TMD measurements in SIDIS with a jet at NLP

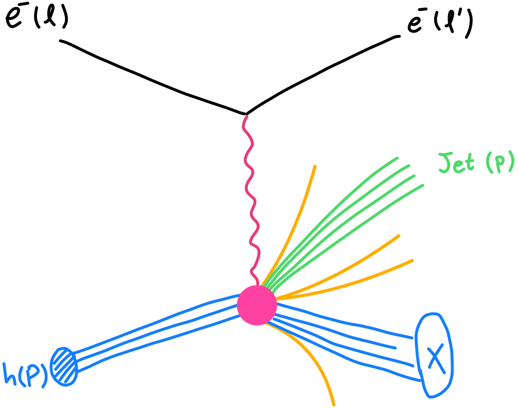
Max Jaarsma, Oscar del Rio, Ignazio Scimemi, Wouter Waalewijn

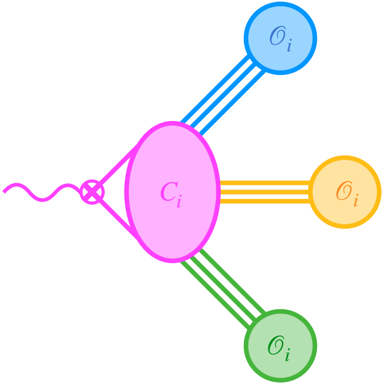
based on: **[arXiv:2507.03072]**

QCD Evolution 2026

El Escorial

What to expect from this talk?

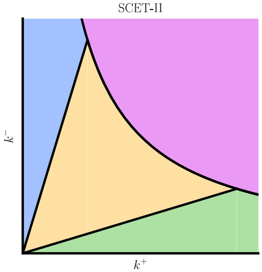


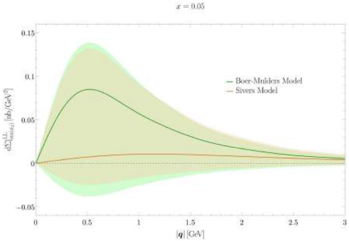


$$F^{g(\phi_J,\phi_S)}_{P_eP_p} = \sum_i H_i \otimes F_i \otimes S_i \otimes J_i$$

Outline

- Motivation
- Introduction
- The SCET-II effective current at NLP
- Factorization of the hadronic tensor
- Soft-collinear overlap subtraction
- Final results + prediction
- Conclusion & outlook

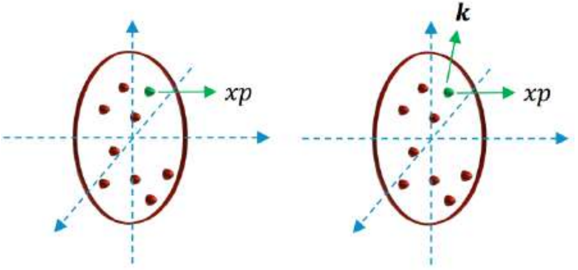




Motivation

Motivation - TMD observables

- Probing the 3D structure of protons
- Sensitivity to a large range of scales
- Tests of universality of factorization
- Spin and azimuthal correlations
- Precision tests of the Standard Model



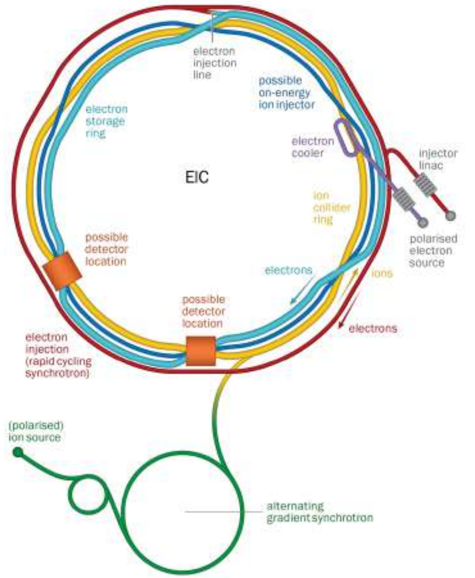
Adapted from seminar slides from J. Gaunt

Motivation - Power corrections in TMD observables

$$q_T \ll Q$$

- Cannot be ignored with improving experimental precision
- Many spin-dependent and azimuthal angle-dependent effects start at NLP
- Improve theoretical understanding of sub-leading power factorization

Ebert, Gao, Stewart (2021)
Vladimirov, Moos, Scimemi (2021)
Gamberg, Kang, Shao, Terry, Zhao (2022)
Talk by Johannes Michel at ESI workshop (2023)
Balitsky (2024)



Motivation - Jet measurements

Jets and TMD observables

The combination of TMD distributions with jet physics provides a powerful framework for probing hadronization and hadronic substructure.

- Jet-related factorization ingredients are perturbatively calculable for $q_T \gg \Lambda_{\text{QCD}}$
- This reduces the number of unknowns in a fit
- Symmetries reduce the number independent terms that appear in the cross section

Introduction to Power Corrections in TMD Observables

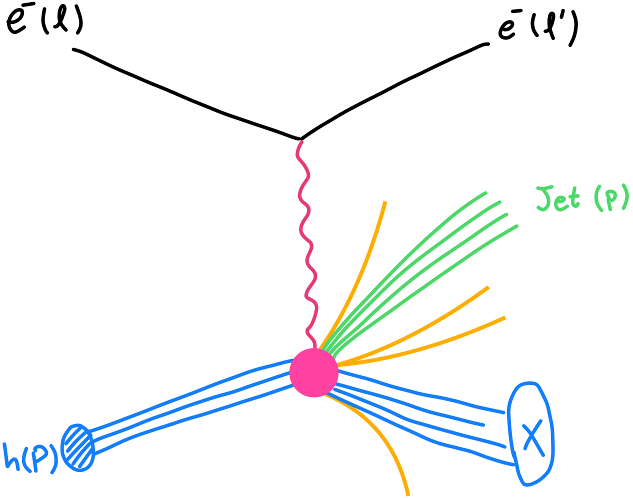
Introduction - SIDIS

- $e^-(\ell) + h(P) \to e^-(\ell') + J(p) + X$
- Process mediated by $\gamma^*(q)$
- Measure the kinematic variables:

$$\mathbf{q}_T^2, \quad x = \frac{Q^2}{2q \cdot P}, \quad y = \frac{q \cdot P}{\ell \cdot P}$$

