Constraining QCD energy-momentum tensor via dispersion relations

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Based on: **PRD 113, 094004** & **PRD 113, 094003**

in collab. with C. Mezrag, D. Binosi, Z.-Q. Yao.

Outline

- Introduction: GPDs, Compton (CFFs) and gravitational form factors (GFFs).
- What is a twist correction?
- Dispersion relation beyond leading twist.
- CFFs $\leftrightarrow$ GFFs *beyond pressure*, and estimations.
- Take aways.

Introduction

Generalized Parton Distributions

GPD

Generalized Parton Distribution ≈ "3D version of a PDF (Parton Distribution Function)." With x the average fraction of the hadron's longitudinal momentum carried by a quark:

$$H_f(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix\bar{p}^+ z^-} \langle p'|\bar{\mathrm{q}}_f(-z/2)\gamma^+ \mathcal{W}[-z/2,z/2]\mathrm{q}_f(z/2)|p\rangle \Big|_{z_\perp = z^+ = 0}$$

$$t = \Delta^2 = (p'-p)^2\,, \quad \xi = -\frac{\Delta n}{2\bar{p}n}\,, \quad \bar{p} = \frac{p+p'}{2}$$

Importance

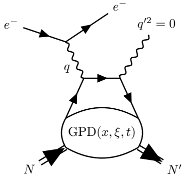
- Connected to **QCD energy-momentum tensor.** GPDs are a way to study **"mechanical" properties** and to address the hadron's spin puzzle (*X. Ji's sum rule**).
- **Tomography:**$ imaging of longitudinal momentum by transverse sections:

$$f(x,\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{4\pi^2}\; e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} H_f(x,0,-\vec{\Delta}_\perp^2)$$

* PRL 78 (1997) 610-613; $ Burkardt, Int. J. Mod. Phys. A 21 (2006) 926-929.

Accessing GPDs: DVCS

- In the 1990s, Müller et al.,[†] Ji[*] and Radyushkin[#] introduced GPDs and deeply virtual Compton scattering (DVCS):



- At LO ($O(\alpha_s^0)$) and LT ($\Lambda/Q^2 \to 0$, $\Lambda \in \{|t|, M^2\}$):

$$\mathcal{H}_{\rm DVCS}(\xi,t) = -\mathrm{PV}\left(\int_{-1}^{1} dx\, \frac{1}{x-\xi} H^{(+)}(x,\xi,t)\right) + \int_{-1}^{1} dx\;\, i\pi\delta(x-\xi) H^{(+)}(x,\xi,t)\,,$$

$$H^{(+)}(x,\xi,t) = H(x,\xi,t) - H(-x,\xi,t)\,.$$

- $\xi = \frac{-n\Delta}{2\bar{p}n}\,, \quad \bar{p} = \frac{p+p'}{2}\,, \quad \Delta = p' - p\,, \quad t = \Delta^2\,.$

[†] Fortsch. Phys. 42 (1994) 101-141. [*] PRD 55 (1997) 7114-7125. [#] PLB 449 (1999) 81-88.

Higher-twist corrections to DVCS off proton: Braun et al., PRD 111 (2025) 7, 076011... and earlier works.

QCD energy-momentum tensor (EMT), $\Theta^{\mu\nu}$

$\Theta^{\mu\nu}$ parameterization $\rightarrow$ gravitational form factors (GFFs):

$$\langle p', s'|\Theta^{\mu\nu}_a(0)|p, s\rangle \overset{\text{spin}-1/2}{=}$$

$$= \bar{u}(p', s')\left\{ \frac{\bar{p}^\mu \bar{p}^\nu}{M} A_a(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C_a(t) + M\eta^{\mu\nu}\bar{C}_a(t) \right.$$

$$\left. + \frac{\bar{p}^{\{\mu} i\sigma^{\nu\}\rho}\Delta_\rho}{4M}\left[A_a(t) + B_a(t)\right] + \frac{\bar{p}^{[\mu} i\sigma^{\nu]\rho}\Delta_\rho}{4M} S_a(t) \right\} u(p, s)\,.$$

$a =$ quarks and gluons.

