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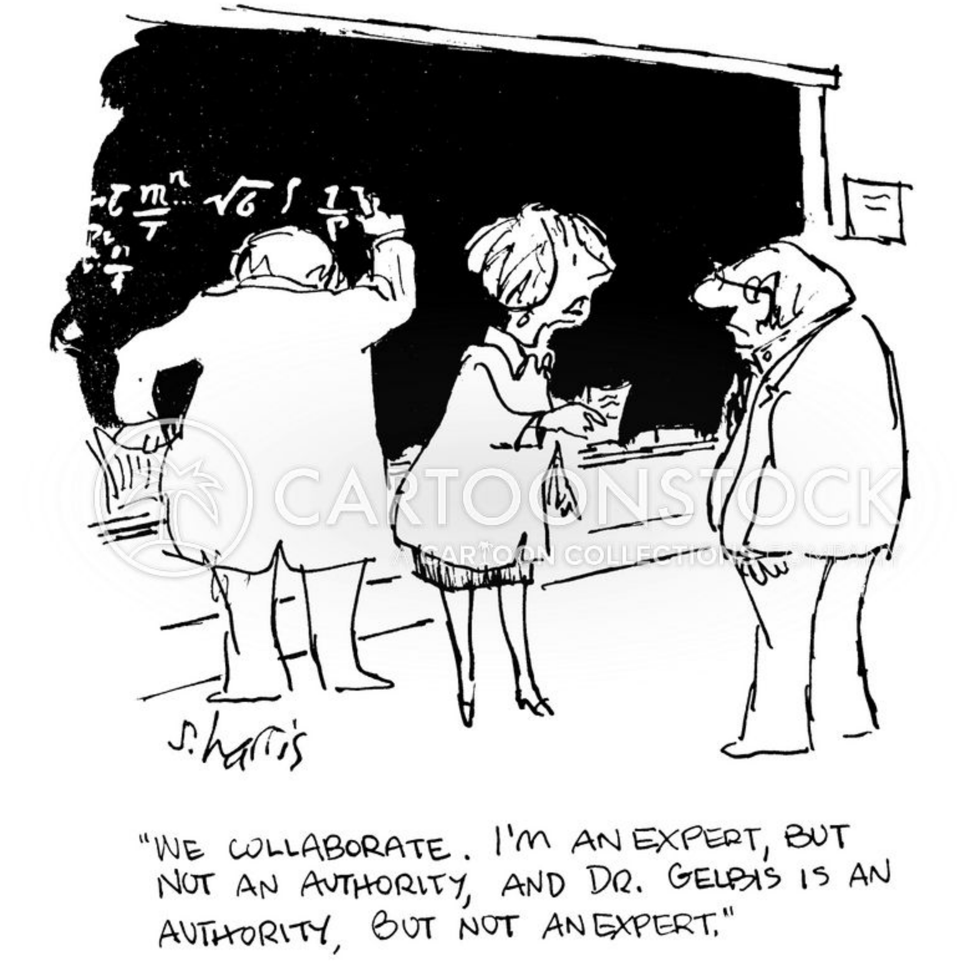
Instituto Nacional de Ciência e Tecnologia
Física Nuclear e Aplicações

From parton distribution functions to quark fragmentation functions



Work in collaboration with

Fernando Serna (Barranquilla, Colombia), Ian Cloët (Argonne, USA),

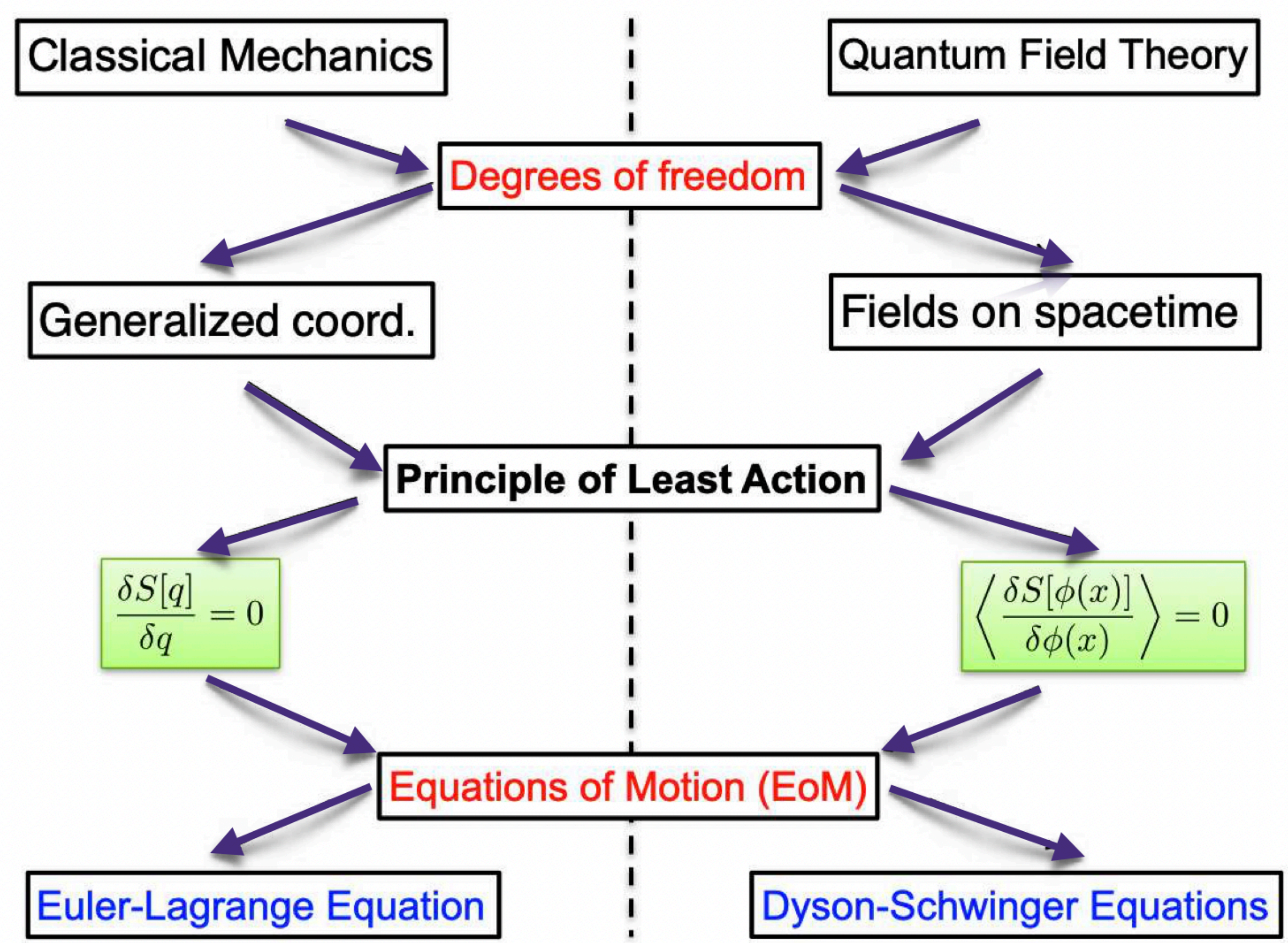
Roberto Silveira, Gustavo Bopsin, Gastão Krein (São Paulo, Brazil)

$$\left(\Box_x + m^2\right) G(x,y) = \delta(x-y)$$

Part I

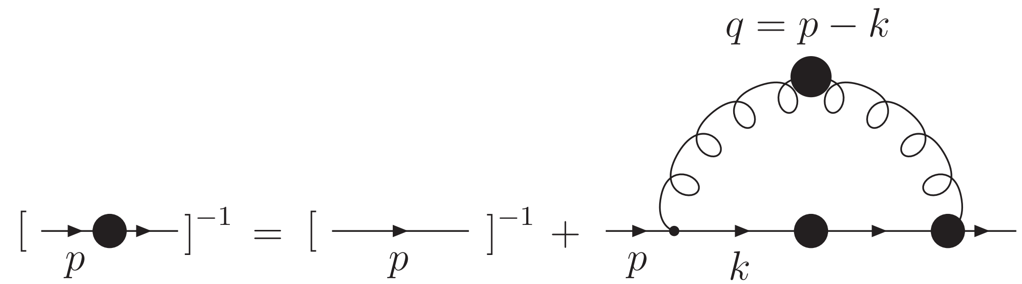
Green functions in a functional approach to QCD

FUNCTIONAL NONPERTURBATIVE APPROACHES TO QCD



Quark-gap equation in QCD

The propagator can be obtained from QCD's gap equation: the Dyson-Schwinger equation (DSE) for the dressed-fermion self-energy, which involves the tower of infinitely many coupled equations.



$$S^{-1}(p) = Z_2(i\gamma\cdot p + m^{\mathrm{bm}}) + \Sigma(p) := i\gamma\cdot p\, A(p^2) + B(p^2)$$

$$\Sigma(p) = Z_1 \int^{\Lambda} \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma^a_\nu(q,p)$$

with the *running* mass function $M(p^2) = B(p^2)/A(p^2)$.

$D_{\mu\nu}$	:	dressed-gluon propagator
$\Gamma^a_\nu(q,p)$	:	dressed quark-gluon vertex
Z_2	:	quark wave function renormalization constant
Z_1	:	quark-gluon vertex renormalization constant

Each satisfies it's own DSE

$$S^{-1}(p)|_{p^2=\zeta^2} = i\gamma\cdot p + m(\zeta)$$

where ζ is the renormalization point.

