NNNLO QCD corrections to Semi-Inclusive Deep-Inelastic Scattering

Haitao Li (Shandong University)

Based on the works

arXiv:2602.06364, arXiv:2602.22972,
arXiv:2603.29673

with Liang Dong, Shen Feng, Jun Gao, Ding Yu Shao,
Yu Jiao Zhu, Hua Xing Zhu

QCD evolution 2026

El Escorial, Spain

Outline

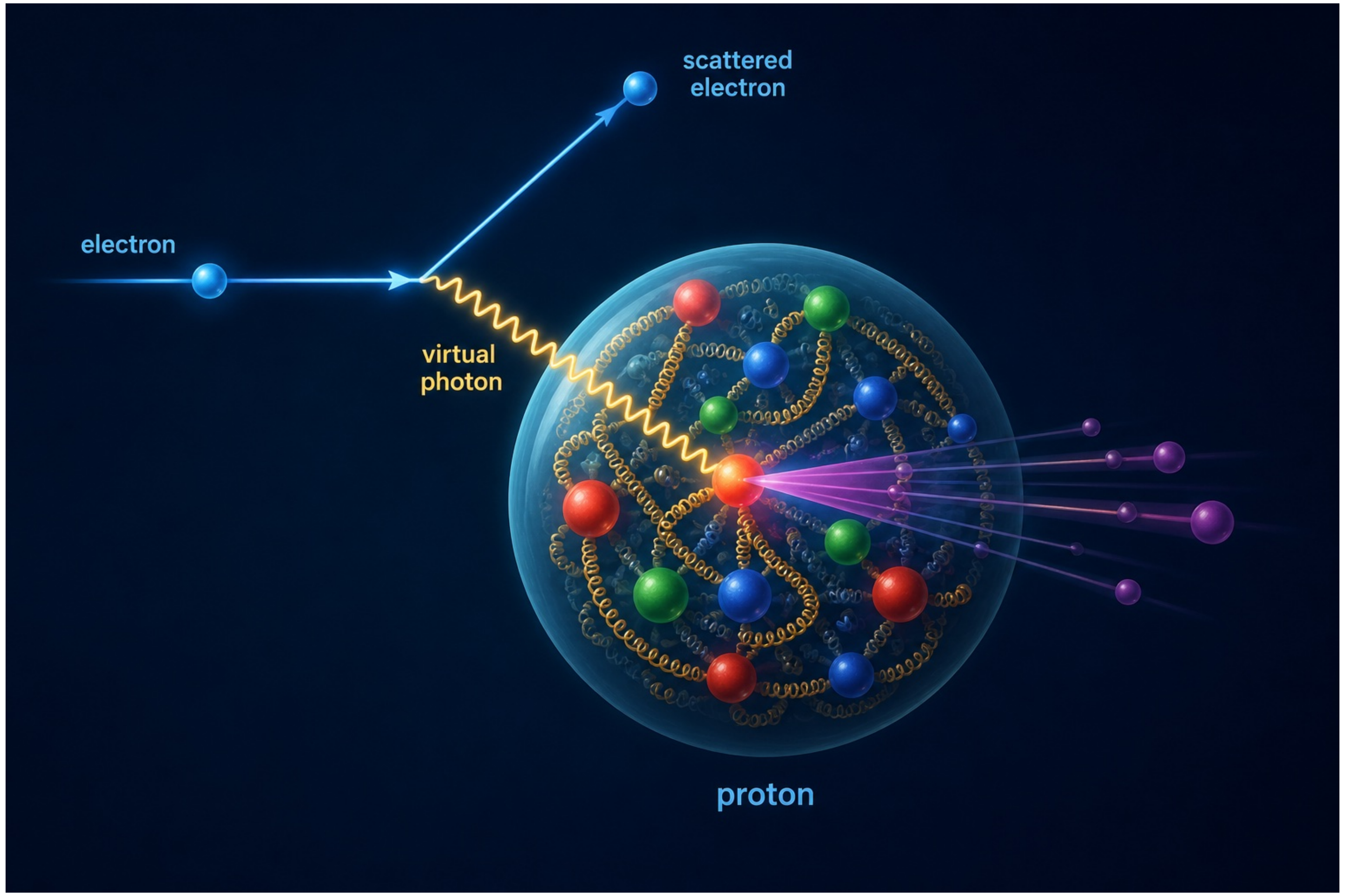
- Introduction
- q_T subtraction for hadron production
- Applications to N3LO QCD corrections to SIDIS
- Summary

Introduction

$$l + N \to l + h + X$$

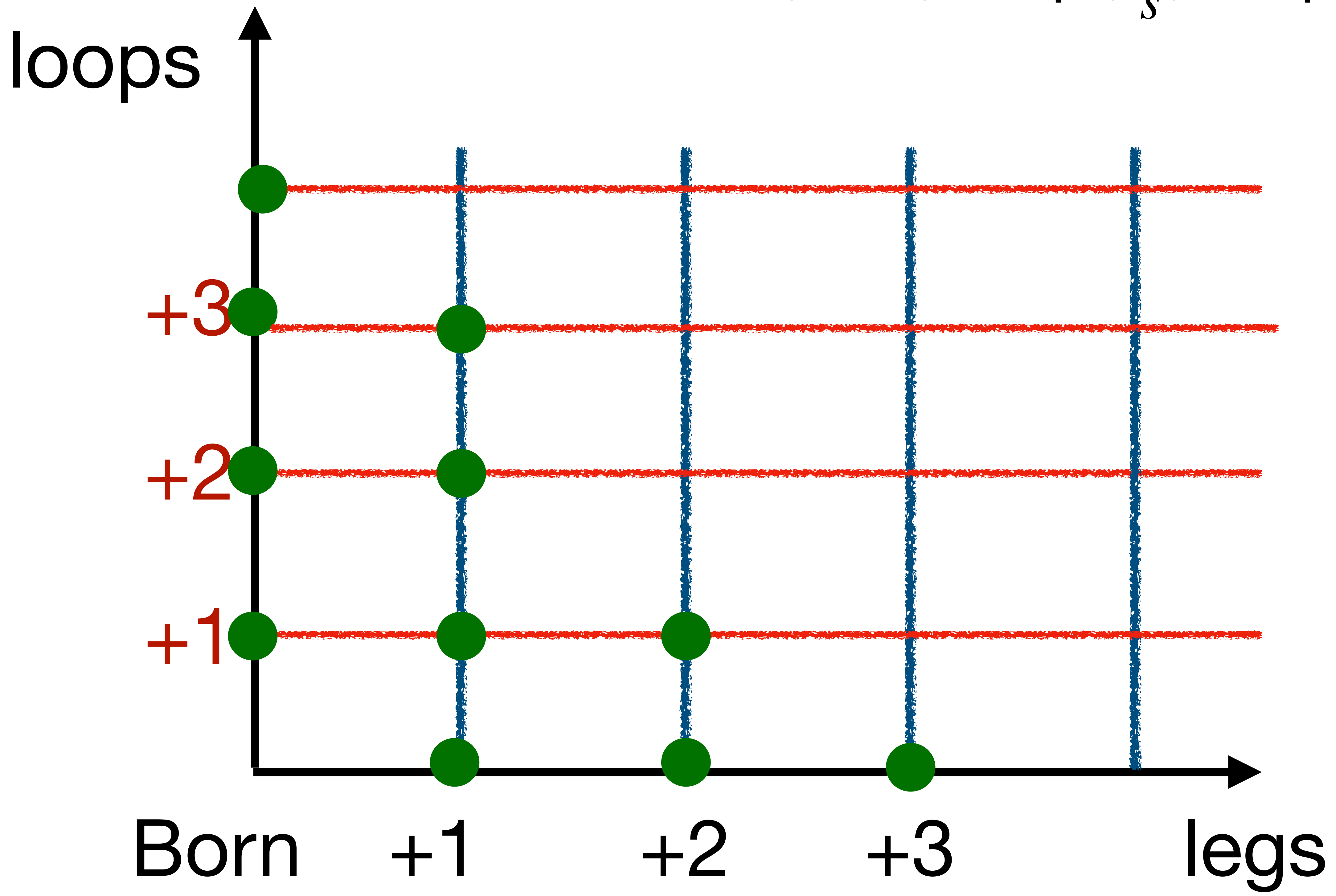
$$\frac{d\sigma^{\ell p \to \ell' h X}}{dx_B\, dQ^2\, dz_h} \propto \sum_q e_q^2\, f_{q/p}(x_B, \mu_F)\, D_{h/q}(z_h, \mu_F)$$

- ❑ Studying the collinear PDFs of nucleon
- ❑ Probing the 3D tomography
- ❑ Unraveling Spin-Orbit correlations
- ❑ Understanding Hadronization Dynamics
- ❑ Advancing Precision pQCD & Factorization



Introduction

$$\sigma = \sigma^{(0)} + \alpha_s \sigma^{(1)} + \alpha_s^2 \sigma^{(2)} + \alpha_s^3 \sigma^{(3)} + \dots$$



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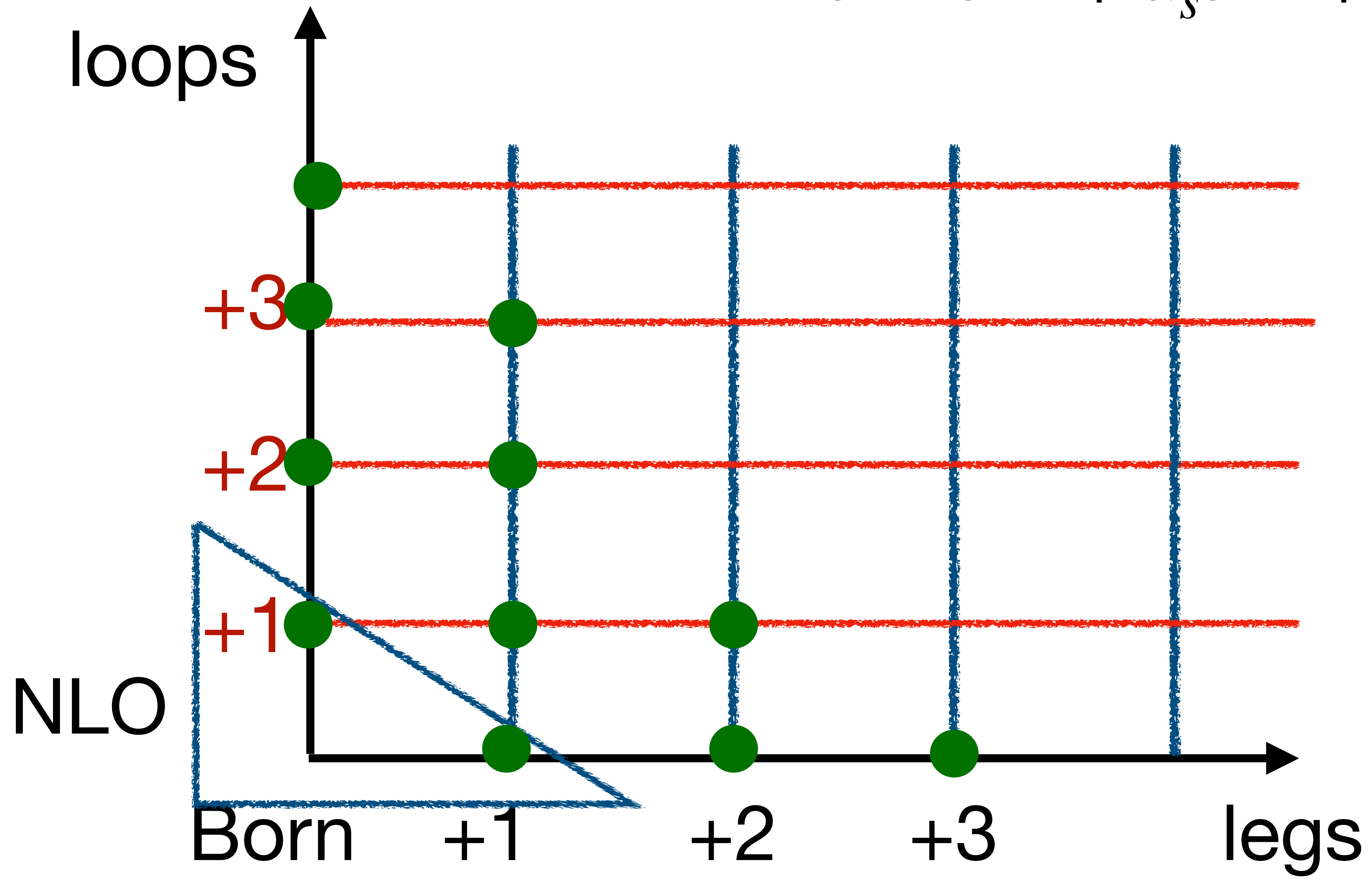

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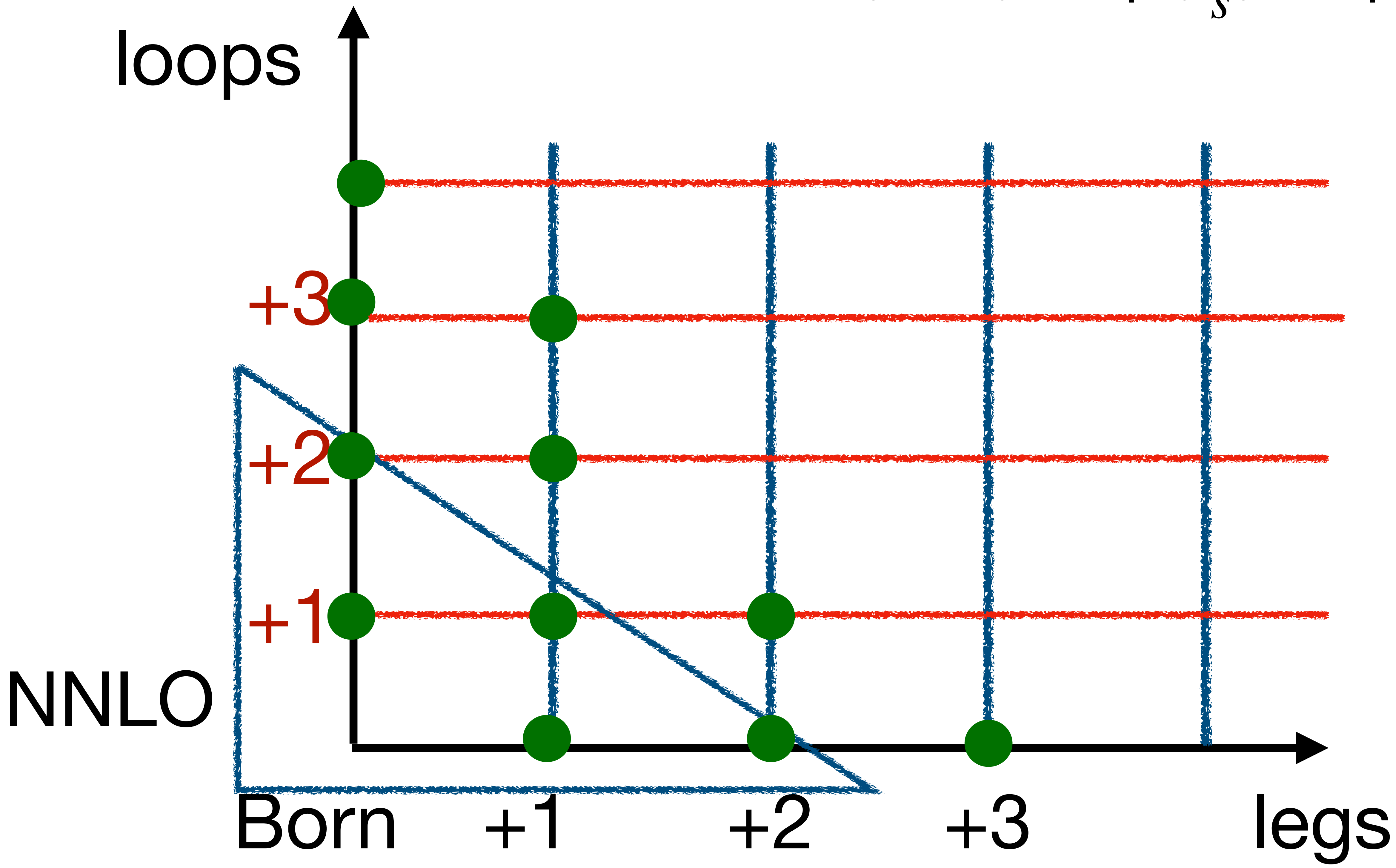

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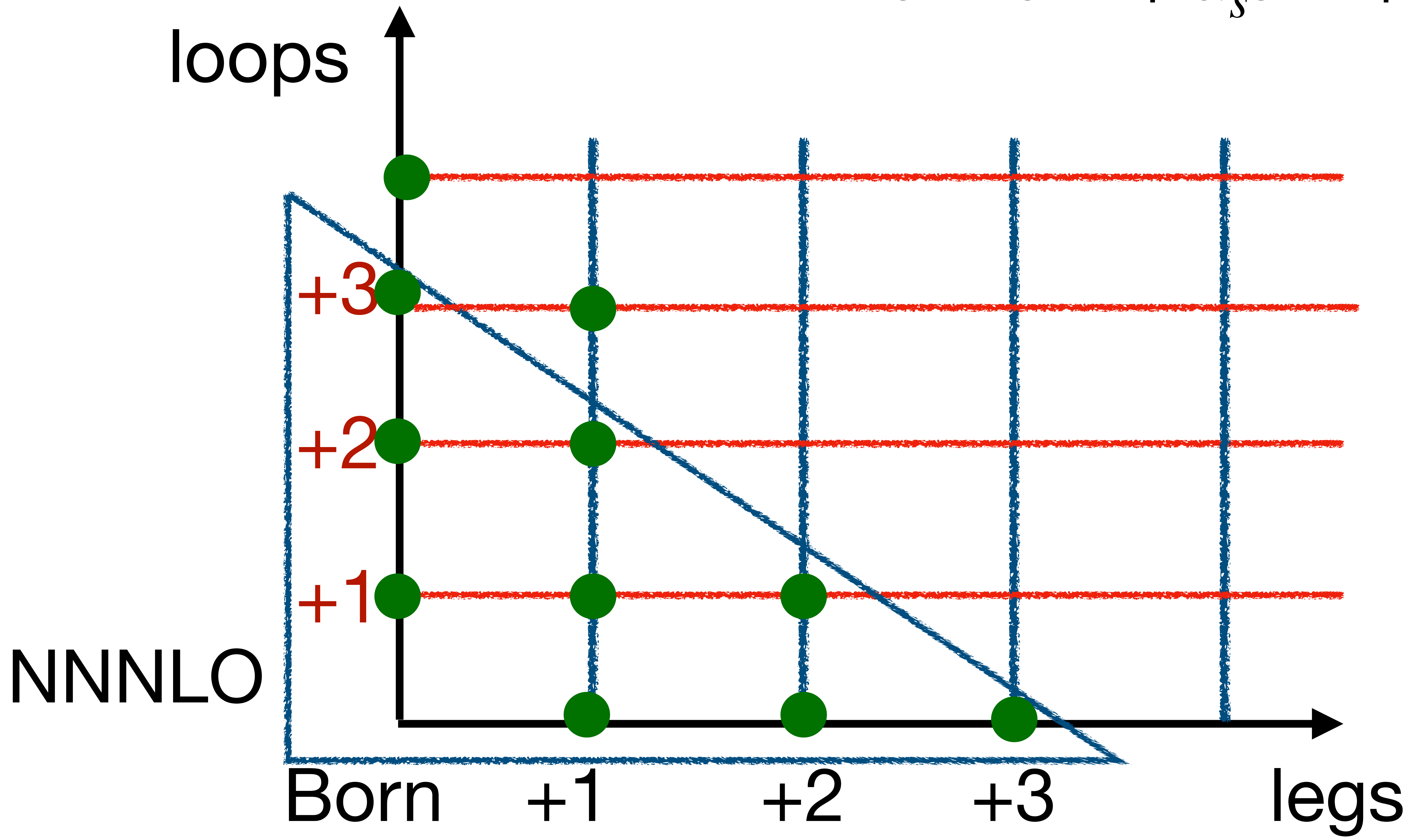

