Quantum Computing Fragmentation Functions

Universidad Complutense de Madrid & IPARCOS

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María Gómez, Tim Hobbs

arxiv: 2510.18869

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FONDO EUROPEO DE DESARROLLO REGIONAL
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MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES
AGENCIA ESTATAL DE INVESTIGACIÓN

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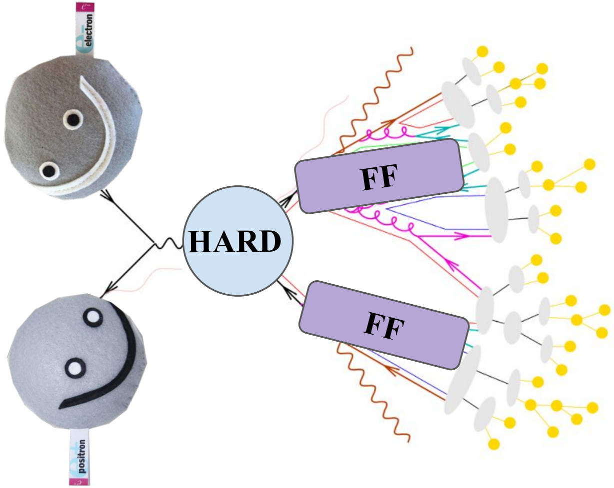
QCD is confined: Hard interactions reconstructed from universal **fragmentation functions (FF)** into hadrons
- the cleanest process is **e+ e- annihilation**



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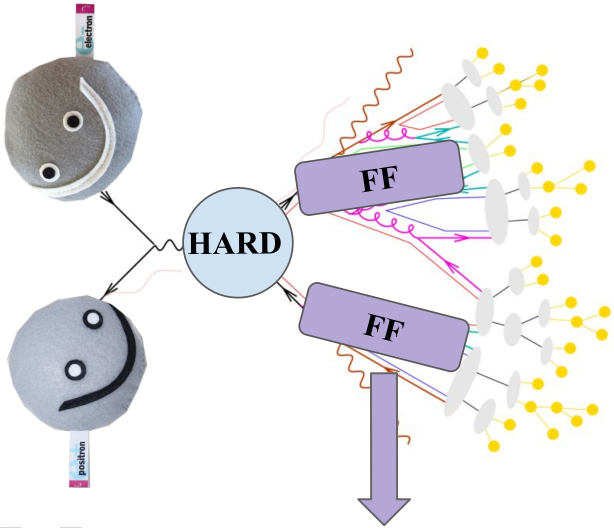

$$D_j^{J/\Psi}(z) \equiv \frac{\mathrm{Tr_m Tr_c}}{3\,N_{c,j}} \sum_X \underbrace{\langle p, j, c | J/\Psi(z,m), X\rangle_{\mathrm{out}}}_{\text{Light cone separation}} \langle J/\Psi(z,m), X|_{\mathrm{out}} | p, j, c\rangle,$$

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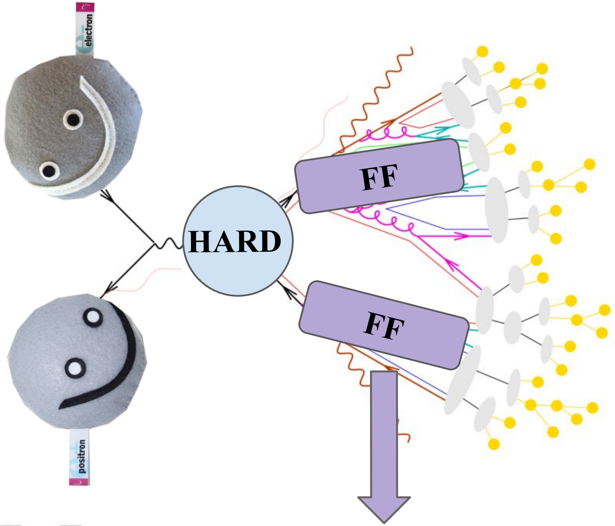

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Summing over X states and introducing light front evolution

$$D_j^{J/\Psi}(z) = \frac{1}{3}\sum_m \frac{1}{N_c}\sum_c \left|\langle \psi_{J/\Psi}(z;m)|e^{-iP^- x^+}|p,j,c\rangle\right|^2$$

1-3

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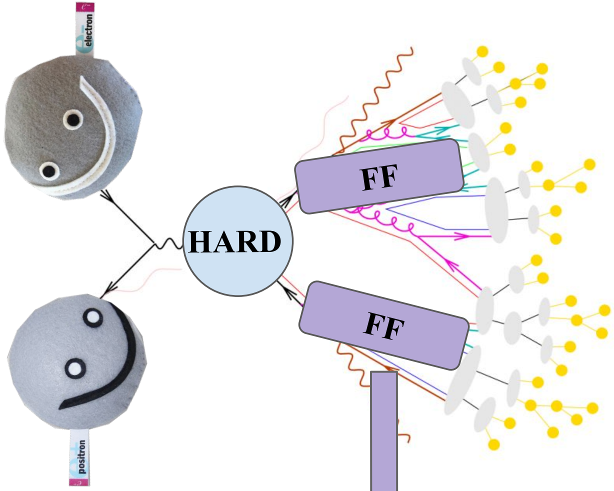

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1-4

In terms of creation and annihilation operators:

$$D_j^{J/\Psi}(z) = \frac{1}{3N_c}\sum_{m,c}\left|\langle 0| a_{zm}^{J/\Psi} e^{-iP^- x^+} b_{pjr}^\dagger |0\rangle\right|^2 \quad (1)$$

doi:10.1103/PhysRevD.109.016006

1. Destruction operator of J/Psi with **momentum fraction z** and **polarization m** $a_{zm}^{J/\Psi}$

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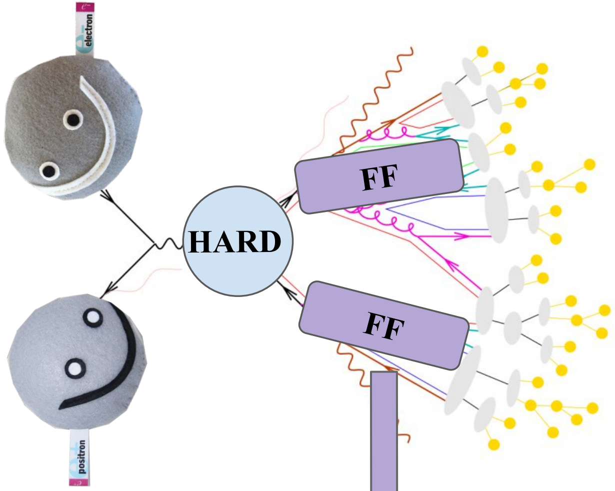

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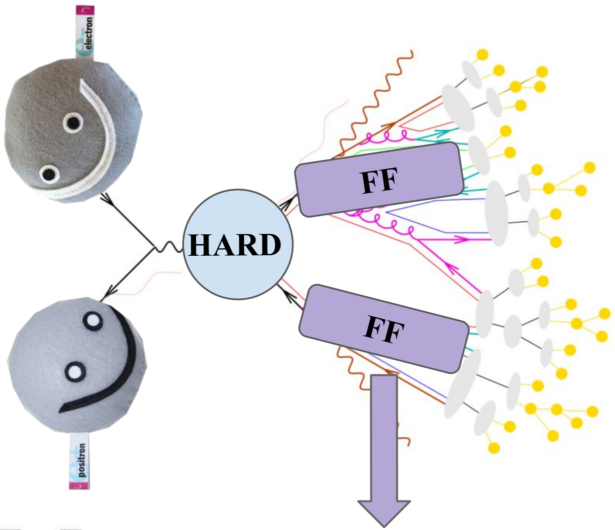

