Resummed azimuthal decorrelation and transverse momentum imbalance of dijets at the LHC

-- resolving Non-global logarithms via the Winner-take-all scheme

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Motivation

3D Tomography

Single-spin asymmetries in dijet production at polarized proton-proton colliders probe 3D proton structure and TMD PDFs. [Boer, Vogelsang '04; Bomhof, Mulders, Vogelsang Yuan '07; Qiu, Vogelsang, Yuan '07 …]

Factorization Violation

Transverse momentum dependent (TMD) factorization is violated in dijet production. [Collins, Qiu '07; Collins '07, Vogelsang, Yuan '07; Rogers, Mulders '10 ...]

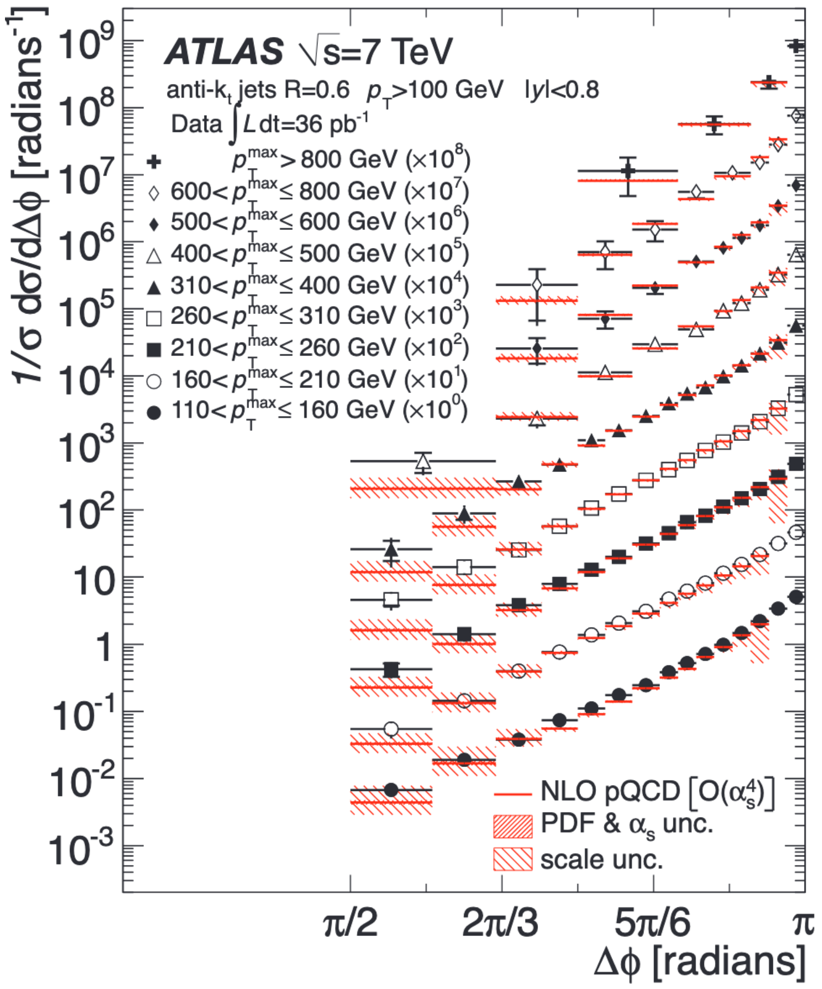
Jet calibration

The transverse momentum imbalance is used, for example, in jet calibration. [The CMS Collaboration '11; The ATLAS Collaboration '12]

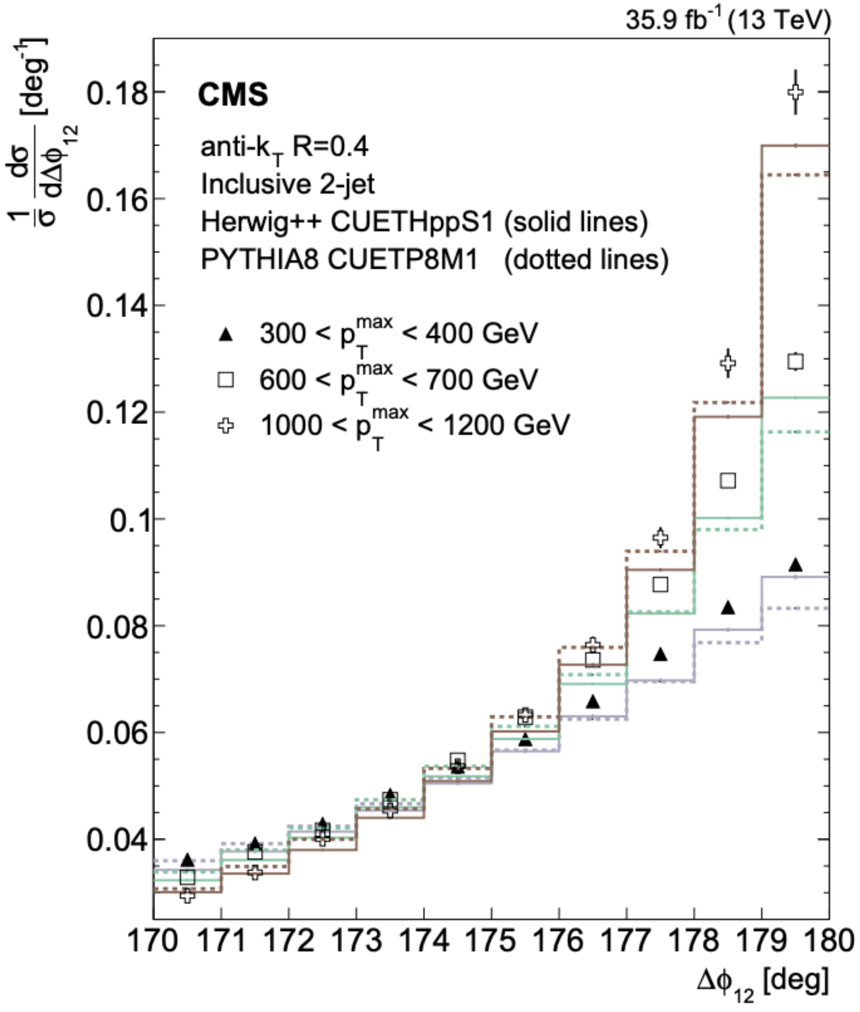
q_T-slicing for multiple jet

By defining the slicing variable with WTA jet momentum, we can generalize q_T-slicing to colored multi-jet systems. [RJF, Rahn, Shao, Waalewijn, Wu '24]

TMD Measurement at LHC



[The ATLAS Collaboration '11]



[The CMS Collaboration '19]

Azimuthal decorrelation $\delta\phi$ and transverse momentum imbalance q_T

Back-to-Back Dynamics

In dijet production, momentum conservation in the transverse plane enforces the back-to-back alignment at the Born level.

Radiation Imbalance

Soft and collinear radiation induces deviations from this topology, generating imbalances in the transverse momentum q_T or azimuthal angle $\delta\phi$.

$$q_T = |\vec{p}_{1,T} + \vec{p}_{2,T}|$$

$$\delta\phi = |\pi - |\phi_1 - \phi_2|| \approx \frac{q_y}{p_{1,T}}$$

All-order resummation is required in the back-to-back limit!

Non-global logarithms (NGLs)

soft radiation sensitive to the jet boundary
Standard E-scheme jet

Non-global logarithms (NGLs)
Complex factorization scheme

TMD predictions are traditionally bottlenecked at **NLL** accuracy due to complex factorization framework.
[Sun, Yuan, Yuan ’14; Chien, Shao, Wu ’19; Gao, Kang, Shao, Terry, Zhang ’23]

Winner-take-all recombination scheme

- **Standard E-scheme** simply sums the four-momenta of all particles within the jet.
- **Winner-take-all (WTA) scheme:** [Bertolini, Chan, Thaler '13]
  Clustering step: for the closest emission pair i and j:

$$p_{T,r} = p_{T,i} + p_{T,j}, \quad \hat{n}_r = \begin{cases} \hat{n}_i, & p_{T,i} \geq p_{T,j} \\ \hat{n}_j, & p_{T,i} < p_{T,j} \end{cases}$$

- The WTA scheme is ***recoil-free***.
- For the azimuthal decorrelation $\delta\phi$, the WTA scheme eliminates non-global logarithms (NGLs) entirely. [Chien, Rahn, Velzen, Shao, Waalewijn, Wu '20; Chien, Rahn, Shao, Waalewijn, Wu '22]
- For the transverse momentum imbalance q_T , the standard NGLs of q_T disappear and the NGLs of jet radius R in the small R limit is isolated in the soft sector. [RJF, Rahn, Shao, Waalewijn, Wu '26]

WTA Transverse Momentum

- Consider a multijet scattering process at LHC: $p(P_a) + p(P_b) \to \sum_{i=1}^{N} J(p_i) + X.$
- The total transverse momentum of the final jets is defined as:

$$\vec{q}_T \equiv \sum_{i=1}^{N} \vec{p}_{i,T}^{\,\mathrm{WTA}} = \sum_{i=1}^{N} \left(\sum_{k \in \text{jet-}i} p_{k,T} \right) \vec{n}_{W_i,T}.$$

$$\vec{q}_T = \sum_{i=1}^{N} \sum_{k \in \text{jet-}i} \left(p_{k,T}\, \vec{n}_{W_i,T} - \vec{p}_{k,T} \right) - \vec{p}_{a,T} - \vec{p}_{b,T} - \vec{p}_{s,T}$$

