Tomography of nucleon through dimeson photoproduction

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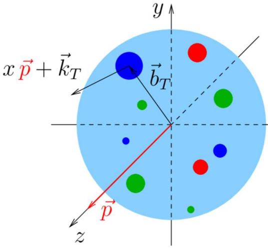
Physical content of Generalized Parton Distributions

- Off-diagonal matrix elements of bilocal operators at light-like separation.

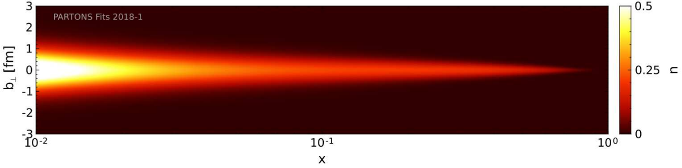
$$\frac{1}{P^+}\bar{u}(p_2)(H^q(x,\xi,t)\gamma^+ + \dots)u(p_1)$$
$$= \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p_2|\bar{q}(-\frac{z}{2})\gamma^+ q(\frac{z}{2})|p_1\rangle_{z_\perp=0,z^+=0}$$



- Connected to impact parameter distribution:

$$q(x,b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-ib_\perp\cdot\Delta_\perp} H^q(x,0,t)$$

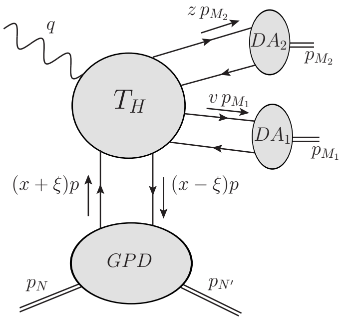
Moutarde, Sznajder, Wagner, 2018



- They give the total angular momentum carried by partons:

$$\int_{-1}^{-1} dx\, x(H^q(x,\xi,0) + E^q(x,\xi,0)) = 2J^q$$

Dimeson photoproduction



Factorisation recently proved in Qiu, Yu, 2023

- Give access both to chiral even and chiral odd GPDs
- Violation of factorization expected for:

$$p\,\gamma \to p\,\pi^+\pi^-$$
$$p\,\gamma \to p\,\rho^+\rho^-$$
$$p\,\gamma \to p\,\pi^0\rho^0$$

Nabeebaccus, Schoenleber, Szymanowski, Wallon arXiv:2311.09146

- Lots of processes to study with no gluon in t channel, for example:

$$p\,\gamma \to n\,\pi^+\rho_0$$
$$p\,\gamma \to p\,\pi_0\pi_0$$
$$p\,\gamma \to n\,\pi_0\rho^+$$
$$p\,\gamma \to n\,\rho_0\rho^+$$
$$p\,\gamma \to p\,\rho_0\rho_0$$

- Leading Order, Leading twist

The goal: the unpolarized differential cross-section for any dimeson photoproduction

Goal: unpolarised differential cross-section of any dimeson photoproduction

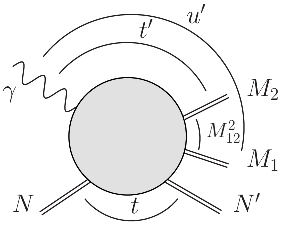
- $2 \to 3$ exclusive process, nucleon slightly deflected

