Hadroproduction of Pseudoscalar Heavy-Quarkonium Mesons via NRFF1.0

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arXiv:2510.10593 [hep-ph]

NRFF1.0: A New Generation of FFs

NRFF1.0 is a public set of collinear Fragmentation Functions (FFs) for quarkonium production at moderate to large p_T.
https://github.com/FGCeliberto/Collinear_FFs

Key Features

- **Framework:** Built on the single-parton fragmentation approximation in a Variable Flavor Number Scheme (VFNS).
- **Consistency:** First set to include **all partonic channels** with consistent flavor threshold treatments.
- **Pseudoscalar Sector:** Construction for η_c and η_b uses fully NLO-accurate NRQCD inputs.

- **Improvement over ZCW19+:** Systematically accounts for heavy quark thresholds during DGLAP evolution via the **HF-NRevo** scheme.
  F. G. Celiberto and M. Fucilla, Eur. Phys. J. C 82, 929 (2022), 2202.12227.

Non-Relativistic QCD (NRQCD)

The NRQCD Lagrangian

The total Lagrangian is constructed from all operators permitted by gauge and Lorentz symmetries:

$$\mathcal{L}_{\text{NRQCD}} = \mathcal{L}_g + \mathcal{L}_l + \mathcal{L}_\psi + \mathcal{L}_\chi + \mathcal{L}_{\psi\chi}$$

where subscripts denote *gluons (g)*, *light quarks (l)*, *Pauli spinors (ψ)*, and *anti-spinors (χ)*.

Expansion at $\mathcal{O}(1/m_Q^2)$

Including the kinetic term $\frac{\mathbf{D}^4}{8m_Q^3}$, the heavy quark sector $\mathcal{L}_\psi$ is:

$$\mathcal{L}_\psi = \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m_Q} + c_F \frac{g\boldsymbol{\sigma}\cdot\mathbf{B}}{2m_Q} + c_D \frac{g[\mathbf{D}\cdot\mathbf{E}]}{8m_Q^2} + \dots \right) \psi$$

NLO NRQCD

Improvements over Leading Order (LO)

- **Scale Stability:** Significantly reduces renormalization and factorization scale dependence.
- **Color-Octet (CO) Mechanisms:** Accounts for $Q\bar{Q}$ pairs produced in CO states that evolve into mesons via soft gluon emission.
- **Universality:** Essential for testing Long-Distance Matrix Elements (LDMEs) across different colliders.

Quarkonium: Factorization and Velocity Scaling

NRQCD Power Counting: Theoretical predictions are organized as double expansions in the strong coupling α_s and the heavy-quark relative velocity v.

- **Velocity Scaling Rules:** Estimate the importance of Long-Distance Matrix Elements (LDMEs).
- **Non-Relativistic Limit:** Valid for $v \ll 1$ (where $v^2 \approx 0.3$ for $c\bar{c}$ and $v^2 \approx 0.1$ for $b\bar{b}$).
- **Predictive Power:** Allows for a systematic truncation of the infinite series of LDMEs.

Table: Leading velocity scaling of LDMEs

State	${}^1S_0^{(1)}$	${}^3S_1^{(1)}$	${}^1S_0^{(8)}$	${}^3S_1^{(8)}$	${}^3P_0^{(8)}$	${}^3P_1^{(8)}$	${}^3P_2^{(8)}$
η_c, η_b	1	–	v_Q^4	v_Q^3	v_Q^4	v_Q^4	v_Q^4
$J/\psi, \Upsilon$	–	1	v_Q^3	v_Q^4	v_Q^4	v_Q^4	v_Q^4

Fock State Expansion and Color-Octet Mechanism

NRQCD Factorization: The $Q\bar{Q}$ pair is produced at short distances in a Fock state $n = {}^{2S+1}L_J^{[a]}$, where $a = 1$ (Singlet) or $a = 8$ (Octet).

- **Transition:** Intermediate CO states evolve into physical CS quarkonia via *non-perturbative* soft gluon emission.
- **Limit:** As $v \to 0$, NRQCD recovers the Color Singlet Model (CSM) for S-wave states.

The Multi-Scale Challenge in Quarkonium Production

The production of heavy quarkonia at high transverse momentum ($q_T \gg m_Q$) involves a duality between

- Short-distance production at low-to-moderate p_T
- Single-parton VFNS at moderate-to-large p_T
- First set of collinear fragmentation functions for the pseudoscalar quarkonium: `https://github.com/FGCeliberto/Collinear_FFs`

Founding Assumptions

1. **Fock State Superposition:** All possible states contribute to the physical meson.
2. **Double Expansion:** Systematic organization in α_s and v.