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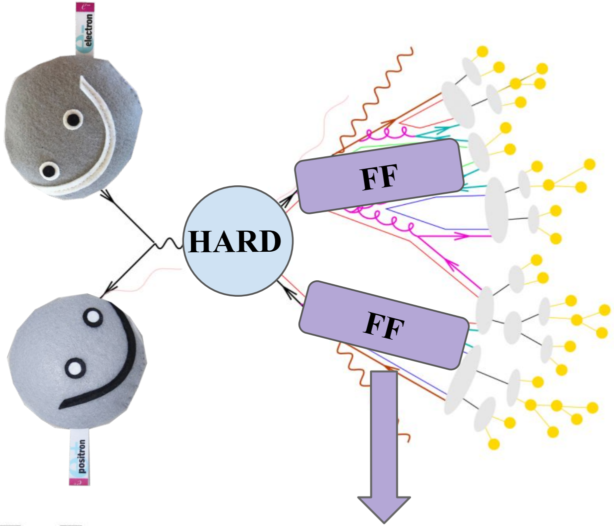

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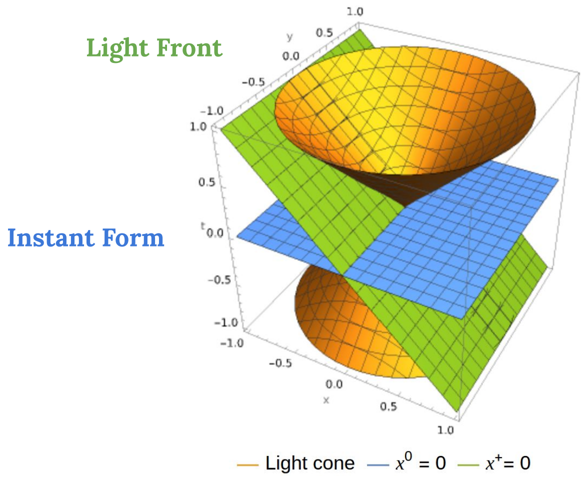
Not a problem for Quantum Computers!

Ab initio QCD on the Light-Front



Coordinates

Instant Form (usual quantisation)	Light Front
(x^0, x^1, x^2, x^3) →	(x^+, x^-, x^1, x^2)
	$x^+ = x^3 + x^0$
	$x^- = x^3 - x^0$
	$x^1,\ x^2 \quad x^\perp$

Energies:

$$p^0 = \sqrt{m^2 + |p|^2} \qquad p^- = \frac{m^2 + |p^\perp|^2}{p^+}$$

Gauges:

$$A^0 = 0 \qquad A^+ = 0$$

Collision axis

(2)

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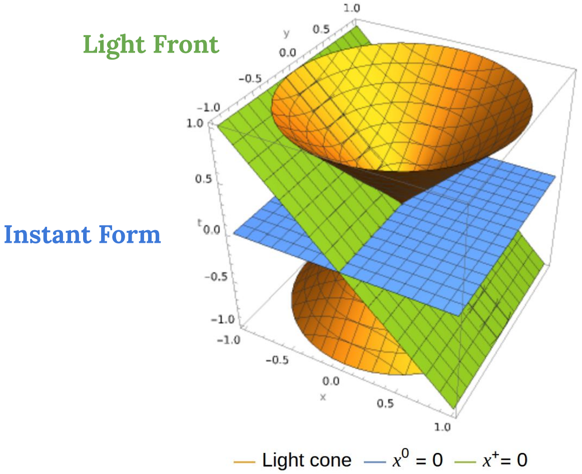

Coordinates

Instant Form (usual quantisation) → **Light Front**

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$x^+ = x^3 + x^0$ ← Collision axis

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Only 2 independent components:

$$\tilde{\mathbf{A}}^- = \frac{1}{\partial^+} 2\partial^\perp \tilde{\mathbf{A}}^\perp$$

From

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi + g\, A^a_\mu J^{\mu,a}_D$$

define LF Hamiltonian by $P^- = \sum \Pi\, \partial_+ A - \mathcal{L}$

(2)

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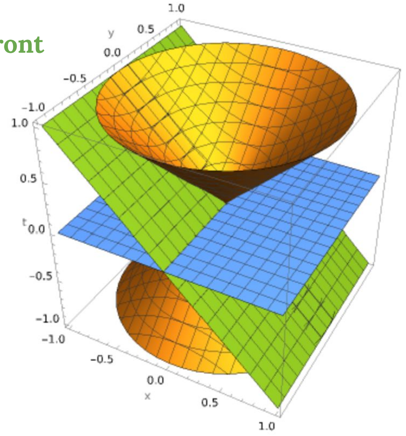

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Collision axis

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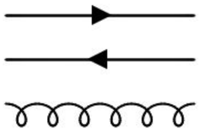
$$P_+ = \frac{1}{2}\int dx_+ d^2x_\perp \left(\bar{\tilde{\Psi}}\gamma^+ \frac{m^2 + (i\nabla_\perp)^2}{i\partial^+} \tilde{\Psi} + \tilde{A}^\mu_a (i\nabla_\perp)^2 \tilde{A}^a_\mu \right) + g\int dx_+ d^2x_\perp\, \tilde{J}^\mu_a \tilde{A}^a_\mu$$
$$+ \frac{g^2}{4}\int dx_+ d^2x_\perp\, \tilde{B}^{\mu\nu}_a \tilde{B}^a_{\mu\nu} + \frac{g^2}{2}\int dx_+ d^2x_\perp \tilde{J}^+_a \frac{1}{(i\partial^+)^2} \tilde{J}^+_a + \frac{g^2}{2}\int dx_+ d^2x_\perp\, \bar{\tilde{\Psi}}\gamma^\mu T^a \tilde{A}^a_\mu \frac{\gamma^+}{i\partial^+}(\gamma^\nu T^b \tilde{A}^b_\nu \tilde{\Psi})\,. \quad (2)$$

Hamiltonian as a dictionary of interactions among **on shell** partons:

doi.org/10.1016/S0370-1573(97)00089-6

Kinetic



Free evolution, just
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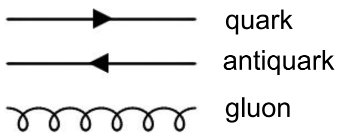

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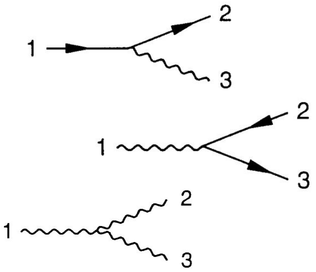

Simple interactions, pair creation and gluon scattering

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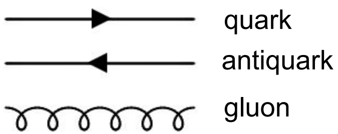

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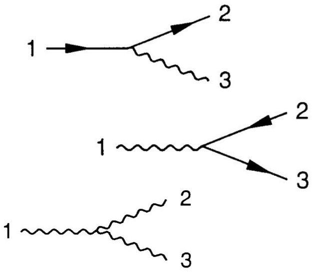
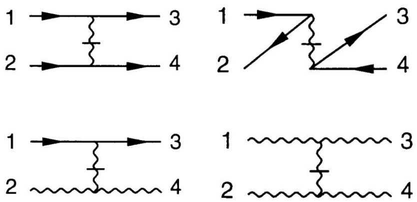