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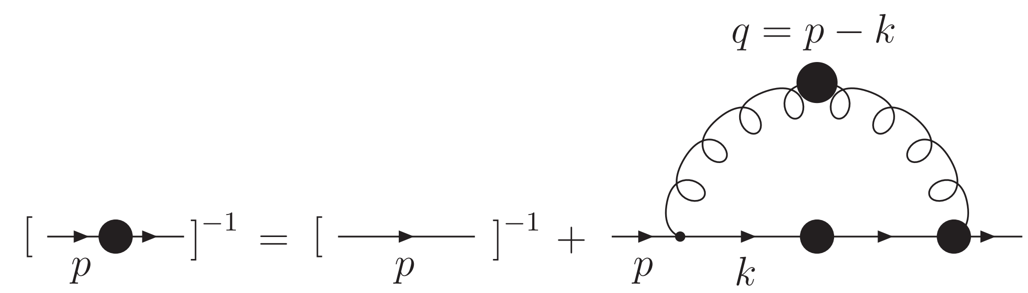

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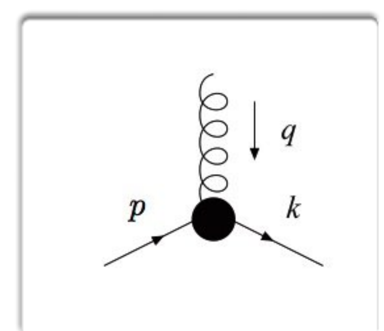

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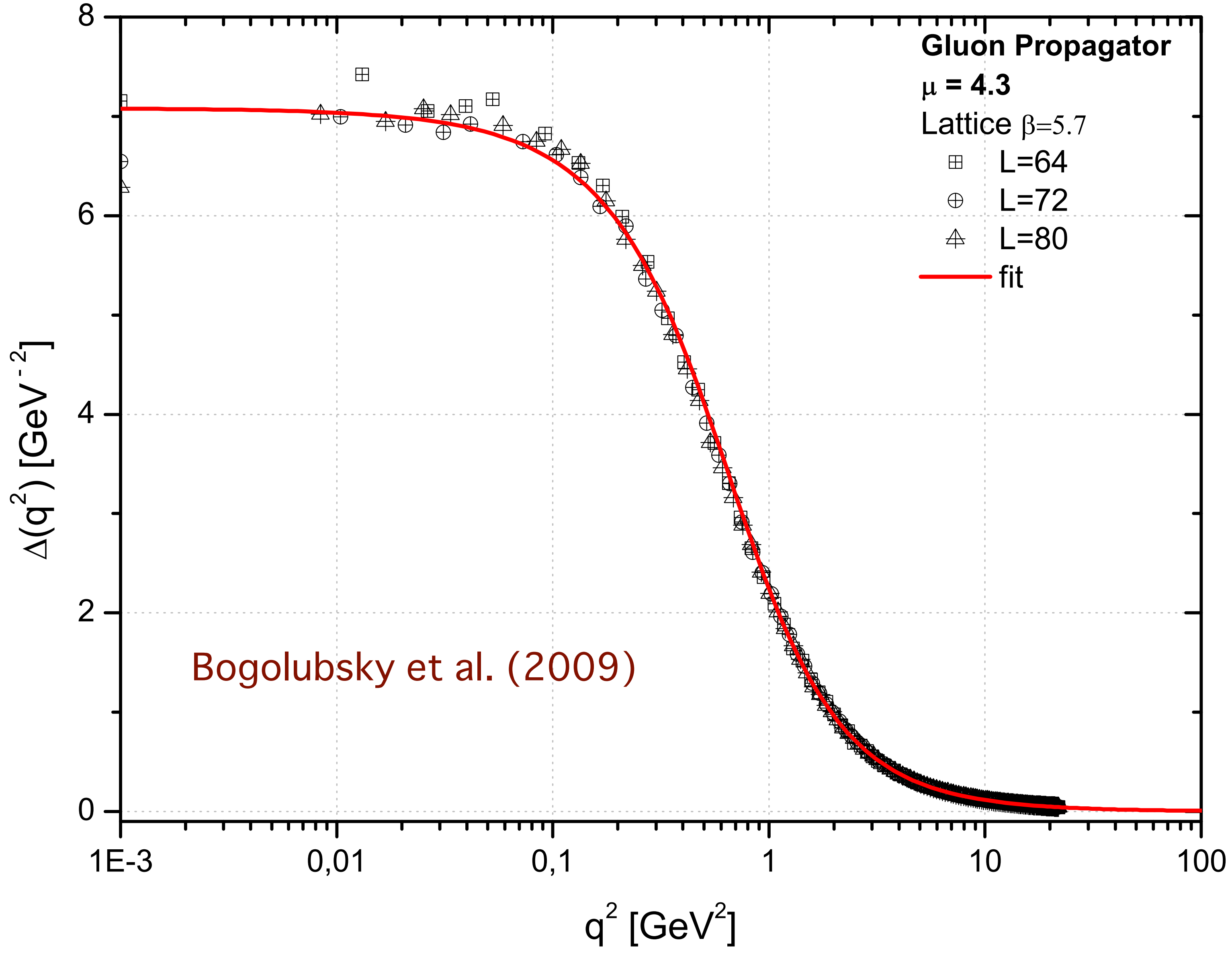
Lattice QCD gluon dressing function

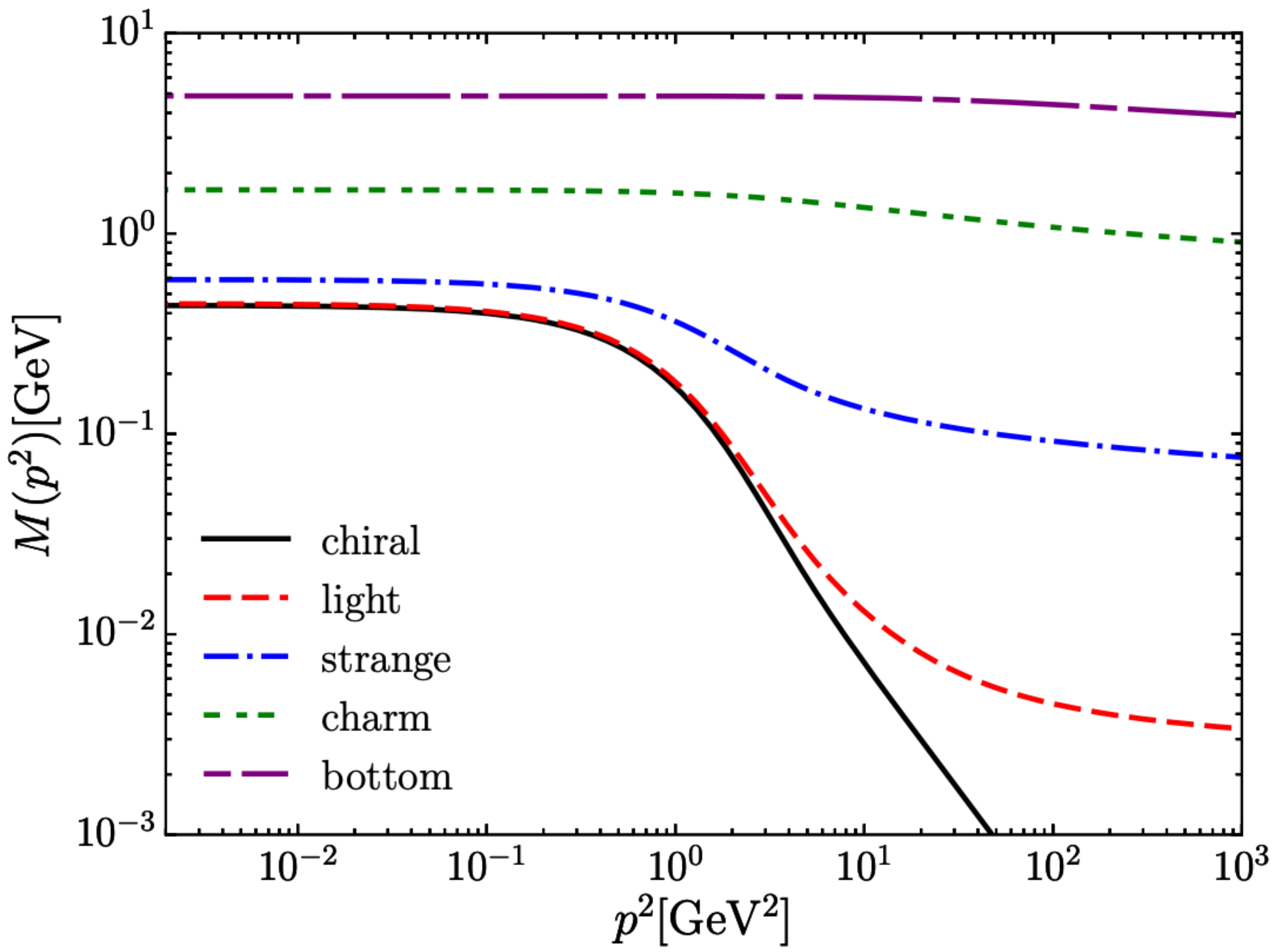


$$\Delta_{\mu\nu}(q) = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Delta(q^2)$$

Landau gauge

Use effective interaction which reproduces Lattice QCD and DSE results for gluon-dressing function: *infrared massive fixed point; ultraviolet massless propagator*.

Flavor dependence of quark mass functions



$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

L. Albino, A. Bashir, B.E., L.X. Gutiérrez Guerrero, E. Rojas PRD 100 (2019)

L. Albino, A. Bashir, B.E., E. Rojas, F. E. Serna, R.C. Silveira, JHEP11 (2021)

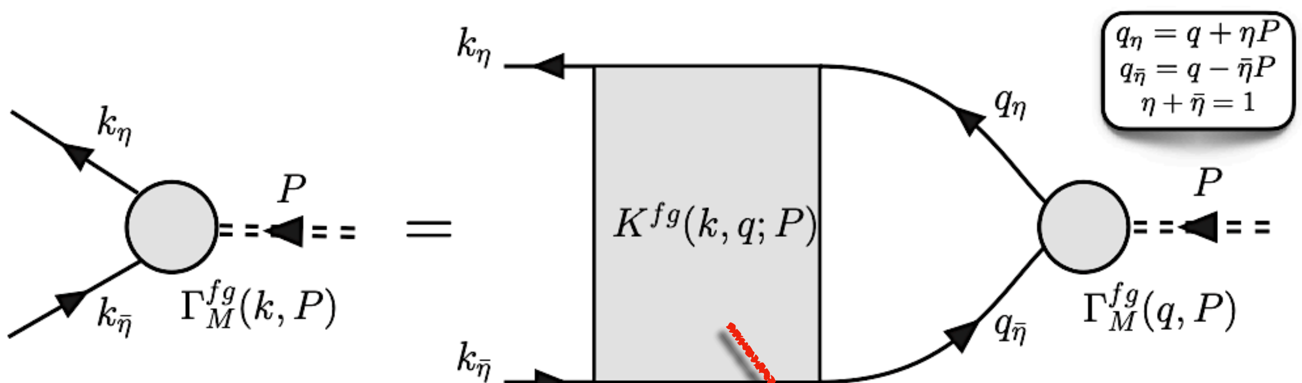
Bethe-Salpeter equation and QCD bound states