NRFF1.0: Theoretical Foundation at μ_0

The initial scale FFs are computed at $\mu_0 = 2m_Q$ using NLO Short-Distance Coefficients (SDCs).

$$D_{i \to \eta_Q}(z, \mu_0) = \frac{\alpha_s^2(\mu_0)}{m_Q^3} \langle \mathcal{O}^{\eta_Q}(^1S_0^{(1)}) \rangle \mathcal{F}_i(z) \tag{1}$$

1. All parton channels are known at NLO
2. Singlet-color
3. HF-NRevo combines elements of hadronic structure with QCD precision (collinear factorization)

NRFF1.0: Theoretical Foundation at μ_0

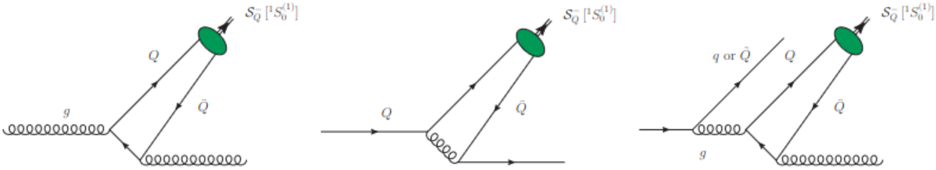


Figure: $S_{\bar{Q}}$ $[^1S_0^{(1)}]$

The HF-NRevo Scheme: Bridging the Thresholds for η_c

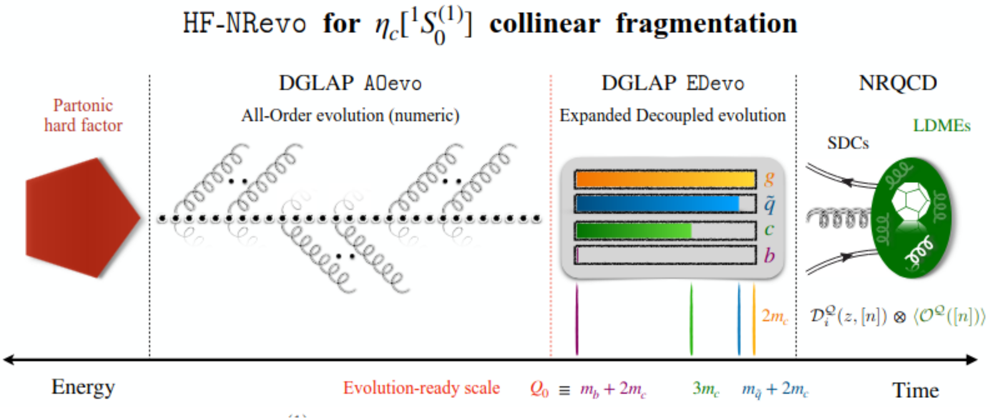


Figure: Evolution workflow within the HF-NRevo scheme

Justification for EDevo Matching

Why is EDevo necessary?

- For η_c and η_b, the transition between flavor schemes is implemented through standard VFNS matching across heavy-quark thresholds, consistently with the perturbative evolution of the fragmentation functions
- **EDevo**: it is crucial to preserve the thresholds, that, in heavy flavor specis, are present in light parton channels (non constituent quarks and gluon).