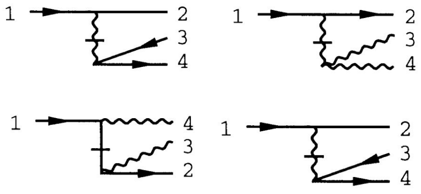
Simple interactions, pair creation and gluon scattering

Seagulls



Instantaneous interactions of 2 particles

Forks



Instantaneous emission/absorption of 3 particles

Encoding I - Creation and annihilation operators

Bosons

$$a_\rho^{(n)\dagger} = \sum_{i=1}^{n} \mathcal{S}_i \cdot \mathbb{P}_0^{(n-i)} \otimes (C_{10} \otimes s_\rho^\dagger)_i \otimes \mathbb{P}_{i-1}^{(i-1)} \quad (3)$$

$$a_\rho^{(n)} = \sum_{i=1}^{n} \mathbb{P}_0^{(n-i)} \otimes (C_{01} \otimes s_\rho)_i \otimes \mathbb{P}_{i-1}^{(i-1)} \cdot \mathcal{S}_i \quad (4)$$

Commutation relations up to boundary term

$$\left[a_\rho^{(n)}, a_\eta^{(n)\dagger}\right] = \delta_{\rho\eta}\, \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)} - \mathcal{S}_n \cdot \left(C_{11} \otimes s_\rho^\dagger s_\eta\right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot \mathcal{S}_n \quad (5)$$

Fermions

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Anticommutation relations up to boundary term

$$\left\{b_\rho^{(n)}, b_\eta^{(n)\dagger}\right\} = \delta_{\rho\eta}\, \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)} + \mathcal{A}_n \cdot \left(C_{11} \otimes s_\rho^\dagger s_\eta\right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot \mathcal{A}_n \quad (8)$$

Details in PhysRevD.110.116018

Encoding II - Single Particle Registers

Bosons

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1. Encode each **single-particle** state as a binary number on **qubit registers**

N states $\to \log_2 N$ qubits

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Gluons have 8 colors (Gell-Mann basis):

$\lambda_1 : (r\bar{b} + b\bar{r})/\sqrt{2}$ $\rightleftarrows$ | 0 | 0 | 0 | $s_1^\dagger = |000\rangle \langle 000|$

(9)

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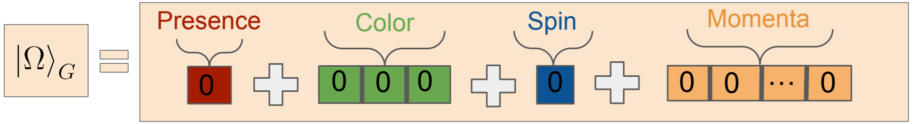
Add spin, momenta (discretized) and **presence**



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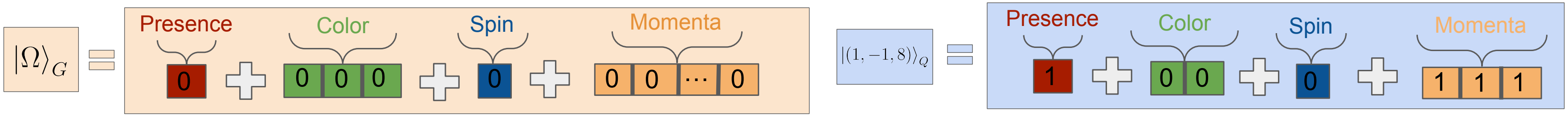

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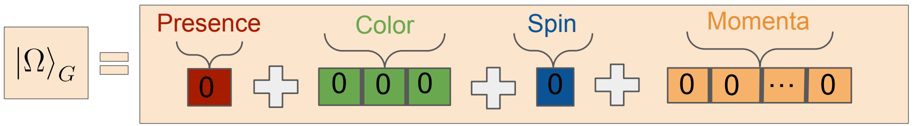

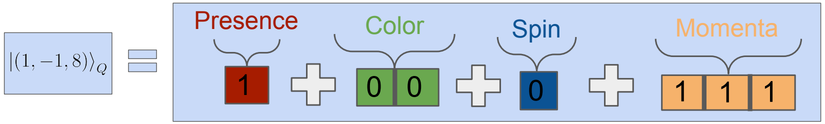

To initiate a register, change modes and presence

$$|(1,\uparrow,(1,0,0))\rangle_G = \left(C_{10} \otimes s^\dagger_{(1,\uparrow,(1,0,0))}\right)_1 |\Omega\rangle_G$$

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N states $\rightarrow \log_2 N$ qubits

Gluons have 8 colors (Gell-Mann basis):

$\lambda_1 : (r\bar{b} + b\bar{r})/\sqrt{2}$ ⇄ 0 0 0 $\quad s_1^\dagger = |000\rangle\langle 000|$

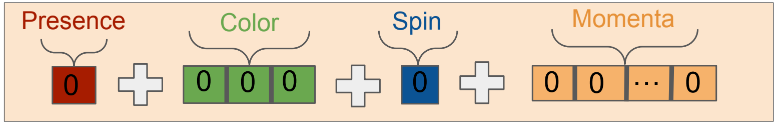
$\lambda_2 : -i(r\bar{b} - b\bar{r})/\sqrt{2}$ ⇄ 0 0 1 $\quad s_2^\dagger = |001\rangle\langle 000|$

... ...

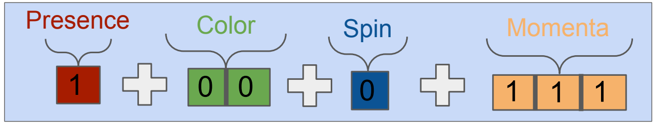
$\lambda_8 : (r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6}$ ⇄ 1 1 1 $\quad s_8^\dagger = |111\rangle\langle 000|$

Add spin, momenta (discretized) and **presence**

$|\Omega\rangle_G =$



$|(1,-1,8)\rangle_Q =$



To initiate a register, change modes and presence

$$|(1,\uparrow,(1,0,0))\rangle_G = \left(C_{10} \otimes s^\dagger_{(1,\uparrow,(1,0,0))}\right)_1 |\Omega\rangle_G = C_{10}|0\rangle \otimes s_1^\dagger |000\rangle \otimes s_\uparrow^\dagger |0\rangle \otimes s^\dagger_{(1,0,0)} |00\ldots0\rangle \quad (9)$$

Encoding III - Multi-particle states

Bosons

$$a_\rho^{(n)\dagger} = \sum_{i=1}^n \mathcal{S}_i \cdot \mathbb{P}_0^{(n-i)} \otimes (C_{10} \otimes s_\rho^\dagger)_i \otimes \mathbb{P}_{i-1}^{(i-1)} \quad (3)$$

$$a_\rho^{(n)} = \sum_{i=1}^n \mathbb{P}_0^{(n-i)} \otimes (C_{01} \otimes s_\rho)_i \otimes \mathbb{P}_{i-1}^{(i-1)} \cdot \mathcal{S}_i \quad (4)$$

Commutation relations up to boundary term

$$\left[a_\rho^{(n)}, a_\eta^{(n)\dagger}\right] = \delta_{\rho\eta} \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)} - \mathcal{S}_n \cdot \left(C_{11} \otimes s_\rho^\dagger s_\eta\right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot \mathcal{S}_n \quad (5)$$

Fermions

$$b_\rho^{(n)\dagger} = \sum_{i=1}^n \mathcal{A}_i \cdot \mathbb{P}_0^{(n-i)} \otimes (C_{10} \otimes s_\rho^\dagger)_i \otimes \mathbb{P}_{i-1}^{(i-1)} \quad (6)$$

$$b_\rho^{(n)} = \sum_{i=1}^n \mathbb{P}_0^{(n-i)} \otimes (C_{01} \otimes s_\rho)_i \otimes \mathbb{P}_{i-1}^{(i-1)} \cdot \mathcal{A}_i \quad (7)$$

Anticommutation relations up to boundary term

$$\left\{b_\rho^{(n)}, b_\eta^{(n)\dagger}\right\} = \delta_{\rho\eta} \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)} + \mathcal{A}_n \cdot \left(C_{11} \otimes s_\rho^\dagger s_\eta\right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot \mathcal{A}_n \quad (8)$$

ρ, η are particle modes, not mesons!

