \gamma^0, v) = \frac{1}{v} \cdot 2 \int d^4k \left(A_2^q + \frac{k^0 + vk^3}{M} A_3^q \right) \delta\left(x_v - \frac{k^3 + vk^0}{vM} \right),$$

$$D^q(x_v, \gamma^3, v) = 2 \int d^4k \left(A_2^q + \frac{k^3 + vk^0}{vM} A_3^q \right) \delta\left(x_v - \frac{k^3 + vk^0}{vM} \right).$$

Note: $(k^3 + vk^0)/(vM) = x_v$ on support of δ, so γ^3 reduces to $A_2^q + x_v A_3^q$. For γ^0 there are two red terms: the overall $1/v$ is exact $(= \sqrt{1 + M^2/P_3^2})$ and cancels with Polyakov's convention or with $v \cdot D^q$; the $(k^0 + vk^3)/M$ coefficient is a kinematic power correction that no redefinition removes.

Momentum-space conclusion

In momentum space, $\Gamma = \gamma^3$ has fewer kinematical finite-v corrections, a faster-convergence candidate in quark models, with the caveat that $\bar{c}^q(0)$ appears in its momentum sum rule.

For numerical calculations: Covariant Parton Model (CPM)as a tool

- Up to this point, the statements were general quark-model results.
- CPM: Covariant extension of Feynman's parton model
- No gauge-field degrees of freedom
- Partons are non-interacting and on shell
- Used here as a numerical tool with input PDFs at $\mu^2 = 4\,\mathrm{GeV}^2$.

Unpolarized CPM correlator

$$A_2^q = 0,$$

$$A_3^q = M\,\delta(k^2 - m_q^2)\,\Theta(P\cdot k) \times \Theta((P-k)^2)\,\mathcal{G}^q(P\cdot k).$$

CPM result for $\bar{c}^q(0)$

In the CPM, the same on-shell correlator used for QPDFs also gives the EMT form factors. Keeping current-quark mass effects,

$$A^q(0) = -4\,\bar{c}^q(0) + \frac{2m_q^2}{M^2}\int d^4k\, A_3^q.$$

For light quarks, $m_q^2 \ll M^2$, this becomes

$$\boxed{\bar{c}^q(0) = -\frac{1}{4}A^q(0)} \qquad \text{(CPM)}.$$

- This is now a CPM result, not part of the general proof.
- The same relation holds in the bag model for $m_q = 0$. [X. D. Ji, W. et al, Phys.Rev.D 56, 5511 (1997), Neubelt, Phys. Rev. D 101 (2020)]
- $\bar{c}^q(0) = 0$ in χQSM [Goeke , et al, Phys.Rev.D 75, (2007), 094021]

The CPM formulas show where finite-v terms enter

Here $\mathcal{G}^q(P\cdot k)$ is the covariant model function that specifies the internal structure of the nucleon in the CPM; all velocity dependence sits in the integration limits and prefactors.

$$D^q(x_v, \gamma^3, v) = 2\pi v x_v M^2 \int_{L(v)}^{M/2} dk\, \mathcal{G}^q(Mk),$$

$$D^q(x_v, \gamma^0, v) = 2\pi M \int_{L(v)}^{M/2} dk\, \mathcal{G}^q(Mk)\left(x_v M v^2 + k(1-v^2)\right),$$

$$L(v) = \frac{v|x_v|M}{1 + v\,\mathrm{sign}(x_v)}.$$

$$D^q(x_v, \gamma^0, v) = v\, D^q(x_v, \gamma^3, v) + (1-v^2)\, 2\pi M \int_{L(v)}^{M/2} dk\, k\, \mathcal{G}^q(Mk).$$

Both expressions become the light-cone PDF when $v \to 1$.

