Computation of NLO GPD evolution equations in the lightcone gauge

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May 14, 2026

Generalized Parton Distributions

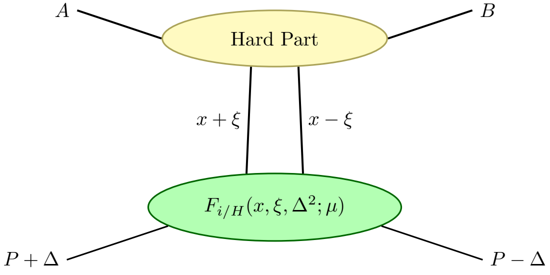
GPDs encode the 3-dimensional information about the hadron structure, give access to the spin of the partons, and the matrix elements of the QCD energy-momentum tensor.

Studied in exclusive processes:

$$N(P+\Delta) + A \longrightarrow N(P-\Delta) + B\,.$$

Factorization: amplitude = convolution of the hard part and GPDs.

Generalized Parton Distributions



$F_{i/H}(x,\xi,\Delta^2;\mu)$

i - parton species,

x - average fraction of the momentum carried by the parton,

ξ - longitudinal momentum transfer ("skewness"),

μ - factorization scale.

Generalized Parton Distributions

Sudakov frame

$$P^\mu = \frac{1}{2} p^+ \left(1,0,0,1\right)^T \ , \qquad (1)$$

$$n^\mu = \left(1,0,0,-1\right)^T \ , \qquad (2)$$

$$n \cdot P = p^+ \ . \qquad (3)$$

"$+$" components of the initial/final hadrons' momenta:

$$n \cdot (P \pm \Delta) = (1 \pm \xi) p^+ \ . \qquad (4)$$

Generalized Parton Distributions

GPDs are defined using the off-diagonal matrix elements of the parton operators on hadron states.

Correlators defining unpolarized bare GPDs in the lightcone gauge ($A^\mu n_\mu = 0$):

$$\hat{F}_{q/H}(x,\xi,\Delta^2;\varepsilon) = \int \frac{dy}{2\pi}\, e^{-ix(n\cdot P)y} \left\langle P-\Delta \middle| \bar{\psi}_q\left(\tfrac{yn}{2}\right) \tfrac{\not{n}}{2} \psi_q\left(-\tfrac{yn}{2}\right) \middle| P+\Delta \right\rangle , \quad (5)$$

$$\hat{F}_{g/H}(x,\xi,\Delta^2;\varepsilon) = \frac{n_\mu n_\nu}{x(n\cdot P)} \int \frac{dy}{2\pi}\, e^{-ix(n\cdot P)y} \left\langle P-\Delta \middle| F_a^{\mu j}\left(\tfrac{yn}{2}\right) F_a^{\nu j}\left(-\tfrac{yn}{2}\right) \middle| P+\Delta \right\rangle . \quad (6)$$

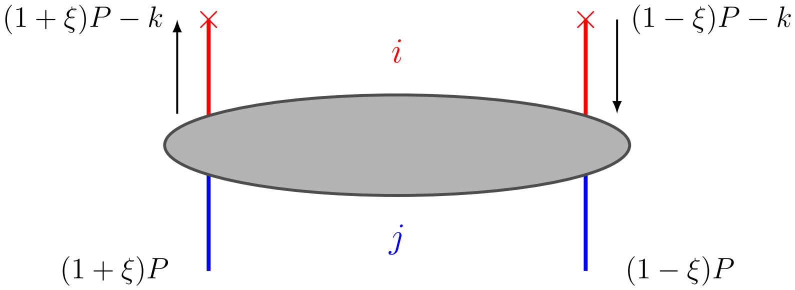
ε - dimensional regularization.

Generalized Parton Distributions

The tensorial decomposition of $\hat{F}_{i/H}$ gives the definition of bare GPDs $\hat{H}_{i/H}, \hat{E}_{i/H}$:

$$\hat{F}_{i/H}(x,\xi,\Delta^2) = \frac{1}{n\cdot P}\Big[\hat{H}_{i/H}(x,\xi,\Delta^2)\bar{u}(P-\Delta)\frac{\not{n}}{2}u(P+\Delta) + \hat{E}_{i/H}(x,\xi,\Delta^2)\bar{u}(P-\Delta)\frac{i\sigma^{\mu\nu}n_\mu\Delta_\nu}{4M}u(P+\Delta)\Big] \quad (7)$$

Factorization theorems are independent of the target H.
$\implies$ One can obtain the evolution equations by computing GPDs on parton targets:



See: Eur.Phys.J.C 82 (2022) 10, 888.

