Next-to-leading Order QCD Factorization for Energy Flow in Electron-ion Collisions at Twist 4

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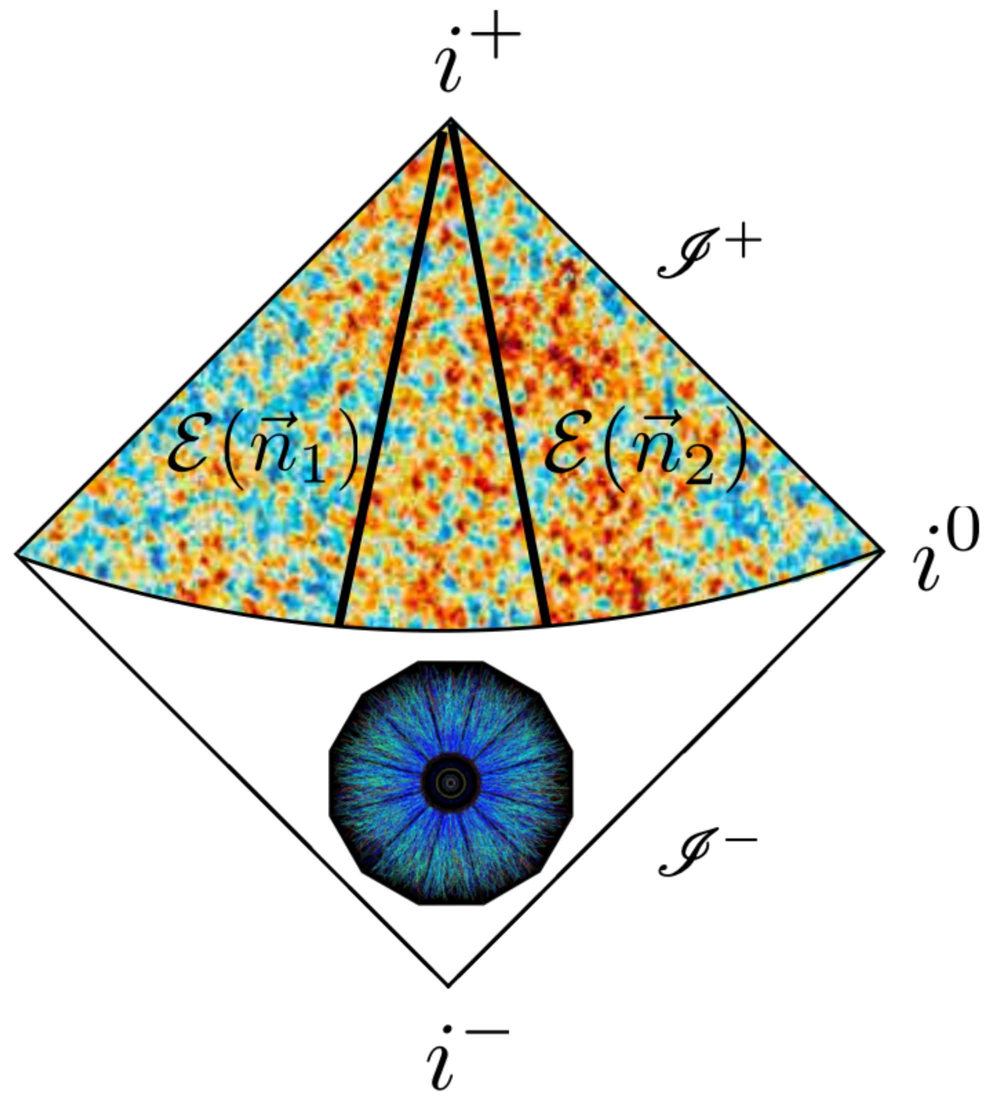
Introduction

- Event-shape observables quantify the geometric & dynamical properties of final-state energy flows.
- Energy-flow operator defined via the light-ray operator $\mathscr{E}(\vec{n})$, a detector level quantity that accumulates energy along direction $\vec{n}$.
- EEC—energy Energy Correlator measures the energy flow between two detectors as a function of opening angle χ. A classic perturbative QCD observable since 1970s.
- High-resolution colliders have measured EEC across collision systems (elementary, cold and hot nuclear matter), where it serves either as a global event-shape observable or a jet-substructure probe of intra-jet energy distribution.

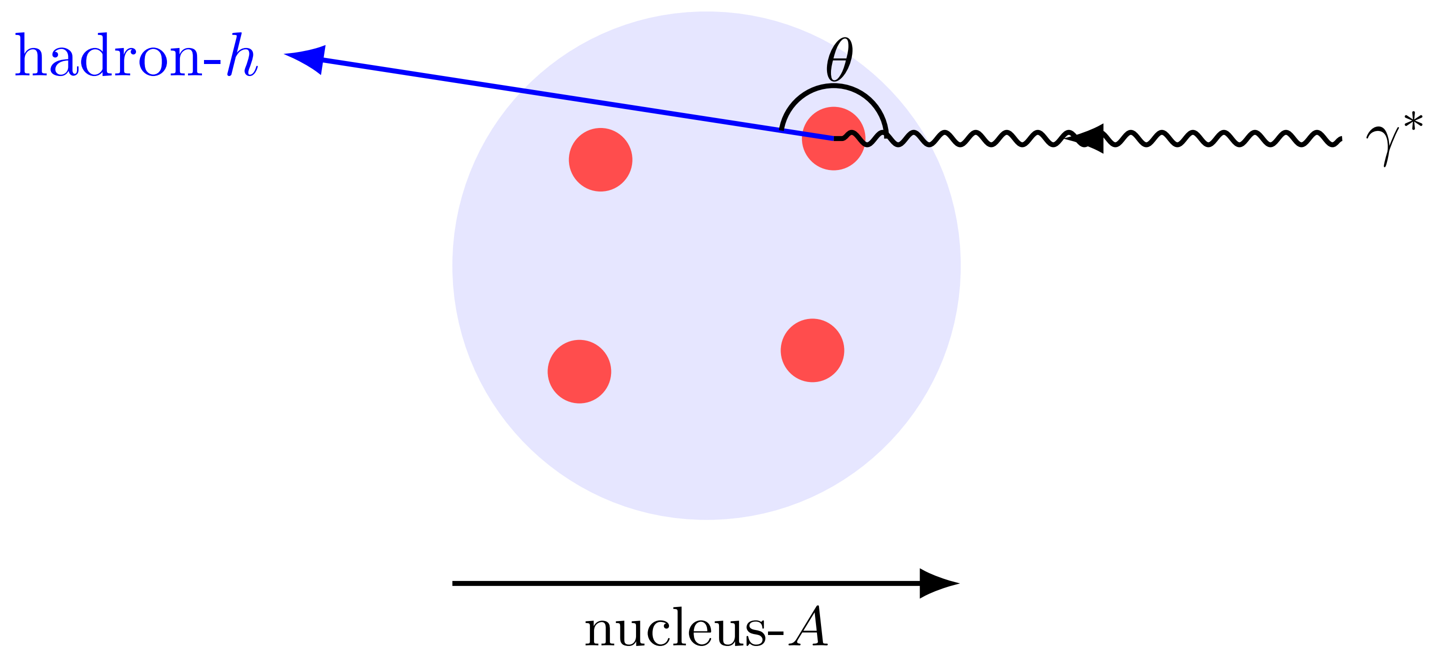


$$\mathscr{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, n^i\, T_{0i}(t, r\vec{n})$$

[Hofmann&Maldacena, 2008]