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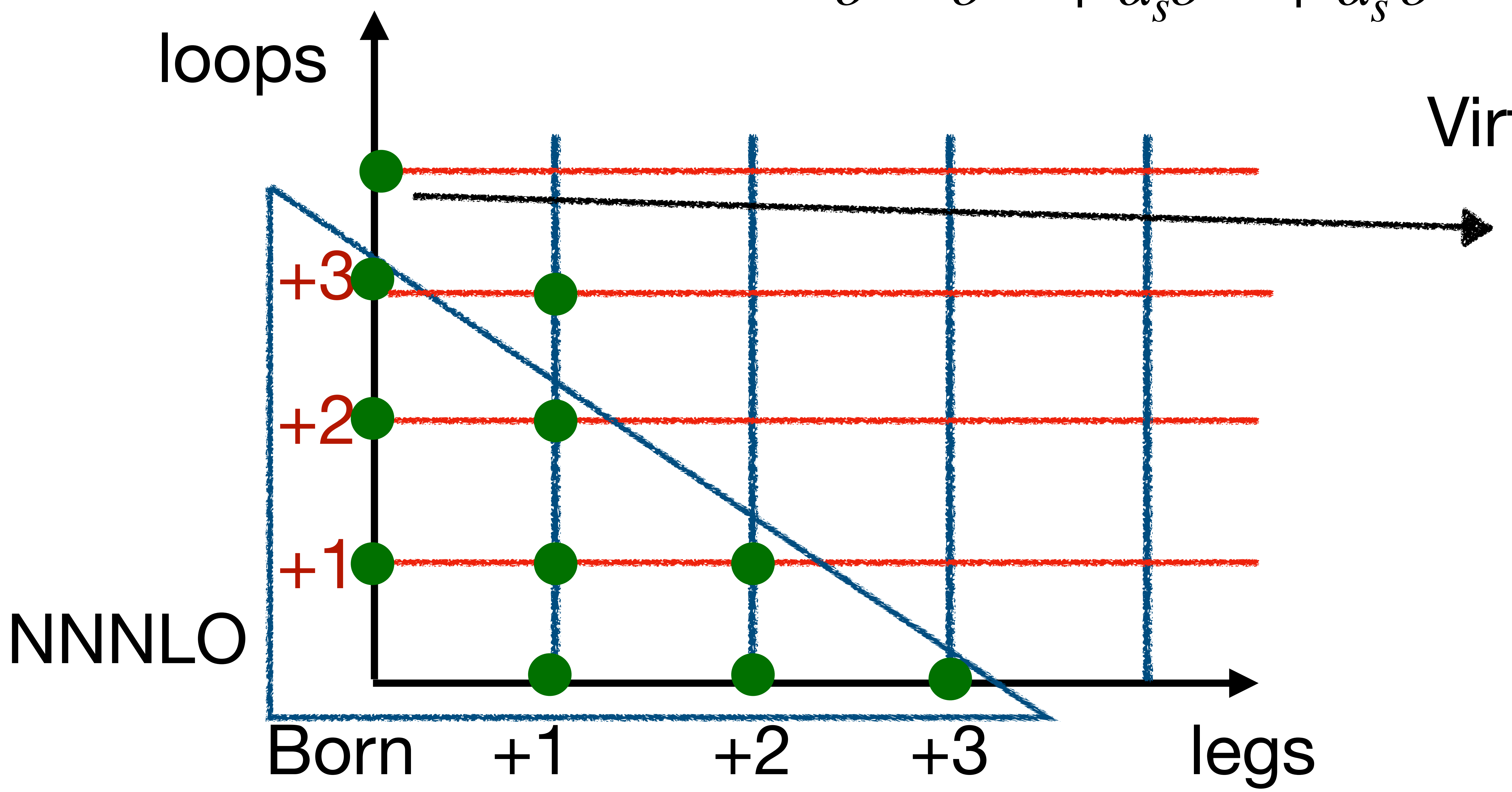

Virtual corrections $\gamma^* + q \to q$

4-loop form factor for massless parton

3-loop form factor for massive quarks

3-loop with mixed QCD and EW corrections

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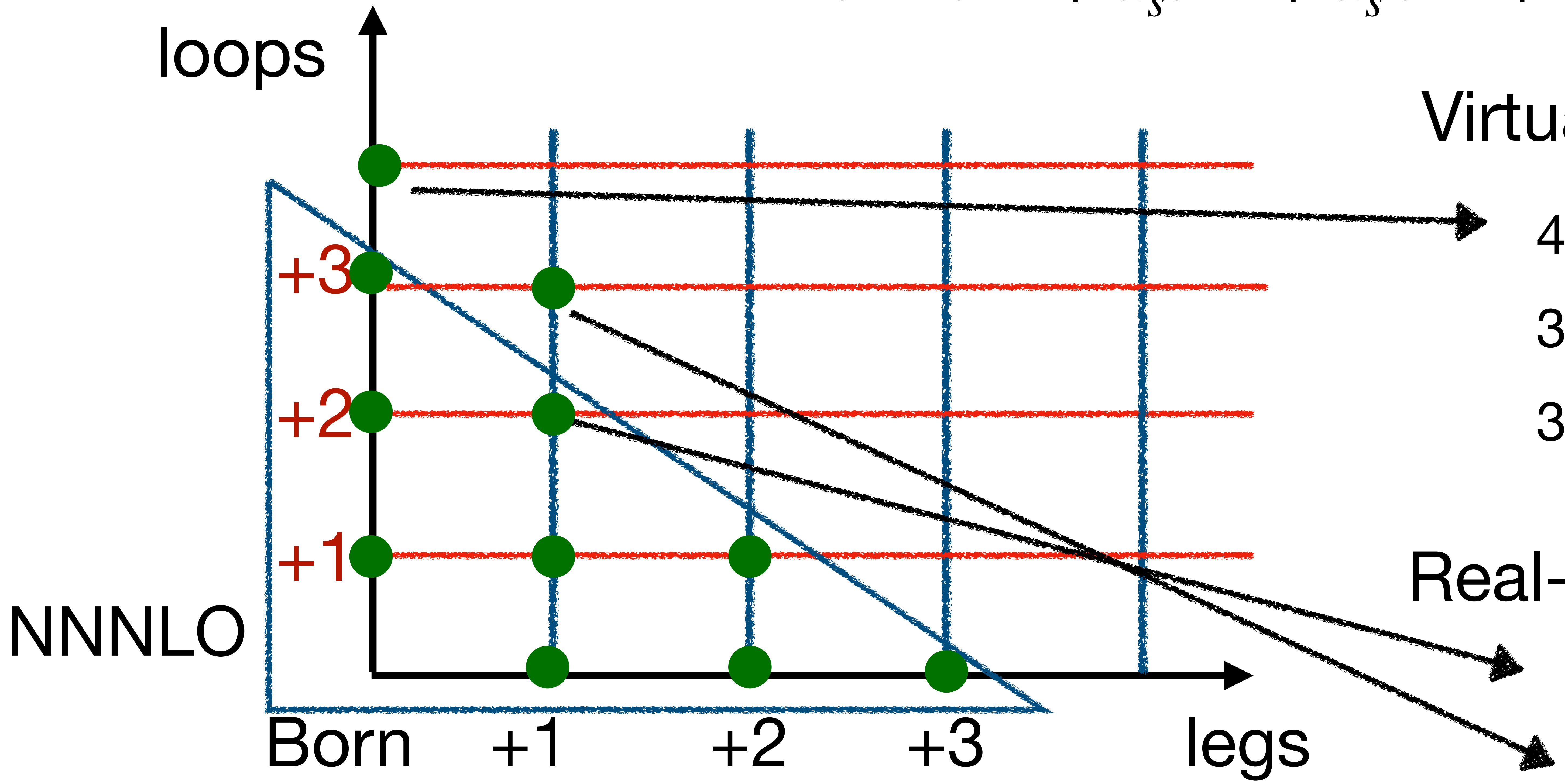

Virtual corrections $\gamma^* + q \to q$

- 4-loop form factor for massless parton
- 3-loop form factor for massive quarks
- 3-loop with mixed QCD and EW corrections

Real-Virtual corrections $\gamma^* + q \to q + g$

- 2-loop matrix element
- 3-loop at LC

hep-ph/0206067, arXiv:0904.2665

arXiv:2307.15405

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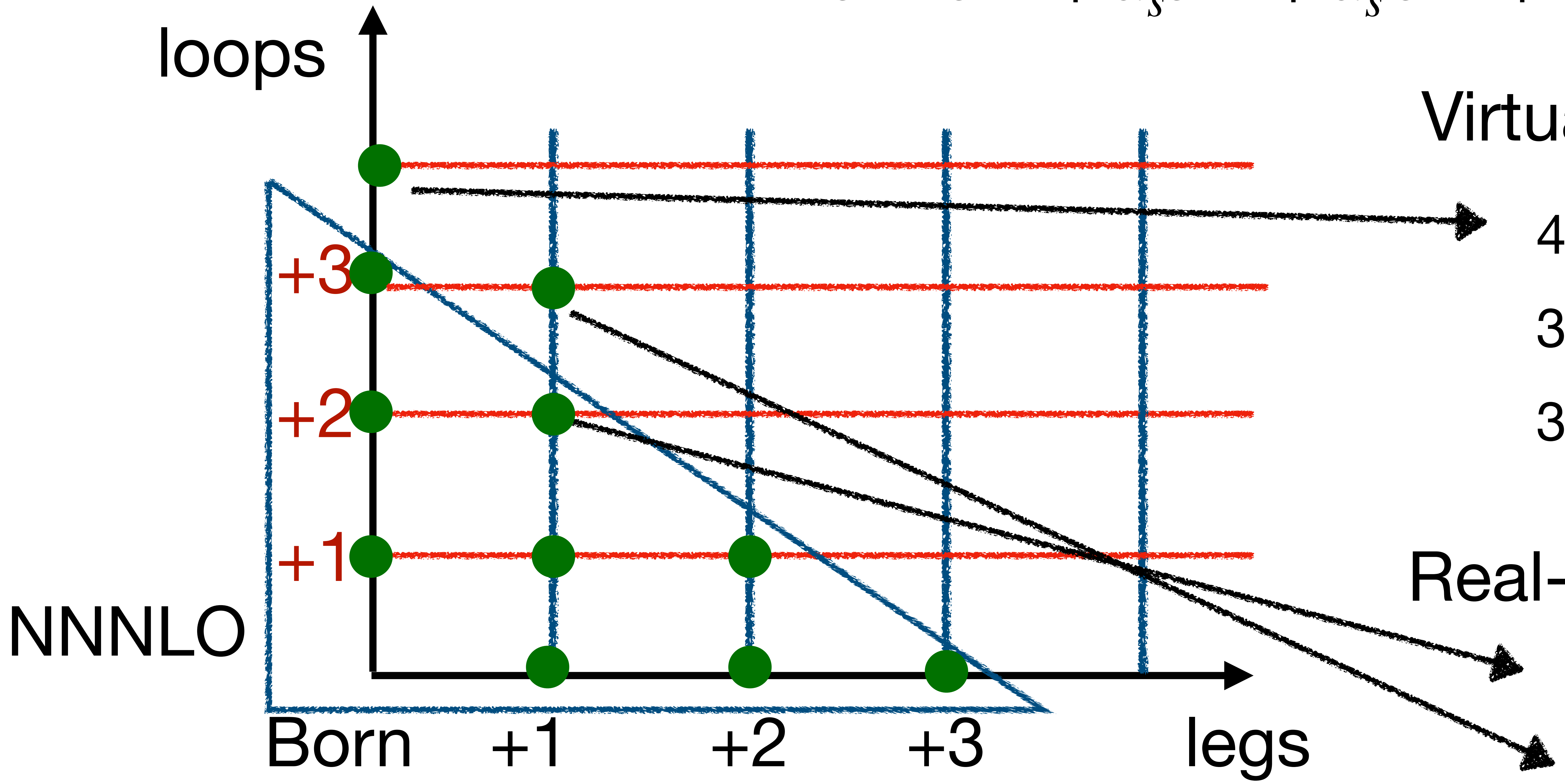

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One-loop and tree amplitude can be generated automatically

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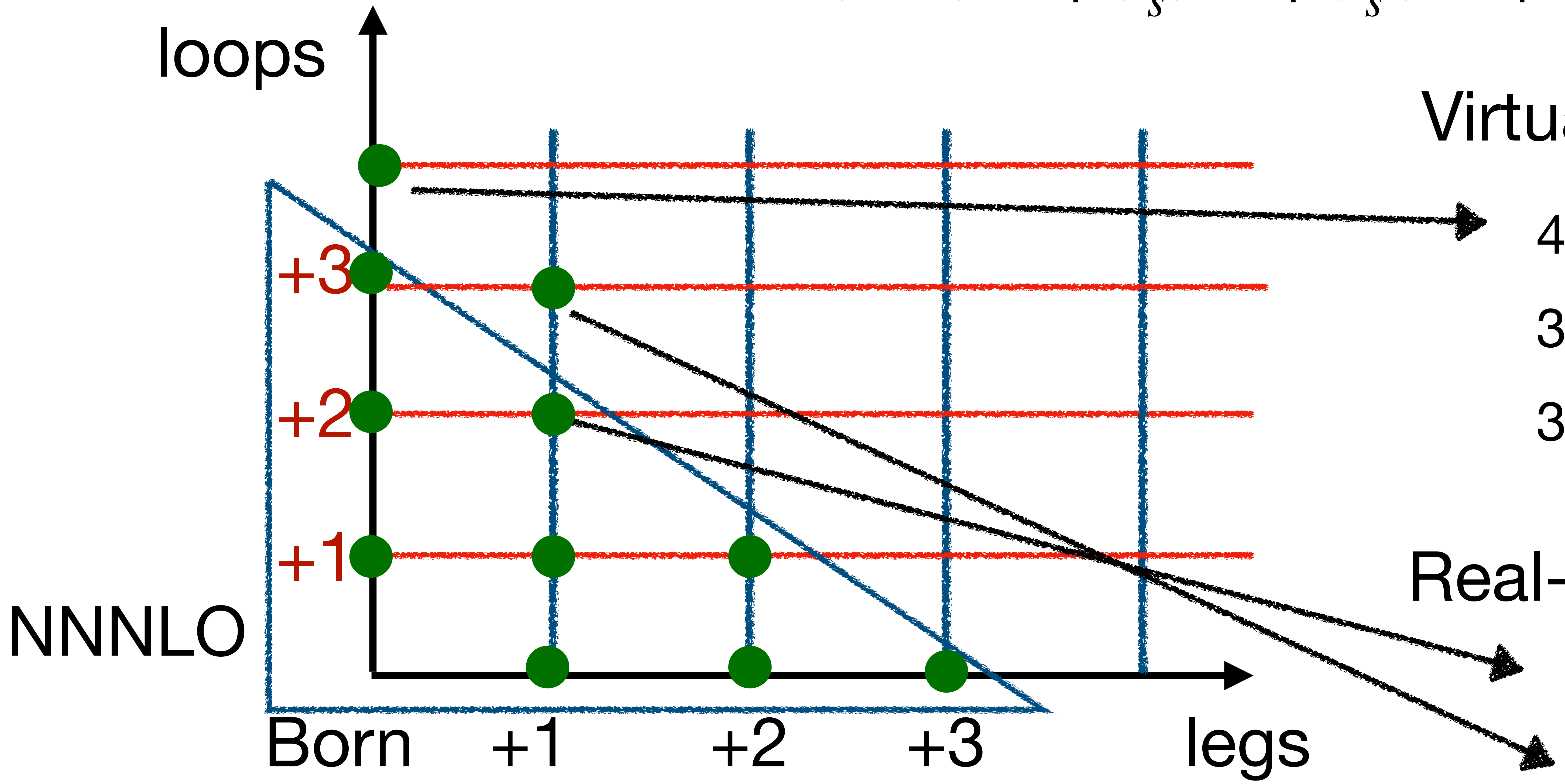

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hep-ph/0206067, arXiv:0904.2665
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Infrared subtraction is still a bottleneck

One-loop and tree amplitude can be generated automatically

Introduction

Modern frontier of precision QCD: N3LO calculations

- **Hadron colliders**
  - pp → H (inclusive and differential from 2015 to 2022).
  - pp → H+j+j (VBF channel, differential from 2016 to 2018).
  - pp → W/Z/γ* (inclusive and differential, from 2020 to 2023).
  - pp → H+V (inclusive, 2022).
  - pp → HH (inclusive and differential in EFT, from 2019 to 2026).
  - pp → γγ (differential, 2026).
- **Lepton colliders**
  - ee → h+X (inclusive, 2025)
  - ee →top pair, (inclusive, 2024)
  - ee → dijet (differential, 2025).
- **DIS**
  - ep → e + jet + X (differential, 2018).
  - **ep → e + h + X (differential, this talk, 2026).**

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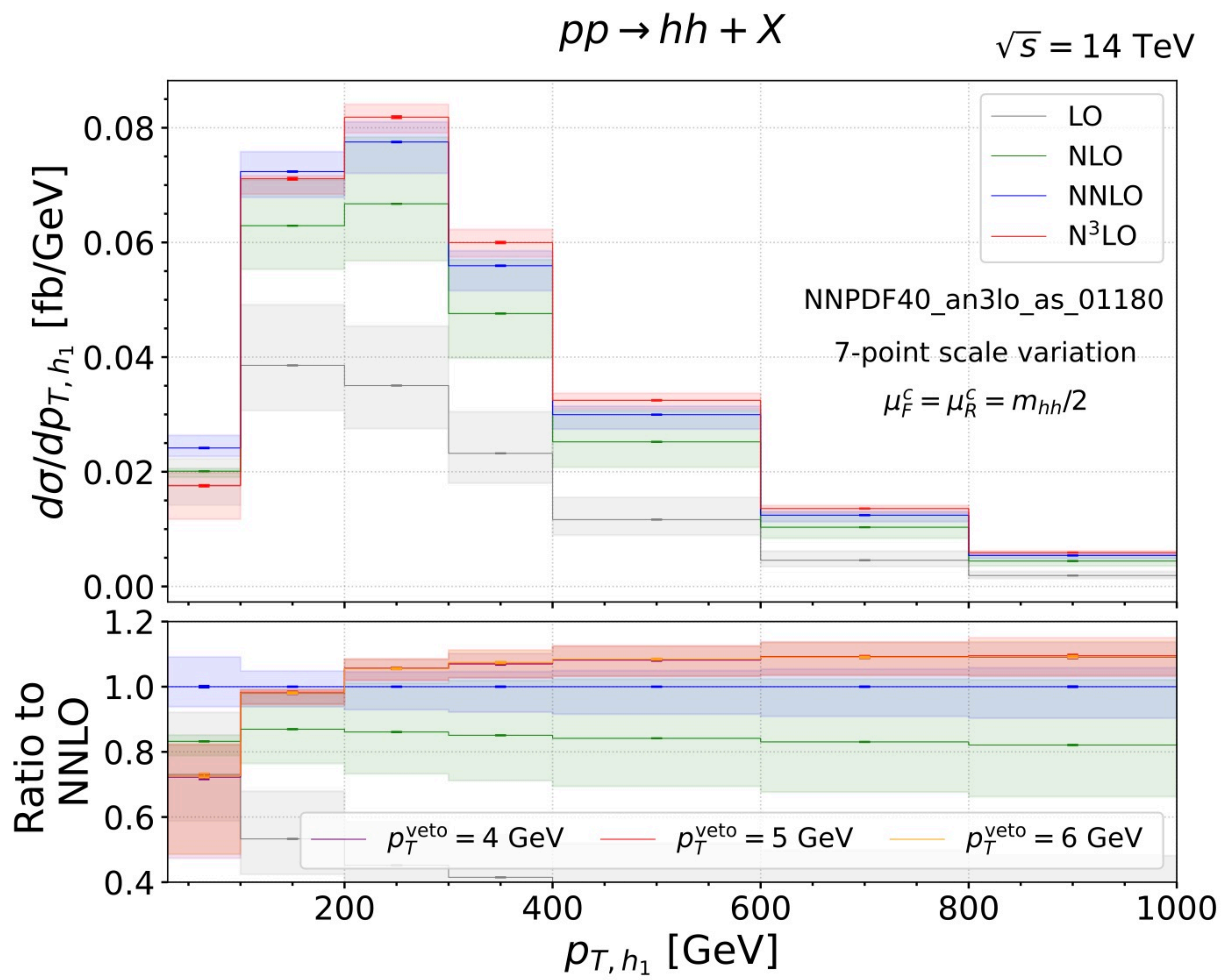