Can we obtain GFFs from GPDs?
Yes, but GPDs are not directly accessible in experiments:

$$\int dx\, x^{1-\mathscr{P}_a} \begin{Bmatrix} H_a(x,\xi,t) \\ E_a(x,\xi,t) \end{Bmatrix} = \begin{Bmatrix} A_a(t) + 4\xi^2 C_a(t) \\ B_a(t) - 4\xi^2 C_a(t) \end{Bmatrix}, \quad \mathscr{P}_a = \begin{cases} 0\,, \text{ if } a = \text{ quark}\,, \\ 1\,, \text{ if } a = \text{ gluon}\,. \end{cases}$$

$$J_a = \frac{A_a(0) + B_a(0)}{2} = \frac{1}{2}\int dx\, x^{1-\mathscr{P}_a}\left[H_a(x,\xi,0) + E_a(x,\xi,0)\right],$$ and other relations.

Can we obtain GFFs from CFFs?
Yes, by dispersion relations beyond LT.

Kinematic higher-twist corrections

Compton tensor (DVCS, vector comp.):

$$T^{\mu\nu} = i \int d^4z \; e^{iq'z} \langle p' | \mathcal{T} \{ j^\mu(z) j^\nu(0) \} | p \rangle = -g_\perp^{\mu\nu} \underbrace{\mathcal{A}^{++}}_{\text{LT}(1+O(\alpha_s)) \oplus |t|/Q^2 \oplus \xi^2 M^2/Q^2}$$

$$+ \frac{1}{|\bar{p}_\perp|^2} \left[p_\perp^\mu p_\perp^\nu - \widetilde{p}_\perp^\mu \widetilde{p}_\perp^\nu \right] \underbrace{\mathcal{A}^{+-}}_{O(\alpha_s \cdot \text{LT}) \oplus |t|/Q^2 \oplus \xi^2 M^2/Q^2}$$

$$+ \frac{\sqrt{2}}{|\bar{p}_\perp|} p_\perp^\mu \left[\frac{1}{Q} q^\nu - \frac{2Q}{\mathbb{Q}^2} q'^\nu \right] \underbrace{\mathcal{A}^{0+}}_{\sqrt{|t|}/Q^2} \,,$$

$$\boxed{\mathcal{A}^{\Lambda\Lambda'} \sim \int dx \, C^{\Lambda\Lambda'}(x/\xi) \, H(x, \xi, t)}$$

Braun, Ji & Manashov, JHEP 03 (2021) 051; and JHEP 01 (2023) 078.

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Braun, Ji & Manashov, JHEP 03 (2021) 051; and JHEP 01 (2023) 078.

Why to go beyond leading twist?

1. Nucleon tomography.
2. Reduce cuts in experimental data.
3. Universality tests $\rightarrow \mathcal{A}^{++}_{\text{DVCS}} \overset{\text{LO, HT}}{\neq} \left(\mathcal{A}^{++}_{\text{TCS}}\right)^*$.
4. Access to gravitational form factors **BEYOND pressure.**

Dispersion relation

Analyticity of the scattering amplitude

Extending the formalism developed in Dutrieux, Meisgny, Mezrag & Moutarde, EPJC 85 (2025) 1, 105.

DVCS: fixed t + momentum conservation: amplitude $= \mathcal{F}(s)$.

Causality $\Rightarrow$ extension to the upper-half of the complex plane: $\mathcal{F}(s) \to \mathcal{F}(s + i0)$.

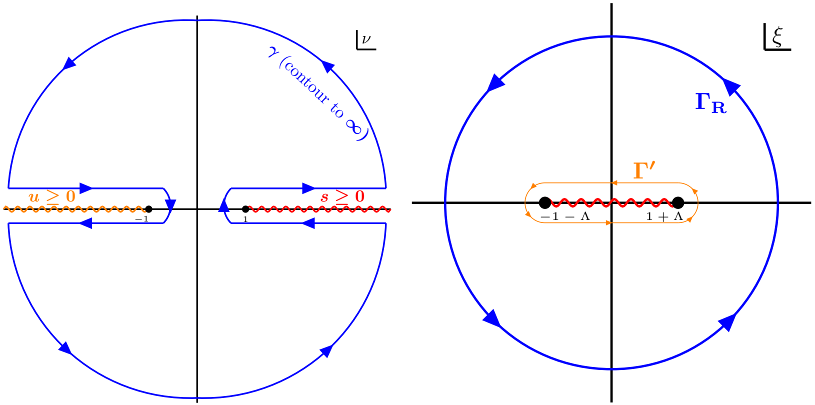
$$s + u = -\mathbb{Q}^2 \left[1 - \frac{2M^2}{\mathbb{Q}^2}\right], \qquad \mathbb{Q}^2 = Q^2 + t\,.$$