One Point Energy Correlator and nuclear modification

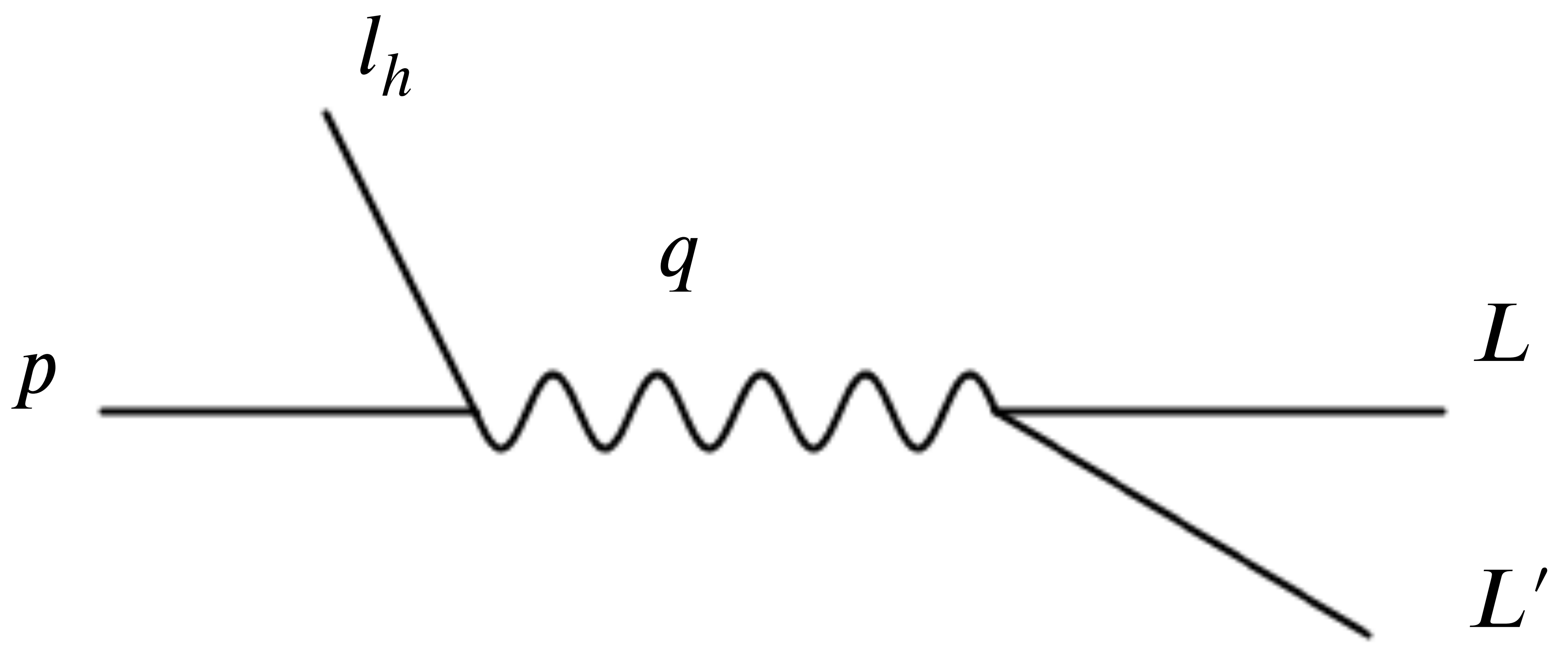
- Correlates energy flow w.r.t. a fixed reference direction (beam axis or jet axis).
- Infrared and collinear safe.
- Comparing e+p vs e+A collisions allows extraction of cold nuclear matter effects, probing in-medium scattering and transverse momentum broadening.



Kinematics

- Consider a lepton L scattering off a large nucleus A.

$$l(L) + A(p) \rightarrow l(L') + h(l_h) + X$$



- The usual SIDIS Lorentz-invariant variable are define as:

$$x_B = \frac{Q^2}{2p\,.\,q} \qquad y = \frac{p\,.\,q}{p\,.\,L} \qquad z_h = \frac{p\,.\,l_h}{p\,.\,q}$$

$$Q^2 = -q^2$$

SIDIS one-one point energy correllator

- Measuring the angular distribution of outgoing hadrons with respect to the incoming nucleus.

$$l(L) + A(p) \to l(L') + h(l_h) + X$$

$$\frac{d\Sigma}{d\cos\theta dx_B dy} = \sum_h \int_0^1 dz_h z_h \left(\frac{d\sigma^h}{d\cos\theta dx_B dy dz_h} \right)$$

[Basham, Brown, Ellis and Love, 1970]
[Meng , Olness , and Soper 1991]

[Li, Makris and Vitev, 2021]

The width of the OPEC

- We want to consider a quantity that measures the transverse spread of the QCD jets which do not depend on how partons fragment into hadrons.

- It is thus natural to use the angular spread of the OPEC as an illustration of an infrared-safe measure of transverse size.

$$\langle \sin^2\theta \rangle = \frac{\int d\cos\theta \, \sin^2\theta \left(d\Sigma / d\cos\theta dx_B dy \right)}{\int d\cos\theta \, \left(d\Sigma / d\cos\theta dx_B dy \right)}$$

[Peccei and Ruckl 1979]

- θ is the angle between the hadron and the beam direction (z-direction).

High Twist Expansion

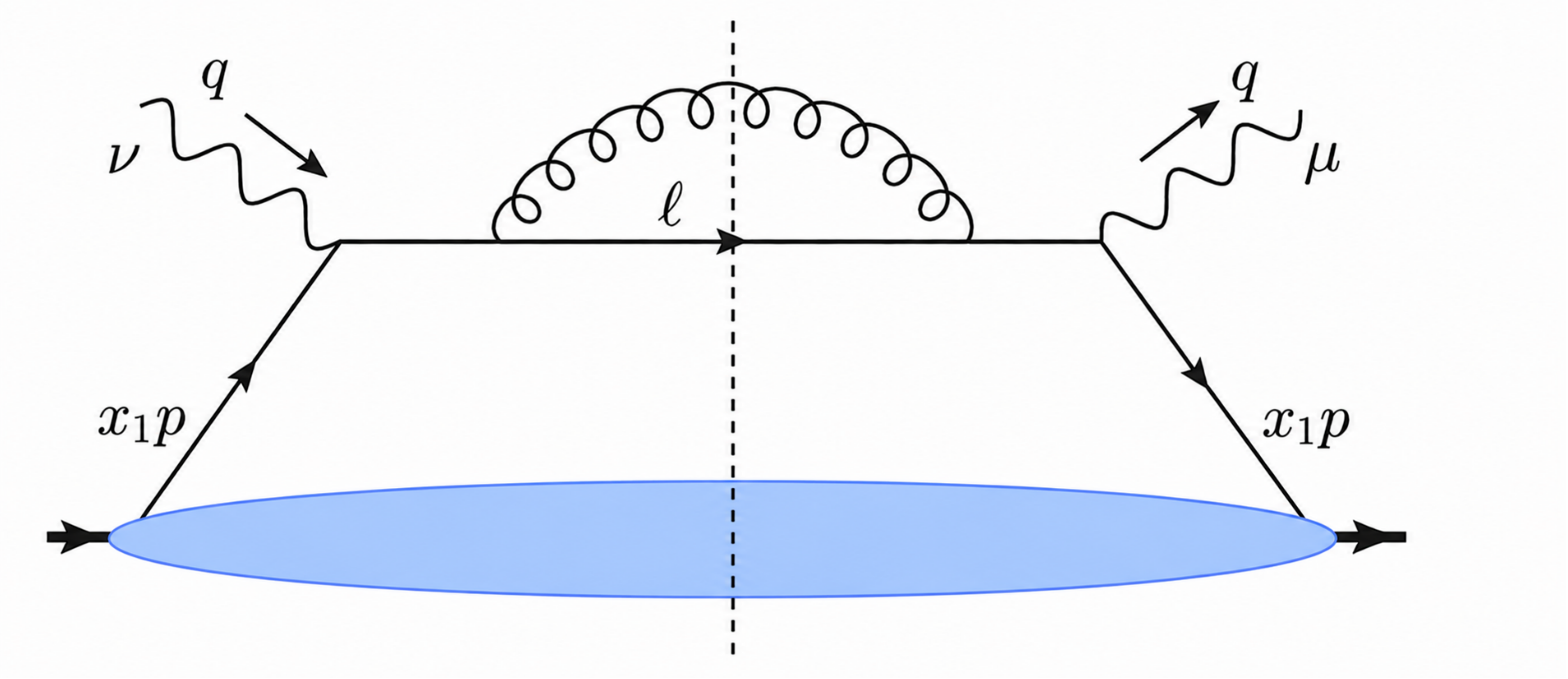
- In a nuclear medium, the outgoing parton in SIDIS may experience additional scatterings with other partons from the nucleus before fragmenting into final observed hadrons.

