Conformal moments of two-loop coefficient functions in DVCS

P. Gotzler

University Regensburg

with V. M. Braun and A. N. Manashov

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- **Deeply Virtual Compton Scattering**: $\gamma^*(q)\ N(p) \longrightarrow \gamma(q')\ N'(p')$

Müller 94, Ji 96, Radyushkin 96

$$\mathcal{A}_{\mu\nu}(q,p,p') = i \int d^4x\, e^{-iqx} \langle p'|T\{j_\mu^{\rm em}(x) j_\nu^{\rm em}(0)\}|p\rangle\,.$$

- **The leading twist approximation**

$$\mathcal{A}_{\mu\nu} = -g^{\perp}_{\mu\nu} V + \epsilon^{\perp}_{\mu\nu} A + \text{power corrections}$$

- **Parametrize amplitudes in terms of Compton Form Factors (CFFs)**
  Diehl 03, Belitsky, Radyushkin 05

$$V = \frac{1}{2P_+}\bar{u}(p')\left[\gamma^+\mathcal{H}(\xi,Q,t) + \frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}\mathcal{E}(\xi,Q,t)\right]u(p).$$
$$A = \frac{1}{2P_+}\bar{u}(p')\left[\gamma^+\widetilde{\mathcal{H}}(\xi,Q,t) + \frac{\Delta_+}{2M}\widetilde{\mathcal{E}}(\xi,Q,t)\right]\gamma_5 u(p).$$

$$\xi = \frac{x_B}{2-x_B} + \mathcal{O}(1/Q^2), \qquad x_B = \frac{Q^2}{2p\cdot q}, \qquad P_\mu = \frac{(p+p')_\mu}{2}, \qquad \Delta_\mu = (q'-q)_\mu$$

CFFs can be factorized in terms of GPDs

GPD factorization

$$\mathcal{H}(\xi,t,Q) = \sum_q \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} C_q(x/\xi, \mu^2/Q^2, \alpha_s(\mu))\, H_q(x,\xi,t,\mu) + \int_{-1}^{1} \frac{\mathrm{d}x}{\xi^2} C_g(x/\xi, \mu^2/Q^2, \alpha_s(\mu))\, H_g(x,\xi,t,\mu),$$

and similarly for $\mathcal{E}$, and also for $\widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}$, but with different Coefficient functions $\widetilde{C}_q$, $\widetilde{C}_g$.

GPDs are normalized to PDFs in forward limit

$$H_q(x,0,0,\mu) = q(x,\mu), \qquad H_g(x,0,0,\mu) = xg(x,\mu)$$

Coefficient functions for quark and gluon

$$C_q = C_q^{(0)} + a_s C_q^{(1)} + a_s^2 C_q^{(2)} + \mathcal{O}(a_s^3),$$

$$C_g = a_s C_g^{(1)} + a_s^2 C_g^2 + \mathcal{O}(a_s), \qquad a_s = \alpha_s/4\pi,$$

LO and NLO CFs

$$C_q^{(0)} = \frac{e_q^2}{2}\left(\frac{1}{z} - \frac{1}{\bar{z}}\right),$$

$$C_q^{(1)} = \frac{e_q^2 C_F}{2z\bar{z}}\left\{-4Lz\ln(\bar{z}) + \bar{z}\ln^2(z) + 3\bar{z}\ln(z) - (z \leftrightarrow \bar{z}) - 3(1-2z)(-L+3)\right\}$$

$$C_g^{(1)} = \frac{\left(\sum_q e_q^2\right)T_F}{4z^2\bar{z}^2}\left\{-2Lz^2\ln(z) - z^2\ln^2(z) + 2z(1+z)\ln(z) + (z \leftrightarrow \bar{z})\right\}$$