1. $s + u < 0$ if $M^2/\mathbb{Q}^2 < 1/2 \Rightarrow$
2. $\exists$ region for both $s, u < 0 \Rightarrow$
3. no particle production & $\mathrm{Im}(\mathcal{F}) = 0$ **(optical theorem)** $\Rightarrow$
4. analytic continuation to the lower-half of the complex plane:

   Schwartz's reflexion principle: $\mathcal{F}(s - i0) = \mathcal{F}^*(s + i0)$.

Domain of (guaranteed) analyticity in ν and ξ

$$s \to \nu \simeq 1/\xi + \Lambda\,, \qquad \Lambda = 2M^2/\mathbb{Q}^2$$



Path γ in ν's complex plane $\to$ Diehl & Ivanov, EPJC 52 (2007) 919-932.

Dispersion relation

- $\mathcal{F} \to \mathcal{H}^{++}(\xi, t) \sim \text{LT} + \text{tw-4} + \cdots$

- ***n*-times subtracted dispersion relation:** $\Lambda = 2M^2/\mathbb{Q}^2$,

$$\sum_{j=0}^{n} h_j^{++} \frac{1}{\xi^j} = \frac{1}{\pi} \mathrm{PV} \int_{-(1+\Lambda)}^{1+\Lambda} d\xi' \, \frac{\mathrm{Im}(\mathcal{H}^{++}(\xi'))}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n + \mathrm{Re}(\mathcal{H}^{++}(\xi))$$

- $|\xi| > 1 \Rightarrow$ GPDs on $x \in (-\xi, \xi) \Rightarrow \underbrace{\mathrm{Im}(\mathcal{H}^{++}(\xi))}_{\text{NLO+tw-4} \in \text{DGLAP region } |x| > |\xi|} = 0$:

$$\sum_{j=0}^{n} h_j^{++} \frac{1}{\xi^j} = \frac{1}{\pi} \mathrm{PV} \int_{-1}^{1} d\xi' \, \frac{\mathrm{Im}(\mathcal{H}^{++}(\xi'))}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n + \mathrm{Re}(\mathcal{H}^{++}(\xi))$$

Subtraction constant and DDs

$$\sum_{j=0}^{n} h_j^{++} \frac{1}{\xi^j} \Rightarrow \boldsymbol{h_0^{++}} = \textbf{subtraction constant of the DR}\,.$$

$$Quarks \rightarrow \begin{Bmatrix} H(x,\xi,t) \\ E(x,\xi,t) \end{Bmatrix} = \iint_{\mathbb{D}} d\beta d\alpha\ \delta(x-\beta-\alpha\xi) \times \begin{Bmatrix} F(\beta,\alpha,t) + \xi D(\alpha,t)\delta(\beta) \\ K(\beta,\alpha,t) - \xi D(\alpha,t)\delta(\beta) \end{Bmatrix}$$

Subtraction constant for **spin-0**

$$h_0^{++}(t) = \int_{-1}^{1} d\alpha\ \left[T_0^{++}(\alpha, t/\mathbb{Q}^2) + \frac{t}{\mathbb{Q}^2} T_1^{++}(\alpha)\right] \boldsymbol{D(\alpha,t)}$$
$$- 4\frac{M^2 - t/4}{\mathbb{Q}^2} \iint_{\mathbb{D}} d\beta d\alpha\ \boldsymbol{F(\beta,\alpha,t)}\beta\, T_1^{++\,(1)}(\alpha)$$

Subtraction constant for **spin-1/2**‡

$$h_0^{++}(t) = \int_{-1}^{1} d\alpha\ \left[T_0^{++}(\alpha, t/\mathbb{Q}^2) + \frac{t}{\mathbb{Q}^2} T_1^{++}(\alpha)\right] \boldsymbol{D(\alpha,t)}$$
$$- 4\frac{M^2}{\mathbb{Q}^2} \iint_{\mathbb{D}} d\beta d\alpha\ \left[\boldsymbol{F(\beta,\alpha,t)} + \frac{t}{4M^2}\boldsymbol{K(\beta,\alpha,t)}\right] \beta\, T_1^{++\,(1)}(\alpha)$$

$T_1^{++\,(n)}(\alpha) = \partial_\alpha^n T_1^{++}(\alpha)$

‡In agreement with previous work: Braun et al., PRD 89 (2014) 7, 074022 $\leftrightarrow$ **NO** connection to GFFs.

