QCD evolution 2026

Simultaneous analysis of TMDs and PDFs

Alexei Prokudin

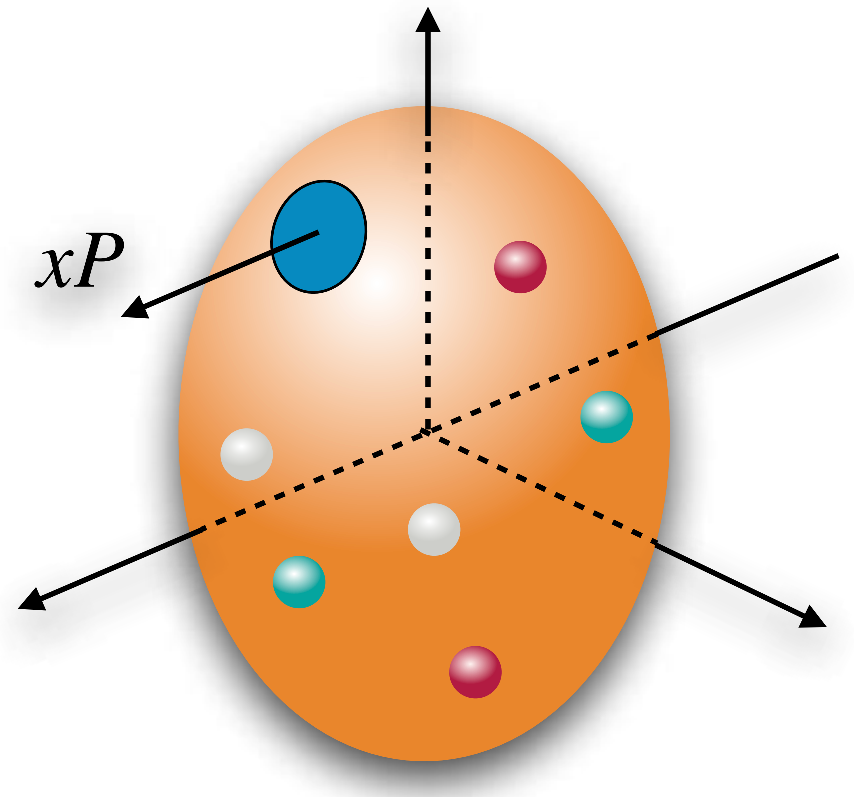
Pennsylvania State University Berks and Jefferson Lab

BASED ON

TMD WORKING GROUP

P. C. Barry, A. Prokudin, T. Anderson, C. Cocuzza, L. Gamberg, W. Melnitchouk, E. Moffat, D. Pitonyak, J.-W. Qiu, N. Sato, A. Vladimirov, and R. Whitehill *2510.13771*

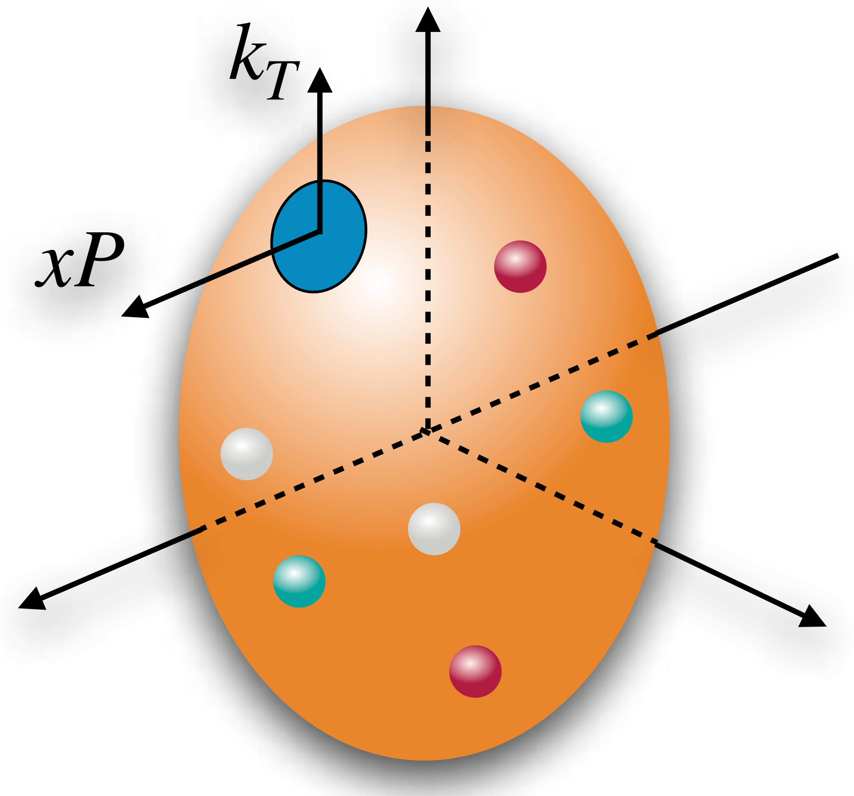
Collinear Parton Distribution Functions (PDFs)



1D

One large scale (Q) sensitive to particle nature of quark and gluons.
PDFs encode the 1D structure of the nucleon, the distribution in x.

Transverse Momentum Dependent Distributions (TMDs)



3D

One large scale (Q) sensitive to particle nature of quark and gluons.
One small scale (k_T) *sensitive to how QCD bounds partons* and to the detailed structure at ~fm distances.
TMDs encode the 3D structure of the nucleon, the distribution in x and k_T (or Fourier conjugate variable b_T).

SEE, E.G., C. LORCÉ, B. PASQUINI, M. VANDERHAEGHEN, JHEP 1105 (11)

TMD handbook: https://inspirehep.net/literature/2650019

TRANSVERSE MOMENTUM DEPENDENT FACTORIZATION

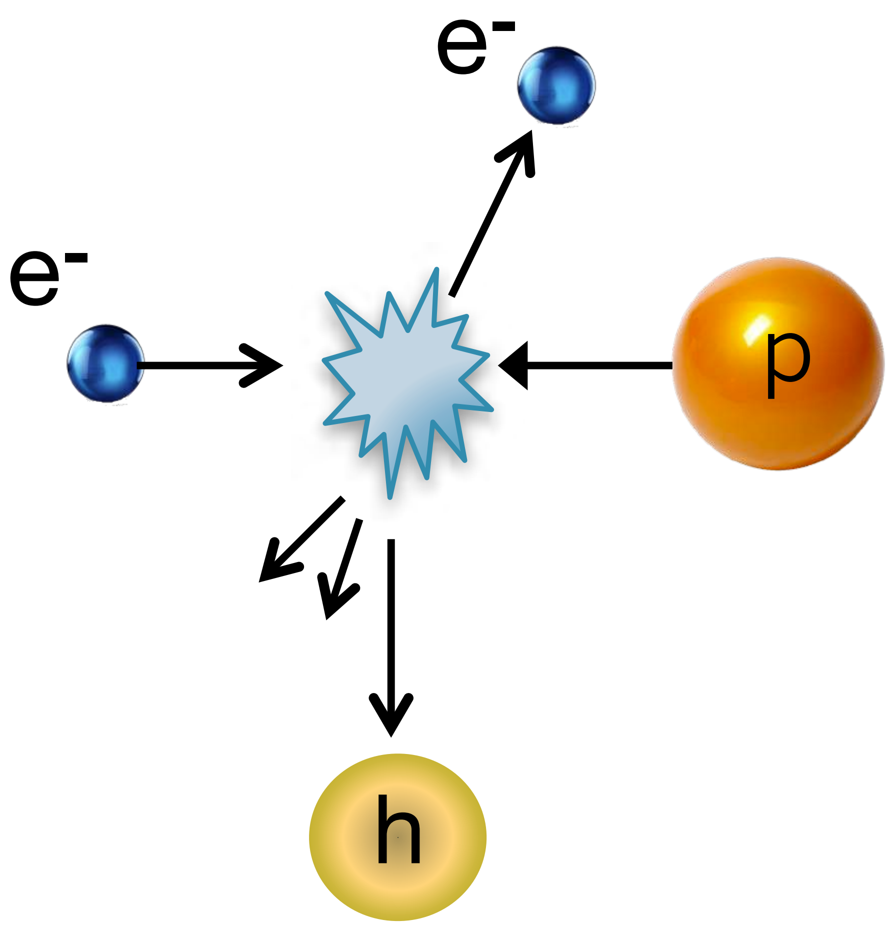
TMD handbook: https://inspirehep.net/literature/2650019

Small scale → $q_T \ll Q$ ← Large scale

The confined motion (k_T dependence) is encoded in TMDs

Semi-Inclusive DIS

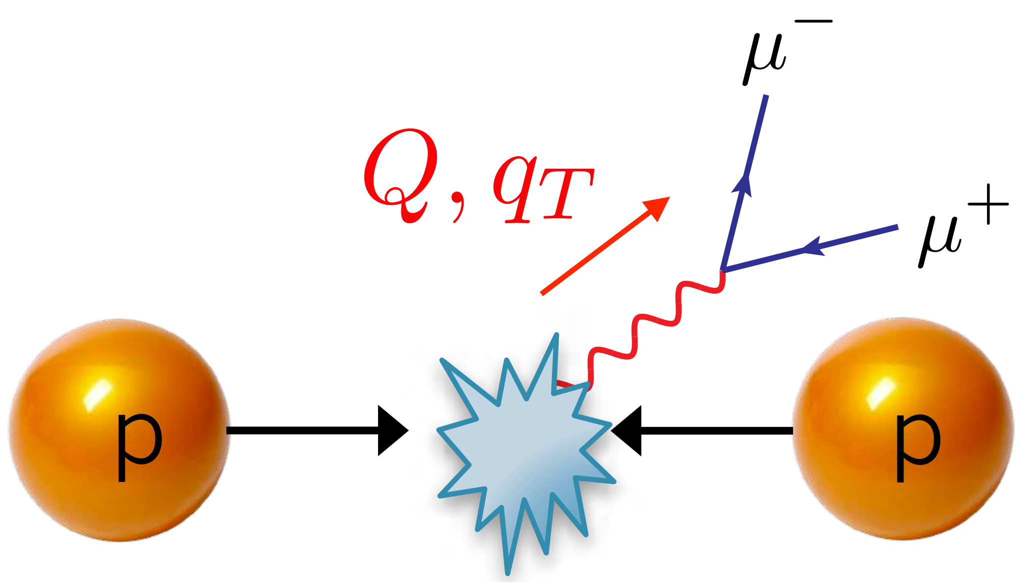
$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(z, k_T)$$



Meng, Olness, Soper (1992)
Ji, Ma, Yuan (2005)
Idilbi, Ji, Ma, Yuan (2004)
Collins (2011)

Drell-Yan

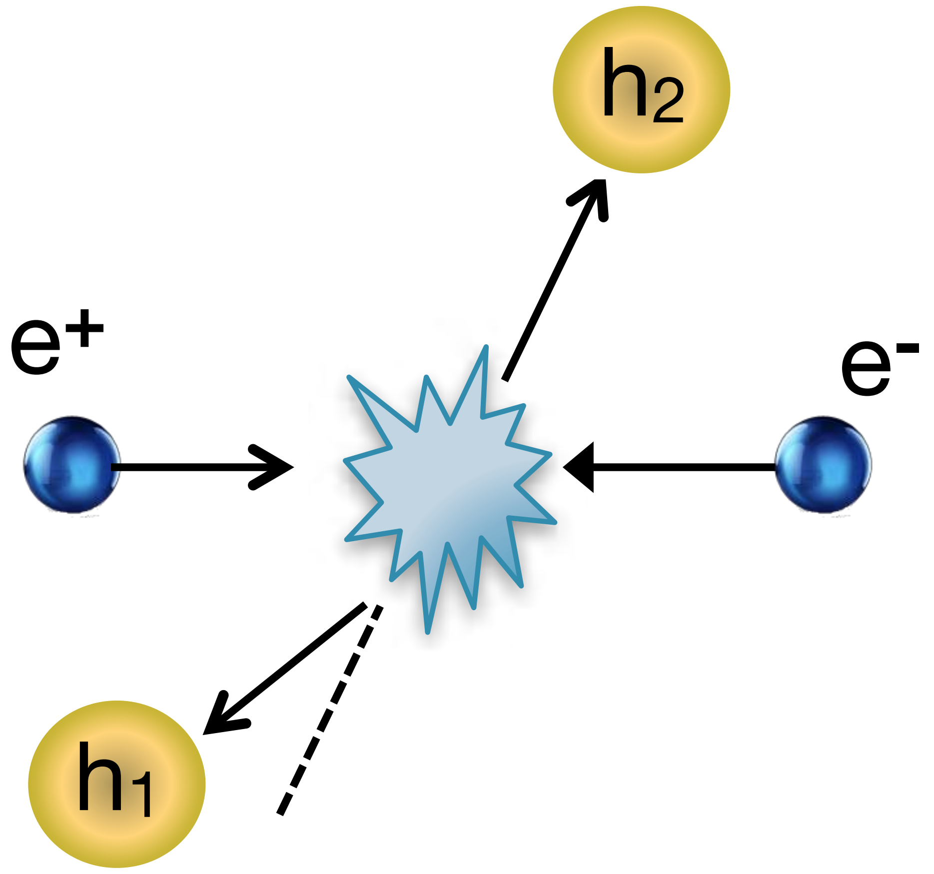
$$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$$



Collins, Soper, Sterman (1985)
Ji, Ma, Yuan (2004)
Collins (2011)

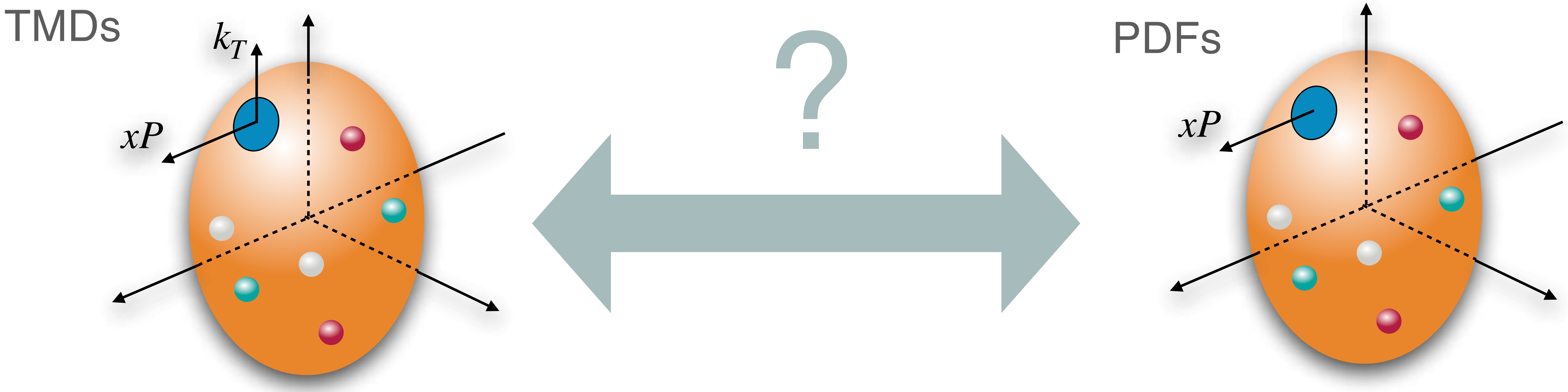
Dihadron in e^+e^-

$$\sigma \sim D_{h_1/P}(z, k_T) D_{h_2/q}(z, k_T)$$



Collins, Soper (1983)
Collins (2011)

WHAT IS THE RELATIONSHIP?



TMDs are related to PDFs, but how?

WHAT IS RELATIONSHIP?

TMD handbook: https://inspirehep.net/literature/2650019

Both PDFs and TMDs are extracted from the data of the same type of inclusive processes (DY, SIDIS, etc) in the corresponding regions of validity of the factorization theorems.