  $\Rightarrow$ 3 independent degrees of freedom

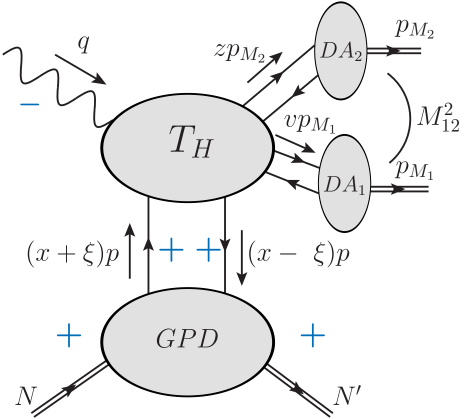
  We choose t, u' and M_{12}^2

- Longitudinal/ transverse vector mesons, and pseudo-scalar mesons

  $\Rightarrow$Vector, Axial or Transverse GPDs



Collinear Factorization of dimeson photoproduction



$$p = \frac{\sqrt{s}}{2}(1,0,0,1) \quad \rightarrow +$$

$$q = \frac{\sqrt{s}}{2}(1,0,0,-1) \quad \rightarrow -$$

$$p_2 = (1-\xi)p$$

$$p_1 = (1+\xi)p \quad \Delta = p_2 - p_1$$

$$p_{M_1} = \alpha q - \frac{p_\perp^2}{\alpha s} p + \boldsymbol{p}_\perp$$

$$p_{M_2} = (1-\alpha) q - \frac{p_\perp^2}{(1-\alpha) s} p - \boldsymbol{p}_\perp$$

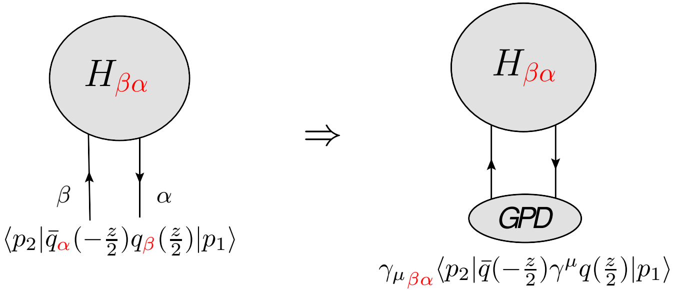
- Hard scale present: $-p_\perp^2, s \gg \Lambda^2_{\text{QCD}}$
- Nucleon is slightly deflected: $-t \sim -\Delta_\perp^2 \ll -p_\perp^2$

Collinear Factorisation $\Rightarrow$
diagrams where the 3 pairs of quarks fly collinearly dominate

How do we get GPDs ?

Fierz Projection in spinor space:

$$\langle p_2|\bar{q}_\alpha(-\frac{z}{2})q_\beta(\frac{z}{2})|p_1\rangle = \frac{1}{4}\gamma_{\mu\,\beta\alpha}\langle p_2|\bar{q}\gamma^\mu q|p_1\rangle + \frac{1}{4}(\gamma^5)\gamma_{\mu\,\beta\alpha}\langle p_2|\bar{q}\gamma^\mu\gamma^5 q|p_1\rangle$$
$$+\frac{1}{4}I_{\beta\alpha}\langle p_2|\bar{q}q|p_1\rangle + \frac{1}{4}\gamma^5_{\beta\alpha}\langle p_2|\bar{q}\gamma^5 q|p_1\rangle + \frac{1}{8}\sigma_{\mu\nu\,\beta\alpha}\langle p_2|\bar{q}\,\sigma^{\mu\nu}q|p_1\rangle$$



What kind of GPD do we get ?

- **Vector GPD:** $H^q, E^q \leftrightarrow \gamma^+$

$$\frac{1}{P^+}\bar{u}(p_2)(H^q(x,\xi,t)\gamma^+ + E^q(x,\xi,t)\frac{i\sigma^{+\mu}\Delta_\mu}{2M})u(p_1)$$
$$= \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p_2|\bar{q}(-\frac{z}{2})\gamma^+ q(\frac{z}{2})|p_1\rangle_{\substack{z_\perp=0\\ z^+=0}}$$

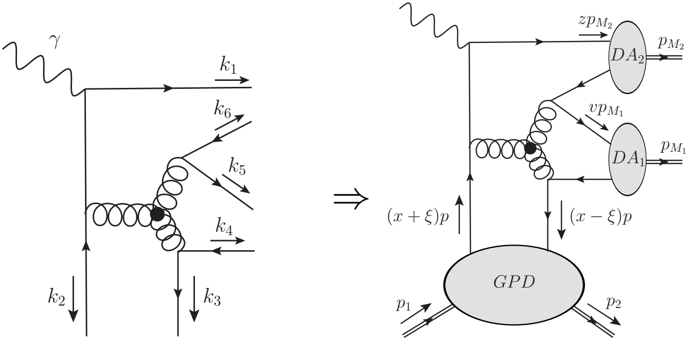
- **Axial GPD:** $\tilde{H}^q, \tilde{E}^q \leftrightarrow \gamma^+\gamma^5$

$$\frac{1}{P^+}\bar{u}(p_2)\left(\tilde{H}^q(x,\xi,t)\gamma^+\gamma^5 + \tilde{E}^q(x,\xi,t)\frac{\gamma^5\Delta^+}{2M}\right)u(p_1)$$
$$= \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p_2|\bar{q}(-\frac{z}{2})\gamma^+\gamma^5 q(\frac{z}{2})|p_1\rangle_{\substack{z_\perp=0\\ z^+=0}}$$