- Include proton spin S
- Measure the following angles:

$$\phi_J, \qquad \phi_S$$



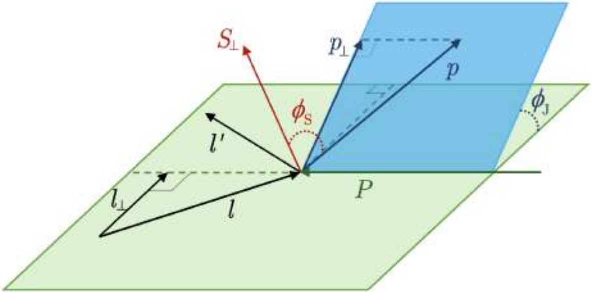
Introduction - SIDIS

- $e^-(\ell) + h(P) \to e^-(\ell') + J(p) + X$
- Process mediated by $\gamma^*(q)$
- Measure the kinematic variables:

$$\mathbf{q}_T^2, \quad x = \frac{Q^2}{2q \cdot P}, \quad y = \frac{q \cdot P}{\ell \cdot P}$$

- Include proton spin S
- Measure the following angles:

$$\phi_J, \qquad \phi_S$$



Introduction - What is the goal?

- Spin- and angular-dependence of cross section can be organized into form factors

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi_J\,\mathrm{d}\phi_S\,\mathrm{d}\mathbf{q}^2} = \frac{\alpha_{\mathsf{em}}^2 y}{8Q^2}\bigg\{F_{UU}\frac{2-2y+y^2}{y^2} + F_{UU}^{\cos\phi_J}\cos\phi_J\frac{2(2-y)\sqrt{1-y}}{y^2} + F_{LU}^{\sin\phi_J}\lambda_e\sin\phi_J\frac{2\sqrt{1-y}}{y} + F_{UL}^{\sin\phi_J}S_\parallel\sin\phi_J\frac{2(2-y)\sqrt{1-y}}{y^2} + \dots\bigg\}$$

- Goal: Find factorized formulas for all form factors

$$F_{P_eP_p}^{g(\phi_J,\phi_S)} = \sum_i H_i \otimes F_i \otimes J_i \otimes S_i$$

Introduction - How do we get there?

- Introduce power counting

$$q_T \sim \lambda Q$$

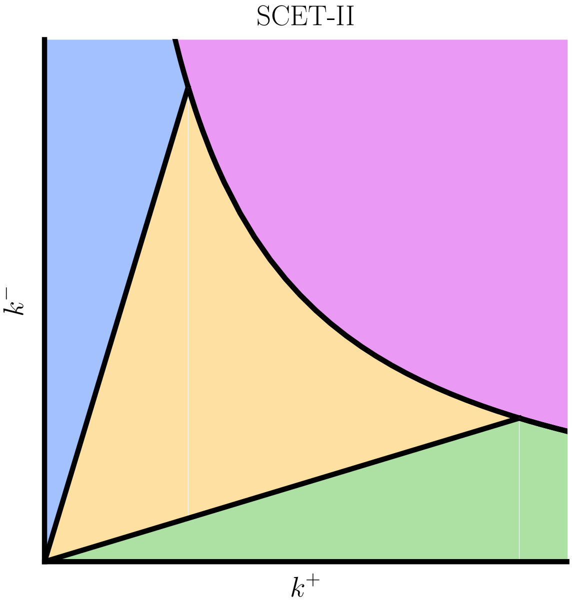
- Identify relevant modes of momentum

$$\text{p-collinear} \sim (\lambda^2, 1, \lambda)$$
$$\text{J-collinear} \sim (1, \lambda^2, \lambda)$$
$$\text{soft} \sim (\lambda, \lambda, \lambda)$$

- Introduce background fields for modes

$$\phi = \phi + \phi + \phi + \phi$$

- Integrate out off-shell modes



Introduction - Recipe

- Start from the full-QCD hadronic tensor

$$W^{\mu\nu} = \int \frac{\mathrm{d}^4 b}{(2\pi)^4}\, e^{+\mathrm{i}q\cdot b}\, \langle P|\, J^\mu(b)\, |J, X\rangle \langle J, X|\, J^\mu(b)\, |P\rangle$$

- Integrate out off-shell modes and insert the effective current

$$J^\mu \to J^\mu_{\text{eff}} = \sum_i C_i \otimes \mathcal{O}_i \otimes \mathcal{O}_i \otimes \mathcal{O}_i$$

- Factorize the matrix elements and reshuffle color, spin and Lorentz indices

$$W^{\mu\nu} = \sum_i \Gamma_i^{\mu\nu}\; H_i \otimes F_i \otimes S_i \otimes J_i$$

The SCET-II Effective Current Operator at NLP

The effective current - What do we mean?

- Hard scattering governed by EM current

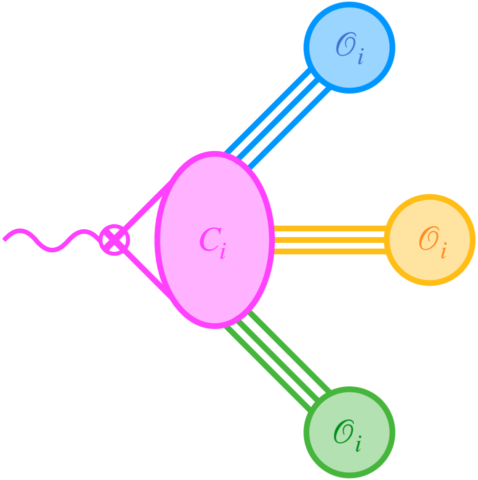
$$J^\mu = \bar{\psi}\gamma^\mu\psi$$

- Integrating out off-shell modes gives J_{eff}

$$J^\mu_{\text{eff}} = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}A\, e^{\mathrm{i}S_{\text{QCD}}[\phi]}\, e^{\mathrm{i}S_{\text{int}}[\phi,\phi,\phi,\phi]}$$
$$\times [\bar{\psi}+\bar{\psi}+\bar{\psi}+\bar{\psi}]\gamma^\mu[\psi+\psi+\psi+\psi]$$

- Effective current can be written in the form

$$J^\mu_{\text{eff}} = \sum_i C_i \otimes \mathcal{O}_i \otimes \mathcal{O}_i \otimes \mathcal{O}_i$$



Ebert, Gao, Stewart (2021)
Vladimirov, Moos, Scimemi (2021)