Connection to gravitational FFs

$\int F(\beta,\alpha)\beta\, T_1^{++\,(1)} \sim$ GFFs, generalized FFs

Example spin-0 case:

$$h_0^{++}(t) = \left(\int D\right) - 4\,\frac{M^2 - t/4}{\mathbb{Q}^2} \underbrace{\iint_{\mathbb{D}} d\beta d\alpha\ \boldsymbol{F(\beta,\alpha,t)}\beta\, T_1^{++\,(1)}(\alpha)}_{\mathcal{I}(t)} .$$

$$\mathcal{I}(t) = \iint_{\mathbb{D}} d\beta d\alpha\ F(\beta,\alpha,t)\beta\ \underbrace{\boldsymbol{T_1^{++\,(1)}(\alpha)}}_{\sum_{n=0}^{\infty} \mathfrak{c}_n \alpha^n,\ R<1}$$

$$\overset{(\star)}{=} \sum_{\substack{n=0 \\ \text{even } n}}^{\infty} \mathfrak{c}_n \iint_{\mathbb{D}} d\beta d\alpha\ F(\beta,\alpha,t)\beta\alpha^n$$

$$= \mathfrak{c}_0 \underbrace{\boldsymbol{A(t)}}_{\textbf{GFF}} + \sum_{\substack{n=2 \\ \text{even } n}}^{\infty} \frac{\mathfrak{c}_n 2^n}{n+1} \underbrace{\boldsymbol{A_{n+2,\,n}(t)}}_{\textbf{genFF}} .$$

Similarly, $\left(\int D\right) \propto C(t)$ upon Gegenbauer d_1 dominance.

($\star$) Lebesgue dominated convergence theorem: Royden & Fitzpatrick, *Real analysis,* 4^{th} ed., Prentice Hall, 2010.

Connection between CFFs and GFFs

85-90% of the correction is on the GFFs → neglect genFFs:

Spin 0, quarks of flavor q

$$h_{0,\,q}^{++}(t,\mathbb{Q}^2) \approx 20C_q(t)\left(1-\frac{t}{3\mathbb{Q}^2}\right) - 4\frac{M^2-\frac{t}{4}}{\mathbb{Q}^2}\mathfrak{c}_0 A_q(t)$$

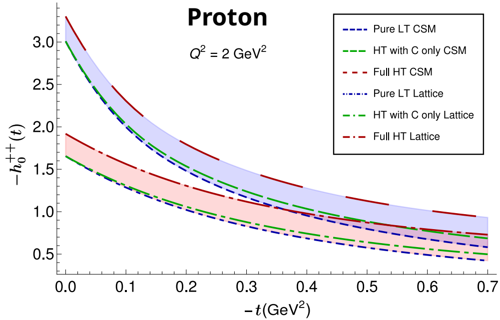
Spin 1/2, quarks of flavor q

$$h_{0,\,q}^{++}(t,\mathbb{Q}^2) \approx 20C_q(t)\left(1-\frac{t}{3\mathbb{Q}^2}\right) - \frac{4M^2\mathfrak{c}_0}{\mathbb{Q}^2}\left[\frac{4M^2-t}{4M^2}A_q(t)+\frac{t}{2M^2}J_q(t)\right]$$

$h_0^{++}(t) = \sum_q e_q^2\, h_{0,\,q}^{++}(t)$
$\mathfrak{c}_0 \approx 0.864$

The **mass term dominates** the power corrections: up to **33% (lattice)** and **25% (CSM)**† at $Q^2 = 2\ \mathrm{GeV}^2$.

