Joint Global PDF Analysis of Experimental and Lattice Pion Data

William Good
In Collaboration with:
Patrick Barry (ANL), Huey-Wen Lin (MSU), Wally Melnitchouk (JLab), Alex NieMiera (MSU), Nobuo Stato (JLab)

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QCD Evolution 2026

Based on arXiv:2507.22730

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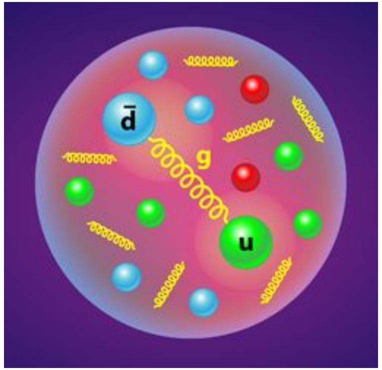
Intro and Motivation

Introduction and Background

- The gluon parton distribution function (PDF) in the pion describes the probability of finding a gluon, carrying a momentum fraction x of the pion's total momentum



APS/Alan Stonebraker

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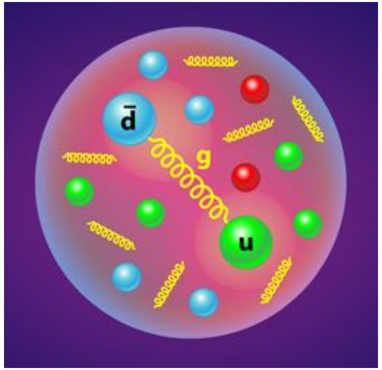

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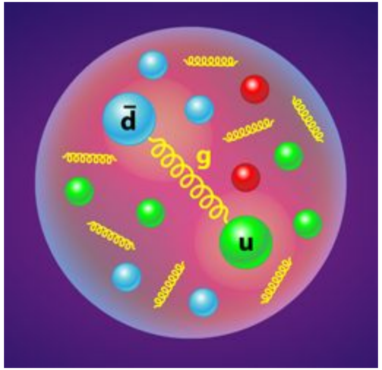

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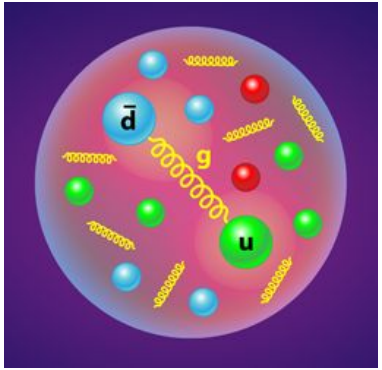

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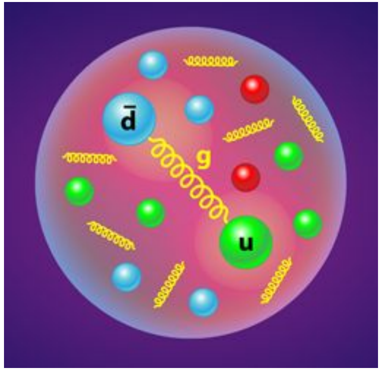

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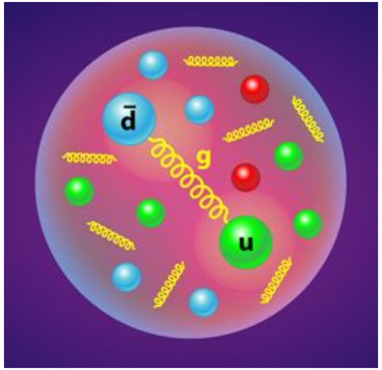
- I will present MSULat and JAM's recent work in constraining the gluonic structure of the pion



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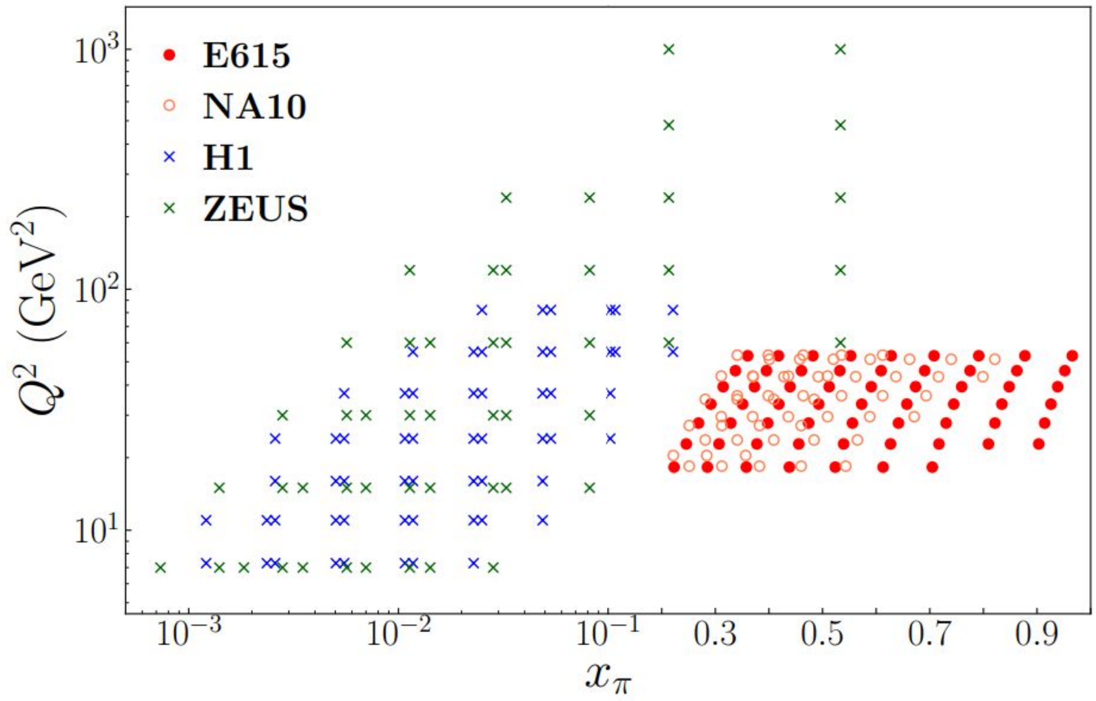

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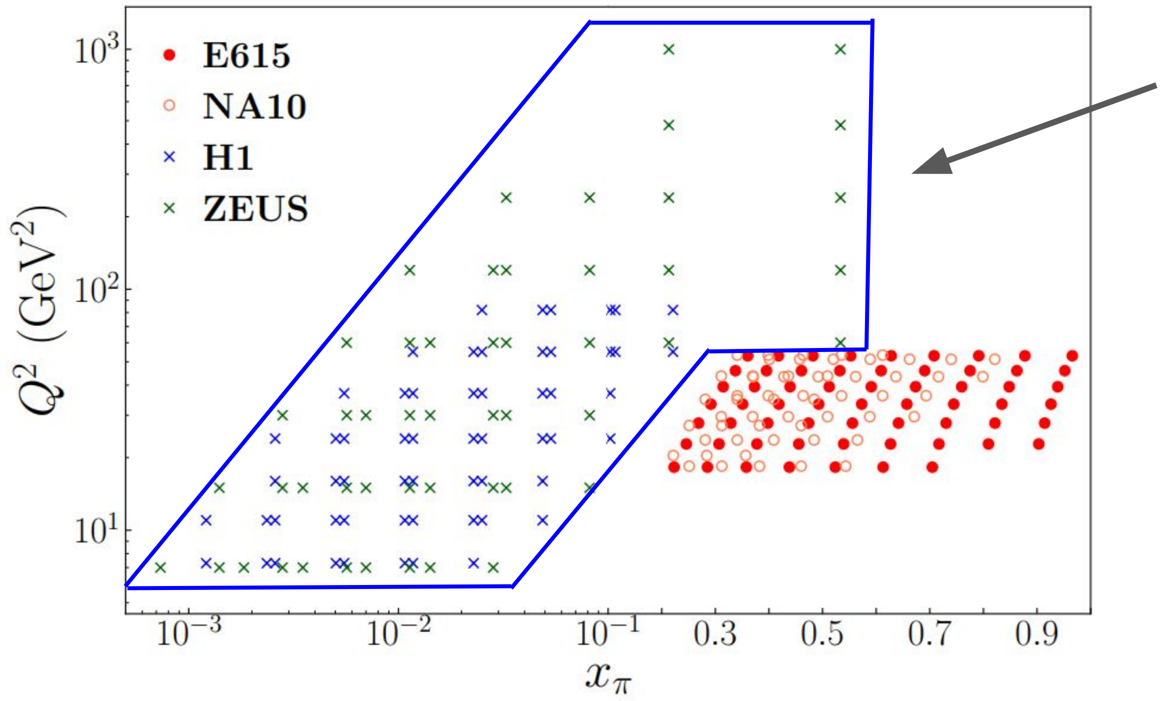