Renormalization of GPDs

GPDs can be renormalized in the $\overline{\text{MS}}$ scheme as follows:

$$F_{i/H}(x,\xi,\Delta^2;\mu) = \sum_j \int_{-1}^{1} \frac{dz}{|z|} Z_{ij}\left(\frac{x}{z},\frac{\xi}{x},\alpha_s(\mu);\varepsilon\right) \hat{F}_{j/H}(z,\xi,\Delta^2;\varepsilon)\,. \tag{8}$$

$\hat{F}_{j/H}$ independent of μ $\implies$ we can obtain the evolution equations of renormalized GPDs in the following form:

$$\frac{d}{d\log\mu} F_{i/H}(x,\xi,\Delta^2;\mu) = \sum_j \int_{-1}^{1} \frac{dz}{|z|} \mathcal{P}_{ij}\left(\frac{x}{z},\frac{\xi}{x},\alpha_s(\mu)\right) F_{j/H}(z,\xi,\Delta^2;\mu)\,. \tag{9}$$

$\mathcal{P}_{ij}$ can be recovered from Z_{ij}.

Known results

- One-loop: Müller et al. Fortschr.Phys., 42:101-141 (1994), Ji Phys.Rev.D 55,7114 (1997), Balitsky, Radyushkin Phys.Lett.B, 413:114-121 (1997).
- Two-loops: Belitsky, Müller, Freund Phys.Lett.B, 461:270-279 (1999), Braun, Manashov, Moch, Strohmaier J. High Energ. Phys. 2019, 191 (2019).
- Three-loop (flavor-nonsinglet): Braun, Manashov, Moch, Strohmaier J. High Energ. Phys. 2017, 37 (2017).

Our work

- Diagram-based computation of GPD evolution at two loops.
- Based on the idea of generalized ladder expansion by Curci, Furmanski and Petronzio, Nuclear Physics B, 175 (1980).
- Performed in the lightcone gauge $\Longrightarrow$ significant reduction of diagrams to consider.
- Currently: working on the flavor non-singlet sector.

Setting up the computation

Lightcone gauge

$$A^\mu n_\mu = 0\,. \tag{10}$$

Gluon propagator:

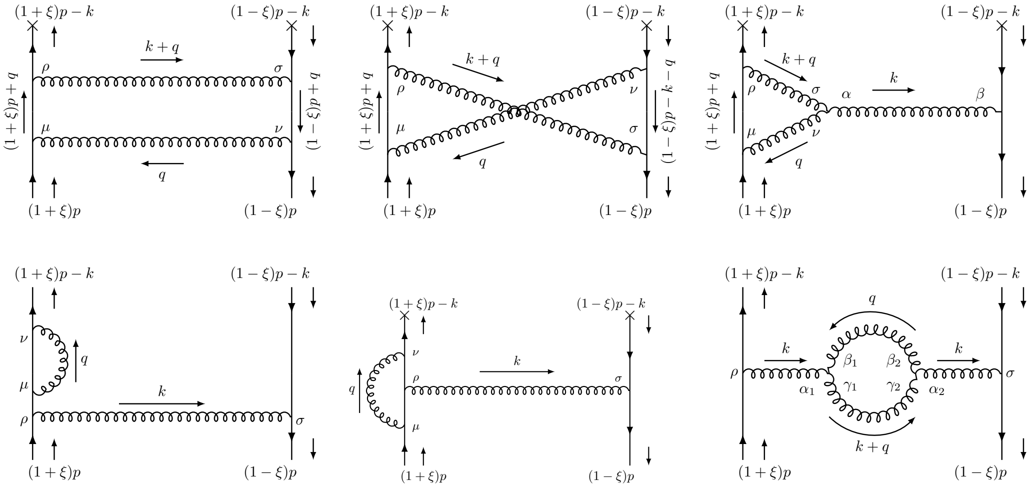
$$D_{\mu\nu}(k) = \frac{1}{k^2 + i\epsilon}\left(-g_{\mu\nu} + \frac{k_\mu n_\nu + k_\nu n_\mu}{n\cdot k}\right)\,. \tag{11}$$

Complications with the gluon propagator, but no Wilson line and ghosts!