Chen, H.T.L., Shao, Wang, 1909.06808, 1912.13001
Chen, Dai, H.T.L., Li, Shao, Wang, 2601.19990

Introduction

NLO QCD corrections to jet production   dipole subtraction, FKS subtraction, et al

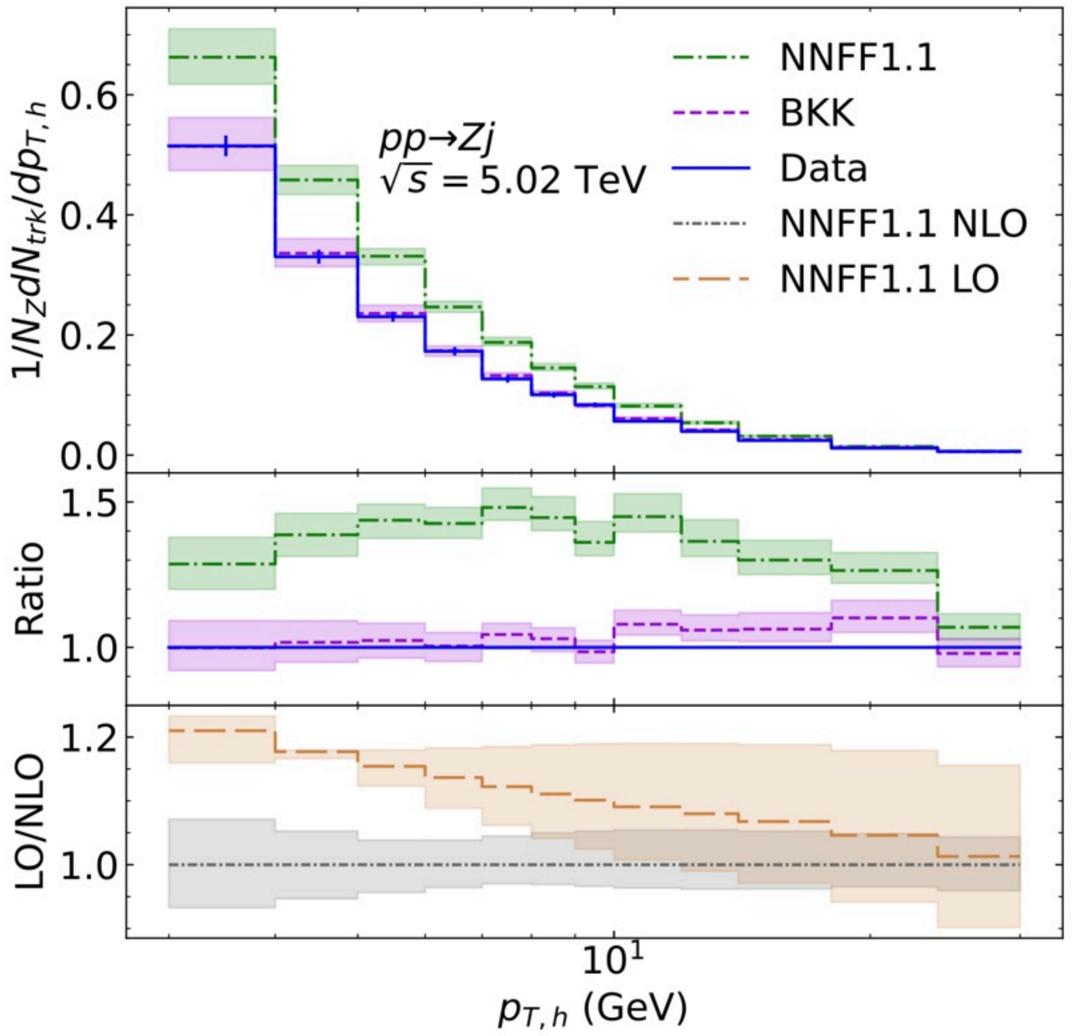
$$\frac{d\sigma}{dF} = \int dPS_m \Big[|M|^2_{B,m} + |M|^2_{V,m} + |\tilde{\mathcal{I}}|^2_m\Big] \delta(\hat{F}(p_m; f_m) - F)$$
$$+ \int dPS_{m+1} \Big[|M|^2_{R,m+1} \delta(\hat{F}(p_{m+1}; f_{m+1}) - F) - |\mathcal{I}|^2_{m+1} \delta(\hat{F}(\tilde{p}_m; \tilde{f}_m) - F)\Big]$$

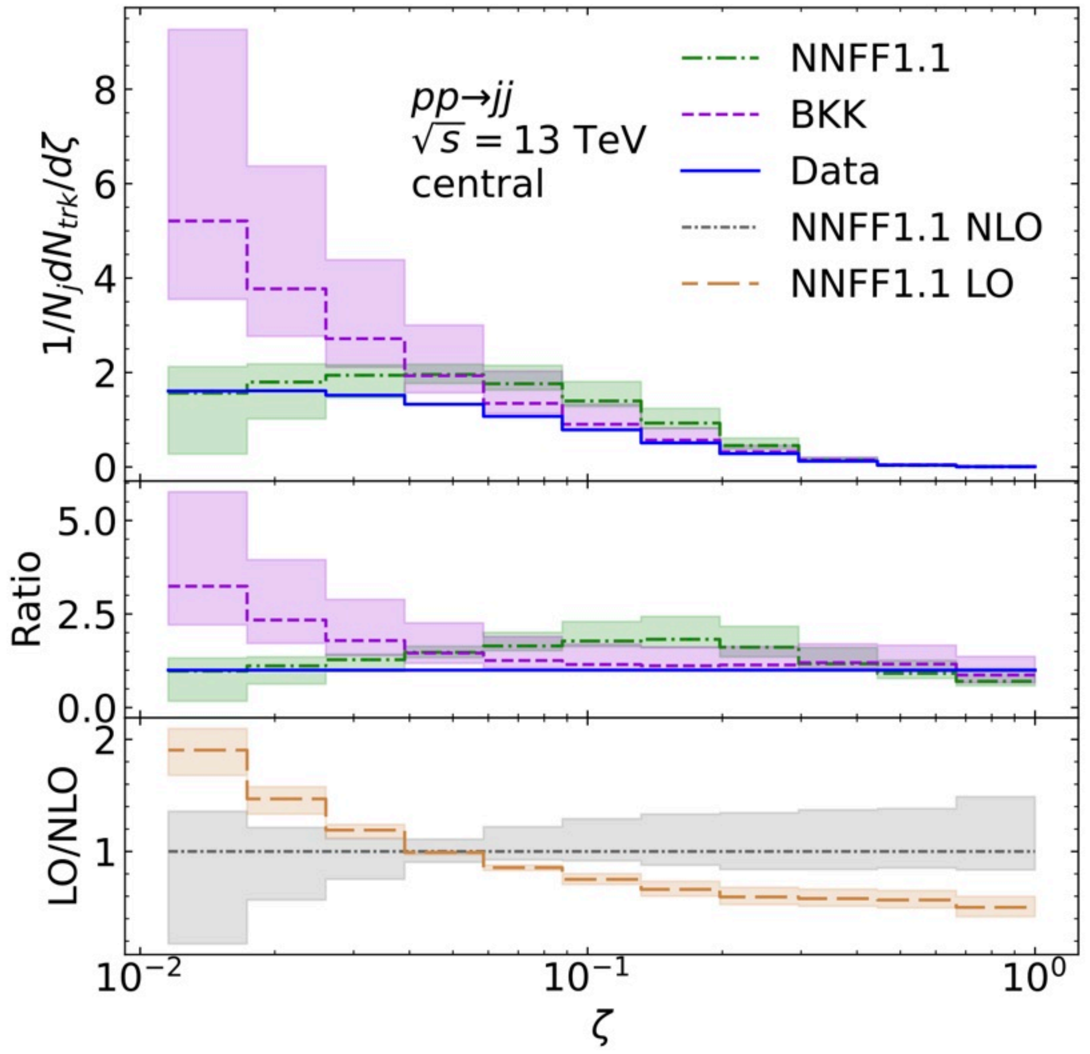
NLO QCD corrections with fragmentation process

Automated calculation at NLO in QCD: FMNLO interfaced to MG5 aMC@NLO

Liu, Shen, Zhou, Gao, 2305.14620

$$\frac{d\sigma}{dp_{T,h}} = \int dx \int dPS_m \Big[|M|^2_{B,m} + |M|^2_{V,m} + |\tilde{\mathcal{I}}|^2_m\Big] \sum_{i=1}^{m} \delta(p_{T,h} - x p_{T,i}) D^0_{h/i}(x)$$
$$+ \int dx \int dPS_{m+1} (\Theta(\lambda - C) + \Theta(C - \lambda)) \Big[|M|^2_{R,m+1} \sum_{i=1}^{m+1} \delta(p_{T,h} - x p_{T,i}) D^0_{h/i}(x)$$
$$- |\mathcal{I}|^2_{m+1} \sum_{\tilde{i}=1}^{m} \delta(p_{T,h} - x \tilde{p}_{T,\tilde{i}}) D^0_{h/\tilde{i}}(x)\Big]$$





Introduction

For Hadron production in DIS

$$l(p_l) + p(P_N) \to l(p_{l'}) + h(P_h) + X \qquad x = \frac{Q^2}{2P_N \cdot q} \quad y = \frac{P_N \cdot q}{P_N \cdot p_l} \quad z = \frac{P_N \cdot P_h}{P_N \cdot q}$$

NNLO results for triple differential distribution over x, y, z

Cross section

$$\frac{\mathrm{d}^3\sigma^h}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} = \frac{4\pi\alpha^2}{Q^2}\left[\frac{1+(1-y)^2}{2y}\mathcal{F}_T^h(x,z,Q^2) + \frac{1-y}{y}\mathcal{F}_L^h(x,z,Q^2)\right],$$

Structure function section

$$\mathcal{F}_i^h(x,z,Q^2) = \sum_{p,p'}\int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}}\int_z^1 \frac{\mathrm{d}\hat{z}}{\hat{z}} f_p\left(\frac{x}{\hat{x}},\mu_F^2\right) D_{p'}^h\left(\frac{z}{\hat{z}},\mu_A^2\right) \times \boxed{C^i_{p'/p}\left(\hat{x},\hat{z},Q^2,\mu_R^2,\mu_F^2,\mu_A^2\right)}, \quad i = T, L\,.$$

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Inclusive analytical hard coefficients obtained at NNLO

Goyal, Moch, Pathak, et al, 2312.17711
Bonino, Gehrmann, Stagnitto, 2401.16281

Neutrino-Nucleon Charged current scattering

Bonino, Gehrmann, Lochner et al, 2504.05376

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NNLO results f

Cross

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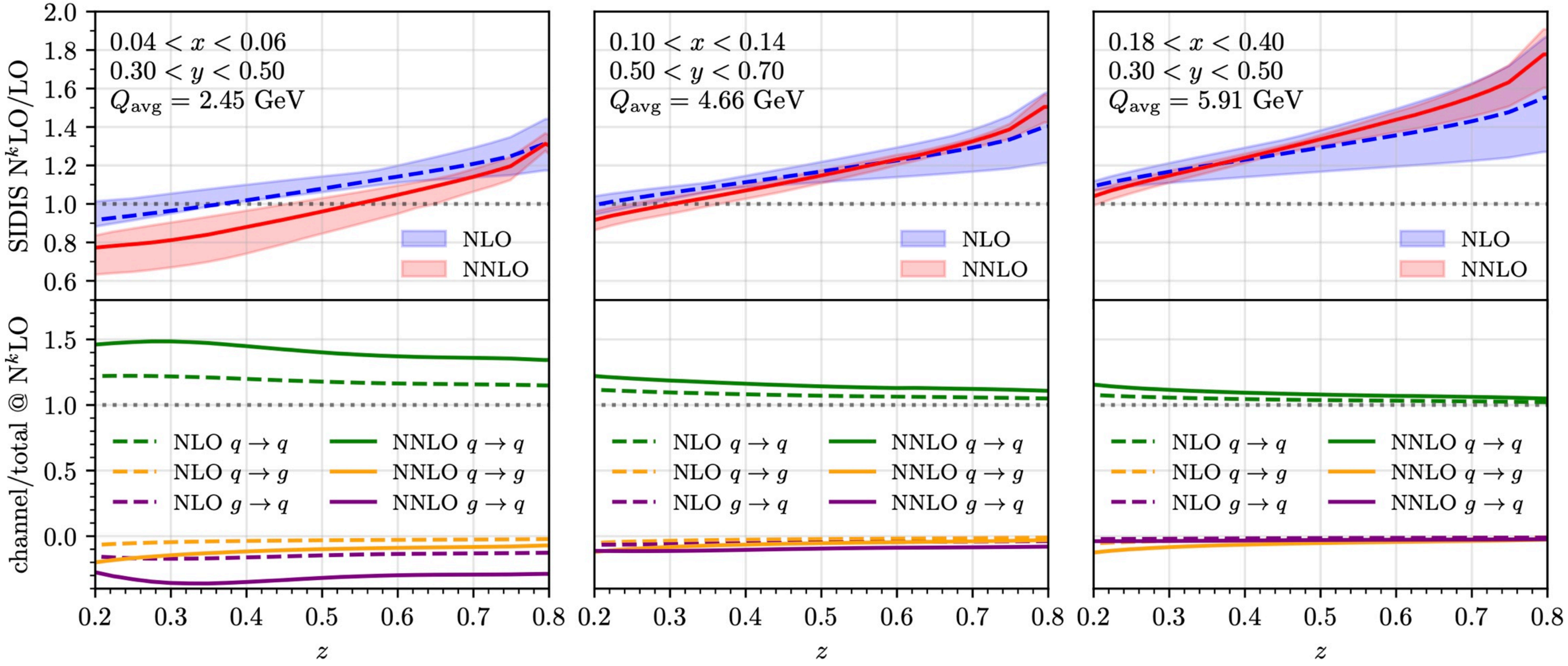

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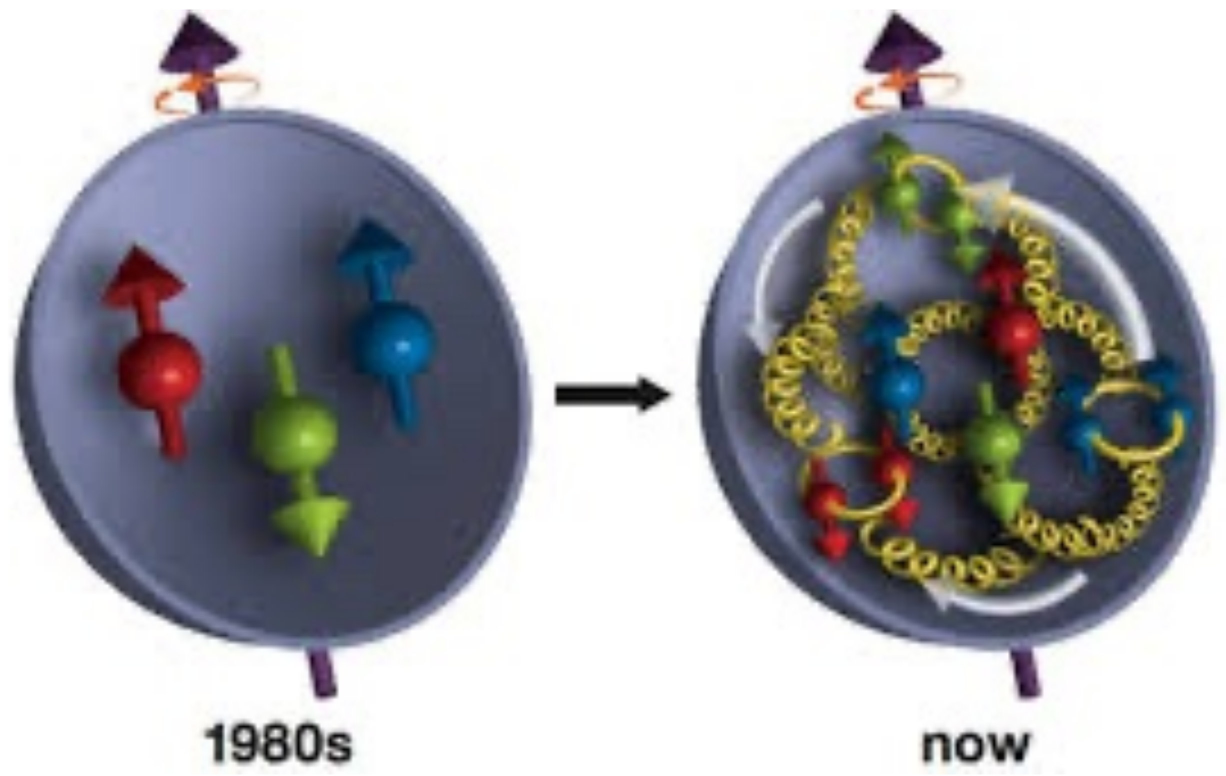
Introduction

Unraveling the Proton Spin Puzzle



$$f(x,Q^2) = f^+(x,Q^2) + f^-(x,Q^2) \qquad \Delta f(x,Q^2) = f^+(x,Q^2) - f^-(x,Q^2)$$

polarized PDFs has been fitted by many groups, such as

BDSSV24, JAMpol, NNPDFpol2.0 et al

$$2g_1^h(x,z,Q^2) = \sum_{p,p'} \int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}} \int_z^1 \frac{\mathrm{d}\hat{z}}{\hat{z}} \Delta f_p\left(\frac{x}{\hat{x}},\mu_F^2\right) \times D_{p'}^h\left(\frac{z}{\hat{z}},\mu_A^2\right) \Delta C_{p'p}\left(\hat{x},\hat{z},Q^2,\mu_R^2,\mu_F^2,\mu_A^2\right).$$