TMDs can be related to PDFs (or other collinear functions for polarized TMDs) via Operator Product Expansion at small values* of b_T

Known up to N3LO

M.A. Ebert, B. Mistlberger and G. Vita, JHEP 09 (2020) 146
M.-x. Luo, T.-Z. Yang, H.X. Zhu and Y.J. Zhu, JHEP 06 (2021) 115

$$\tilde{f}_f(x,b;\mu,\zeta) = \sum_{f'} \int_x^1 \frac{dy}{y} \tilde{C}_{f\to f'}(y,b,\mu,\zeta) f_{f'}\left(\frac{x}{y},\mu\right) + O(b^2)$$

TMDs can be related to PDFs via integral relations (TMMs)

Studied up to N3LO

$$\int^{\mu} d^2k_T f(x,k_T;\mu,\zeta) = f(x;\mu)$$

O. del Rio, AP, I. Scimemi, A.Vladimirov Phys.Rev.D 110 (2024)

A. Bacchetta, AP, Nucl.Phys.B 875 (2013) 536-551
M. A. Ebert, J. K. L. Michel, I. W. Stewart and Z. Sun, JHEP 07 (2022) 129
J. O. Gonzalez-Hernandez, T. Rainaldi, T. C. Rogers Phys.Rev.D 107 (2023) 9, 094029

* I will use b or b_T in this presentation as in the literature they are used to denote the same variable for TMDs

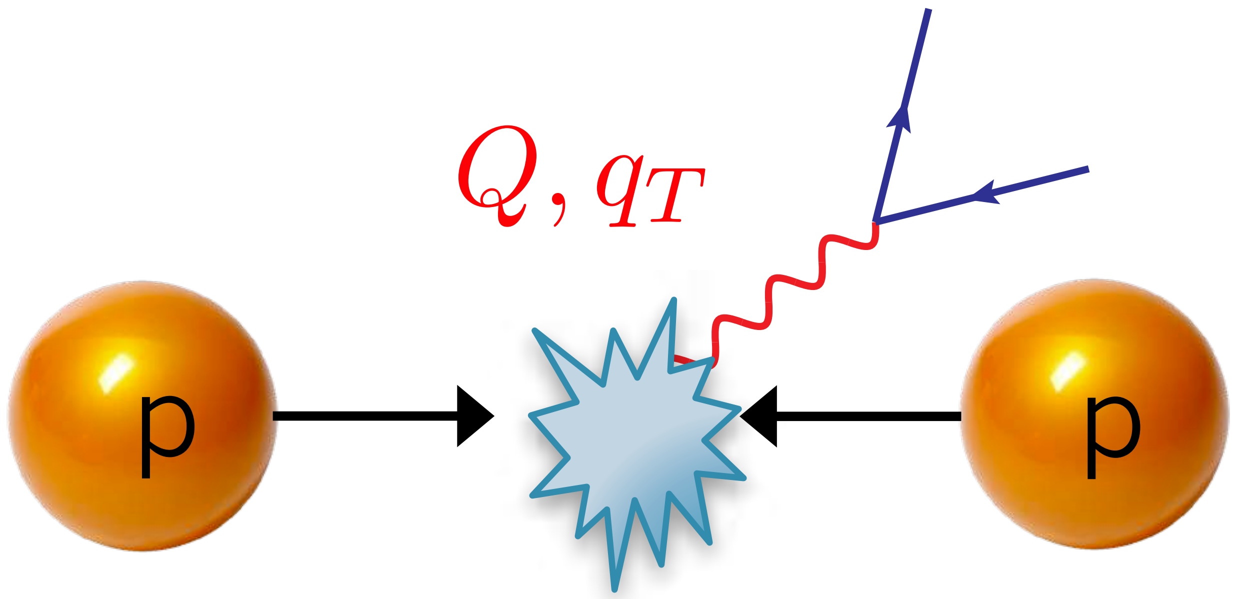
TMD FITS OF UNPOLARIZED DATA

TMD handbook: https://inspirehep.net/literature/2650019

	Framework	W+Y	HERMES	COMPASS	DY	Z boson	W boson	N of points
KN 2006 hep-ph/0506225	LO-NLL	W	✘	✘	✔	✔	✘	98
QZ 2001 hep-ph/0506225	NLO-NLL	W+Y	✘	✘	✔	✔	✘	28 (?)
RESBOS resbos@msu	NLO-NNLL	W+Y	✘	✘	✔	✔	✘	>100 (?)
Pavia 2013 arXiv:1309.3507	LO-PM	W	✔	✘	✘	✘	✘	1538
Torino 2014 arXiv:1312.6261	LO-PM	W	✔ (separately)	✔ (separately)	✘	✘	✘	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO-NNLL	W	✘	✘	✔	✔	✘	223
EIKV 2014 arXiv:1401.5078	LO-NLL	W	1 (x,Q^2) bin	1 (x,Q^2) bin	✔	✔	✘	500 (?)
SIYY 2014 arXiv:1406.3073	NLO-NLL	W+Y	✘	✔	✔	✔	✘	200 (?)
Pavia 2017 arXiv:1703.10157	LO-NLL	W	✔	✔	✔	✔	✘	8059
SV 2017 arXiv:1706.01473	NNLO-NNLL	W	✘	✘	✔	✔	✘	309
BSV 2019 arXiv:1902.08474	NNLO-NNLL	W	✘	✘	✔	✔	✘	457
Pavia 2019 arXiv:1912.07550	NNLO-N3LL	W	✘	✘	✔	✔	✘	353
SV 2019 arXiv:1912.06532	NNLO-N3LL	W	✔	✔	✔	✔	✘	1039
MAP pion 2022 arXiv:2210.01733	NLO-N3LL	W	✘	✘	✔	✘	✘	138
MAP 2022 arXiv:2206.07598	NNLO-N3LL-	W	✔	✔	✔	✔	✘	2031
JAM 2023 arXiv: 2302.01192	NLO-NNLL	W	✘	✘	✔	✘	✘	608
ART 2023 arXiv:2305.07473	N3LO-N4LL	W	✘	✘	✔	✔	✔	627
RESBOS arXiv:2311.09916	NNLO-N3LL	W+Y	✘	✘	✔	✔	✘	384
Aslan at al 2024 arXiv:2401.14266	NLO-NLL	W	✘	✘	✔	✘	✘	130
MAP 2025 arXiv:2502.04166	NNLO-N3LL	W	✘	✘	✔	✔	✘	482
ART 2025 arXiv:2503.11201	N3LO-N4LL	W	✔	✔	✔	✔	✔	1269
JAM 2025 arXiv: 2510.13771	NLO-NNLL	W	✘	✘	✔	✔	✘	436+4279
KSZZ 2026 arXiv:2604.14133	N3LO-N4LL	W	✘	✘	✔	✔	✘	465

Is it possible to combine the full body of data used in the extractions of TMDs and PDFs, respectively, and to perform a global QCD analysis of both distributions simultaneously?

DRELL-YAN PROCESS



At small values of q_T Drell-Yan cross section reads:

$$\frac{d^3\sigma}{dQ^2 dy dq_T^2} = \frac{4\pi^2\alpha_{\rm em}^2}{9Q^2 s}\mathcal{P}\sum_j c_j^2(Q) H_{j\bar{j}}^{\rm DY}(\mu, Q)\int\frac{d^2\boldsymbol{b}_T}{(2\pi)^2}e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}\tilde{f}_{q_j/\mathcal{N}}(x_1, b_T;\mu,\zeta)\tilde{f}_{\bar{q}_j/\mathcal{N}}(x_2, b_T;\mu,\zeta),$$

Fiducial factor

Hard factor

Electroweak couplings

TMD for the beam

TMD for the target

To match the NLO precision of collinear PDFs by JAM Collaboration we use **NLO + N2LL** accuracy in our analysis: PDFs are at NLO $\mathcal{O}(\alpha_S^2)$, hard factor at $\mathcal{O}(\alpha_S)$, Operator Product Expansion at $\mathcal{O}(\alpha_S)$, anomalous dimension of TMD at $\mathcal{O}(\alpha_S^2)$ and, Collins Soper-Kernel at $\mathcal{O}(\alpha_S^2)$, and Cusp anomalous dimension at $\mathcal{O}(\alpha_S^3)$. We use ζ prescription for TMDs

DRELL-YAN DATASETS

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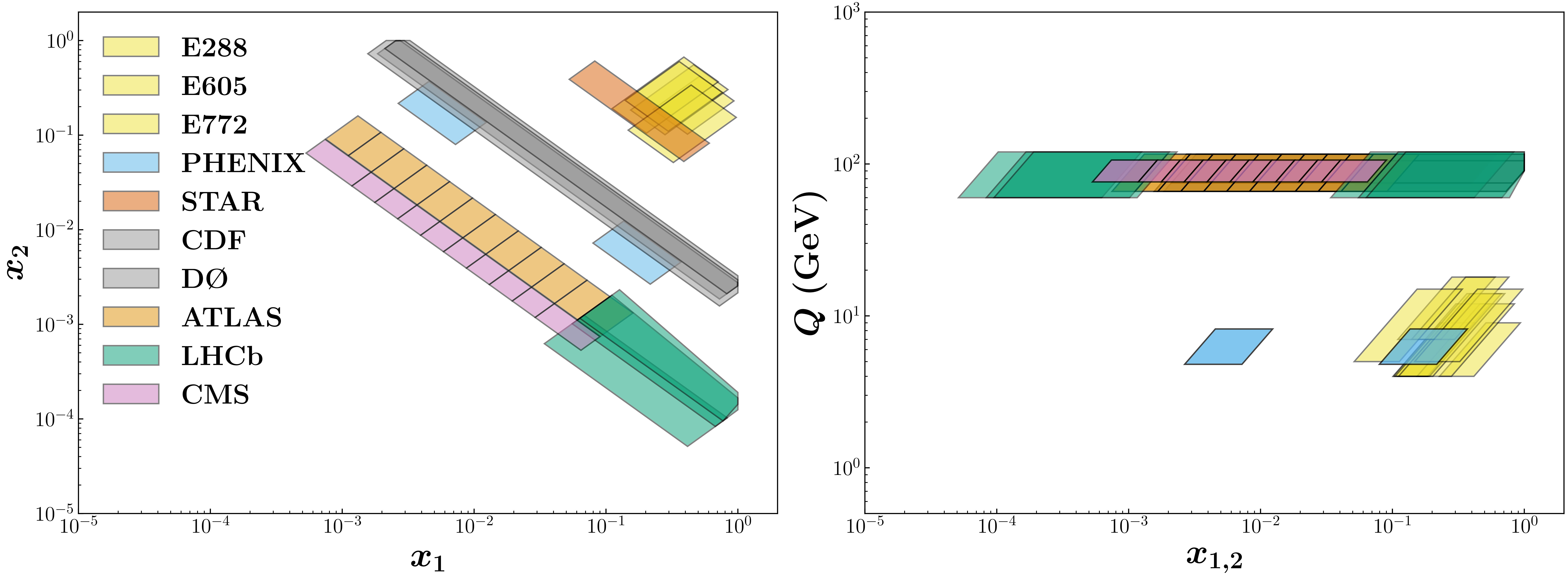
Data from low energy fixed target DY (20-40 GeV). Collider data from RHIC, PHENIX and STAR at 200 and 510 GeV, Fermilab, CDF and D0 at 1.8 TeV, 1.96 TeV, and LHC data from CMS, LHCb, ATLAS at 7, 8,13 TeV



DATASETS AND KINEMATICS

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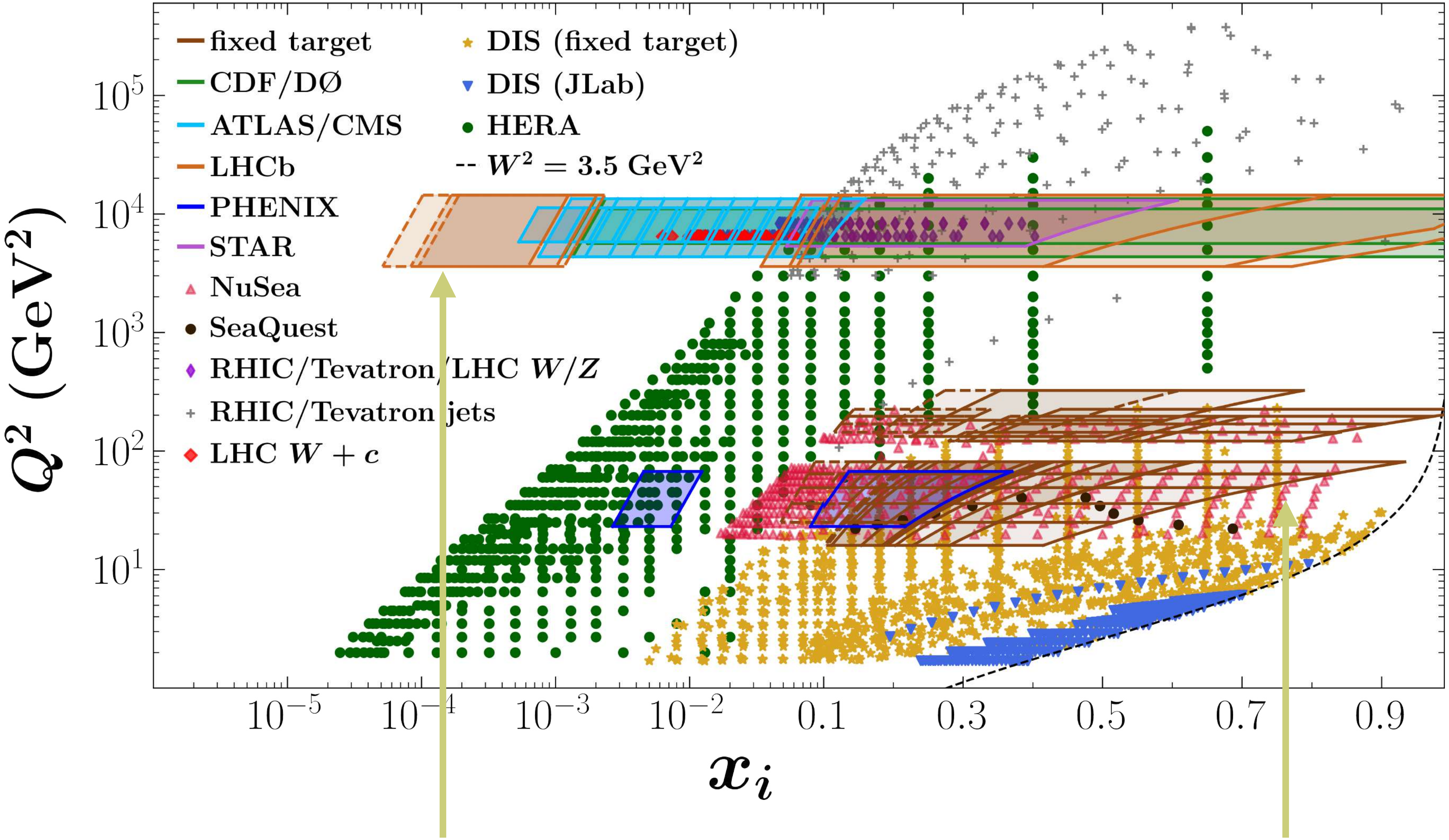
Symbols are for the data relevant to collinear extraction, regions are for TMD related data

JAM Collaboration (PDF Analysis Group) Trey Anderson et al. Phys.Rev.D 112 (2025) 9, 094011
Jefferson Lab Angular Momentum (JAM) C. Cocuzza et al. Phys.Rev.Lett. 127 (2021) 24, 242001



Precise data at small and intermediate x

Data at large-x

NON PERTURBATIVE MODELS FOR TMDS AND PDFS

Fit $\lambda_1$and λ_2 in this functional form for these flavors: $u, d, \bar{u}, \bar{d}$ and $sea = s, \bar{s}, c, \bar{c}, b, \bar{b}$. Total of 10 parameters

$$F^f(x, b) = \frac{1}{\cosh\left(\lambda_1^f(1-x) + \lambda_2^2 x)b\right)}$$

Moos, Scimemi, Vladimirov), Zurita: JHEP 05 (2024)

Collinear PDFs are parametrized and the input scale to the charm quark mass, $m_c = 1.28$ GeV for $u_v, d_v, \bar{u}, \bar{d}, s, \bar{s}, g$

$$f(x; m_c) = Nx^\alpha(1-x)^\beta(1 + \gamma\sqrt{x} + \delta x)$$

JAM Collaboration (PDF Analysis Group),T. Anderson et al. Phys.Rev.D 112 (2025) 9, 09401

heavy flavors are generated via evolution. We have 35 total parameters for collinear PDFs

NON PERTURBATIVE MODEL FOR COLLINS-SOPER KERNEL

Collins-Soper kernel can be expressed as

$$\mathscr{D}(b_T;\mu) = \mathscr{D}_{\text{pert}}(b^*;\mu^*) + \int_{\mu^*}^{\mu} \frac{d\mu'}{\mu'}\Gamma(\mu') + \mathscr{D}_{\text{NP}}(b_T)$$

Moos, Ignazio Scimemi, Alexey Vladimirov), Zurita: JHEP 05 (2024)

For the non perturbative Collins-Soper kernel we use

$$\mathscr{D}_{NP} = bb^* \left(c_0 + c_1 \ln \frac{b^*}{b_{NP}} \right),$$ where $b^*(b_T) = \frac{b_T}{\sqrt{1 + b_T^2/B_{\text{NP}}^2}}$, $\mu^* = \frac{2e^{-\gamma_E}}{b^*(b_T)}$ and

$B_{\text{NP}} = 1.5 \text{ GeV}^{-1}$

The perturbative part of the kernel $\mathscr{D}_{\text{pert}}(b^*;\mu^*)$ is calculated at $\mathcal{O}(\alpha_S^2)$

This parametrization ensures evolution and perturbative convergence, c_0, c_1 are free parameters.