Principal value regularization

$$\frac{1}{n\cdot k} \longrightarrow \frac{1}{2}\left(\frac{1}{n\cdot k + i\delta} + \frac{1}{n\cdot k - i\delta}\right)\,. \tag{12}$$

Setting up the computation



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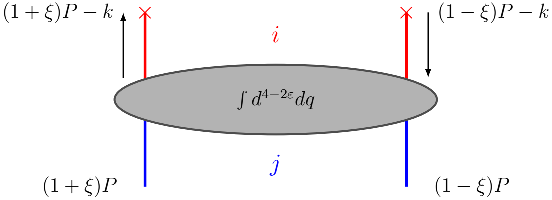

$k^{\pm} = \frac{1}{2}(k^0 \pm k^3)$

k - "external" loop momentum

q - "internal" loop momentum

1. $\int dy\, e^{-ix(nP)y} \ldots \implies$ we get $\delta\big(k^+ - (1-x)p^+\big)$ which allows to easily compute $\int dk^+$.

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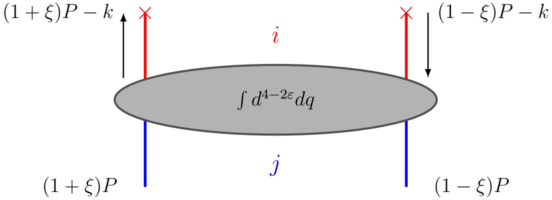

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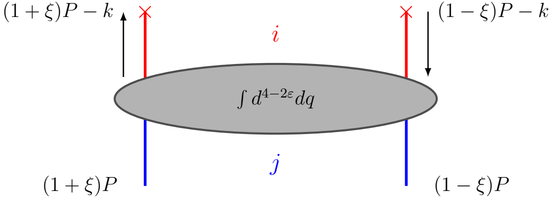

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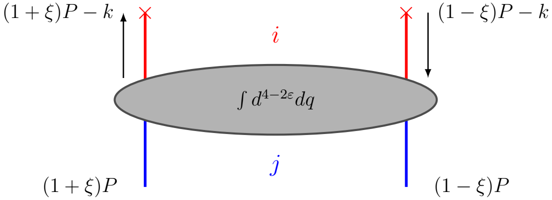

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2. Compute the integrals $\int d^{4-2\varepsilon}q$.
3. Compute the integrals $\int dk^-$.
4. **The UV-divergence shows up after integrating over $d^{2-2\varepsilon}k_\perp$.**

Integration over $d^{4-2\varepsilon}q$

We get at most 4 standard propagators $\left((q-l_i)^2+i\epsilon\right)^{-1}$
and 2 eikonal propagators $(n\cdot(q-l_j)\pm i\delta)^{-1}$ (l_i - combinations of the momenta p and k).

Partial fraction decomposition: reduce terms with 4 propagators to a sum of terms with at most 3 standard and 1 eikonal propagators.

$$\frac{1}{nq}\frac{1}{n(q-l)} = \frac{1}{nl}\left[\frac{1}{n(q-l)} - \frac{1}{nq}\right]. \tag{13}$$

Integration over $d^{4-2\varepsilon}q$

Let $T_j = (q - l_j)^2 + i\epsilon$, $j \in \{0, 1, 2, 3\}$. There are 2 possible forms of PFD:

Case 1 (non-planar box)

$$\sum_{j=0}^{3} a_j T_j = 1 \quad \Longrightarrow \quad \prod_{i=0}^{3} T_i^{-1} = \sum_{j=0}^{3} a_j \prod_{\substack{i=0 \\ i \neq j}}^{3} T_i^{-1}\,, \tag{14}$$

Case 2 (planar box)

$$\sum_{j=0}^{3} a_j T_j = 0 \quad \Longrightarrow \quad \prod_{i=0}^{3} T_i^{-1} = -\frac{1}{a_0 T_0^2} \sum_{j=1}^{3} a_j \prod_{\substack{i=1 \\ i \neq j}}^{3} T_i^{-1}\,. \tag{15}$$

a_j are independent of q.