Details in PhysRevD.110.116018

Encoding III - Multi-particle states

2. Define multi-particle states combining registers

No particles

$$\Omega \otimes \cdots \otimes \Omega \otimes \Omega$$

Encoding III - Multi-particle states

2. Define multi-particle states combining registers

No particles

1 particle

2 particles

How to distinguish? Use projectors!

Encoding III - Multi-particle states

2. Define multi-particle states combining registers

No particles

1 particle

2 particles

How to distinguish? Use projectors!

Introduce projectors to select registers:

$$\mathbb{P}_i^{(n)} = \overbrace{(C_{00} \otimes i)_n \otimes \cdots \otimes (C_{00} \otimes i)_{i+1} \otimes \underbrace{(C_{11} \otimes i)_i \otimes \cdots \otimes (C_{11} \otimes i)_1}_{i \text{ particles}}}^{n \text{ registers}} \quad (10)$$

Encoding III - Multi-particle states

2. Define multi-particle states combining registers

No particles

1 particle

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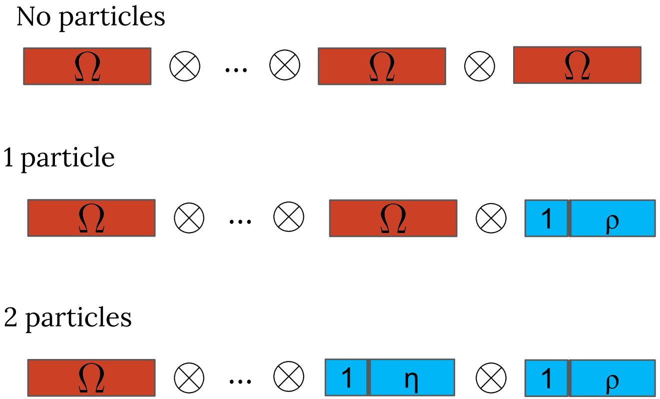
Combine projectors and set operators to create particles on specific registers

$$\left[\left(C_{10}\otimes s_\eta^\dagger\right)_2 \otimes \mathbb{P}_1^{(1)} = \right] a_{\eta,2}^\dagger \; \Omega \otimes \Omega = 0$$

Encoding III - Multi-particle states

2. Define multi-particle states combining registers



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$$\left[\left(C_{10}\otimes s_\eta^\dagger\right)_2 \otimes \mathbb{P}_1^{(1)} =\right] a^\dagger_{\eta,2} \; \boxed{\Omega} \otimes \boxed{\Omega} = 0$$

$$a^\dagger_{\eta,2} \; \boxed{\Omega} \otimes \boxed{1 \mid \rho} = \boxed{1 \mid \eta} \otimes \boxed{1 \mid \rho}$$

Encoding III - Multi-particle states

Bosons

$$a_\rho^{(n)\dagger} = \sum_{i=1}^n \mathcal{S}_i \cdot \mathbb{P}_0^{(n-i)} \otimes (C_{10} \otimes s_\rho^\dagger)_i \otimes \mathbb{P}_{i-1}^{(i-1)} \quad (3)$$

$$a_\rho^{(n)} = \sum_{i=1}^n \mathbb{P}_0^{(n-i)} \otimes (C_{01} \otimes s_\rho)_i \otimes \mathbb{P}_{i-1}^{(i-1)} \cdot \mathcal{S}_i \quad (4)$$

Commutation relations up to boundary term

$$\left[a_\rho^{(n)}, a_\eta^{(n)\dagger}\right] = \delta_{\rho\eta}\, \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)} - \mathcal{S}_n \cdot \left(C_{11} \otimes s_\rho^\dagger s_\eta\right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot \mathcal{S}_n \quad (5)$$

Fermions

$$b_\rho^{(n)\dagger} = \sum_{i=1}^n \mathcal{A}_i \cdot \mathbb{P}_0^{(n-i)} \otimes (C_{10} \otimes s_\rho^\dagger)_i \otimes \mathbb{P}_{i-1}^{(i-1)} \quad (6)$$

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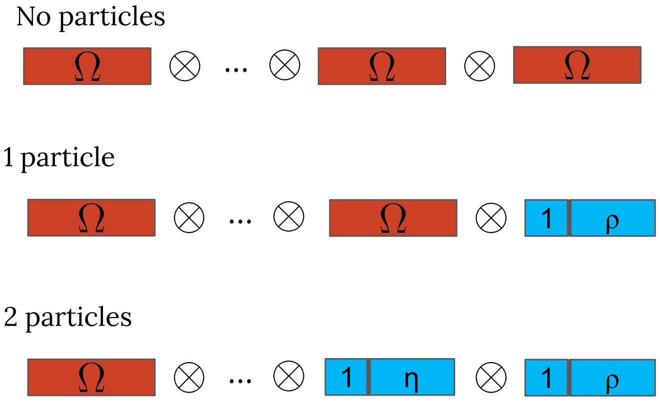
Anticommutation relations up to boundary term

$$\left\{b_\rho^{(n)}, b_\eta^{(n)\dagger}\right\} = \delta_{\rho\eta}\, \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)} + \mathcal{A}_n \cdot \left(C_{11} \otimes s_\rho^\dagger s_\eta\right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot \mathcal{A}_n \quad (8)$$

Details in PhysRevD.110.116018

Encoding III - Multi-particle states

2. Define multi-particle states combining registers

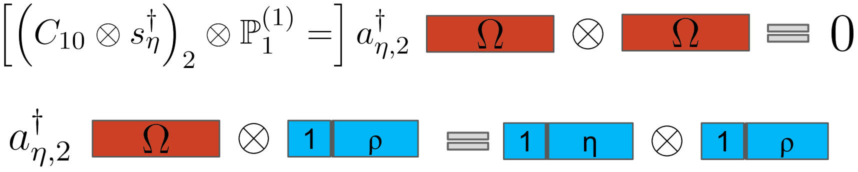


How to distinguish? Use presence!