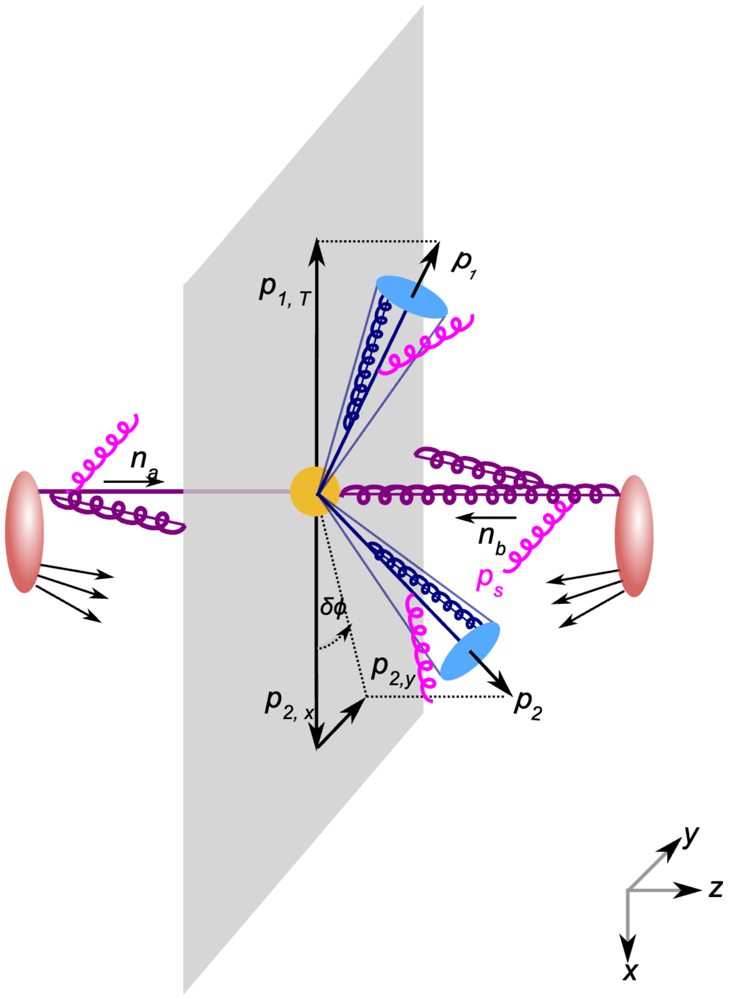
$$= - \sum_{i=1}^{N} \sum_{k \in \text{jet-}i} \vec{p}_{k,\perp}^{\,(i)} - \vec{p}_{a,T} - \vec{p}_{b,T} - \vec{p}_{s,T},$$

with $\vec{p}_{k,\perp}^{\,(i)} \equiv \vec{p}_{k,T} - p_{k,T}\, \vec{n}_{W_i,T}, \quad k \in \text{jet-}i.$

$$\vec{p}_{k,\perp}^{\,(i)} \cdot \vec{n}_{W_i,T} = 0 + \mathcal{O}\left(p_{k,T} \left(\phi_k - \phi_i^{\mathrm{WTA}} \right)^2 \right)$$

→ In-cone contribution $\vec{\mathrm{p}}_{\mathrm{k},\perp}^{(\mathrm{i})}$ is perpendicular to the jet and the beam axis at leading power.

Azimuthal decorrelation $\delta\phi \equiv |\pi - |\phi_1^{\rm WTA} - \phi_2^{\rm WTA}||.$



- In the back-to-back limit $(\delta\phi \ll 1)$ $\quad \delta\phi \approx \dfrac{q_y}{p_T}$
  - **hard**: $\quad p_h^\mu \sim p_T(1,1,1),$
  - $n_{a,b}$**-collinear**: $\quad p_c^\mu \sim p_T\,(\delta\phi^2, 1, \delta\phi)_{n_i\bar{n}_i},$
  - **soft**: $\quad p_s^\mu \sim p_T\,(\delta\phi, \delta\phi, \delta\phi),$
  - $n_{1,2}$**-collinear**: $\quad p_c^\mu \sim p_T\,(\delta\phi^2, 1, \delta\phi)_{n_i\bar{n}_i}.$
- SCET-II Factorization formula:

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}\eta_1\mathrm{d}\eta_2\mathrm{d}p_T\mathrm{d}q_y} = \sum_{ijk\ell}\frac{x_a x_b}{16\pi\hat{s}^2}\frac{2p_T}{1+\delta_{k\ell}}\mathcal{H}_{ij\to k\ell,JI}(p_T,\eta_1-\eta_2,\mu)$$
$$\times\int_{-\infty}^{\infty}\frac{\mathrm{d}b_y}{2\pi}e^{ib_yq_y}\boldsymbol{S}_{ijk\ell,IJ}(b_y,\eta_1,\eta_2,\mu,\nu)\mathcal{J}_k\,(b_y,\omega_1,\mu,\nu)\,\mathcal{J}_\ell\,(b_y,\omega_2,\mu,\nu)$$
$$\times\, B_{i/p}\,(x_a,b_y,\omega_a,\mu,\nu)\,B_{j/p}\,(x_b,b_y,\omega_b,\mu,\nu)$$

Resummation formula for $\boldsymbol{\delta\phi}$

RRG: Jet & beam ν_i → Soft ν_s

RG: Hard μ_h → Soft, Jet & beam μ_f

Rapidity **R**enormalization **G**roup [Chiu, Jain, Neill, Rothstein '11]

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}\eta_1\,\mathrm{d}\eta_2\,\mathrm{d}p_T\,\mathrm{d}q_y} = \sum_{ijk\ell}\frac{x_a x_b}{16\pi\hat{s}^2}\frac{2p_T}{1+\delta_{k\ell}}\int_0^\infty \frac{\mathrm{d}b_y}{\pi}\cos\left(b_y q_y\right)\prod_{i=a,b,1,2}\left(\frac{\nu_s}{\nu_i}\right)^{\Gamma^i_\nu(b_y,\mu_f)}$$
$$\times\sum_{KK'}\exp\left\{\int_{\mu_h}^{\mu_f}\frac{\mathrm{d}\mu}{\mu}\left[\gamma_\mathrm{cusp}(\alpha_s)\left(C_H\ln\frac{\hat{s}}{\mu^2}+\lambda_K+\lambda^*_{K'}\right)+2\gamma_H(\alpha_s)\right]\right\}$$
$$\times\boldsymbol{\mathcal{H}}_{ij\to k\ell,KK'}(p_T,\eta_1-\eta_2,\mu_h)\boldsymbol{S}_{ijk\ell,K'K}(b_y,\eta_1,\eta_2,\mu_f,\nu_s)$$
$$\times B_{i/p}\left(x_a,\omega_a,b_y,\mu_f,\nu_a\right)B_{j/p}\left(x_b,\omega_b,b_y,\mu_f,\nu_b\right)$$
$$\times\mathcal{J}_k\left(b_y,\omega_1,\mu_f,\nu_1\right)\mathcal{J}_\ell\left(b_y,\omega_2,\mu_f,\nu_2\right)$$
$$\times\exp\left[-S^i_\mathrm{NP}(b_y,Q_0,\omega_a)-S^j_\mathrm{NP}(b_y,Q_0,\omega_b)\right].$$

Transverse momentum imbalance q_T

$$\vec{q}_T \equiv \sum_{i=1}^{N} \vec{p}_{i,T}^{\,\mathrm{WTA}} = -\sum_{i=1}^{N} \sum_{k\in\text{jet-}i} \vec{p}_{k,\perp}^{\,(i)} - \vec{p}_{a,T} - \vec{p}_{b,T} - \vec{p}_{s,T}$$

- SCET Factorization formula:

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}\eta_1\,\mathrm{d}\eta_2\,\mathrm{d}p_T\,\mathrm{d}q_T} = \sum_{ijk\ell} \frac{x_a x_b}{16\pi \hat{s}^2} \frac{2p_T}{1+\delta_{k\ell}} \mathcal{H}_{ij\to k\ell,JI}(p_T, \eta_1-\eta_2, \mu)\; q_T \int_0^{2\pi} \mathrm{d}\phi_q \int \frac{\mathrm{d}^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}$$
$$\times\, \boldsymbol{S}_{ijk\ell,IJ}(\vec{b}_T, \eta_1, \eta_2, R, \mu, \nu)\,\mathcal{J}_k\,(b_y, \omega_1, \mu, \nu)\;\mathcal{J}_\ell\,(b_y, \omega_2, \mu, \nu)$$
$$\times\, B_{i/p}(x_a, \omega_a, b_T, \mu, \nu)\; B_{j/p}(x_b, \omega_b, b_T, \mu, \nu).$$

- Only soft function is new: Outside the jet ($\vec{q}_T$), inside the jet (q_y).