Integration over $d^{4-2\varepsilon}q$

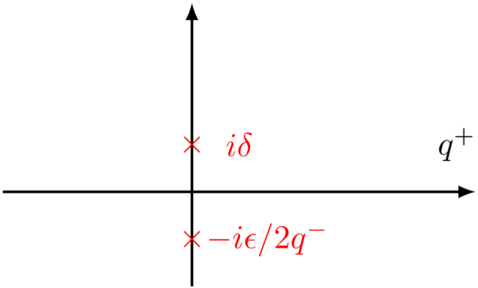
Momentum integrals: a general structure

- 2 or 3 propagators $(q-l_i)^2+i\epsilon$.
- At most one eikonal propagator $\frac{1}{n(q-l_j)}$.
- (In some terms) $q^\mu, q^\mu q^\nu$ in the numerator.

Within the dimensional regularization everything can be written using power functions and hypergeometric functions up to $\mathcal{O}(\varepsilon^0)$.

Pinched singularities due to the eikonal propagator:

$$\frac{1}{nq - i\delta}\frac{1}{q^2 + i\epsilon}\Big|_{q_\perp^2=0} = \frac{1}{q^+ - i\delta}\frac{1}{2q^+q^- + i\epsilon}\,. \tag{16}$$



We get pinched singularities as $|\delta| \to 0$.

$\to \frac{1}{\varepsilon}\log\delta,\ \log\delta,\ \log^2\delta$ singularities need to cancel out in the final result.

Integration over dk^-

Let us write

$$k^- = \frac{k_\perp^2}{2p^+} y \,, \qquad y \in \mathbb{R} \,. \tag{17}$$

The general structure of the integrals

$$\int dy \, \mathcal{F}(y; b, c) \, y^n \prod_j \frac{1}{y - a_j + i\epsilon \, \mathrm{sgn}(a_j)} \,, \tag{18}$$

where

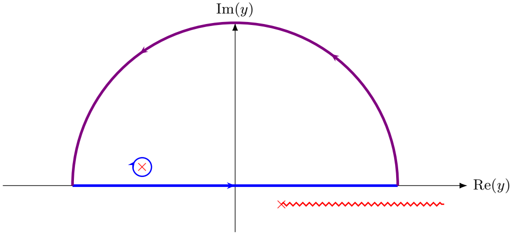
$$\mathcal{F}(y; b, c) = (1 - yb - i\epsilon)^{-\varepsilon} \, {}_2F_1 \left(1 + \varepsilon, -\varepsilon; 1 - \varepsilon; \frac{cy}{by - 1 + i\epsilon} \right) \,. \tag{19}$$

a, b, c - combinations of x, ξ.

The integrand can be written as a sum of:

$$\frac{1}{y - a_j + i\epsilon\,\mathrm{sgn}(a_j)}\,\mathcal{F}(y;b,c)\,.$$

The whole expression: $\sim y^{-2-\varepsilon}$,
individual terms: $\sim y^{-1-\varepsilon}$.



Contour integration: cannot neglect the integrals over the arcs (the violet path) at the level of individual integrals, but they cancel out when summed $\implies$ we can safely remove them.
The whole result can be written using:

$$\mathcal{I}(a,b,c) = \int_{\mathbb{R}} dy\,\frac{1}{y - a + i\epsilon\,\mathrm{sgn}(a)}\mathcal{F}(y;b,c)\; - \left[\int_{\text{arc}} \ldots dy\right]. \tag{20}$$

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Thank you

Backup: Integration over dk^-

The function $\mathcal{F}(y;b,c)$ is given by:

$$\mathcal{F}(y;b,c) = (1 - yb - i\epsilon)^{-\varepsilon}\,{}_2F_1\left(1+\varepsilon, -\varepsilon; 1-\varepsilon; \frac{cy}{by - 1 + i\epsilon}\right)\,. \tag{21}$$

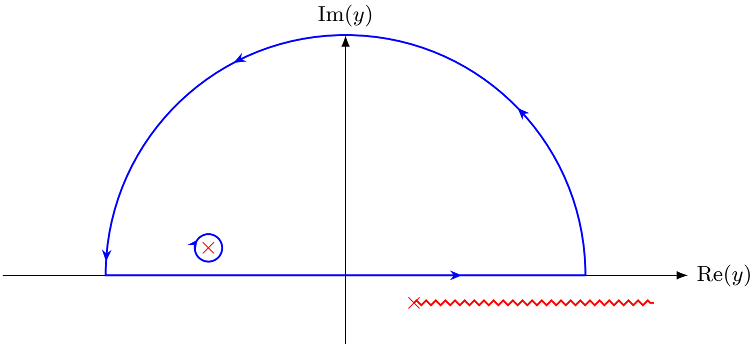
Special cases: $b = 0$ and $c = 0$. Note that ${}_2F_1(\alpha, \beta; \gamma; 0) = 1$.