ANALYSIS SET UP

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We perform the analysis in four steps:

- **baseline PDF** — an analysis of PDFs only
- **baseline TMD** — an analysis of TMDs with fixed PDFs. Low energy DY, RHIC, and Tevatron data
- **baseline TMD+LHC** — an analysis of TMDs with fixed PDFs, **baseline TMD** data plus LHC data
- **JAM25$_{\text{TMD+PDF}}$** — combined analysis of TMDs and PDFs

In each step we generate around 1000 replicas and perform Bayesian inference

The following cuts are used:

$$\frac{q_T}{Q} < 0.2,\ W^2 > 3.5\ (\text{GeV}^2),\ Q > m_c = 1.28\ (\text{GeV})$$

RESULTS

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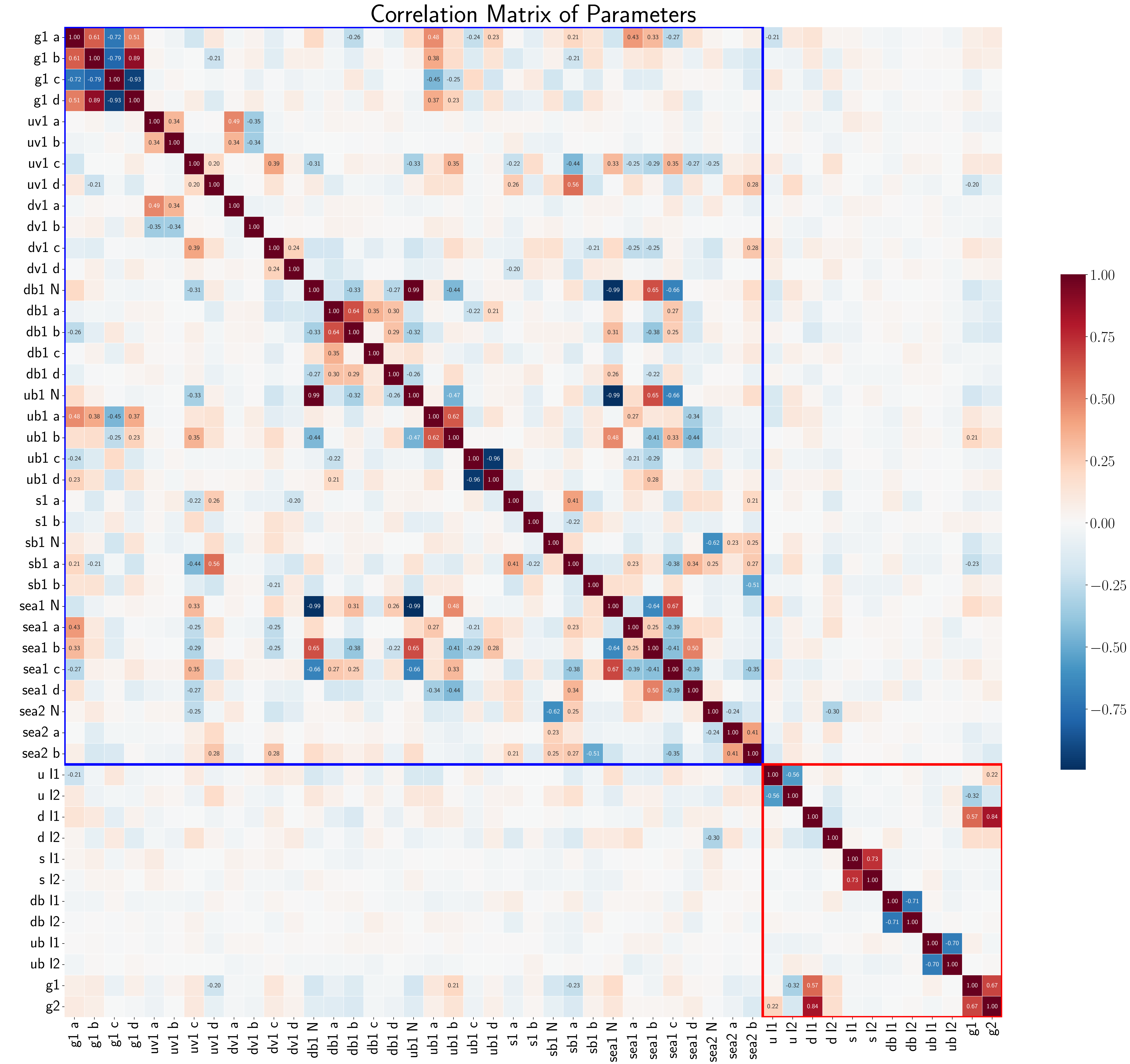
Collinear				
Process	Experiment	N_{pts}	χ^2/N_{pts} (Z-score) PDF only	**TMD+PDF**
DIS	fixed target	2501	1.02 (0.82)	**1.02 (0.59)**
	HERA	1185	1.27 (6.01)	**1.27 (6.11)**
DY	E866, 906	205	1.27 (2.54)	**1.26 (2.50)**
$W\ \ell$ a.	LHC, RHIC	70	0.80 (-1.20)	**0.78 (-1.35)**
W ch. a.	CDF, D0	27	1.12 (0.51)	**1.13 (0.53)**
Z rap.	CDF, D0	56	1.09 (0.55)	**1.11 (0.60)**
Inc. jets	CDF, D0, STAR	198	1.00 (0.00)	**0.98 (-0.13)**
$W + c$	ATLAS, CMS	37	0.65 (-1.66)	**0.62 (-1.84)**
Total		4279	1.10 (4.31)	1.09 (4.14)

TMD				
Process	Experiment	N_{pts}	TMD only	**TMD+PDF**
DY	E288, 605, 772	224	1.36 (3.44)	**1.31 (3.02)**
	CDF, D0	80	1.06 (0.45)	**1.11 (0.71)**
	STAR, PHENIX	12	1.15 (0.47)	**1.20 (0.60)**
	ATLAS	30	2.07 (3.29)	**1.78 (2.55)**
	CMS	64	1.18 (1.03)	**0.92 (-0.39)**
	LHCb	26	0.53 (-1.97)	**0.50 (-2.12)**
Total		436	1.27 (3.72)	1.20 (2.76)
Total		**4715**		**1.10 (4.79)**

Combined TMD and PDF extraction improves on the quality of description of the data while keeping collinear datasets described well.



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The analysis does not find strong correlation of PDF (blue rectangle) and TMD (red rectangle) parameters

Pearson coefficients are shown

$$r = \frac{\mathrm{Cov}(X,Y)}{\sigma_X \sigma_Y} \in [-1,1]$$

RESULTS

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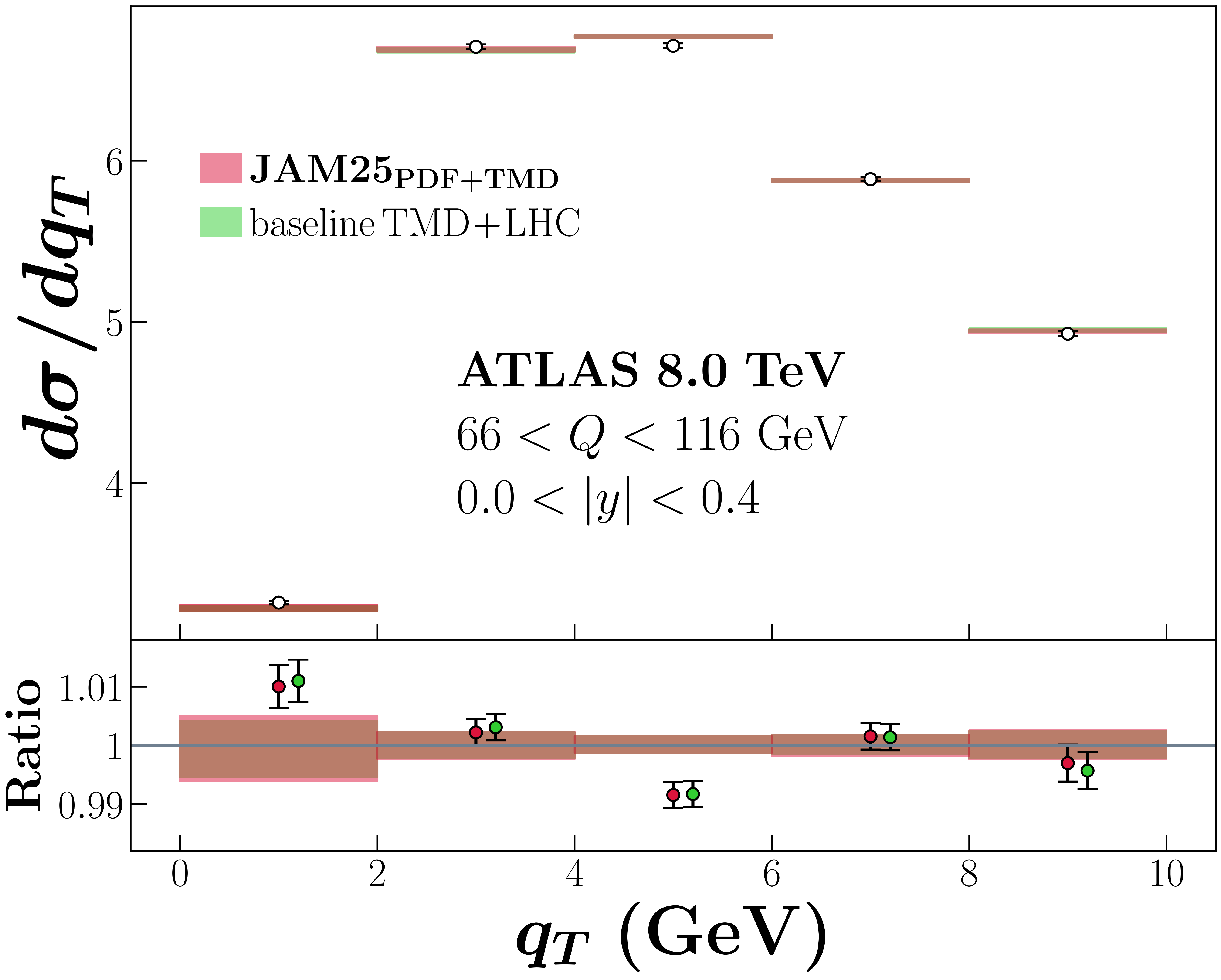

RESULTS COLLINEAR PDFS

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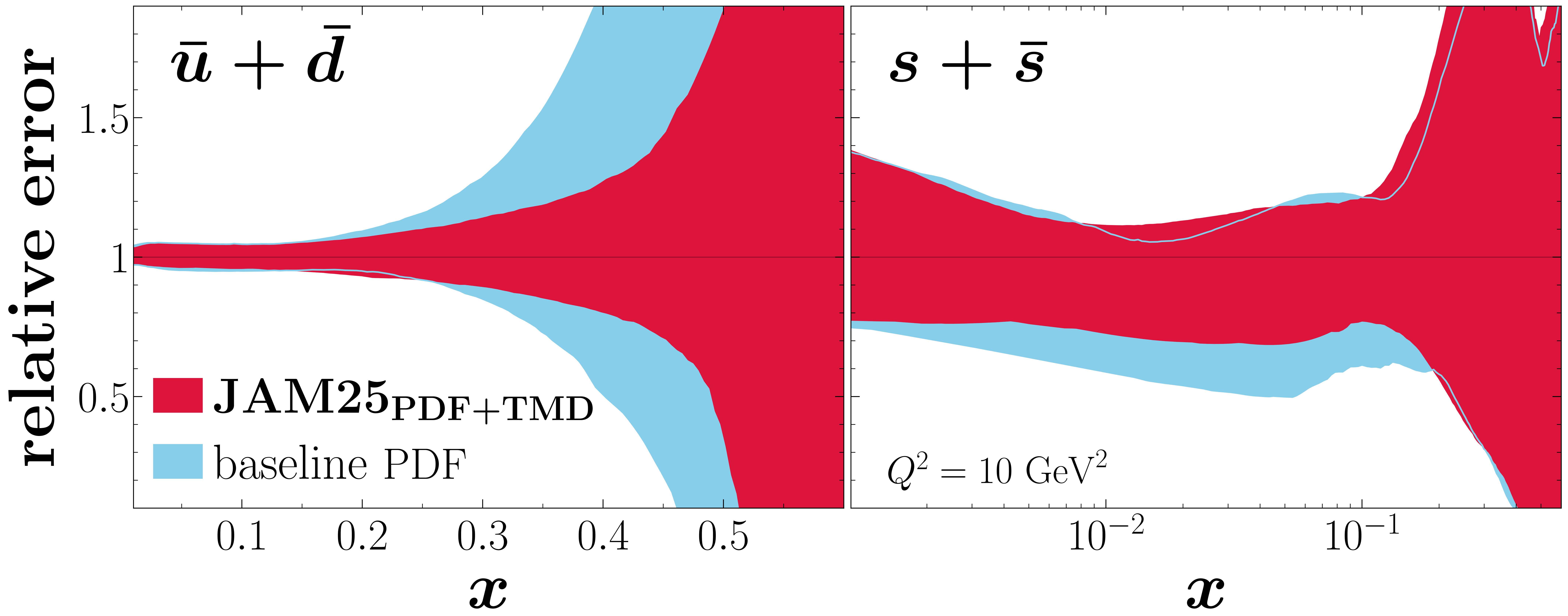

The combined analysis results in the improvement for $\bar{d} + \bar{u}$ in large x region due to the inclusion of fixed target DY data and improvement of $s + \bar{s}$ due to the inclusion of the LHC data.

Shown here is the 95% confidence interval.

RESULTS COLLINEAR PDFS

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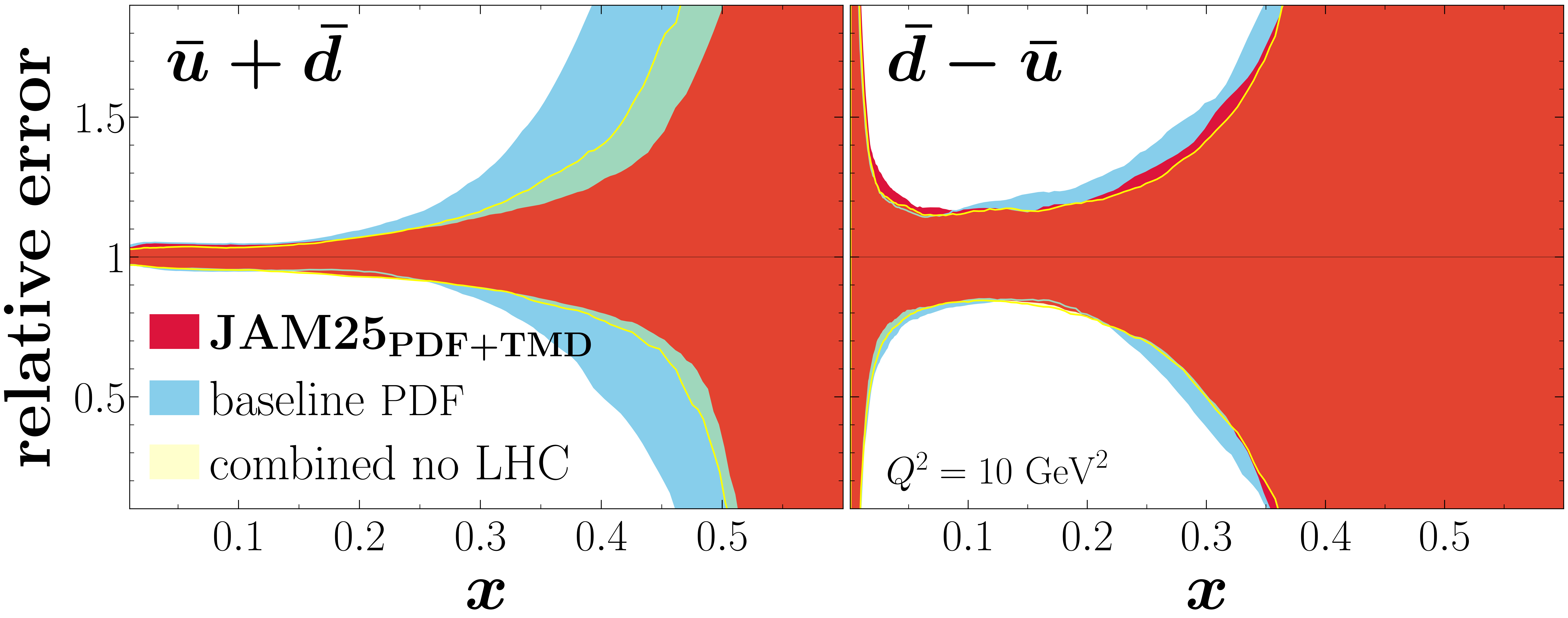
- **Exclude LHC data to estimate the impact of fixed target DY**



The combined analysis results in the improvement for $\bar{d} + \bar{u}$ in large x region due to the inclusion of fixed target DY consistent with the isoscalar nature of the target. Notice that $\bar{d} - \bar{u}$ is not impacted.

RESULTS COLLINEAR PDFS

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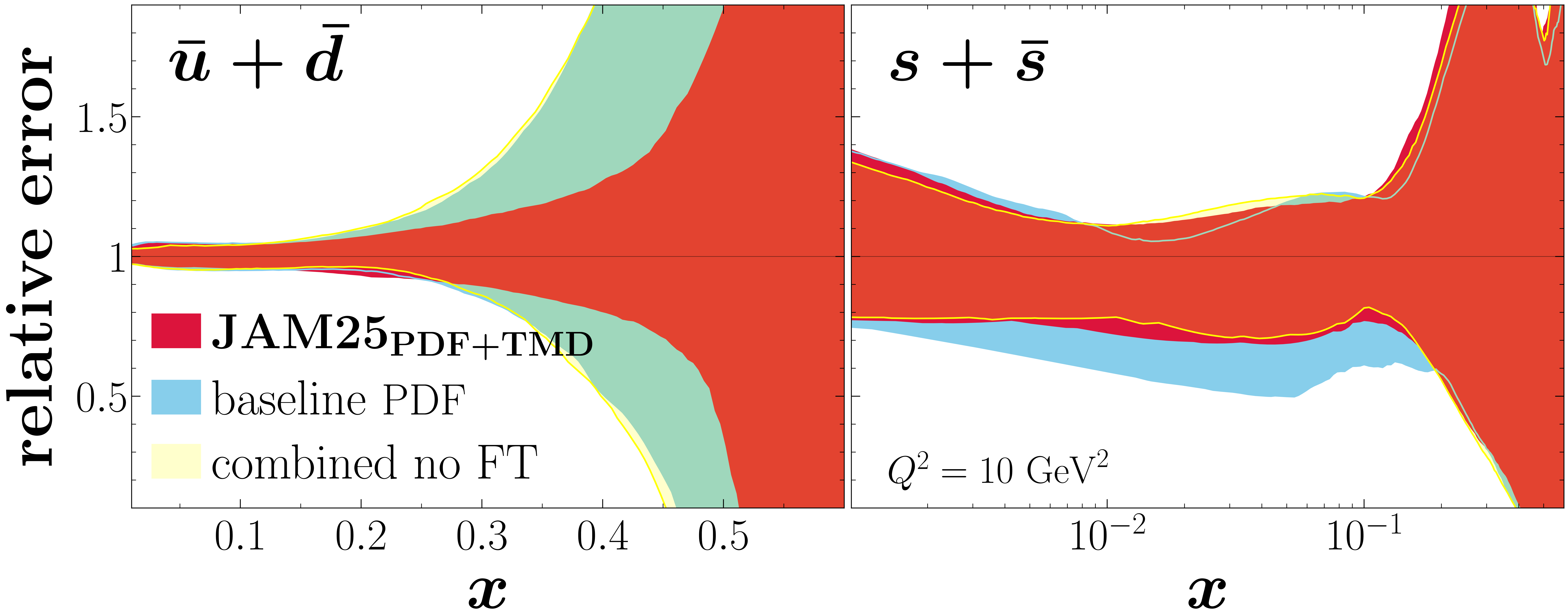
- Exclude fixed target DY data to estimate the impact of LHC data



The improvement of $s + \bar{s}$ due to the inclusion of the LHC data.