Ji, Osborne, 98, Noritzsch, 04

$$z = \tfrac{1}{2}(1-x), \qquad L = \ln(Q^2/\mu^2), \qquad \bar{z} = 1 - z.$$

Braun, Ji, Schönleber, 2022

$$C_q^{(2)} = \frac{1}{2z\bar{z}}\left[e_q^2 C_F\left(C_F C_{NS}^{(F)} + C_A C_{NS}^{(A)} + \beta_0 C_{NS}^{(\beta_0)}\right) + \left(\sum_{q'} e_{q'}^2\right) T_F C_F C_{PS}\right]$$

$$C_g^{(2)} = \frac{\left(\sum_q e_q^2\right)}{4z^2\bar{z}^2} T_F\left(C_F C_g^{(F)} + C_A C_g^{(A)}\right).$$

$H_{\vec{m}}$ – Harmonic Polylogarithms

$$\begin{aligned} C_{\mathrm{PS}} = L^2\Big[&- 8(z-1)H_{1,0} + 4H_1\left(4z^2-5z+1\right) + 4H_0 z(4z-3) - 8H_2 z + 8\zeta_2 z\Big] \\ &- 8L\Big[z(4z-3)(H_2 - H_{0,0}) + (z-1)(4z-1)(H_{1,1} - H_{1,0}) - 2zH_{2,1} \\ &+ 2(z-1)(H_{1,0,0} - H_{1,1,0}) - \left(2\zeta_2 - 3\right)H_1(z-1) - 3H_0 z + 2H_3 z - \zeta_2 z(4z-3)\Big] \\ &+ 16(z-1)\left(-H_{1,0,0,0} + H_{1,1,0,0} + \tfrac{1}{2}H_{1,1,1,0}\right) + 8z(4z-3)(H_{0,0,0} + H_{2,1}) \\ &+ 8(z-1)(4z-1)\left(H_{1,0,0} + H_{1,1,1}\right) - 4(z-1)(4z+5)H_{1,1,0} \\ &+ 16z\left(-H_{2,1,1} + \tfrac{3}{2}H_{0,0} + H_{3,1} - zH_{2,0}\right) - 8\left(\zeta_2 - 3\right)(z-1)H_{1,1} \\ &- 16(z-1)^2 H_{1,2} + 4(1-z)H_{1,0} - 8H_0 z\left(2\zeta_2 z + 5\right) - 4H_3 z(4z-9) - 8H_4 z \\ &- 4H_1(z-1)\left(2\left(\zeta_3 - 5\right) + \zeta_2(4z+5)\right) - 4H_2 z + 4z\left(2\zeta_2^2 + \zeta_2 - 3\zeta_3(4z+1)\right). \end{aligned}$$

Mellin-Barnes representation for CFFs

Müller, 2006, Müller, Kumerički, Passek-Kumerički 2007

$$\mathcal{F} = \int_C \frac{dj}{2\xi} \frac{(i\xi)^{-j}}{\cos\frac{\pi j}{2}} C_{F,j}(Q^2/\mu^2, a_s(\mu))\, F_j(\xi, t, \mu), \qquad \mathcal{F} \in \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$$

$$C_{q,N} = 2\int_0^1 dz\, z\bar{z}\, G_{N-1}^{(3/2)}(1-2z)\, C_q(z) = 1 + a_s\, C_{q,N}^{(1)} + a_s^2\, C_{q,N}^{(2)} + \ldots$$

$$C_{g,N} = 6\int_0^1 dz\, z^2\bar{z}^2 G_{N-2}^{(5/2)}(1-2z)\, C_g(z) = a_s C_{g,N}^{(1)} + a_s^2 C_{g,N}^{(2)} + \ldots$$

$N = j-1$ - Lorentz spin of local operator $G_j^{(\alpha)}$ - Gegenbauer Polynomials

$F_j(\xi, t, \mu)$ - Conformal moment of GPD F $C_{F,j}(Q^2/\mu^2)$ - Conformal moment CF

