Connecting
Baryon Light-Front Wave Functions
to Quasi-TMD Correlators in Lattice QCD

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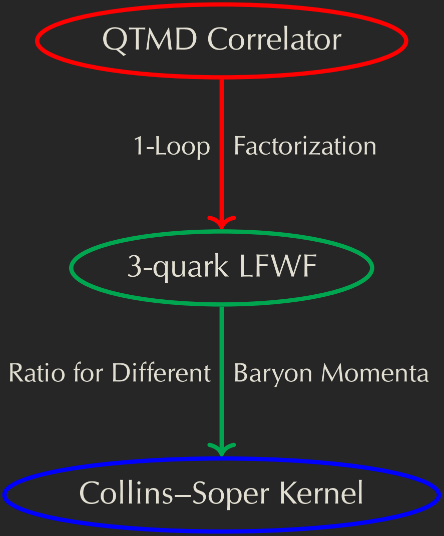
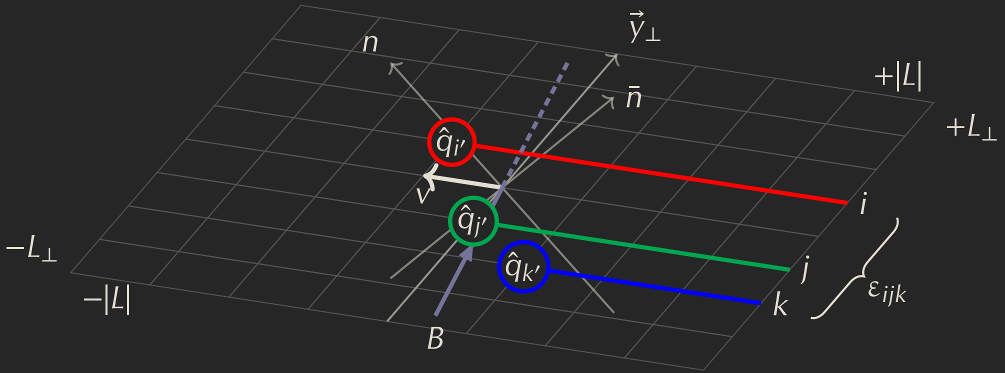
arXiv:2603.08405, in print in Phys. Rev. D


QTMD Correlator
1-Loop Factorization
3-quark LFWF
Ratio for Different Baryon Momenta
Collins–Soper Kernel



$\vec{y}_\perp$
n
$\bar{n}$
$+|L|$
$+L_\perp$
$\hat{q}_{i'}$
v
$\hat{q}_{j'}$
$\hat{q}_{k'}$
i
j
k
ε_{ijk}
$-L_\perp$
$-|L|$
B

$$\widetilde{\Omega}_{3q;B}\left(\left\{y_q = l_q v + b_q\right\}\right) = \langle 0|\varepsilon_{ijk} J_i(y_1)\, J_j(y_2)\, J_k(y_3)\,|B\rangle$$

$$J(y) = [Lv + b, lv + b]\, q(lv + b)$$

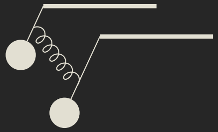
$$q \mapsto \psi + q_{\bar{n}} + q_v \qquad A^\mu \mapsto B^\mu + A^\mu_{\bar{n}} + A^\mu_v$$

$$(\partial^+, \partial^-, \partial_\perp)(q_{\bar{n}}, A_{\bar{n}}) \lesssim \big(\quad 1, \lambda^2, \lambda\big) P^+ (q_{\bar{n}}, A_{\bar{n}})$$
$$(\partial^+, \partial^-, \partial_\perp)(q_v, A_v) \lesssim \big(\lambda^2 \quad, \lambda^2, \lambda\big) P^+ (q_v, A_v)$$

$$\lambda = (M/P^+) \ll 1$$

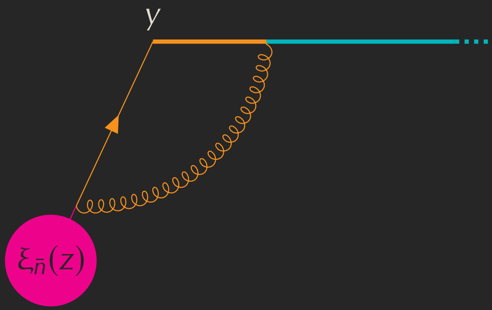
$$q_{\bar{n}} = \left(\frac{1}{2}\gamma^-\gamma^+ + \frac{1}{2}\gamma^+\gamma^-\right) q_{\bar{n}} := \xi_{\bar{n}} + \eta_{\bar{n}} \overset{\text{EOM}}{\Longrightarrow} \eta_{\bar{n}} \sim \lambda \xi_{\bar{n}}$$