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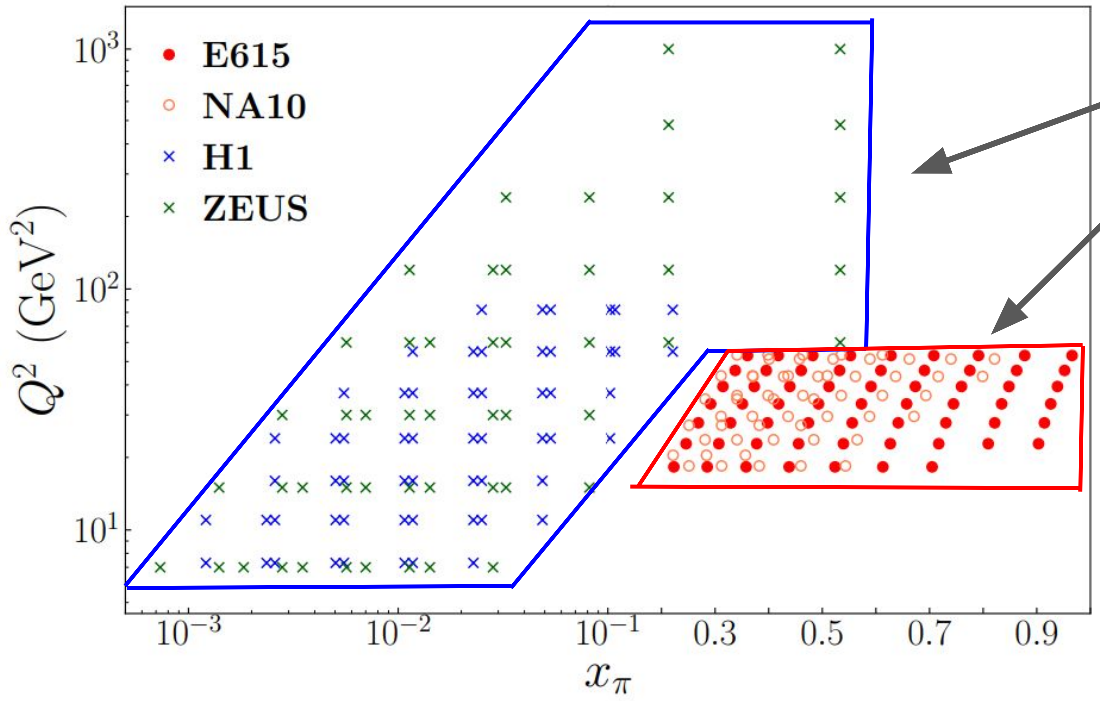

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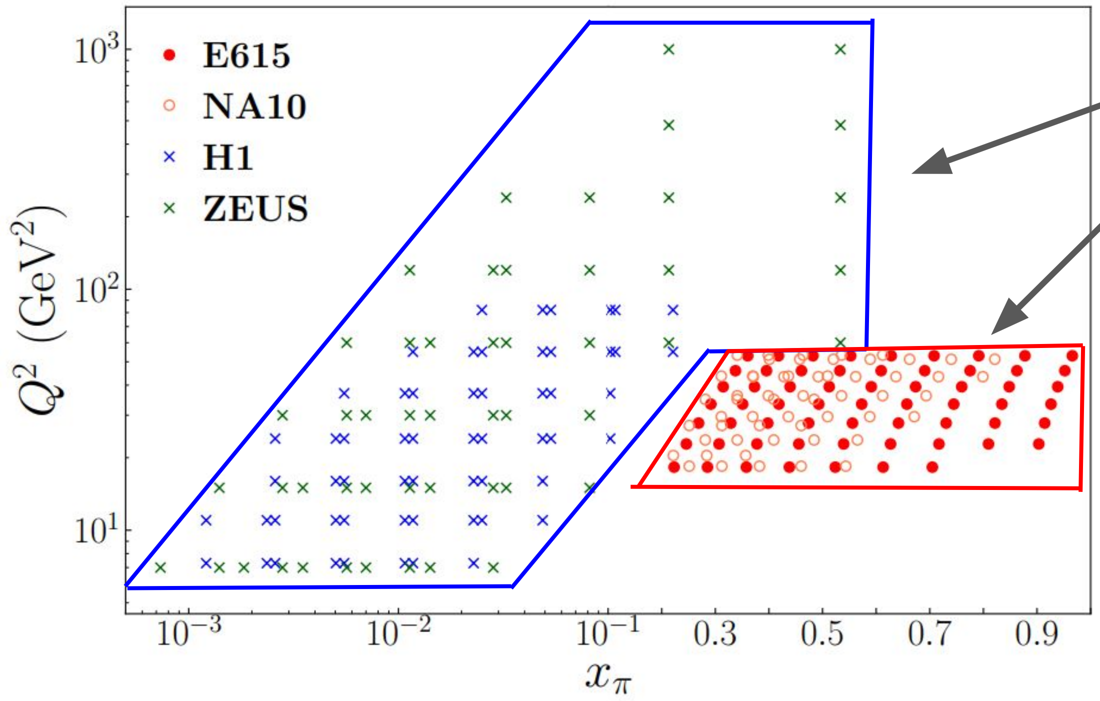

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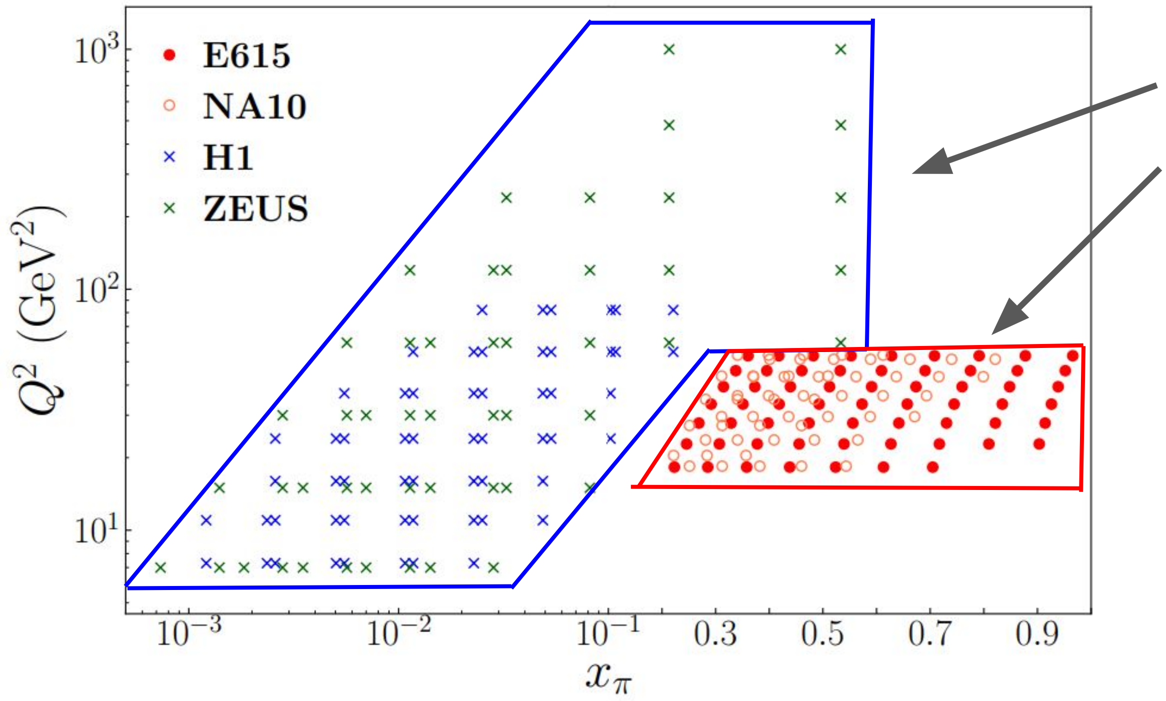

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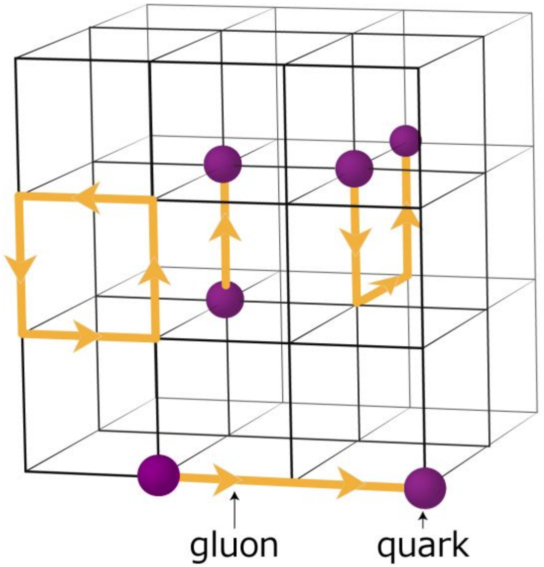
Lattice should compliment the LN data well because it has more constraining power on the intermediate-x range!

Framework and Methodology

Lattice QCD



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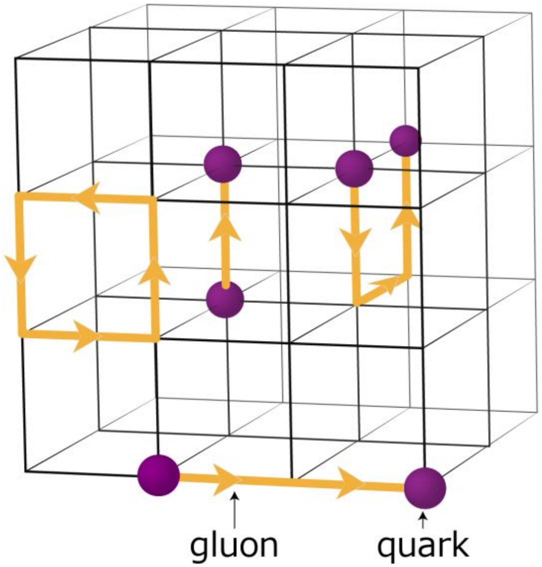

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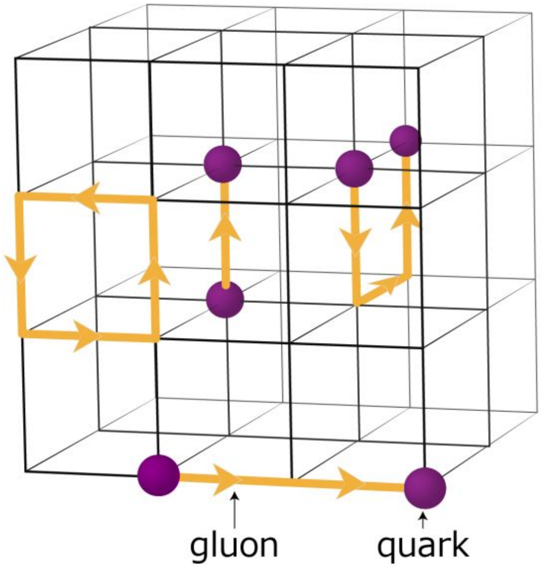