† **See today's talk by J. Rodríguez-Quintero**



Take aways

- Analyticity of scattering amplitude $\leftrightarrow$ DR at all orders.
- Kin. twist-4:
  $\mathbf{Re}\mathcal{H}^{++} - \int \mathbf{Im}\mathcal{H}^{++}/(\xi' - \xi) \sim$ **GFFs** $\rightarrow$ **direct link between CFFs and GFFs BEYOND pressure.**
- **Large HT effects** on the DR with current data $\rightarrow$ further challenging previous extractions of proton's pressure.
- **DR provides new benchmarks** for GFFs obtained from continuum and lattice QCD.

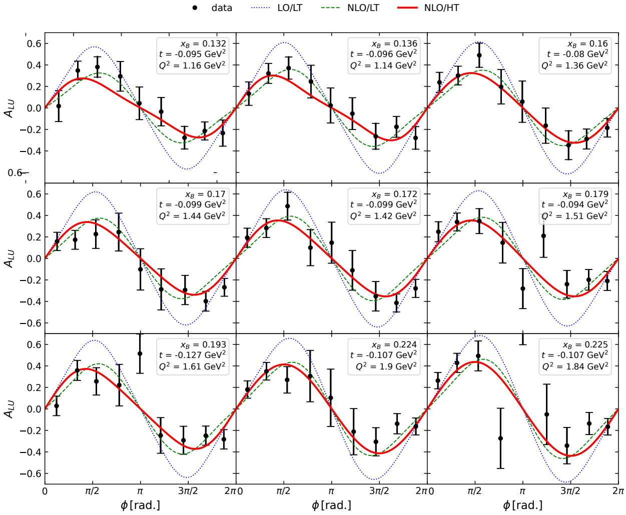
Thank you!

BONUS:

Impact of twist corrections on data analysis

DVCS off ^{4}He: JLab6 data vs our model

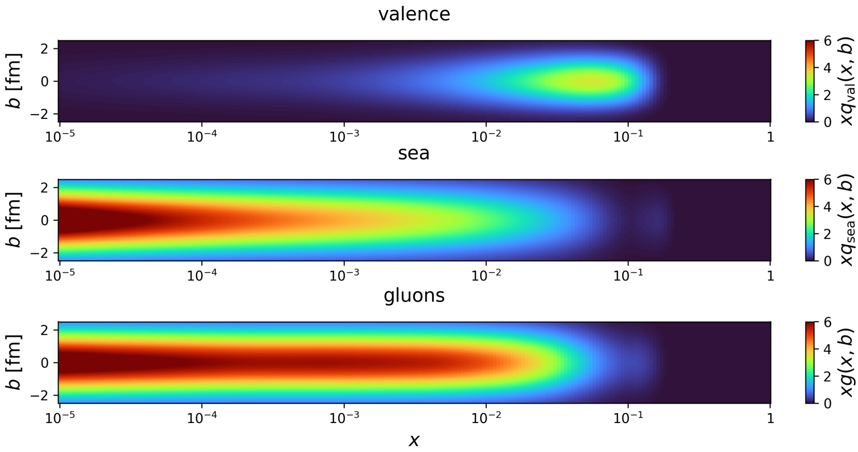
- 1st GPD fit based on kinematic HT and NLO (+ gluon transversity).



PRELIMINARY

Tomography of ^{4}He

- 1st helium tomography.



PRELIMINARY

Complementary slides

LT NLO coefficient functions

$$T_{\rm LT}^{++}(x/\xi) = \left[C_0(x/\xi) + C_1(x/\xi) + \ln\left(\frac{Q^2}{\mu_{\rm F}^2}\right) C_{\rm coll}(x/\xi)\right] - (x/\xi \to -x/\xi)\,,$$

$$C_0(x/\xi) = -\frac{\xi}{x+\xi-i0}\,,$$

$$C_1(x/\xi) = \frac{\alpha_S C_F}{4\pi}\frac{\xi}{x+\xi-i0}\left[9 - 3\frac{x+\xi}{x-\xi}\ln\left(\frac{x+\xi}{2\xi}-i0\right) - \ln^2\left(\frac{x+\xi}{2\xi}-i0\right)\right],$$

$$C_{\rm coll}(x/\xi) = \frac{\alpha_S C_F}{4\pi}\frac{\xi}{x+\xi-i0}\left[-3 - 2\ln\left(\frac{x+\xi}{2\xi}-i0\right)\right],$$

$+$ gluon terms.