The effective current - Standard SCET vs. background field

Standard approach

- Match QCD onto SCET-I current with

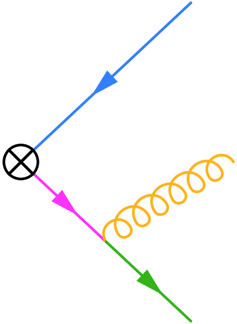
$$\text{hc} \sim (\lambda, 1, \lambda^{\frac{1}{2}})$$
$$\text{s} \sim (\lambda, \lambda, \lambda)$$

- Match hard-collinear operators onto collinear and soft operators.
- Matching is carried out by calculating matrix elements of the operators in partonic external states

Our approach

- Simultaneously integrate out all off-shell modes
- Happens on an operator level with position-space Feynman rules
- Effective operator is calculated directly, no matching involved

The effective current - Example of calculation



- Integrate out off-shell mode

$$J^\mu_{\text{eff}} \supset \bar{\psi}\gamma^\mu \int \mathrm{d}^d w \frac{\Gamma(2-\epsilon)}{2\pi^{d/2}} \frac{\slashed{w}}{(-w^2+\mathrm{i}0)^{2-\epsilon}} g\slashed{A}(w)\psi(w)$$

- Multipole expand

$$\Delta(w)A(w)\psi(w) \to \Delta(w)A(w^+\bar{n})\psi(w^-n) + \dots$$

- Apply Field equations to keep operator basis minimal

$$J^\mu_{\text{eff}} \supset \bar{\psi}\gamma^\mu \frac{1}{\mathrm{i}\partial^-} gA^-\psi + n^\mu \bar{\psi}\left(\slashed{A}_T - \frac{\mathrm{i}\slashed{\partial}_T}{\mathrm{i}\partial^-} gA^-\right)\frac{1}{\mathrm{i}\partial^+}\psi$$

The effective current - Match to SCET building blocks

- Match onto familiar SCET building blocks

$$\chi = W^\dagger \frac{\gamma^-\gamma^+}{2}\psi \qquad \mathcal{A}_T^\rho = W^\dagger\left[\mathrm{i}D_T^\rho, W\right]$$

- This can be done order-by-order

$$\mathcal{A}_T^\rho = gA_T^\rho - \frac{\mathrm{i}\partial_T^\rho}{\mathrm{i}\partial^-}gA^- + \mathcal{O}(g^2)$$

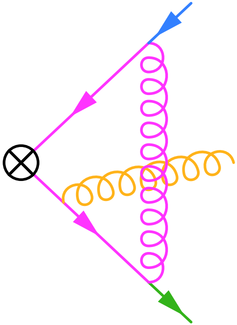
- Two sets of building block operators for the soft fields

$$\Psi_n = S_n^\dagger\psi \qquad \mathcal{A}_{T,n}^\rho = S_n^\dagger\left[\mathrm{i}D_T^\rho, S_n\right]$$

$$\Psi_{\bar{n}} = S_{\bar{n}}^\dagger\psi \qquad \mathcal{A}_{T,\bar{n}}^\rho = S_{\bar{n}}^\dagger\left[\mathrm{i}D_T^\rho, S_{\bar{n}}\right]$$

Bauer, Pirjol, Stewart (2001)

The effective current - Higher orders



- At higher orders, loops result in convolutions along the lightcone

$$C(\{x^+\},\{y^+,y^-\},\{z^-\}) \\ \times \mathcal{O}(x^+\bar{n})\mathcal{O}_n(y^-n)\mathcal{O}_{\bar{n}}(y^+\bar{n})\mathcal{O}(z^-n)$$

- Convolutions involving Soft operators can arise from hard-collinear loops.

- Wilson coefficients can be constrained from symmetries

The effective current - Summary of operators

- Usual LP operator $\bar{\chi}\gamma_T^\mu S_n^\dagger S_{\bar{n}}\chi$
- Transverse derivative of LP building block $\bar{\chi} S_n^\dagger S_{\bar{n}} n^\mu \frac{\mathrm{i}\partial\!\!\!/_T}{\mathrm{i}\partial^+}\chi$
- Two building blocks operators $\bar{\chi} S_n^\dagger S_{\bar{n}} \frac{n^\mu}{\mathrm{i}\partial^+}\mathcal{A}\!\!\!/_T\chi$
- $\bar{q}qg$ configuration $\bar{\chi}\gamma_T^\mu S_n^\dagger S_{\bar{n}}\mathcal{A}\!\!\!/_T S_{\bar{n}}^\dagger S_n \frac{\gamma^-}{\mathrm{i}\partial^-}\chi$

The effective current - Summary of operators

- Soft gluon operator (constrained by $\partial_\mu J^\mu = 0$) $\bar{\chi} S_n^\dagger S_{\bar{n}} \mathcal{A}^\rho_{T,\bar{n}} \frac{n^\mu}{i\partial^+} \chi$
- Soft gluon operator (constrained by RPI) $\bar{\chi} \gamma_T^\mu S_n^\dagger S_{\bar{n}} \frac{1}{i\partial^-} \mathcal{A}^\rho_{T,\bar{n}} \frac{i\partial_T^\rho}{i\partial^+} \chi$
- Soft gluon operator (unconstrained) $\bar{\chi} \gamma_T^\mu S_n^\dagger S_{\bar{n}} \frac{i\not{\partial}_T}{i\partial^-} \not{\mathcal{A}}_{T,\bar{n}} \frac{i\partial_T^\rho}{i\partial^+} \chi$
- Soft quark operator (unconstrained) $\bar{\chi} \gamma_T^\mu S_n^\dagger S_{\bar{n}} \not{\mathcal{A}}_T \frac{\gamma^-}{i\partial^-} \Psi_{\bar{n}}$

The effective current - Constraints from conservation

- Demand that the EM current is conserved up to power corrections

$$\mathrm{i}\partial_\mu J^\mu_{\text{eff}}(x) = 0 + \mathcal{O}(\lambda^{\frac{7}{2}})$$

- Results in the following constraints

$n^\mu \bar\chi S_n^\dagger S_{\bar n} \frac{\mathrm{i}\not\partial_T}{\mathrm{i}\partial^+}\chi$ 🔗 $\bar\chi \gamma_T^\mu S_n^\dagger S_{\bar n}\chi$