Semi-inclusive hadron production in longitudinally polarized DIS

NNLO analytical hard coefficients

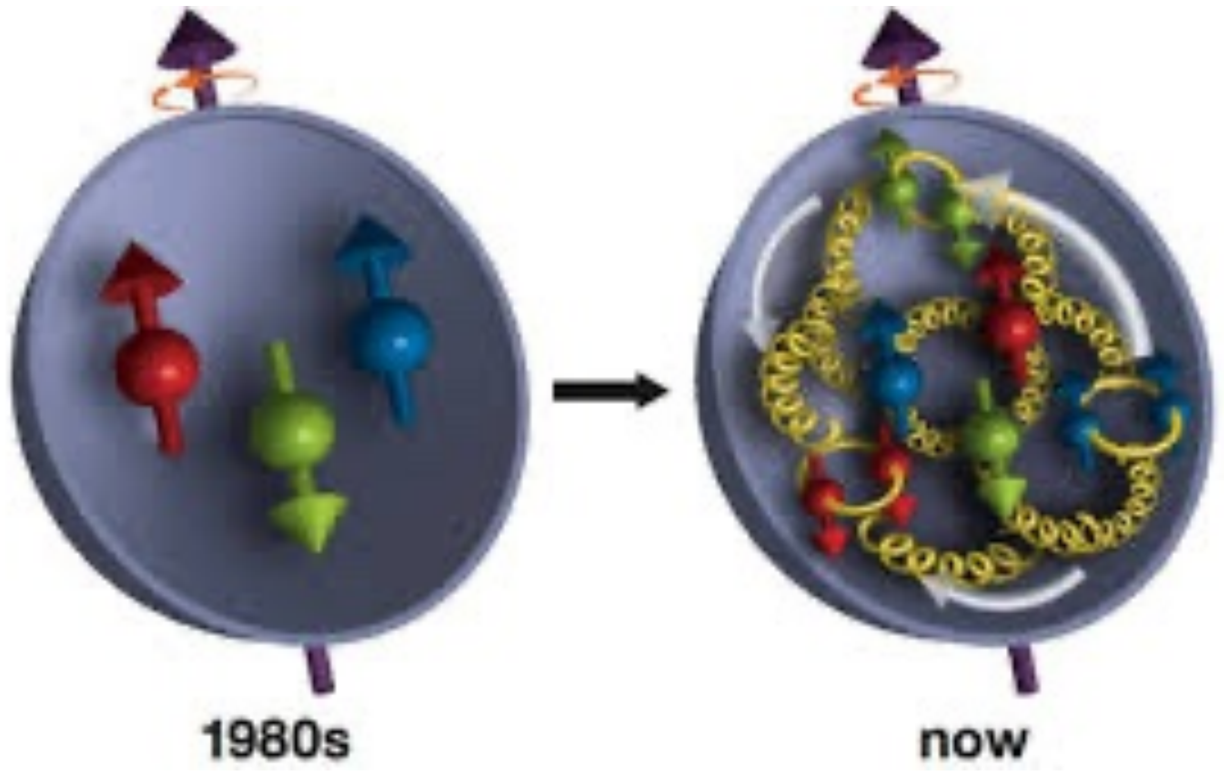
Bonino,Gehrmann, Lochner, Schonwald, 2404.08597, 2510.00100
Goyal, Lee, Moch, et al, 2404.09959, 2412.19309

NNLO QCD⊗QED corrections for unpolarized and polarized

Goyal, Lee, Moch, et al, 2510.18872

Introduction

Unraveling the Proton Spin P...



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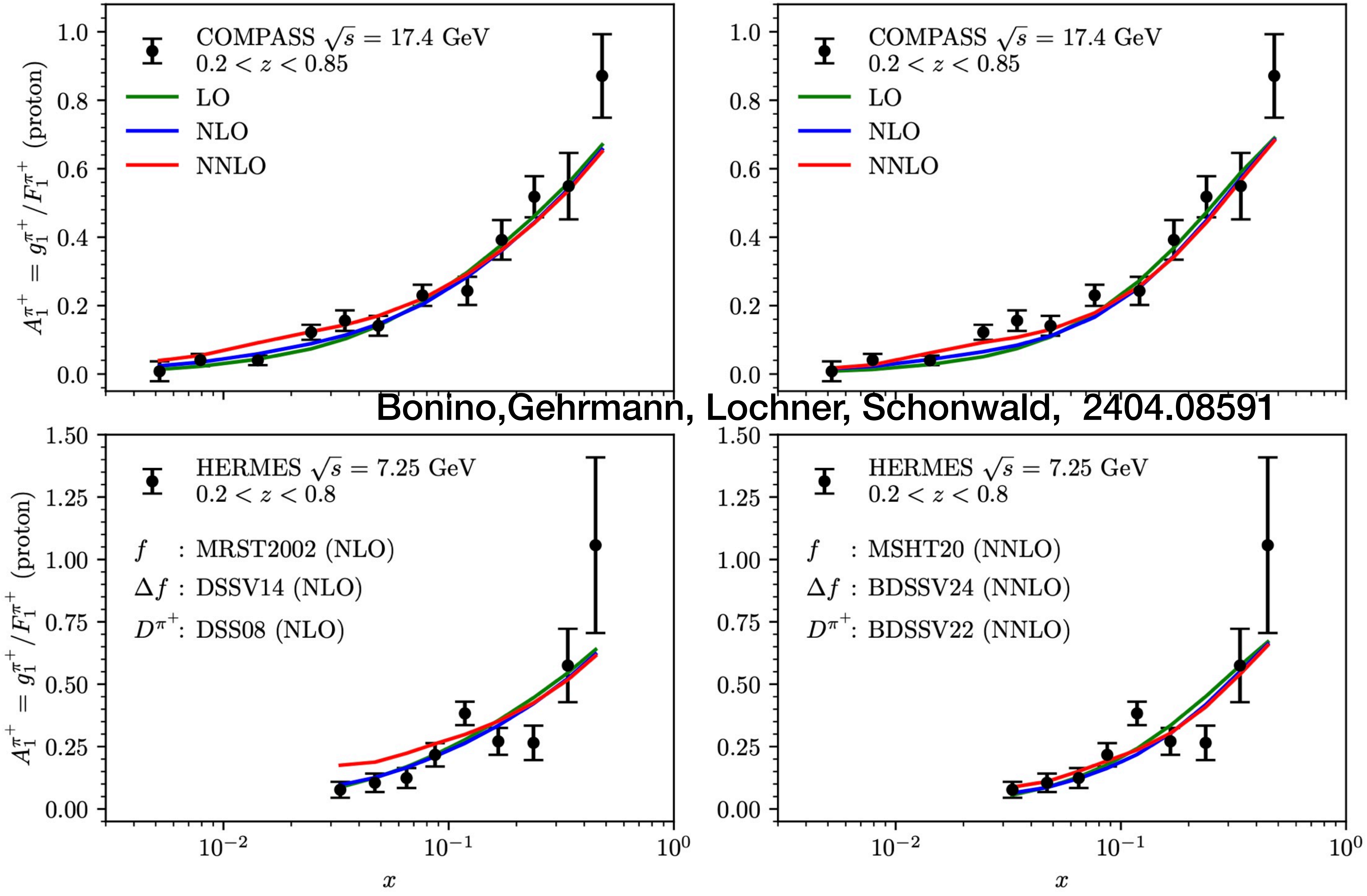

Bonino,Gehrmann, Lochner, Schonwald, 2404.08591

$\mu_A^2)$.

Semi-inclusive hadron producti...

NNLO analytical hard coefficier...

NNLO QCD⊗QED corrections f...

Outline

- Introduction
- **q_T subtraction for hadron production**
- Applications to N3LO QCD corrections to SIDIS
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q_T subtraction for hadron production

q_T subtraction method induced in 2007

Catani, Grazzini, hep-ph/0703012

$$\frac{d\sigma}{d\mathcal{O}} = \int_0^{q_{\mathrm{T,cut}}} dq_T \frac{d\sigma}{dq_T d\mathcal{O}} + \int_{q_{\mathrm{T,cut}}}^{q_{\mathrm{T,max}}} dq_T \frac{d\sigma}{dq_T d\mathcal{O}}$$

For example, $p_1 + p_2 \to \mathrm{Higgs} + X$

cross section with $0 \sim q_{T,h} < q_{\mathrm{cut}}$

$$d\sigma^F_{(N)NLO} = \mathcal{H}^F_{(N)NLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+\mathrm{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO}\right]$$

Unresolved: obtained from factorized cross section as convolutions TMD beam functions, soft functions, hard functions.

cross section with $q_{T,h} > q_{\mathrm{cut}}$

$$d\sigma^F_{(N)NLO}\big|_{q_T \neq 0} = d\sigma^{F+\mathrm{jets}}_{(N)LO}$$

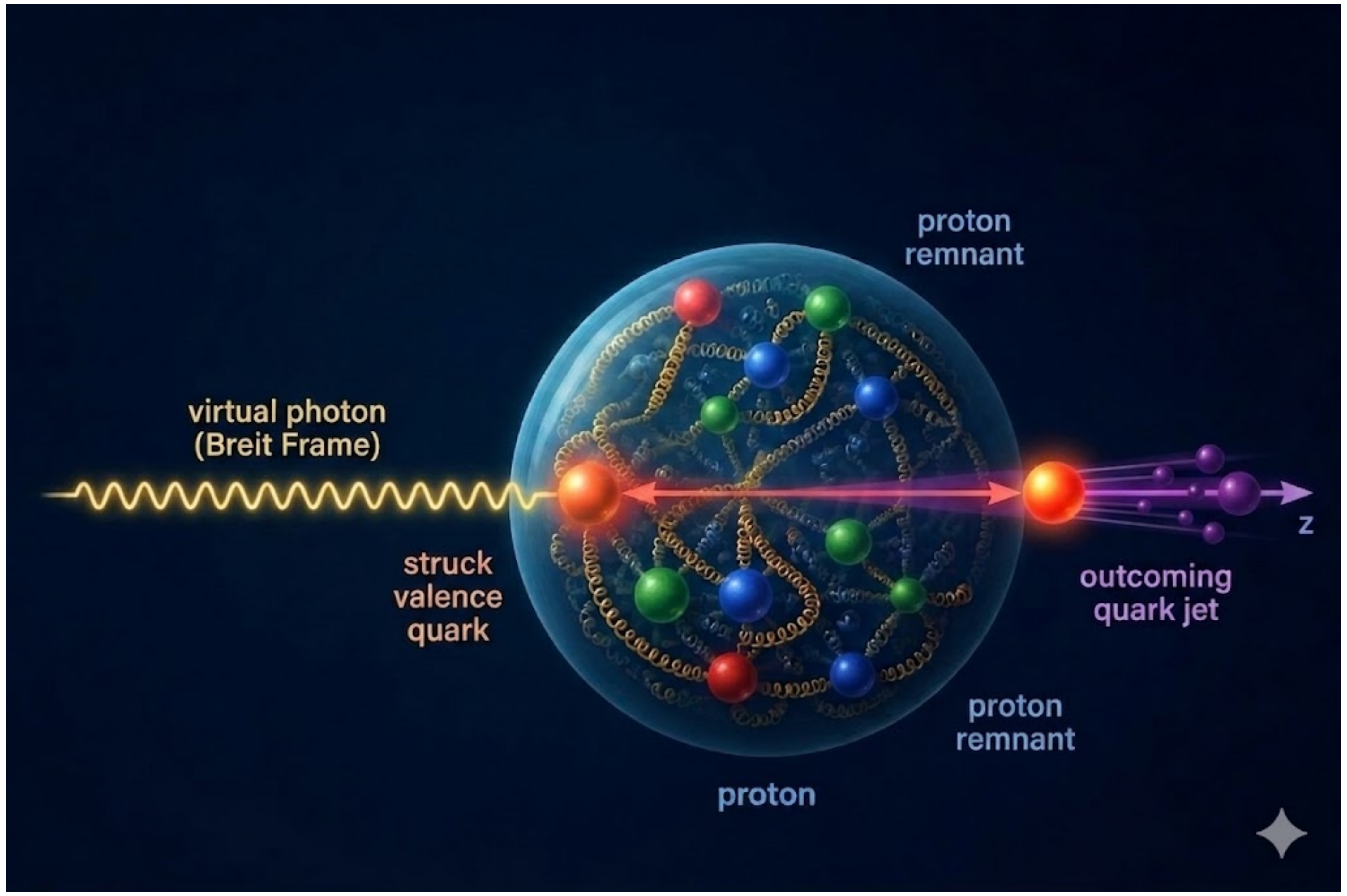
Resolved: NNLO corrections to h+X requires NLO corrections to Higgs +jet+X

Only successfully applied to colorless or massive final state

qT subtraction for hadron production

q_T subtraction method extension to hadron production

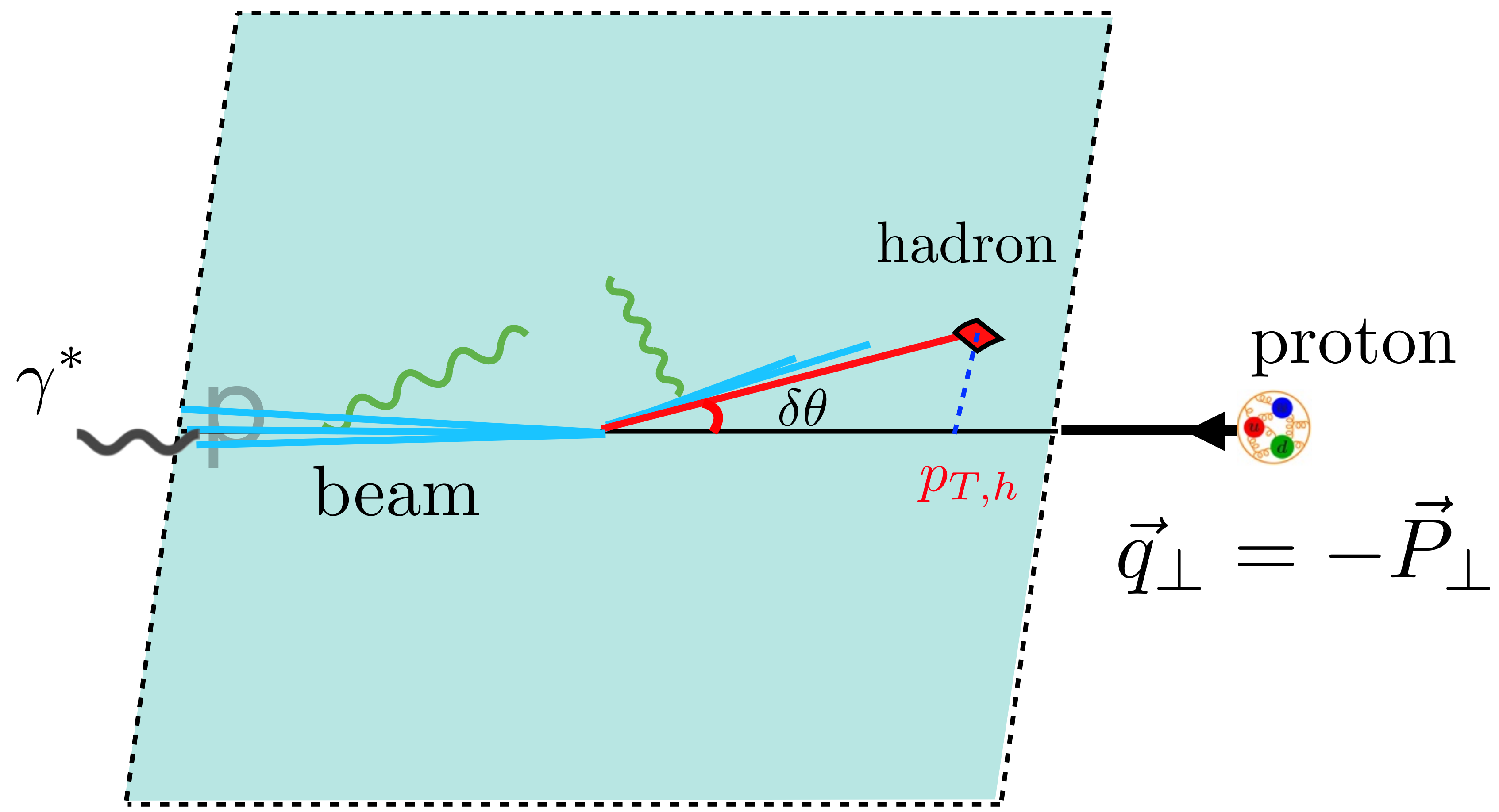
$$\sum_{p,p'}\int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}}\int_z^1 \frac{\mathrm{d}\hat{z}}{\hat{z}} f_p\left(\frac{x}{\hat{x}},\mu_F^2\right) D_{p'}^h\left(\frac{z}{\hat{z}},\mu_A^2\right) \times C^i_{p'/p}\left(\hat{x},\hat{z},Q^2,\mu_R^2,\mu_F^2,\mu_A^2\right)$$



- ❑ Hadron q_T can be not used as resolution scale
- ❑ To deal with IR singularities, we need to design slicing using the information at parton level
- ❑ In Breit frame, if we cross the final state quark to initial state, it becomes Drell-Yan processes
- ❑ A natural choice, is the transverse momentum of the virtual photon in the hadron frame
- ❑ Equivalently, the final state parton's q_T can be used

Gao, H.T.L., Zhu, Zhu, 2602.06364

q_T subtraction for hadron production



$\vec{q}_\perp = -\vec{P}_{\perp,h}/z$

$\delta\theta \propto q_T$

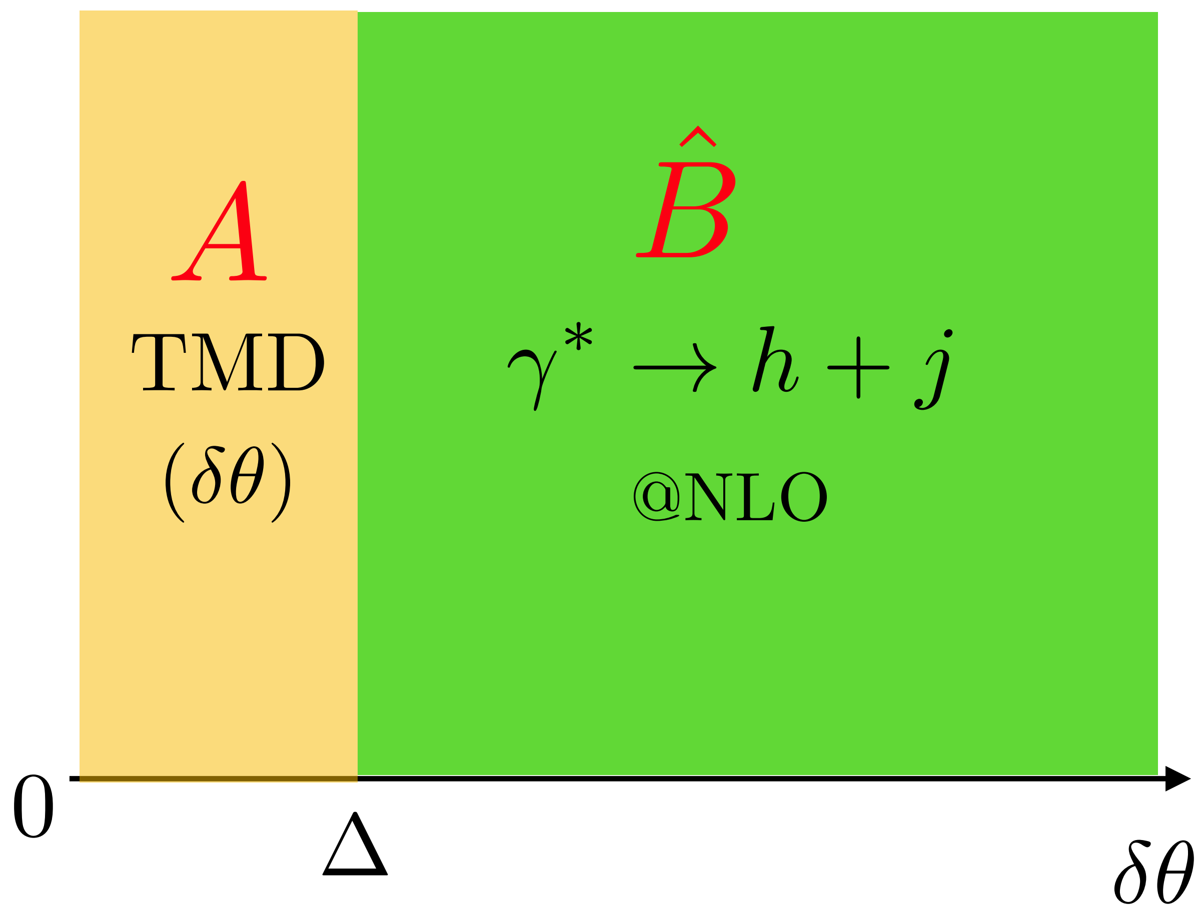
$$\frac{d\sigma}{d\mathcal{O}} = \int_0^{q_{\mathrm{T,cut}}} dq_T \frac{d\sigma}{dq_T d\mathcal{O}} + \int_{q_{\mathrm{T,cut}}}^{q_{\mathrm{T,max}}} dq_T \frac{d\sigma}{dq_T d\mathcal{O}}$$