Small-R refactorization of q_T-soft function ($q_T/p_T \ll R \ll 1$)

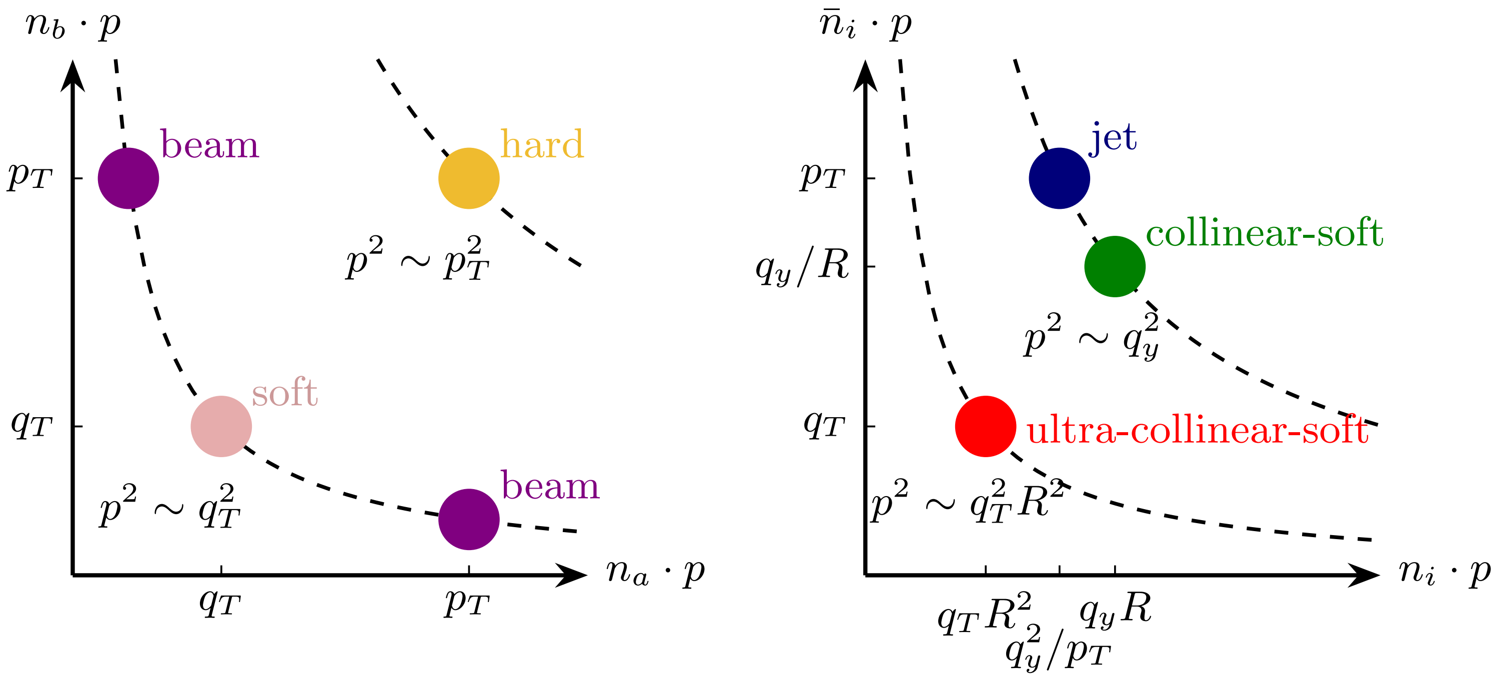
- In small-R limit, the q_T-soft function refactorizes into global and "collinear-soft" contributions, up to power corrections in the jet radius.,

$$\boldsymbol{S}_{ijk\ell}(\vec{b}_T, R, \eta_1, \eta_2, \mu, \nu) = \boldsymbol{S}^{\text{global}}_{ijk\ell}(\vec{b}_T, \eta_1, \eta_2, \mu, \nu) \boxed{S_k(\vec{b}_T, \eta_1, R, \mu, \nu)} S_\ell(\vec{b}_T, \eta_2, R, \mu, \nu) + \mathcal{O}(R^{2n}).$$

Encoding the collinear-soft and ultra-collinear-soft modes

- **Non-Global Logarithms:** $\mathrm{S}_k(\mathrm{S}_\ell)$ contains NGLs of the jet radius.
- Calculation Method 1: These can be computed using the **multiplicity-interacting EFT framework**. [Becher, Neubert, Rothen, Shao '15, '16; Becher, Pecjak, Shao '16; Larkoski, Moult, Neill '15]
- Calculation Method 2: Boosting along the jet axis reduces $\mathrm{S}_k(\mathrm{S}_\ell)$ to the **hemisphere soft function**, separating the dependence:
  - **In-cone:** depends on $b_\perp$
  - **Out-of-cone:** depends on b_-

SCET modes for q_T distribution



(i = 1 or 2)

- **hard:** $p_h^\mu \sim p_T(1,1,1)$,
- $n_{a,b}$**-collinear:** $p_{c_i}^\mu \sim (q_T^2/p_T, p_T, q_T)_{n_i\bar{n}_i}$,
- $n_{1,2}$**-collinear:** $p_{c_i}^\mu \sim (q_y^2/p_T, p_T, q_y)_{n_i\bar{n}_i}$.
- **soft:** $p_s^\mu \sim q_T\,(1,1,1)$,
- $n_{1,2}$**-collinear-soft:** $p_{cs}^\mu \sim |q_y|/R\,(R^2,1,R)_{n_i\bar{n}_i}$,
- $n_{1,2}$**-ultra-collinear-soft:** $p_{ucs}^\mu \sim q_T\,(R^2,1,R)_{n_i\bar{n}_i}$.

Refactorization of S_i

- For the momentum k of ultra-collinear-soft mode, we have $k^- \gg k^+, k_\perp$.
- Under this hierarchy, the Fourier exponent associated with the out-of-cone measurement admits the power expansion

$$\underbrace{\vec{b}_T \cdot \vec{k}_T}_{\text{beam coordinates}} = -b \cdot k \approx \underbrace{-\frac{1}{2} b^+ k^-}_{\text{jet coordinates}} ,$$

with $b^\mu = (0, b_x, b_y, 0)$ and $b^+ = -b_x n_{i,x}$.

- Momentum decomposition along the jet axis: $p^\mu = (n_i \cdot p) \frac{\bar{n}_i^\mu}{2} + (\bar{n}_i \cdot p) \frac{n_i^\mu}{2} + p_\perp^\mu \equiv (n_i \cdot p,\ \bar{n}_i \cdot p,\ p_\perp^\mu)_{n_i \bar{n}_i}$

- Applying the factorization framework for non-global observables in the small R limit: [Becher, Neubert, Rothen, Shao '15, '16; Becher, Pecjak, Shao '16]

$$S_i(\vec{b}_T, \eta_i, R, \mu, \nu) = \sum_{m_i=0}^{\infty} \prod_{j=1}^{m_i} \int \frac{d\Omega(\vec{u}_j)}{4\pi} \frac{1}{d_i} \mathrm{Tr}\left[\boldsymbol{\mathcal{S}}^{\mathrm{cs}}_{m_i}(\{n_i, \bar{n}_i, \underline{u}\}, b_y, R, \mu, \nu) \boldsymbol{\mathcal{S}}^{\mathrm{ucs}}_{m_i}(\{n_i, \bar{n}_i, \underline{u}\}, b_x, R, \mu) \right]$$

$$:= \sum_{m_i=0}^{\infty} \left\langle \boldsymbol{\mathcal{S}}^{\mathrm{ucs}}_{m_i}(\{n_i, \bar{n}_i, \underline{u}\}, b_x, R, \mu) \otimes \boldsymbol{\mathcal{S}}^{\mathrm{cs}}_{m_i}(\{n_i, \bar{n}_i, \underline{u}\}, b_y, R, \mu, \nu) \right\rangle .$$

Collinear-soft and ultra-collinear-soft functions

$$\mathcal{S}^{\rm cs}_{m_i}(\{n_i,\bar n_i,\underline{u}\},b_y,R,\mu,\nu) =$$

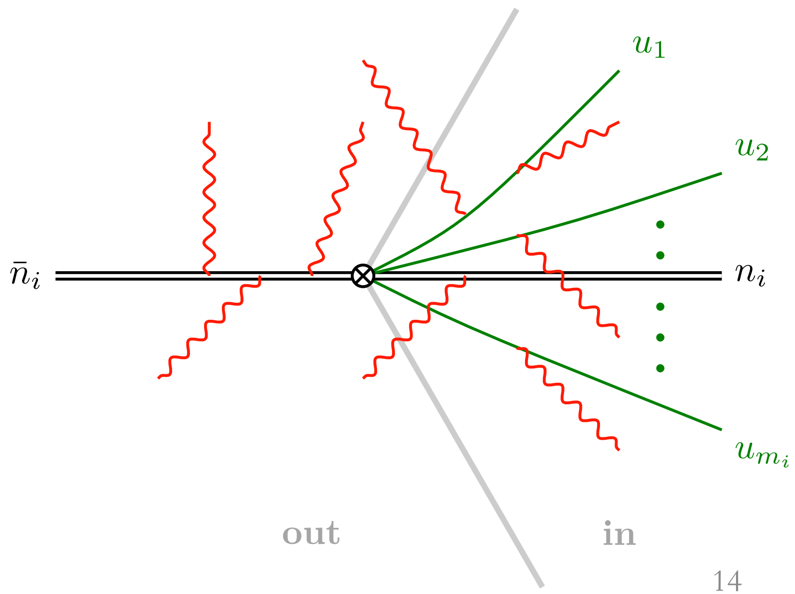
$$\prod_{j=1}^{m_i}\int \frac{\mathrm{d}E_j E_j^{d-3}}{(2\pi)^{d-2}} \left|\mathcal{M}^{cs}_{m_i}(n_i,\bar n_i,\{\underline{p}\})\right\rangle \left\langle \mathcal{M}^{cs}_{m_i}(n_i,\bar n_i,\{\underline{p}\})\right| \exp\left(ib_y\sum_{k=1}^{m_i} p_{k,y}\right)\Theta^i_{\rm in}(\{\underline{p}\}),$$