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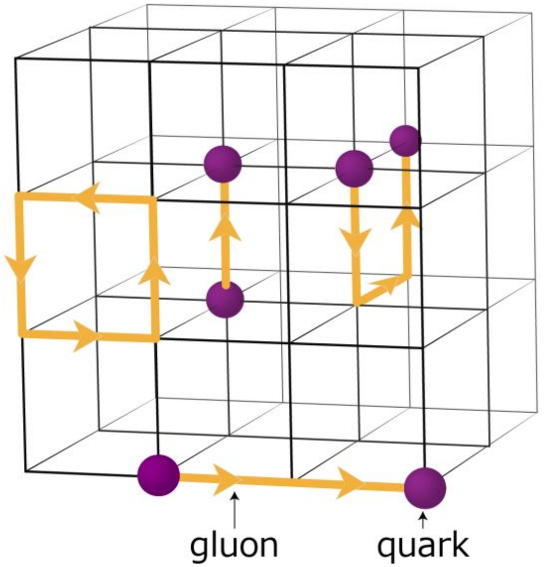

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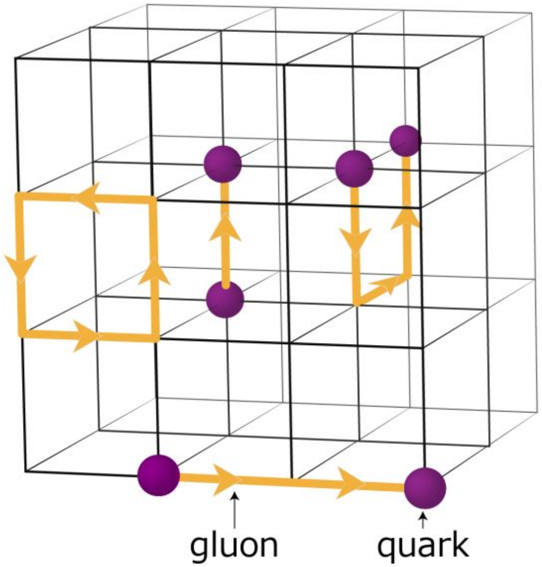

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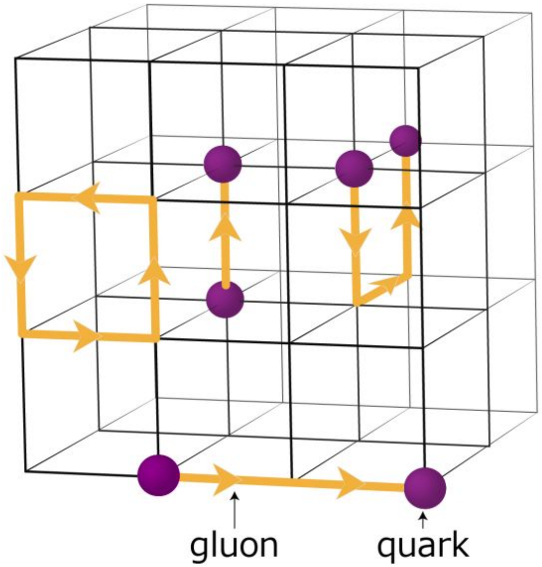

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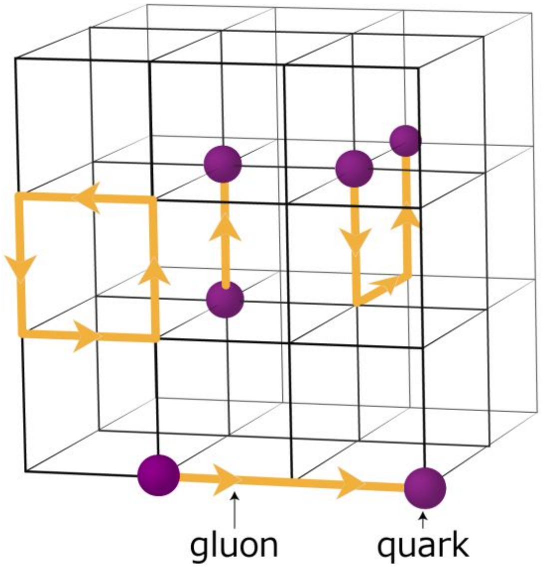

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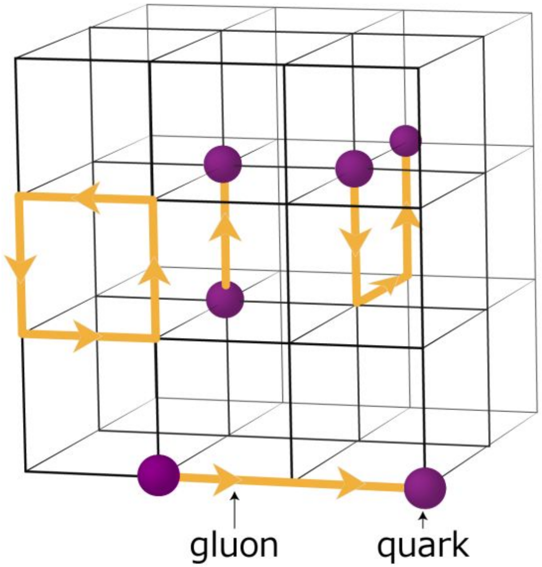

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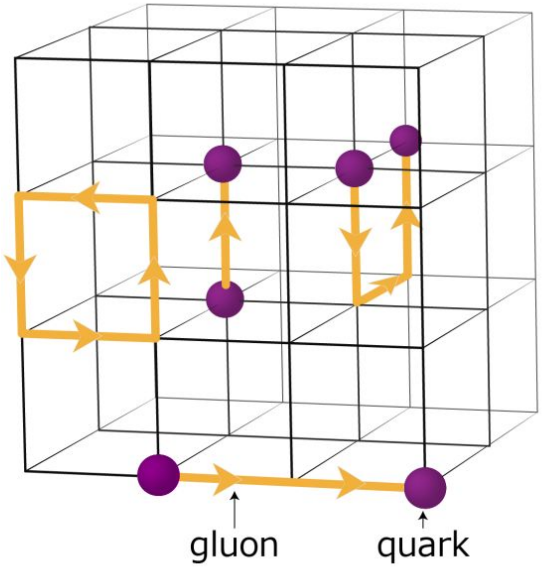

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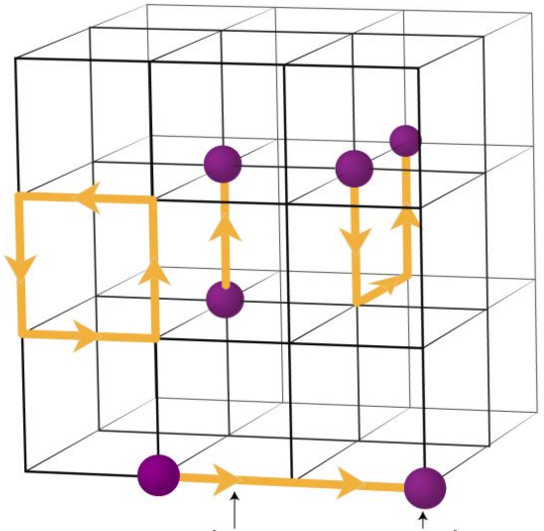

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  - Because of limited signal at long distances, the pseudo-PDF approach has been applied more widely to gluon PDFs than LaMET, and we focus on the pseudo-PDF approach, as it allows the lattice data to be treated just like experimental data

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Pseudo-PDF for Global Fitting

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Perturbative matching kernels: $R_{gg}(x\nu, z^2) + R_r(x\nu, z^2)$, $R_{gq}(x\nu, z^2)$

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Lattice data

Non-perturbative PDFs and moments

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- Although it is a little jumbled up, we have a perfect relationship to use for global fitting, which is sensitive to the gluon and sea quark PDFs!

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Same form as in: **P. C. Barry, et al., PRD 105:114051 (2022)**

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Same form as in: **P. C. Barry, et al., PRD 105:114051 (2022)**

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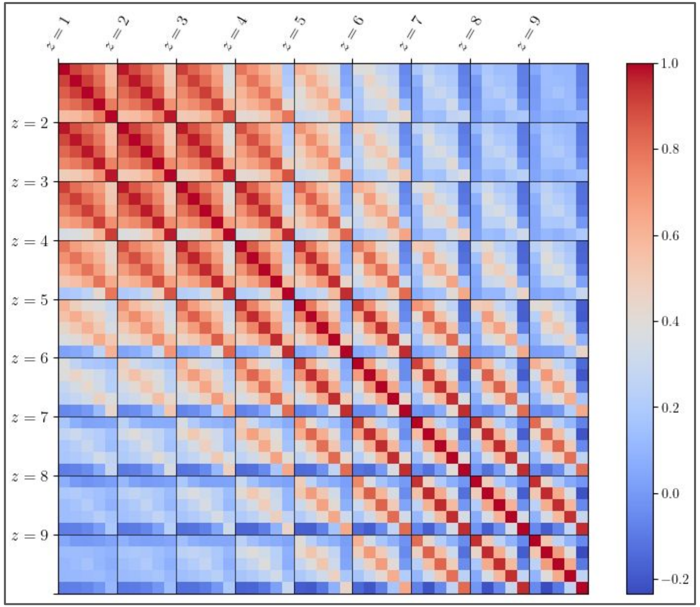

Correlations Are Important

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$$\chi^2_{\mathrm{Lat}} = (\mathbf{d} - \mathbf{t})^T \mathbf{\Sigma}^{-1} (\mathbf{d} - \mathbf{t})$$