RESULTS COLLINEAR PDFS

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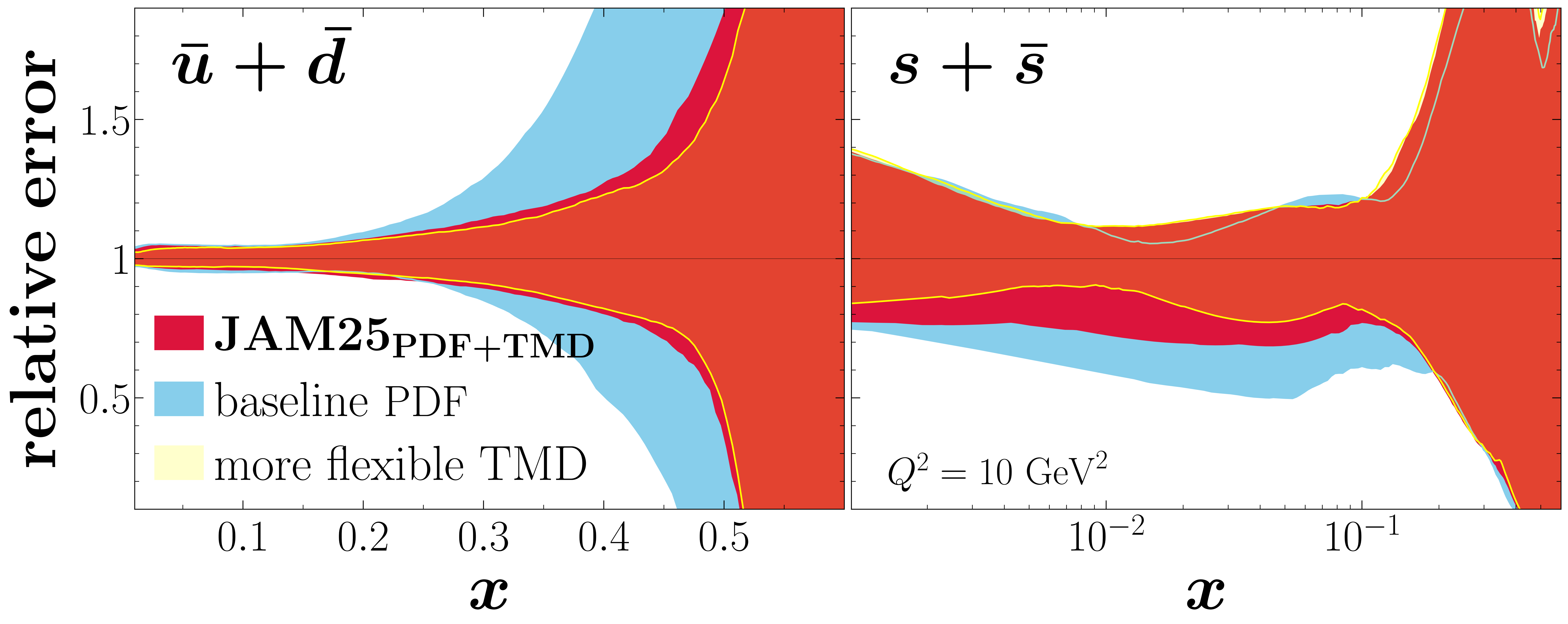
Maybe the rigidify of TMD parametrizations make this improvement?

$$F^f(x,b) = \frac{1}{\cosh\left(\lambda_1^f(1-x)^{\lambda_f^3} + \lambda_2^2 x)b\right)}$$



No strong influence of results on TMD parametrization.

RESULTS COLLINEAR PDFS

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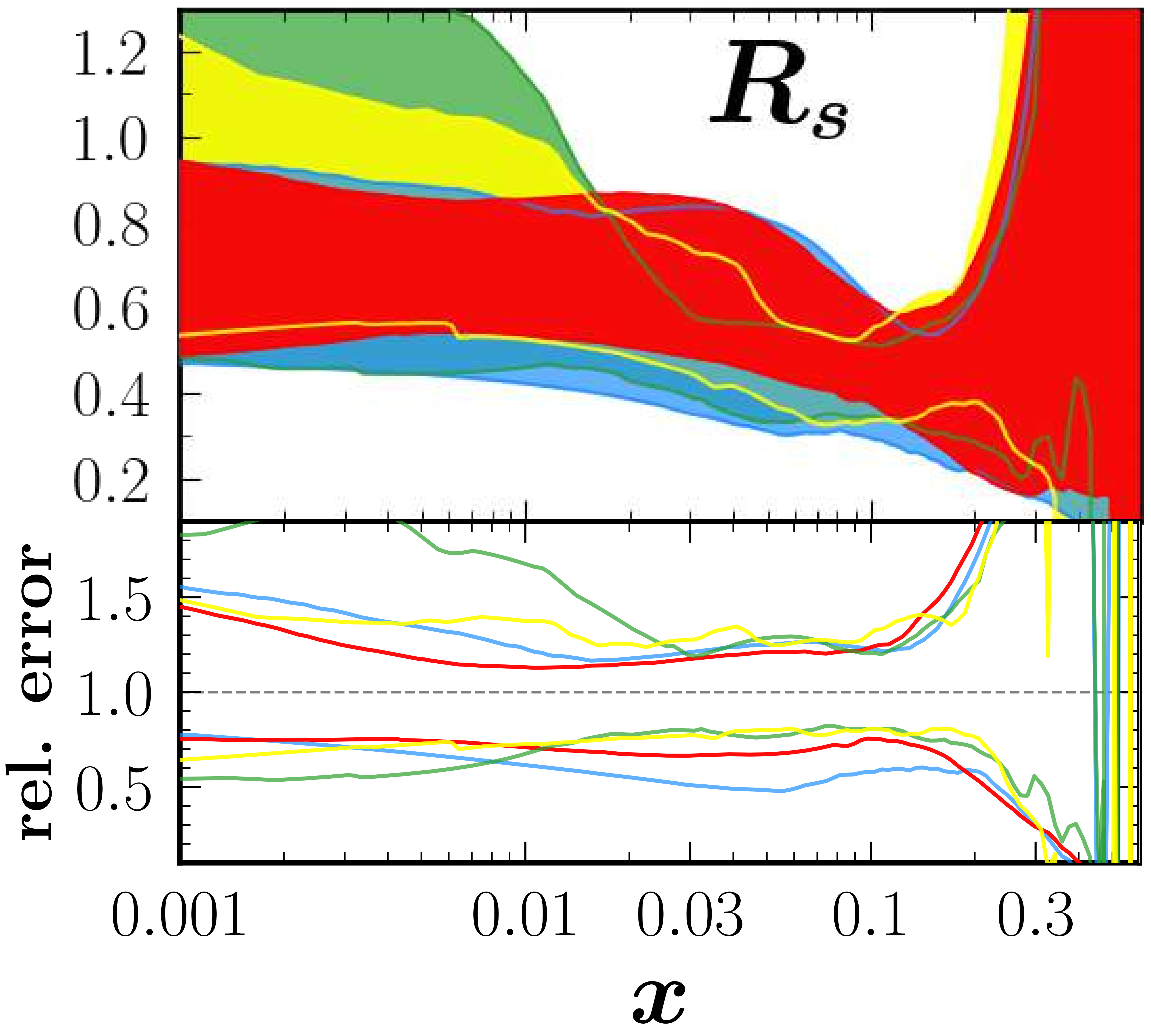
Is the improvement consistent with findings of the analyses that include LHC integrated data?



baseline
JAM25$_{\text{PDF+TMD}}$
NNPDF3.0
NNPDF3.1

$$R_s \equiv \frac{s + \bar{s}}{\bar{u} + \bar{d}}$$

The reduction in uncertainty is consistent with the inclusion of the collinear LHC data as observed by NNPDF. This speaks to the consistency of the two independent theoretical frameworks.

RESULTS TMDS

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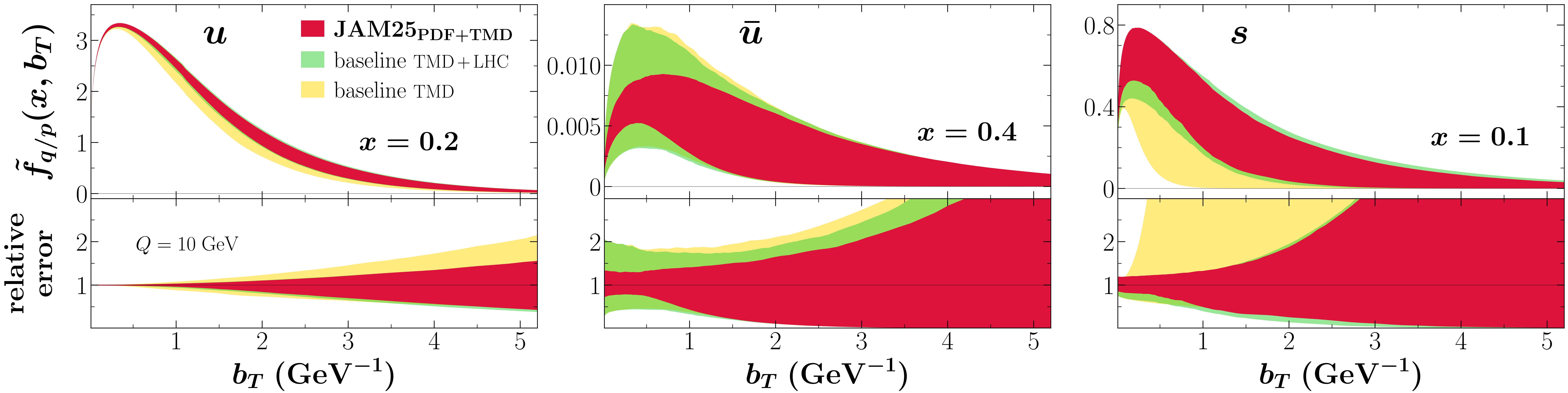

Combined fit results in the improvement of the extraction of TMDs. The LHC data are important for u, d, s quarks. Fixed target DY data improves $\bar{u}, \bar{d}$ quarks at large x

RESULTS TMDS

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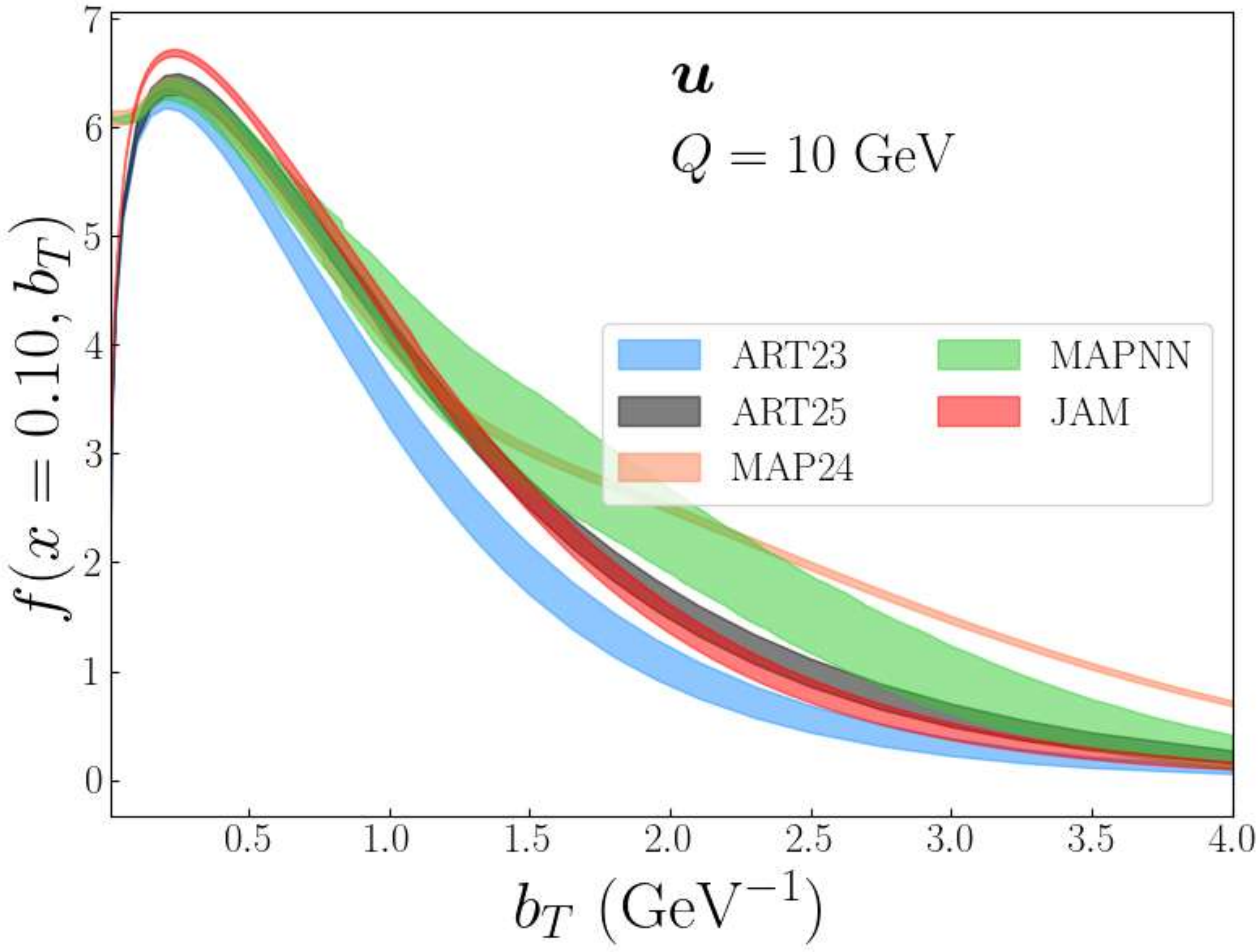
Our results are in agreement with results from other groups.



ART23: V. Moos, I. Scimemi, A. Vladimirov, P. Zurita JHEP 05 (2024) 036
ART25: V. Moos, I. Scimemi, A. Vladimirov, P. Zurita e-Print: 2503.11201
MAP24: MAP Collaboration, A. Bacchetta, JHEP 08 (2024) 232
MAPNN: MAP Collaboration, A. Bacchetta, Phys.Rev.Lett. 135 (2025) 2, 021904

RESULTS TMDS

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We perform *closure* tests to assess robustness of the framework:

- Fix PDFs and recover TMDs using TMD pseudo data only
- Fix TMDs and recover PDFs using pseudo PDF and TMD data
- Open all and recover TMDs and PDFs using pseudo PDF and TMD data

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RESULTS TMDS

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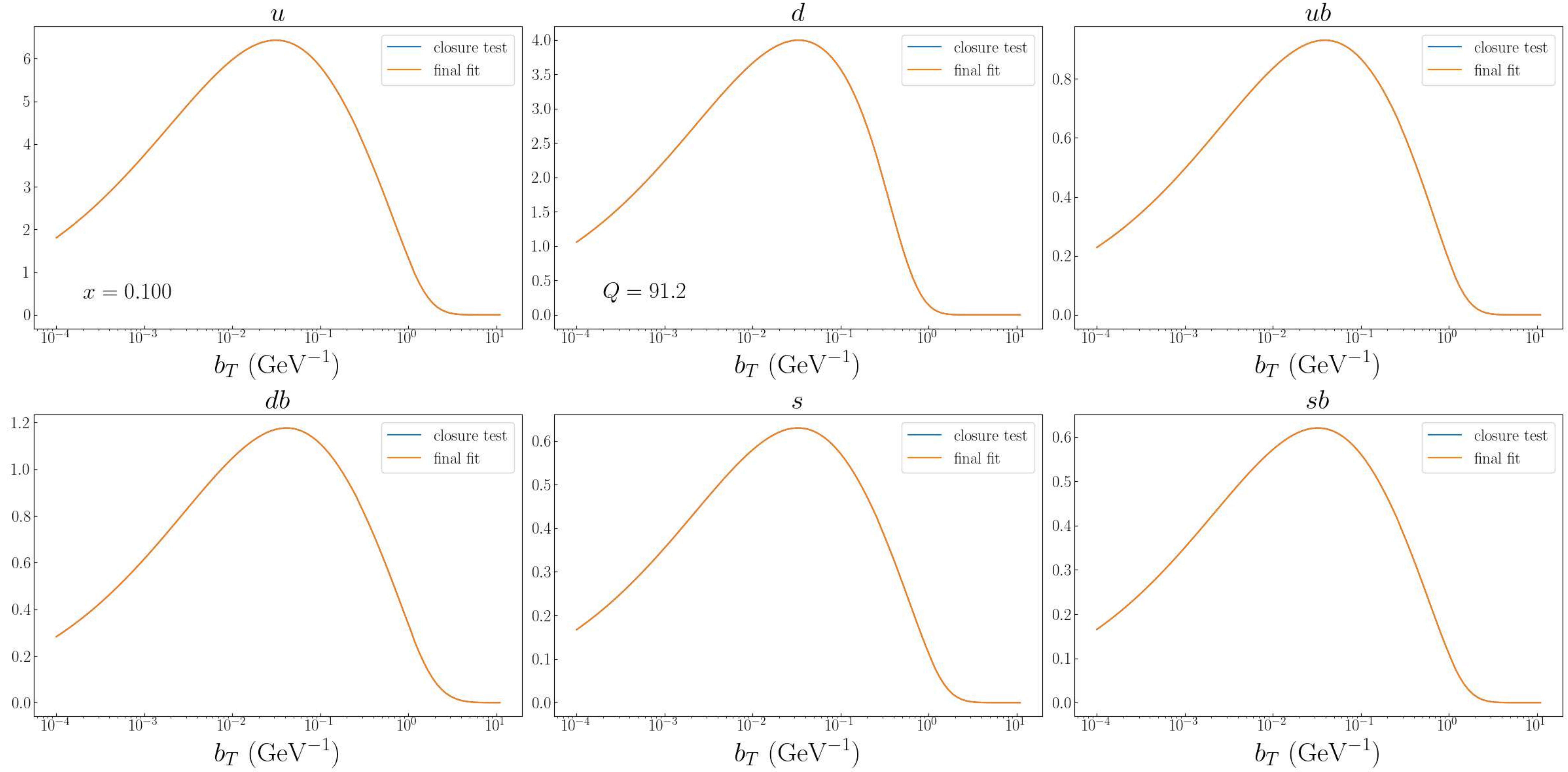
We perform closure tests to assess robustness of the framework:

- Fix PDFs and recover TMDs using TMD pseudo data only



Perfect consistency in TMD sector

RESULTS TMDS

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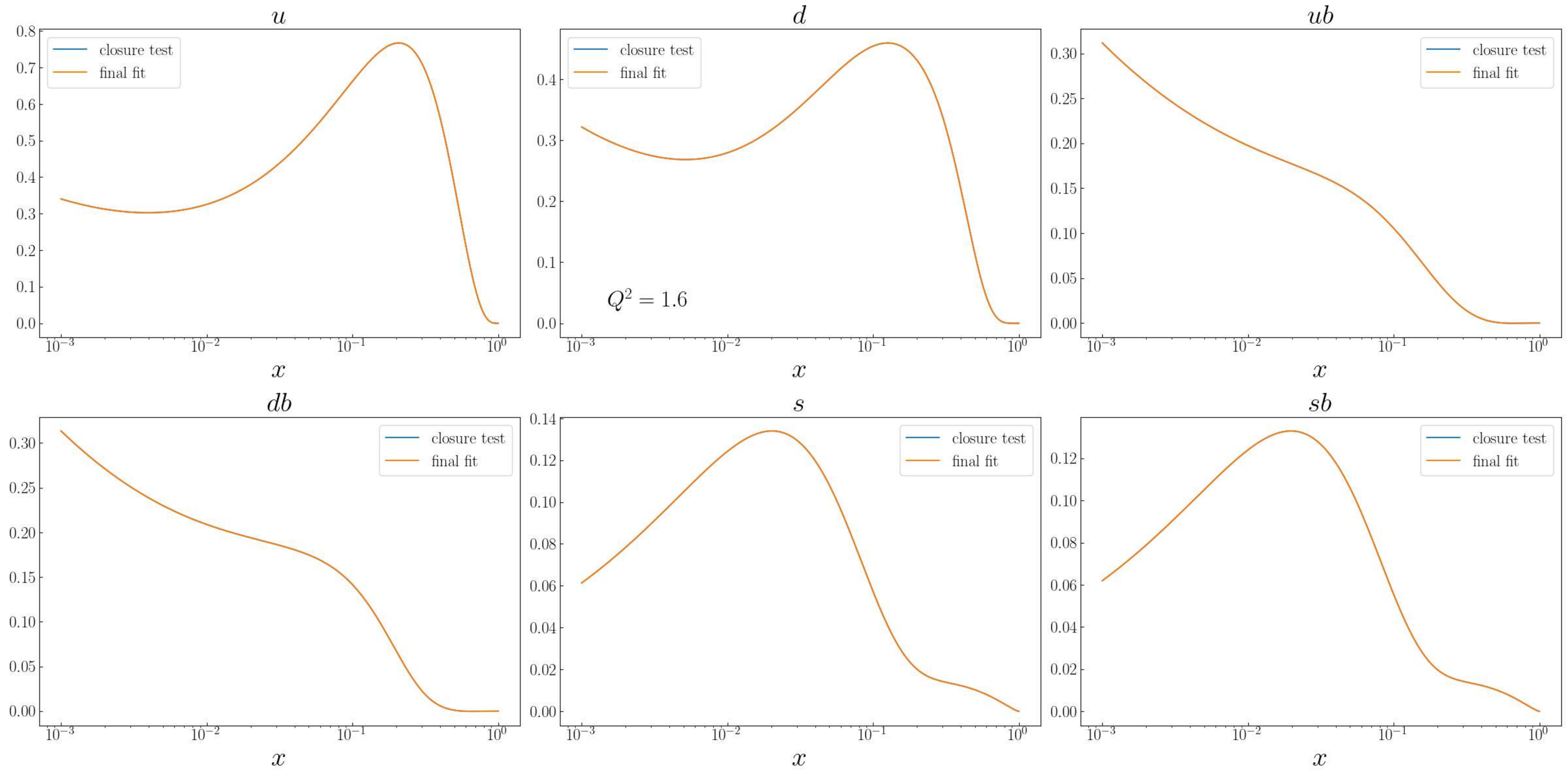
We perform closure tests to assess robustness of the framework:

- Fix TMDs and recover PDFs using pseudo PDF and TMD data



Perfect consistency in PDF sector

RESULTS TMDS

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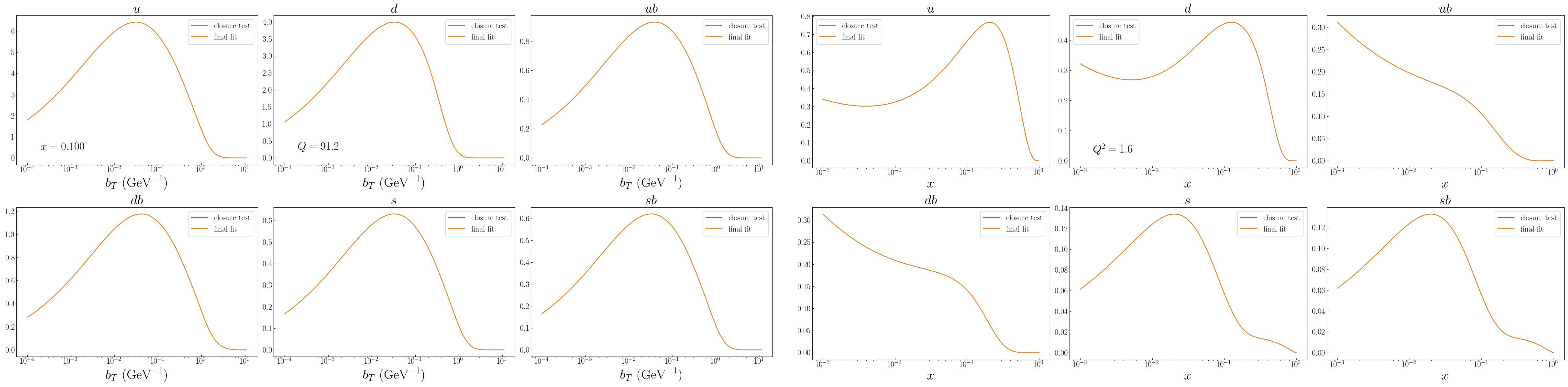
We perform closure tests to assess robustness of the framework:

- Open all and recover TMDs and PDFs using pseudo PDF and TMD data



We demonstrated consistency across these three complementary closure tests and absence of biases hidden in the restricted tests therefore strengthening the reliability of our framework.

RESULTS COLLINS-SOPER KERNEL

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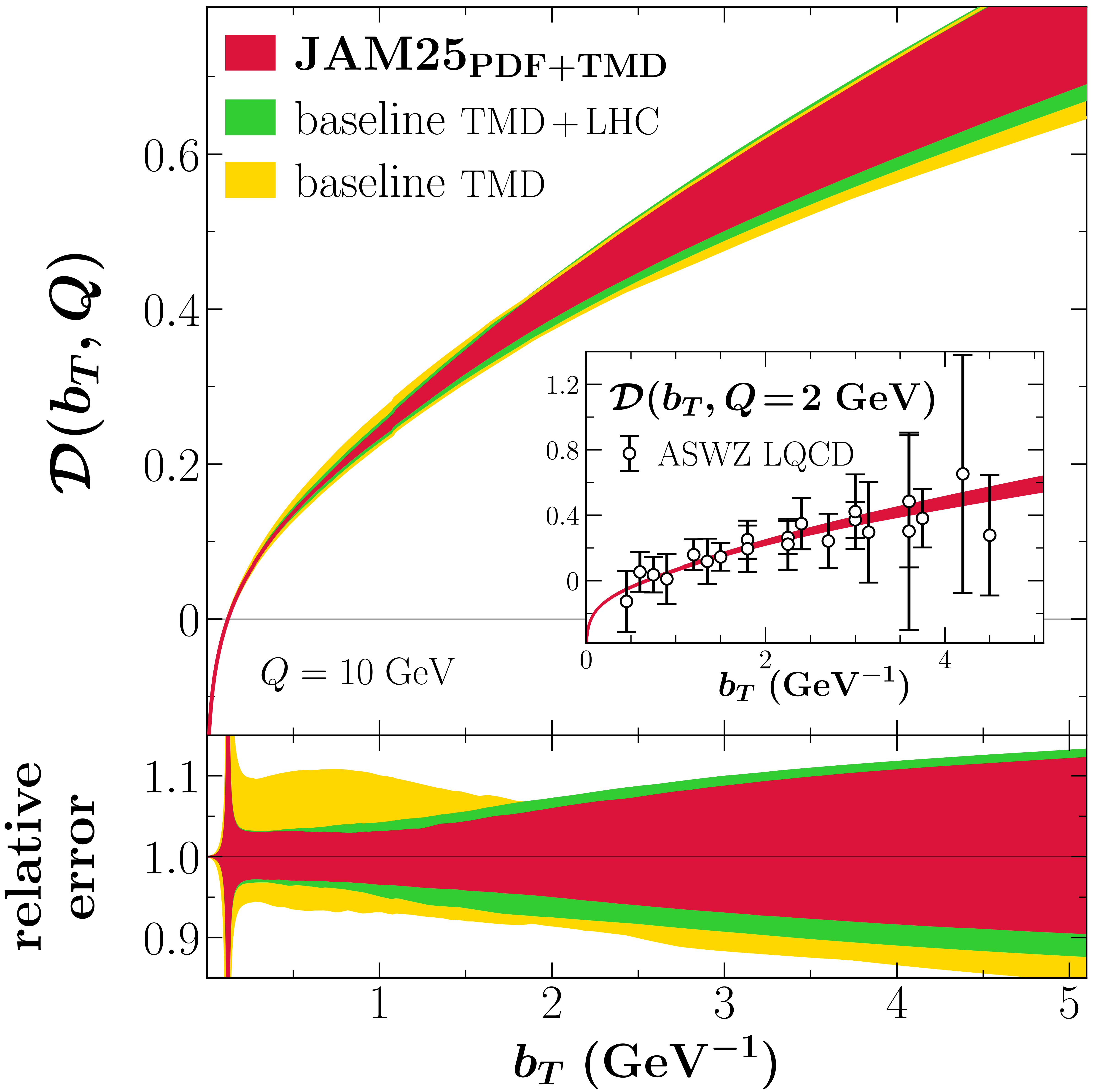

Collins Soper kernel is an important ingredient of the analysis. The LHC data is particularly impactful on its extraction.

Compared to lattice results from

A. Avkhadiev, P. E. Shanahan, M. L. Wagman, and Y. Zhao, Phys. Rev. Lett. 132, 231901 (2024)

RESULTS COLLINS-SOPER KERNEL

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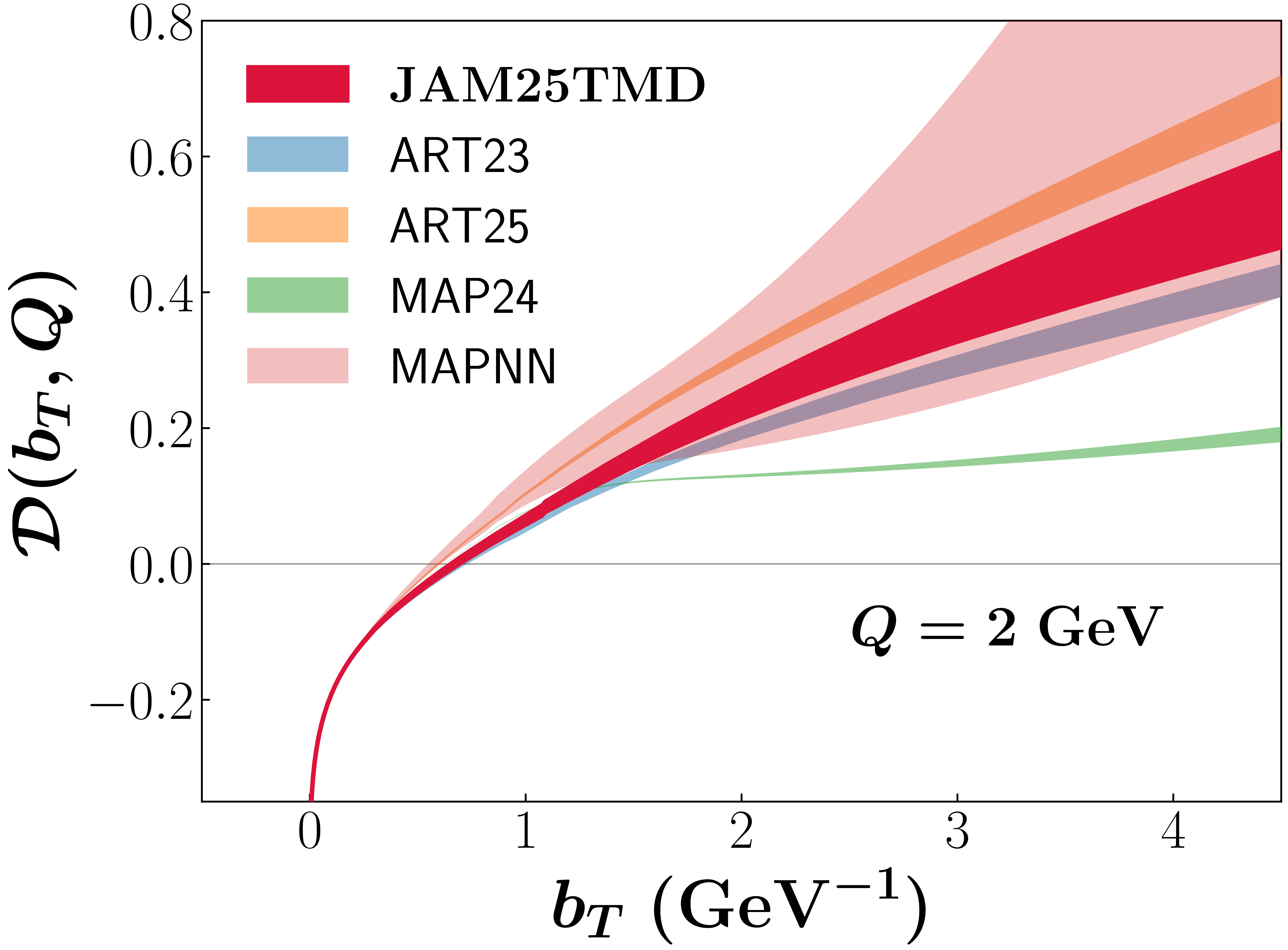


Collins Soper kernel compares well to existing extractions.

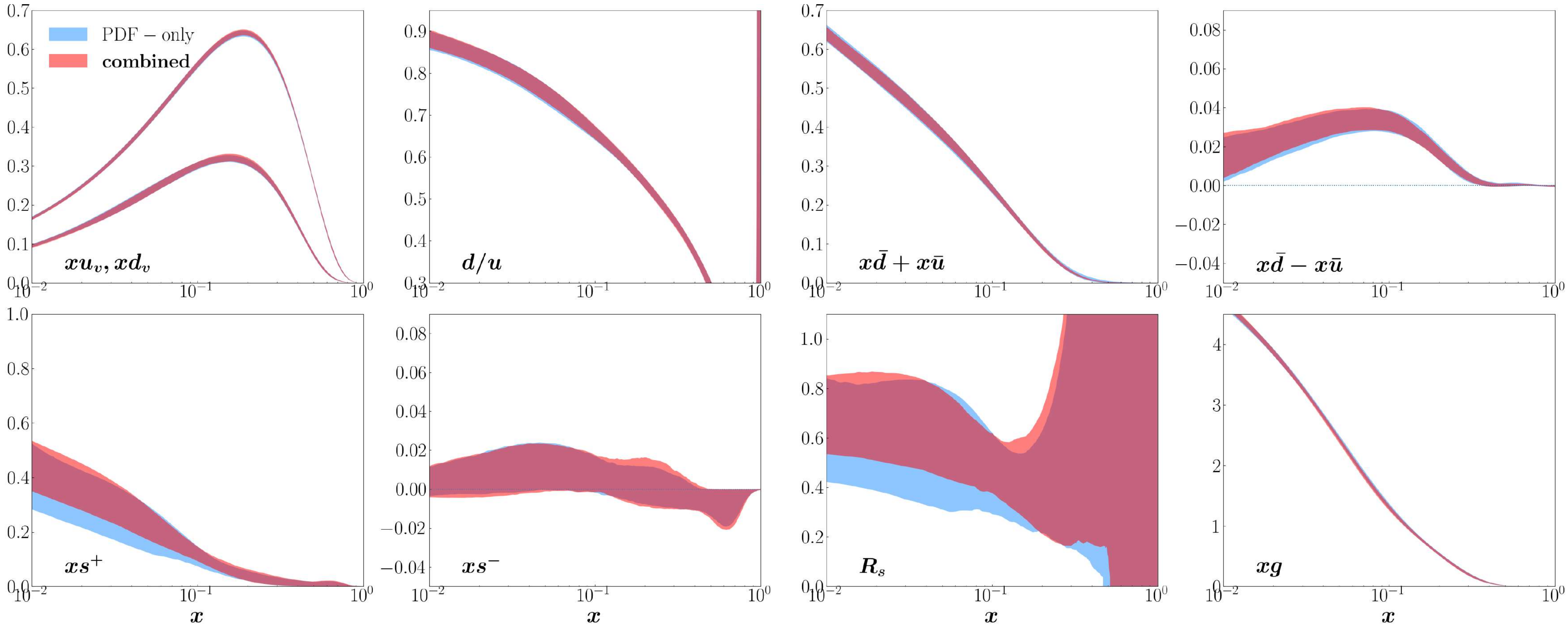
ART23: V. Moos, I. Scimemi, A. Vladimirov, P. Zurita JHEP 05 (2024) 036
ART25: V. Moos, I. Scimemi, A. Vladimirov, P. Zurita e-Print: 2503.11201
MAP24: MAP Collaboration, A. Bacchetta, JHEP 08 (2024) 232
MAPNN: MAP Collaboration, A. Bacchetta, Phys.Rev.Lett. 135 (2025) 2, 021904