Collinear-soft amplitudes: $\left|\mathcal{M}^{cs}_{m_i}(n_i,\bar n_i,\{\underline{p}\})\right\rangle = \langle\{\underline{p}\}|S(n_i)S^\dagger(\bar n_i)|0\rangle$

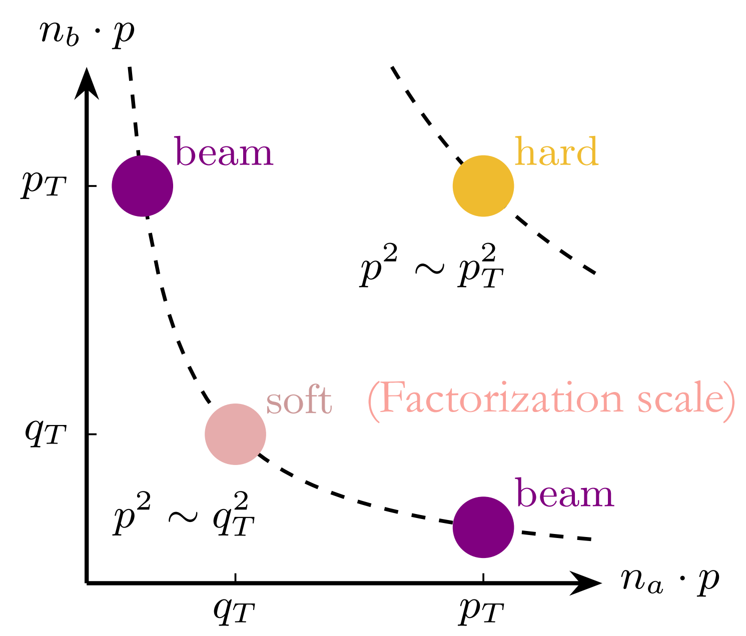
In-cone constraint: $\Theta^i_{\rm in}(\{\underline{p}\}) = \prod_{j=1}^{m_i}\Theta\left(R_i^2 - \frac{n_i\cdot p_j}{\bar n_i\cdot p_j}\right), \quad R_i = \frac{R}{2\cosh\eta_i}.$

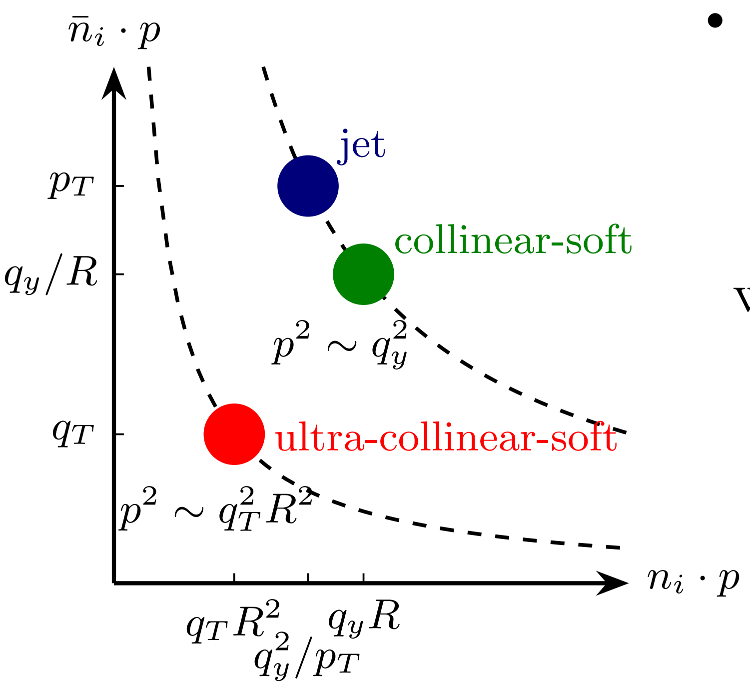
$$\mathcal{S}^{\rm ucs}_{m_i}(\{n_i,\bar n_i,\underline{u}\},b_x,R,\mu) = \sum\!\!\!\!\!\!\!\int_{X_{ucs}} \langle 0|U^\dagger_{m_i}(u_{m_i})\cdots U^\dagger_1(u_1)U^\dagger_b(\bar n_i)U^\dagger_a(n_i)|X_{ucs}\rangle$$

$$\langle X_{ucs}|U_a(n_i)U_b(\bar n_i)U_1(u_1)\cdots U_{m_i}(u_{m_i})|0\rangle \exp\left(-\frac{i}{2}b^+\sum_{j\notin \text{jet}-i} k_j^-\right)$$



RRG and global RG evolutions





- Two sets of RRG evolution:

$$\prod_{i=ab}\left(\frac{\nu_s}{\nu_i}\right)^{\Gamma_\nu^i(b_T,\mu_f)} \quad \prod_{i=12}\left(\frac{\nu_{cs,i}}{\nu_i}\right)^{\Gamma_\nu^i(b_y,\mu_{cs})}$$

with the rapidity scales

$$\nu_i \sim \omega_i\,(i=a,b,1,2), \quad \nu_s \sim b_0/b_T,$$

$$\nu_{cs,i} \sim b_0/(|b_y|R_i)\,(i=1,2).$$

- Typical scales in RG evolution: $\mu_h \sim 2p_T, \quad \mu_b \sim \mu_s \sim b_0/b_T, \quad \mu_{cs} \sim \mu_j \sim b_0/|b_y|, \quad \mu_{ucs} \sim R\,b_0/(2|b_x|)$.
- The RG evolution of the hard function is analogous to that for the q_y distribution.
- The RG evolution of the jet function from μ_j to $\mu_f \sim b_0/b_T$ should be included to resum the large logarithms $\ln(|b_y|/b_T)$ that arise in the limit $\sin\phi_b \to 0$.

Non-global RG evolution

- The evolution matrix for the collinear-soft and ultra-collinear-soft functions:

$$\prod_{i=1,2} \exp\left[\int_{\mu_{cs}}^{\mu_f} \frac{\mathrm{d}\mu}{\mu}\Gamma_i^{\mathrm{cs}} + \int_{\mu_{ucs}}^{\mu_f} \frac{\mathrm{d}\mu}{\mu}\Gamma_i^{\mathrm{ucs}}\right] U_{\mathrm{NG}}^i(\mu_{ucs}, \mu_{cs}).$$

- We resum the leading non-global logarithms $\ln R$,

$$U_{\mathrm{NG}}^i(\mu_{ucs}, \mu_{cs}) \overset{\mathrm{LL}_R}{=} \sum_{m=0}^{\infty} \langle \mathbf{1} \,\hat{\otimes}\, \boldsymbol{U}_{m0}(\{n_i, \bar{n}_i, \underline{u}\}, \mu_{ucs}, \mu_{cs})\rangle$$

- Dasgupta–Salam parametrization [Dasgupta, Salam, '01]

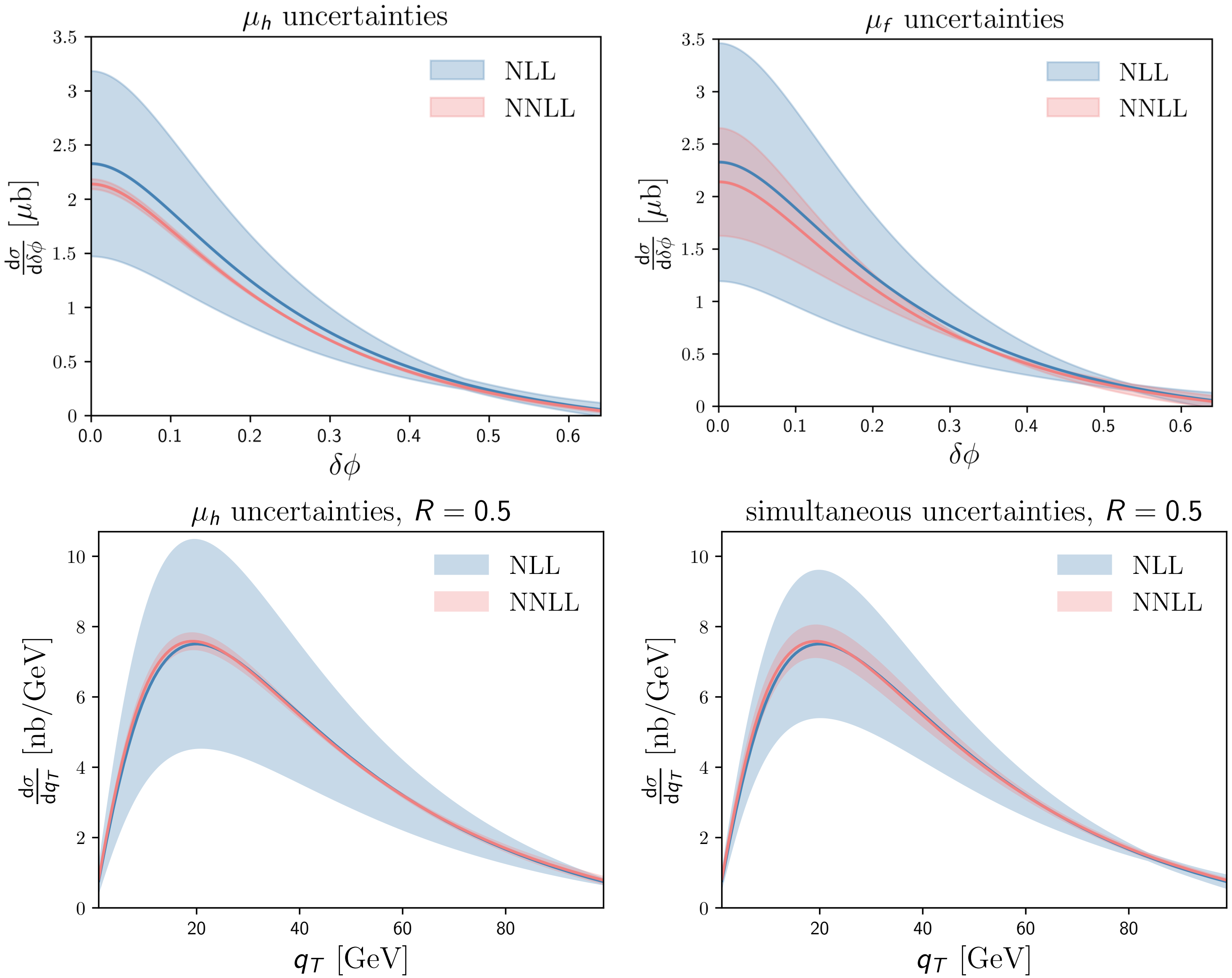
$$U_{\mathrm{NG}}^i(\mu_{ucs}, \mu_{cs}) \approx \exp\left(-C_A C_i \frac{\pi^2}{3} u^2 \frac{1+(au)^2}{1+(bu)^c}\right), \quad u = \frac{1}{\beta_0} \ln \frac{\alpha_s(\mu_{ucs})}{\alpha_s(\mu_{cs})},$$