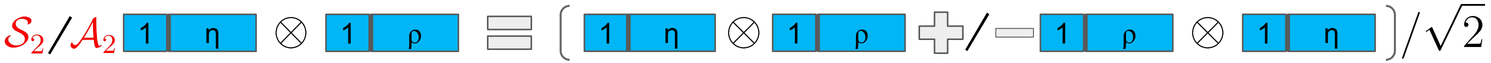
Introduce projectors to select registers:

$$\mathbb{P}_i^{(n)} = \overbrace{(C_{00} \otimes i)_n \otimes \cdots \otimes (C_{00} \otimes i)_{i+1} \otimes \underbrace{(C_{11} \otimes i)_i \otimes \cdots \otimes (C_{11} \otimes i)_1}_{i \text{ particles}}}^{n \text{ registers}} \quad (10)$$

Combine projectors and set operators to create particles on specific registers



3. Add (anti)symmetrizers for (fermion)bosons



$$\mathcal{S}_j = \left(\mathcal{I} + \mathcal{P}_{(j)(j-1)} + \cdots + \mathcal{P}_{(j)(1)}\right) / \sqrt{j} \quad (11) \qquad \mathcal{A}_j = \left(\mathcal{I} - \mathcal{P}_{(j)(j-1)} - \cdots - \mathcal{P}_{(j)(1)}\right) / \sqrt{j} \quad (12)$$

Encoding IV - Commutation relations revisited

Bosons

$$a_\rho^{(n)\dagger} = \sum_{i=1}^n \mathcal{S}_i \cdot \mathbb{P}_0^{(n-i)} \otimes (C_{10} \otimes s_\rho^\dagger)_i \otimes \mathbb{P}_{i-1}^{(i-1)}$$

$$a_\rho^{(n)} = \sum_{i=1}^n \mathbb{P}_0^{(n-i)} \otimes (C_{01} \otimes s_\rho)_i \otimes \mathbb{P}_{i-1}^{(i-1)} \cdot \mathcal{S}_i$$

Commutation relations up to boundary term

$$\left[a_\rho^{(n)}, a_\eta^{(n)\dagger}\right] = \delta_{\rho\eta}\, \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)} - \mathcal{S}_n \cdot \left(C_{11} \otimes s_\rho^\dagger s_\eta\right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot \mathcal{S}_n$$

Fermions

$$b_\rho^{(n)\dagger} = \sum_{i=1}^n \mathcal{A}_i \cdot \mathbb{P}_0^{(n-i)} \otimes (C_{10} \otimes s_\rho^\dagger)_i \otimes \mathbb{P}_{i-1}^{(i-1)}$$

$$b_\rho^{(n)} = \sum_{i=1}^n \mathbb{P}_0^{(n-i)} \otimes (C_{01} \otimes s_\rho)_i \otimes \mathbb{P}_{i-1}^{(i-1)} \cdot \mathcal{A}_i$$

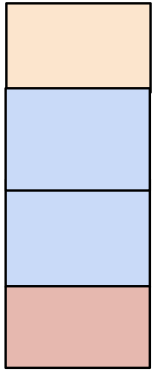
Anticommutation relations up to boundary term

$$\left\{b_\rho^{(n)}, b_\eta^{(n)\dagger}\right\} = \delta_{\rho\eta}\, \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)} + \mathcal{A}_n \cdot \left(C_{11} \otimes s_\rho^\dagger s_\eta\right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot \mathcal{A}_n$$

Details in PhysRevD.110.116018

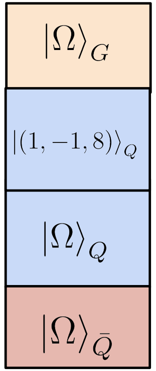
Algorithm I - Set up and exponentiation

1. Minimal: **up to** 2 quarks, 1 antiquark and 1 gluon



Algorithm I - Set up and exponentiation

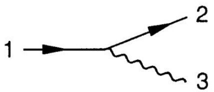
1. Minimal: **up to** 2 quarks, 1 antiquark and 1 gluon
2. Start with quark of max momenta



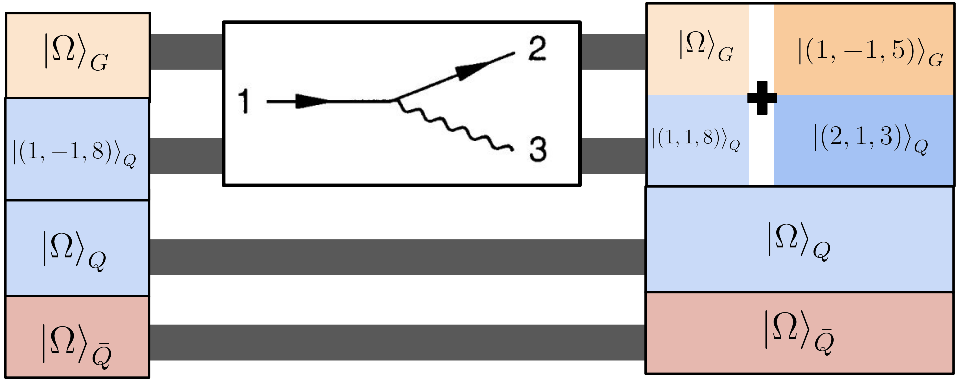
Algorithm I - Set up and exponentiation

1. Minimal: **up to** 2 quarks, 1 antiquark and 1 gluon
2. Start with quark of max momenta
3. Encode and exponentiate

$$V_1 = \int V(1\,;\,2,\,3)\, a_3^\dagger\, b_2^\dagger\, b_1 + h.c.$$



$$= \sum_{123} V(1;2,3) \Bigg(\underbrace{\mathcal{S}_{j_b} \cdot \mathbb{P}_0^{(n_b - j_b)} \cdot \left(C_{10} \otimes s_3^\dagger\right)_{j_b} \otimes \mathbb{P}_{j_b-1}^{(j_b-1)}}_{\text{gluons}} \bar{\otimes} \underbrace{\mathcal{A}_{j_f} \cdot \mathbb{P}_0^{(n_f-j_f)} \cdot \left(C_{11}\otimes s_2^\dagger s_1\right)_{j_f} \otimes \mathbb{P}_{j_f-1}^{(j_f-1)} \cdot \mathcal{A}_{j_f} + h.c.}_{\text{fermions}} \Bigg) \quad (13)$$

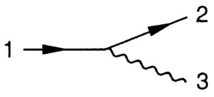


7-3

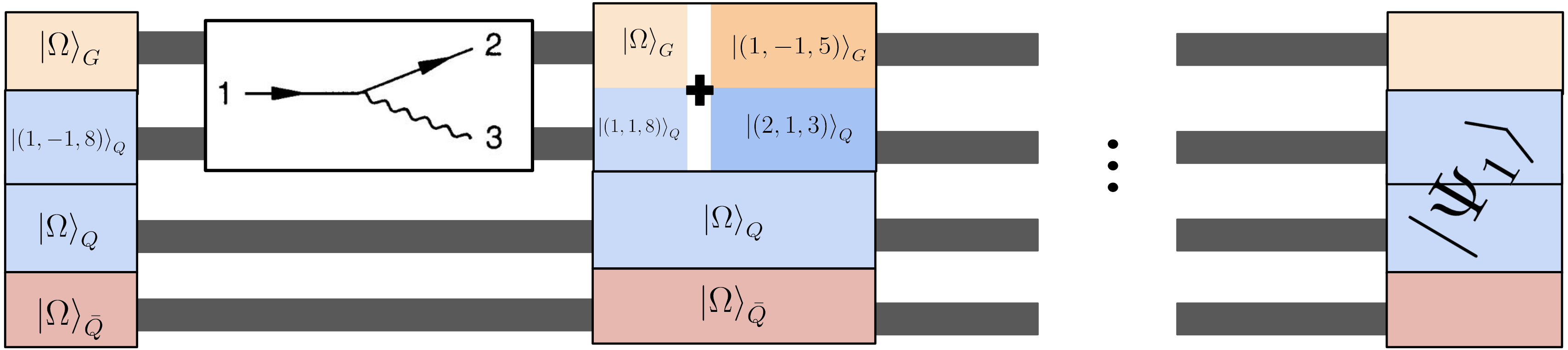
Algorithm I - Set up and exponentiation

1. Minimal: **up to** 2 quarks, 1 antiquark and 1 gluon
2. Start with quark of max momenta
3. Encode and exponentiate
4. Repeat for all matrix elements to evolve **1 Trotter step**

$$V_1 = \int V(1\,;\,2,\,3)\, a_3^\dagger\, b_2^\dagger\, b_1 + h.c.$$

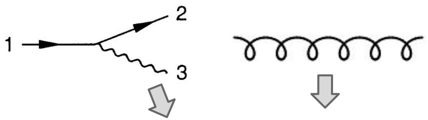


$$= \sum_{123} V(1;2,3) \left(\underbrace{\mathcal{S}_{j_b} \cdot \mathbb{P}_0^{(n_b-j_b)} \cdot \left(C_{10} \otimes s_3^\dagger\right)_{j_b} \otimes \mathbb{P}_{j_b-1}^{(j_b-1)}}_{\text{gluons}} \bar{\otimes} \underbrace{\mathcal{A}_{j_f} \cdot \mathbb{P}_0^{(n_f-j_f)} \cdot \left(C_{11} \otimes s_2^\dagger s_1\right)_{j_f} \otimes \mathbb{P}_{j_f-1}^{(j_f-1)} \cdot \mathcal{A}_{j_f} + h.c.}_{\text{fermions}} \right) \quad (13)$$