CONCLUSIONS

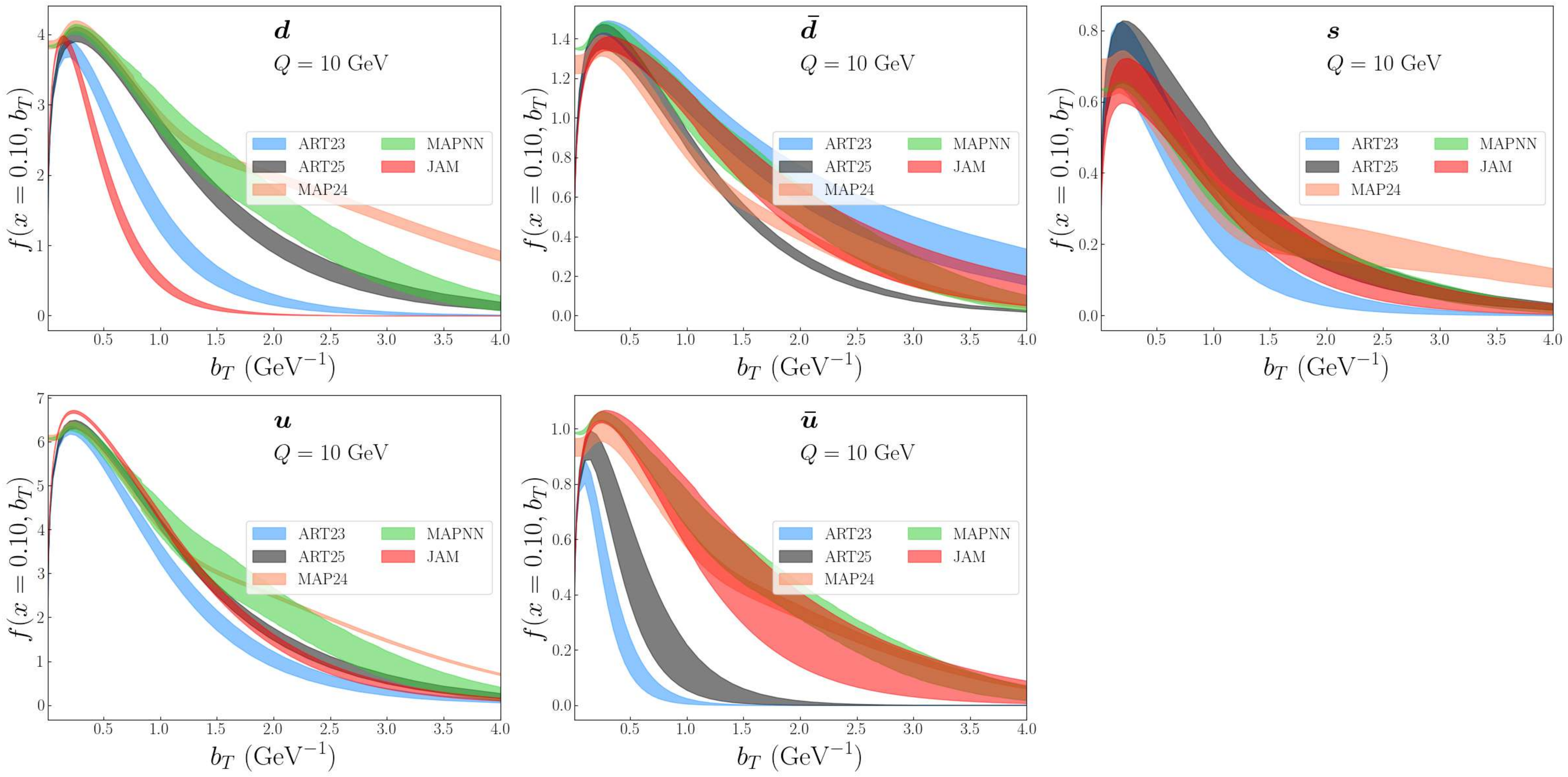
- A combined analysis of TMDs and PDFs is possible and leads to refinements in both TMDs and PDFs
- We are planning on performing a comparative study of prescriptions for the solution of TMD evolution equations: CSS, Qiu-Zhang, ζ prescription
- We are planning on studying collinear and TMD fragmentation functions and on inclusion of Semi-Inclusive Deep Inelastic Scattering data

BACKUP

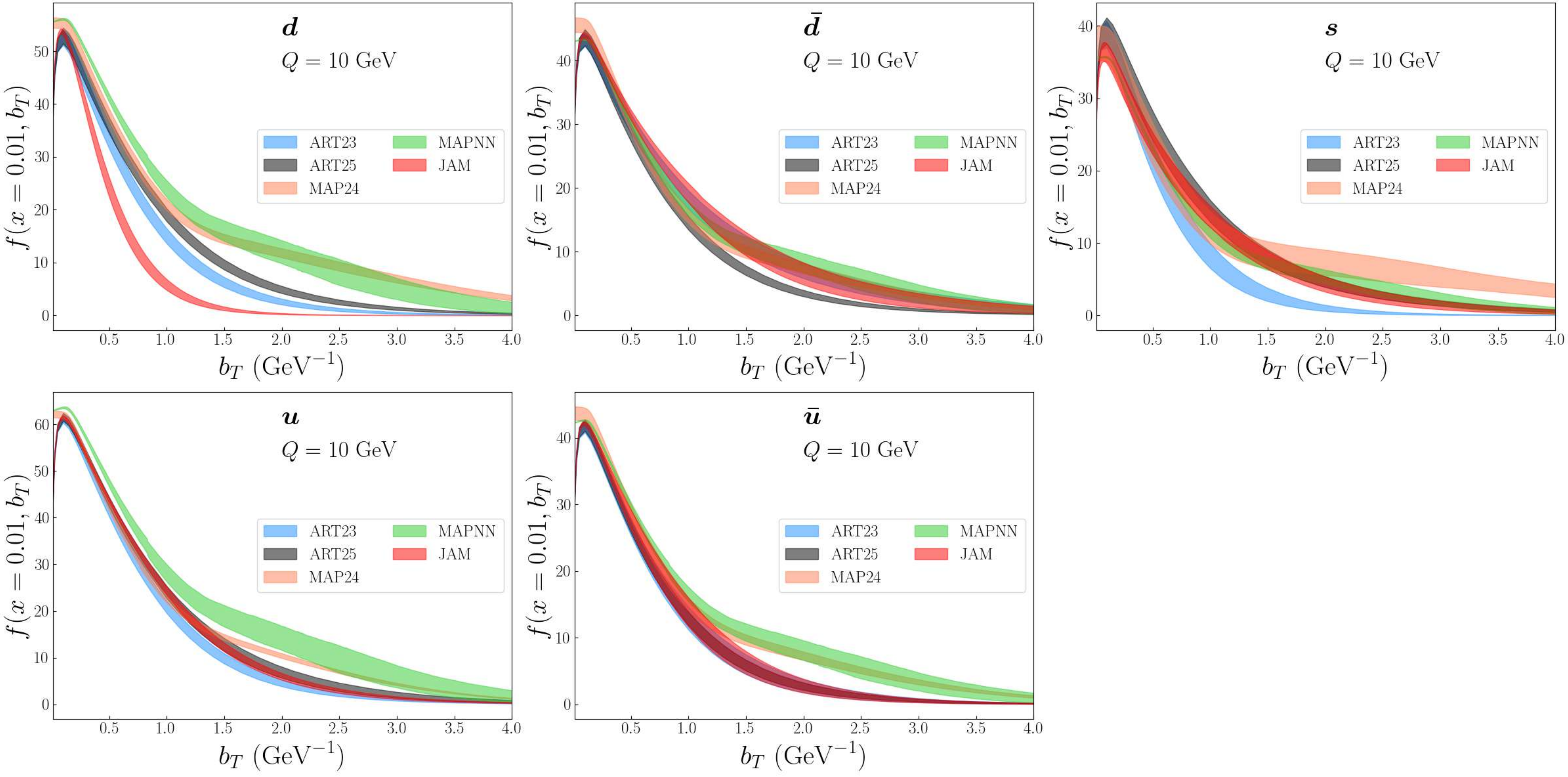
PDFS FROM JAM25$_{\text{TMD+PDF}}$



TMDS FROM JAM25$_{\text{TMD+PDF}}$



TMDS FROM JAM25$_{\text{TMD+PDF}}$



COLLINEAR DATA

Phenomenological framework — The collinear data used in this analysis includes DIS data from BCDMS [64], NMC [65, 66], SLAC [67], Jefferson Lab [68–71] and HERA [72], DY lepton-pair production from the Fermilab E866 NuSea [73] and E906 SeaQuest [74] experiments, $W^{\pm}$-lepton asymmetries from CMS [75–78] and LHCb [79, 80] at the LHC and STAR [81] from RHIC, reconstructed Z/γ^{*} and $W^{\pm}$ asymmetries from the Tevatron [81–85], $W+$charm production [86–88] from the LHC, and jet production data from the Tevatron [89, 90] and RHIC [91]. A cut on DIS data of $W^2 > 3.5$ GeV^2 allows sensitivity to quark distributions in the large-x region.

TMDS IN ζ-PRESCRIPTION

I. Scimemi and A. Vladimirov, JHEP 08, 003 (2018)

Collins-Soper evolution equations:

$$\mu^2 \frac{d}{d\mu^2} F(x,b;\mu,\zeta) = \frac{\gamma_F(\mu,\zeta)}{2} F(x,b;\mu,\zeta)$$

$$\zeta \frac{d}{d\zeta} F(x,b;\mu,\zeta) = -\mathcal{D}(b,\mu) F(x,b;\mu,\zeta)$$

$$\mathcal{D} = -\tilde{K}/2 \qquad \gamma_F(\mu,\zeta) = \Gamma_{cusp}(\mu) \ln\left(\frac{\mu^2}{\zeta}\right) - \gamma_V(\mu)$$

One can derive:

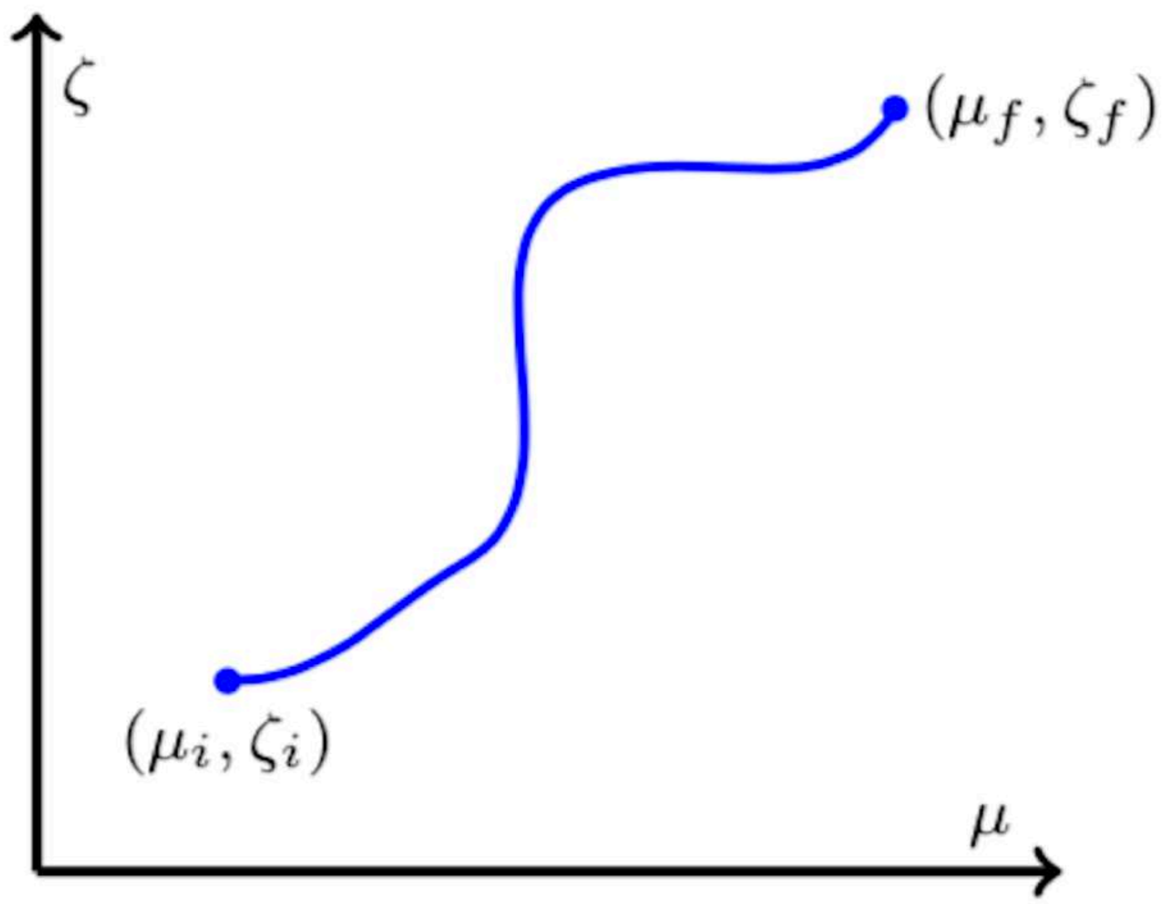
$$\zeta \frac{d}{d\zeta} \gamma_F(\mu,\zeta) = -\Gamma(\mu)$$

$$\mu \frac{d}{d\mu} \mathcal{D}(\mu,b) = \Gamma(\mu)$$

$$\zeta \frac{d}{d\zeta} \gamma_F(\mu,\zeta) = -\mu \frac{d}{d\mu} \mathcal{D}(\mu,b)$$

The solution:

$$F(x,b;\mu_f,\zeta_f) = \exp\left[\int_P \left(\gamma_F(\mu',\zeta')\frac{d\mu'}{\mu'} - \mathcal{D}(b,\mu')\frac{d\zeta'}{\zeta'}\right)\right] F(x,b;\mu_i,\zeta_i)$$



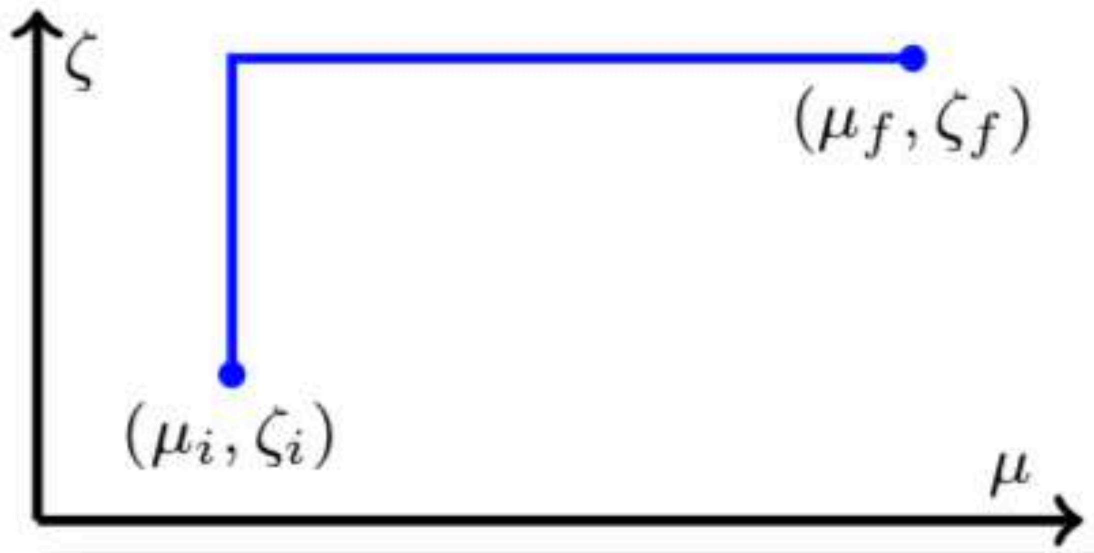
The evolution factor is path independent

(in principle)

TMDS IN ζ-PRESCRIPTION

I. Scimemi and A. Vladimirov, JHEP 08, 003 (2018)

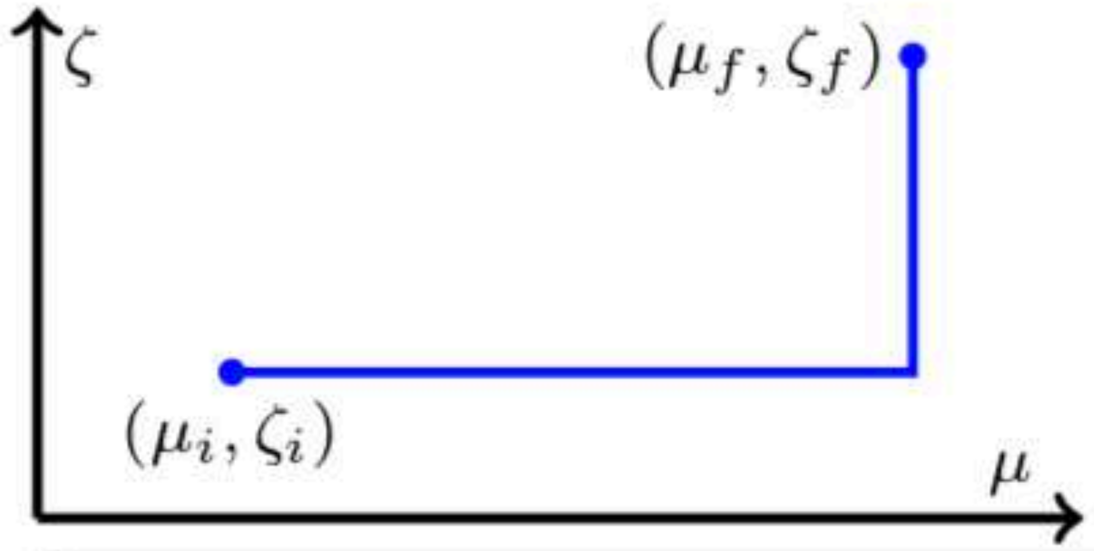
COUPLED EVOLUTION OF TMD AND TRUNCATION OF THE PERTURBATIVE SERIES



Solution 1

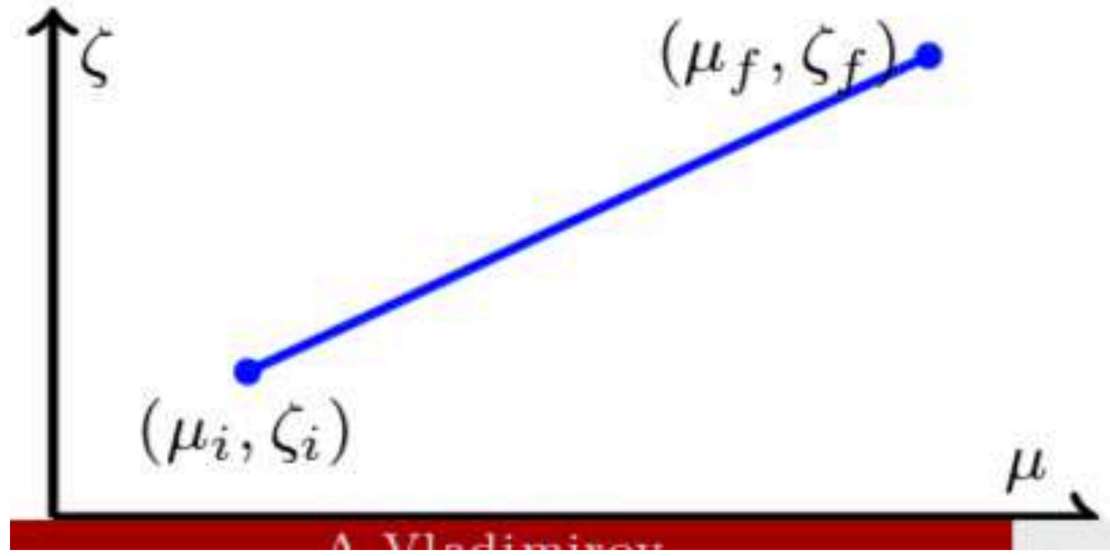
$$\ln R = \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F(\mu, \zeta_f) - \mathcal{D}(\mu_i, b) \ln\left(\frac{\zeta_f}{\zeta_i}\right)$$

given in [Collins' textbook]