Quarkonium FFs from HF-NRevo evolution - LDME uncertainties

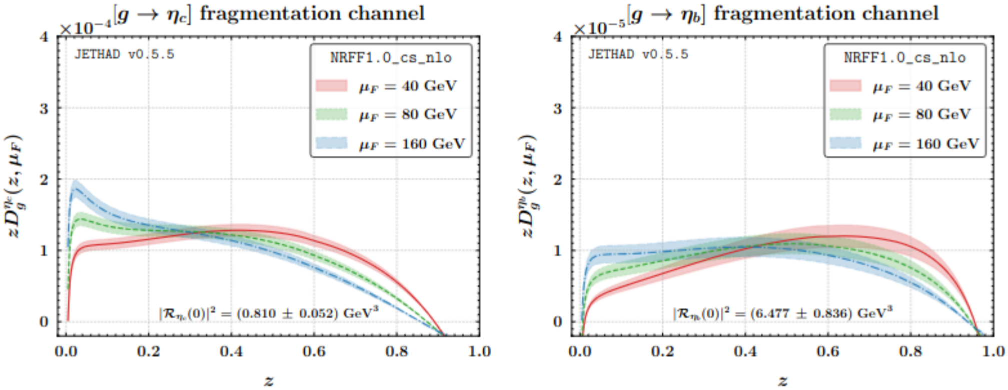


Figure: Gluon channel

Quarkonium FFs from HF-NRevo evolution - LDME uncertainties

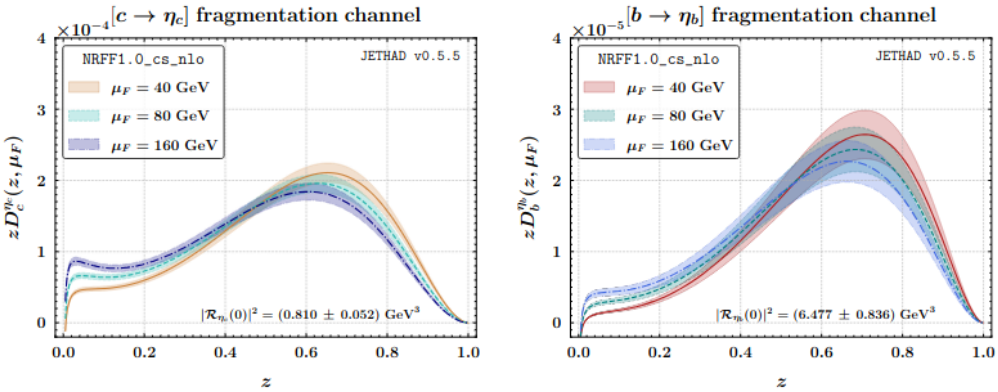
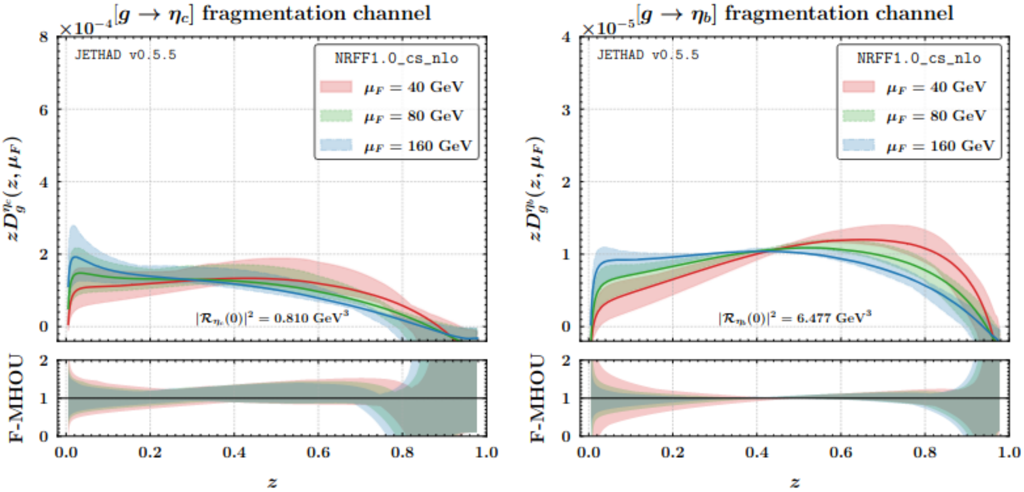
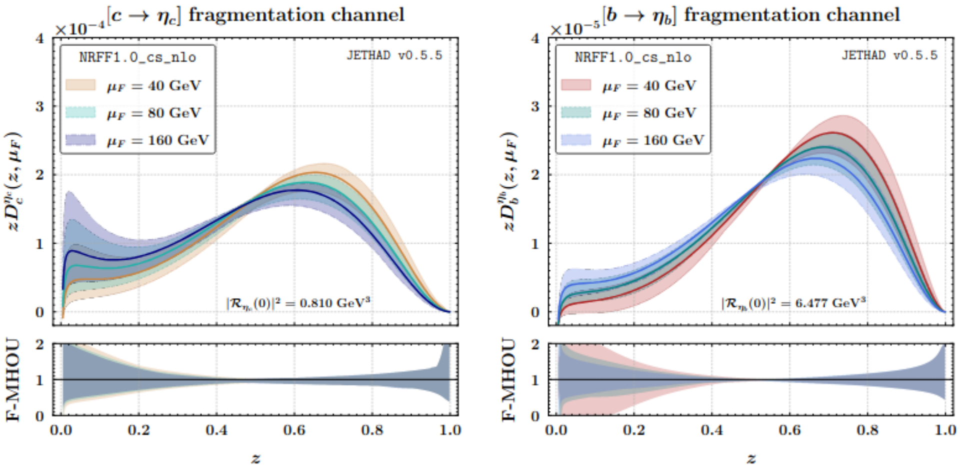