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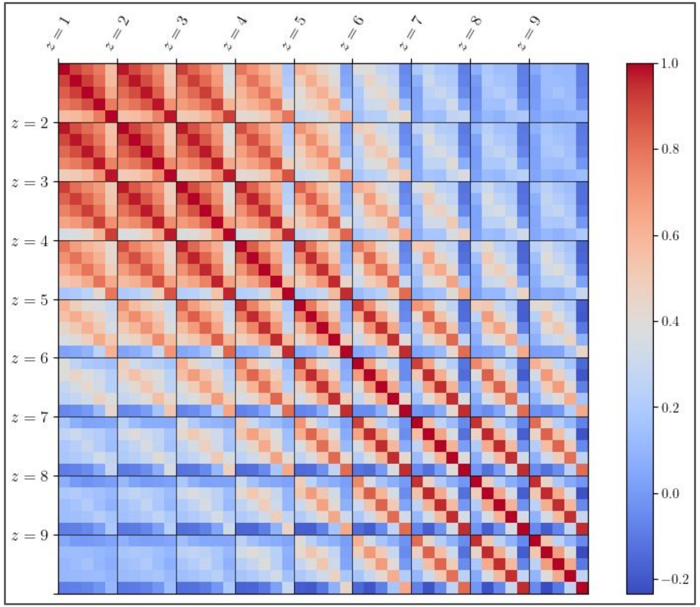

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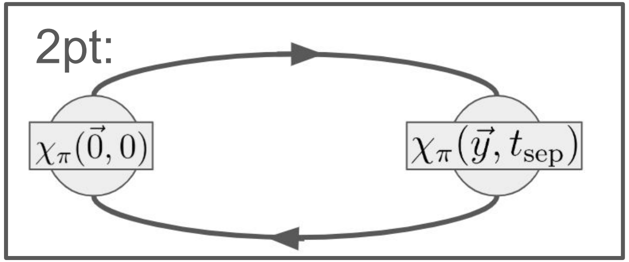

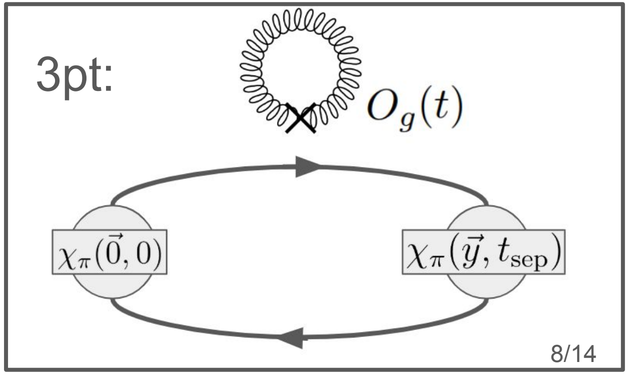

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$$M(\nu = zP_z, z^2) = \langle \pi(P_z)|O_g(z)|\pi(P_z)\rangle$$

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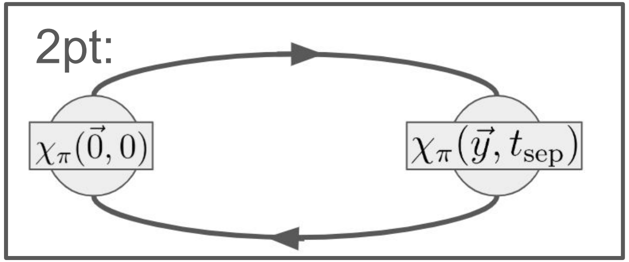

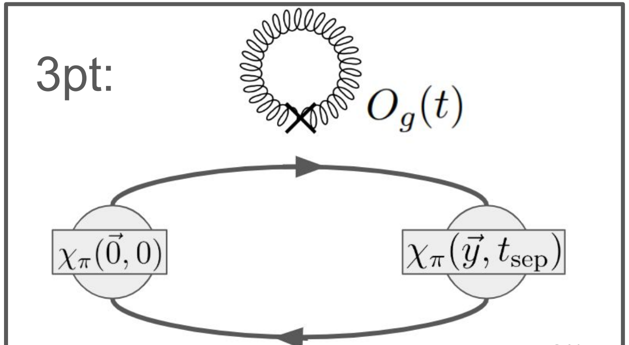

Results

Lattice and Experimental Data Fits

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- We take the $\mathcal{O}(800)$ replicas of original experiment only PDF fits, resample the experimental and lattice data and minimize $\chi^2_{\text{Expt}} + \chi^2_{\text{Lat}}$ with and without the systematic terms: $z^2 B_1(\nu) + \frac{1}{z} P_1(\nu)$

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- We consider only the fit with systematic terms, moving forward

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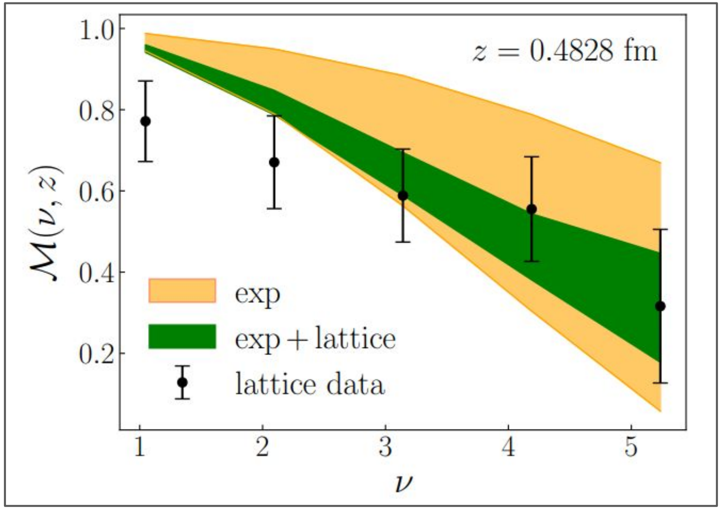

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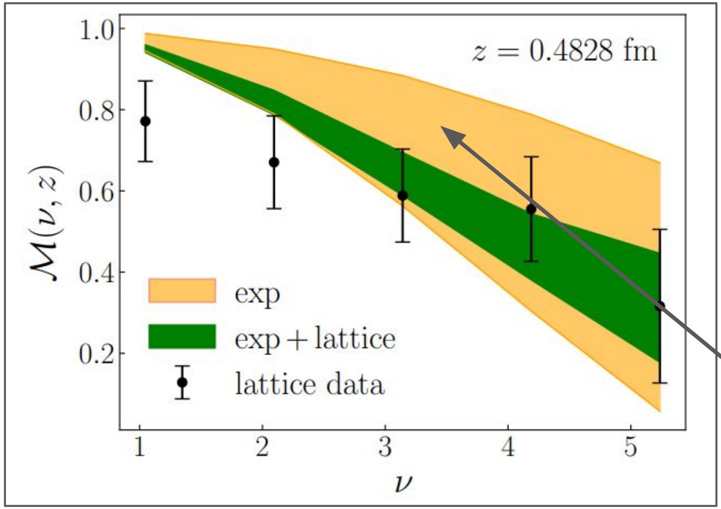

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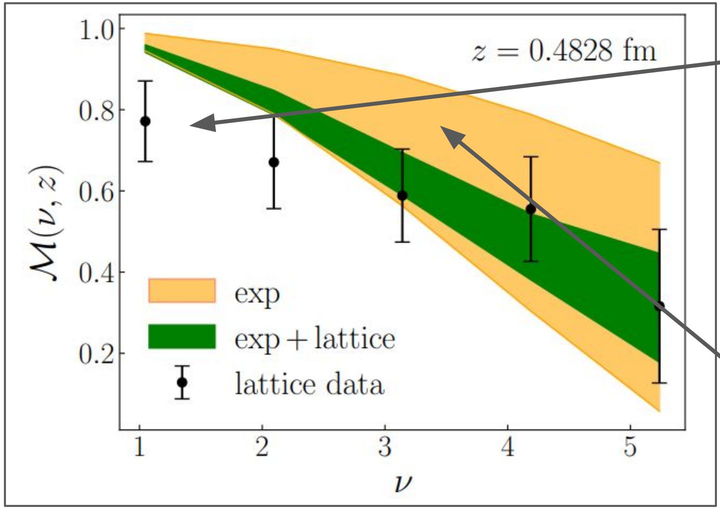

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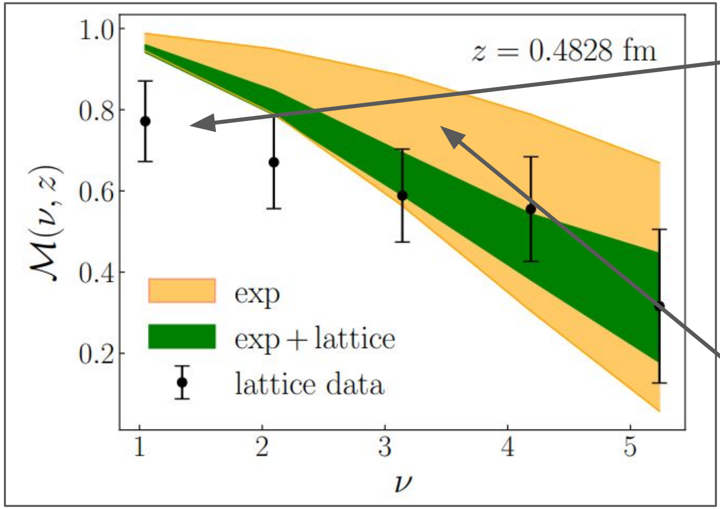

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- This is a correlated fit, so we have to look at agreement in another light…