- **Transverse GPD:** $H_T^q, \tilde{H}_T^q, E_T^q, \tilde{E}_T^q \leftrightarrow \sigma^{+j}$

$$\frac{1}{P^+}\bar{u}(p_2)\left(H_T^q(x,\xi,t)\sigma^{+j} + \dots\right)u(p_1)$$
$$= \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p_2|\bar{q}(-\frac{z}{2})\sigma^{+j} q(\frac{z}{2})|p_1\rangle_{\substack{z_\perp=0\\ z^+=0}}$$

Creation of diagrams using FeynArts



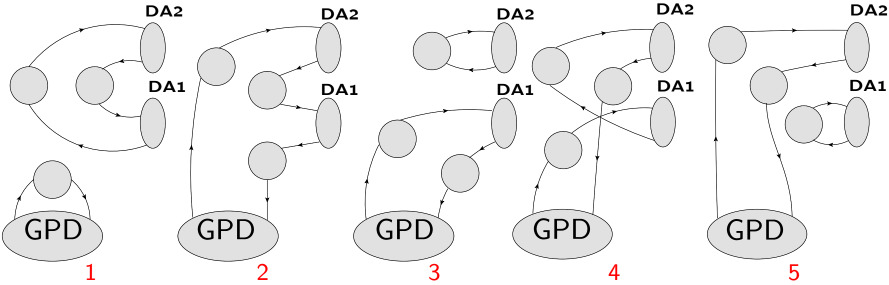
$$\frac{\boxed{\bar{u}(k1)}\, \not{\epsilon}(q1)(\not{k}2+\not{k}3+\not{k}4+\not{k}5+\not{k}6)\gamma_\mu \,\boxed{u(k2)\bar{u}(k3)}\, (\not{k}3+\not{k}4+2\not{k}5+2\not{k}6)\, \boxed{v(k4)\bar{u}(k5)}\, \gamma^\mu \,\boxed{v(k6)}}{(k3+k4)^2(k5+k6)^2(k3+k4+k5+k6)^2(k2+k3+k4+k5+k6)^2}$$

$$\Downarrow$$

$$\frac{\mathrm{Tr}\left(\not{\epsilon}(q1)(\not{k}2+\not{k}3+\not{k}4+\not{k}5+\not{k}6)\gamma^\mu \,\boxed{GPD}\, (\not{k}3+\not{k}4+2\not{k}5+2\not{k}6)\, \boxed{DA1}\, \gamma_\mu \,\boxed{DA2}\right)}{(k3+k4)^2(k5+k6)^2(k3+k4+k5+k6)^2(k2+k3+k4+k5+k6)^2}$$

Sorting and storage of diagrams

120 diagrams before Fierz projection grouped into 5 topologies



14 combinations of Dirac structures ⇒ 1304 projected diagrams stored

{diag, Dirac struct, top, photon} ↑   Prefactor of the projected diagram ↑

$$\{41,1,1,\{1,4\}\} \to \frac{-eg^4\left(\mathrm{Nc}^2-1\right)\left(\overline{\varepsilon}_q\cdot\overline{p}_\perp\right)\left(\xi\left(-2\alpha^2 z+\alpha v(2z-1)+2\alpha z^2+\alpha-2vz+v-z\right)+(\alpha-1)x(2\alpha z-1)\right)\mathrm{f}(\{41,1,1,\{1,4\}\},\mathrm{N1},\mathrm{N2},\{\mathrm{M1},\mathrm{t1}\},\{\mathrm{M2},\mathrm{t2}\})}{128(\alpha-1)\mathrm{Nc}^3 s^2 v\xi^3(z-1)z(v-\alpha(v+z-1))(\alpha(v+z-1)-vz)\left(-\frac{\xi(\alpha(v+z-1)-2vz+v)}{\alpha(v+z-1)-v}+\imath\epsilon+x\right)}$$