- Observe that $G_N^{(3/2)}(z) := 2z\bar{z}G_{N-1}^{(3/2)}(1-2z)$ are eigenfunctions of $\mathrm{SL}(2,\mathrm{R})$ invariant operators $\mathbb{H}$:

$$\int_0^1 dz' \mathbb{H}(z,z') G_N^{(3/2)}(z') = E_N G_N^{(3/2)}(z).$$

- If $M_N[f] = \int_0^1 dz f(z) G_N^{(3/2)}(z)$ is Gegenbauer moment of a function $f(z)$, then the Gegenbauer moment of

$$f^{\mathbb{H}}(z) = \int_0^1 dz' f(z') \mathbb{H}(z',z)$$

is given by the product

$$M_N[f^{\mathbb{H}}] = M_N[f] E_N.$$

- Can use this to construct a basis of functions $f^{\mathbb{H}}$ with known Gegenbauer moments.
- Explicit expressions for integral kernels $\mathbb{H}(z',z)$ difficult $\Rightarrow$ Position Space!

- Action of $\mathrm{SL}(2,\mathrm{R})$ invariant operator on a function of two position space variables

$$[\mathbb{H}^{(h)}\varphi](w_1,w_2) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta\, h(\tau)\, \varphi(w_{12}^{\alpha}, w_{21}^{\beta}), \quad w_{12}^{\alpha} = w_1\bar{\alpha}+w_2\alpha,\, \tau = \alpha\beta/(\bar{\alpha}\bar{\beta})$$

- Eigenfunctions satisfy $[\mathbb{H}^{(h)}\Psi_N^p](w_1,w_2) = E_N^{(h)}\Psi_N^p(w_1,w_2)$ and can be chosen as

$$\Psi_N^p(w_1,w_2) = \int_0^1 dz\, e^{-ip(w_1 z+w_2\bar{z})} G_N^{(3/2)}(z).$$

- Eigenvalues are $E_N^{(h)} = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta\, h(\tau)(1-\alpha-\beta)^{N-1}$ and do not depend on p.
- Integrating over p one obtains eigenfunctions of the form

$$\Psi_N^{(f)}(w_1,w_2) = \int_0^1 dz f(w_1 z + w_2\bar{z}) G_N^{(3/2)}(z).$$

- Applying $\mathbb{H}^{(h)}$ and sending $w_1 \to 1$, $w_2 \to 0$, get

$$f^h(z) = [\mathbb{H}^{(h)} f](z) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta\, h(\tau)\, f(z\bar{\alpha}+\bar{z}\beta).$$ **HyperInt, E.Panzer, 2014**

- Gegenbauer moments of f and f^h are related as

$$M_N[f^h] = E_N^{(h)}\, M_N[f].$$

- Want to find Gegenbauer moments of functions

$$F_m^{\pm}(z) = \frac{1}{z}H_m(z) \pm \frac{1}{\bar{z}}H_m(\bar{z}), \quad \bar{F}^{\pm}(z) = \frac{1}{\bar{z}}H_m(z) \pm \frac{1}{z}H_m(\bar{z}),$$

where $m = \{m_1, ..., m_t\}, m_k \in \{0, 1\}$ and H_m are HPL.

- Define $M_m^{\pm}(N) = M_N[F_m^{\pm}]$, $\bar{M}_m^{\pm}(N) = M_N[\bar{F}_m^{\pm}]$ are nonzero for odd (even) N.
- Calculate Gegenbauer moments for functions of weight one, $m = 0$: Apply $\mathbb{H}^{(1)}$

$$\mathbb{H}^{(1)} F_{\{\}}^{\pm} = \mathbb{H}^{(1)} \left(\frac{1}{z} \pm \frac{1}{\bar{z}} \right) = - \left(\frac{\ln z}{\bar{z}} \pm \frac{\ln \bar{z}}{z} \right) = -\bar{F}_{\{0\}}^{\pm}(z).$$