Solution 2

$$\ln R = \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F(\mu, \zeta_i) - \mathcal{D}(\mu_f, b) \ln\left(\frac{\zeta_f}{\zeta_i}\right)$$



Solution 3

$$\ln R = \int_0^1 \Big(\gamma_F(\mu(t), \zeta(t)) \frac{\mu_f - \mu_i}{(\mu_f - \mu_i)t + \mu_i} - \mathcal{D}(\mu(t), b) \frac{\zeta_f - \zeta_i}{(\zeta_f - \zeta_i)t + \zeta_i}\Big) dt$$

TMDS IN ζ-PRESCRIPTION

I. Scimemi and A. Vladimirov, JHEP 08, 003 (2018)

Collins-Soper evolution equations:

$$\mu^2 \frac{d}{d\mu^2} F(x,b;\mu,\zeta) = \frac{\gamma_F(\mu,\zeta)}{2} F(x,b;\mu,\zeta)$$

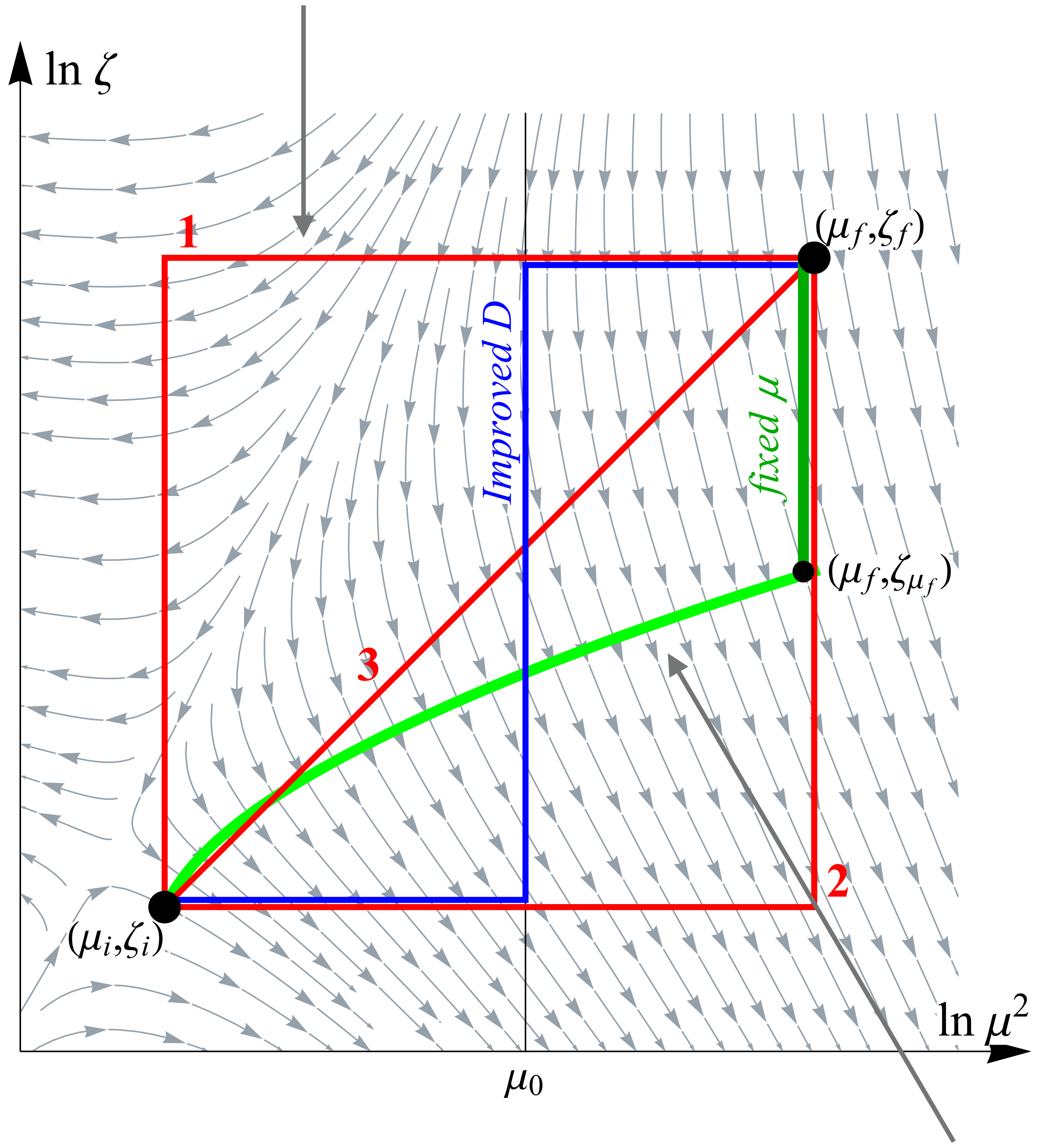
$$\zeta \frac{d}{d\zeta} F(x,b;\mu,\zeta) = -\mathcal{D}(b,\mu) F(x,b;\mu,\zeta)$$

$$\mathcal{D} = -\tilde{K}/2 \qquad \gamma_F(\mu,\zeta) = \Gamma_{cusp}(\mu) \ln\left(\frac{\mu^2}{\zeta}\right) - \gamma_V(\mu)$$

The solution:

$$F(x,b;\mu_f,\zeta_f) = \exp\left[\int_P \left(\gamma_F(\mu',\zeta')\frac{d\mu'}{\mu'} - \mathcal{D}(b,\mu')\frac{d\zeta'}{\zeta'}\right)\right] F(x,b;\mu_i,\zeta_i)$$

Standard TMD formalism



ζ-prescription

Equipotential lines exist in the field $\mathbf{E} \equiv (\gamma_F(\mu,\zeta)/2, -\mathscr{D}(b,\mu))$

$$\Gamma_{\text{cusp}}(\mu) \ln\left(\frac{\mu^2}{\zeta_\mu(b)}\right) - \gamma_V(\mu) = 2\mathcal{D}(b,\mu)\frac{d\ln\zeta_\mu(b)}{d\ln\mu^2}$$

TMDs do not evolve along those lines $\quad \mu^2 \frac{d}{d\mu^2} F(x,b;\mu,\zeta_\mu(b)) = 0$

TMDS IN ζ-PRESCRIPTION

I. Scimemi and A. Vladimirov, JHEP 08, 003 (2018)

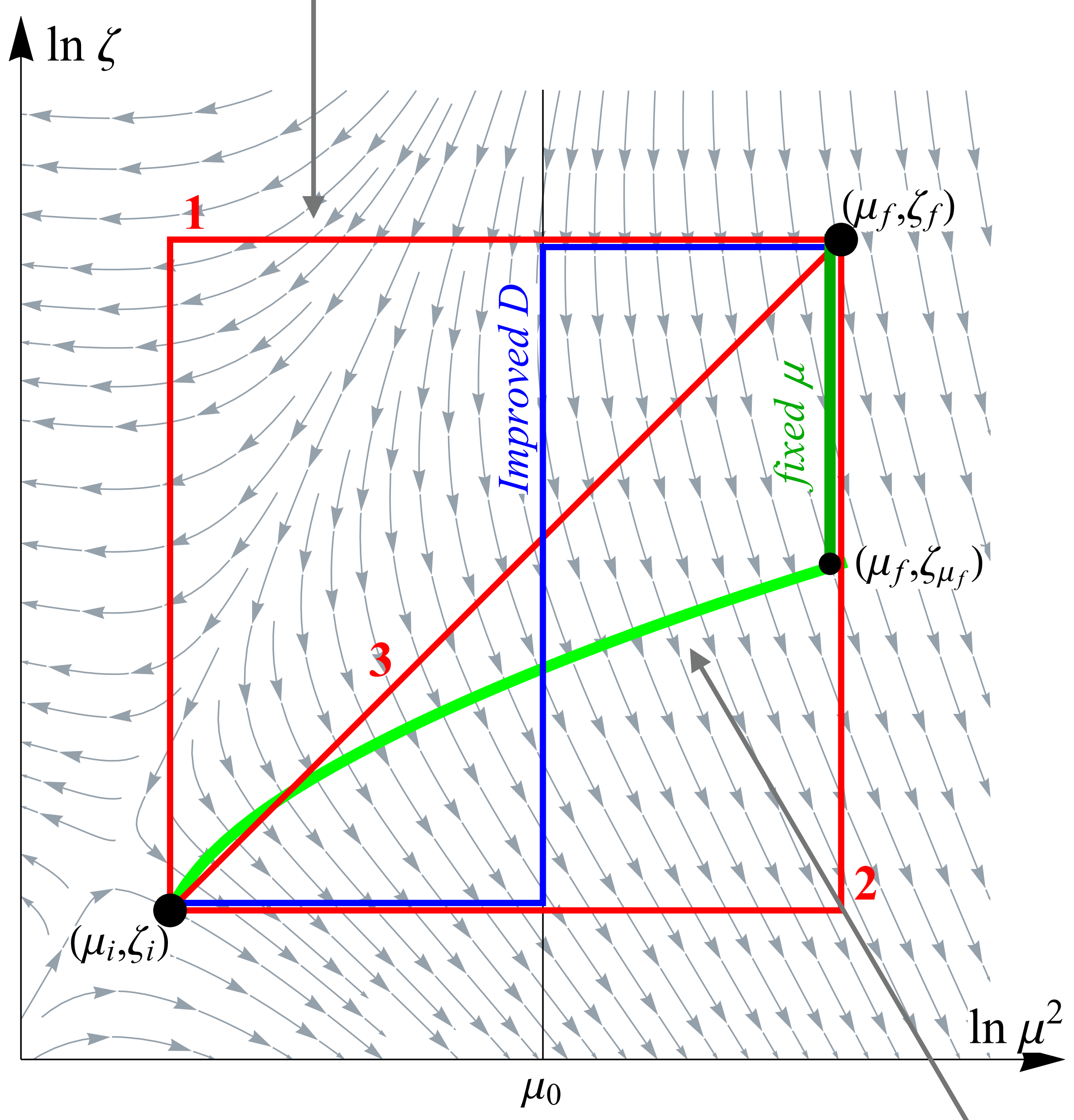
Collins-Soper evolution equations:

$$\mu^2 \frac{d}{d\mu^2}\widetilde{F}(x,b;\mu,\zeta) = \frac{\gamma_F(\mu,\zeta)}{2}\widetilde{F}(x,b;\mu,\zeta)$$

$$\zeta\frac{\partial}{\partial\zeta}\widetilde{F}(x,b;\mu,\zeta) = -\mathcal{D}(b,\mu)\widetilde{F}(x,b;\mu,\zeta)$$

$$\mathcal{D} = -\tilde{K}/2$$

Standard TMD formalism



ζ-prescription

The solution:

$$F(x,b;\mu,\zeta) = \exp\left[\int_P \left(\gamma_F(\mu',\zeta')\frac{d\mu'}{\mu'} - \mathcal{D}(b,\mu')\frac{d\zeta'}{\zeta'}\right)\right] F(x,b;\mu_0,\zeta_0)$$

Equipotential lines exist in the field $\mathbf{E} \equiv (\gamma_F(\mu,\zeta)/2, -\mathscr{D}(b,\mu))$

TMDs do not evolve along those lines $\quad \mu^2 \frac{d}{d\mu^2} F(x,b;\mu,\zeta_\mu(b)) = 0$

TMDS IN ζ-PRESCRIPTION

I. Scimemi and A. Vladimirov, JHEP 08, 003 (2018)

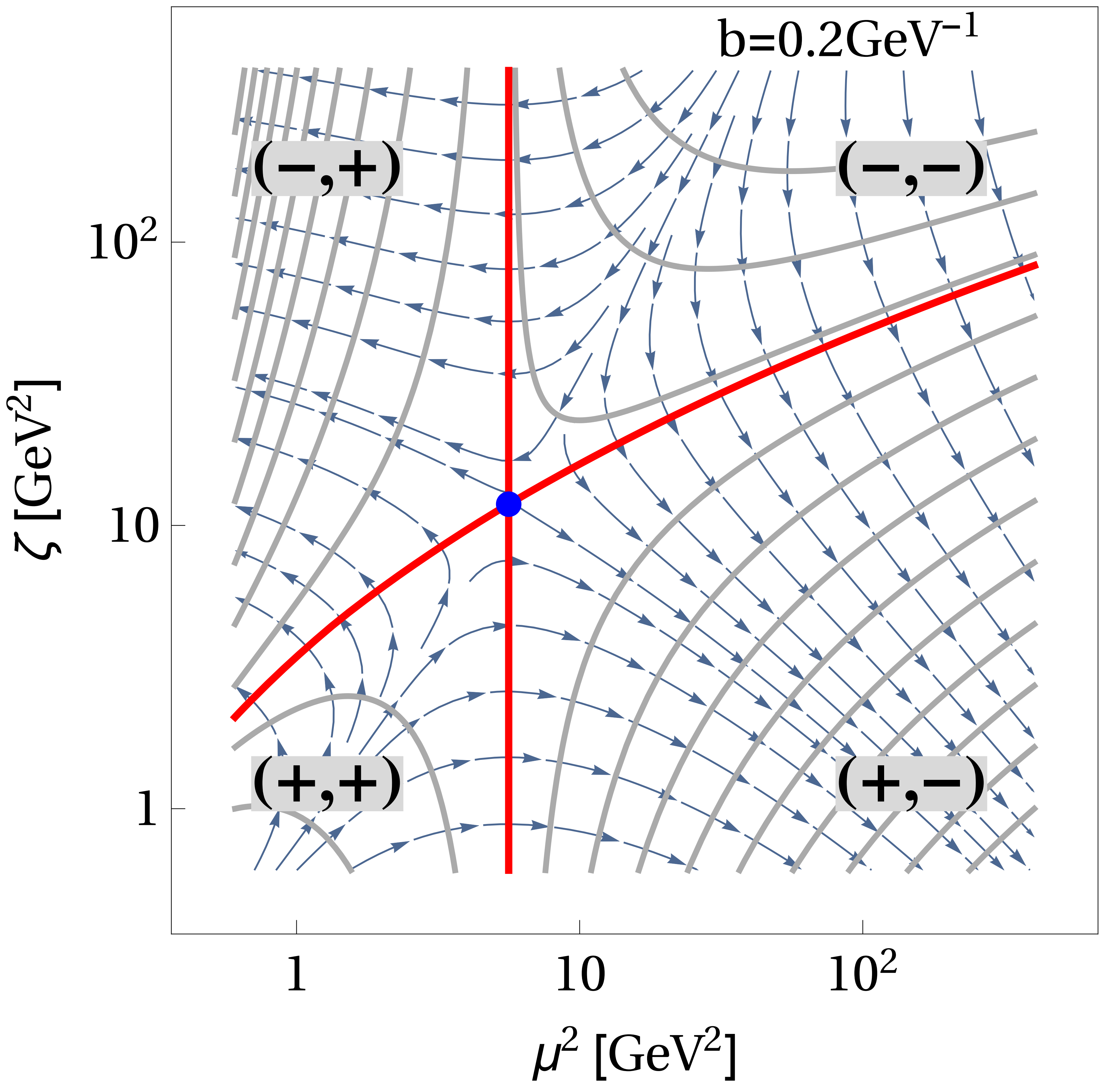
One particular line is of interest, the saddle point (μ_0, ζ_0):

$$\mathcal{D}(b, \mu_0) = 0,$$

$$\Gamma_{\text{cusp}}(\mu_0) \ln\left(\frac{\mu_0^2}{\zeta_0}\right) - \gamma_V(\mu_0) = 0.$$

TMD on this line is called the optimal TMD and it is has no scale dependence $F(x, b)$