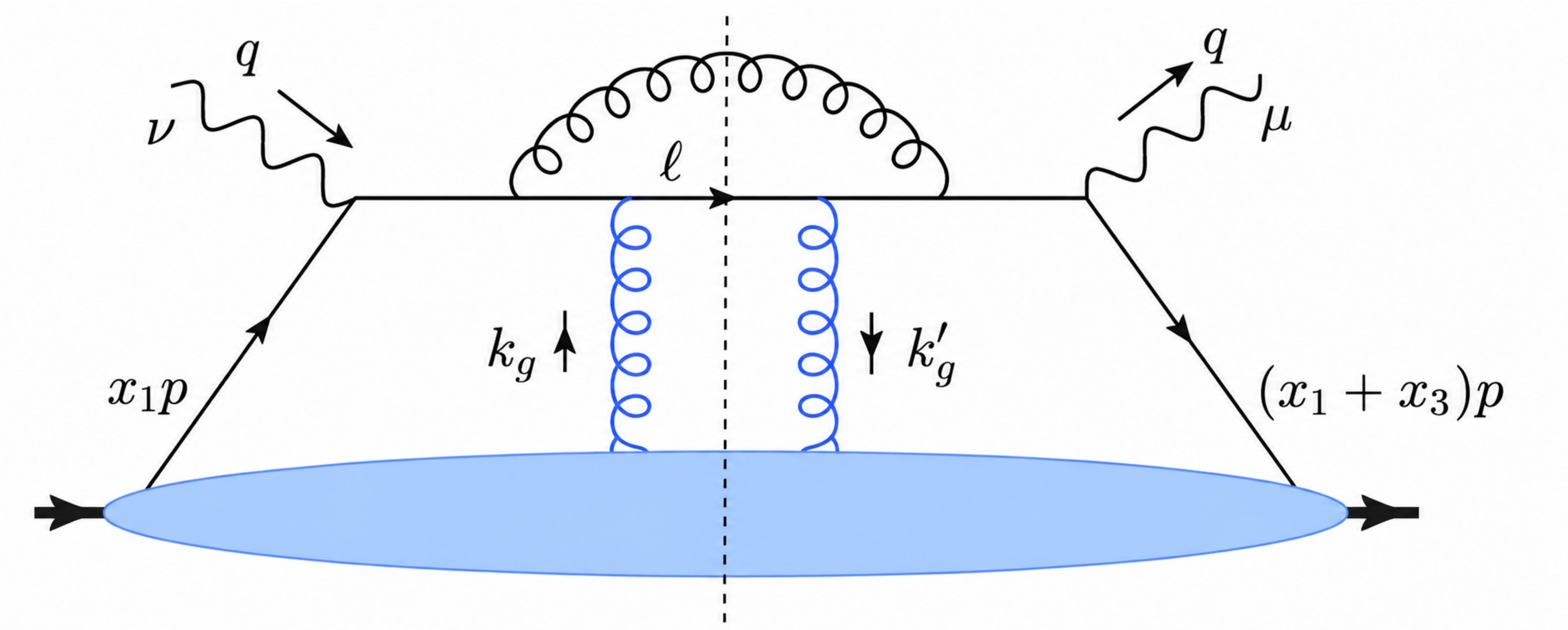
$$d\sigma = d\sigma^S + d\sigma^D + \ldots$$

[Luo, Qiu, Sterman 1990's]

- Single scattering does not lead to a significant modification for the production rate from e+ p to e+ A collisions, except for a mild A dependence from nuclear parton distribution functions.

- Double scattering is usually power suppressed $\sim 1/Q^2$ by the hard scale of the process, though it is enhanced by the nuclear size $\sim A^{1/3}$ (when two partons come from different nucleons inside the nucleus).