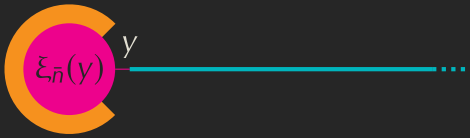
$$b \lesssim \frac{1}{M} \quad \Rightarrow$$

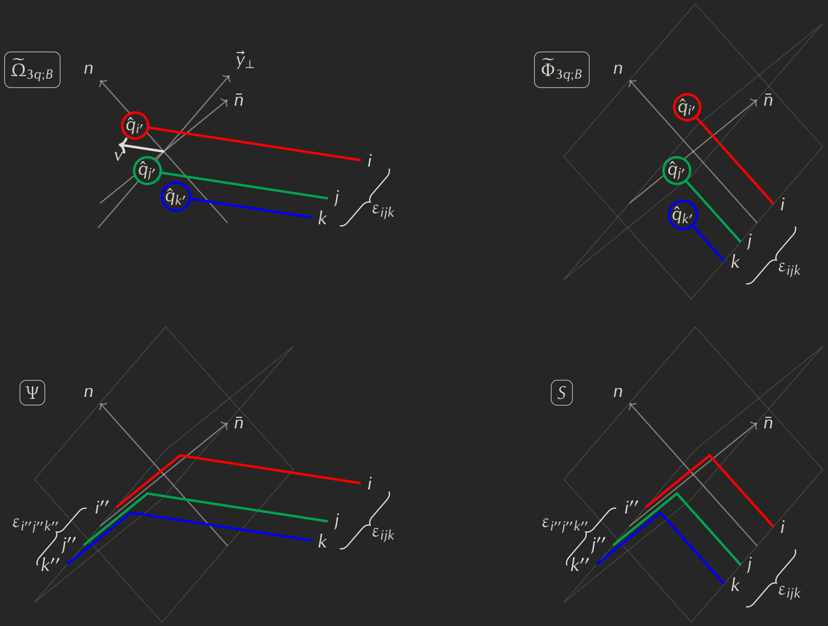


$$\sim \lambda^2$$

$$J(y) = [Lv + b, lv + b]\, q(y) \mapsto [Lv + b, lv + b]_v\, \hat{C}\, \xi_{\bar{n}}(y)$$

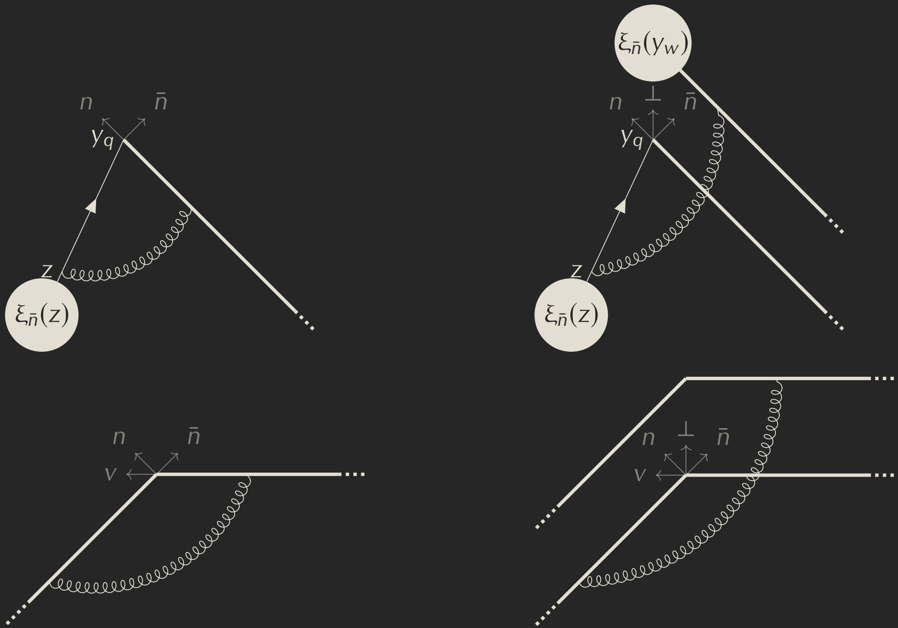





$\widetilde{\Omega}_{3q;B}$
n
$\vec{y}_\perp$
$\bar{n}$
$\hat{q}_{i'}$
v
$\hat{q}_{j'}$
$\hat{q}_{k'}$
i
j
k
ε_{ijk}
$\widetilde{\Phi}_{3q;B}$
n
$\bar{n}$
$\hat{q}_{i'}$
$\hat{q}_{j'}$
$\hat{q}_{k'}$
i
j
k
ε_{ijk}
Ψ
n
$\bar{n}$
i
j
k
ε_{ijk}
i''
j''
k''
$\varepsilon_{i''j''k''}$
S
n
$\bar{n}$
i
j
k
ε_{ijk}
i''
j''
k''
$\varepsilon_{i''j''k''}$