A $\hat{B}$

TMD factorization

all components ready up to three loops

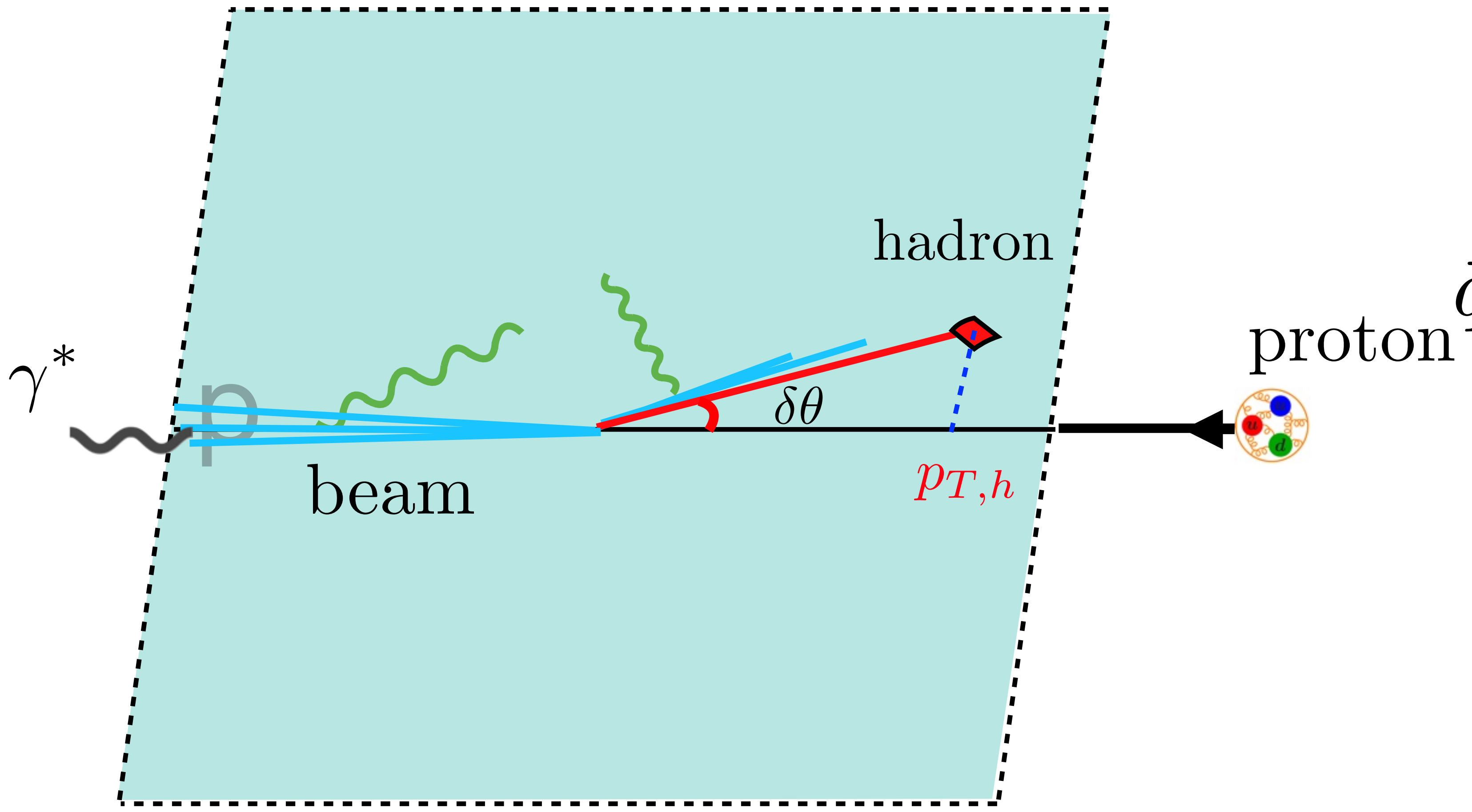
fixed order calculation of h+jet production with one order lower



A new method was proposed to calculation fully differential SIDIS cross sections

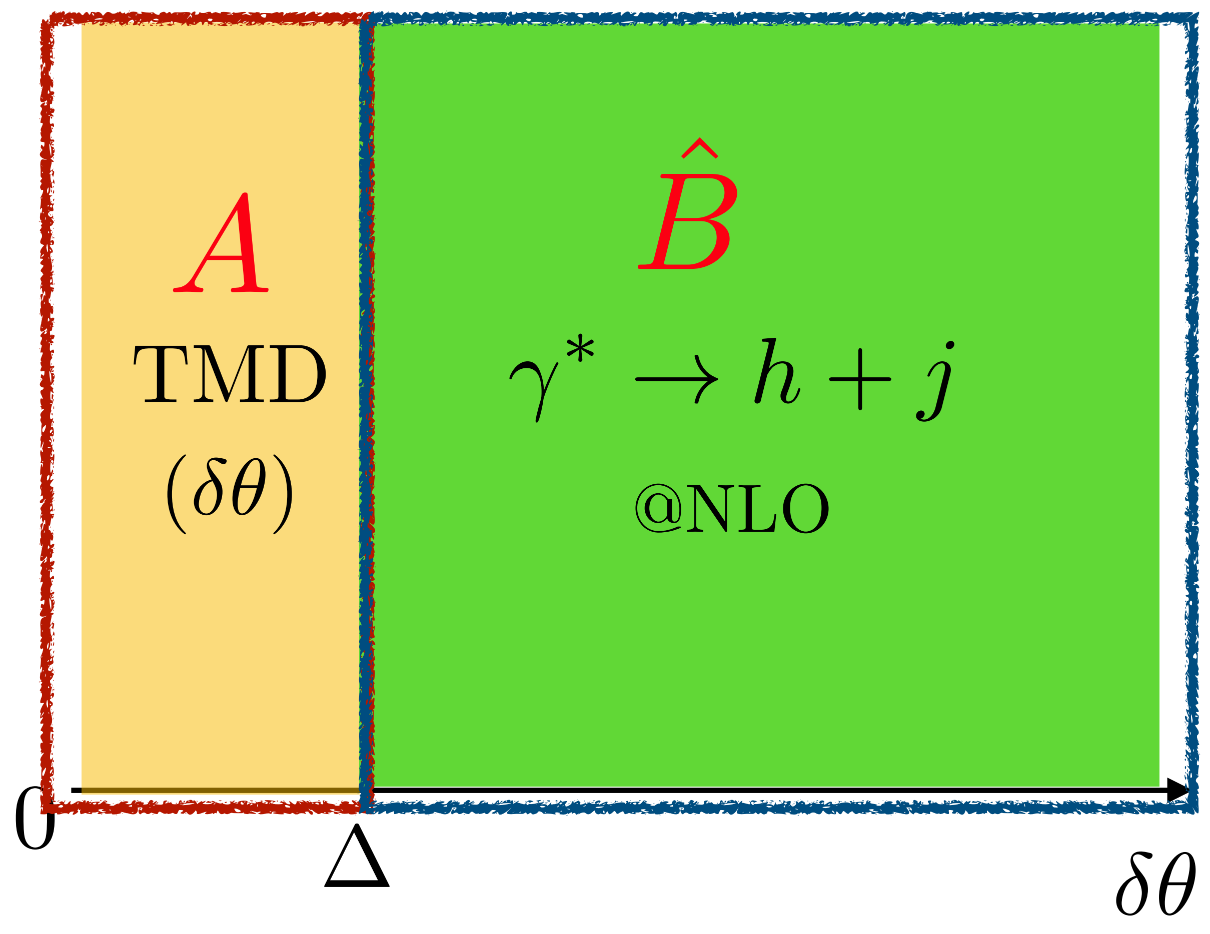
Gao, H.T.L., Zhu, Zhu, 2602.06364

q_T subtraction for hadron production



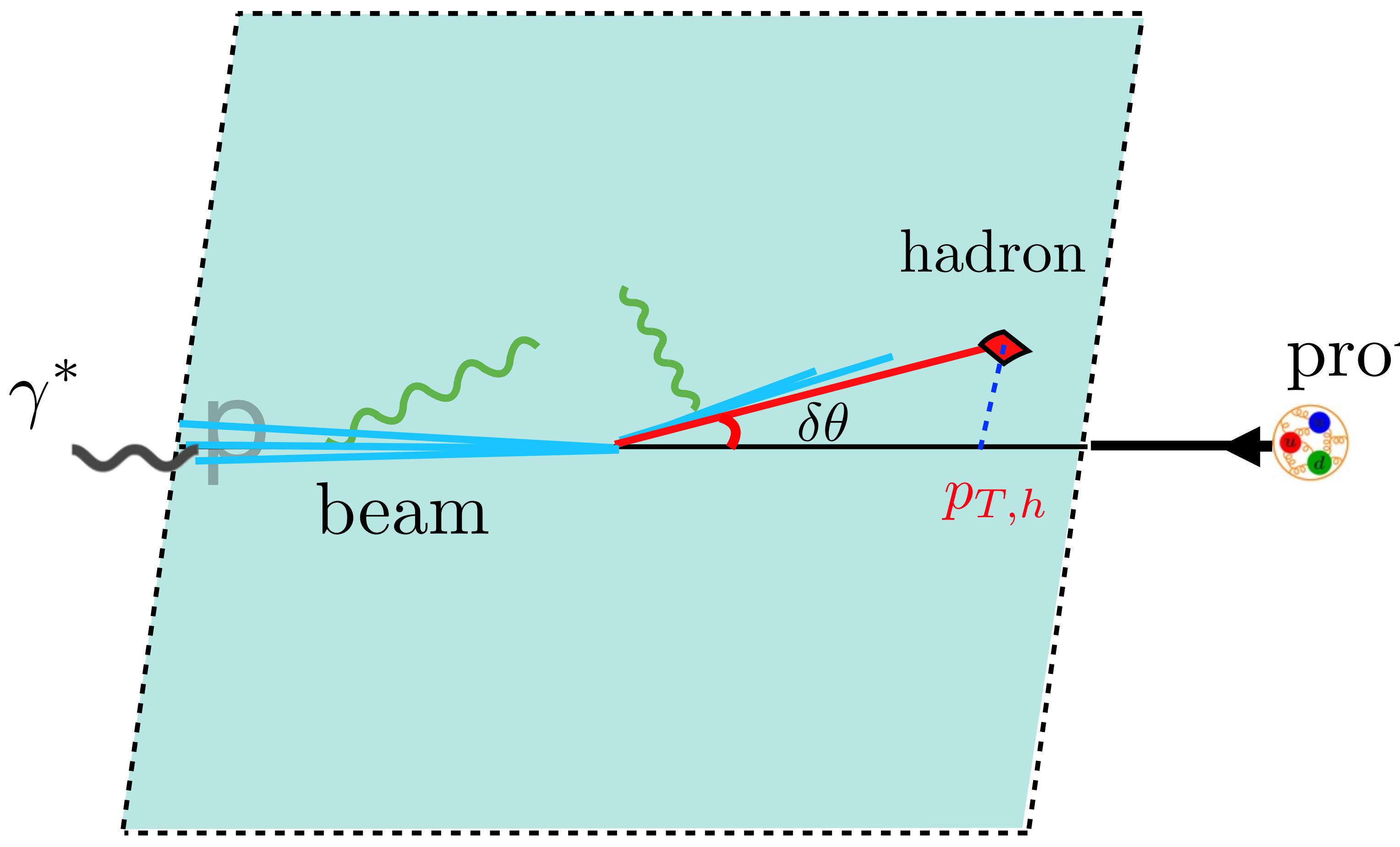
$$\vec{q}_\perp = -\vec{P}_{\perp,h}/z$$

$$\frac{d\sigma^A}{dx\,dy\,dz\,d^2\vec{P}_{hT}} \propto \int \frac{d^2\vec{b}_\perp}{4\pi^2} e^{-i\vec{P}_{hT}\cdot\vec{b}_\perp/z} \sum_i H_{ei\to ei}(Q) \times \mathcal{B}_{i/p}(x,\vec{b}_\perp)\,\mathcal{D}_{h/i}(z,\vec{b}_\perp/z)\,S_{qq}(\vec{b}_\perp)\,[1+\mathcal{O}(\Delta)]$$



SIDIS@NNLO

q_T subtraction for hadron production



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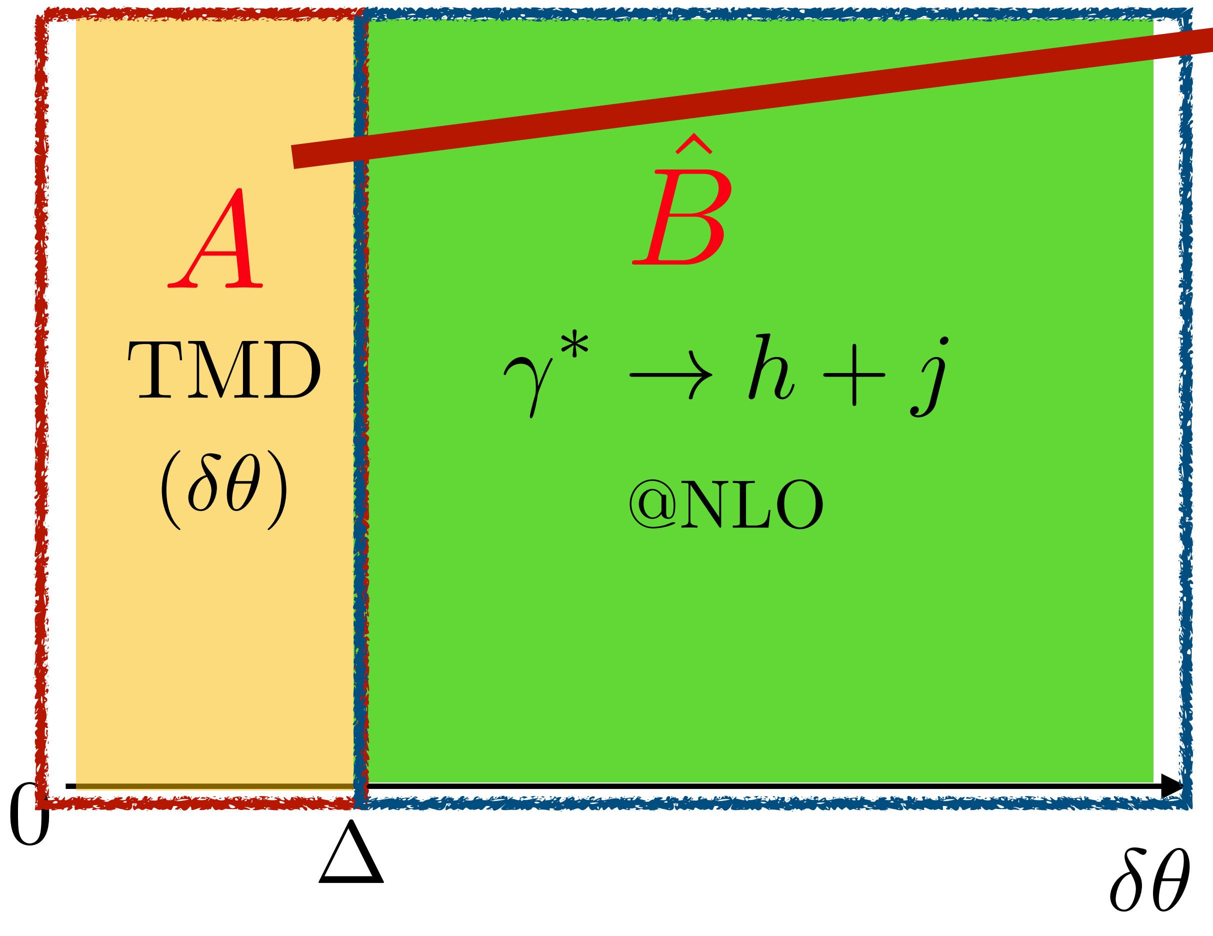
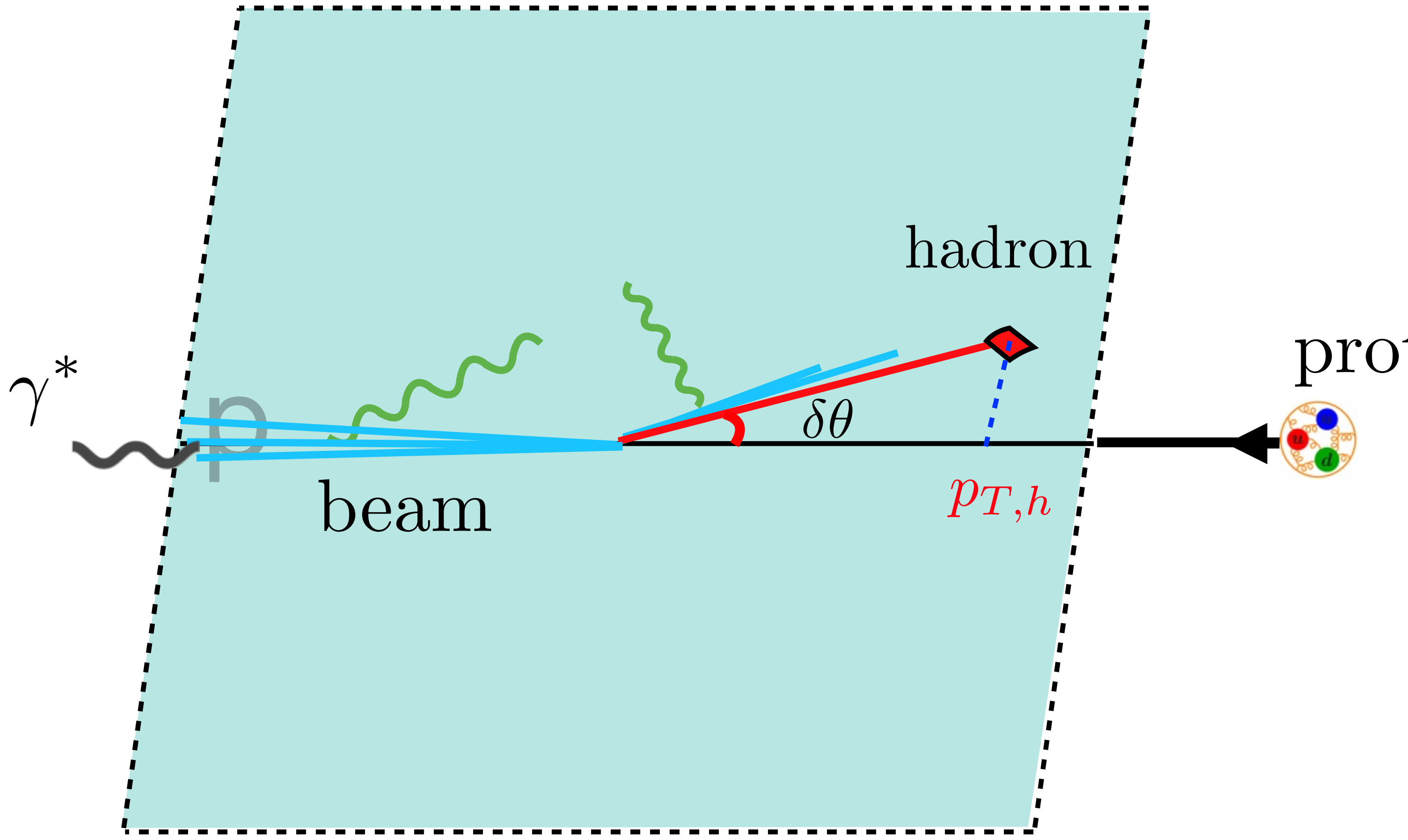