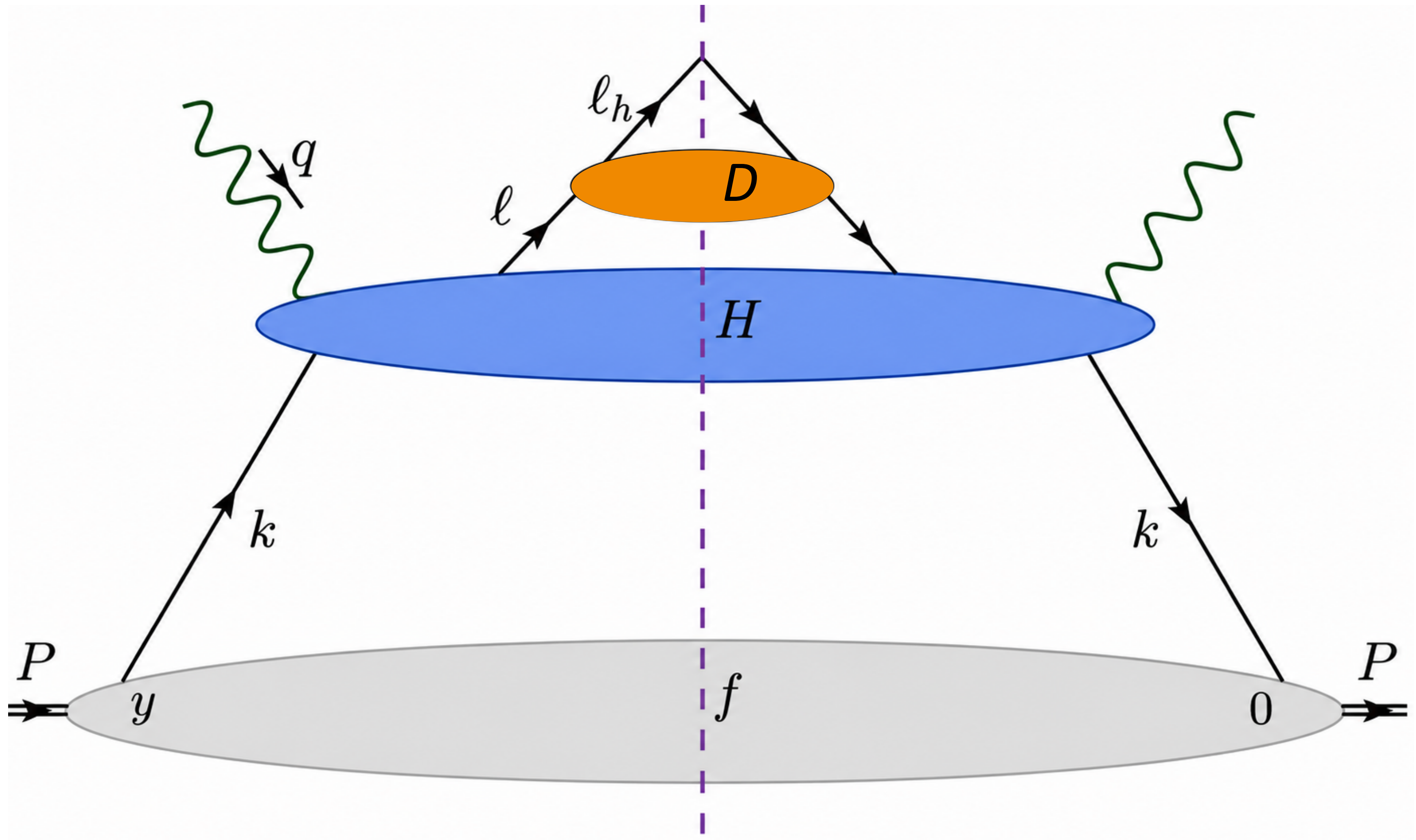




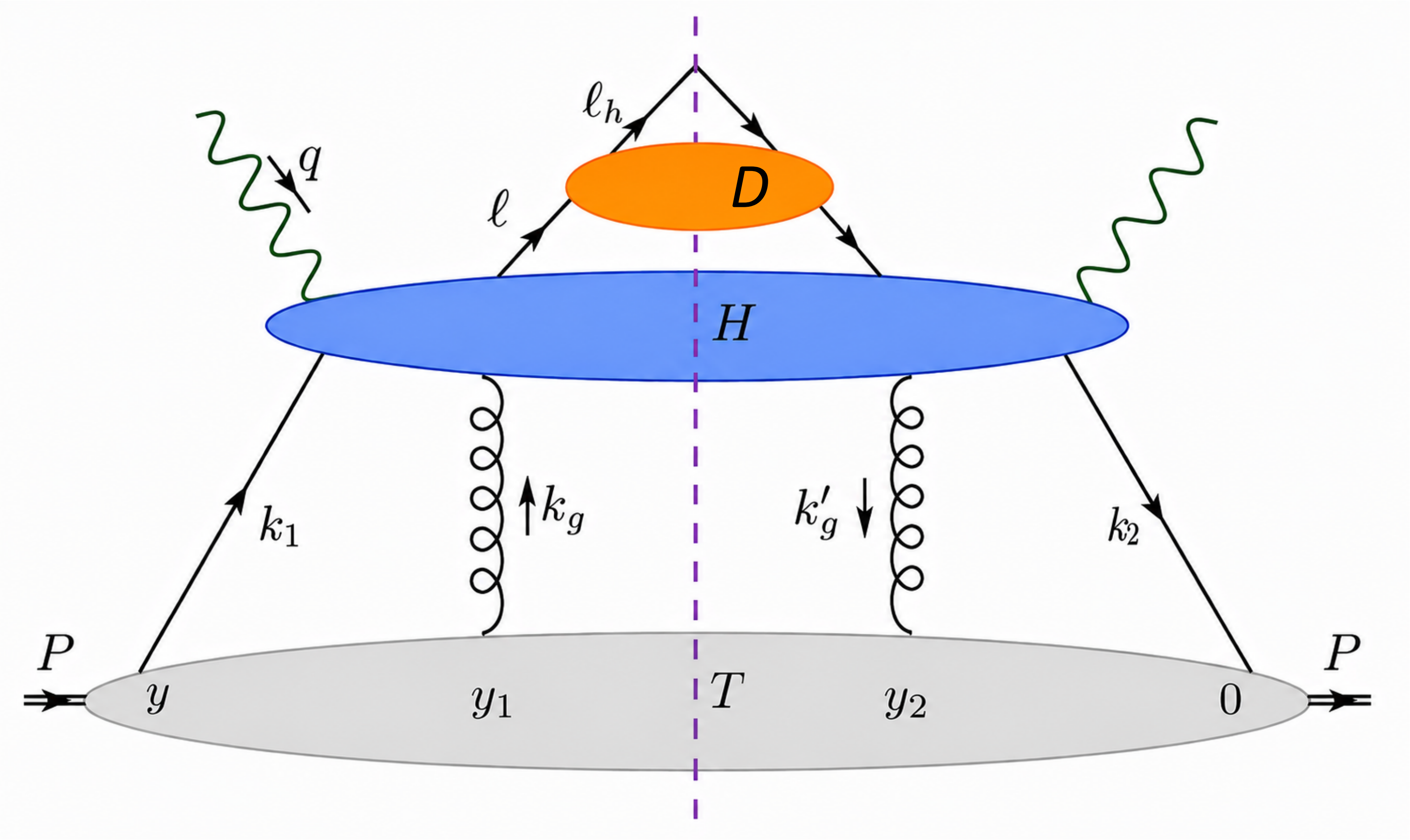
Twist-2 (leading twist)

- At NLO, the twist-2 cross section factorizes

$$d\sigma^S_{\gamma^*+A\to h+X} \sim \sum_{abc} f_{a/A} \otimes d\hat{\sigma}^S_{\gamma^*+a\to b+c} \otimes D_{b/c\to h}$$



Twist-4 (higher twist)



- One carries out a collinear expansion of hard parts and reorganizes the final results in terms of power corrections, where the second-order expansion gives rise to the twist-4 contribution.

$$d\sigma^D_{\gamma^*+A\to h+X} \sim \sum_{abc} T_{ag} \otimes d\hat{\sigma}^D_{\gamma^*+a\to b+c} \otimes D_{b/c\to h}$$

$$k_g = x_2 P + k_T$$
$$k'_g = (x_2 - x_3)P + k_T$$

$$\hat{H}(k_1, k_g, k'_g) \sim \hat{H}(k_1, k_g, k'_g)\Big|_{k_T=0} + k_T^2 \frac{\partial^2 \hat{H}(k_1, k_g, k'_g)}{\partial k_T^2}\Big|_{k_T=0} + \ldots \qquad k_T \ll \ell_{hT}$$

Broadening measurements

- The transverse momentum broadening of the hadron

$$\Delta\langle \ell_{hT}^2 \rangle \equiv \langle \ell_{hT}^2 \rangle_{eA} - \langle \ell_{hT}^2 \rangle_{ep},$$

[Kang, Wang,Wang and Xing 2014]

- measures the difference between the average squared transverse momentum of hadrons produced on a nuclear target and those on a proton target.

- Double scatterings will then lead to a nuclear enhancement in the average squared transverse momentum for the observed hadron produced in e+ A collisions.

- The leading contribution is

$$\Delta\langle \ell_{hT}^2 \rangle \approx \frac{d\langle \ell_{hT}^2 \sigma^D \rangle}{d\mathbb{PS}} \Bigg/ \frac{d\sigma}{d\mathbb{PS}}$$

$$\frac{d\langle \ell_{hT}^2 \sigma^D \rangle}{d\mathbb{PS}} \equiv \int d\ell_{hT}^2 \, \ell_{hT}^2 \frac{d\sigma^D}{d\mathbb{PS} d\ell_{hT}^2}$$

The width of OPEC

- We introduce the broadening of the width of the OPEC

$$\Delta\langle \sin^2\theta\rangle \equiv \langle \sin^2\theta\rangle_{eA} - \langle \sin^2\theta\rangle_{ep} \approx \frac{d\langle \sin^2\theta\,\Sigma^D\rangle/dx_B dy}{d\Sigma/dx_B dy}$$

- Requires less experimental input than that needed in $\Delta\langle \ell_{hT}^2\rangle$, because it doesn't depend on the fragmentation function.

$$\sum_h \int_0^1 dz_h z_h D_{b/c\to h}(z_h, \mu_f^2) = 1$$

Final-state divergence

Quark Fragmentation

- The energy flow carried by the quarks or by the gluons is not separately infrared insensitive, but their sum is.

- The quark fragmentation contribution to the NLO $sin^2\theta$- weighted cross section at twist -4 has the following final state divergence,