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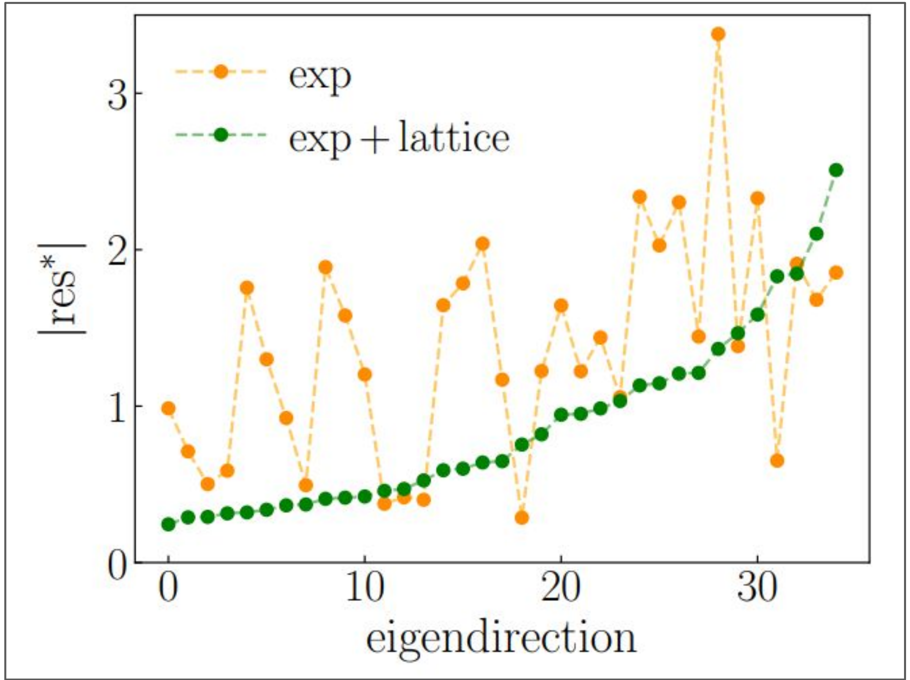

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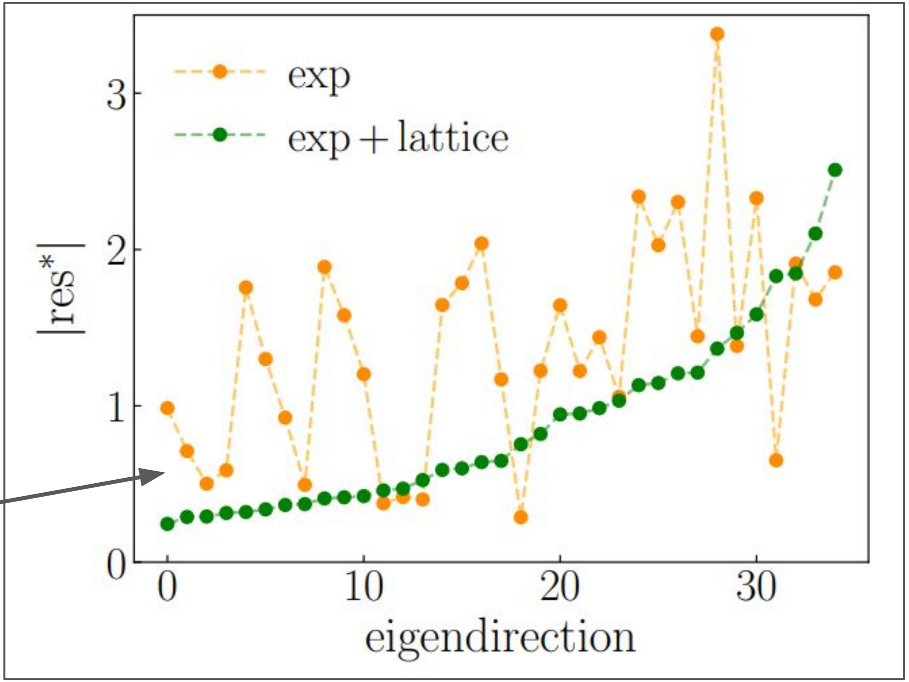

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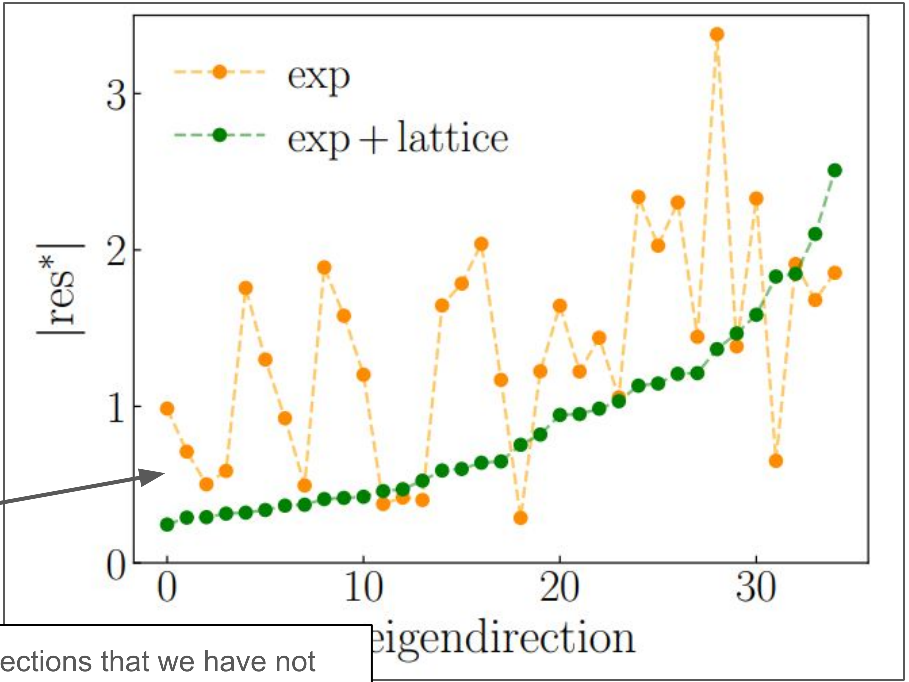
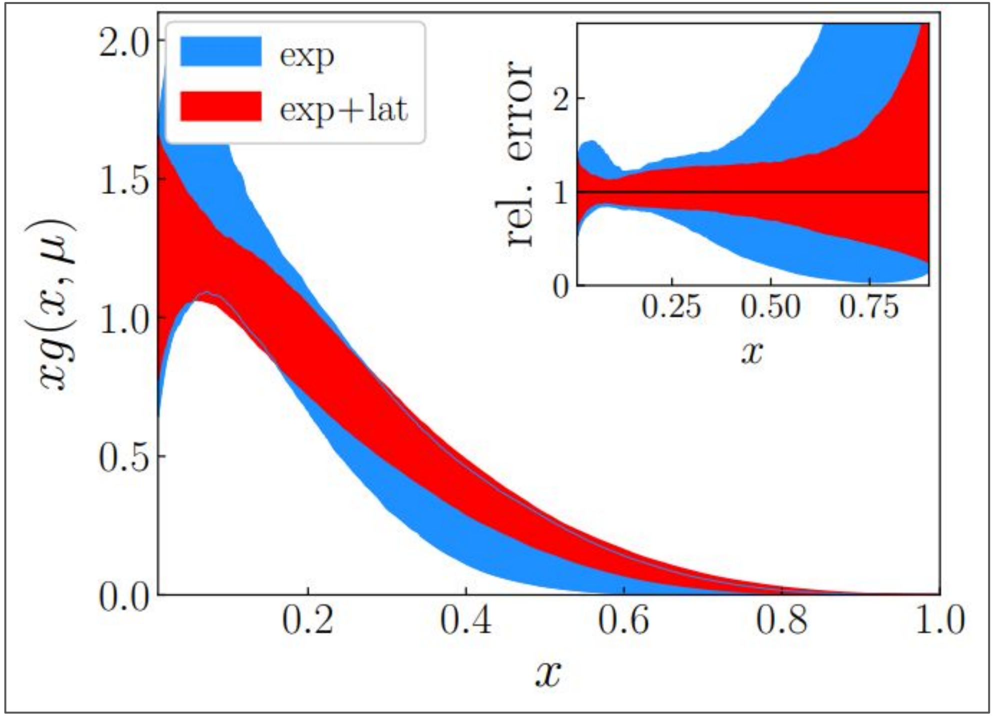


Note: The physical kinematics are so evenly distributed across the eigendirections that we have not found any way to interpret what agreement and disagreement in the eigenspace means physically

PDFs


exp
exp+lat
$xg(x,\mu)$
2.0
1.5
1.0
0.5
0.0
0.2
0.4
0.6
0.8
1.0
x
rel. error
2
1
0
0.25
0.50
0.75
x

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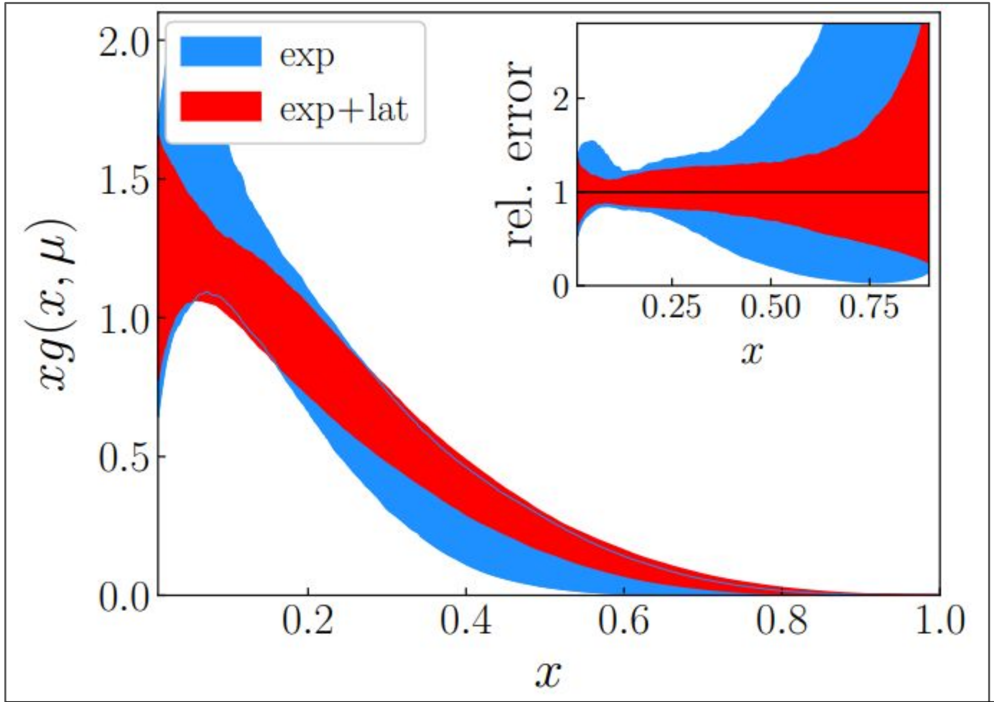