$n^\mu \bar\chi S_n^\dagger S_{\bar n} \not{\mathcal{A}}_{T,\bar n} \frac{1}{\mathrm{i}\partial^+}\chi$ 🔗 $\bar\chi \gamma_T^\mu S_n^\dagger S_{\bar n}\chi$

$\frac{\bar n^\mu}{\mathrm{i}\partial^-}\bar\chi \mathcal{A}_T S_n^\dagger S_{\bar n}\chi$ 🔗 $\bar\chi \mathcal{A}_T S_n^\dagger S_{\bar n}\frac{n^\mu}{\mathrm{i}\partial^+}\chi$

The effective current - Constraints from RPI

- RPI: Choice of frame should not matter too much

$$\begin{array}{rll} \text{I:} & n \to n + \Delta_T & \bar{n} \to \bar{n} \\ \text{II:} & n \to n & \bar{n} \to \bar{n} + \bar{\Delta}_T \\ \text{III:} & n \to n\, e^{\alpha} & \bar{n} \to \bar{n}\, e^{-\alpha} \end{array}$$

- This connects the following operators

$n^\mu \bar{\chi} S_n^\dagger S_{\bar{n}} \frac{\mathrm{i}\not{\partial}_T}{\mathrm{i}\partial^+} \chi$ 🔗 $\bar{\chi} \gamma_T^\mu S_n^\dagger S_{\bar{n}} \chi$

$\bar{\chi} \gamma_T^\mu S_n^\dagger S_{\bar{n}} \frac{1}{\mathrm{i}\partial^-} \mathcal{A}_{T,\bar{n}}^\rho \frac{\mathrm{i}\partial_T^\rho}{\mathrm{i}\partial^+} \chi$ 🔗 $\bar{\chi} \gamma_T^\mu S_n^\dagger S_{\bar{n}} \chi$

The effective current - Summary

- Complete result for the SCET-II effective current at NLP
- Only 5 independent Wilson coefficients
- Constraints to be cross-checked

$$C_1 \otimes \bar{\chi}\gamma_T^\mu S_n^\dagger S_{\bar{n}}\chi$$

$$C_1 \otimes \bar{\chi} S_n^\dagger S_{\bar{n}} \slashed{A}_{\bar{n},T} \frac{n^\mu}{\mathrm{i}\partial^+}\chi$$

$$C_1 \otimes \bar{\chi}\gamma_T^\mu S_n^\dagger S_{\bar{n}} \frac{1}{\mathrm{i}\partial^-} A^\rho_{\bar{n},T} \frac{\mathrm{i}\partial_T^\rho}{\mathrm{i}\partial^+}\chi$$

$$C_2 \otimes \left(\frac{\bar{n}^\mu}{\mathrm{i}\partial^-} - \frac{n^\mu}{\mathrm{i}\partial^+}\right)\bar{\chi}\slashed{\mathcal{A}}_T S_n^\dagger S_{\bar{n}}\chi$$

$$C_3 \otimes \bar{\chi}\gamma_T^\mu S_n^\dagger S_{\bar{n}} \frac{\mathrm{i}\slashed{\partial}_T}{\mathrm{i}\partial^-} \slashed{A}_{\bar{n},T} \frac{1}{\mathrm{i}\partial^+}\chi$$

$$C_4 \otimes \bar{\chi}\gamma_T^\mu\gamma^- S_n^\dagger S_{\bar{n}} \slashed{A}_T \frac{1}{\mathrm{i}\partial^-}\Psi_{\bar{n}}$$

$$C_5 \otimes \bar{\chi}\gamma_T^\mu\gamma^- S_n^\dagger S_{\bar{n}} \slashed{A}_T S_{\bar{n}}^\dagger S_n \frac{1}{\mathrm{i}\partial^-}\chi_{\bar{n}}$$

Factorization of the Hadronic Tensor

Hadronic Tensor - Intermediate result

- The object of interest:

$$W^{\mu\nu} = \int \frac{\mathrm{d}^4 b}{(2\pi)^4} \, e^{+\mathrm{i}q\cdot b} \, \langle P | \, J^\mu(b) \, | J, X \rangle \, \langle J, X | \, J^\mu(b) \, | P \rangle$$

- Direct insertion of the effective current results in large amount of terms

- Many terms vanish because of:
  1. Conservation of fermion number
  2. C, P and T symmetries
  3. Boost invariance of the vacuum

Hadronic Tensor - Intermediate result

- Boost invariance of the vacuum implies

$$0 = \langle 0 | \frac{1}{\mathrm{i}\partial^-} \mathcal{A}^{\rho}_{T,\bar{n}}(b_T)\, S^{\dagger}_{\bar{n}} S_n(b_T)\, |X\rangle\langle X|\, S^{\dagger}_{n} S_{\bar{n}}(0)\, |0\rangle$$

$$0 = \langle 0 | \frac{\mathrm{i}\partial^{\sigma}_T}{\mathrm{i}\partial^-} \mathcal{A}^{\rho}_{T,\bar{n}}(b_T)\, S^{\dagger}_{\bar{n}} S_n(b_T)\, |X\rangle\langle X|\, S^{\dagger}_{n} S_{\bar{n}}(0)\, |0\rangle$$

$$0 = \langle 0 |\, S^{\dagger}_{n} S_{\bar{n}}(b_T) \left[\frac{1}{\mathrm{i}\partial^-}\gamma^- \Psi_{\bar{n}}(b_T)\right]_{\beta} |X\rangle\langle X| \left[\frac{1}{\mathrm{i}\partial^-}\bar{\Psi}_{\bar{n}}(0)\gamma^-\right]_{\gamma} S^{\dagger}_{\bar{n}} S_n(0)\, |0\rangle$$