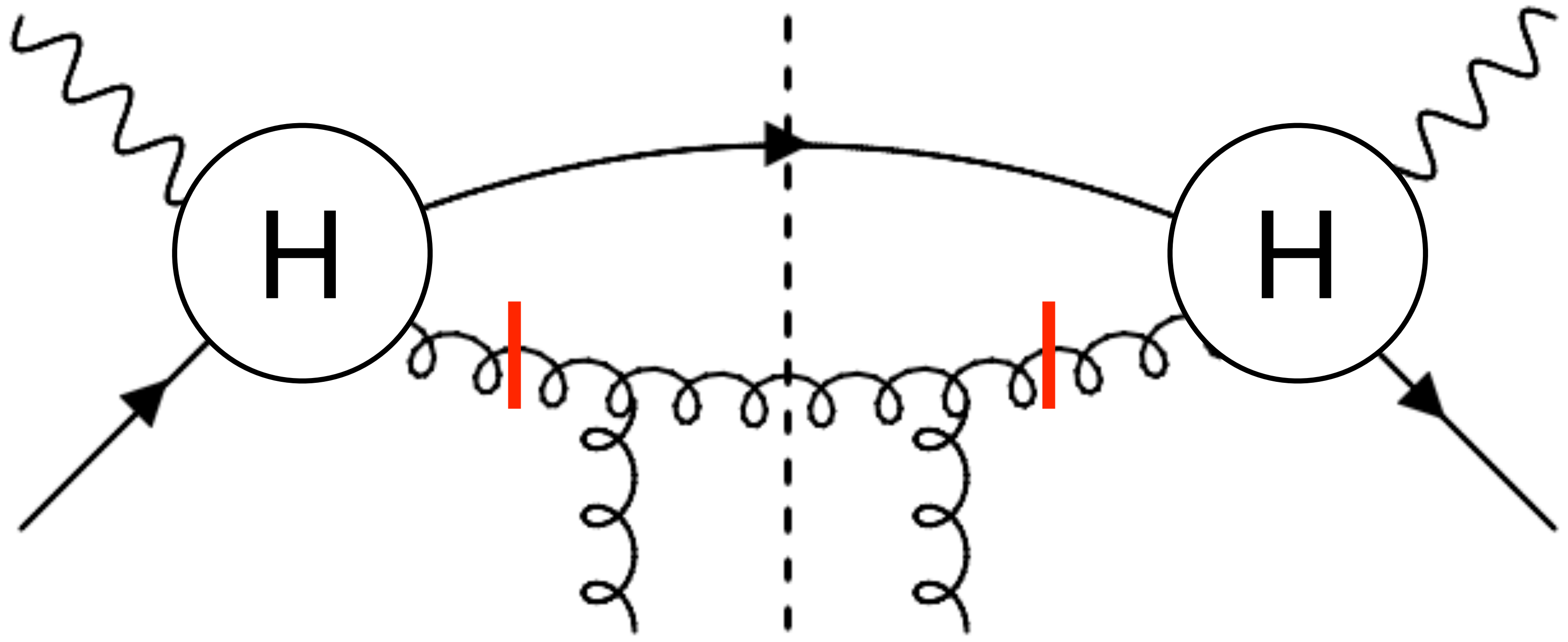
$$C_F \int_{z_h}^{1} \frac{dz_q}{z_q} D_{q\to h}(z_q) \left\{ \frac{-1}{\epsilon} \delta(1-\hat{x}) P_{qq}(\hat{z}_q) \right\}$$ [Kang, Wang,Wang and Xing 2014]

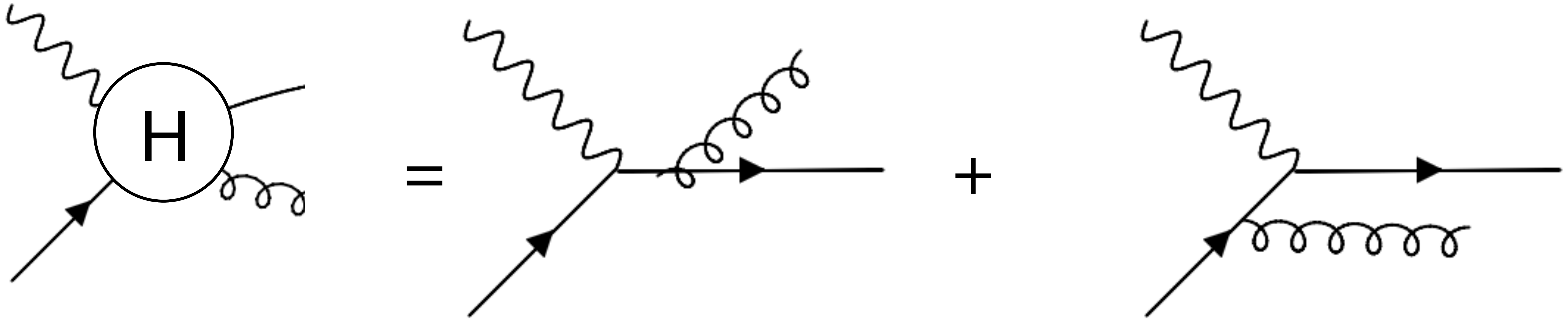
- In order to be inclusive over the final state, we need to calculate the contribution from gluon fragmentation.

Gluon fragmentation

soft-soft scattering

$$I_g^{ss} = C_A \left\{ \frac{-1}{\epsilon} \delta(1-\hat{x}) P_{gq}(\hat{z}_g) \right\}$$

