QCD–Gravity Duality and Discrete BFKL Evolution in Multi-Regge Kinematics

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QCD evolution 2026, El Escorial,
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May 15, 2026

Outline

1. Double copy structure
   - QCD process
   - Gravity process
   - Double-copy structure in the MRK
2. Discrete BFKL
   - Introduction to the discrete BFKL
   - Walk expansion and Reggeization
   - Anomalous dimension
3. Conclusions and outlook

Procedure - Mathematica libraries

- FeynArts draws the Feynman Diagrams[1]

[1] Hahn 2001.
[2] Shtabovenko et al. 2025.
[3] Latosh 2025.

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- FeynCalc computes the amplitudes[2]
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- Working on the compatibility between FeynGrav and FeynArts

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QCD diagrams: $qq' \to qq'g$

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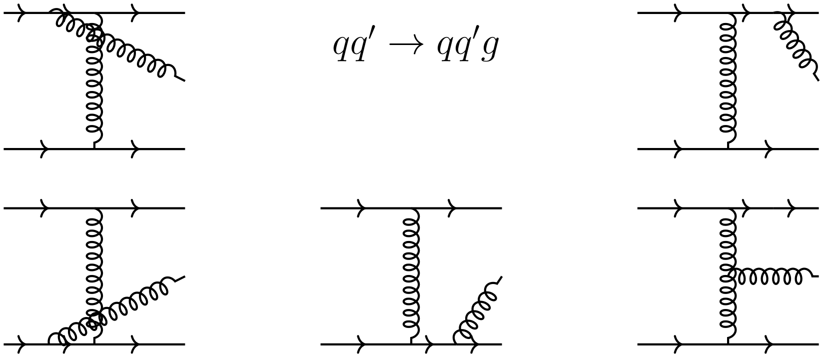

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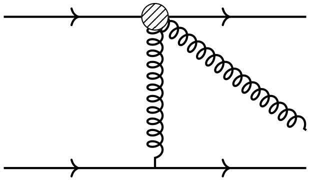
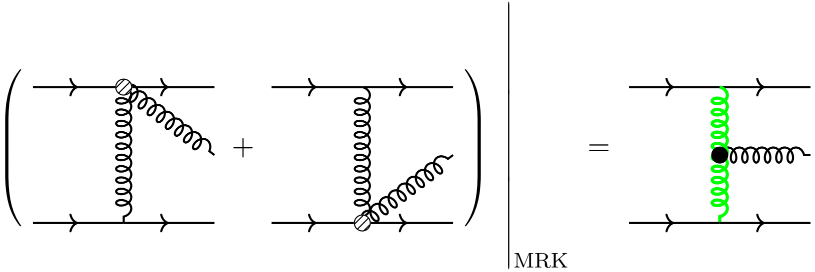
And with them, an effective vertex pops up



$\propto \quad \overline{u}\gamma u \overline{u}\Lambda u$

By combining the two GI vertices one obtains the full amplitude



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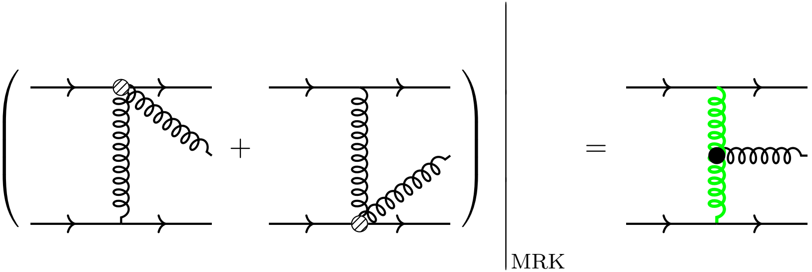
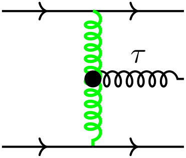


Where we can define the effective vertex



$$\propto \qquad \Gamma^{\tau}_{\chi\sigma} = i\alpha_s g_{\chi\sigma}\Omega^{\tau}$$