Finite v widens support and causes leaking

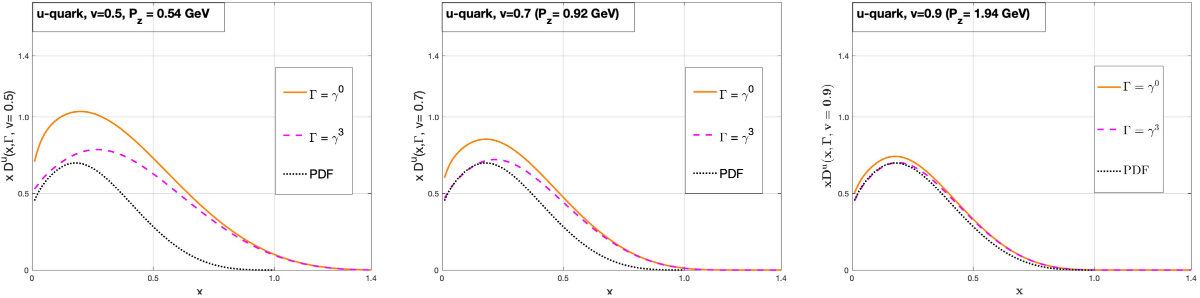
$$D^q(x_v, \Gamma, v) \neq 0 \quad \text{if} \quad -\frac{1+v}{2v} \leq x_v \leq \frac{1+v}{2v} \xrightarrow[v\to 1]{} -1 < x < 1.$$

Interpretation

At finite v, $\mathcal{G}^q(P\cdot k)$ leaks into negative x_v and $\mathcal{G}^{\bar{q}}(P\cdot k)$ leaks into positive x_v; the clean quark/antiquark separation is recovered only as $v \to 1$.

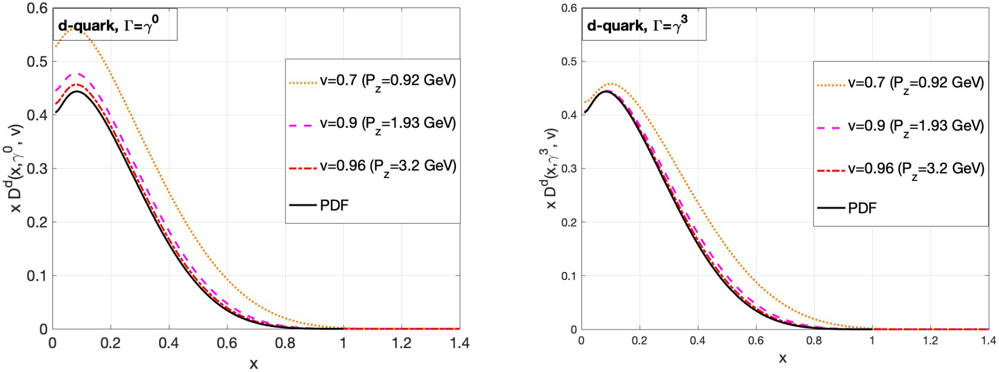
Numerical comparison of $\Gamma = \gamma^0$ and γ^3: u-quark

nucleon velocity v	0.2	0.5	0.7	0.9	1
P_z [GeV]	0.19	0.54	0.92	1.94	∞
$x_{v,\max}$	3	1.50	1.21	1.06	1



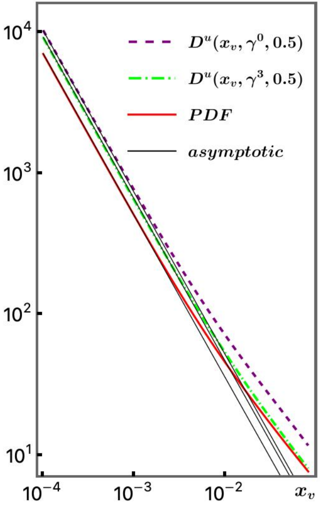
Takeaway: for $10^{-2} \lesssim x_v \lesssim 0.2$–$0.3$, $\Gamma = \gamma^3$ approaches the PDF faster than $\Gamma = \gamma^0$; at larger x_v, the two QPDFs agree with each other before reaching the PDF.

Convergence at multiple velocities: *d*-quark



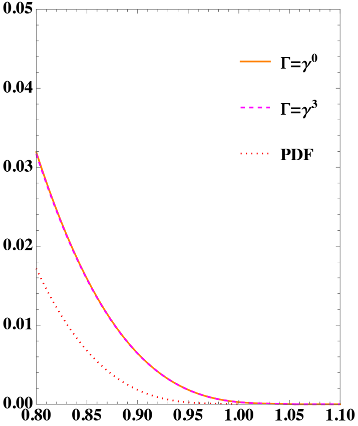
Flavor number and momentum sum rules are satisfied; at $P_z \sim 3\,\mathrm{GeV}$ the QPDF is close to the PDF for both Γ choices.

Small-x_v behavior: analytic CPM control



- The CPM gives analytic results down to asymptotically small x_v.
- Lattice QCD is limited at small $|x_v|$ by finite Fourier-transform range.
- The faster γ^3 convergence at intermediate x_v need not persist to the small-x_v region.