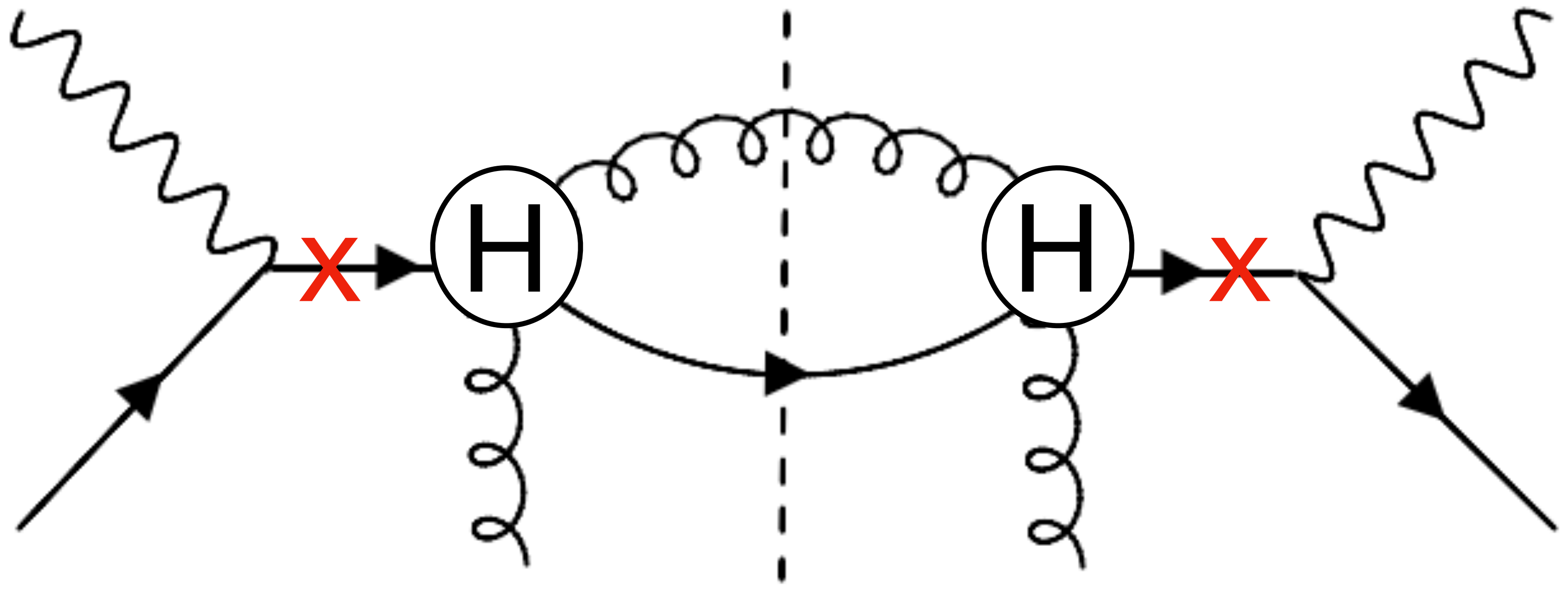


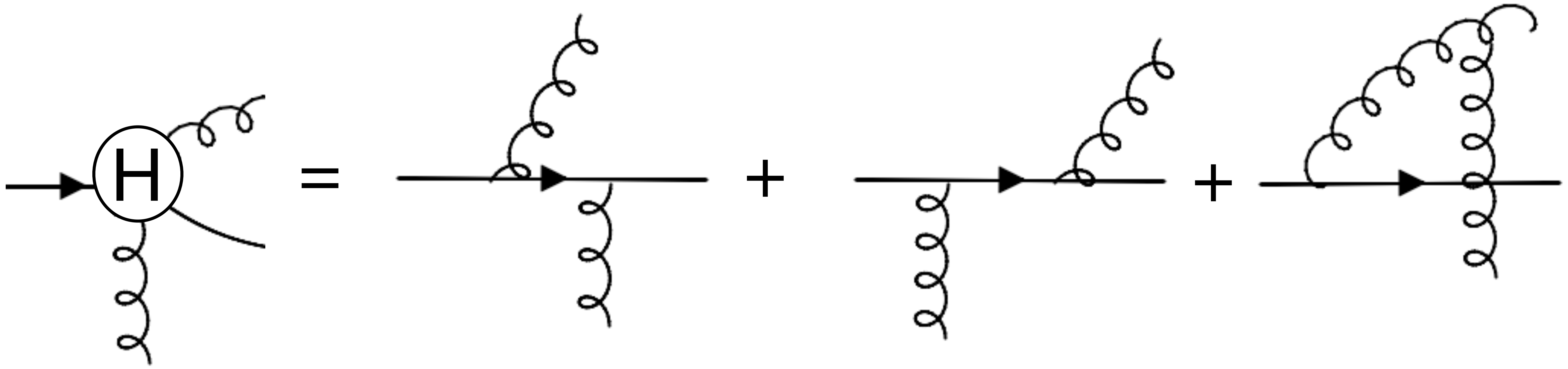


Gluon fragmentation

hard-hard scattering

$$I_g^{hh} = C_A \left\{ \frac{-1}{\epsilon} \delta(1-\hat{x}) P_{gq}(\hat{z}_g) \left[1 - \hat{z}_g + \frac{C_F}{C_A} \hat{z}_g^2 \right] \right\}$$

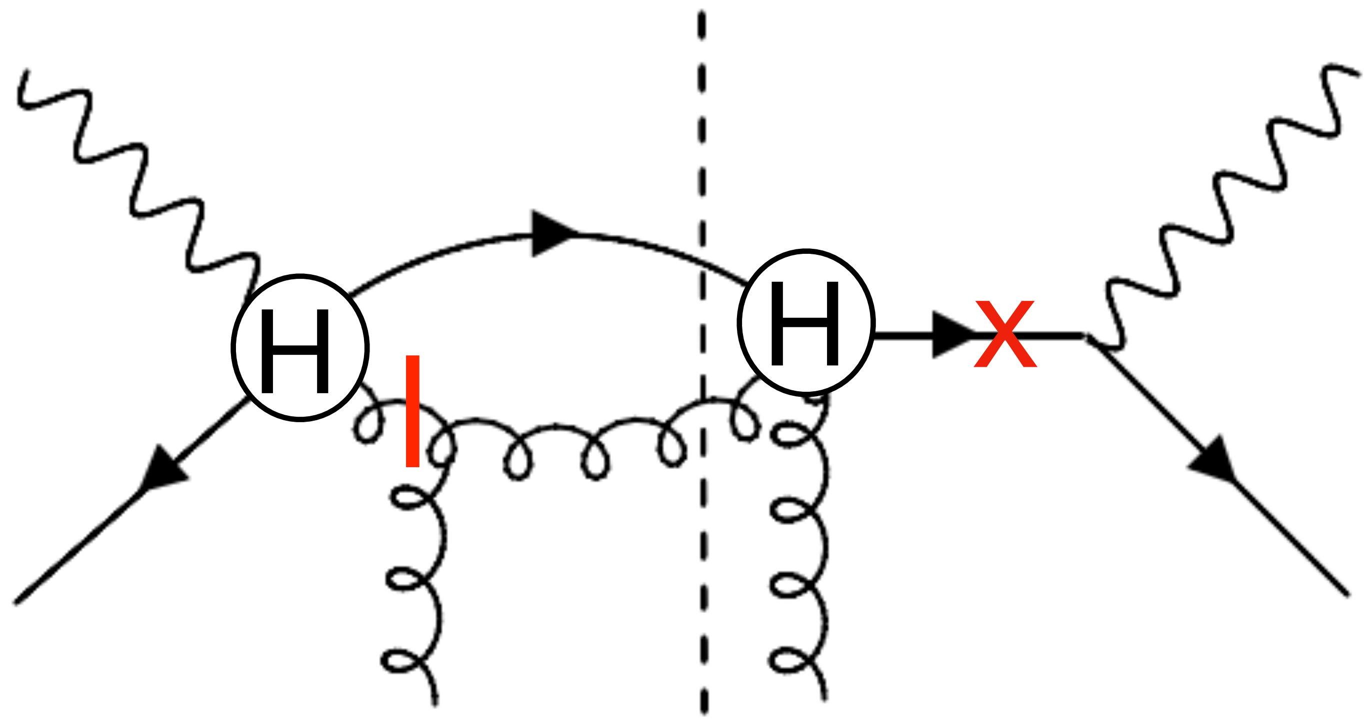




Gluon fragmentation

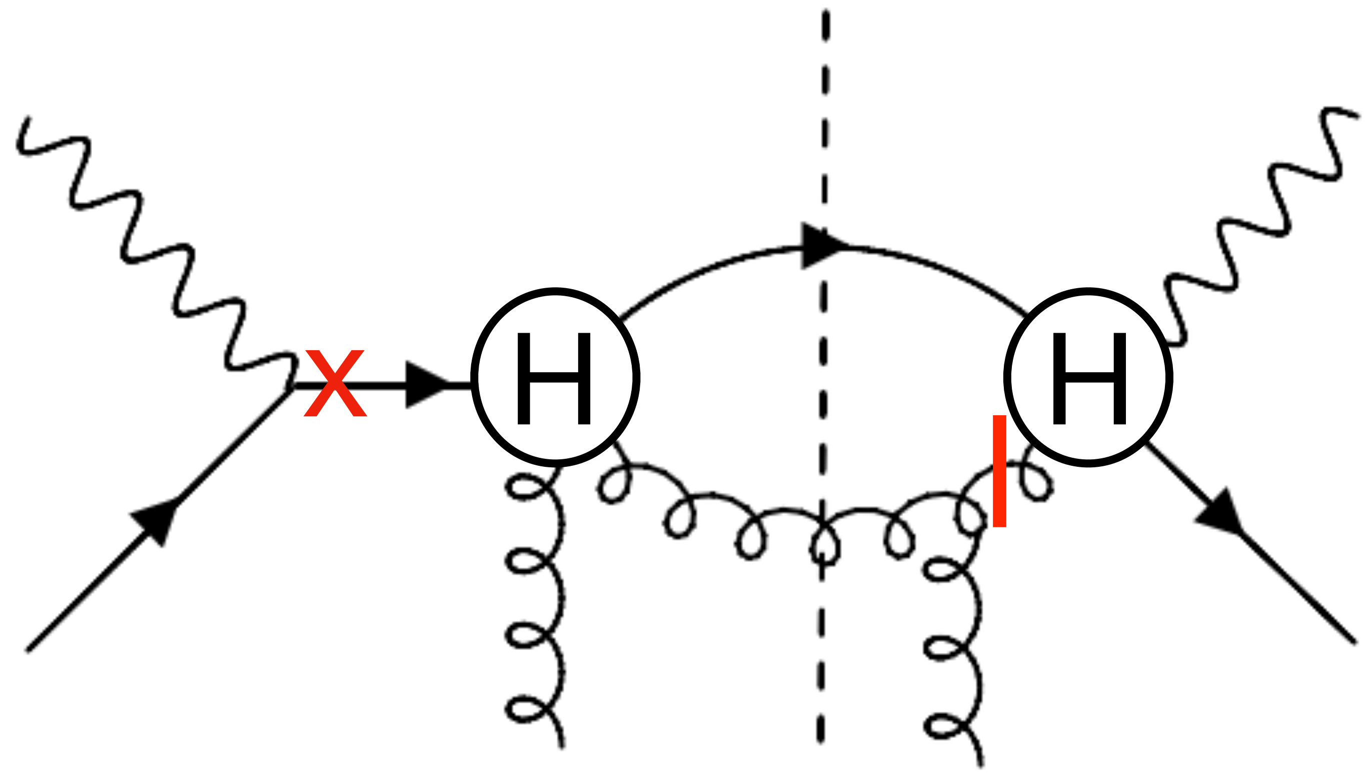
soft-hard scattering

$$I_g^{sh} = \frac{C_A}{2}\left\{\frac{-1}{\epsilon}\delta(1-\hat{x})P_{gq}(\hat{z}_g)\left[\hat{z}_g - 2\right]\right\}$$