Integration over x, z,v

$$\frac{eg^4 v(z-1)(2\alpha(z-1)+1)\left(\varepsilon_q\cdot p_\perp\right)f(\{110,1,2,\{5,6\}\},\mathrm{N1},\mathrm{N2},\{\mathrm{M1},\mathrm{t1}\},\{\mathrm{M2},\mathrm{t2}\})}{81\alpha s\xi(s-\alpha s)(\alpha(-z)+\alpha+z-1)(\imath\epsilon+x-\xi)(-\imath\epsilon+x+\xi)}$$

$\Downarrow$ Partial Fraction over x

$$\frac{eg^4 v(2\alpha(z-1)+1)\left(\varepsilon_q\cdot p_\perp\right)f(\{110,1,2,\{5,6\}\},\mathrm{N1},\mathrm{N2},\{\mathrm{M1},\mathrm{t1}\},\{\mathrm{M2},\mathrm{t2}\})}{162(\alpha-1)^2\alpha s^2\xi^2(\imath\epsilon+x-\xi)} - \frac{eg^4 v(2\alpha(z-1)+1)\left(\varepsilon_q\cdot p_\perp\right)f(\{110,1,2,\{5,6\}\},\mathrm{N1},\mathrm{N2},\{\mathrm{M1},\mathrm{t1}\},\{\mathrm{M2},\mathrm{t2}\})}{162(\alpha-1)^2\alpha s^2\xi^2(-\imath\epsilon+x+\xi)}$$

$$\left(\frac{1}{x-\xi\pm\imath\epsilon} = \mathrm{p.v.}\frac{1}{x-\xi} \mp \imath\pi\delta(x-\xi)\right)$$

$\Downarrow$ (N1=p, N2=n, {M1,t1}={ρ^+,$\|$},{M2,t2}={ρ^0,$\|$})

$$\frac{eg^4(v-1)v^2(z-1)z(2\alpha(z-1)+1)\varepsilon_q\cdot p_\perp(\mathrm{Qd\,H}^d(-\xi)-\mathrm{Qd\,H}^d(x)+\mathrm{Qu}\,(\mathrm{H}^u(-\xi)-\mathrm{H}^u(x)))}{9(\alpha-1)^2\alpha\xi^2 s^2(\xi+x)} + \frac{eg^4(v-1)v^2(z-1)z(2\alpha(z-1)+1)\varepsilon_q\cdot p_\perp(-\mathrm{Qd\,H}^d(\xi)+\mathrm{Qd\,H}^d(x)+\mathrm{Qu}\,(\mathrm{H}^u(x)-\mathrm{H}^u(\xi)))}{9(\alpha-1)^2\alpha\xi^2 s^2(x-\xi)}$$

$$+\frac{eg^4(v-1)v^2(z-1)z\left(\log\left(\frac{1-\xi}{\xi+1}\right)-i\pi\right)(2\alpha(z-1)+1)\varepsilon_q\cdot p_\perp(\mathrm{Qd\,H}^d(\xi)+\mathrm{Qu\,H}^u(\xi))}{9(\alpha-1)^2\alpha\xi^2 s^2} - \frac{eg^4(v-1)v^2(z-1)z\left(\log\left(\frac{\xi+1}{1-\xi}\right)+i\pi\right)(2\alpha(z-1)+1)\varepsilon_q\cdot p_\perp(\mathrm{Qd\,H}^d(-\xi)+\mathrm{Qu\,H}^u(-\xi))}{9(\alpha-1)^2\alpha\xi^2 s^2}$$