Figure: Quark channel

Quarkonium FFs from HF-NRevo evolution - MHOUs



Quarkonium FFs from HF-NRevo evolution - MHOUs



Factorization scale dependence

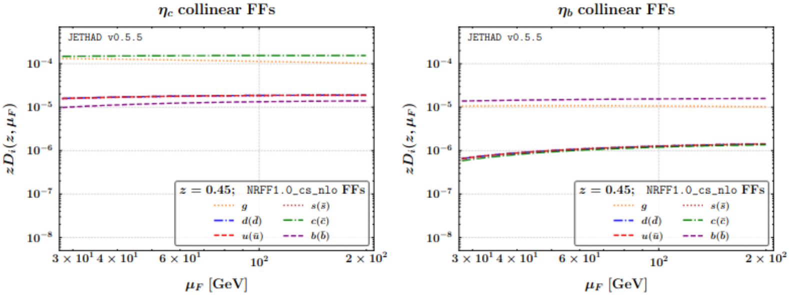


Figure: Factorization scale dependence

Natural Stabilization → **F. G. Celiberto, Acta Phys. Polon. Supp. 16, 41 (2023), 2211.11780**

Hybrid Factorization (HyF) Framework

Concept: Combines High-Energy (BFKL) and Collinear factorization to describe processes at NLL/NLO+ accuracy.

- **High-Energy Resummation:** Incorporates BFKL Green's functions with singly off-shell emission functions (forward production impact factors).

The differential cross section is expressed as a Fourier expansion:

$$\frac{(2\pi)^2 d\sigma}{dy_1 dy_2 d|\vec{q}_{T1}| d|\vec{q}_{T2}| d\phi_1 d\phi_2} = \mathcal{C}_0 + 2\sum_{n=1}^{\infty} \cos(n\phi)\mathcal{C}_n \tag{2}$$

where $\phi \equiv \phi_1 - \phi_2 - \pi$.

High-Energy Resummation (HyF)

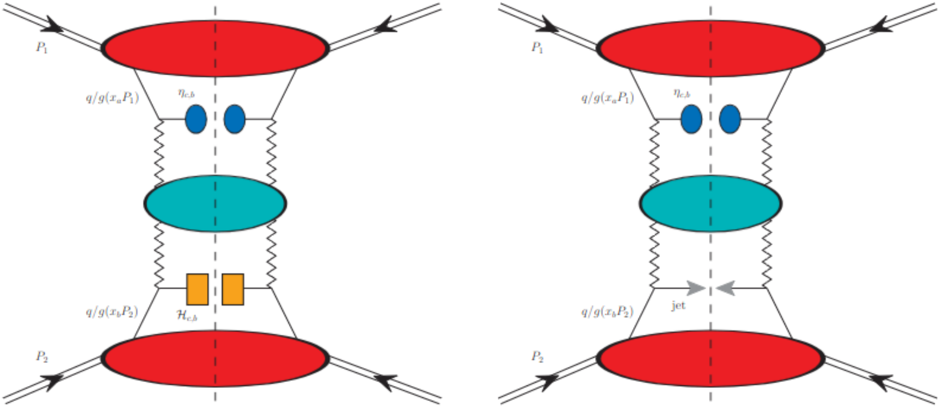


Figure: Hybrid factorization for the semi-inclusive hadroproduction

High-Energy Resummation (HyF) - JETHAD

At very high energies, we encounter logarithms ΔY that can destabilize NLO results

$$\sigma_{\text{Hybrid}} = \sigma_{\text{NLO}} + \sigma_{\text{NLL res}} \tag{3}$$

Hybrid factorization for the semi-inclusive hadroproduction of a η_c and η_b quarkonium accompanied by a hadron or a light jet.
F.G. Celiberto, Eur. Phys. J. C 81 (2021) 8, 691 [2008.07378 [hep-ph]]
F.G. Celiberto, Phys. Rev. D 105 (2022) 11, 114008 [2204.06497 [hep-ph]]