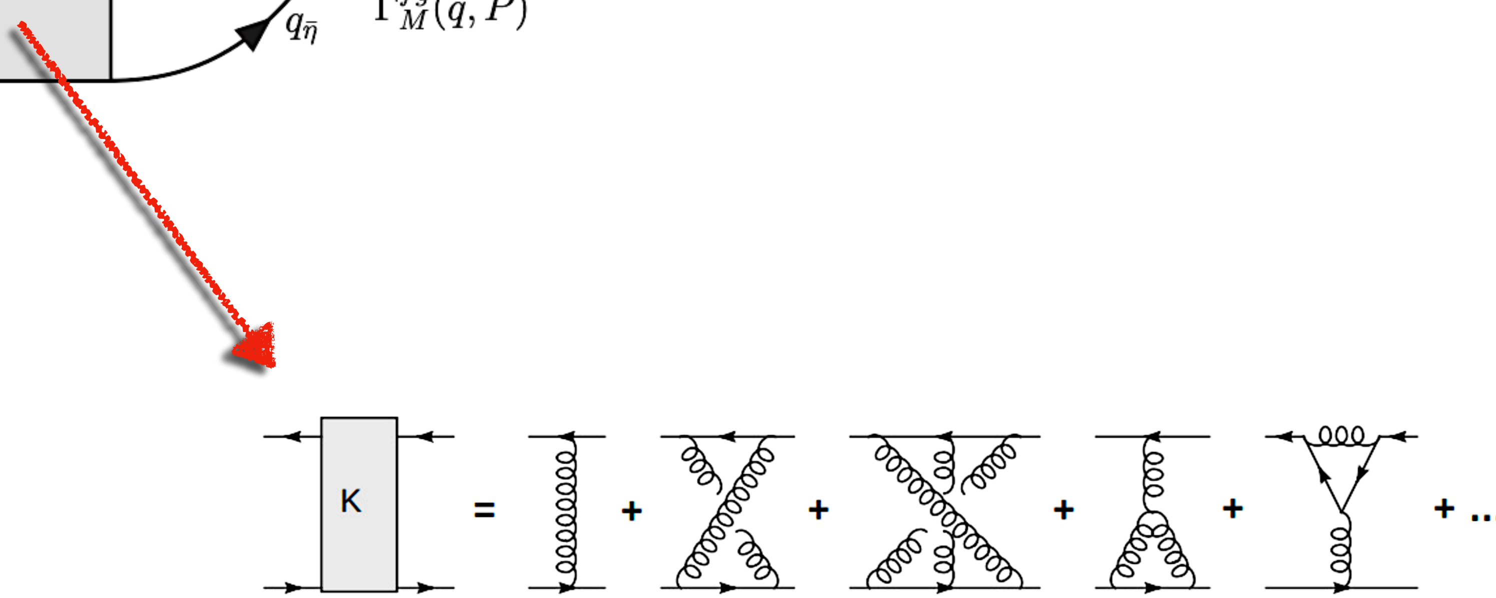
$$\Gamma_M^{fg}(k,P) = \int^{\Lambda} \frac{d^4q}{(2\pi)^4} K_{fg}(k,q;P) S_f(q_\eta) \Gamma_M^{fg}(q,P) S_g(q_{\bar{\eta}})$$

$K_{fg}(q,k;P)$ = Quark-antiquark scattering kernel

$S_f(q_\eta)$ = Dressed quark propagator

$\Gamma_M^{fg}(k,P)$ = Meson's Bethe-Salpeter Amplitude (BSA)





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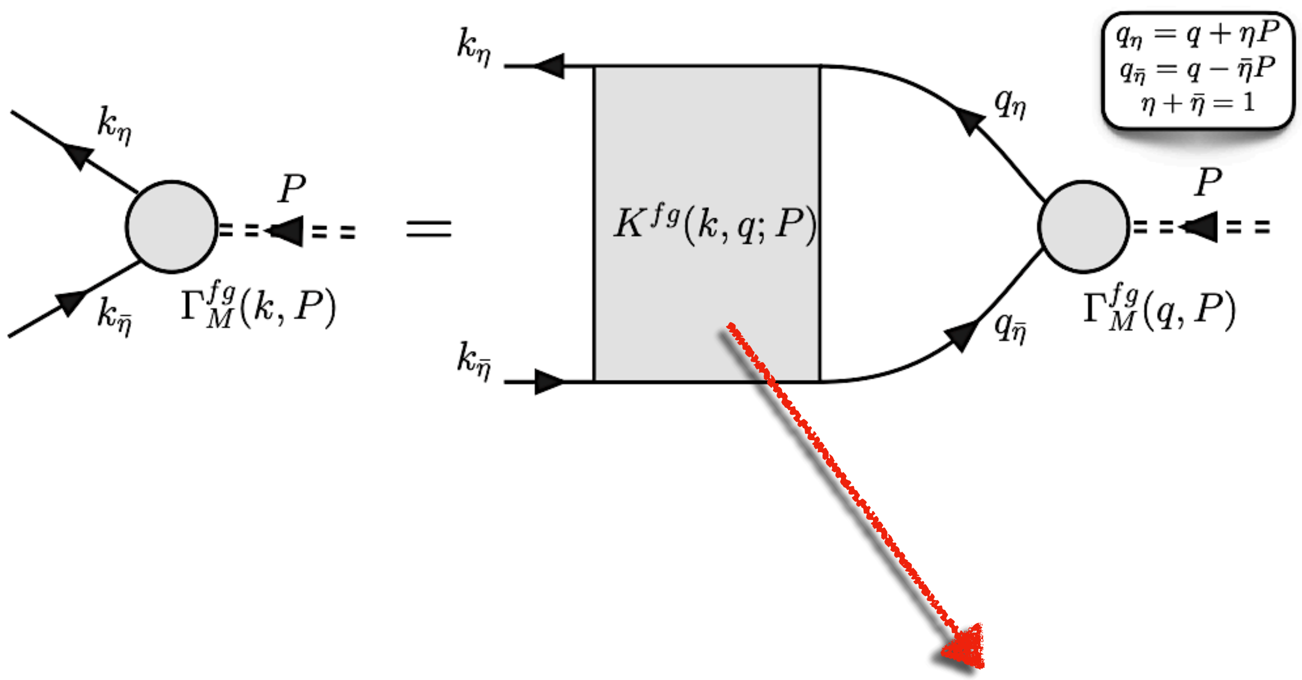
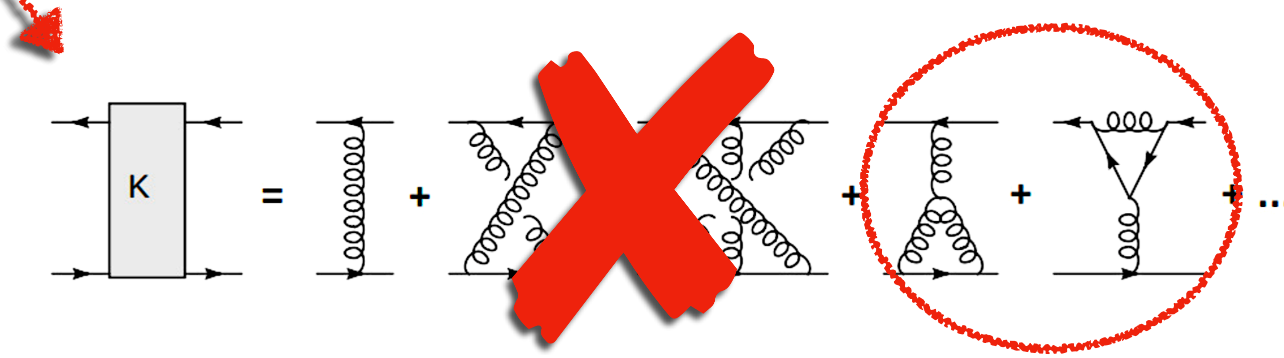
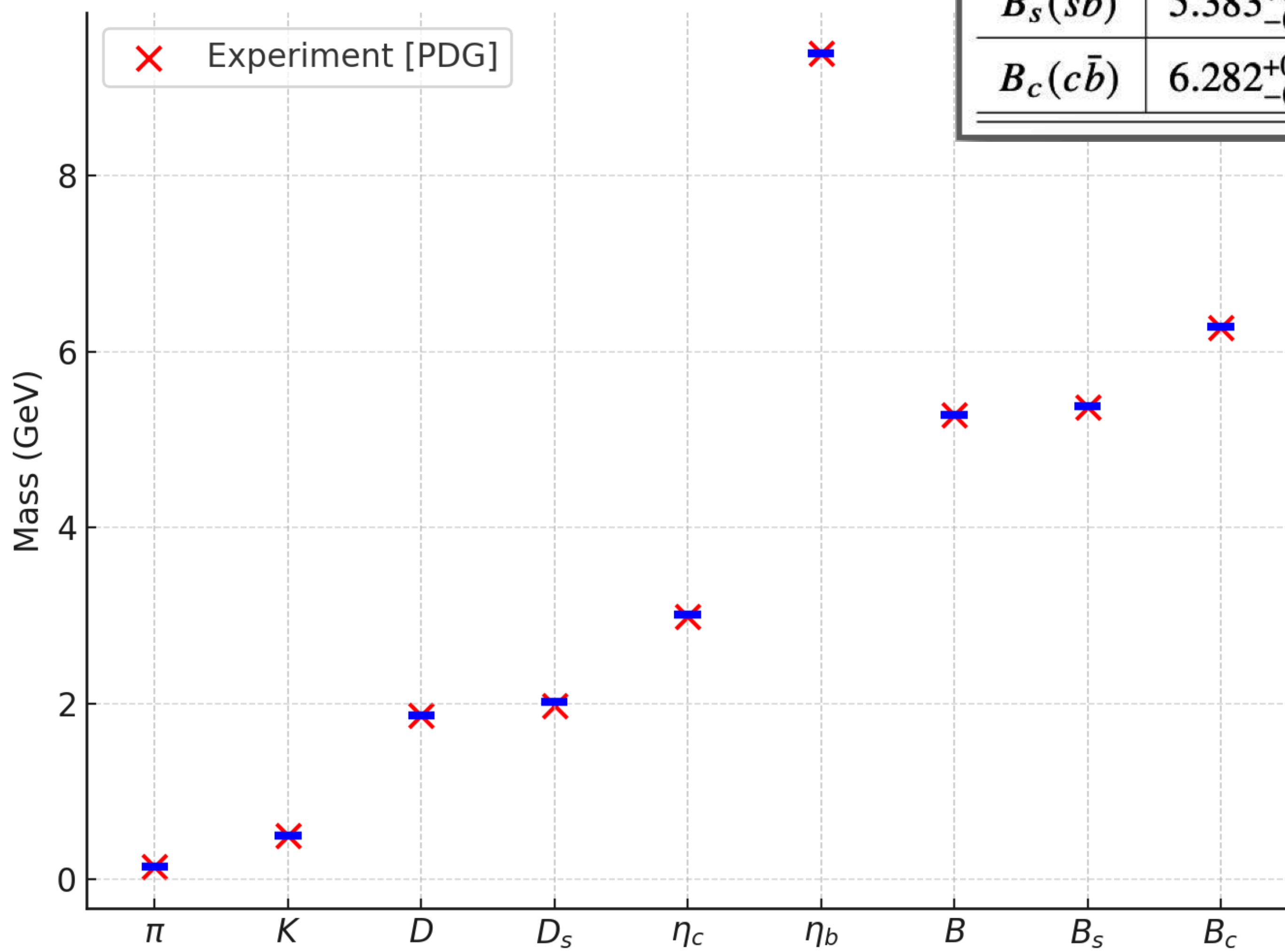