We perform a numerical integration in x, v, z in the end

A more sophisticated case

$$\frac{e g^4 \bar{p}_\perp . \bar{\epsilon}_{q\perp} \left(\alpha\xi\left(-4v^2+8vz-8z^2+4z-1\right)+2\xi(2v-1)(v-z)+\alpha x(2z-1)\right) f(\{255,1,3,\{1,6\}\})}{108\bar{\alpha}^2 s^2 (v-1) v (\mathrm{i}\epsilon+x-\xi)(-\mathrm{i}\epsilon+x+\xi)(\bar{\alpha}v+\alpha z)(\bar{\alpha}v+\alpha z+1)\left(\frac{\xi(v(\alpha+2z-1)-\alpha z)}{\bar{\alpha}v+\alpha z}+x-\mathrm{i}\epsilon\right)\left(\frac{\xi-\xi v(\alpha-2z+1)+(\alpha-2)\xi z}{-\bar{\alpha}v-\alpha z+1}+x+\mathrm{i}\epsilon\right)}$$

$$=$$

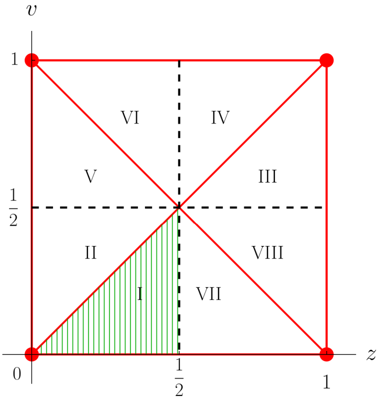
$$-\frac{(v-z)P_1(v,z)}{432\bar{\alpha}^2 s^2 (v-1)^2 v (z-1)(-\alpha v+v+\alpha z-1)\left((v-z)^2+\mathrm{i}\epsilon\frac{(\bar{\alpha}v+\alpha z)(\bar{\alpha}v+\alpha z-1)}{\bar{\alpha}\alpha\xi}\right)(\alpha\xi v-\xi z(\alpha+v-1))\left(\mathrm{i}\epsilon+x+\frac{\xi-\xi v(\alpha-2z+1)+(\alpha-2)\xi z}{(\alpha-1)v-\alpha z+1}\right)}$$

$$+\frac{(v-z)P_2(v,z)}{432\bar{\alpha}^2 s^2 (v-1) v^2 z\left((v-z)^2+\mathrm{i}\epsilon\frac{(\bar{\alpha}v+\alpha z)(\bar{\alpha}v+\alpha z+1)}{\bar{\alpha}\alpha\xi}\right)(\xi v(\alpha+z-1)-\alpha\xi z)\left(-\mathrm{i}\epsilon+x+\frac{\xi(v(\alpha+2z-1)-\alpha z)}{(\alpha-1)v-\alpha z}\right)}$$

+ 2 other terms

Partial Fraction $\Rightarrow$ spurious divergences at $z=1$, $z=0$, $v=1$, $v=0$, $v=z$, $v=1-z$

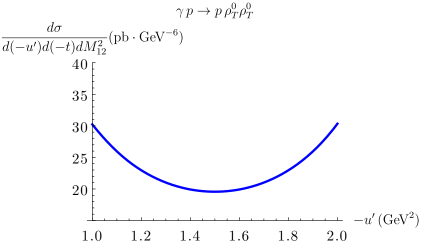
How to deal with spurious divergences

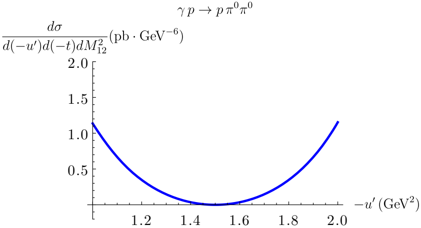


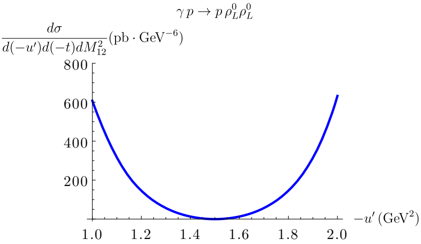
$$\int_0^1 dz \int_0^1 dv\, \mathcal{A}(z,v) = \int_0^{\frac{1}{2}} dz \int_0^z dv \Big(\underset{\mathrm{I}}{\mathcal{A}(z,v)} + \underset{\mathrm{II}}{\mathcal{A}(v,z)} + \underset{\mathrm{III}}{\mathcal{A}(1-v,1-z)}$$