Hadronic Tensor - Intermediate result

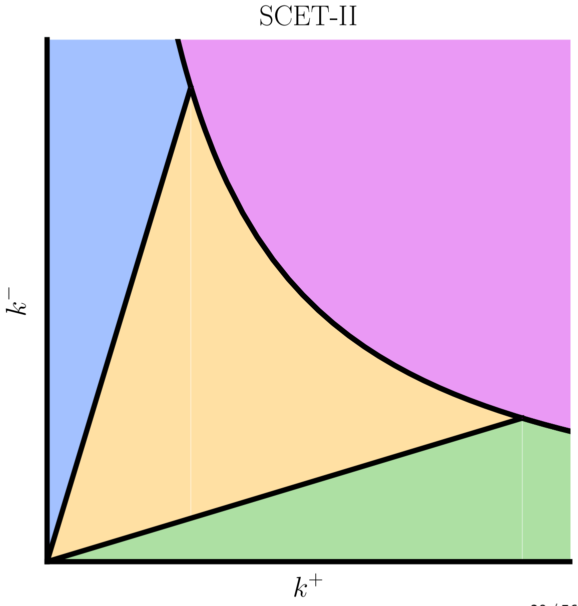
Leading-power	$\langle\bar{\chi}\vert\chi\rangle\langle\chi\vert\bar{\chi}\rangle\langle S_n^\dagger S_{\bar{n}}\vert S_{\bar{n}}^\dagger S_n\rangle$
Multipole correction	$\langle\bar{\chi}\vert\chi\rangle\langle\chi\vert\bar{\chi}\rangle\langle \mathrm{i}\partial^+ S_n^\dagger S_{\bar{n}}\vert S_{\bar{n}}^\dagger S_n\rangle$
∂_T correction	$\langle \mathrm{i}\partial_T\bar{\chi}\vert\chi\rangle\langle\chi\vert\bar{\chi}\rangle\langle S_n^\dagger S_{\bar{n}}\vert S_{\bar{n}}^\dagger S_n\rangle$
$\mathcal{A}_T$ correction	$\langle\bar{\chi}\mathcal{A}_T\vert\chi\rangle\langle\chi\vert\bar{\chi}\rangle\langle S_n^\dagger S_{\bar{n}}\vert S_{\bar{n}}^\dagger S_n\rangle$
$\mathcal{A}_T$ correction	$\langle\bar{\chi}\vert\chi\rangle\langle\chi\vert\bar{\chi}\rangle\langle S_n^\dagger S_{\bar{n}}\mathcal{A}_{T,\bar{n}}\vert S_{\bar{n}}^\dagger S_n\rangle$

Hadronic Tensor - Rapidity divergences

- Factorization ingredients contain rapidity divergences due to separation in rapidity
- Cancel between collinear, soft and anti-collinear
- Structure of cancellation more complicated at NLP

$$\langle P|\,\bar{\chi}\not{\partial}_T\chi\,|P\rangle$$

$$\langle P|\,\bar{\chi}\not{\mathcal{A}}_T\chi\,|P\rangle$$

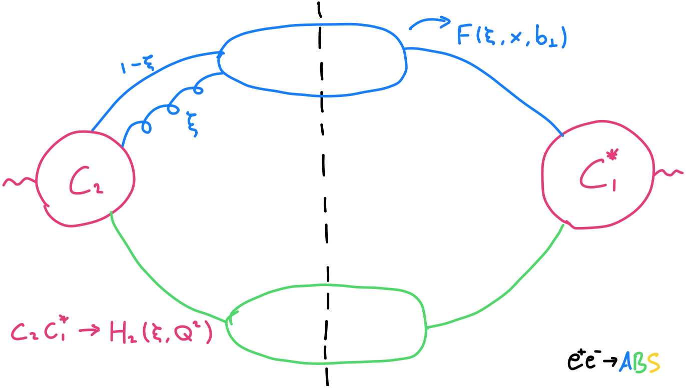

SCET-II

Ebert, Gao, Stewart (2021)
Vladimirov, Moos, Scimemi (2021)

Hadronic Tensor - Endpoint divergences

$$C_2 \otimes \bar{\chi} \mathcal{A}_T S_n^\dagger S_{\bar{n}} \chi \quad \Rightarrow \quad \int \mathrm{d}\xi \, H_2(\xi, \dots) \, F(\xi, \dots) \, S(\dots) \, J(\dots)$$

$$\Uparrow \qquad\qquad \Uparrow$$



$$F(\xi, \dots) \sim \frac{1}{\xi}$$

$$H_2(\xi, \dots) \sim \ln \xi$$

Liu, Neubert (2019)
Vladimirov, Moos, Scimemi (2021)
Talk by Johannes Michel at ESI workshop (2023)
Beneke, Garny, Jaskiewicz, Strohm, Szafron, Vernazza (2022)
Liu, Mecaj, Neubert, Wang (2022)

Hadronic Tensor - What to do with these divergences?

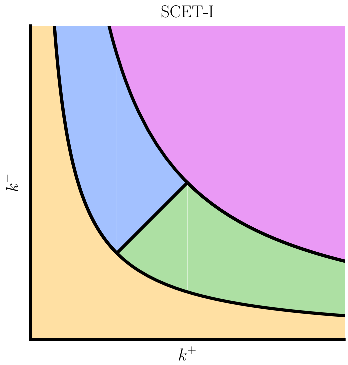
- Individual factorization ingredients contain rapidity divergences
- Convolutions between NLP ingredients result in endpoint divergences
- All divergences cancel in the cross section

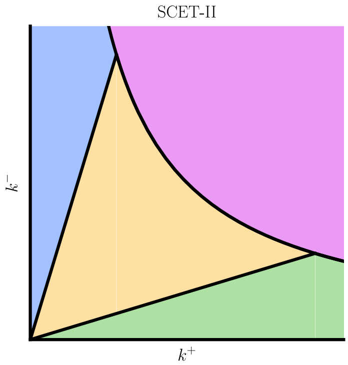
Goal

Redefine ingredients such that all ingredients and convolutions are manifestly finite

Soft-Collinear Overlap (0-bin) Subtraction

Overlap subtraction - What's the deal?





- Soft and (anti-)collinear modes are separated by their virtualities
- Overlap region vanishes in dimensional regularization

- Soft and (anti-)collinear modes are separated by their rapidities
- Overlap region vanishes for many rapidity regulators, but not always

Manohar, Stewart (2006)
Lee, Sterman (2006)
Idilbi, Mehen (2007)

Overlap subtraction - Why do we need a proper formalism?

- At leading power, some rapidity regulators always result in vanishing overlap regions

Overlap subtraction

If the overlap always vanishes, why bother setting up a formalism?