- Eigenvalues are $E_N^{(1)} = 1/N/(N+1)$, so that

$$\bar{M}_{\{0\}}^{\pm} = -\frac{1}{N(N+1)}.$$

- A special invariant operator $\widehat{\mathbb{H}}$ $(h(\tau) = \delta_+(\tau))$ has eigenvalue $2S_1(N)$ and

$$\widehat{\mathbb{H}} F_{\{\}}^{\pm}(z) = -\frac{\ln z + \ln \bar{z}}{z\bar{z}} = -F_{\{0\}}^{\pm}(z) - \bar{F}_{\{0\}}^{\pm}(z)$$

which gives

$$M_{\{0\}}^{\pm}(N) + \bar{M}_{\{0\}}^{\pm}(N) = -2S_1(N).$$

For higher weight apply operators $\mathbb{H}^{(1,\delta_+)}$ again to weight one functions and so forth:

- Weight 1: kernels $1, \delta_+ \leftrightarrow \frac{1}{N(N+1)}, 2S_1(N)$.
- Weight 2:
  $1 \otimes 1, \delta_+ \otimes \delta_+, 1 \otimes \delta_+, \bar{\tau} \leftrightarrow \frac{1}{N^2(N+1)^2}, (2S_1(N))^2, \frac{2S_1(N)}{N(N+1)}, (-1)^N[2S_{-2}(N) + \zeta_2]$
- Iteratively to desired weight. At weight four need a 16 independent basis functions, equivalently invariant operators.
- Kernels needed at higher weight are linear combinations of

$$h(\tau) = \left\{ \bar{\tau} H_m(\tau), \frac{\bar{\tau}}{\tau} H_m(\tau) \right\}$$

$\Rightarrow$ Express CFs as linear combinations of basis functions with known Gegenbauer moments.

Conformal moment of pure singlet contribution

$$
\begin{aligned}
C^{(2)}_{q,\mathrm{PS}} = & \frac{32}{(N-1)(N+2)}\left(2S_1^2 - S_{-2} - 4S_1 - \zeta_2 + 3\right) + H\Big(-32S_1S_{-2} - 16(2S_{-2,1} - S_{-3}) \\
& - 48S_1^2 - 24S_{-2} + 144S_1 - 16\zeta_2 S_1 - 8\zeta_3 + 4\zeta_2 - 84 \\
& - H\left(32S_1^2 - 16S_{-2} - 160S_1 - 8\zeta_2 + 140\right) + 4H^2(16S_1 - 33) - 40H^3\Big) \\
& + 16L\left(-\frac{4(S_1-1)}{(N-1)(N+2)} + H\left(2S_{-2} + \left(2H+3\right)S_1 + \zeta_2 - 2H^2 - 5H - \frac{9}{2}\right)\right) \\
& + 4L^2\left(\frac{4}{(N-1)(N+2)} - 2H^2 - 3H\right),
\end{aligned}
$$

$S_a =$ Harmonic Sums, $H = \frac{1}{N(N+1)}$

What's different for the gluon?

- Gegenbauer Moments calculated as

$$M_N[f] = \int_0^1 dz f(z) G_N^{(5/2)}(z), \quad G_N^{(5/2)}(z) = 6(z\bar{z})^2 G_{N-2}^{(5/2)}(1-2z).$$

- CFs are combinations

$$G_m^{\pm}(z) = \frac{1}{z^2} H_m(z) \pm \frac{1}{\bar{z}^2} H_m(\bar{z}), \quad \bar{G}_m^{\pm}(z) = \frac{1}{\bar{z}^2} H_m(z) \pm \frac{1}{z^2} H_m(\bar{z}).$$

- Invariant kernels can be written as $(1-\alpha-\beta)h(\tau)$, so that action and eigenfunctions are given by

$$[\mathbb{H}^{(h)}\varphi](w_1,w_2) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta\, (1-\alpha-\beta) h(\tau) \varphi(w_{12}^{\alpha}, w_{21}^{\beta})$$

$$\Psi_N^p(w_1,w_2) = \int_0^1 dz e^{-ip(w_1 z + w_2 \bar{z})} G_N^{(5/2)}(z)$$

$$f^h(z) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta\, (1-\alpha-\beta) h(\tau) f(w_{12}^{\alpha}, w_{21}^{\beta}).$$