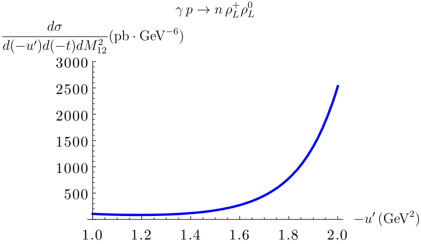
$$+ \underset{\mathrm{IV}}{\mathcal{A}(1-z,1-v)} + \underset{\mathrm{V}}{\mathcal{A}(1-v,z)} + \underset{\mathrm{VI}}{\mathcal{A}(z,1-v)} + \underset{\mathrm{VII}}{\mathcal{A}(1-z,v)} + \underset{\mathrm{VIII}}{\mathcal{A}(v,1-z)} \Big),$$

At JLab $S_{\gamma N} = 20$ GeV^2 $M_{12}^2 = 3$ GeV^2

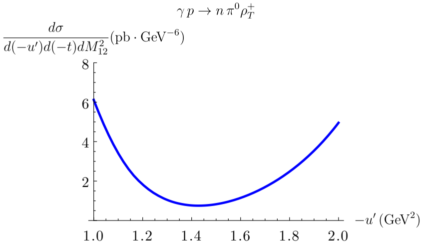


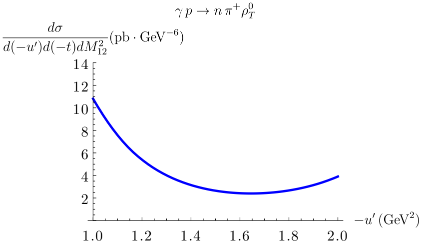


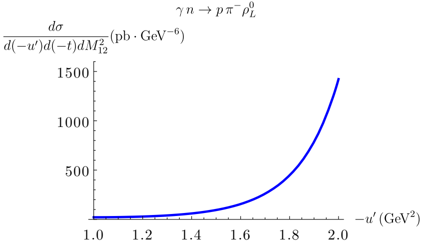


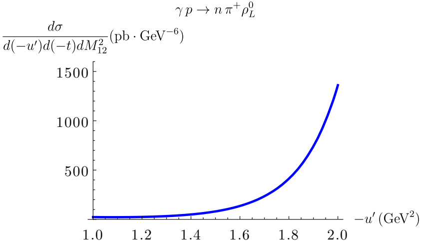


At JLab $S_{\gamma N} = 20\ \text{GeV}^2 \quad M_{12}^2 = 3\ \text{GeV}^2$

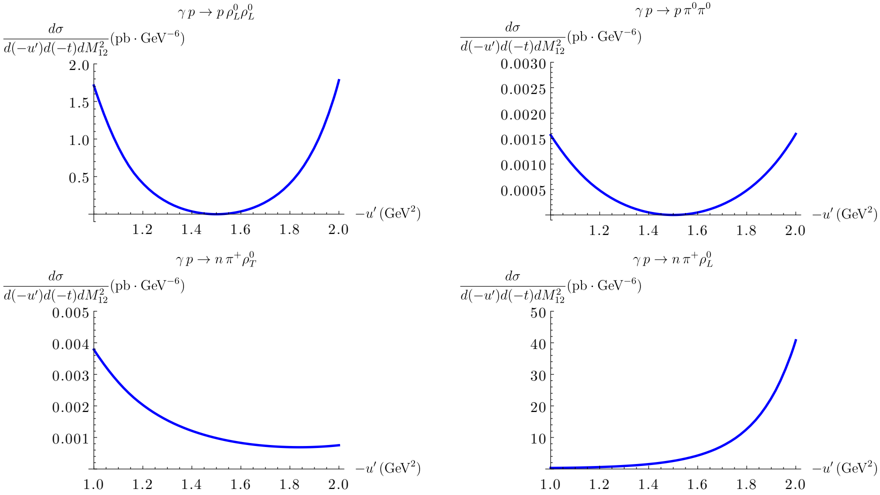




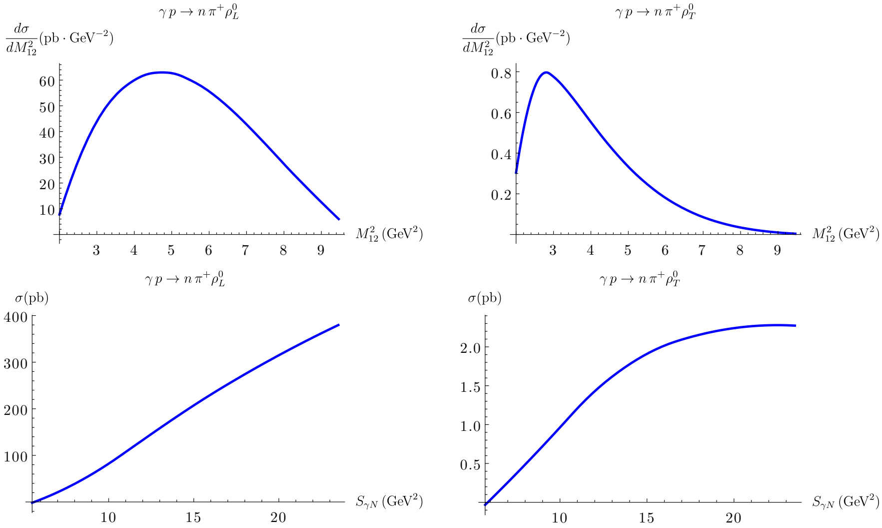




At EIC $S_{\gamma N} = 2000$ GeV^2 $M_{12}^2 = 3$ GeV^2



At JLab, CLAS 12 experiment, $E_{beam} = 12$ GeV



Conclusion

- Dimeson photoproduction is a rich channel that gives access to both chiral even and chiral odd generalized parton distributions
- 26 processes can now be studied automatically.
- The cross-sections are much larger than those for γM photoproduction
- We can consider various kinematics such as those at JLab, Compass, LHC and EIC corresponding to ξ values between 10^{-5} and 10^{-1}.
- We plan to compute the polarisation asymmetries and to test different models of GPDs and DAs.