Large-x_v behavior is also controlled



$$x_{v,\mathrm{max}} = \frac{1+v}{2v}, \qquad \lim_{v\to 1} x_{v,\mathrm{max}} = 1.$$

$$D^a(x_v,\Gamma,v) = c_L\, v x_v \left(1 - \frac{x_v}{x_{v,\mathrm{max}}}\right)^N + \cdots,$$
$$\Gamma = \gamma^0, \gamma^3.$$

Seen also in Lattice and model calculations

Both Dirac choices have the same large-x_v asymptotic behavior near the endpoint away from PDF.

Conclusion and outlook

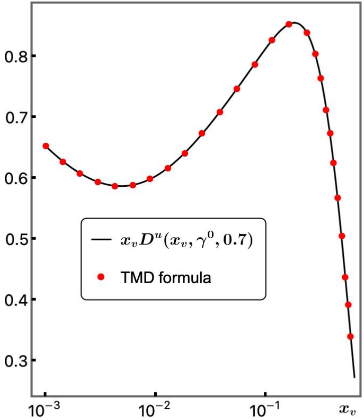
- Quark models can treat QPDFs consistently. $v \to 1$ limit, flavor number sum rule, and momentum sum rule are all satisfied.
- The $\Gamma = \gamma^3$ momentum sum rule exposes $\bar{c}^q(0)$, the EMT form factor for non-conservation of an individual constituent contribution.
- Individual $\bar{c}^q(t)$ can be nonzero; the total EMT is conserved by cancellation among all constituents (in QCD: quark and gluon sectors).
- CPM provides a clean numerical tool for finite-v QPDFs, including small-x_v, large-x_v, and Γ-dependence.
- These results give concrete guidance for model calculations of QPDFs.
- Next steps: polarized QPDFs and subleading twist

Thank you!

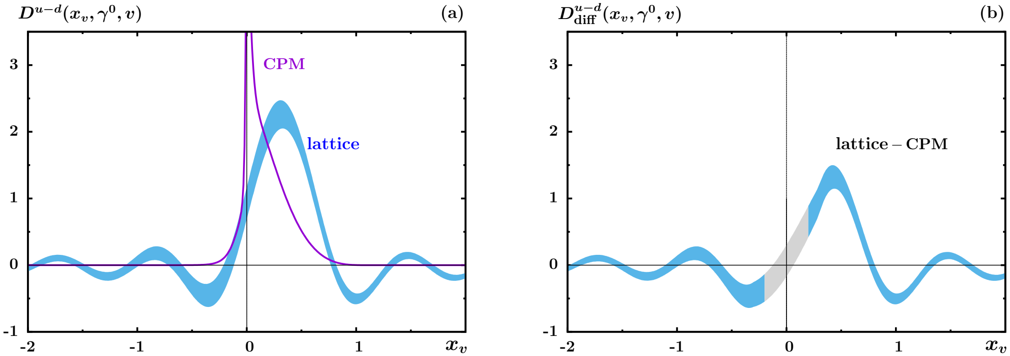
Radyushkin formula test

$$D^q(x_v, \gamma^0, v) = P^0 \int_{-1}^{1} dx \int_{-\infty}^{\infty} dk_1 \times f_1^q\left(x, k_1^2 + (x_v - x)^2 P_z^2\right) .$$

- Eq. (13) relates the $\Gamma = \gamma^0$ QPDF to the model TMD.
- The paper uses this as a numerical cross-check and consistency test.
- Test shown: $x_v D^u(x_v, \gamma^0, v)$ at $v = 0.7$.
- Direct CPM calculation and Eq. (13) agree within numerical accuracy.



Comparison to lattice QCD



- Comparison: isovector QPDF $(D^u - D^d)(x_v, \gamma^0, v)$ at $\mu^2 = 4\,\mathrm{GeV}^2$.
- Lattice point used in the paper: $P_z = 1.38\,\mathrm{GeV}$, corresponding to $v = 0.827c$
- Difference plotted as

$$D^{u-d}_{\mathrm{diff}}(x_v, \gamma^0, v) = (D^u - D^d)(x_v, \gamma^0, v)\Big|_{\mathrm{lattice}} - (D^u - D^d)(x_v, \gamma^0, v)\Big|_{\mathrm{CPM}}.$$

- Shaded region $|x_v| < 0.3$: caution from finite Fourier-transform range; in the cited lattice setup $\nu_{\mathrm{max}} = 6.5$.