Phenomenological Analaysis

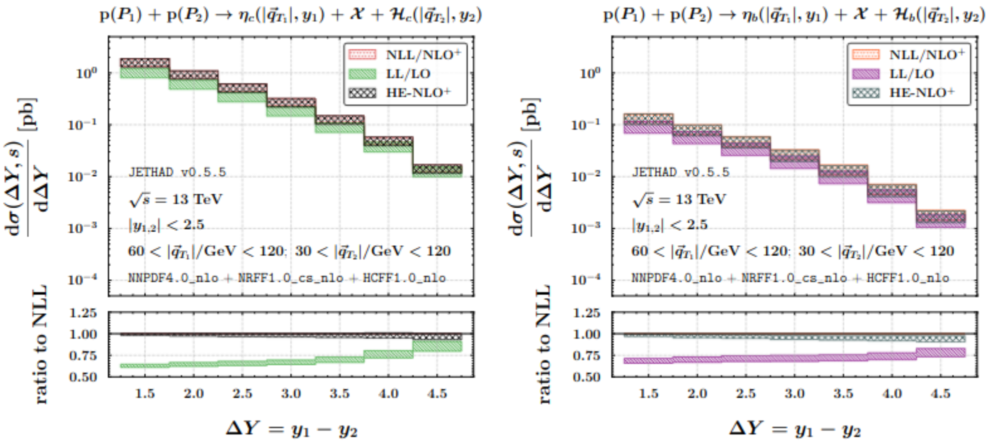


Figure: Rapidity distributions for the semi-inclusive hadroproduction

Outlook: The Next Generation

- **Vector States:** The HF-NRevo scheme is currently being extended to J/ψ and Υ, where the color-octet contributions are more complex.
- **TMD Evolution & Matching:** Future work involves moving beyond collinear factorization to Transverse Momentum Dependent (TMD) fragmentation.
- **Exotic States:** This framework provides the consistent basis for studying the fragmentation of tetraquarks and other exotics.
- Investigating how **intrinsic charm** manifests itself in high-energy hadronic processes.
- The HyF framework can be extended to cover **single-inclusive emissions of quarkonium states**
- We plan to further develop the NRFF1.0 sets by performing a **systematic uncertainty analysis**

Thank you!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Phenomenological Analysis

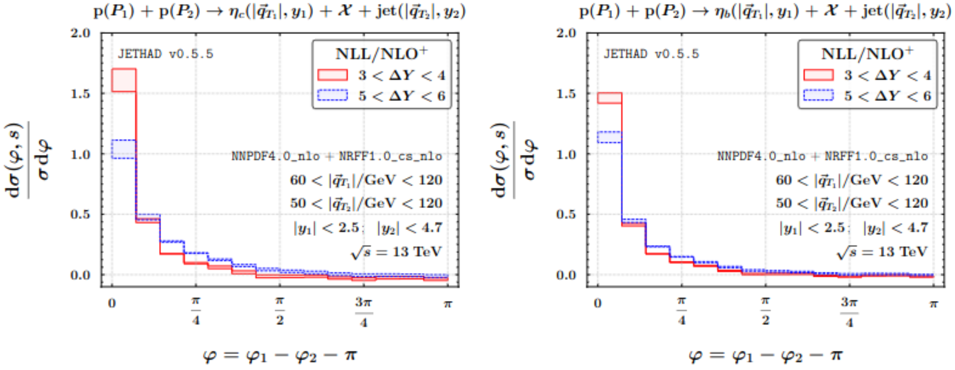

Figure: Angular multiplicities for the semi-inclusive hadroproduction

Phenomenological Analysis

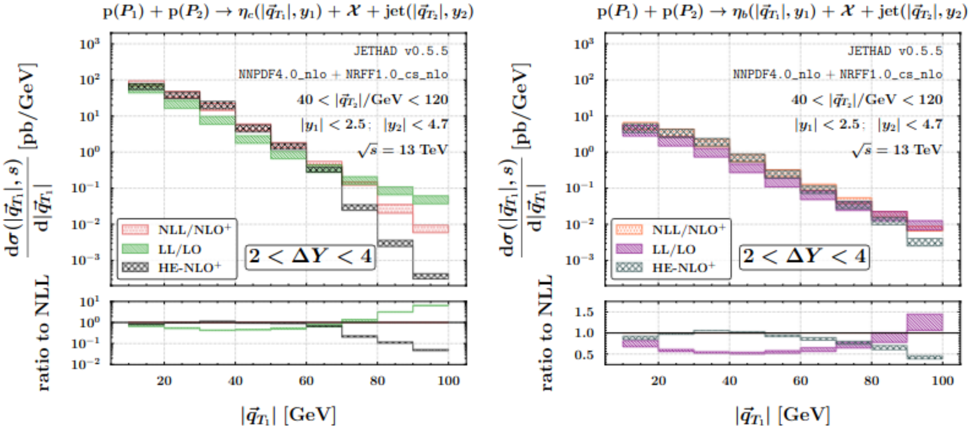

Figure: Transverse momentum distributions for the semi-inclusive hadroproduction