For more details, see arXiv:2605.03880 [hep-ph]

What do we choose for $H(x,\xi,t), \tilde{H}(x,\xi,t), H_T(x,\xi,t)$?

A GPD model has to satisfy:

- Forward limit: $H^q(x,\xi=0,t=0) = q(x)\Theta(x) - \bar{q}(-x)\Theta(-x)$
- Reduction to elastic form factors: $\int_{-1}^{1} dx H^q(x,\xi,t) = F_1^q(t)$
- Polynomiality:

$$\int_{-1}^{1} dx x^m H^q(x,\xi,t) = \sum_{i=0}^{\lfloor \frac{m}{2} \rfloor} (2\xi)^{2i} A^q_{i,m}(t) + mod(m,2)(2\xi)^{m+1} C_{m+1}(t)$$

see Mezrag: Introductory lecture on Generalised Parton Distributions

But still many possibilities.

Generalized Parton Distribution modelling

We stick to Radyushkin Double Distribution Ansatz

$$H^q(x,\xi,t) = \int_{\{|\beta|+|\alpha|\leq 1\}} d\beta\, d\alpha\, \delta(\beta+\xi\alpha-x) F^q(\beta,\alpha,t) = \mathrm{Radon}[F]$$

(we discard the D-term)

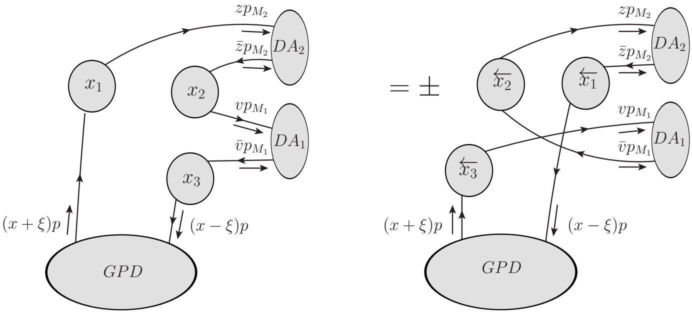
- $F^q(\beta,\alpha,t) = (\Pi(\beta,\alpha) q(\beta)\Theta(\beta) - \Pi(-\beta,\alpha)\bar{q}(-\beta)\Theta(-\beta))\, \frac{C^2}{(t-C)^2}$
- $\tilde{F}^q(\beta,\alpha,t) = (\Pi(\beta,\alpha)\Delta q(\beta)\Theta(\beta) - \Pi(-\beta,\alpha)\Delta\bar{q}(-\beta)\Theta(-\beta))\, \frac{C^2}{(t-C)^2}$
- $F_T^q(\beta,\alpha,t) = (\Pi(\beta,\alpha)\delta q(\beta)\Theta(\beta) - \Pi(-\beta,\alpha)\delta\bar{q}(-\beta)\Theta(-\beta))\, \frac{C^2}{(t-C)^2}$