Hard-Soft scattering

$$I_g^{hs} = \frac{C_A}{2}\left\{ \frac{-1}{\epsilon}\delta(1-\hat{x})P_{gq}(\hat{z}_g)\left[\hat{z}_g - 2\right] \right\}$$



Final state divergence

- Adding them together, the gluon fragmentation gives

$$C_F \int_{z_h}^{1} \frac{dz_g}{z_g} D_{g\to h}(z_g) \left\{ \frac{-1}{\epsilon} \delta(1-\hat{x}) P_{qg}(\hat{z}_g) \right\} .$$

- Remember, we had for the quark fragmentation

$$C_F \int_{z_h}^{1} \frac{dz_q}{z_q} D_{q\to h}(z_q) \left\{ \frac{-1}{\epsilon} \delta(1-\hat{x}) P_{qq}(\hat{z}_q) \right\}$$

IR-safety

Twist-4

- We show that the final state divergence cancels when one adds gluon and quark contribution.

$$I_{T4} = C_F \int_{z_h}^{1} \frac{dz_q}{z_q} D_{q\to h}(z_q) \left\{ \frac{-1}{\epsilon} \delta(1-\hat{x}) P_{qq}(\hat{z}_q) \right\} + C_F \int_{z_h}^{1} \frac{dz_g}{z_g} D_{g\to h}(z_g) \left\{ \frac{-1}{\epsilon} \delta(1-\hat{x}) P_{qg}(\hat{z}_g) \right\}$$

Using $\sum_h \int_0^1 dz_h z_h D_{b\to h}(z_h, \mu_f^2) = 1$

We get
$$\sum_h \int_0^1 dz_h\, z_h\, I_{T4} = \int_0^1 d\hat{z}_q\, \hat{z}_g P_{qq}(\hat{z}_q) + \int_0^1 d\hat{z}_g\, \hat{z}_g P_{gq}(\hat{z}_g)$$

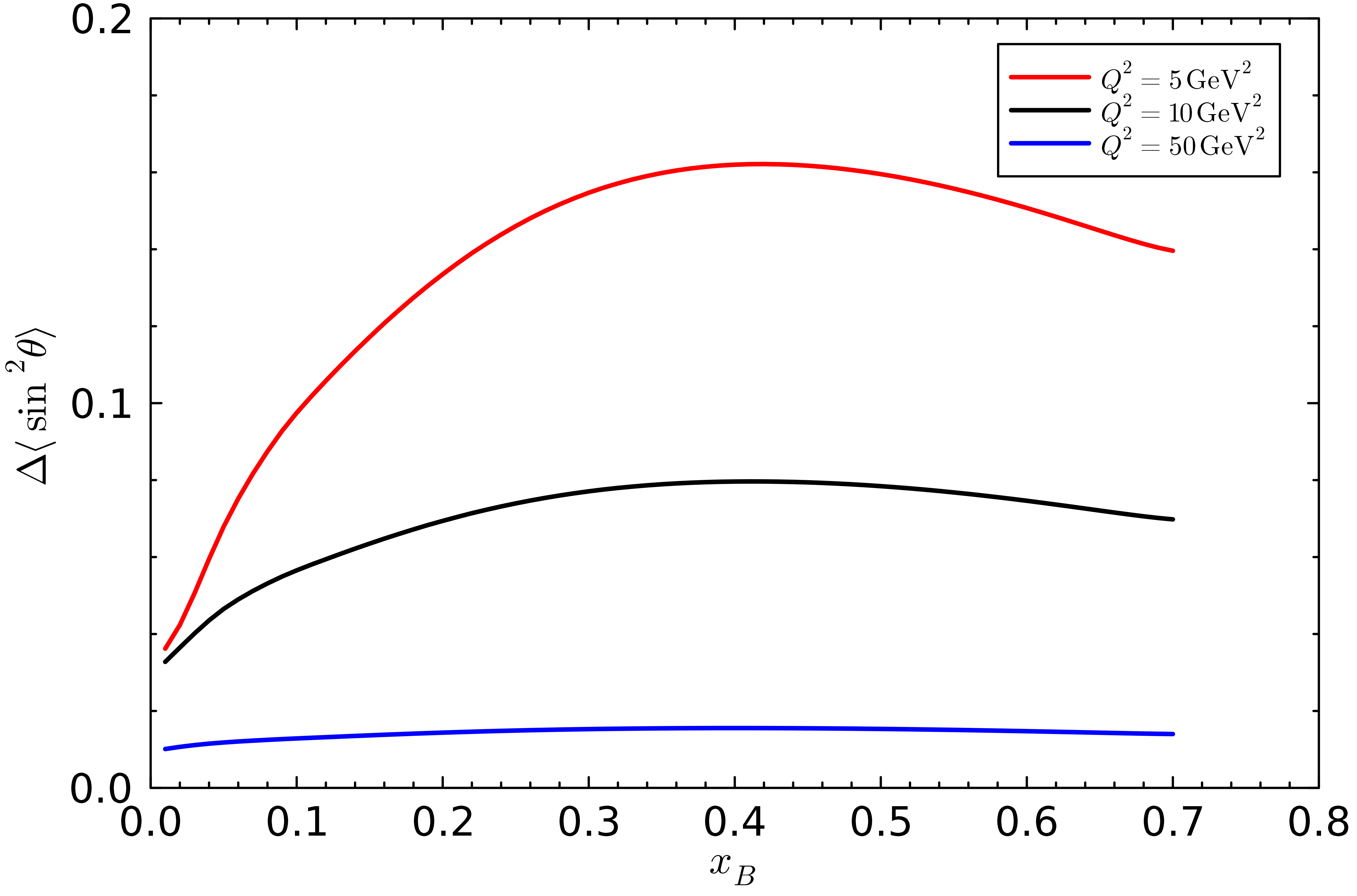
$$= \int_0^1 d\hat{z} \left(\hat{z}_q + (1-\hat{z}_q) \right) P_{qq}(\hat{z}_q) = \int_0^1 P_{qq}(\hat{z}_q) = 0$$

Results (soft-soft)

The width of OPEC Vs. x_B

The width of OPEC at twist-4 as defined by $\Delta\langle\sin^2\theta\rangle \equiv \langle\sin^2\theta\rangle_{eA} - \langle\sin^2\theta\rangle_{ep}$for different value of Q^2 for lead Pb^{208} with $\hat{q} = 0.02\,\mathrm{GeV}^2/\mathrm{fm}.$