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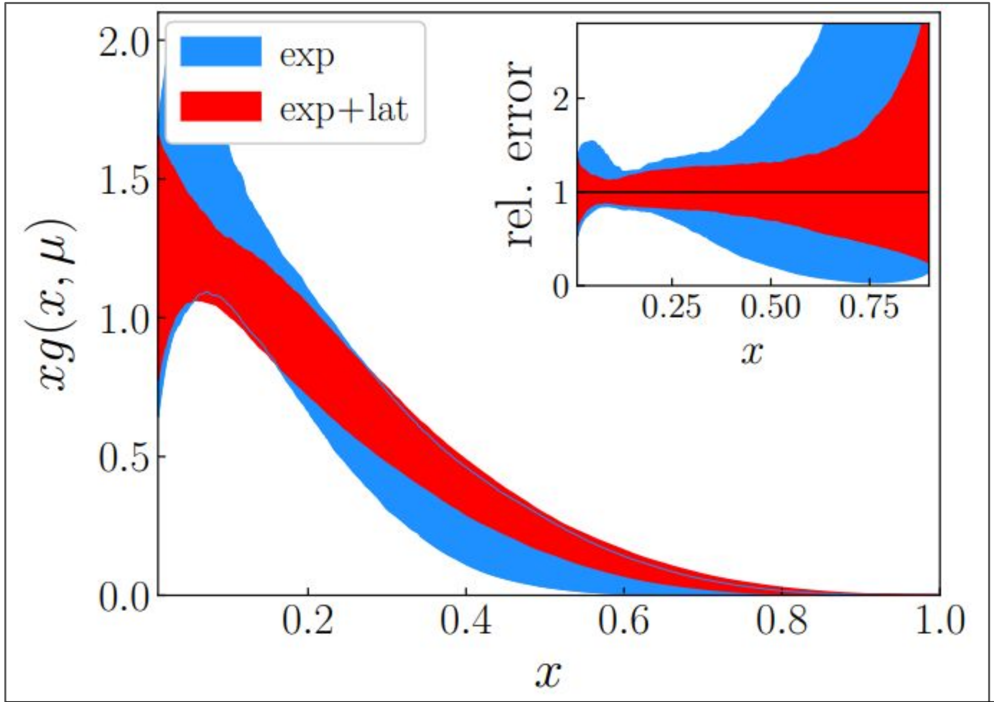

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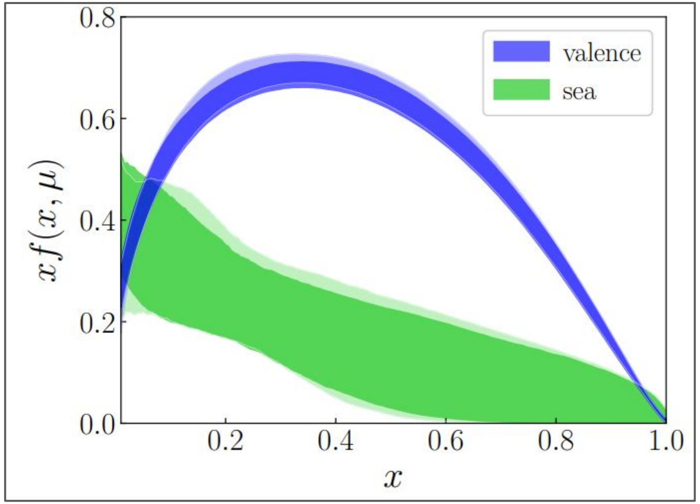

Again, the quarks are not changed very much (lighter color is expt. only, darker is expt + lat)

Gluon Behavior in the Pion vs the Proton

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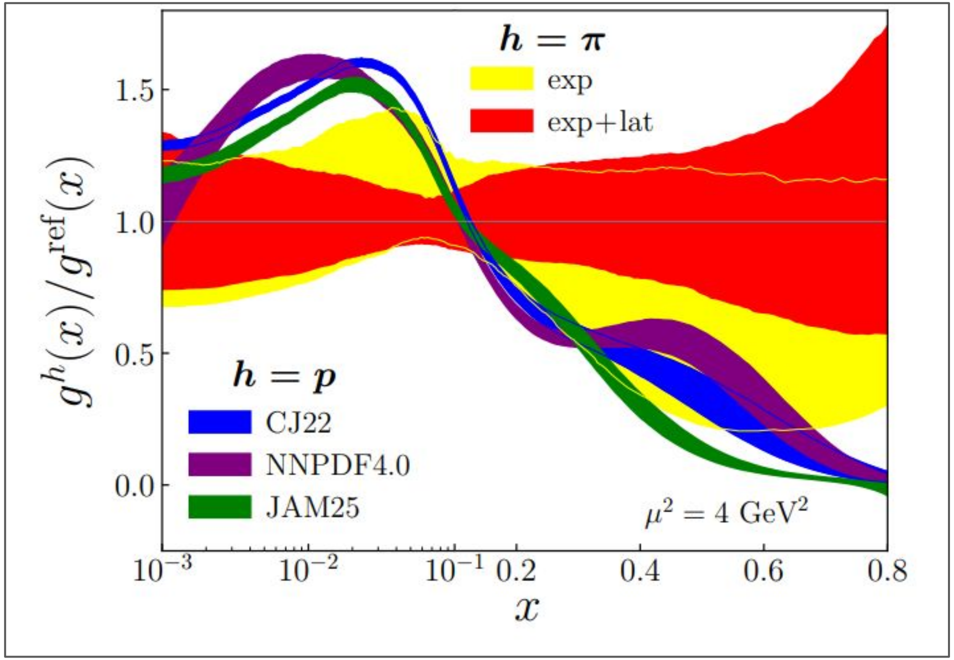

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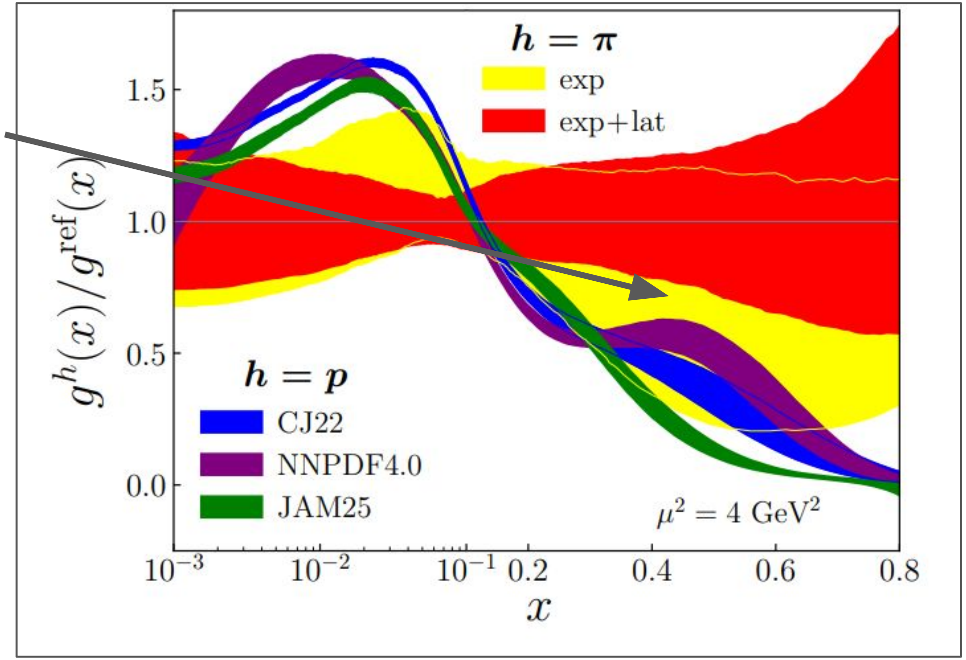

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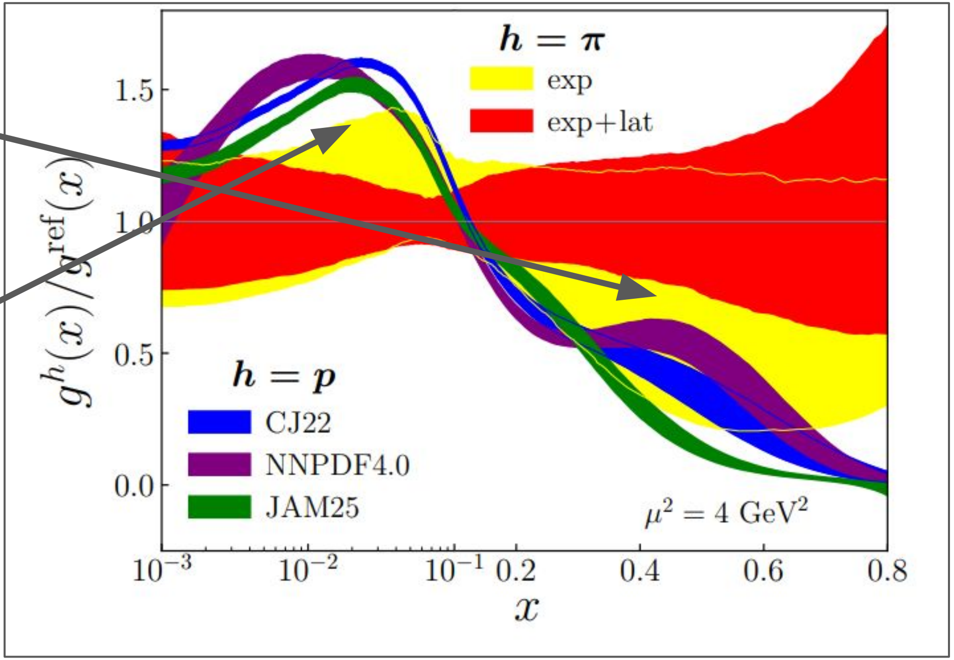