SIDIS@NNLO

- ❑ Hard function
  - Baikov,Chetyrkin,Smirnov,Smirnov,Steinhauser, 0902.3519
  - Lee, Smirnov, Smirnov, 1001.2887
  - Gehrmann, Glover, Huber, Ikizlerli, Studerus, 1004.3653
- ❑ TMD beam function
  - Luo, Yang, Zhu, Zhu, 1912.05778
  - Luo, Yang, Zhu, Zhu, 2012.03256
  - Ebert, Mistlberger, Vita, 2006.05329
- ❑ Soft function
  - Li, Zhu, 1604.01404
  - Zhu, 2009.08919

q_T subtraction for hadron production



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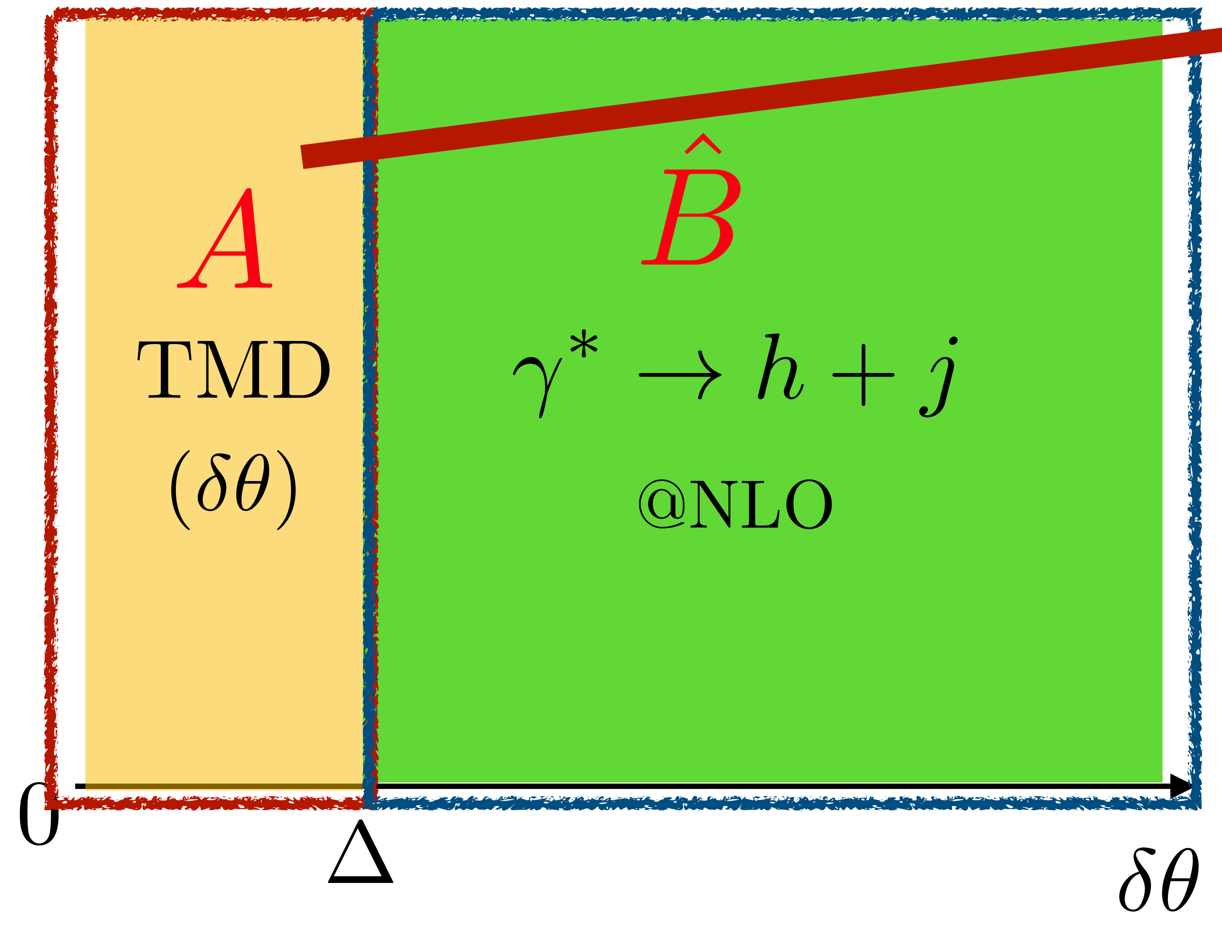

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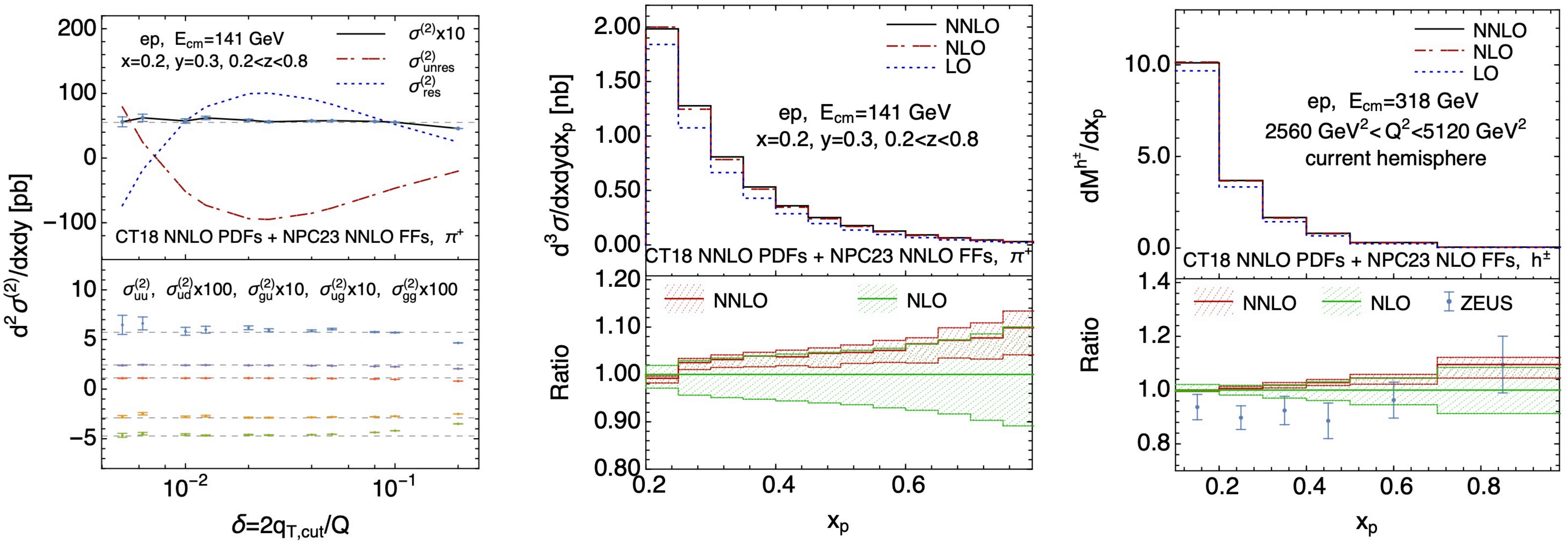
$\hat{B}$: An in-house code base on FMNLO method

q_T subtraction for hadron production



Gao, H.T.L., Zhu, Zhu, 2602.06364

Outline

- Introduction
- q_T subtraction for hadron production
- **Applications to N3LO QCD corrections to SIDIS**
- Summary

Applications to N3LO

q_T slicing for SIDIS at N3LO

$$\frac{d\sigma}{d\mathcal{O}} = \boxed{\int_0^{q_{\mathrm{T,cut}}} dq_T \frac{d\sigma}{dq_T d\mathcal{O}}} + \boxed{\int_{q_{\mathrm{T,cut}}}^{q_{\mathrm{T,max}}} dq_T \frac{d\sigma}{dq_T d\mathcal{O}}}$$

Unresolved @ 3 loops

Hard ⊗ TMDPDF ⊗ TMDFF

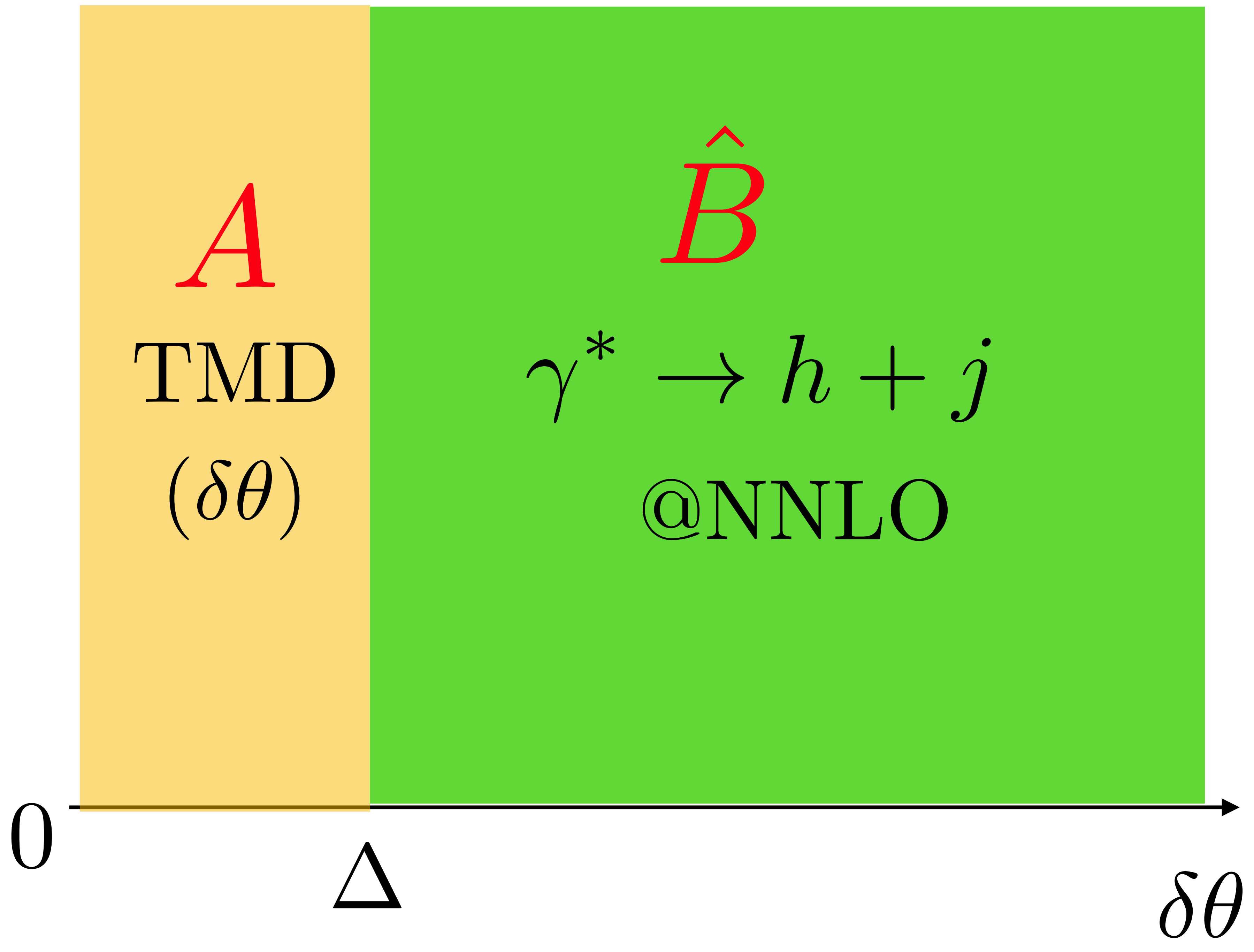
Part A
Ready

Resolved @ NNLO

NNLO corrections to harton + Jet

Dong, Fang, Gao, H.T.L., Zhu, 2602.22972

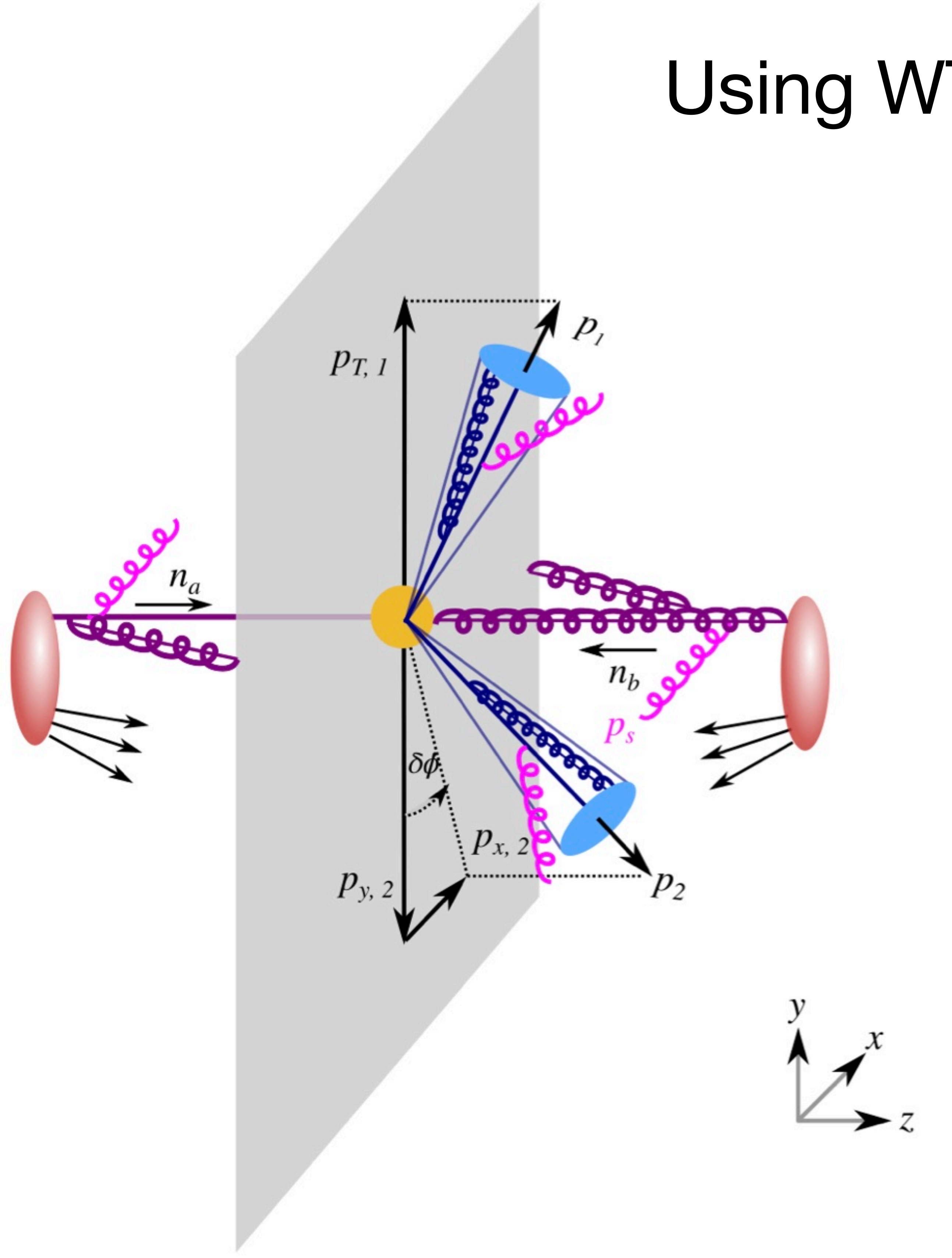
see Shen Fang's talk on Tue.



qT subtraction for hadron production

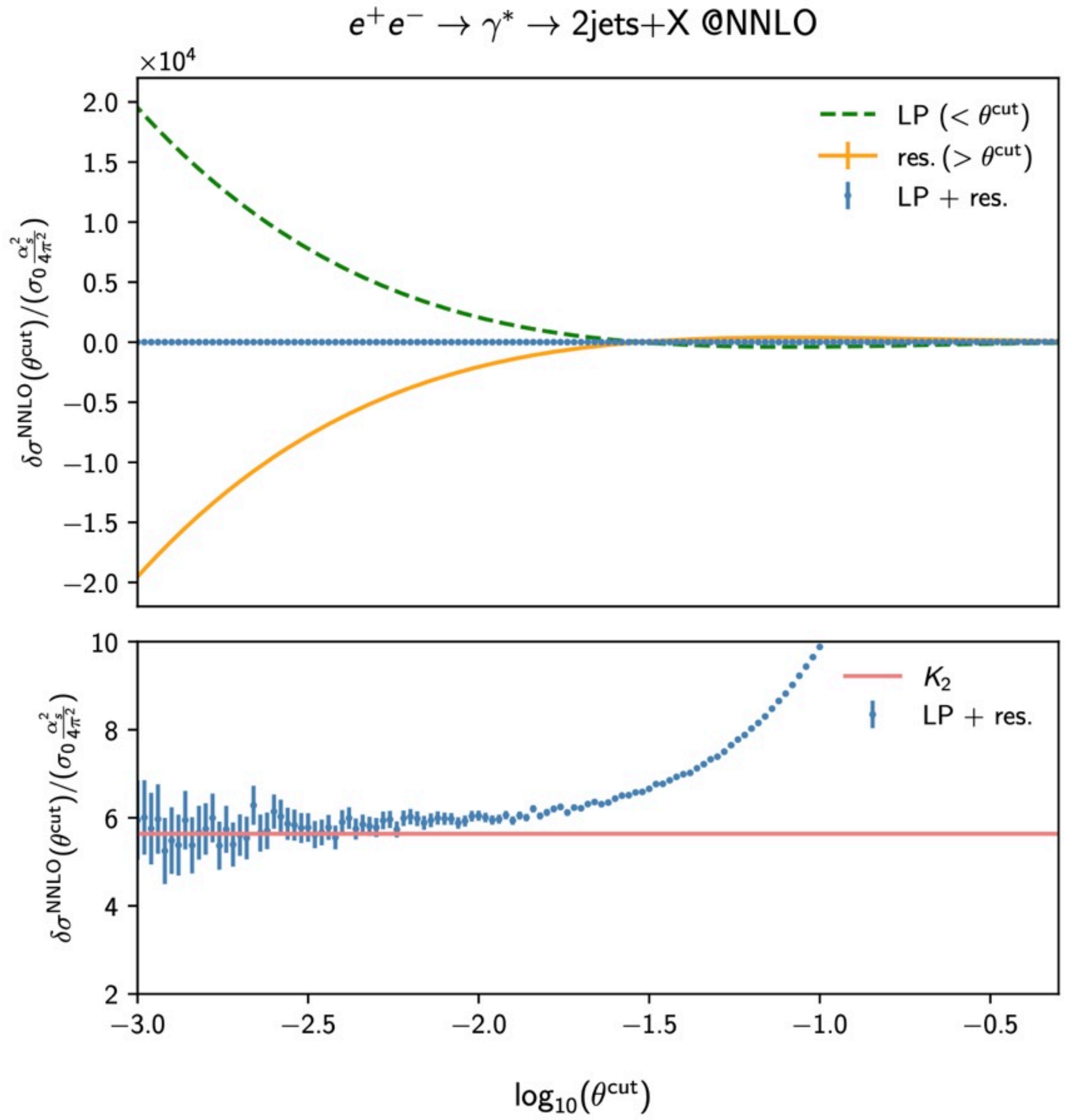
A extension of qT subtraction method to multijet production **Fu, Rahn, Shao, Waalewijn, Wu, 2412.05358**

See Rong-Jun Fu's talk on Tue.