Treated with kernel beyond RL introducing flavor dependence



Pseudoscalar Meson Spectrum

	M_P	M_P^{exp}	ϵ_{M_P} [%]	f_P	$f_P^{\text{exp/lQCD}}$	ϵ_{f_P} [%]
$\pi(u\bar{d})$	0.140	0.138	1.45	$0.094^{+0.001}_{-0.001}$	0.092(1)	2.17
$K(u\bar{s})$	0.494	0.494	0	$0.110^{+0.001}_{-0.001}$	0.110(2)	0
$D(c\bar{d})$	$1.867^{+0.008}_{-0.004}$	1.864	0.11	$0.144^{+0.001}_{-0.001}$	0.150 (0.5)	4.00
$D_s(c\bar{s})$	$2.015^{+0.021}_{-0.018}$	1.968	2.39	$0.179^{+0.004}_{-0.003}$	0.177(0.4)	1.13
$\eta_c(c\bar{c})$	$3.012^{+0.003}_{-0.039}$	2.984	0.94	$0.270^{+0.002}_{-0.005}$	0.279(17)	3.23
$\eta_b(b\bar{b})$	$9.392^{+0.005}_{-0.004}$	9.398	0.06	$0.491^{+0.009}_{-0.009}$	0.472(4)	4.03
$B(u\bar{b})$	$5.277^{+0.008}_{-0.005}$	5.279	0.04	$0.132^{+0.004}_{-0.002}$	0.134(1)	4.35
$B_s(s\bar{b})$	$5.383^{+0.037}_{-0.039}$	5.367	0.30	$0.128^{+0.002}_{-0.003}$	0.162(1)	20.5
$B_c(c\bar{b})$	$6.282^{+0.020}_{-0.024}$	6.274	0.13	$0.280^{+0.005}_{-0.002}$	0.302(2)	10.17



F. Serna, R. Correa da Silveira, J.J. Cobos Martínez, B.E., E. Rojas, Eur. Phys. J. C 80 (2020)

Vector Meson Spectrum

	M_V	M_V^{exp}	ϵ_{M_V} [%]	f_V	$f_V^{\text{exp/lQCD}}$	ϵ_{f_V} [%]
$\rho(u\bar{u})$	0.730	0.775	5.81	0.145	0.153(1)	5.23
$\phi(s\bar{s})$	1.070	1.019	5.20	0.187	0.168(1)	11.31
$K^*(u\bar{s})$	0.942	0.896	5.13	0.177	0.159(1)	11.32
$D^*(c\bar{d})$	2.021	2.009	0.60	0.165	0.158(6)	4.43
$D_s^*(c\bar{s})$	2.169	2.112	2.70	0.205	0.190(5)	7.90
$J/\psi(c\bar{c})$	3.124	3.097	0.87	0.277	0.294(5)	5.78
$\Upsilon(b\bar{b})$	9.411	9.460	0.52	0.594	0.505(4)	17.62

F. Serna, R. Correa da Silveira, B.E., PRD Letters 106 (2022)

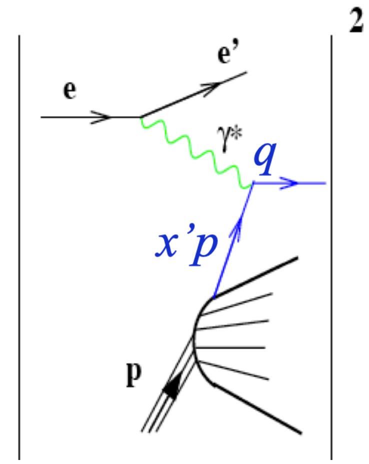
Part II

Fragmentation Functions

Deep Inelastic Scattering and Parton Distribution Functions

- Assumes fast moving hadron appears as a jet of partons moving in same direction and sharing its total momentum.
- DIS cross section is an incoherent sum of elastic scattering cross sections off individual partons.
- Parton model should work perfectly for $Q^2 \to \infty$ where coupling constant vanishes.

$$d\sigma \equiv \sum_i \int dx_i q_2^2 \left| \cdots \right|^2$$

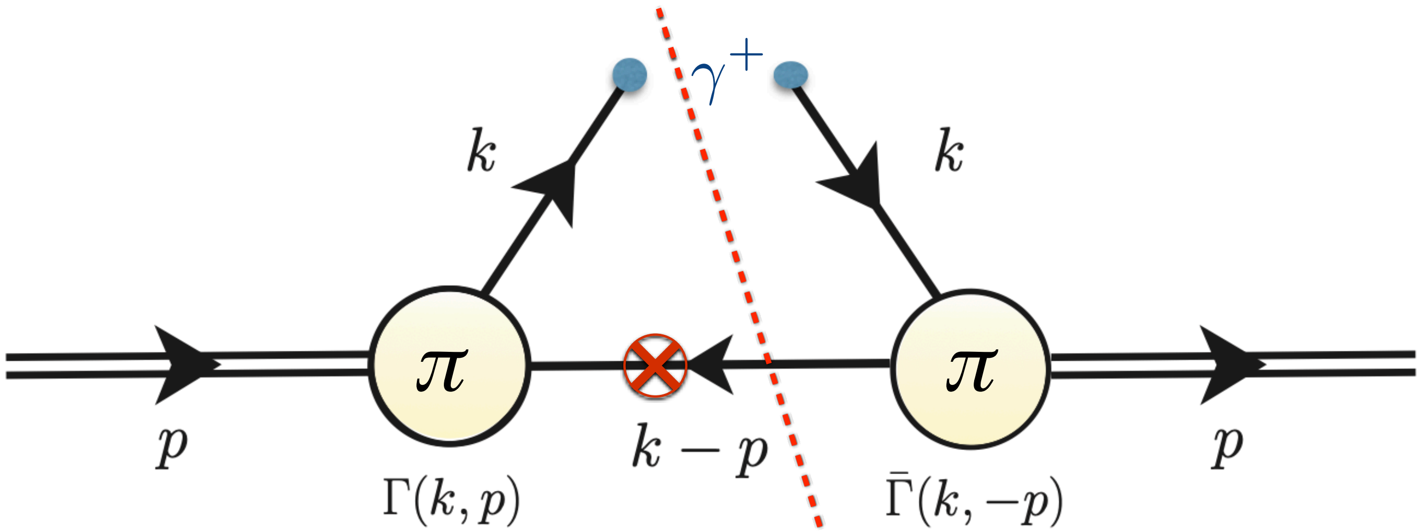


$$\mathrm{d}\sigma = \sum_i \mathrm{d}\hat{\sigma}(\hat{s}, \hat{t}, \hat{u})\; f_i(x)\mathrm{d}x$$

Parton momentum distribution is defined as $f_i(x) \equiv \dfrac{dP_i}{dx}$, where $\sum_i \int_0^1 f_i(x)\, dx = 1$

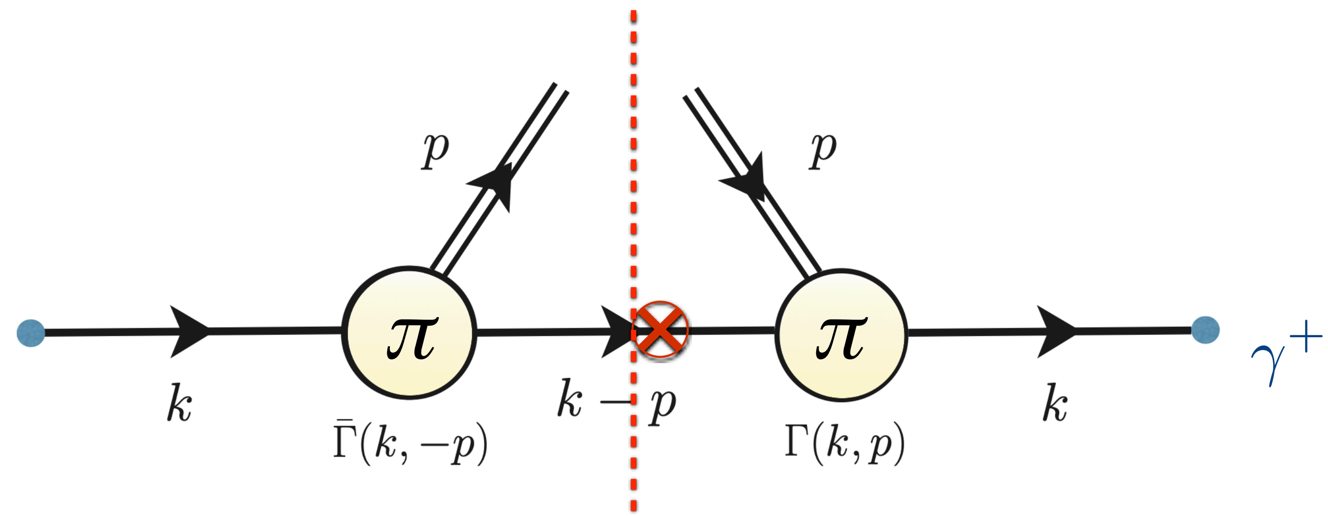
Defines probability to find a parton with light-front momentum fraction $x = \dfrac{k^+}{P^+}$ of the hadron.