Gravitational diagrams

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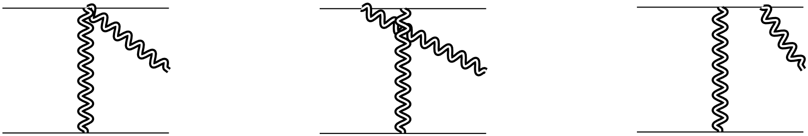

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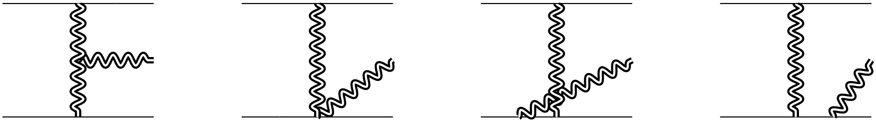

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We obtain the total amplitude

$$\mathcal{M} = \mathcal{M}^{\mu\nu}\epsilon_{\mu\nu}$$

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By checking the Steinmann relations we can see that this should not be valid, and we need to use[5]

$$\mathcal{M}^{\mu\nu} = \Omega^{\mu}\Omega^{\nu} - \mathcal{N}^{\mu}\mathcal{N}^{\nu}.$$

[5] Lipatov 1982.

Discrete Hamiltonian

As shown in the BFKL equation for forward scattering, i.e.,

$$\frac{\partial \phi(Q^2, Y)}{\alpha \partial Y} = \int_0^\infty \frac{dq^2}{|q^2 - Q^2|} \left(\phi(q^2, Y) - \frac{2 \min(q^2, Q^2)}{q^2 + Q^2} \phi(Q^2, Y) \right)$$

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$$\frac{\partial}{\alpha \partial Y} |\phi^{(N)}\rangle = \hat{\mathcal{H}}_N |\phi^{(N)}\rangle; \qquad \hat{\mathcal{H}}_N = A - 2D,$$

where

$$A_{ij} = \begin{cases} 1/|i-j|, & i \neq j, \\ 0, & i = j \end{cases} \qquad D_{ij} = H_{i-1} \delta_{ij}$$

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Norm bounds and Rayleigh quotient

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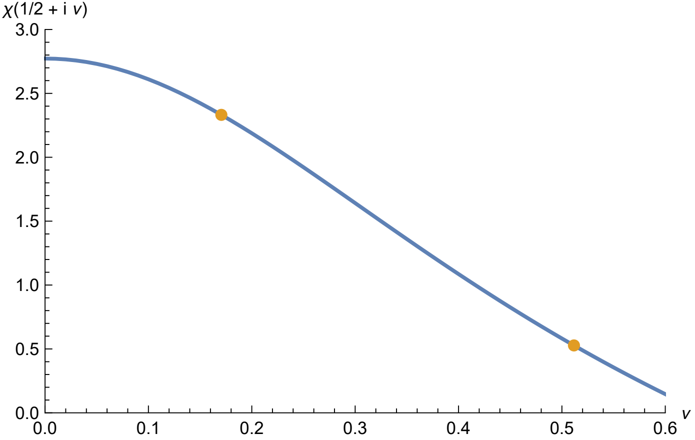
By imposing hard-wall conditions on the boundary we obtain

$$\lambda_n(\hat{\mathcal{H}}_N) \approx 4\log(2) - 14\zeta(3)\left(\frac{n\pi}{L}\right)^2$$


χ(1/2 + i v)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
v

Vertex-dressed resolvent

Starting at a site N_0 and looking at a site n, the evolution is given by

$$d_n^{(r)} = \langle e_n, (A - 2D)^r e_{N_0} \rangle.$$

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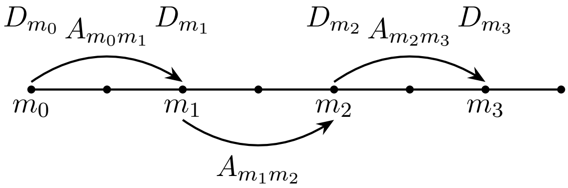