NLO and higher-twist coefficient functions:
$C_i \to \mathrm{Im}(C_i) \propto \theta(x-\xi),\ \delta(x-\xi)$, accessing therefore:

$$\begin{cases} \text{DGLAP:} & |x| > |\xi| \ \checkmark \\ \text{ERBL:} & |x| < |\xi| \ \times \end{cases}$$

Belitsky, Müller, PLB 417 (1998) 129; Ji, Osborne, PRD 58 (1998) 094018; Ji, Osborne, PRD 57 (1998) 1337; Mankiewicz, Piller, Stein, Vänttinen, Weigl, PLB 425 (1998) 186; Pire, Szymanowski & Wagner, PRD 83 (2011) 034009.

Double DVCS' $\mathcal{A}^{++} =$ LT + tw-4 + O(tw-6), at LO

Spin-0 target:

$$\mathcal{H}^{++} = \mathcal{A}^{++} = \int_{-1}^{1} dx \left\{ -\left(1 - \frac{t}{2\mathbb{Q}^2} + \frac{t(\xi-\rho)}{\mathbb{Q}^2}\partial_\xi\right) \frac{H^{(+)}}{x-\rho+i0} \right.$$
$$+ \frac{t}{\xi\mathbb{Q}^2}\left[\mathbb{P}_{(\mathrm{i})} + \mathbb{P}_{(\mathrm{ii})} - \frac{\xi L}{2} + \frac{\widetilde{\mathbb{P}}_{(\mathrm{iii})} - \widetilde{\mathbb{P}}_{(\mathrm{i})}}{2}\right.$$
$$\left. - \frac{\xi}{x+\xi}\left(\ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right) - \frac{\xi+\rho}{2\xi}\ln\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right) - \widetilde{\mathbb{P}}_{(\mathrm{i})}\right)\right] H^{(+)}$$
$$- \frac{t}{\mathbb{Q}^2}\partial_\xi\left[\left(\mathbb{P}_{(\mathrm{i})} + \mathbb{P}_{(\mathrm{ii})} - \frac{\xi L}{2}\right.\right.$$
$$\left.\left. - \frac{\xi}{x+\xi}\left(\ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right) - \frac{\xi+\rho}{2\xi}\ln\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right) - \widetilde{\mathbb{P}}_{(\mathrm{i})}\right)\right) H^{(+)}\right]$$
$$\left. + \frac{\xi^2\bar{p}_\perp^2}{\mathbb{Q}^2} 2\xi\partial_\xi^2\left[\left(\mathbb{P}_{(\mathrm{i})} + \mathbb{P}_{(\mathrm{ii})} - \frac{\xi L}{2} + \frac{\widetilde{\mathbb{P}}_{(\mathrm{iii})} + \widetilde{\mathbb{P}}_{(\mathrm{i})}}{2}\right) H^{(+)}\right]\right\}$$
$$+ O(\mathrm{tw\text{-}6})\,.$$

- $\xi^2\bar{p}_\perp^2 = \xi^2 M^2 - t\left(\xi^2 - 1\right)/4\,.$
- **All amplitudes in VMF, Pire, Sznajder & Wagner, PRD 111 (2025) 7, 074034.**

- Coefficient functions of $\mathcal{A}^{++}$:

$$\mathbb{P}_{(\mathrm{i})}(x/\xi,\rho/\xi) = \frac{\xi-\rho}{x-\xi}\mathrm{Li}_2\left(-\frac{x-\xi}{\xi-\rho+i0}\right),$$

$$\widetilde{\mathbb{P}}_{(\mathrm{i})}(x/\xi,\rho/\xi) = -\frac{\xi-\rho}{x-\xi}\ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right),$$

$$\mathbb{P}_{(\mathrm{ii})}(x/\xi,\rho/\xi) = \frac{\xi-\rho}{x+\xi}\left[\mathrm{Li}_2\left(-\frac{x-\xi}{\xi-\rho+i0}\right)-(x\to-\xi)\right],$$

$$\widetilde{\mathbb{P}}_{(\mathrm{iii})}(x/\xi,\rho/\xi) = -\frac{\xi+\rho}{x+\xi}\ln\left(\frac{x-\rho+i0}{-\xi-\rho+i0}\right),$$