PDF from a cut diagram



- PDFs can be computed from cut diagrams related to the imaginary part of the forward Compton amplitude (optical theorem, unitarity).
- Cuts put intermediate quarks on-shell, allowing for a probabilistic interpretation.
- In our approach the quark propagators are dressed, nonperturbative functions of the momentum, and the Bethe-Salpeter wave functions of the mesons are Poincaré invariant.
- Pseudoscalar mesons have internal structure with *S*- and *P*-wave orbital angular momentum.

Drell-Levy-Yan relation



- Fragmentation functions describe the hadronization of highly energetic partons with nearly longitudinal momentum.

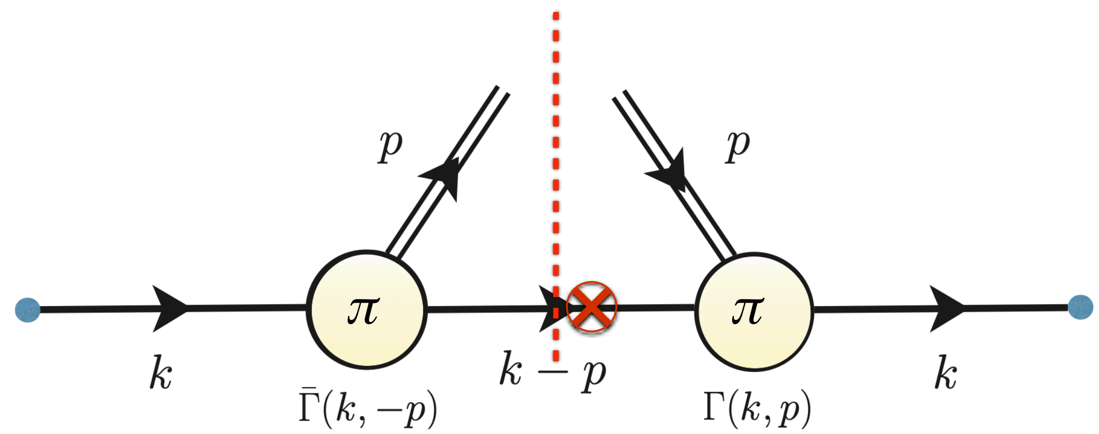
- Elementary fragmentation function for *physical* $z = 1/x \leq 1$ can be related to PDF for *unphysical* $x > 1$ using charge conjugation and crossing symmetry for matrix elements:

$$d_q^\pi(z) = \frac{z}{6}\, f_q^\pi(x) \qquad \text{(DLY relation)}$$

- Represents a probability density and can also be obtained from a cut diagram:

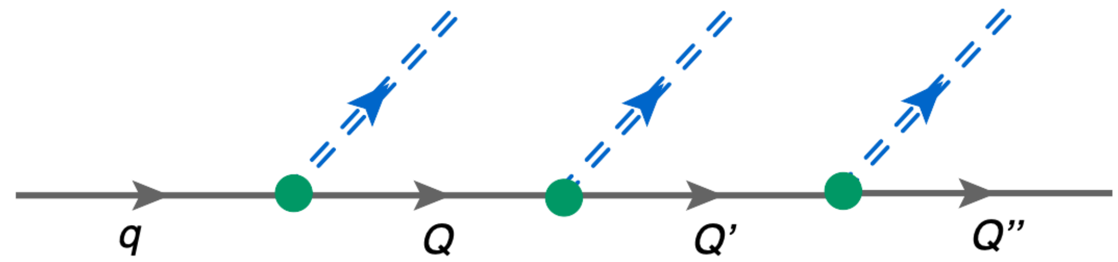
$$d_q^\pi(z) = \frac{(1+\tau_q\tau_\pi)iN_c z}{12}\int\frac{d^4k}{(2\pi)^4}\,\delta\Big(k^+ - \tfrac{p^+}{z}\Big)\,\mathrm{Tr}_\mathrm{D}\Big[S(k)\gamma^+ S(k)\bar\Gamma_\pi(k,-p)\,S_\mathrm{os}(k-p)\Gamma_\pi(k,p)\Big]$$

Beyond DLY



- DLY relation only useful to derive lowest order elementary fragmentation function.
- Based on the assumption that PDF can be continued analytically beyond $x = 1$.
- We know that the Q^2 evolution equations lead to soft (integrable) singularities at $x = 1$
- DLY is still useful to relate the kernels of the evolution equations for the distribution and fragmentation functions at LO, J. Blumlein, V. Ravindran and W. L. van Neerven (2000).
- At NLO the relation between kernels is violated, M. Stratmann and W. Vogelsang (1998).
- In a single fragmentation process momentum and isospin sum rules cannot be satisfied.
- In the quark jet-model formulated by Field and Feynman the meson observed in a semi-inclusive process is one among many others which form a jet.

Jet equations



- Use multiplicative ansatz for the total fragmentation function, see derivation in T. Ito, W. Bentz, I. C. Cloët, A. Thomas & K. Yazaki (2009)

  ⇒ resembles integral ansatz by Fields and Feynman.

- Allows to derive integral (jet) equations for the cascade of emitted mesons:

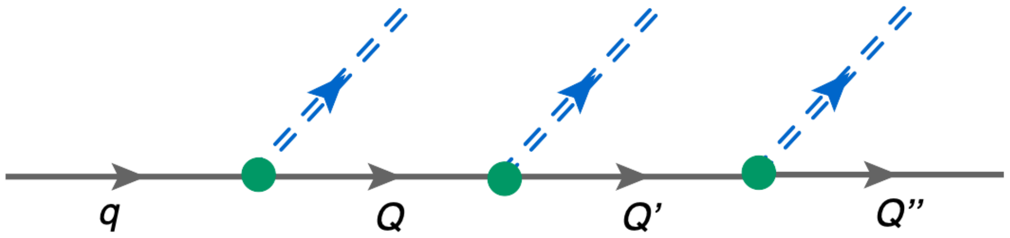
$$D_q^m(z) = d_q^m(z) + \sum_Q \int_z^1 \frac{dy}{y}\, d_q^Q\!\left(\frac{z}{y}\right) D_Q^m(y)\,, \quad d_q^Q(z) = d_q^m(1-z)\big|_{m=q\bar{Q}}$$

- For instance, probability for finding a pion with light-front momentum fraction z in the jet is described by the jet function $D_q^\pi(z)$ summing over all possible fragmentations.

- Initial work took into account a cascade of charged and neutral pions.

- Currently being extended to include kaons, heavy mesons and vector mesons.

Jet equations

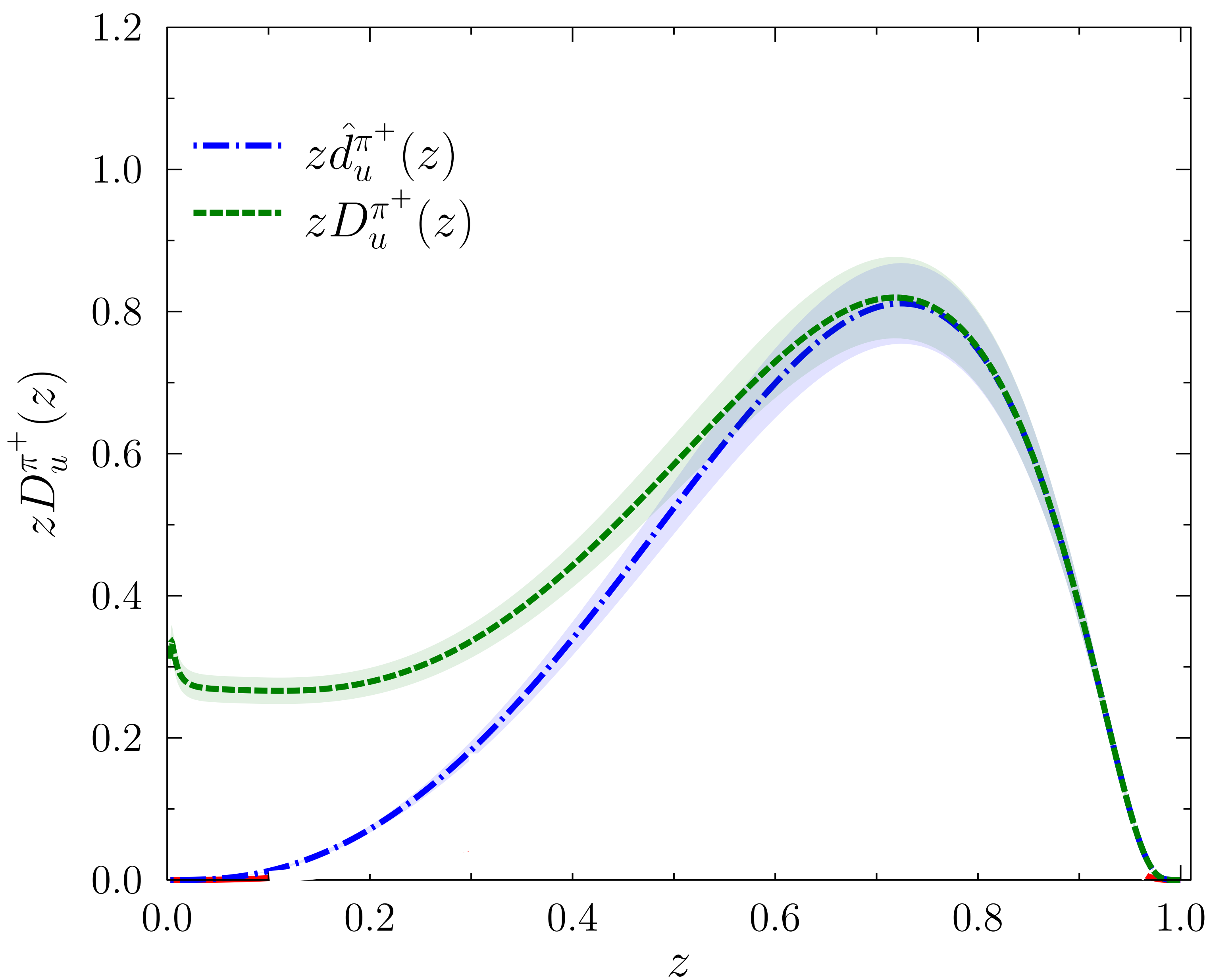


$$D_q^m(z)dz = d_q^m(z)dz + \sum_Q \int_z^1 d_q^Q\left(\frac{z}{y}\right) D_Q^m(y)\frac{dy}{y}dz$$

Probability to create a meson carrying the light-cone momentum fraction *z+dz of initial quark*.