with $a = 0.85 C_A, b = 0.86 C_A, c = 1.33.$

NNLL+LL$_R$ Resummation formula for q_T

$$
\begin{aligned}
&\frac{\mathrm{d}^5\sigma}{\mathrm{d}\eta_1\,\mathrm{d}\eta_2\,\mathrm{d}p_T\,\mathrm{d}q_T\,\mathrm{d}\phi_q} \\
&= \sum_{ijk\ell} \frac{x_a x_b}{16\pi\hat{s}^2}\frac{2p_T}{1+\delta_{k\ell}} q_T \int_0^{2\pi}\mathrm{d}\phi_b \int_0^\infty \frac{b_T\,\mathrm{d}b_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T} \\
&\times \sum_{KK'} \exp\left\{\int_{\mu_h}^{\mu_f}\frac{\mathrm{d}\mu}{\mu}\left[\gamma_{\rm cusp}(\alpha_s)\left(C_H\ln\frac{\hat{s}}{\mu^2}+\lambda_K+\lambda_{K'}^*\right)+2\gamma_H(\alpha_s)\right]\right\} \\
&\times \prod_{p=ab}\left(\frac{\nu_s}{\nu_p}\right)^{\Gamma_\nu^p(b_T,\mu_f)} \prod_{p=12}\left(\frac{\nu_{cs,p}}{\nu_p}\right)^{\Gamma_\nu^p(b_y,\mu_{cs})} \\
&\times \prod_{p=12}\exp\left[\int_{\mu_{cs}}^{\mu_f}\frac{\mathrm{d}\mu}{\mu}\left(\Gamma^{J_p}+\Gamma_p^{\rm cs}\right)+\int_{\mu_{ucs}}^{\mu_f}\frac{\mathrm{d}\mu}{\mu}\Gamma_p^{\rm ucs}\right]U_{\rm NG}^p(\mu_{ucs},\mu_{cs}) \\
&\times \boldsymbol{\mathcal{H}}_{ij\to k\ell,KK'}(p_T,\eta_1-\eta_2,\mu_h)\boldsymbol{\mathcal{S}}^{\rm global}_{ijk\ell,K'K}(\vec{b}_T,\eta_1,\eta_2,\mu_f,\nu_s) \\
&\times \prod_{p=12}\sum_{m_p=0}^{\infty}\left\langle \boldsymbol{\mathcal{S}}^{\rm ucs}_{m_p}(\{n_p,\bar{n}_p,\underline{u}\},\vec{b}_T,R,\mu_{ucs})\otimes\boldsymbol{\mathcal{S}}^{\rm cs}_{m_p}(\{n_p,\bar{n}_p,\underline{u}\},b_y,R,\mu_{cs},\nu_{cs,p})\right\rangle \\
&\times B_{i/p}(x_a,b_T,\omega_a,\mu_f,\nu_a)\,B_{j/p}(x_b,b_T,\omega_b,\mu_f,\nu_b)\,\mathcal{J}_k(b_y,\omega_1,\mu_j,\nu_1)\,\mathcal{J}_\ell(b_y,\omega_2,\mu_j,\nu_2) \\
&\times \exp\left[-S_{\rm NP}^i(b_T,\omega_a,\nu_a)-S_{\rm NP}^j(b_T,\omega_b,\nu_b)\right].
\end{aligned}
$$

NLL and NNLL Resummation results



- b_*-prescription:

$$b_{T,*} = \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}}, \quad b_{y,*} = \frac{|b_y|}{\sqrt{1 + b_y^2/b_{\max}^2}}, \quad b_{x,*} = \frac{|b_x|}{\sqrt{1 + b_x^2/b_{\max}^2}}$$

- Scales in $\delta\phi$ resummation:

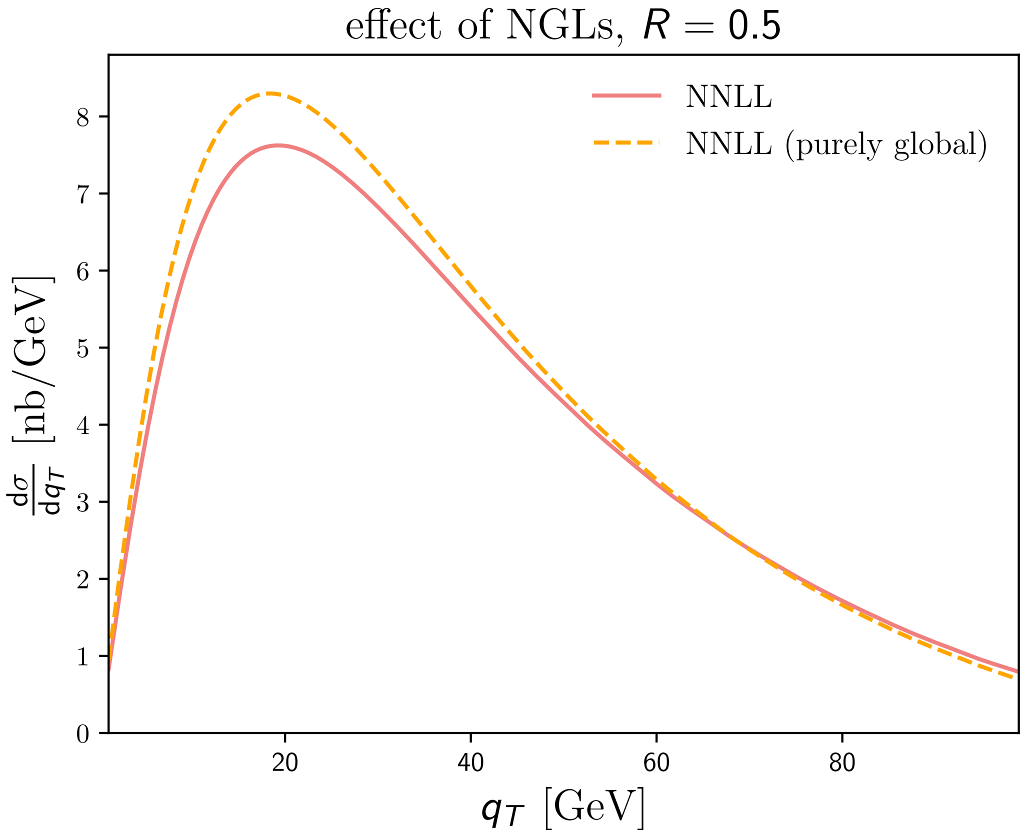
$$\mu_h = v_h \cdot 2p_T \qquad \mu_f = \min\{v_f \cdot \mu_{b_{y,*}}, \mu_h\}$$

- Scales in q_T resummation:

$$\mu_s = \mu_b = \mu_f = \min\left\{v_f \frac{b_0}{b_{T,*}}, \mu_h\right\}, \quad \mu_j = \mu_{cs} = v_f \frac{b_0}{b_{y,*}},$$

$$\mu_h = v_h \cdot 2p_T \qquad \mu_{ucs} = \min\left\{v_f \frac{Rb_0}{2b_{x,*}}, \mu_{cs}\right\}.$$

Effects of NGLs



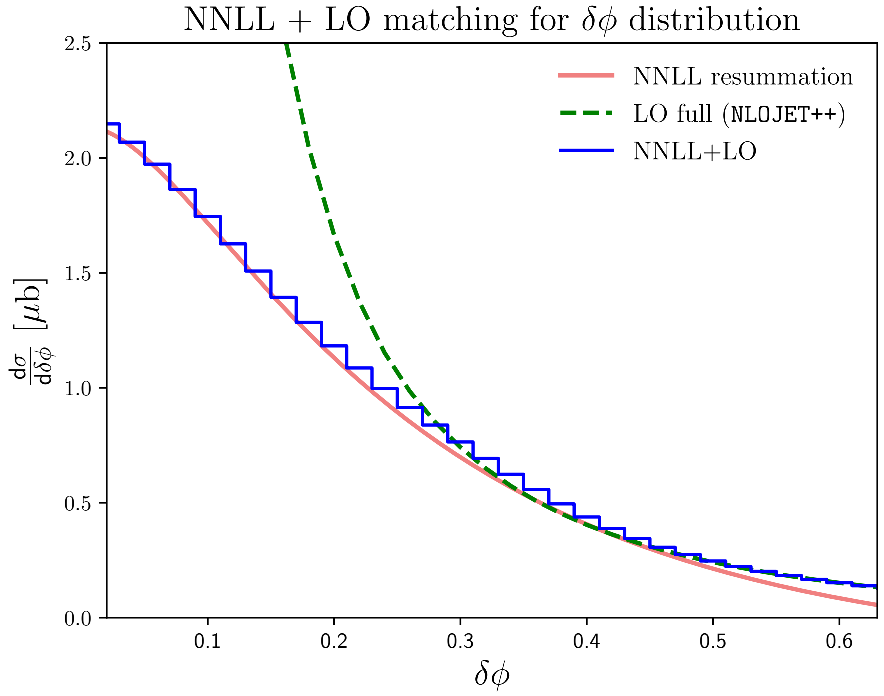
- The comparison reveals that the inclusion of the resummation of NGLs leads to a noticeable **suppression** of the cross-section in the peak region.