$$\Omega_{3q;B}\Big(\{y_q^+, x_q, b_q\}\Big) \to \frac{\Psi(\{b_q\})\, C(x_1)\, C(x_2)\, C(x_3)\, \boldsymbol{\Phi_{3q;B}}(\{x_q, b_q\})}{S(\{b_q\})}$$

$$x_q = \frac{p_q^+}{P^+} \quad x_1 + x_2 + x_3 = 1$$


n
n̄
y_q
z
ξ_n̄(z)
ξ_n̄(y_w)
n
⊥
n̄
y_q
z
ξ_n̄(z)
n
n̄
v
n
⊥
n̄
v

$$Z_{\Phi 1} \leftrightarrow \epsilon\text{-poles}, \ln(\delta^+) \qquad R_{\Phi 1} \leftrightarrow \ln\left(-(b_w - b_q)^2\right)\ln(\delta^+)$$

$$Z_{\Psi 1} \leftrightarrow \epsilon\text{-poles}, \ln(\delta^-) \qquad R_{\Psi 1} \leftrightarrow \ln\left(-(b_w - b_q)^2\right)\ln(\delta^-)$$

$$S \leftrightarrow \epsilon\text{-poles}, \ln(\delta^+\delta^-), \ln\left(-(b_w - b_q)^2\right)\ln(\delta^+\delta^-)$$

$$\Omega_{3q;B}(\mu) = \Psi_{\text{sub}}\left(\mu, \{\bar{\zeta}_q\}\right) C_0(x_1)\, C_0(x_2)\, C_0(x_3)\, \boldsymbol{\Phi}_{\mathbf{sub,3}\boldsymbol{q};\boldsymbol{B}}\left(\mu, \{\zeta_q\}\right)$$

$$\zeta \sim (xP^+)^2 \qquad \ln(xP^+), \ln^2(xP^+) \leftrightarrow C_0(x) \in \mathbb{C}$$

$$\mu^2 \frac{\partial}{\partial \mu^2} \Phi_{\text{sub}}\left(\mu, \{\zeta_q\}\right) = \sum_{q'=1}^{3} \gamma_{\Phi 1}\left(x_{q'}, \frac{\zeta_{q'}}{\mu^2}\right) \Phi_{\text{sub}}\left(\mu, \{\zeta_q\}\right)$$

$$\zeta_q \frac{\partial}{\partial \zeta_q} \Phi_{\text{sub}}\left(\mu, \{\zeta_q\}\right) = -\frac{1}{4} \sum_{\substack{w=1 \\ w \neq q}}^{3} \mathcal{D}\left(\left(b_q - b_w\right)^2, \mu\right) \Phi_{\text{sub}}\left(\mu, \{\zeta_q\}\right)$$

$$\frac{\Phi_{\text{sub}}\left(\mu, \{\zeta_q\}\right)}{\Phi_{\text{sub}}\left(\mu_0, \{\zeta_{q,0}\}\right)} = \prod_{q=1}^{3} \left(\frac{\zeta_q}{\zeta_{q,0}}\right)^{-\frac{1}{4}\sum_{\substack{w=1\\ w\neq q}}^{3} \mathcal{D}\left((b_q-b_w)^2,\mu\right)} \exp \int \frac{d\mu^2}{\mu^2} \gamma_{\Phi 1}$$

$$\frac{\Omega(\mu; P_1)}{\Omega(\mu; P_2)} = \left(\frac{P_1^+}{P_2^+}\right)^{-\frac{1}{2}\sum_{\substack{q,w=1\\ w\neq q}}^{3} \mathcal{D}\left((b_q-b_w)^2,\mu\right)} \prod_{q=1}^{3} \frac{C_0\left(x_q; P_1^+\right)}{C_0\left(x_q; P_2^+\right)}$$

Future Prospects

- Verify factorization beyond 1-loop
- Study higher Fock components
- Collaborate with the Lattice QCD community for actual simulations