Creation of meson further down the quark cascade after a splitting to a quark Q with light-cone momentum fraction y.

$u \to \pi$ fragmentation functions



R. C. da Silveira, F. E. Serna and B. E., PRC (2025)

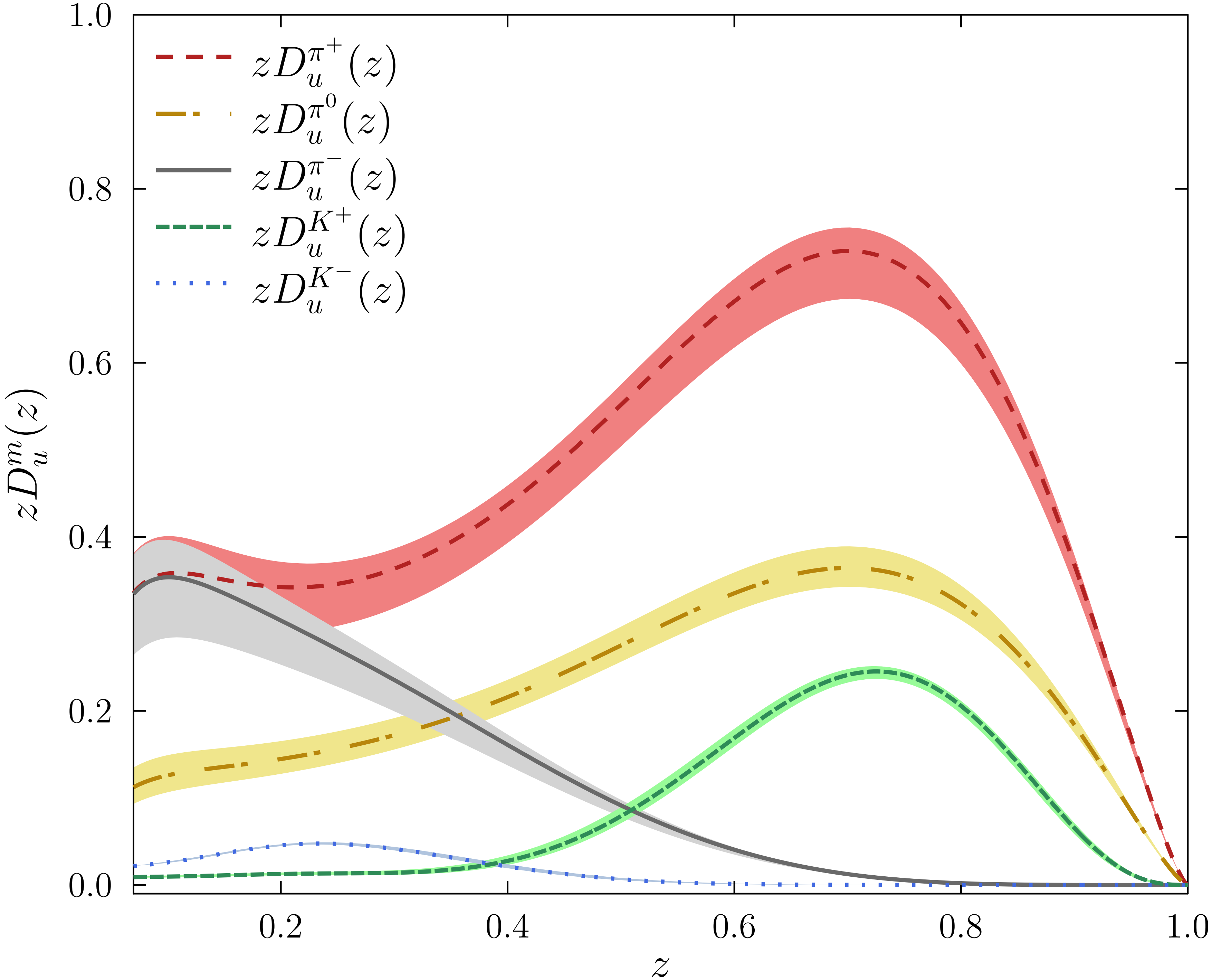
Normalized fragmentation function satisfies isospin sum rule:

$$\sum_m \int_0^1 \hat{d}_q^m(z)\, dz = 1$$

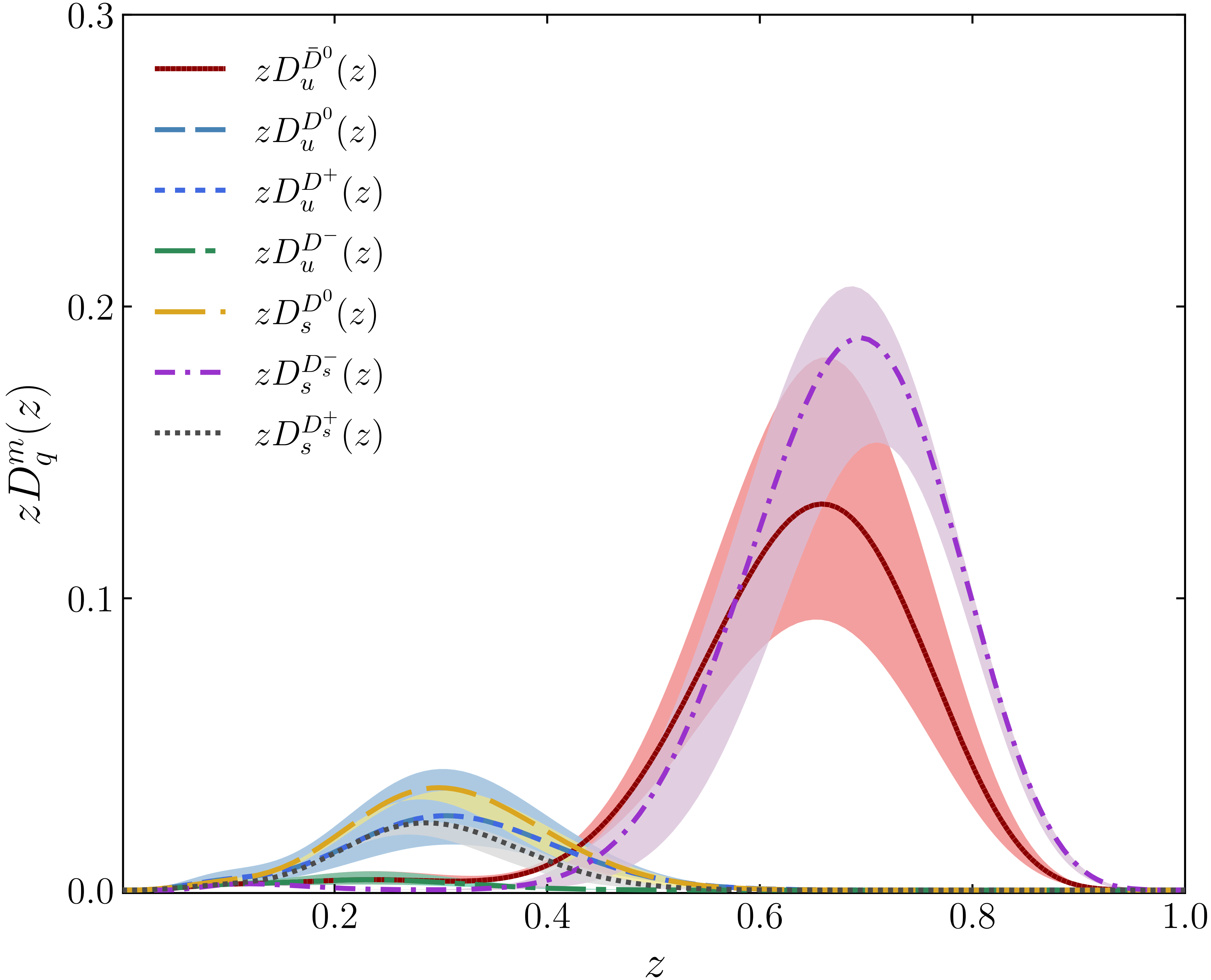
$$D_u^\pi(z) \xrightarrow{z\to 1} \hat{d}_q^\pi(z)$$

Limit in which quark gives all its momentum to initial pion.