Using WTA jet, they proposed subtraction scheme

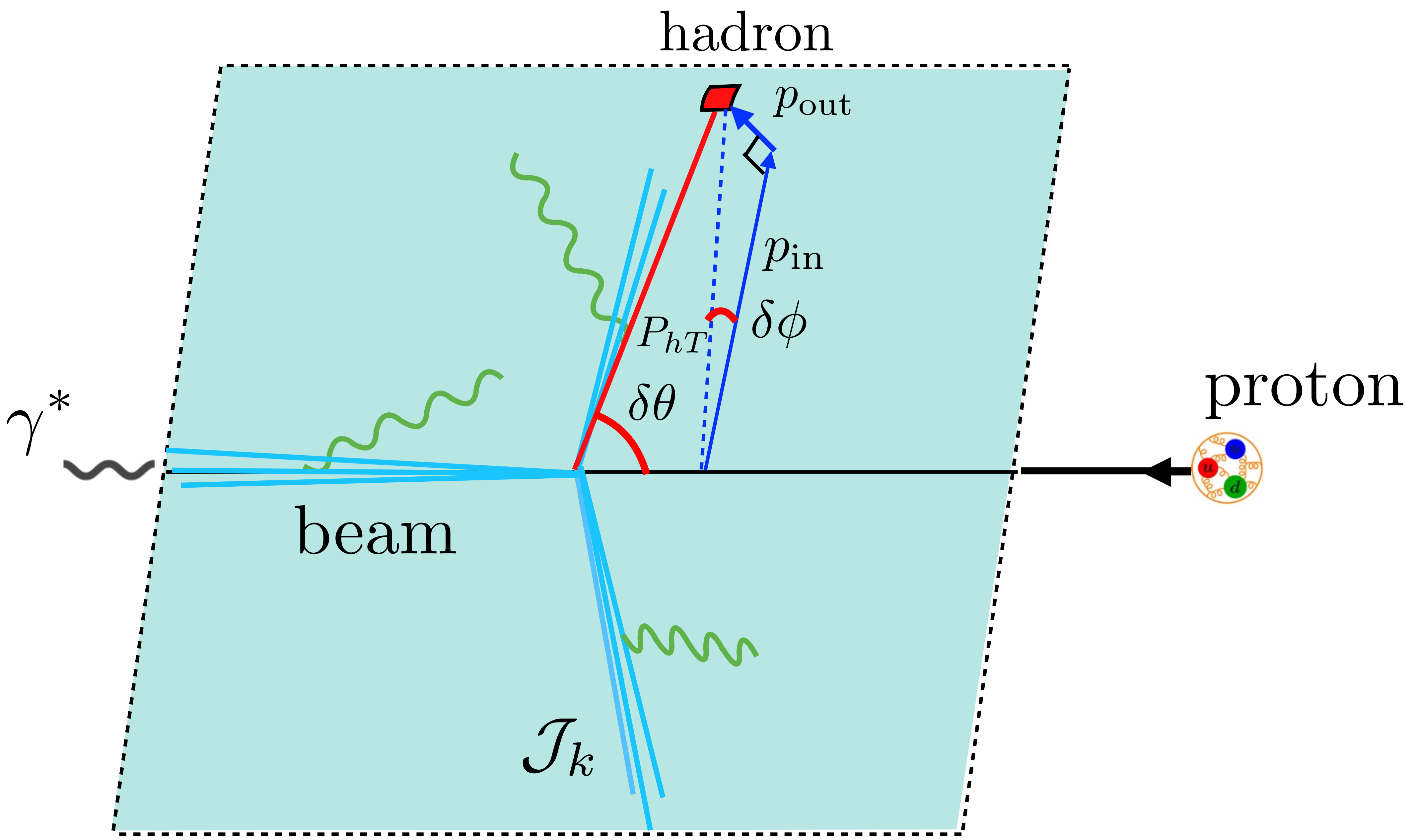
$$\frac{\mathrm{d}\sigma_{\mathrm{LP}}}{\mathrm{d}p_{T,1}\,\mathrm{d}\eta_1\,\mathrm{d}\eta_2\,\mathrm{d}q_T} = q_T\int\frac{\mathrm{d}^2\vec{b}_T}{2\pi}\,J_0(q_T|\vec{b}_T|)\sum_{i,j,k,\ell} B_i(x_a,\vec{b}_T)\,B_j(x_b,\vec{b}_T)\,\mathscr{J}_k(b_x) \times \mathscr{J}_\ell(b_x)\,\mathrm{tr}\big[\hat{\mathcal{H}}_{ij\to k\ell}(p_{T,1},\eta_1-\eta_2)\,\hat{S}_{ijk\ell}(\vec{b}_T,\eta_1,\eta_2,R)\big].$$



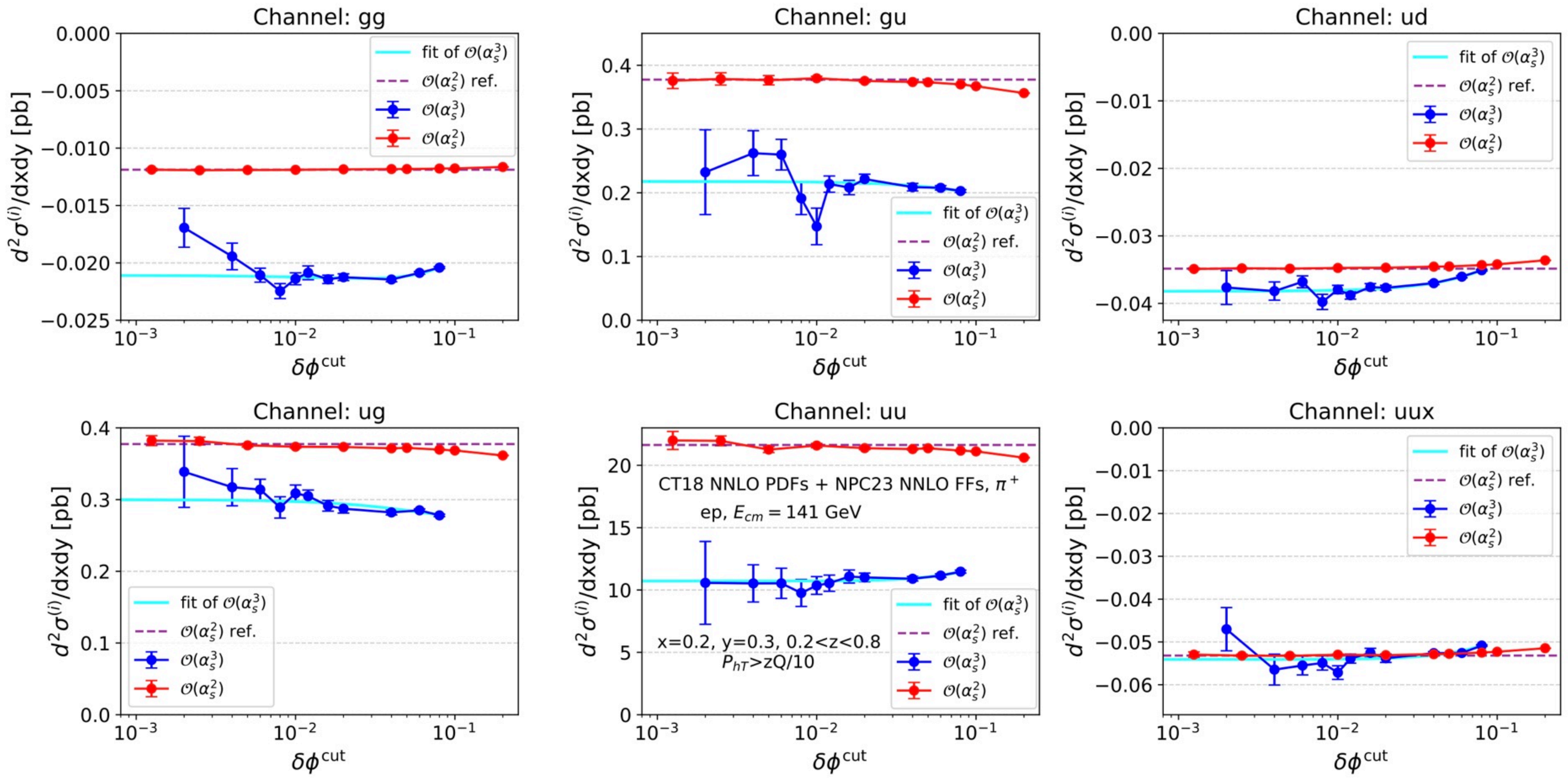
Two loop quark jet function was extracted

Fang, Gao, H.T.L., Shao, 2409.09248

Applications to N3LO



$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{O}} = \int_0^{\delta\phi^{\rm cut}} \mathrm{d}\delta\phi \frac{\mathrm{d}\sigma}{\mathrm{d}\delta\phi\, \mathrm{d}\mathcal{O}} + \int_{\delta\phi^{\rm cut}}^{\delta\phi^{\rm max}} \mathrm{d}\delta\phi \frac{\mathrm{d}\sigma}{\mathrm{d}\delta\phi\, \mathrm{d}\mathcal{O}}$$

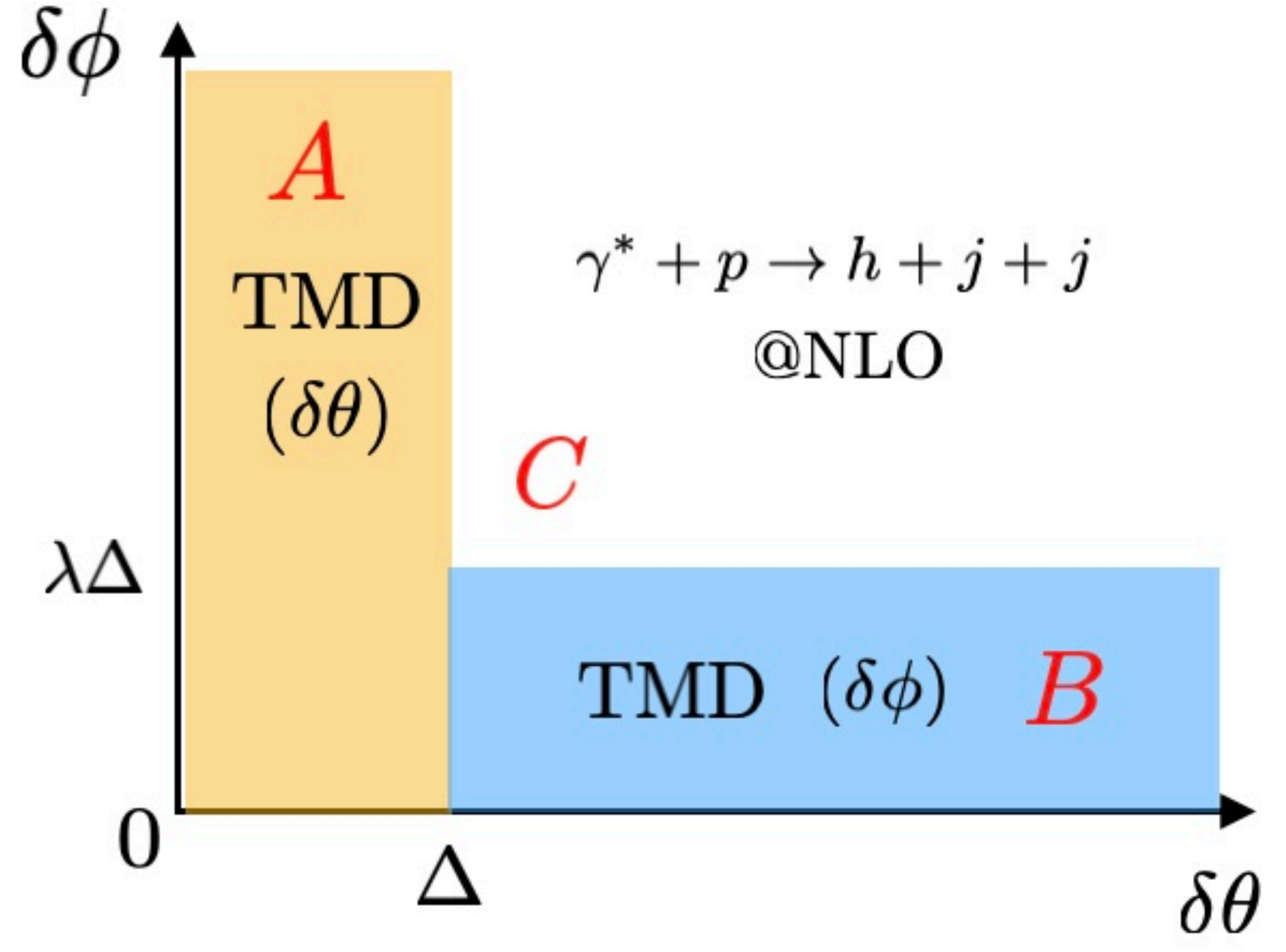


Demonstrates numerical stability and cutoff independence across all channels.

Dong, Fang, Gao, H.T.L., Zhu, 2602.22972, also see Shen Feng' talk

Applications to N3LO

A new method of **two-dimensional transverse momentum subtraction** for SIDIS



- ☐ subtraction with hadron q_T
- ☐ subtraction with azimuthal angle correction between hadron and jet

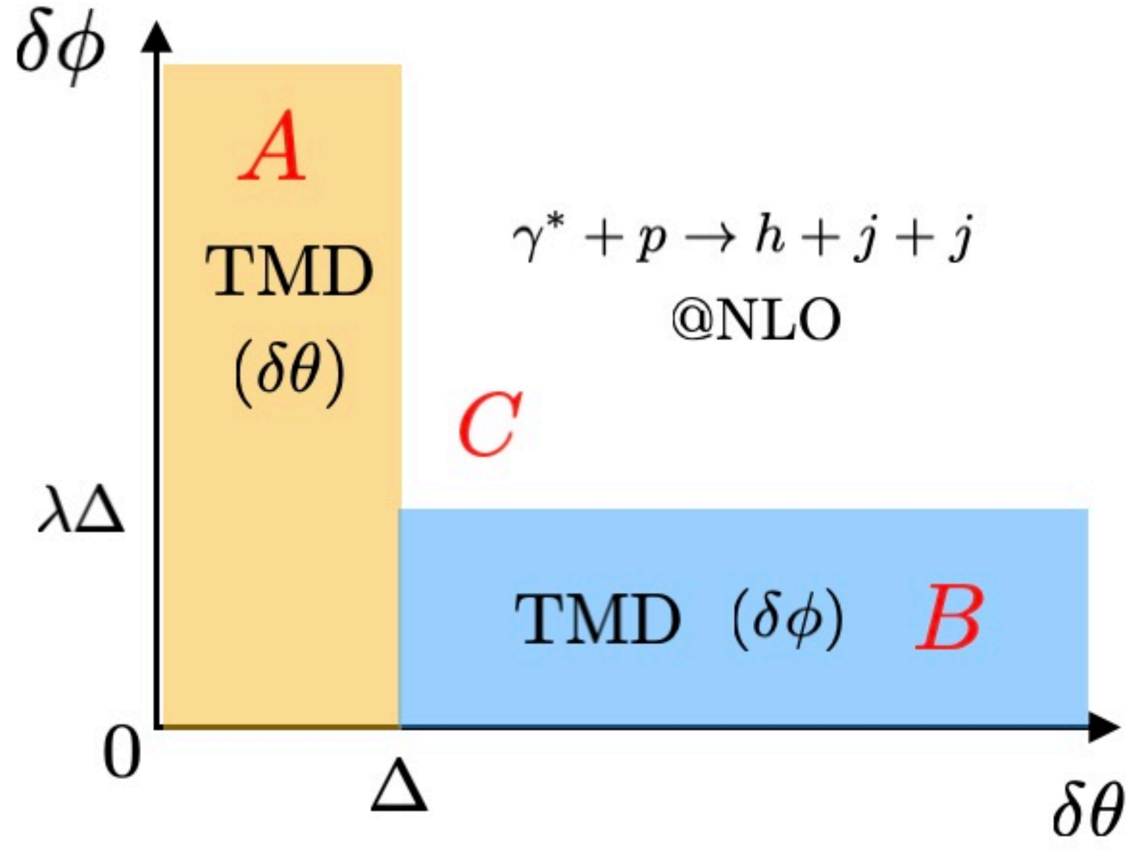
Dong, Fang, Gao, H.T.L., Shao, Zhu, Zhu, 2603.29673