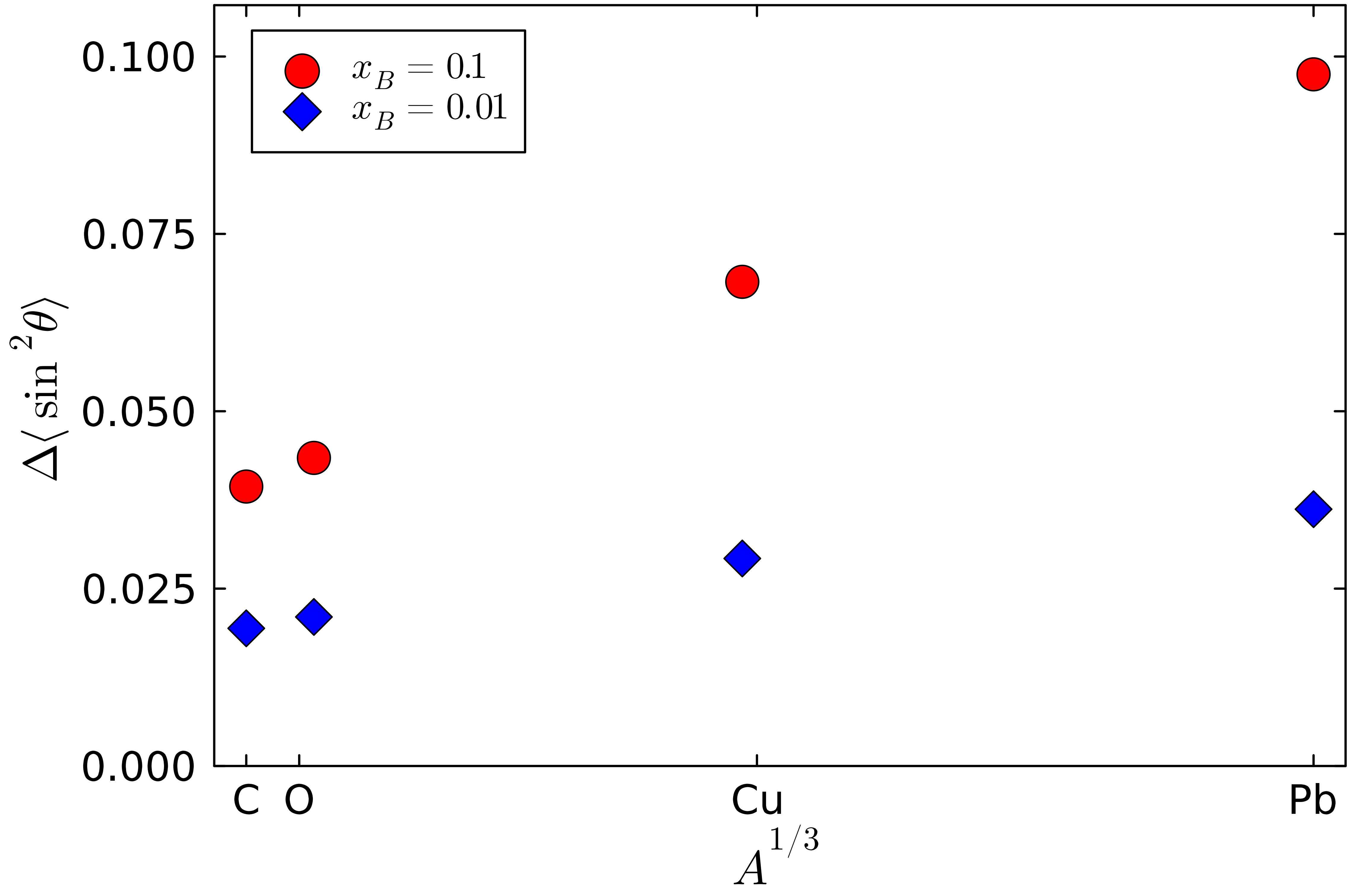
$$T_{qg}(x,0,0;\mu^2) \approx \frac{9\,R_A}{8\pi^2\,\alpha_s} f_{q/A}(x,\mu^2)\,\hat{q}_0$$



Results (soft-soft)

The width of OPEC Vs. $A^{1/3}$

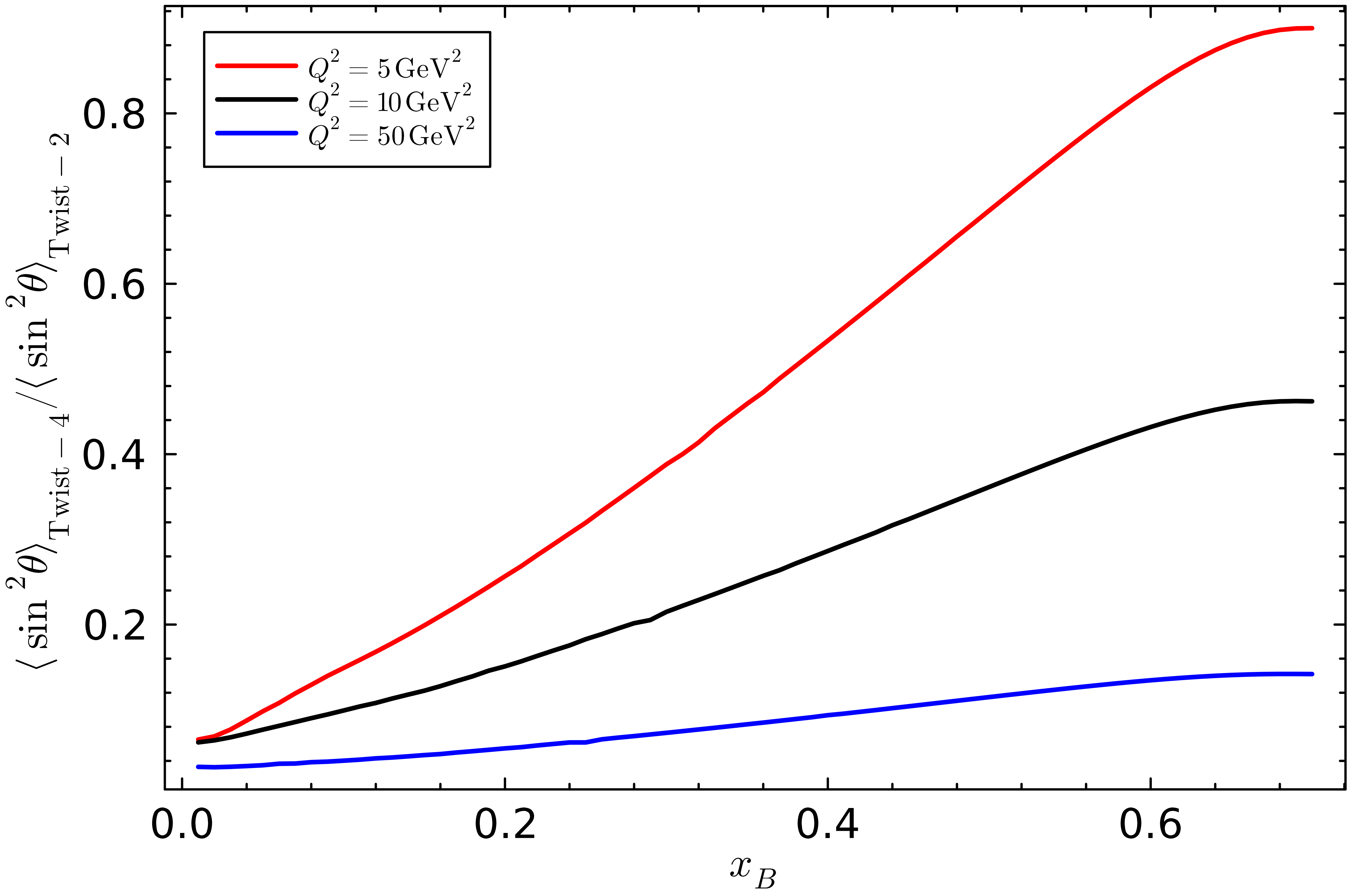
The width of OPEC at twist-4 as defined by $\Delta\langle\sin^2\theta\rangle \equiv \langle\sin^2\theta\rangle_{eA} - \langle\sin^2\theta\rangle_{ep}$ for different value of x_B with $\hat{q} = 0.02\,\mathrm{GeV}^2/\mathrm{fm}$.



Result (soft-soft)

Ratio of twist-4 to twist-2

The ratio of the width of OPEC at twist-4 to the width of OPE at twist-2 for different values of Q^2 for lead Pb^{208} with $\hat{q} = 0.02\,\mathrm{GeV}^2/\mathrm{fm}$.



Conclusion

- We calculated the twist-4 contribution to the width of the one-point energy correlator (OPEC) in SIDIS at NLO and demonstrated that it is an infrared and collinear-safe observable.
- We showed that the OPEC width provides a direct measure of hadronic jet broadening, depending only on hard coefficients and the twist-4 quark-gluon correlation function.
- A complete analysis, including a realistic model for the off-diagonal components of the twist-4 quark-gluon correlation function, would allow for a full evaluation of the remaining contributions.

THANKS FOR LISTENING