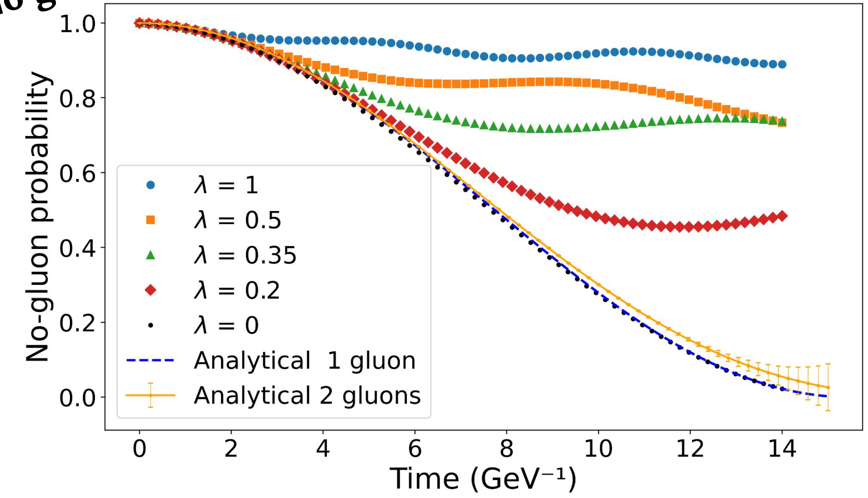
Algorithm II - Halting protocol

Truncations induce oscillations, focus on gluon scattering:



$$\exp\{-i\Delta t(V_1 + \lambda E_c)\}$$

No gluons



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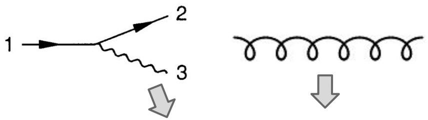

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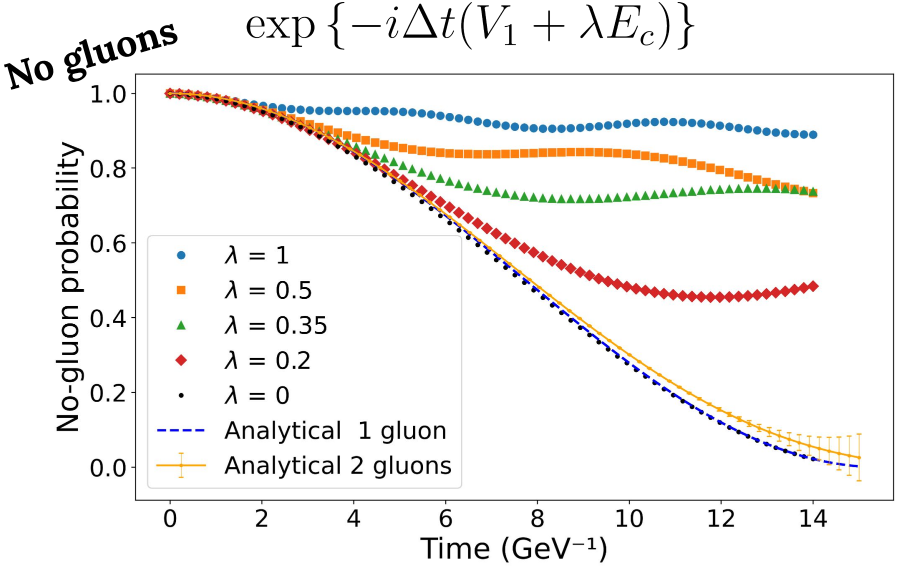
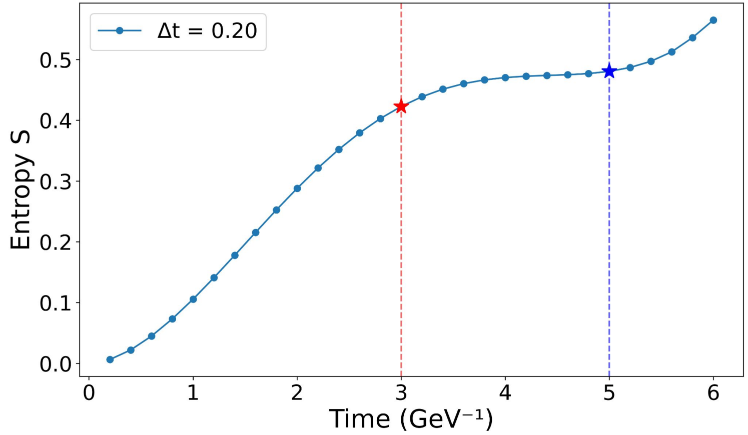


We expect the entropy to increase monotonically:

$$S = \sum_i -p_i \log(p_i)$$



30 Trotter Steps for a total evolution time of 6 GeV^{-1}

Algorithm III - Measurement

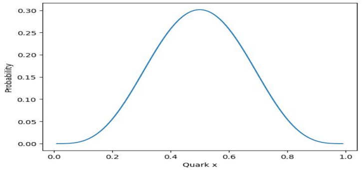
5. Measure J/Ψ wavefunction

An ansatz
$$a^{\dagger J/\Psi}_{z,m} = \sum \sqrt{\frac{\chi_0(x)}{x\,(z-x)}} \quad b^{\dagger}_{xp,i,s} d^{\dagger}_{(z-x)p,j,r} \qquad (14)$$

doi.org/10.1140/epjc/s10052-022-10988-5

- Simple momentum fraction dependence

$$\chi_0 = \frac{1}{\sqrt{N}} x^{\beta/2} (z-x)^{\alpha/2}$$



$$\beta = 4m_{\bar{q}}^2/\kappa \qquad \alpha = 4m_q^2/\kappa$$

P^+	$\alpha_s(P^+)$	m_c	x_i	κ
10	0.18	1.27	$k/N, k=1,\ldots,N$	1.34

Momentum, mass and \kappa in GeV

Algorithm III - Measurement

5. Measure J/Ψ wavefunction

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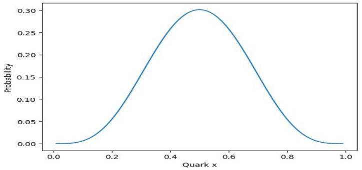

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Momentum, mass and \kappa in GeV

- Color singlet

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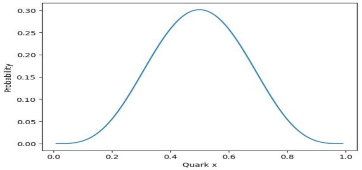

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Momentum, mass and \kappa in GeV