Problem: we don't know precisely Δq and δq

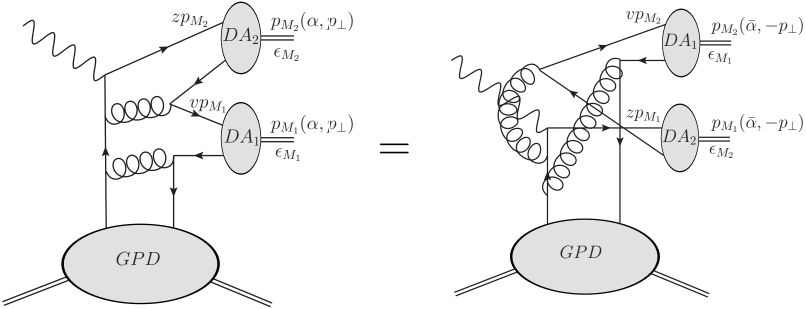
Charge conjugation symmetry

$$\mathrm{Tr}\left(GPD\ x_3 DA_1 x_2 DA_2 x_1\right) = (-1)^{\sharp\gamma^5}\,\mathrm{Tr}\left(GPD\ \overleftarrow{x_1}\ DA_2\ \overleftarrow{x_2}\ DA_1\ \overleftarrow{x_3}\right)$$

$\overleftarrow{x_i}$ means that the order of the γ matrices inside x_i is reversed.



Meson exchange symmetry



$$
\begin{aligned}
i\mathcal{M}(\gamma N \to N' M_1 M_2) &= \sum_i C_i(\alpha)\, T_i = \sum_i C_i'(\bar{\alpha})\, T_i' \Big|_{p_\perp \to -p_\perp,\, \epsilon_{M_1} \leftrightarrow \epsilon_{M_2}} \\
&= i\mathcal{M}(\gamma N \to N' M_2 M_1)\Big|_{p_\perp \to -p_\perp,\, \epsilon_{M_1} \leftrightarrow \epsilon_{M_2}}
\end{aligned}
$$

Isospin symmetry

$$H^{ud} = H_p^u - H_p^d = H^{du}$$

$$H_n^u = H_p^d \quad \text{and} \quad H_n^d = H_p^u$$

$$\Downarrow$$

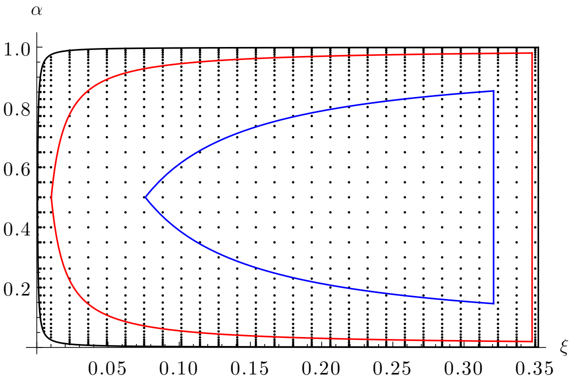
$$\gamma p \to p M_1 M_2 \overset{Q_u \leftrightarrow Q_d}{=} \gamma n \to n \bar{M}_1 \bar{M}_2$$

$$\gamma p \to n M_1 M_2 \overset{Q_u \leftrightarrow Q_d}{=} \gamma n \to p \bar{M}_1 \bar{M}_2$$

at amplitude level.

Parametrisation in α and ξ

$$|\overline{\mathcal{M}}|^2 = \frac{f_1^2 f_2^2}{s^3} g(\alpha, \xi)$$



$E_{beam} = 1000$ GeV $E_{beam} = 50$ GeV $E_{beam} = 7$ GeV