$u \to \pi, K$ fragmentation functions



$u \to D$ and $s \to D_{(s)}$ fragmentations



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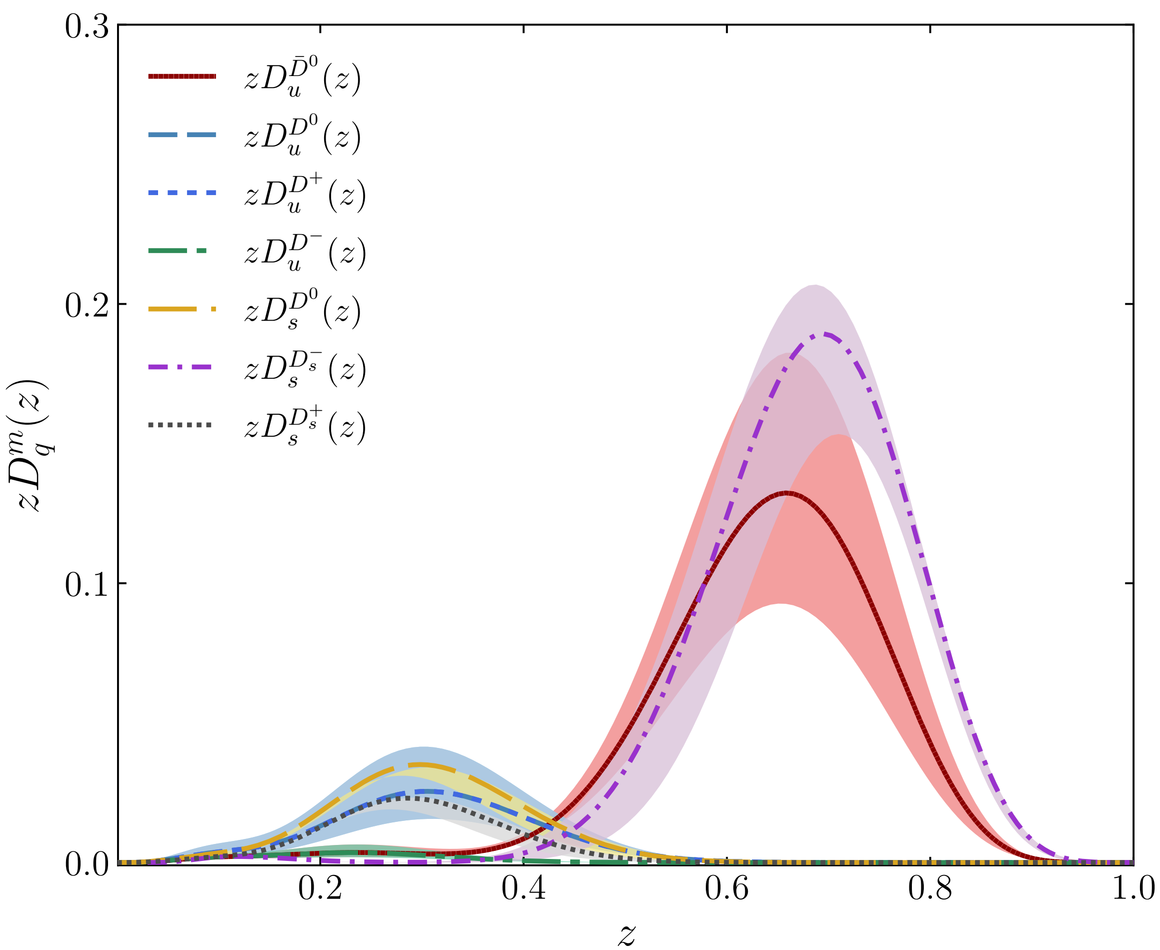
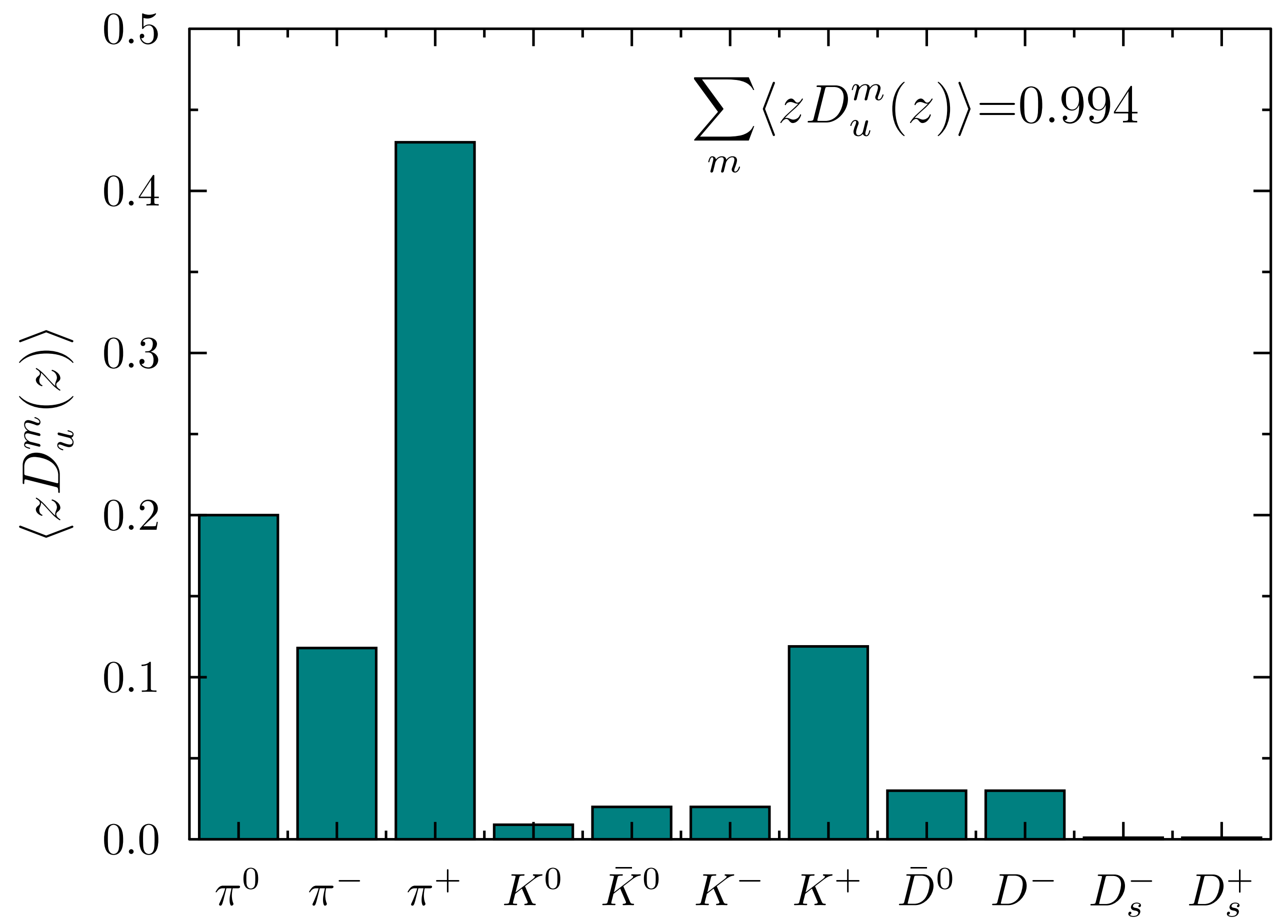
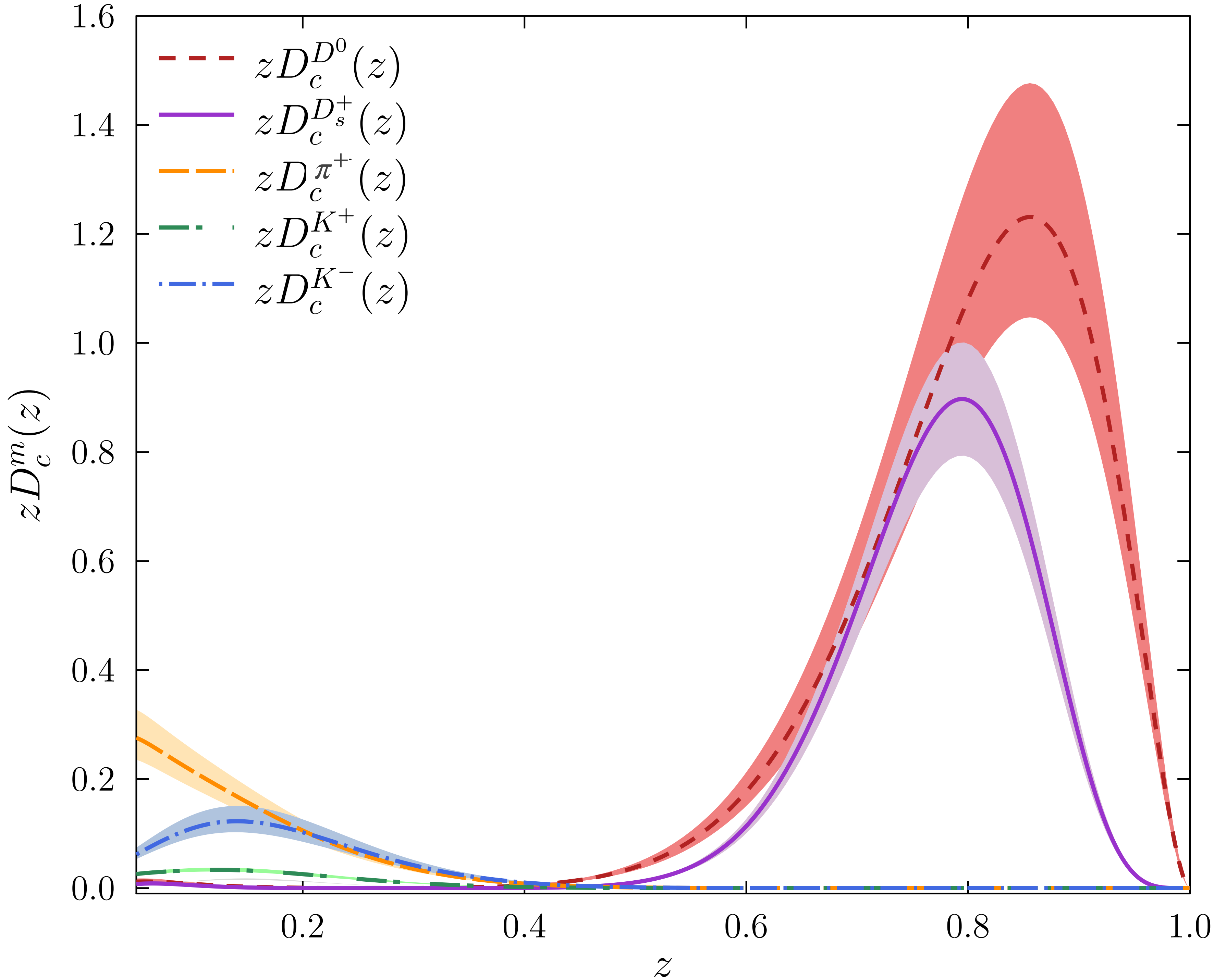
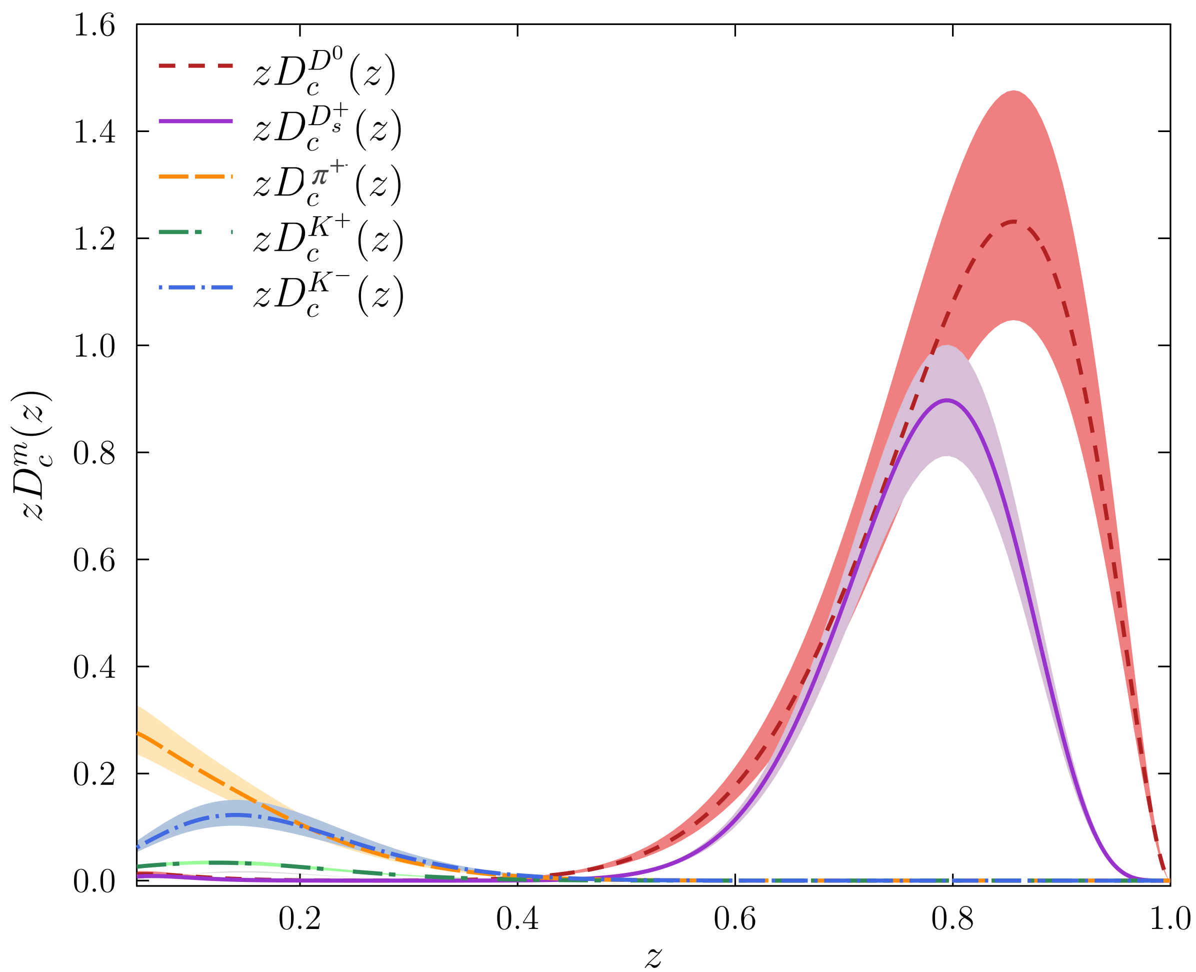