- Color singlet
- Polarization m

Algorithm III - Measurement

5. Measure J/Ψ wavefunction

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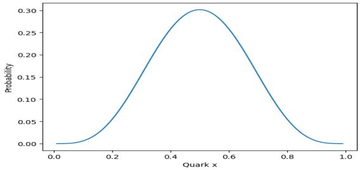

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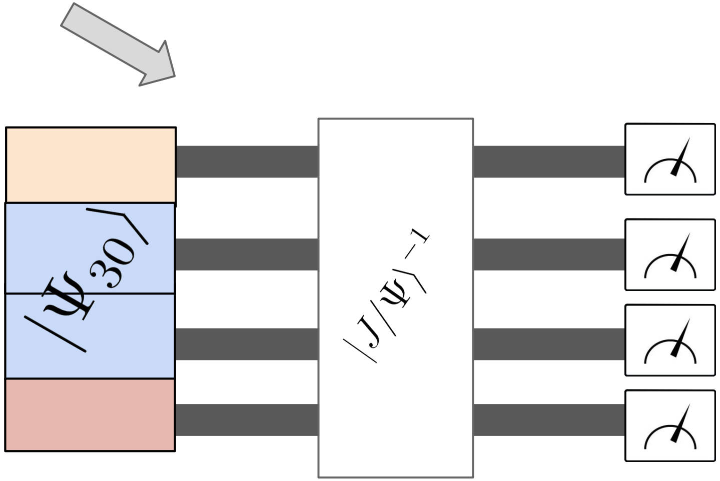


Algorithm III - Measurement

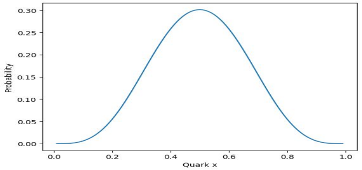
5. Measure J/Ψ wavefunction

An ansatz $a_z^{\dagger}$ Should be obtained from Hamiltonian! $x)_{p,j,r}$ (14)

doi.org/10.1140/epjc/s10052-022-10988-5

- Simple momentum fraction dependence

$$\chi_0 = \frac{1}{\sqrt{N}} x^{\beta/2} (z-x)^{\alpha/2}$$



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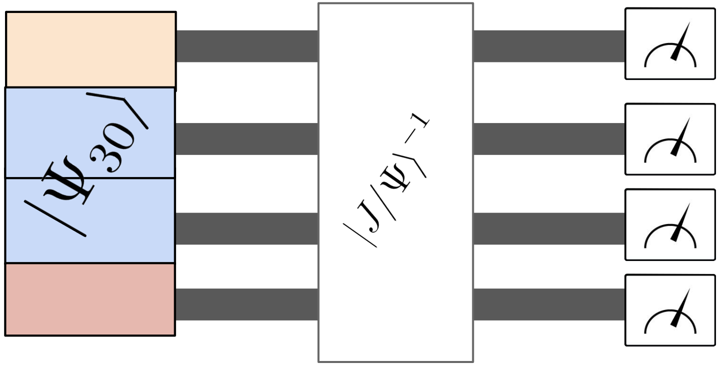
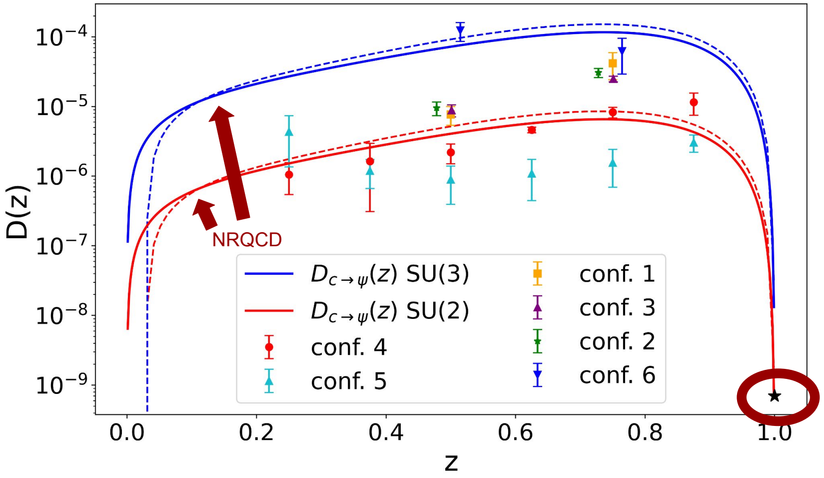


Results & conclusions

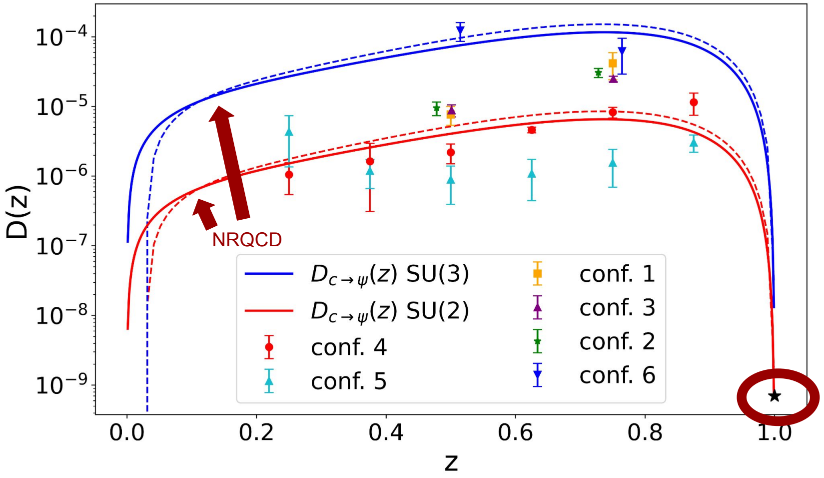


10.1103/PhysRevD.48.4230, 10.1103/PhysRevD.100.014005

Run configuration	Terms	N_c	N grid	N_{qubits}	Δt (GeV^{-1})	ϵ	Runtime (min)
1	up to g	2	4	25	0.2	0	1.5
2	all	2	4	25	0.1	0	22
3	all	2	4	25	0.2	0.02	15
4	up to g	2	8	29	0.2	0	110
5	all	2	8	29	0.2	0.02	1800
6	up to g	3	4	29	0.25	0	120

@ Proteus UGR

Results & conclusions



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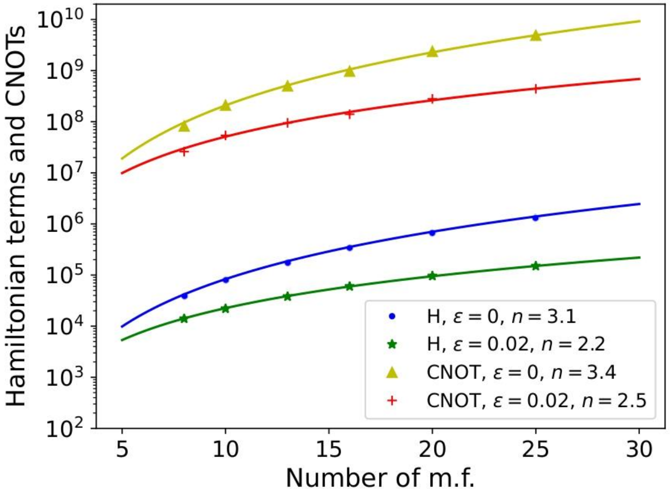
$$C(N; a, n) = a \cdot N^n$$