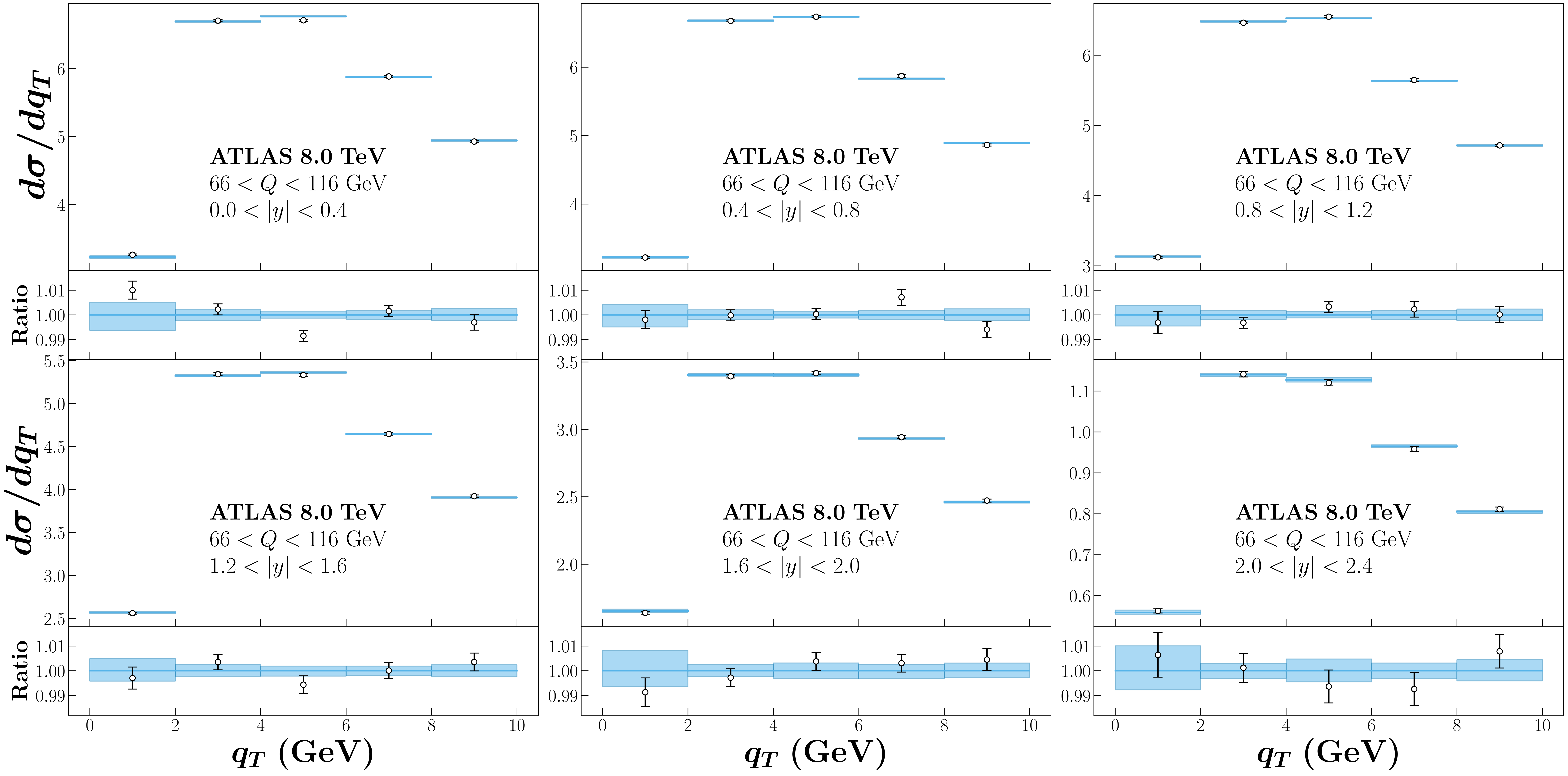
The evolution becomes a multiplicative factor



$$F(x, b; \mu, \zeta) = \left(\frac{\zeta}{\zeta_\mu(b)}\right)^{-\mathcal{D}(b,\mu)} F(x, b)$$

RESULTS

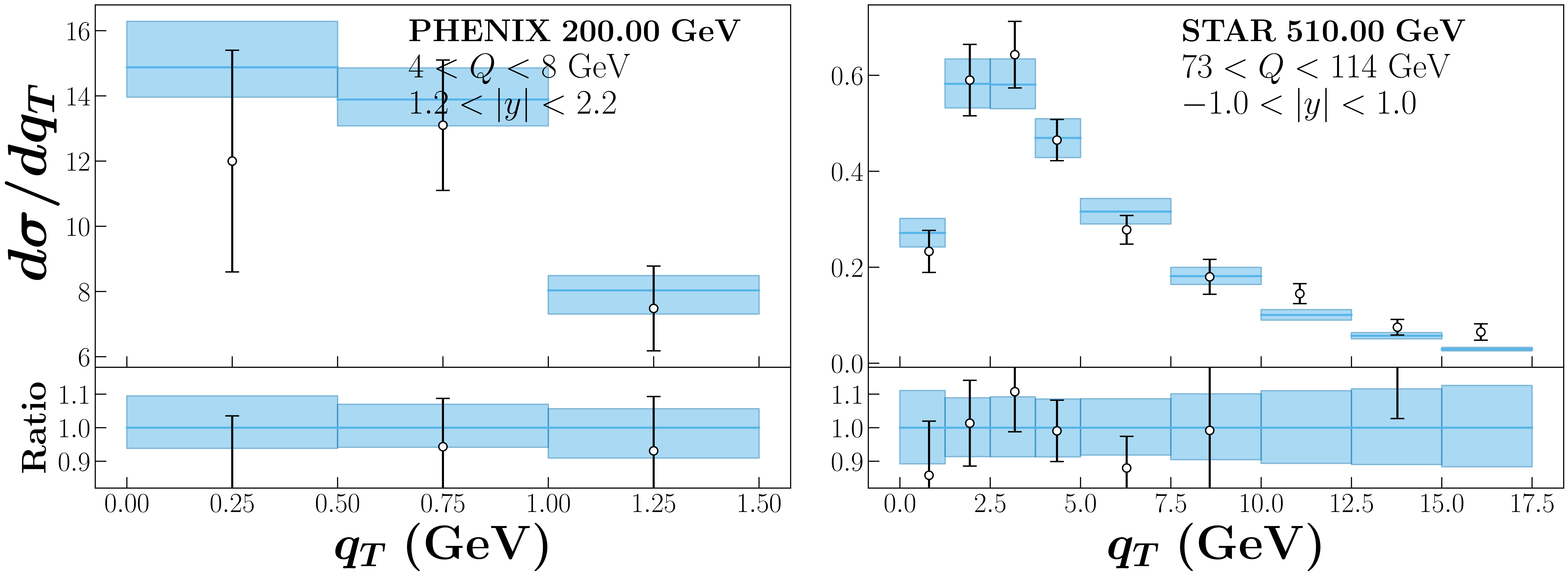
P. C. Barry et al 2510.1377



ATLAS data are in bins of rapidity, very sensitive to the shape of PDFs

RESULTS

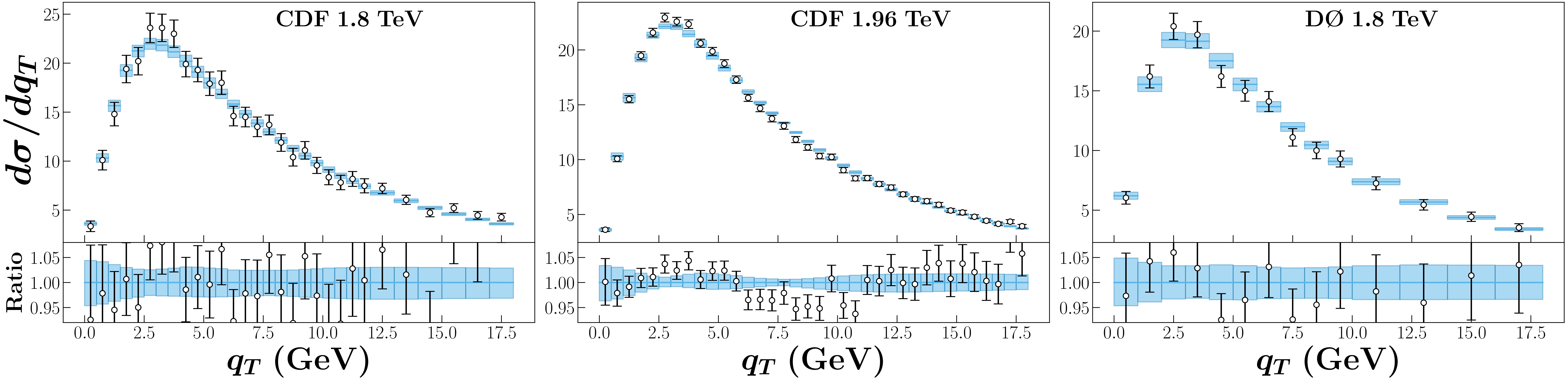
P. C. Barry et al 2510.1377



RHIC data are integrated in rapidity

RESULTS

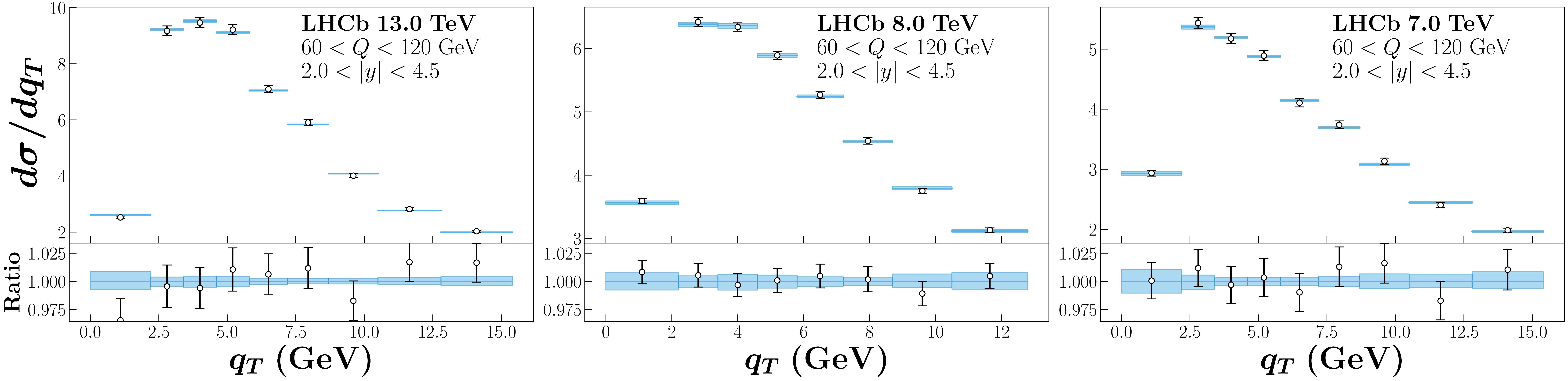
P. C. Barry et al 2510.1377



TEVATRON data are integrated in rapidity

RESULTS

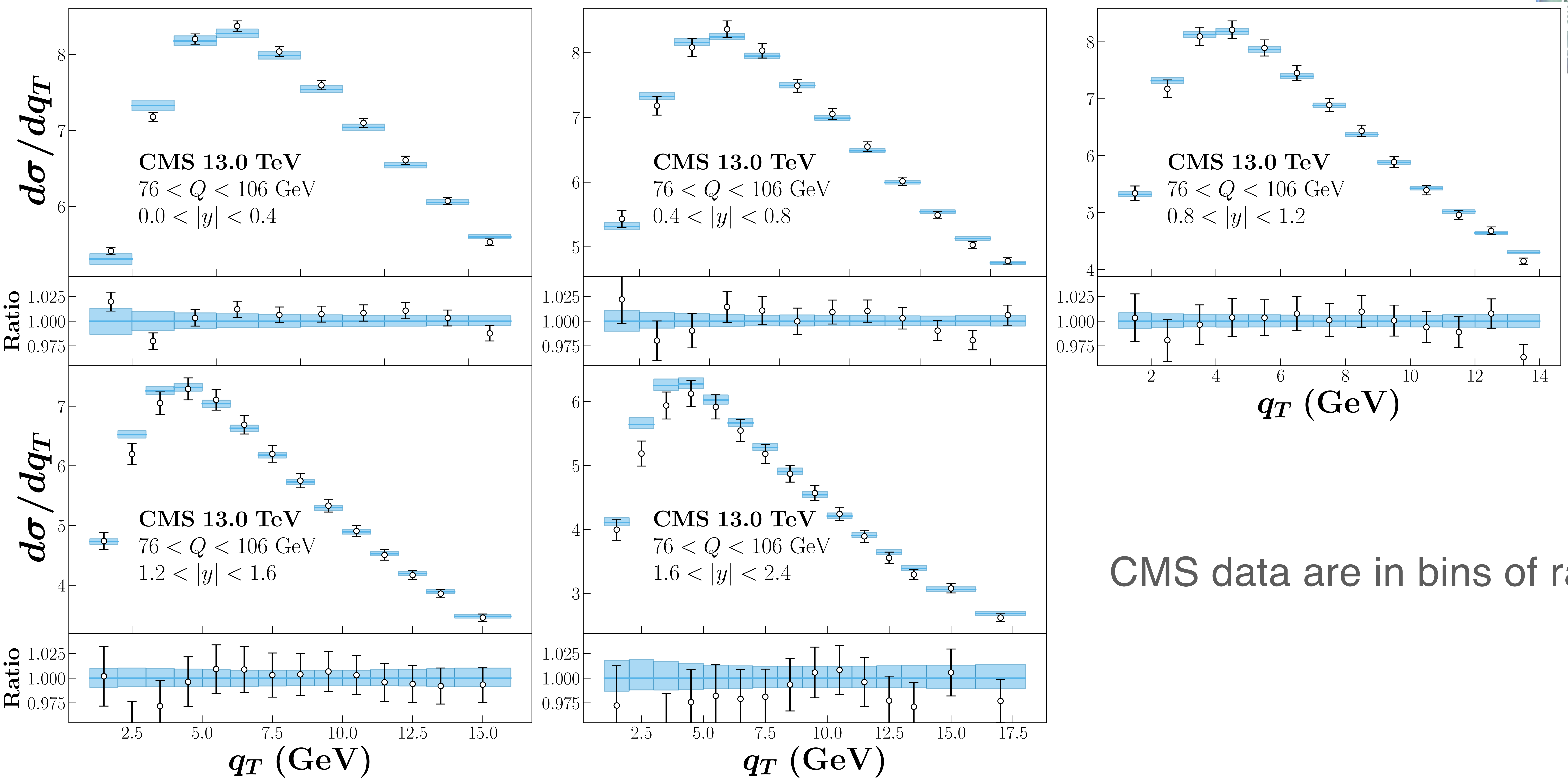
P. C. Barry et al 2510.1377



LHCb data are integrated in rapidity

RESULTS

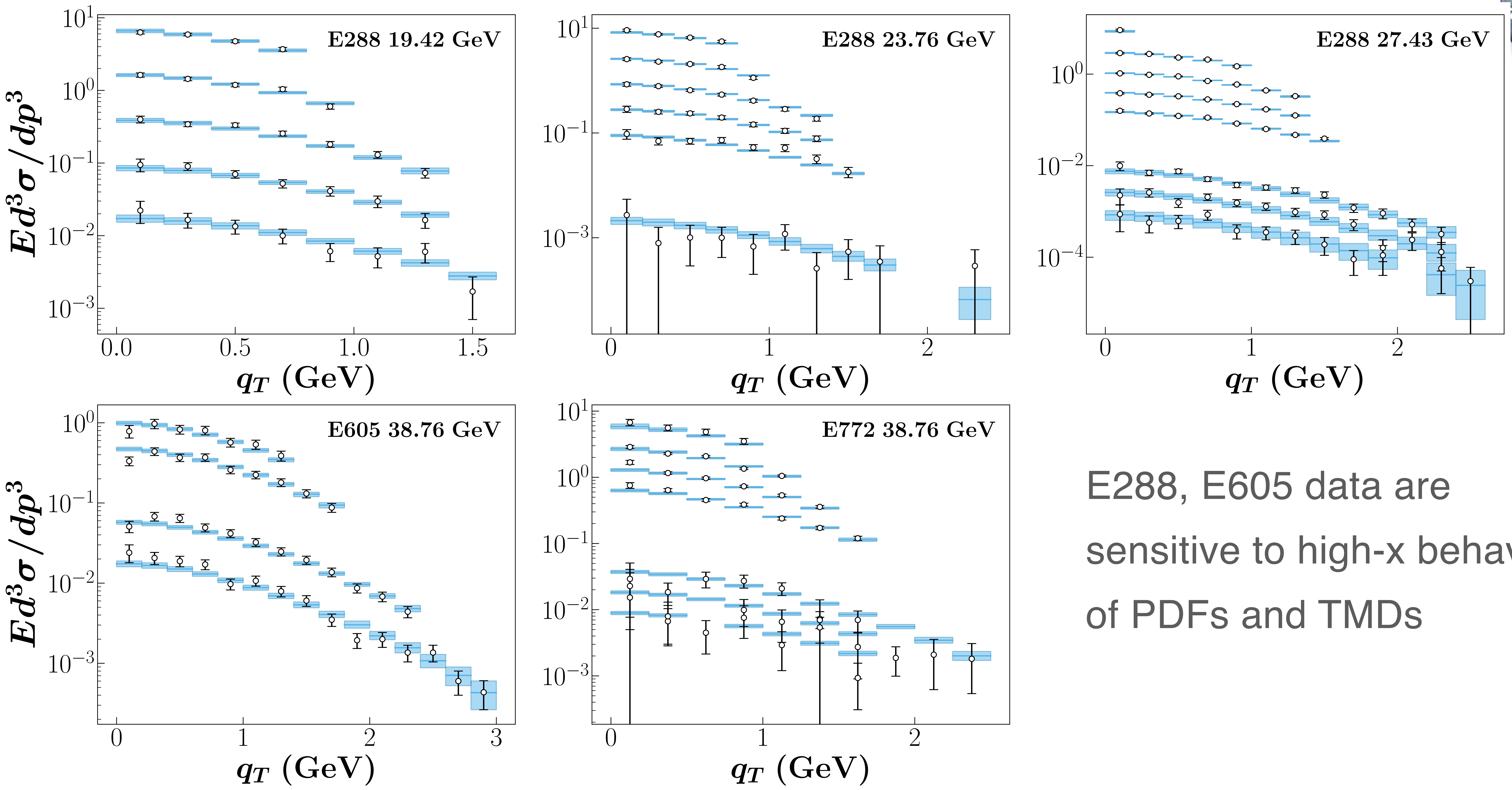
P. C. Barry et al 2510.1377



CMS data are in bins of rapidity

RESULTS

P. C. Barry et al 2510.1377



E288, E605 data are sensitive to high-x behavior of PDFs and TMDs