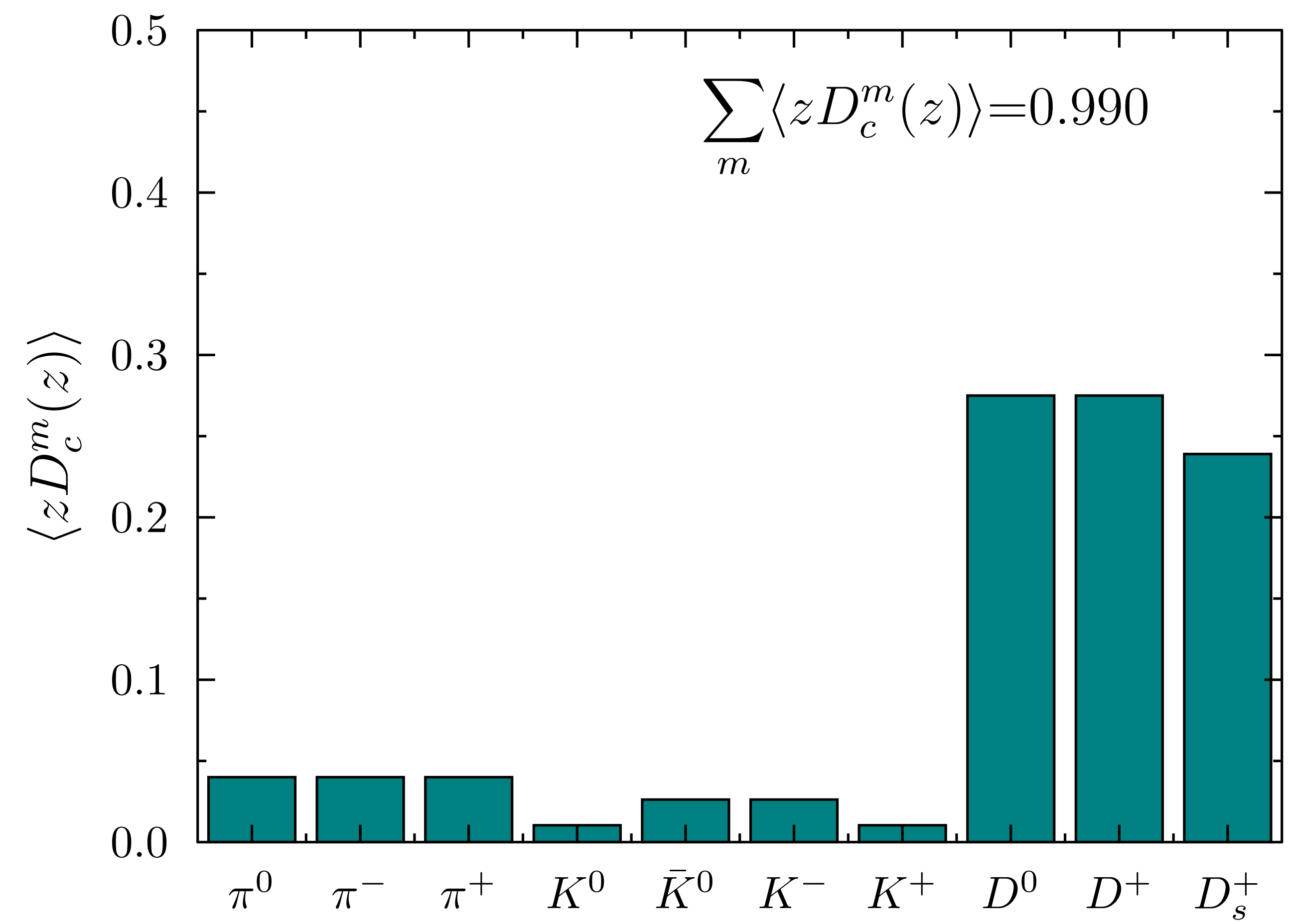


$c \to \pi, K, D$ fragmentation functions

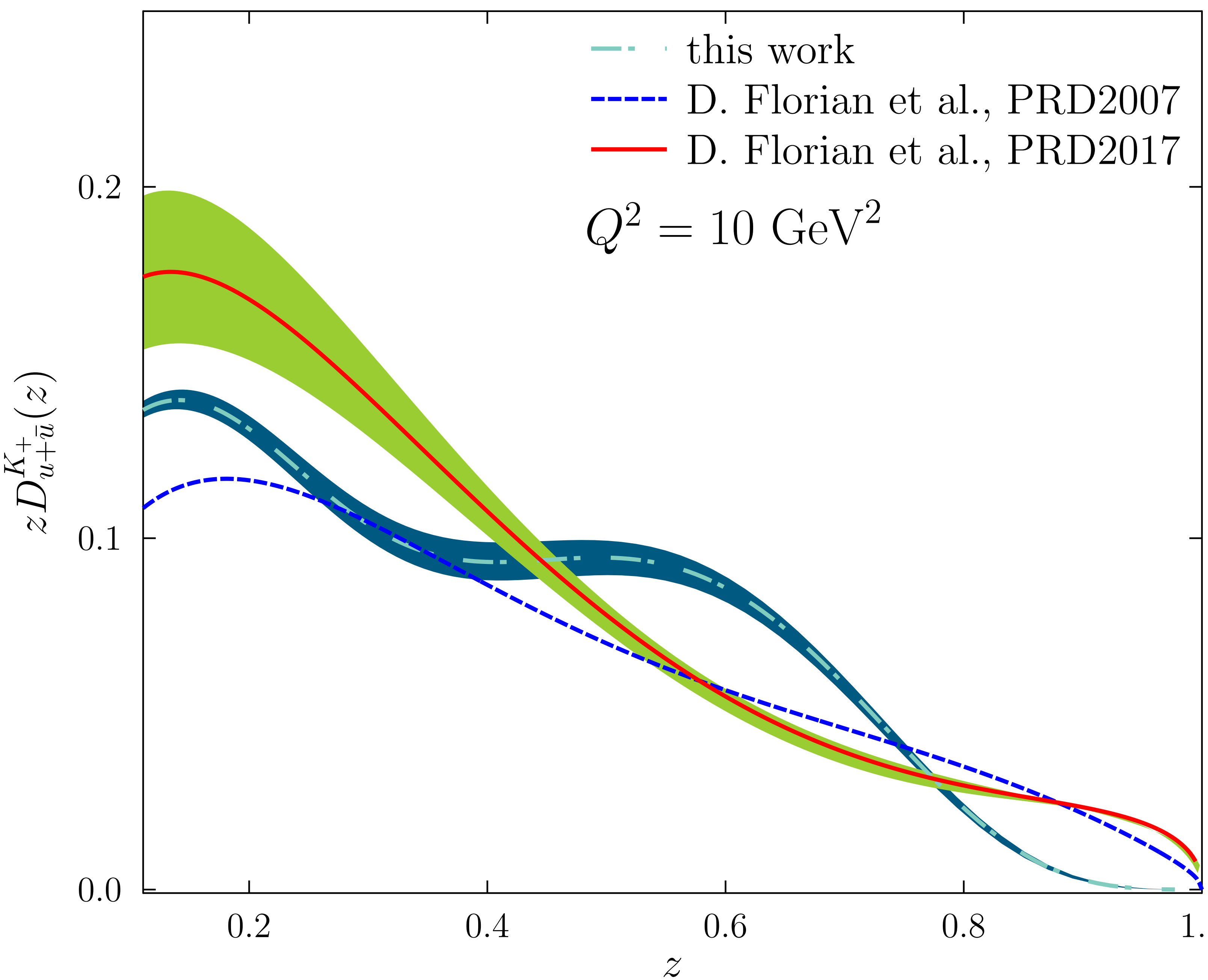


$c \to \pi, K, D$ fragmentation functions





Global analysis of $\bar{u} + u \to K^+$ fragmentation



Final remarks

- Elementary quark-to-meson fragmentations are obtained from cut diagrams and fed into jet equations.
- Resulting fragmentation functions satisfy isospin and momentum sum rules.
- Strong suppression of the fragmentation of light quarks into heavy mesons.
- *In progress:* inclusion of light and heavy vector mesons and quarkonia in the jet equations and (hopefully) baryons in the near future.