How to integrate

$$\int dy\, \frac{1}{y - a + i\epsilon\,\mathsf{sgn}(a)}\,(1 - yb - i\epsilon)^{-\varepsilon}\,{}_2F_1\left(1+\varepsilon, -\varepsilon; 1-\varepsilon; \frac{cy}{by - 1 + i\epsilon}\right)\,? \tag{22}$$

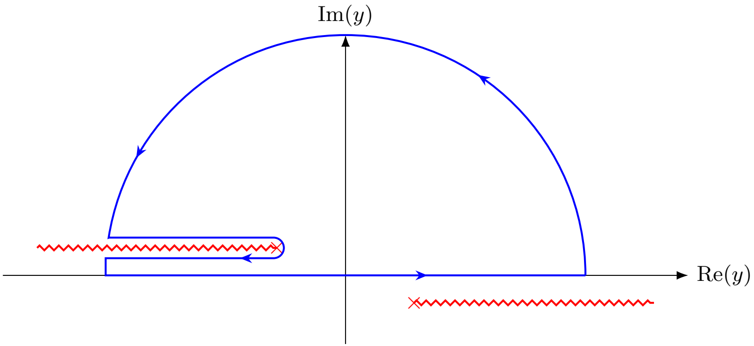
Backup: Integration over dk^-

$b = 0$ or $c/b < 1 \implies \mathcal{F}$ has a branch cut only on one half of the complex half-plane.
Close the contour avoiding the branch cut, use the Cauchy theorem.



Backup: Integration over dk^-

$b \neq 0$ and $c/b > 1 \implies \mathcal{F}$ has branch cuts on both half-planes.
Pick one half-plane, and close the contour avoiding the branch cut.

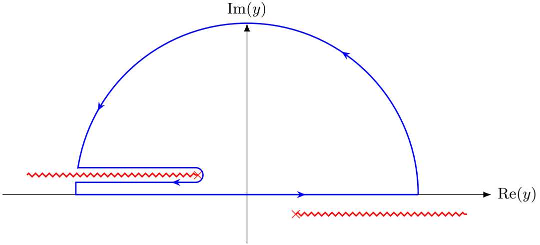


Backup: Integration over dk^-

Integral around the branch cut $\longrightarrow$ integral of the discontinuity along the real axis.

$$\mathrm{Disc}\big[f(z)\big] = \lim_{\epsilon\to 0^+}\big[f(z+i\epsilon) - f(z-i\epsilon)\big]\,. \tag{23}$$

$\mathrm{Disc}\big[\mathcal{F}(y;b,c)\big] \longrightarrow$ a product of two power functions $(y-y_1)^\alpha(y-y_2)^\beta$.



Backup: the hypergeometric function

$$ {}_2F_1\left(a,b;c;z\right) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(b-c)} \int_0^1 dt\, t^{b-1}(1-t)^{c-b-1}(1-zt)^{-a}\,. \tag{24}$$

There is a branch cut for $z \in [1, +\infty)$.
In our case, the Pfaff transformations (see: https://dlmf.nist.gov/15.8) allow one to obtain $a = 1$.

$$\begin{aligned} \text{Disc}\, {}_2F_1\left(1,b;c;z\right) &= \frac{\Gamma(c)}{\Gamma(b)\Gamma(b-c)} \int_0^1 dt\, t^{b-1}\,(1-t)^{c-b-1} \text{Disc}\,(1-zt)^{-1} \\ &= \frac{\Gamma(c)}{\Gamma(b)\Gamma(b-c)} \int_0^1 dt\, t^{b-1}\,(1-t)^{c-b-1}\, 2\pi i \delta(1-zt)\,. \end{aligned} \tag{25}$$