$$L = \int_0^1 dw\,\frac{-4}{x-\xi-w(x+\xi)}\int_0^1 du\,\ln\left(1+\frac{\bar{u}[x-\xi-w(x+\xi)]}{\xi-\rho+i0}\right)C_{\bar{u},\bar{u}w},$$

$$C_{\bar{u},v} = \ln\left(\frac{\bar{u}-v}{1-v}\right)+\frac{1}{1-v}.$$

- From the DDVCS result:

$$\begin{cases}\rho\to\xi\Rightarrow \text{DVCS}^{\#},\\ \rho\to-\xi(1-2t/Q'^2)\Rightarrow \text{TCS to twist-4 accuracy.}\end{cases}$$

#DVCS for spin-0 target was already computed in: Braun, Ji & Manashov, JHEP 01 (2023) 078.

D-term at LO/NLO + LT precision

Parameterization of the D-term **(→ connected to pressure):**

$$D_q(\alpha, t, \mu^2) = (1-\alpha^2) \sum_{\text{odd } n} d_{n,\,q}(t,\mu^2) \underbrace{C_n^{(3/2)}(\alpha)}_{\text{Gegenbauer poly.}} ,$$

$$D_g(\alpha, t, \mu^2) = \frac{3}{2}(1-\alpha^2)^2 \sum_{\text{odd } n} d_{n,\,g}(t,\mu^2) C_{n-1}^{(5/2)}(\alpha) ,$$

$$d_{n,\,a}(t,\mu^2) = d_{n,\,a}(0,\mu^2) \cdot (1 - t[\mathrm{GeV}^2]/0.8^2)^{-3} , \quad a \in \{q, g\} .$$

LO h_0^{++}, $n=1$, radiative gluons
$d_{1,\,uds}(0,\mu_0^2)$ free (only)

$d_{1,\,uds}(0, 2\ \mathrm{GeV}^2) =$	-0.6 ± 1.1
$d_{1,\,g}(0, 2\ \mathrm{GeV}^2) =$	-0.8 ± 1.5
$d_{1,\,c}(0, 2\ \mathrm{GeV}^2) =$	-0.003 ± 0.005

NLO h_0^{++}, $n=1$, free gluons
$d_{1,\,uds}(0,\mu_0^2)$, $d_{1,\,g}(0,\mu_0^2)$ free

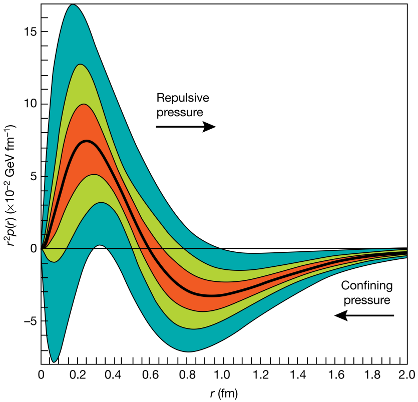
$d_{1,\,uds}(0, 2\ \mathrm{GeV}^2) =$	-1.1 ± 7.7
$d_{1,\,g}(0, 2\ \mathrm{GeV}^2) =$	-6 ± 78
$d_{1,\,c}(0, 2\ \mathrm{GeV}^2) =$	-0.02 ± 0.27

Results from Dutrieux, Meisgny, Mezrag & Moutarde, EPJC 85 (2025) 1, 105.

Pressure at leading twist

$$\text{At LT: } h_0^{++} = \left(\int D\right) \propto C(t) \mapsto \textit{pressure}$$

With a certain GPD model and data from JLab's Hall B:



From Burkert, Elouadrhiri & Girod, Nature 557, 396–399 (2018).

With the *same data set and ANNs* as the plot (Kumerizčki, Nature 570, E1–E2 (2019)): **h_0^{++} is compatible with zero.**

With a *global analysis* of DVCS and *ANNs* (Dutrieux et al., EPJC 81, 300 (2021)): **$d_1 \propto C(t)$ is compatible with zero.** Conclusion persists at NLO.

$$D_q(\alpha, t, \mu^2) = (1-\alpha^2) \times \sum_{\text{odd } n} d_{n,q}(t,\mu^2) \underbrace{C_n^{(3/2)}(\alpha)}_{\text{Gegenbauer poly.}}$$