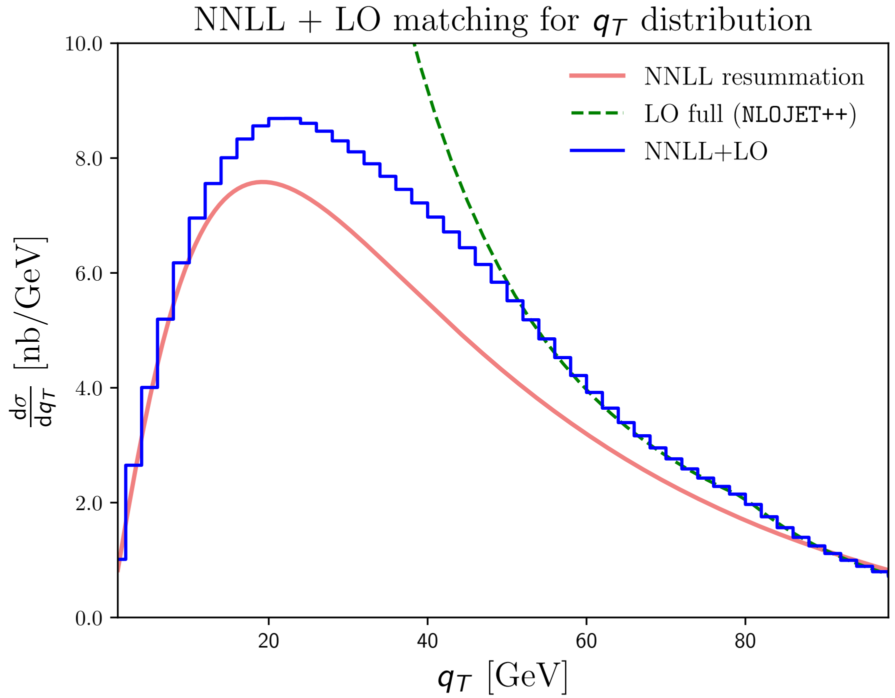
NNLL+LO Matching

$$\mathrm{d}\sigma_{\mathrm{match}}(\mathrm{NNLL}+\mathrm{LO}) = (1 - t(\mathcal{O}))\left(\mathrm{d}\sigma(\mathrm{NNLL}) + U_H^{\mathrm{eff}}(\mathcal{O})\ \mathrm{d}\sigma(\mathrm{LO\ non\text{-}singular})\right) + t(\mathcal{O})\ \mathrm{d}\sigma(\mathrm{LO\ full})$$

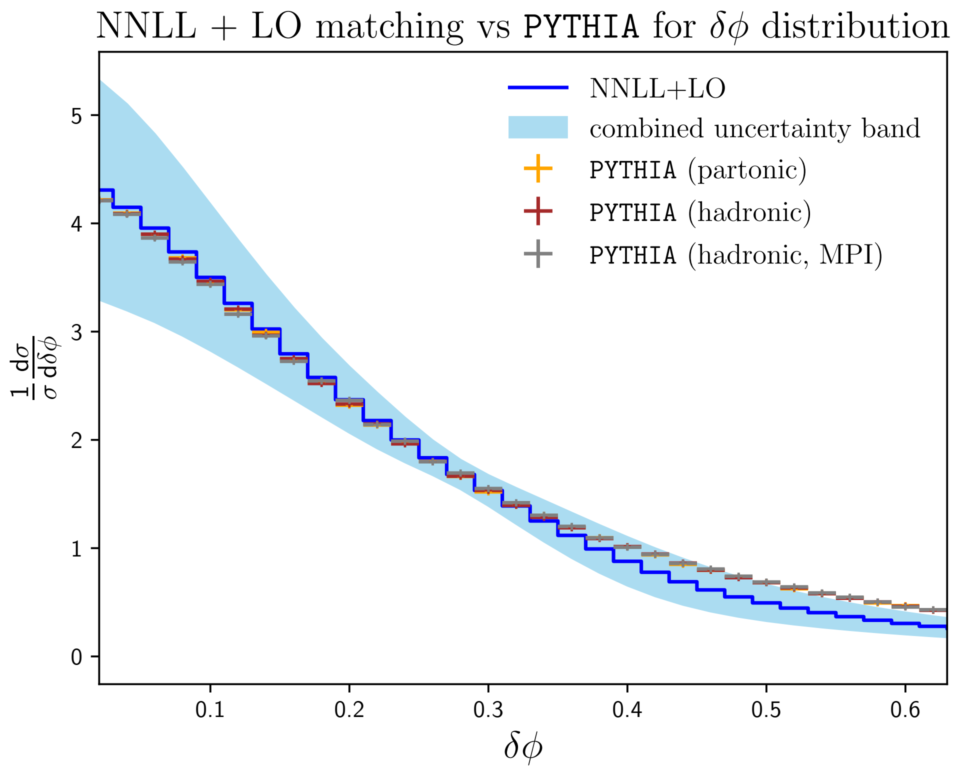
- The effective evolution matrix, $U_H^{\mathrm{eff}}(\mathcal{O}) = \exp\left[\int_{2p_T}^{\mu_{\mathcal{O}}} \frac{\mathrm{d}\mu}{\mu}\left(\frac{\alpha_s(2p_T)}{\pi} 4C_A \ln\frac{4p_T^2}{\mu^2}\right)\right],$

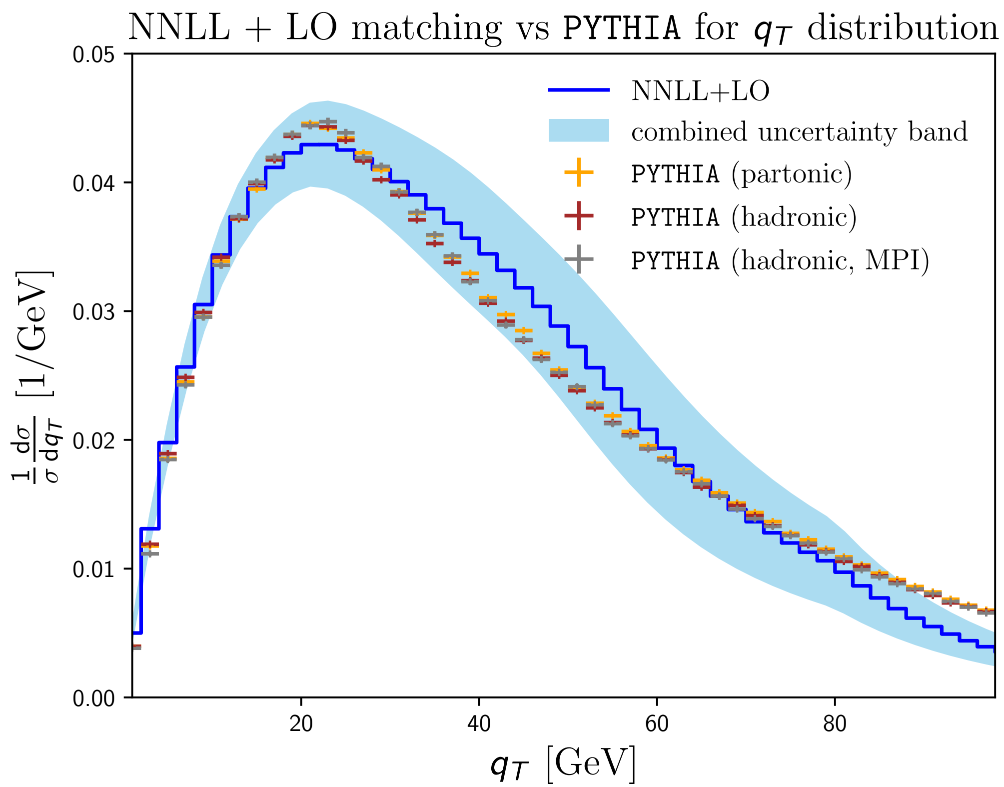
  is used to suppress the divergent non-singular terms in the infrared regime.
- The transition function $t(\mathcal{O})$ acts as a smooth switch, transitioning from 0 in the resummation-dominated region to 1 in the fixed-order region.