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Ji, PRL 74:1071 (1995)

Gluon Behavior in the Pion vs the Proton

Anderson, *et al.* arXiv 2501.00665 (2024)
Hou, *et al.* (CTEQ) PRD 103(1):014013 (2021)
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- We compare the gluon PDF of the pion and several proton studies
- Lattice data allows us to definitively say that the pion has a higher gluon density in the large-x than the proton
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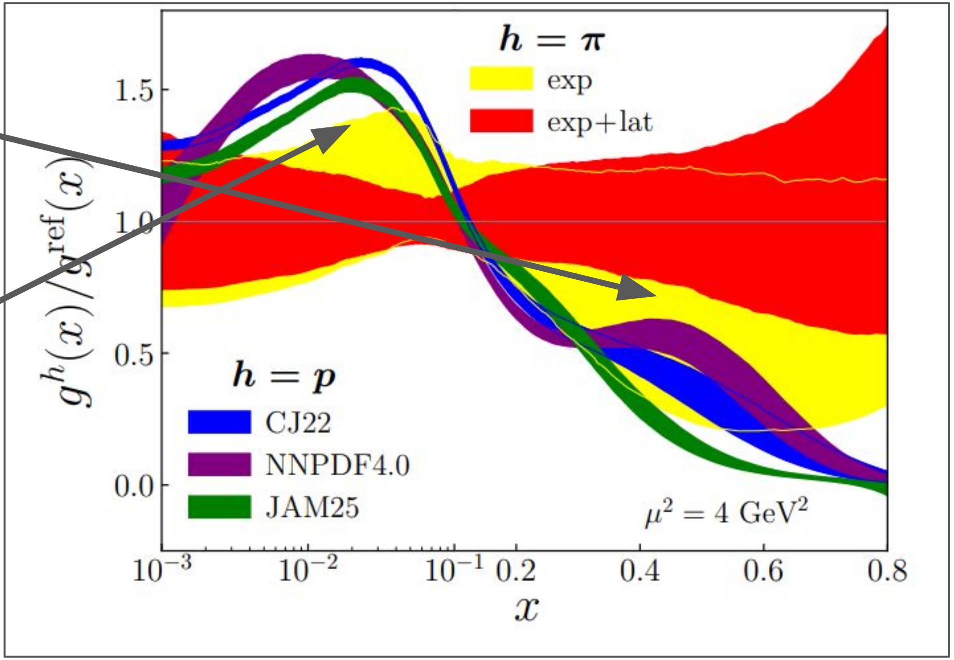

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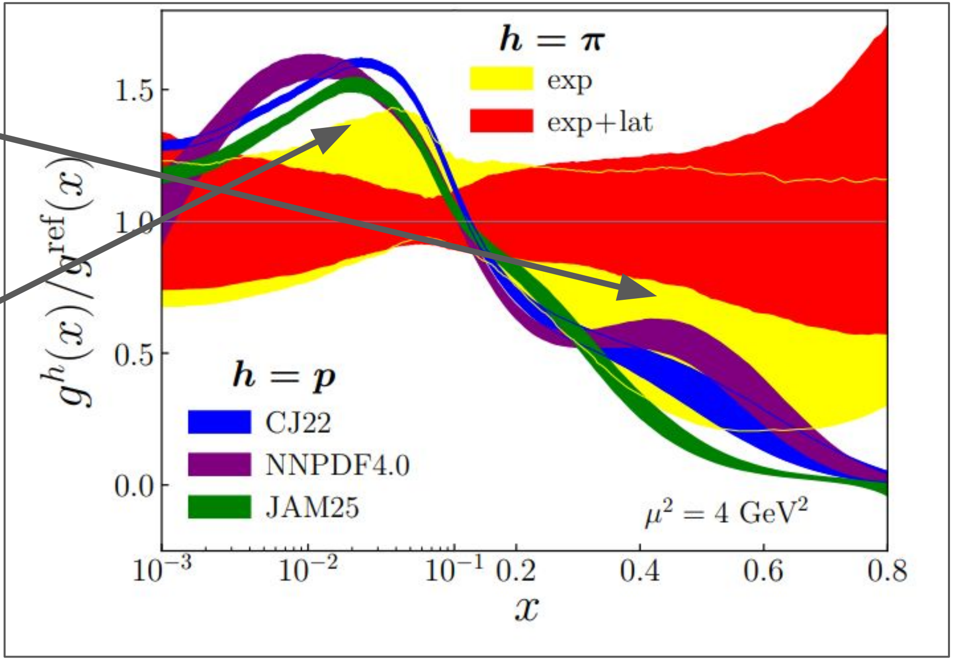

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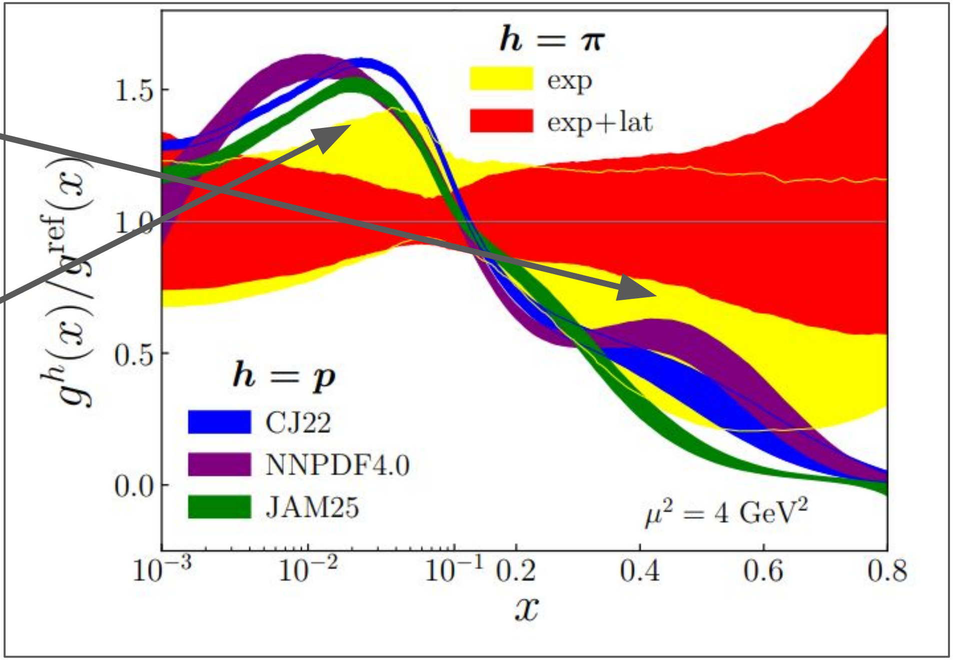

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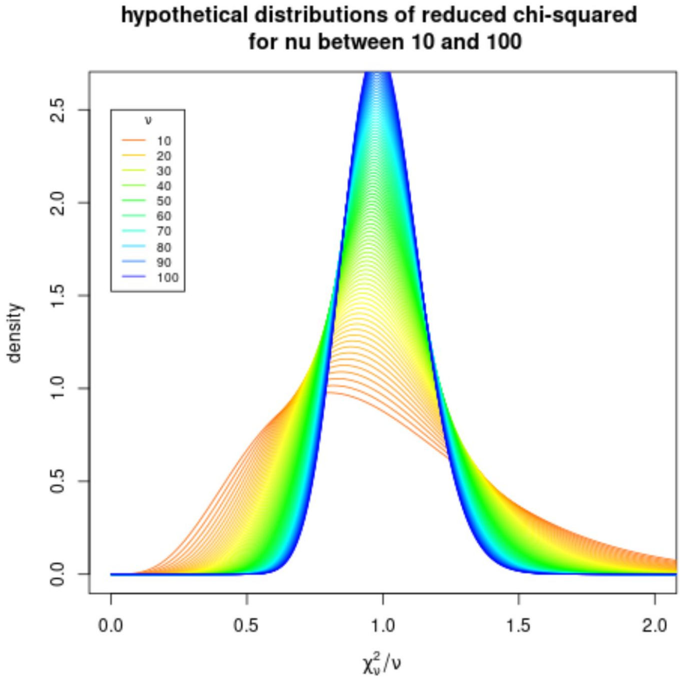
- It would be interesting to understand this from the underlying physics
- Lattice systematics like pion mass and lattice spacing should be controlled further in future studies

Backup

Z-Score

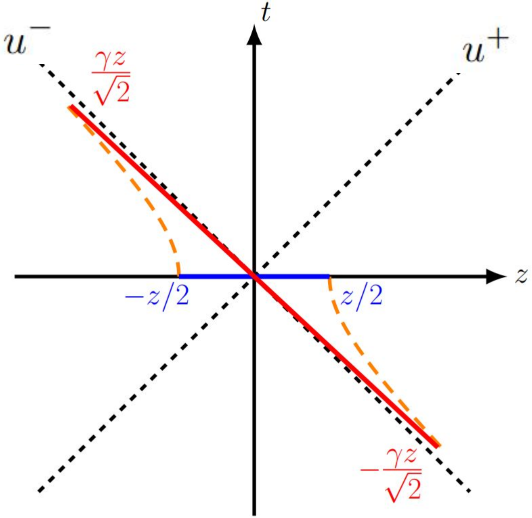
- The chi^2/dof distribution changes width as the dof changes, so chi^2/dof is not necessarily the full picture
- The Z-score gives the number of standard deviations, your value of chi^2/dof is away from 1
- Ex. A chi^2/dof of 1.5 is reasonable with 10 points, but not so much 100 points