Reggeization

With this, we obtain

$$d_n^{(r)} = \sum_{j=0}^{r} (-2)^{r-j} \sum_{(m_0,\ldots,m_j)} \left[h_{r-j}(H_{m_0-1}, \ldots, H_{m_j-1}) \prod_{i=0}^{j-1} \frac{1}{|m_i - m_{i+1}|} \right]$$

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When the sum over m's means for all the paths connecting n and N_0. By introducing a generating function, we can write

$$\mathcal{G}_n(z) = \sum_{j=0}^{\infty} z^j \sum_{(m_0,\ldots,m_j)} \left(\prod_{i=0}^{j-1} \frac{1}{|m_i - m_{i+1}|} \right) \left(\prod_{l=0}^{j} \frac{1}{1 + 2zH_{m_i-1}} \right)$$

From discrete to continuum

Instead of writting

$$d_n^{(r)} = (A - 2D)d^{(r-1)}; \qquad \mathcal{G}^{(r)}(m/n; x) = \frac{d_m^{(r)}}{n(m/n - x)}$$

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one can write

$$\mathcal{G}^{(r+1)}(1; x) = \frac{1}{x}\int_0^x \frac{\mathcal{G}^{(r)}(y; x)}{y} dy + \frac{1}{1-x}\int_x^1 \frac{\mathcal{G}^{(r)}(y; x)}{1-y} dy$$

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And from this, one can obtain the differential equation

$$\frac{d}{dx}\mathcal{F}^{(r+1)}(x) = \left(\frac{A_0}{x} + \frac{A_1}{1-x}\right)\mathcal{F}^{(r)}(x),$$

where $\mathcal{G}^{(r)}(1; x) \equiv \mathcal{F}^{(r)}(x)$.

Single log structure

Following the HPL structure, we can write

$$\mathcal{F}^{(r)}(x) = \sum_{|\tau|=r-1} c_\tau^{(r)} H_\tau(x)$$

[7] Del Duca et al. 2014.

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The Gluon's green function can be written as

$$\tilde{\phi}_n(\omega) = \sum_{r\geq 0} \frac{\alpha^r}{\omega^{r+1}} d_n^{(r)} \qquad \tilde{\phi}_n(\omega) \propto \sum_{r\geq 0} \sum_{|\omega|=r-1} \frac{\alpha^r}{\omega^{r+1}} c_\omega^{(r)} H_\omega(x)$$

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The single-log and single-valued[7] version is given by

$$\Pi_{SV}(\tilde{\phi}_n) \propto \log(x).$$

[7] Del Duca et al. 2014.

Anomalous dimension coefficients

By writting the anomalous dimension expansion as

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one finds the following coefficients

n	c_1^{n+1}
0	1
1	0
2	0
3	$2\zeta_3$
4	0
5	$2\zeta_5$
$\vdots$	$\vdots$

Results and conclusions

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- Discrete BFKL
  1. Pomeron intercept
  2. Evolution and Reggeization
  3. Knizhnik–Zamolodchikov and Harmonic Polylogarithms
  4. Anomalous dimension expansion

Further research

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Thank you for your attention!

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Steinmann relations

- We cannot have a double discontinuity
- No double poles
- Two channels cannot overlap

More on duality relations

There are other works in the duality relation[8]

$$A = \sum_i \frac{n_i c_i}{d_i}; \qquad \tilde{A} = \sum_i \frac{\tilde{n}_i c_i}{d_i}$$

$$n_i - n_j = n_k; \qquad c_i - c_j = c_k;$$

$$A_{\text{grav}} = \sum_i \frac{n_i \tilde{n}_i}{d_i}$$

[8] Bern et al. 2019.

Gravity BFKL

There are works for the Gravity BFKL equation[9]

- We can ask if we can do something similar
- Would the KZ structure also appear?
- Work in progress

[9] Raj et al. 2025.