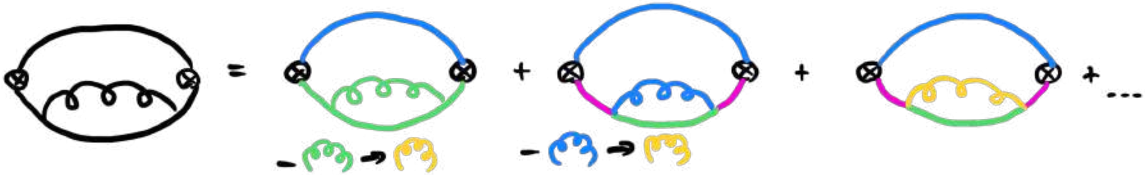
1. Formulating factorization in a regulator-independent way
2. Overlap regions at sub-leading power do not necessarily vanish

Overlap subtraction - Method of regions treatment

- Method of regions: sum over all regions and subtract the overlaps
  Beneke, Smirnov (1998)

- From a regions point of view, the convolution in ξ is part of the full loop integral

- The limit $\xi \to 0$ corresponds to the collinear gluon becoming soft



Overlap subtraction - The Field Theoretical Way

1. Split off a soft component from the collinear fields

$$\phi(x) \to \phi(x) + \phi(x)$$

2. Remove all leading-power soft-collinear overlap interactions by field redefinition

$$\psi(x) \to S_n(x)\psi(x) \qquad A^\mu(x) \to S_n(x)A^\mu(x)S_n^\dagger(x)$$

3. Replace all collinear operators in the factorized formula by

$$\chi \to S_{\bar{n}}^\dagger S_n \chi + \Psi_{\bar{n}} \qquad \mathcal{A}_T^\mu \to S_{\bar{n}}^\dagger S_n \mathcal{A}_T^\mu S_n^\dagger S_{\bar{n}} + \mathcal{A}_{T,\bar{n}}^\mu$$

4. Solve for the pure-collinear matrix element by inverting the transformation

Idilbi, Mehen (2007)

Overlap subtraction - Example at leading-power

3. Apply overlap transformation to collinear operators

$$\langle P|\,\bar\chi\,|X\rangle\,\gamma^-\,\langle X|\,\chi\,|P\rangle \to \langle P|\,\bar\chi\,|X\rangle\,\gamma^-\,\langle X|\,\chi\,|P\rangle\,\frac{1}{N_c}\mathrm{tr}\Big[\langle 0|\,S_n^\dagger S_{\bar n}\,|X\rangle\,\langle X|\,S_{\bar n}^\dagger S_n\,|0\rangle\Big]$$

4. Invert transformation to obtain pure collinear transformation

$$\langle P|\,\bar\chi\,|X\rangle\,\gamma^-\,\langle X|\,\chi\,|P\rangle_{\text{pure}} = \frac{\langle P|\,\bar\chi\,|X\rangle\,\gamma^-\,\langle X|\,\chi\,|P\rangle}{\frac{1}{N_c}\mathrm{tr}\Big[\langle 0|\,S_n^\dagger S_{\bar n}\,|X\rangle\,\langle X|\,S_{\bar n}^\dagger S_n\,|0\rangle\Big]}$$

- This subtraction leads to the definition of physical TMD PDFs by

$$B\otimes B\otimes S\to\frac{B}{S}\otimes\frac{B}{S}\otimes S\equiv f\otimes f$$

Overlap subtraction - Example at next-to-leading-power

Overlap subtraction

At next-to-leading power also additive terms play a role

- Of particular importance for $\bar{\chi}\mathcal{A}\chi$ correlation functions

$$\langle P|\,\bar{\chi}\mathcal{A}(b_T)\chi(0)\,|P\rangle \to \langle P|\,\bar{\chi}\mathcal{A}(b_T)\chi(0)\,|P\rangle \tfrac{1}{N_c}\mathrm{tr}\left[\langle 0|\,S_n^\dagger S_{\bar{n}}(b_T)S_{\bar{n}}^\dagger S_n(0)\,|0\rangle\right]$$
$$+ \langle P|\,\bar{\chi}(b_T)\chi(0)\,|P\rangle \tfrac{1}{N_c}\mathrm{tr}\left[\langle 0|\,S_n^\dagger S_{\bar{n}}(b_T)\mathcal{A}(b_T)S_{\bar{n}}^\dagger S_n(0)\,|0\rangle\right]$$

- Leads to subtraction term in definition of physical TMD PDFs

$$F_2 = \frac{F_2^{(0)}}{\sqrt{S}} - \frac{F_1 S_2}{\sqrt{S}}$$

Overlap subtraction - Effect on endpoint divergences

- Subtraction of the overlap should remove the leading $\xi \to 0$ behavior

$$F_2^{(0)}(\xi, \dots) \sim \frac{1}{\xi} + \mathcal{O}(\xi^0)$$

- At leading order the subtracted TMD PDF behaves as

$$F_2(\xi) = \frac{F_2^{(0)}}{\sqrt{S}} - \frac{F_1 S_2}{\sqrt{S}} \sim \mathcal{O}(\xi^0)$$

- Convolution in ξ is manifestly finite

$$\int \mathrm{d}\xi \, H_2(\xi, \dots) \, F(\xi, \dots) \; = \text{finite}$$

Liu, Mecaj, Neubert, Wang (2021)

Overlap subtraction - The final hadronic tensor

The Hadronic Tensor at NLP

There are 3 types of contributions to the hadronic tensor at NLP

Leading-power	$H_1 \otimes J_1 \otimes F_1$
Kinematic correction	$H_1 \otimes J_1 \otimes \partial_T^* F_1$
Higher-twist correction	$H_2 \otimes J_1 \otimes F_2$

Final Results for the SIDIS Form Factors

Final results - Form factors

- Two form factors do not receive a correction at this order in q_T/Q,

$$F_{UU,T} = f_1 \otimes J_1 \qquad\qquad F_{LL} = g_1 \otimes J_1$$

- Power corrections show up in angular modulations

$$F_{UU}^{\cos\phi_J} = -\frac{2|\mathbf{q}|}{Q}\Big\{\mathcal{J}_{0,0}\big[f_1 J_1\big] + \mathcal{J}_{1,1}\big[f_1 J_1'\big] + \text{Re}\Big(\mathcal{J}_{1,1}^{(2)}\big[f_1 J_2\big]\Big) + \text{Im}\Big(\mathcal{J}_{1,1}^{(2)}\big[f_2^{\perp} J_1\big]\Big)\Big\},$$

$$F_{LU}^{\sin\phi_J} = \frac{2|\mathbf{q}|}{Q}\Big\{\text{Im}\Big(\mathcal{J}_{1,1}^{(2)}\big[f_1 J_2\big]\Big) - \text{Re}\Big(\mathcal{J}_{1,1}^{(2)}\big[f_2^{\perp} J_1\big]\Big)\Big\},$$