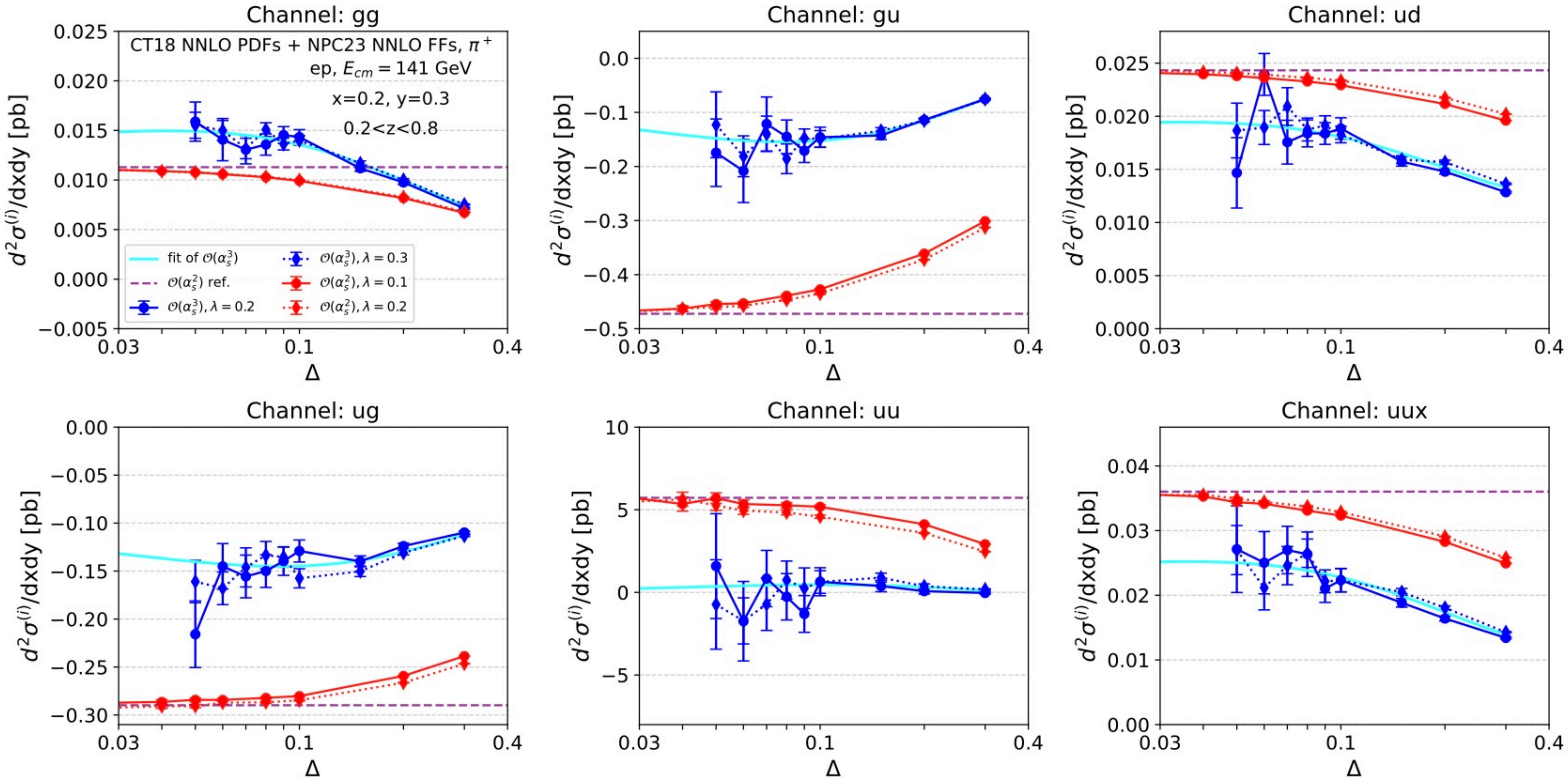
$$\frac{d\sigma}{d\mathcal{O}} = \underbrace{\int_0^{\Delta} d\delta\theta \frac{d\sigma^A}{d\delta\theta\, d\mathcal{O}}}_{\text{A}} + \int_{\Delta}^{\delta\theta^{\max}} d\delta\theta \times \left(\underbrace{\int_0^{\lambda\Delta} d\delta\phi \frac{d\sigma^B}{d\delta\theta\, d\delta\phi\, d\mathcal{O}}}_{B} + \underbrace{\int_{\lambda\Delta}^{\delta\phi^{\max}} d\delta\phi \frac{d\sigma^C}{d\delta\theta\, d\delta\phi\, d\mathcal{O}}}_{\text{C}} \right)$$

- ☑ **Manages the complex pattern of overlapping and nested infrared and collinear singularities that arise in N3LO calculations**
- ☑ **Dramatically reduces computational and analytic complexity**
- ☑ **Preserves exact final-state kinematics to allow arbitrary fiducial cuts**

Applications to N3LO

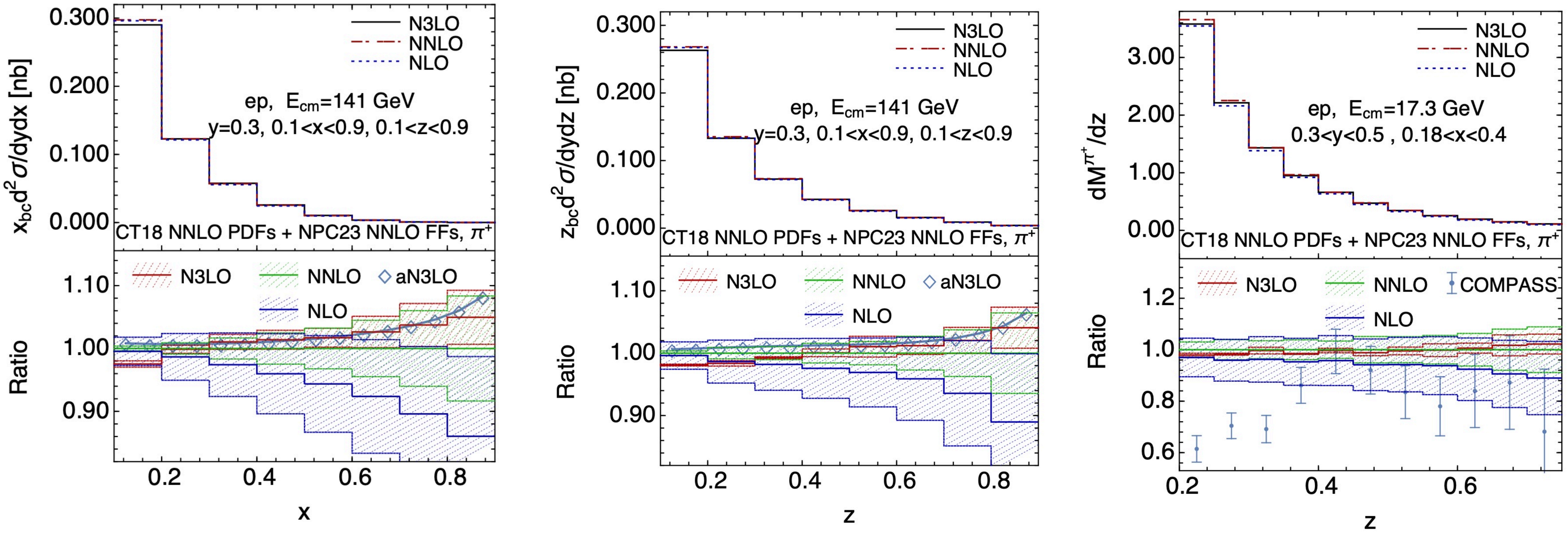
Validations to h+X @ N3LO





Numerical Stability and Cutoff Independence at N3LO: power corrections relatively small

Applications to N3LO



- A clear, systematic reduction in theoretical scale uncertainties is observed
- The N3LO corrections introduce a modest but visible shape adjustment

Applications to N3LO

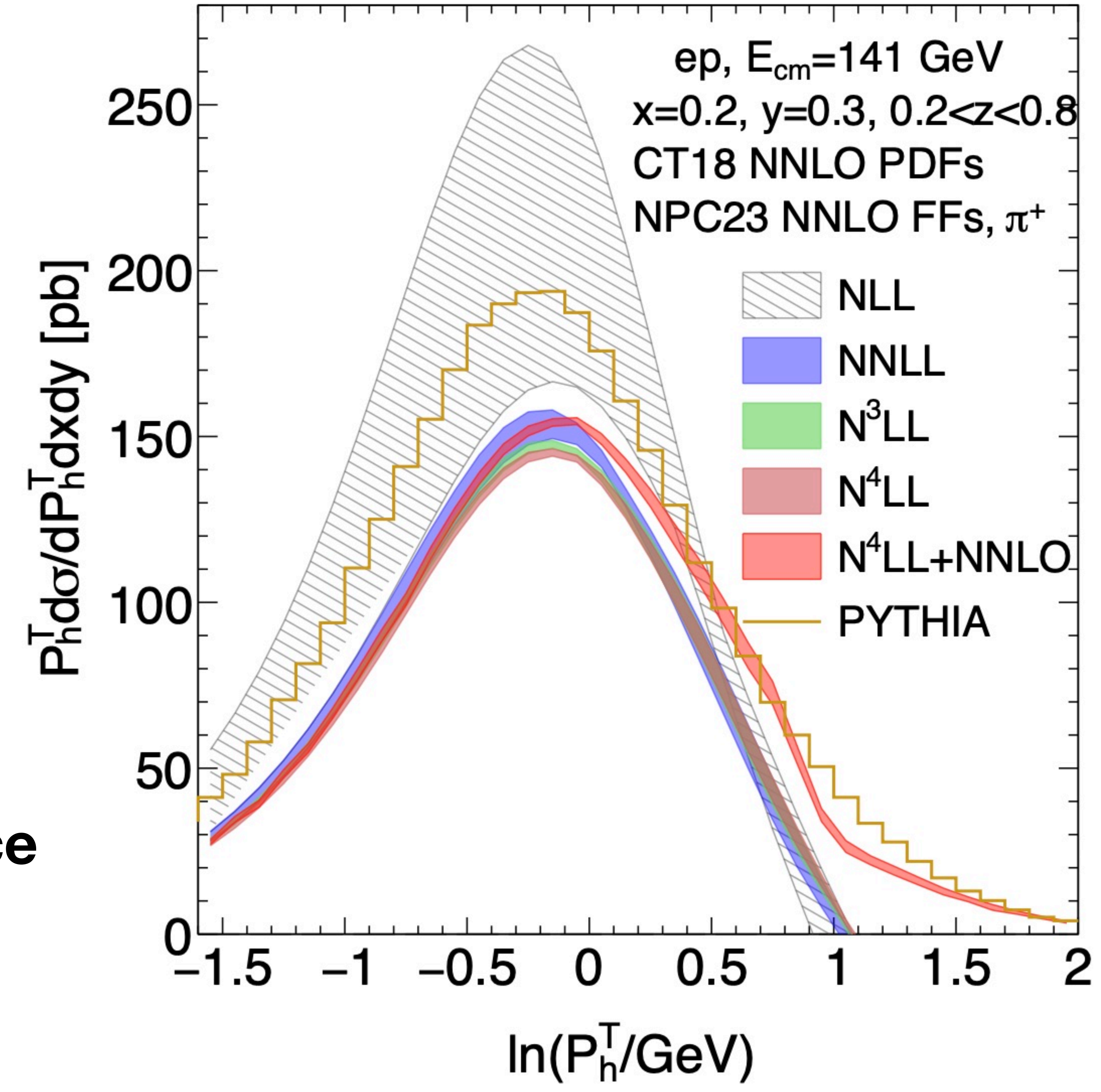
N4LL resummation

$$\frac{1}{\sigma_0}\frac{d\sigma_{\ell+N\to\ell'+h+X}}{d^2\vec{q}_\perp dx dy dz} \simeq \sum_q H_q(Q^2,\mu)\times\int\frac{d^2\vec{b}_\perp}{(2\pi)^2}e^{i\vec{b}_\perp\cdot\vec{q}_\perp} f_1^q(x,b_\perp,\xi_0^n,\mu_0^n)\times D_1^q\left(z,\frac{b_\perp}{z},\xi_0^{\bar{n}},\mu_0^{\bar{n}}\right)$$
$$\times\prod_i e^{-2K^i_{\rm cusp}(\mu_0^i,\mu)+A_H^i(\mu_0^i,\mu)}\times\left(\frac{\xi^i}{{\mu_0^i}^2}\right)^{A^i_{\rm cusp}(\mu_0^i,\mu)}\times\left(\frac{\sqrt{\xi^i}}{\sqrt{\xi_0^i}}\right)^{K^i(b_\perp,\mu_0^i)}$$

Matching for fixed order by

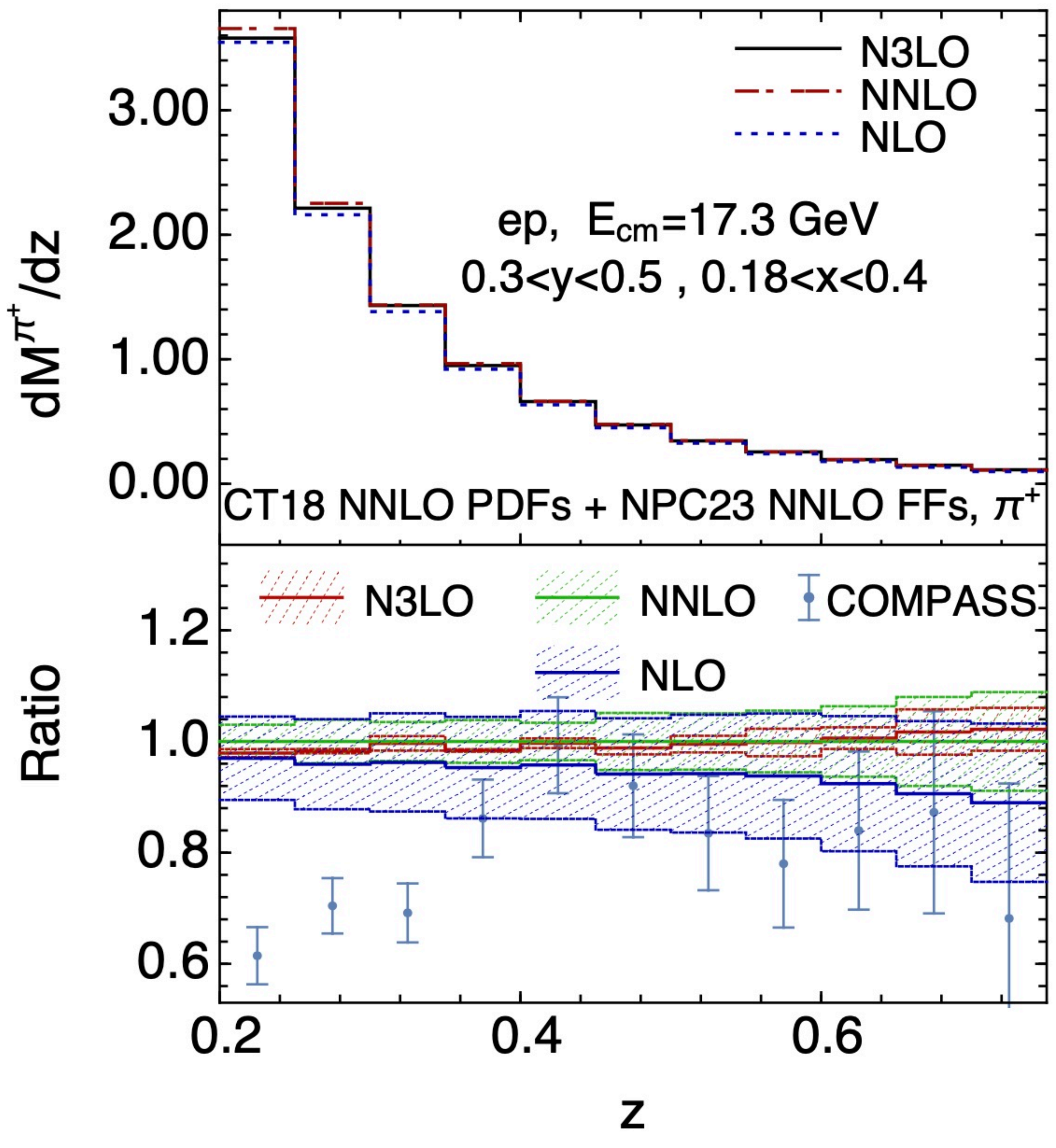
$$d\sigma_{\rm NNLO+N^4LL} = (1-f)\times d\sigma_{\rm NNLO} + f\times d\sigma_{\rm N^4LL+NS}$$

- ❑ NLL results agree with PYTHIA
- ❑ The predictions demonstrate excellent **perturbative convergence**
- ❑ Fixed order matching are important even in the Sudakov region



Summary

- SIDIS provides unprecedented access to the 3D tomographic structure of the nucleon
- We present **a new method** for calculating fully differential SIDIS cross sections at higher orders in QCD.
- We propose a novel method of **two-dimensional transverse momentum subtraction** for calculations of identified hadron production.
- This enabled the first fully differential calculations of SIDIS at both **NNLO** and **NNNLO** in QCD.
- Provide new theoretical tools for the study of the three-dimensional internal structure of nucleons



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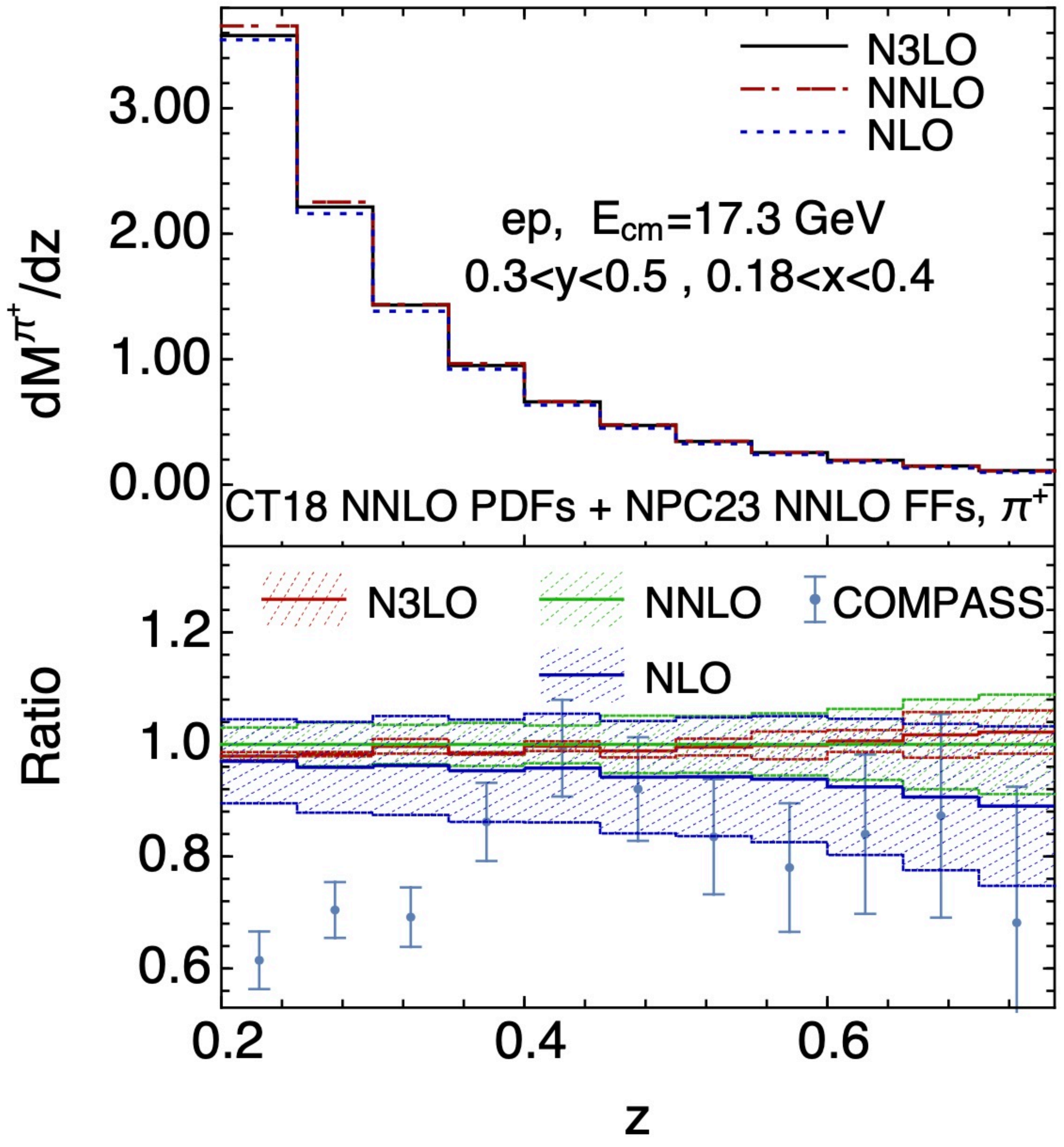

Thank you!