Ji, PRL 110:262002 (2013)
Ji, Sci. China Phys. Mech. Astron. 57:1407 (2014)
Radyushkin, PRD 96:034025 (2017)

PDFs from the Lattice

$$u^{\pm} = \frac{u^0 \pm u^3}{\sqrt{2}}$$



PDFs are formally defined by light-cone correlations of quarks (or gluons):

$$f(x) = \int dz^- e^{ixP^+z^-} \langle P|\bar{\psi}\left(-\frac{z}{2}\right)\gamma^+ W\left(-\frac{z}{2}, +\frac{z}{2}\right)\psi\left(+\frac{z}{2}\right)|P\rangle$$

where the Wilson line is required in order to maintain gauge invariance,

$$W(z,0) = \mathcal{P}\exp\left[-ig\int_0^z dz' A^z(z')\right]$$

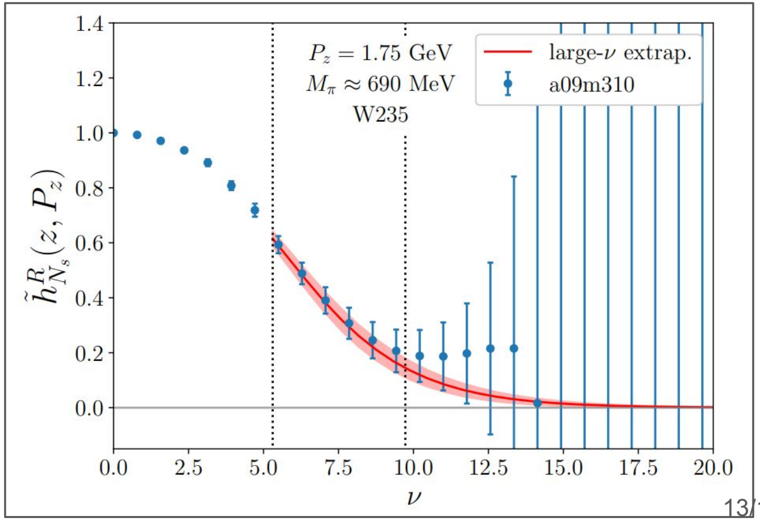
We can only compute Euclidean correlations on the lattice, but loosely speaking, the Euclidean correlators approach the light cone at short distances or large momenta

- This observation forms the foundation for two related frameworks: **Large Momentum Effective Theory (LaMET),** which uses the large-momentum limit, and the **pseudo-PDF method**, which is based on a short-distance expansion.

Quasi-PDF Long Distance Signal Challenge

Gao, *et al.* PRD 109(0):094506 (2024)
Ji, *et al.* NPB. 964:115311 (2021)

- We define the bare gluon PDF matrix elements measured at a lattice spacing a as $h^B(z, P_z, 1/a) = \langle P_z | O_g(z) | P_z \rangle$ $\quad O_g(z) = F^{ti}(z)\mathcal{W}(z,0)F^t_{\ i}(0) - F^{ij}(z)\mathcal{W}(z,0)F_{ij}(0)$
- The Wilson line, $\mathcal{W}(z,0)$, on the lattice has the effect of introducing an exponential decay to the bare matrix elements $e^{-\delta m z}$, where δm is a scheme dependent mass renormalization factor
- What this means is that the signal to noise ratio of the bare matrix elements decays exponentially. When we renormalize, especially in the gluon case, we get very large errors in the long distance
- We rely heavily on trusting theoretically motivated long distance extrapolations:



Pseudo- and Quasi-PDF Methods

Ji, PRL 110:262002 (2013).
Ji, Sci. China Phys. Mech. Astron. 57:1407 (2014).
Radyushkin PRD 96:034025 (2017)

Balitsky *et al.*, PLB 808:135621, 2020.

$$\mathscr{M}(\nu, z^2) = \int_0^1 dx \frac{xg(x,\mu^2)}{\langle x \rangle_g} R_{gg}(x\nu, z^2\mu^2) + \mathcal{O}(z^2 m^2, z^2 \Lambda_{\mathrm{QCD}})$$

Spatially separated matrix elements (MEs) $\langle h(P_z)|O_g(z)|h(P_z)\rangle$ from the lattice

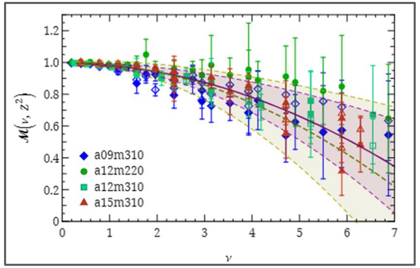
Some choice of renormalization

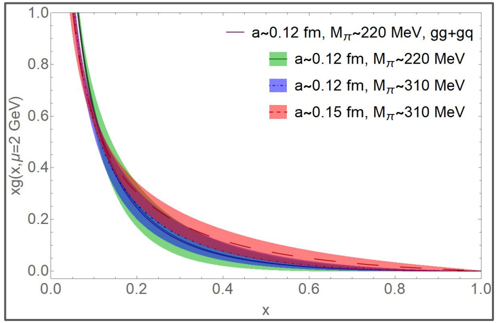
Renormalized MEs $h^{\mathrm{R}}(z, P_z)$

Short distance (small-z^2) matching

Fit PDF model $\frac{xg(x)}{\langle x \rangle_g} = Nx^\alpha(1-x)^\beta$

Pseudo-PDF Method





- PDFs are defined as the Fourier transform of light-front correlators which can't be measured on a Euclidean lattice
- Both methods give ways to connect spatially separated correlators to the light-cone PDFs

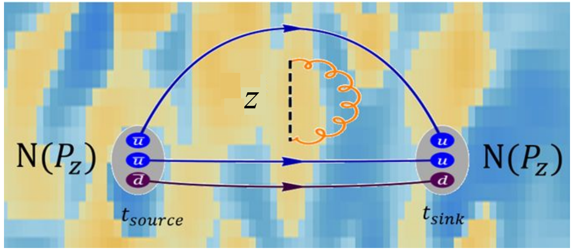
Fourier transform

Quasi-PDF $\tilde{g}(x, P_z)$

Quasi-PDF Method

Large momentum matching

$xg(x)/\langle x \rangle_g$ Typically reliable around $x \in [0.2, 0.8]$

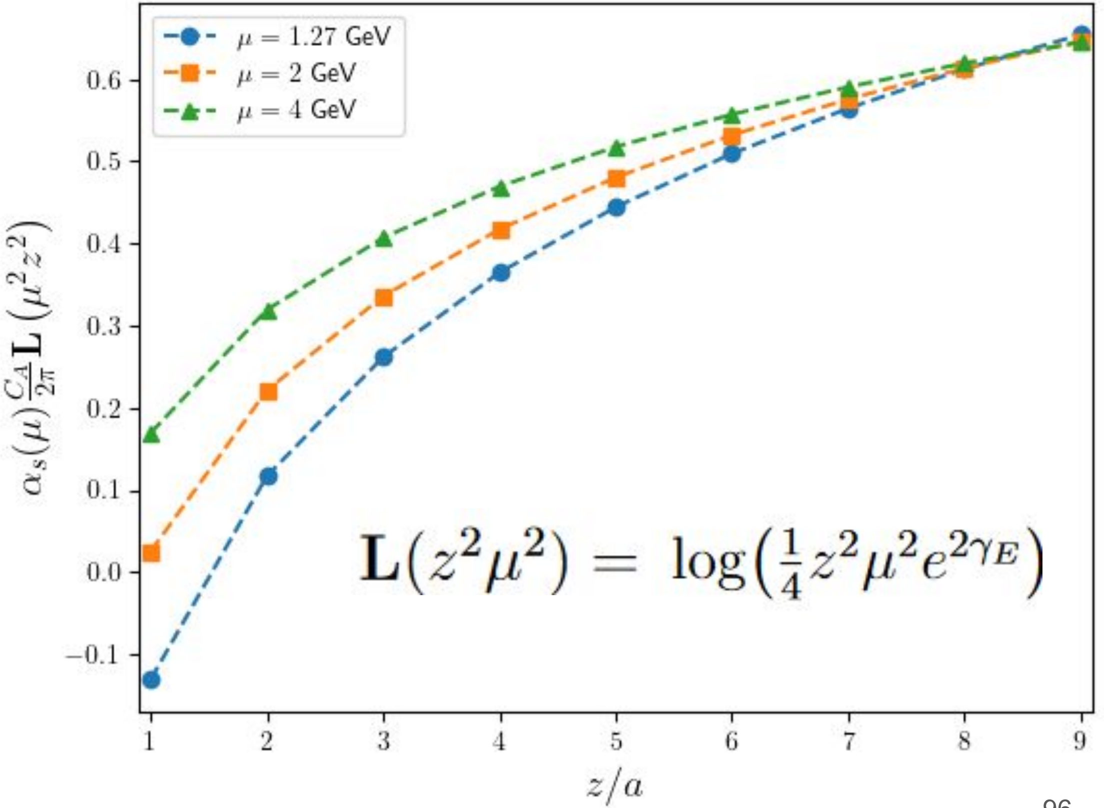


$$x\tilde{g}(x, P^z, \mu) = \int_{-1}^{1} dy F_{gg}(x, y, \mu) y g(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{(xP_z)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{((1-x)P_z)^2}\right)$$

High-z Validity

$$R_{gg}(y, z^2\mu^2) = \cos y - \frac{\alpha_s}{2\pi} C_A \left\{ \left[\ln\left(z^2\mu^2 \frac{e^{2\gamma_E}}{4} \right) \right] R_B(y) + R_L(y) + R_C(y) \right\}$$

- We can plot the leading coefficient in the matching kernel expansion, which grows with z
- The term that multiplies this coefficient is O(1)
- This supports that we need the higher twist effects, but we aren't breaking our perturbative expansion



z-Cut Justification

- Attempting a fit with z = 1a and 2a results in a large χ^2/N_{pts} with a very large spread: 5.2(17)

- The small-z have large discretization effects!