Comparison with PYTHIA 8.3





- Both the azimuthal decorrelation $\delta\phi$ and the transverse momentum imbalance q_T are robust against hadronization and multi-parton interactions (MPI).

Conclusions

- WTA scheme eliminates standard non-global logarithms (NGLs), simplifying the factorization;
- Small-R Limit: Exploiting $R \ll 1$ allows the soft function to be **refactorized** into global, collinear-soft, and ultra-collinear-soft modes, enabling the calculation of NGLs of R;
- Joint resummation involving NNLL accuracy for global logarithms and LL accuracy for the small-R NGLs.

THANK YOU!

Backup: Combination of the collinear-soft and ultra-collinear-soft functions

- The perturbative expansion of S_i is organized as

$$S_i(b_\perp, b^+) = 1 + \frac{Z_\alpha \alpha_s(\mu)}{4\pi} S_i^{(1)}(b_\perp, b^+) + \left(\frac{Z_\alpha \alpha_s(\mu)}{4\pi}\right)^2 S_i^{(2)}(b_\perp, b^+) + \mathcal{O}(\alpha_s^3)$$

- NLO S_i: $S_i^{(1)}(b_\perp, b^+) = \langle \boldsymbol{S}_0^{\mathrm{ucs}(1)} \boldsymbol{S}_0^{\mathrm{cs}(0)} \rangle + \langle \boldsymbol{S}_0^{\mathrm{ucs}(0)} \boldsymbol{S}_0^{\mathrm{cs}(0)} \rangle + \langle \boldsymbol{S}_1^{\mathrm{ucs}(0)} \otimes \boldsymbol{S}_1^{\mathrm{cs}(1)} \rangle = \langle \boldsymbol{S}_0^{\mathrm{ucs}(1)} \rangle + \langle \mathbf{1} \otimes \boldsymbol{S}_1^{\mathrm{cs}(1)} \rangle$

with: $\langle \mathbf{1} \otimes \boldsymbol{S}_1^{\mathrm{cs}(1)}(b_\perp) \rangle = C_i \left(\frac{\mu |b_\perp|}{b_0}\right)^{2\epsilon} \left(\frac{\nu |b_\perp| R_i}{b_0}\right)^{\eta} h_{\mathrm{in}}, \quad \langle \boldsymbol{S}_0^{\mathrm{ucs}(1)}(b^+) \rangle = C_i \left(\frac{i\, b^+ \mu}{b_0 R_i}\right)^{2\epsilon} s_{\mathrm{out}},$

$$h_{\mathrm{in}} = \frac{2}{\epsilon^2} - \frac{4}{\eta\epsilon} - \frac{\pi^2}{6} + \left(-\frac{\eta}{\epsilon^3} + \frac{\pi^2 \eta}{12\epsilon} - \frac{\zeta_3}{3}\eta - \frac{\pi^2 \epsilon}{3\eta} - \frac{4\zeta_3}{3}\epsilon - \frac{17\pi^4}{1440}\eta\epsilon - \frac{4\zeta_3 \epsilon^2}{3\eta} - \frac{3\pi^4}{80}\epsilon^2 - \frac{\pi^4 \epsilon^3}{40\eta}\right)$$

$$s_{\mathrm{out}} = -\frac{2}{\epsilon^2} - \frac{\pi^2}{2} + \left(-\frac{14\zeta_3}{3}\epsilon - \frac{7\pi^4}{48}\epsilon^2\right)$$

Backup: Combination of the collinear-soft and ultra-collinear-soft functions

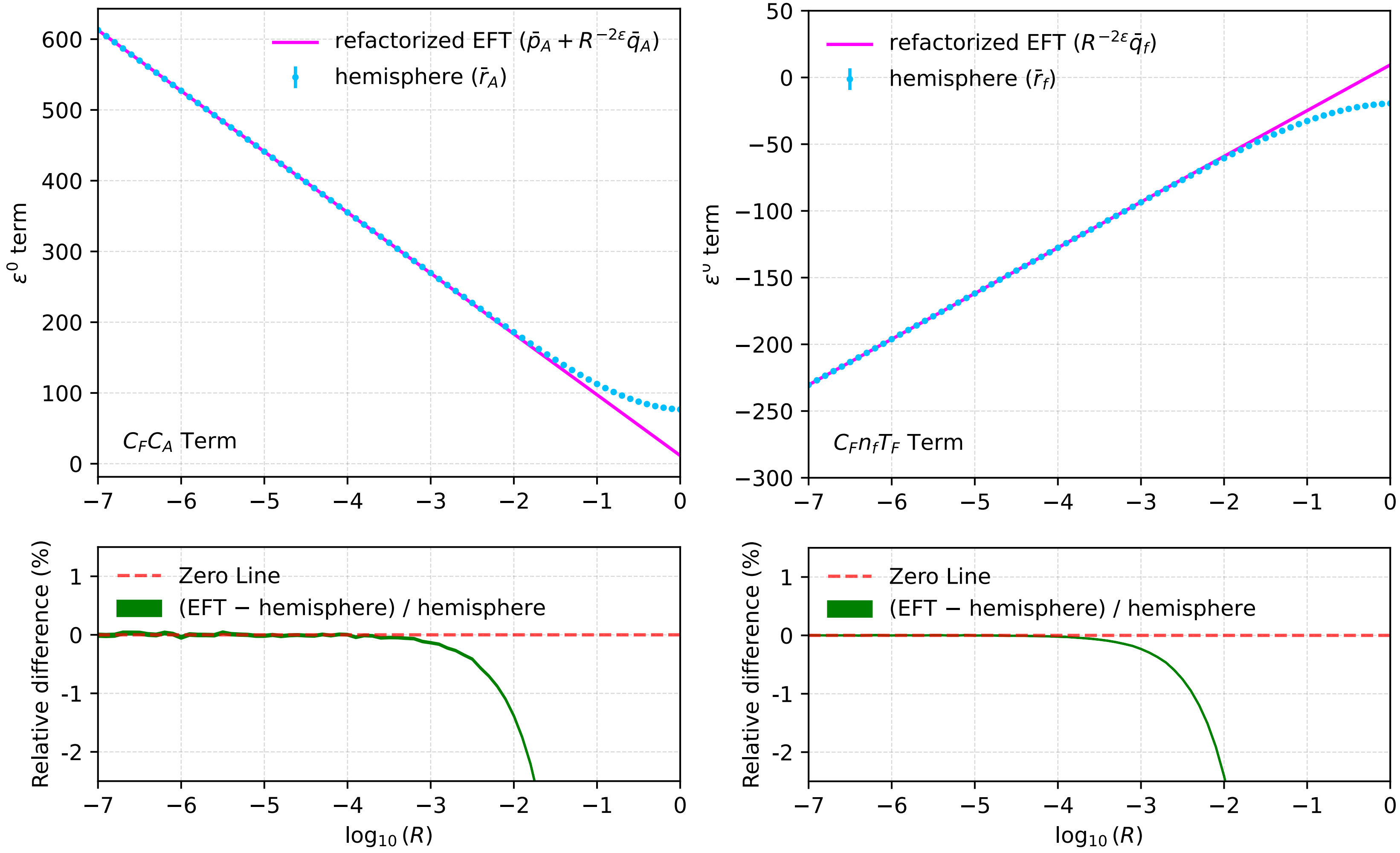
- NNLO S_i:

$$\begin{aligned} S_i^{(2)}(b_\perp, b^+) &= \left(\frac{\mu|b_\perp|}{b_0}\right)^{4\epsilon}\left(\frac{\nu|b_\perp|R_i}{b_0}\right)^{\eta}\omega^2\left[\omega^2\left(\frac{\nu|b_\perp|R_i}{b_0}\right)^{\eta} C_i^2\frac{h_{\rm in}^2}{2} + C_iC_A(h_A + v_A^{\rm in}) + C_in_fT_Fh_f\right] \\ &+ \left(\frac{ib^+\mu}{b_0R_i}\right)^{4\epsilon}\left[C_i^2\frac{s_{\rm out}^2}{2} + C_iC_A(g_A + v_A^{\rm out}) + C_in_fT_Fg_f\right] \\ &+ \left(\frac{ib^+\mu}{b_0R_i}\right)^{4\epsilon}\left[C_iC_A(s_A - g_A - v_A^{\rm out}) + C_in_fT_F(s_f - g_f)\right] \\ &+ \left(\frac{\mu|b_\perp|}{b_0}\right)^{2\epsilon}\left(\frac{ib^+\mu}{b_0R_i}\right)^{2\epsilon}\left[\left(\frac{\nu|b_\perp|R_i}{b_0}\right)^{\eta}\omega^2C_i^2h_{\rm in}s_{\rm out} + C_iC_Ap_A\right]. \end{aligned}$$

  - Double-real ultra-collinear-soft emissions, where one emission is in-cone and unobserved, while the other is emitted outside the jet cone and measured:

$$q_A = s_A - g_A - v_A^{\rm out} = \left(-\frac{2}{3} + \frac{22\pi^2}{9} - 4\zeta_3\right)\frac{1}{\epsilon} + \frac{40}{9} - \frac{134\pi^2}{27} + \frac{44\zeta_3}{3} + \frac{8\pi^4}{45}, \quad q_f = s_f - g_f = \left(\frac{4}{3} - \frac{8\pi^2}{9}\right)\frac{1}{\epsilon} - \frac{68}{9} + \frac{64\pi^2}{27} - \frac{16\zeta_3}{3}.$$