Final results - Phenomenological prediction

- Focus on one particular form factor, $F_{UL}^{\sin\phi_J}$, obtained by

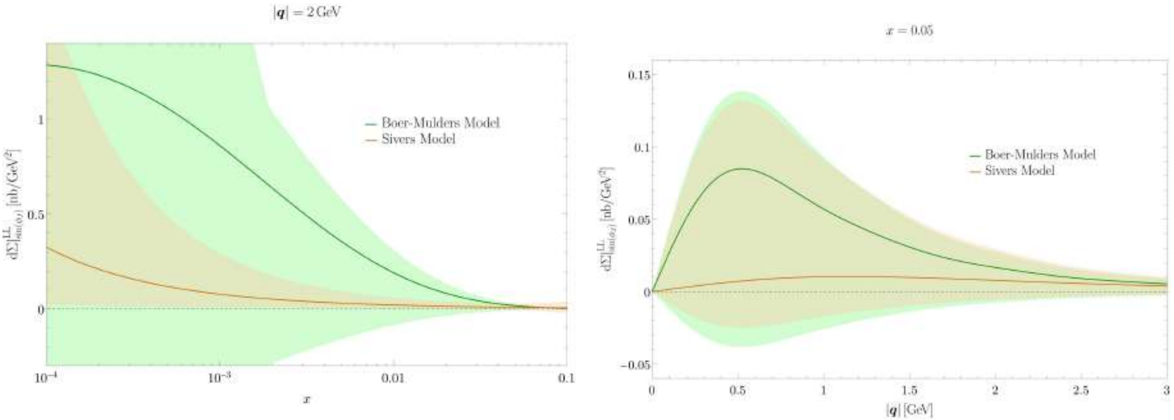
$$\mathrm{d}\Sigma\big|_{\sin\phi_J} \equiv \int_0^{2\pi} \mathrm{d}\phi_J \, \frac{\sin\phi_J}{\pi} \int_0^{2\pi} \mathrm{d}\phi_S \int_{0.01}^{0.95} \mathrm{d}y \; \frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi_J\,\mathrm{d}\phi_S\,\mathrm{d}\mathbf{q}^2}\Bigg|_{\lambda_e,\,|S_\perp|=0}$$

- Jet functions can be calculated perturbatively

- Use a model (Gaussian in ξ) for the twist-3 TMD PDFs, we study 2:
  - Boer-Mulders model: $F_2 \sim h_1^\perp$
  - Sivers model: $F_2 \sim f_{1T}$

- Fits for these functions are available in the literature
  Piloneta, Vladimirov (2024)
  Bury, Prokudin, Vladimirov (2021)

Final results - Phenomenological prediction



- q_T dependence of the two models agree within error bands
- Large bands reflect the uncertainties in the non-perturbative TMD parameterization

Conclusions

Conclusions

- Constructed the SCET-II effective EM current to NLP
- Constructed the hadronic tensor for jet production in SIDIS
- Formulated 0-bin subtraction on an operator level
- After subtraction, only 3 types of contributions remained
  - Leading power F_1
  - Kinematic correction $\partial_T F_1$
  - Higher-twist correction F_2
- Derived factorization formulas for the form factors
- Phenomenological prediction for $F_{UL}^{\sin\phi_J}$

Thank you for your attention!

- Constructed the SCET-II effective EM current to NLP

- Constructed the hadronic tensor for jet production in SIDIS

- Formulated 0-bin subtraction on an operator level

- After subtraction, only 3 types of contributions remained
    - Leading power F_1
    - Kinematic correction $\partial_T F_1$
    - Higher-twist correction F_2

- Derived factorization formulas for the form factors

- Phenomenological prediction for $F_{UL}^{\sin\phi_J}$

Back-up

Introduction - What types of power corrections are there?

- Power corrections from kinematical variables

$$-g_T^{\mu\nu} L_{\mu\nu} = +2Q^2 \frac{2 - 2y + y^2}{y^2} - 4|\mathbf{q}|Q \cos(\phi_J) \frac{(2-y)\sqrt{1-y}}{y^2} + \mathcal{O}(|\mathbf{q}|^2)$$

- Power corrections from sub-leading derivative operators

$$\langle P| \bar{\chi}(b_T) \partial_T^\rho \chi(0) |P\rangle \Rightarrow \partial_T f_1(x, b_T)$$

- Power corrections from sub-leading field operators

$$\langle P| \bar{\chi}(b_T) \mathcal{A}_T \chi(0) |P\rangle \Rightarrow f_2(x, \xi, b_T)$$

Back-up slide: Position-space integration

$$\int \mathrm{d}w^+ \, \mathrm{d}w^- \, \frac{\mathcal{O}(w^+\bar{n})\mathcal{O}(w^-n)}{[-2w^+w^- + \mathrm{i}0]^\alpha} = \frac{-w^{1-\alpha}\mathrm{i}\pi}{\Gamma(\alpha)\Gamma(1-\alpha)} \cdots \frac{1}{(\mathrm{i}\partial^-)^\alpha}\mathcal{O}\frac{1}{(\mathrm{i}\partial^+)^\alpha}\mathcal{O}$$

Back-up slide: Modified transverse derivative

$$\partial_T^* = \partial_T + \tfrac{1}{2}\partial_T K(b_T) \ln\left(\frac{\zeta}{\bar{\zeta}}\right)$$

Back-up slide: Models for F_2

- Boer-Mulders model

$$g_{2L,\oplus/\ominus}^{\perp,q/\bar{q}}\left(x,\xi,\mathbf{b}^2\right) \simeq \frac{M^2|\mathbf{b}|^2}{\sqrt{2}\,\pi^{3/2}}\left[\Theta(\xi-x_1)-\Theta(\xi-x_2)\right]e^{-\xi^2/2}h_1^{\perp}\left(x,\mathbf{b}^2\right)$$

- Sivers model

$$h_{1;f\leftarrow h}^{\perp}(x,\mathbf{b}^2) = A_f N_f \frac{(1-x)x^{\beta_q}\,(1+\epsilon_q x)}{n(\beta_q,\epsilon_q)}\exp\left(-\frac{r_0+x\,r_1}{\sqrt{1+r_2\,x^2\,\mathbf{b}^2}}\mathbf{b}^2\right)$$