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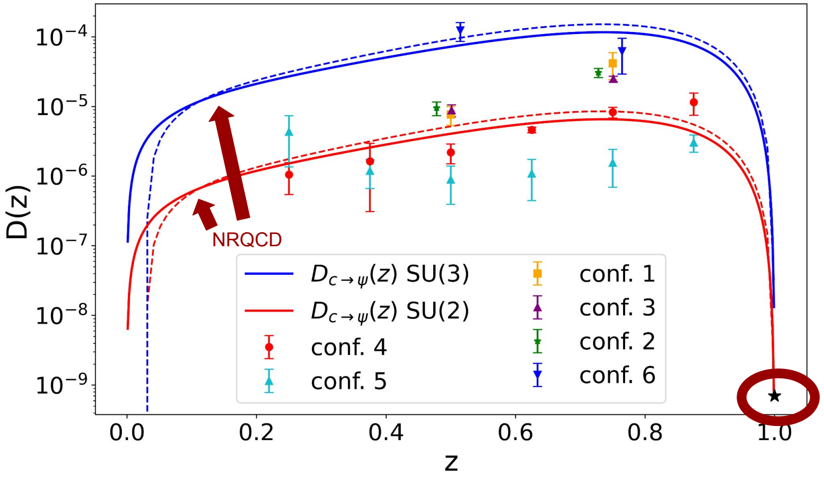
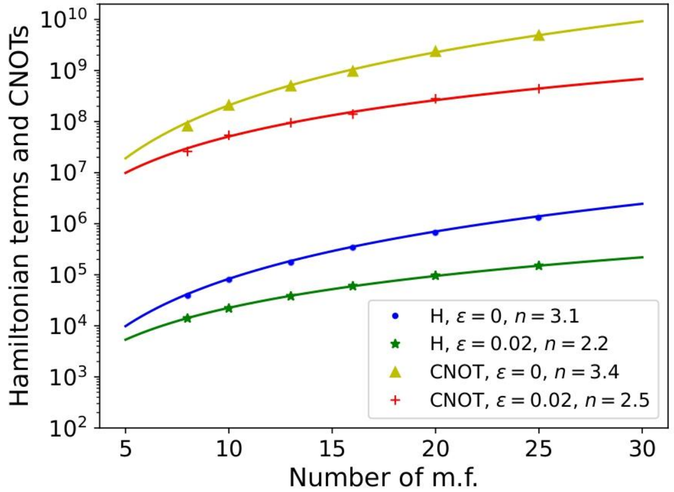
3	all	2	4	25	0.2	0.02	15
4	up to g	2	8	29	0.2	0	110
5	all	2	8	29	0.2	0.02	1800
6	up to g	3	4	29	0.25	0	120

Results & conclusions



10.1103/PhysRevD.48.4230, 10.1103/PhysRevD.100.014005

$$C(N;a,n) = a \cdot N^n$$



Run configuration	Terms	N_c	N grid	N_{qubits}	Δt (GeV^{-1})	ϵ	Runtime (min)
1	up to g	2	4	25	0.2	0	1.5
2	all	2	4	25	0.1	0	22
3	all	2	4	25	0.2	0.02	15
4	up to g	2	8	29	0.2	0	110
5	all	2	8	29	0.2	0.02	1800
6	up to g	3	4	29	0.25	0	120

- **End to end** simulation of fragmentation in LF QCD with 2 quarks, 1 antiquark, and 1 gluon.
- 29 qubits $\Longrightarrow$ 17 GB of wavefunction
- Total number of gates $\sim 10^8$ far from today$\sim 10^3$ gates, but improving fast!

Thank you for your attention!

Quantum Computing Fragmentation Functions

Universidad Complutense de Madrid & IPARCOS

JJ Gálvez*, Felipe J. Llanes, Nicolás Martínez,
María Gómez, Tim Hobbs

arxiv: 2510.18869

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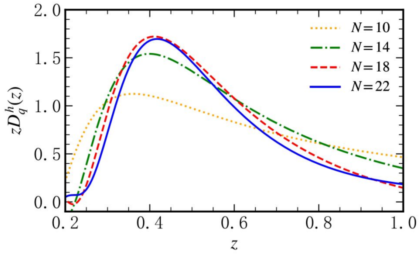
Backup: Algorithms for quantum simulations of hadron physics

- (2011) Real-time scattering of scalar QFT efficient by the JLP algorithm
  https://arxiv.org/pdf/1111.3633

- (2024) Fragmentation has been studied with 1+1D NJL model https://arxiv.org/abs/2406.05683



- See https://arxiv.org/abs/2510.26293 for (our) review

Preparations for Quantum Computing in Hadron Physics

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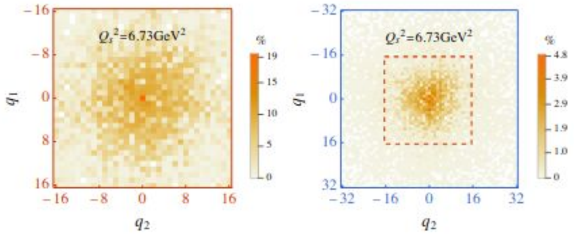
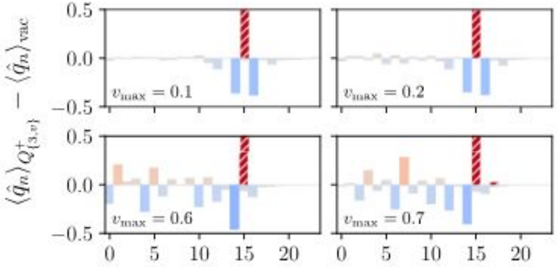
- Other works:

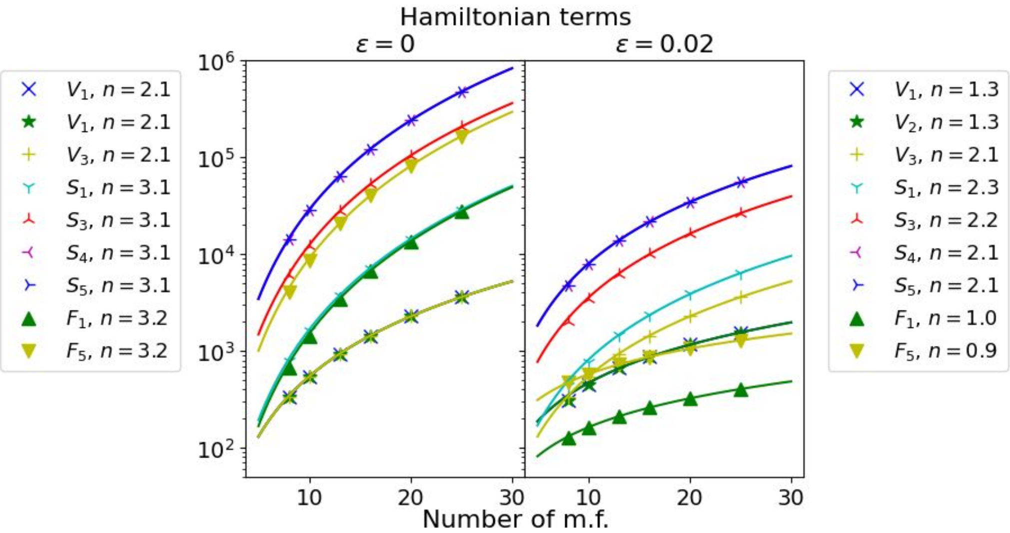
1. Medium induced jet broadening doi:10.1103/PhysRevD.106.074013



2. Energy loss on dense media doi:10.1103/PhysRevC.111.015202



Backup: More about scaling



Backup: Fragmentation at z = 1