Backup: Refactorization of the hemisphere calculation and its power corrections



The comparison demonstrates that the two approaches are consistent in the small radius limit $R \ll 1$.

$$\bar{r}_A = \bar{p}_A + R^{-2\epsilon}\bar{q}_A + \mathcal{O}(\epsilon^0 R \ln R, \epsilon^0 R)$$
$$= \frac{2\pi^2}{3\epsilon^2} - \frac{2\left(3 - 11\pi^2 + 6\pi^2 \ln 2 - 18\zeta_3\right)}{9\epsilon} + \left[\left(\frac{4}{3} - \frac{44\pi^2}{9} + 8\zeta_3\right)\ln R\right.$$
$$\left. + \frac{40}{9} + \frac{7\pi^4}{15} + \frac{\pi^2}{27}\left(-134 + 36\ln^2 2\right) + \frac{44\zeta_3}{3} - 16\zeta_3 \ln 2\right] + \mathcal{O}(\epsilon^0 R \ln R, \epsilon^0 R),$$

$$\bar{r}_f = R^{-2\epsilon}\bar{q}_f + \mathcal{O}(\epsilon^0 R \ln R, \epsilon^0 R)$$
$$= \left(\frac{4}{3} - \frac{8\pi^2}{9}\right)\frac{1}{\epsilon} + \frac{8}{9}\left(-3 + 2\pi^2\right)\ln R + \frac{4}{27}\left(-51 + 16\pi^2 - 36\zeta_3\right) + \mathcal{O}(\epsilon^0 R \ln R, \epsilon^0 R).$$

Back-up: Anomalous dimensions

- The standard form of the RG equations of collinear-soft and ultra-collinear-soft functions can be derived as,

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\boldsymbol{\mathcal{S}}_m^{\mathrm{cs}}(\{n_i,\bar{n}_i,\underline{u}\},b_y,R,\mu,\nu) = \sum_{l=0}^{m}\boldsymbol{\Gamma}_{ml}^{\mathrm{cs}}\,\boldsymbol{\mathcal{S}}_l^{\mathrm{cs}}(\{n_i,\bar{n}_i,\underline{u}\},b_y,R,\mu,\nu)$$
$$= \sum_{l=0}^{m}\left(\Gamma^{\mathrm{cs}}\delta_{ml}\mathbf{1}+\hat{\boldsymbol{\Gamma}}_{ml}\right)\boldsymbol{\mathcal{S}}_l^{\mathrm{cs}}(\{n_i,\bar{n}_i,\underline{u}\},b_y,R,\mu,\nu),$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\boldsymbol{\mathcal{S}}_l^{\mathrm{ucs}}(\{n_i,\bar{n}_i,\underline{u}\},b_x,R,\mu) = -\sum_{m=l}^{\infty}\boldsymbol{\mathcal{S}}_m^{\mathrm{ucs}}(\{n_i,\bar{n}_i,\underline{u}\},b_x,R,\mu)\hat{\otimes}\boldsymbol{\Gamma}_{ml}^{\mathrm{ucs}}$$
$$= \sum_{m=l}^{\infty}\boldsymbol{\mathcal{S}}_m^{\mathrm{ucs}}(\{n_i,\bar{n}_i,\underline{u}\},b_x,R,\mu)\hat{\otimes}\left(\Gamma^{\mathrm{ucs}}\delta_{ml}\mathbf{1}-\hat{\boldsymbol{\Gamma}}_{ml}\right).$$

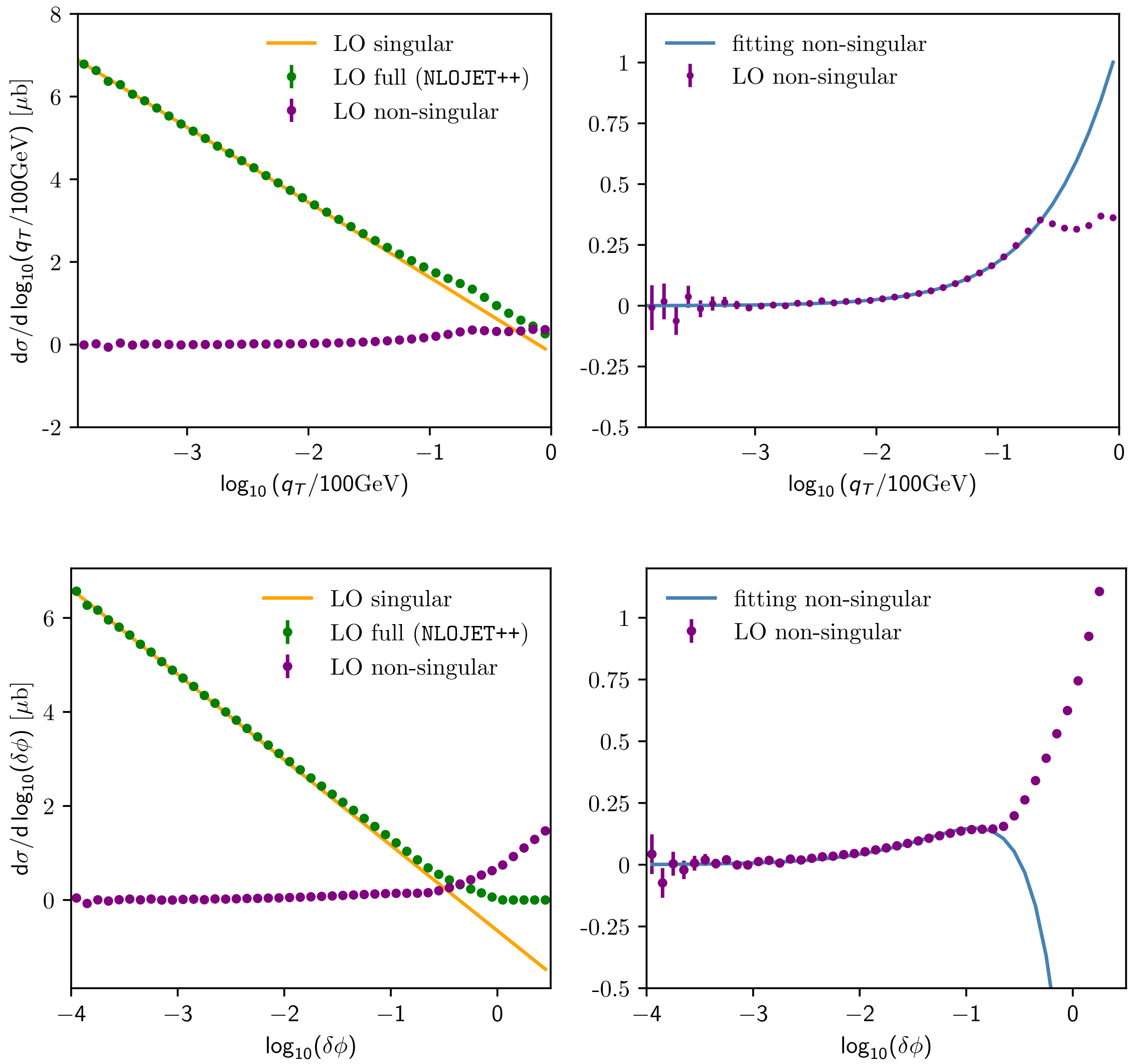
- Their combination S_i, evolves under the RG without any residual multiplicity mixing,

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}S_i(\vec{b}_T,\eta_i,R,\mu,\nu) = \left(\Gamma^{\mathrm{cs}}+\Gamma^{\mathrm{ucs}}\right)S_i(\vec{b}_T,\eta_i,R,\mu,\nu).$$

- The two-loop global anomalous dimensions for the collinear-soft and ultra-collinear-soft functions are

$$\Gamma^{\mathrm{cs}}(\alpha_s,\{n_i,\bar{n}_i\},R,\mu,\nu) = 2C_i\gamma_{\mathrm{cusp}}(\alpha_s)\ln\frac{\mu}{\nu R_i},$$
$$\Gamma^{\mathrm{ucs}}(\alpha_s,\{n_i,\bar{n}_i\},b_x,R,\mu) = -C_i\gamma_{\mathrm{cusp}}(\alpha_s)\left[\ln\frac{4\mu^2b_x^2}{b_0^2R^2}-i\pi\mathrm{Sign}(b_xn_{i,x})\right],$$

Validation of factorization and Power corrections



- The non-singular contribution $(B_1 + B_0 \ln \mathcal{O})$ ($\mathcal{O} = \delta\phi$ or q_T) retains a logarithmic divergence as $\mathcal{O} \to 0$, despite being suppressed relative to the leading singular terms:

$$\frac{d\sigma(\text{LO full})}{d\log_{10}\mathcal{O}} \simeq \frac{d\sigma(\text{LO singular})}{d\log_{10}\mathcal{O}} + \frac{d\sigma(\text{power corrections})}{d\log_{10}\mathcal{O}}$$

$$= (A_1 \ln \mathcal{O} + A_0) + (B_1 \mathcal{O} + B_0 \mathcal{O} \ln \mathcal{O}),$$

$$\frac{d\sigma(\text{LO full})}{d\mathcal{O}} \simeq \frac{A_1 \ln \mathcal{O} + A_0}{\mathcal{O}} + (B_1 + B_0 \ln \mathcal{O}).$$

- This logarithmic enhancement is characteristic of exclusive or jet-dependent observables. [Ebert, Tackmann '19; Salam, Slade '21; Grazzini, Wiesemann '17]