Hadronic Tensor - Intermediate result

$$
\begin{aligned}
\left[\mathcal{W}^{\mu\nu}\right]^{(2,2)}_{A} = {} & (\gamma^{\mu}_{T})_{\alpha\beta}(\gamma^{\nu}_{T})_{\gamma\delta} \int \frac{\mathrm{d}^2 b}{(2\pi)^2}\, e^{+\mathrm{i} q_T\cdot b_T} \\
& \times \int \mathrm{d}y^+\, \mathrm{d}y^-\, \mathrm{d}z^+\, \mathrm{d}z^-\, e^{\mathrm{i}(q^- z^+ + q^+ z^- - q^- y^+ - q^+ y^-)} C_1(y^+, y^-)\, C_1^*(z^+, z^-) \\
& \times \int \frac{\mathrm{d}b^+}{2\pi}\, e^{+\mathrm{i}q^- b^+}\, \langle P|\, \bar{\chi}_{i,\alpha}(b_T + b^+ \bar{n})\, |X\rangle\langle X|\, \chi_{l,\delta}(0)\, |P\rangle \\
& \times \int \frac{\mathrm{d}b^-}{2\pi}\, e^{+\mathrm{i}q^+ b^-}\, \langle 0|\, \chi_{j,\beta}(b_T + b^- n)\, |p, X\rangle\langle p, X|\, \bar{\chi}_{k,\gamma}(0)\, |0\rangle \\
& \times \langle 0|\, \left[S_n^{\dagger} S_{\bar{n}}(b_T)\right]_{ij} |X\rangle\langle X|\, \left[S_{\bar{n}}^{\dagger} S_n(0)\right]_{kl} |X\rangle \,.
\end{aligned}
$$

About our jets

- E^n recombination scheme:

$$\mathbf{P}_{\text{Jet}}^{E^n} = \frac{k_1^0 + k_2^0}{(k_1^0)^n + (k_2^0)^n}\left[(k_1^0)^{n-1}\mathbf{k}_1 + (k_2^0)^{n-1}\mathbf{k}_2\right] + \mathcal{O}\left(\frac{Q_T^2}{Q^2}Q_T\right)$$

- WTA recombination scheme (E^∞)

Bertolini, Chan, Thaler (2014)

$$\mathbf{P}_{\text{Jet}}^{\text{WTA}} = \Theta(k_1^0 - k_2^0)\,\frac{k_1^0 + k_2^0}{k_1^0}\,\mathbf{k}_1 + \Theta(k_2^0 - k_1^0)\,\frac{k_1^0 + k_2^0}{k_2^0}\,\mathbf{k}_2$$

- Energy-weighting enables soft-collinear factorization

— Refs

Overlap subtraction - Example at next-to-leading-power

$$
\begin{aligned}
&\left[\mathcal{F}^{\text{bare}}_{q,21}(x,\xi,b_T,\zeta)\right]_{\delta\alpha} \\
&= \mathrm{i}q^+ \int \frac{\mathrm{d}b_1^-}{2\pi}\frac{\mathrm{d}b_2^-}{2\pi}\, e^{-\mathrm{i}\bar{\xi}b_1^-\,q^+}\, e^{-\mathrm{i}\xi b_2^-\,q^+} \\
&\qquad \times \Bigg\{ \frac{\langle P|\left[\bar{\chi}(b_T+b_1^-n)\not{\mathcal{A}}_T(b_T+b_2^-n)\right]_{i,\alpha}|X\rangle\langle X|\chi_{i,\delta}(0)|P\rangle}{\sqrt{S(b_T,\zeta(\delta^+/q^+)^2}\,)} \\
&\qquad\qquad - \frac{\langle P|\left[\bar{\chi}(b_T+b_1^-n)\,\gamma_T^\rho\right]_{i,\alpha}|X\rangle\langle X|\chi_{i,\delta}(0)|P\rangle}{\sqrt{S(b_T,\zeta(\delta^+/q^+)^2}\,)} \\
&\qquad\qquad\quad \times \frac{1}{N_c}\mathrm{tr}\left[\langle 0|\,S_{\bar{n}}^\dagger S_n(b_T)\,\mathcal{A}^\rho_{T,n}(b_T+b_2^-n)\,|X\rangle\langle X|\,S_n^\dagger S_{\bar{n}}(0)\,|0\rangle\right] \Bigg\}
\end{aligned}
$$

Overlap subtraction - Effect on rapidity divergences

- $\mathrm{i}\partial_T \chi$ correction contains rapidity divergences not cancelled by $\sqrt{S}$

$$
\begin{aligned}
&\langle P|\, \mathrm{i}\partial_T^\rho \bar{\chi}_\alpha(b_T + b^+ \bar{n})\, |X\rangle \, \langle X|\, \chi_\delta(0)\, |P\rangle \to \\
&\langle P|\, \mathrm{i}\partial_T^\rho \bar{\chi}_\alpha(b_T + b^+ \bar{n})|X\rangle\langle X|\chi_\delta(0)|P\rangle\langle 0|\left[S_n^\dagger S_{\bar{n}}(b_T)\right]|X\rangle\langle X|\left[S_{\bar{n}}^\dagger S_n(0)\right]|0\rangle \\
&\quad + \langle P|\bar{\chi}_\alpha(b_T + b^+ \bar{n})|X\rangle\langle X|\chi_\delta(0)|P\rangle\langle 0|\mathrm{i}\partial_T^\rho\left[S_n^\dagger S_{\bar{n}}(b_T)\right]|X\rangle\langle X|\left[S_{\bar{n}}^\dagger S_n(0)\right]|0\rangle \,.
\end{aligned}
$$

- Overlap subtraction introduces additional term involving $\mathrm{i}\partial_T S_n$

$$
\mathcal{A}^\rho_{T,n} S_n^\dagger S_{\bar{n}} = -\frac{1}{2}\mathrm{i}\partial_T^\rho\left[S_n^\dagger S_{\bar{n}}\right] + \frac{1}{2}\left[S_n^\dagger S_{\bar{n}} \mathcal{A}^\rho_{T,\bar{n}} + \mathcal{A}^\rho_{T,n} S_n^\dagger S_{\bar{n}}\right] .
$$

- Combining the $\mathcal{A}_T$ correction, the above identity, and using C and P symmetry, one arrives at a manifestly finite expression

$$
\partial_T B \otimes B \otimes S + \frac{1}{2} B \otimes B \otimes \partial_T S \to \partial_T^* \left(\frac{B}{\sqrt{S}}\right) \otimes \frac{B}{\sqrt{S}}
$$