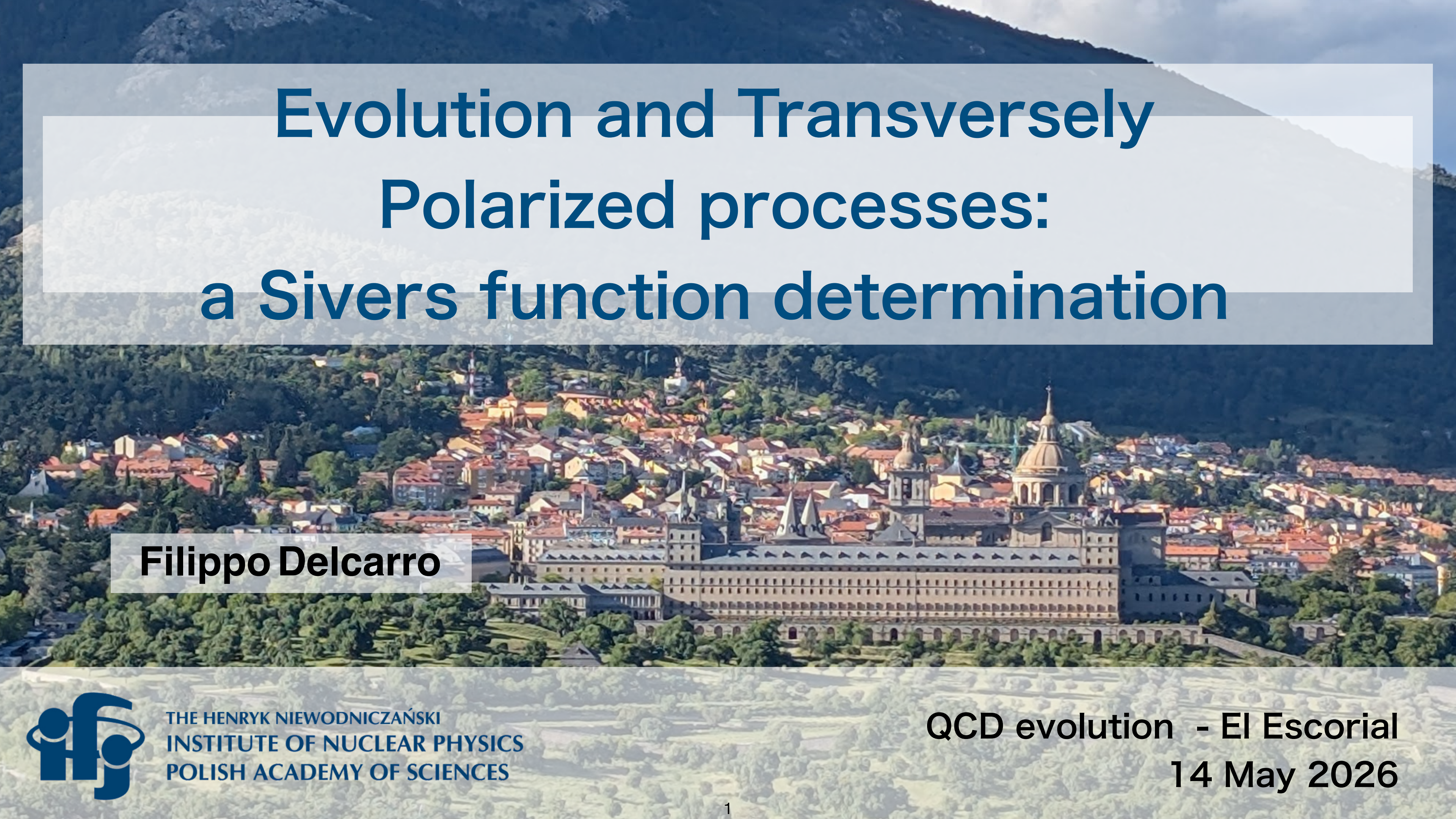

Evolution and Transversely Polarized processes: a Sivers function determination

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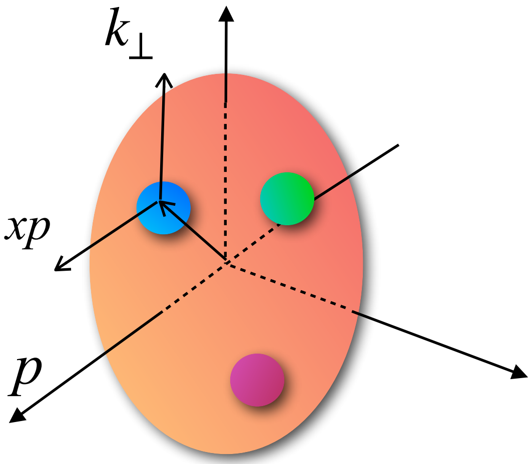
POLISH ACADEMY OF SCIENCES

QCD evolution - El Escorial
14 May 2026

Transverse Momentum Distributions: TMD PDF



quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

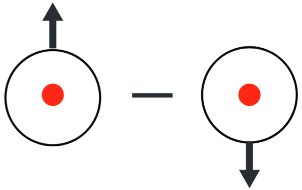
$f_{1T}^\perp$ → Sivers function

dependence on:

longitudinal momentum fraction x

transverse momentum $k_\perp$

energy scale



Spin and quark motion correlation

Open issues

spin budget of hadrons

missing contributions from elementary constituents

not yet quantified

SSAs in hadron reactions

- not vanishing as expected with increasing energy
- correlation with parton dynamics

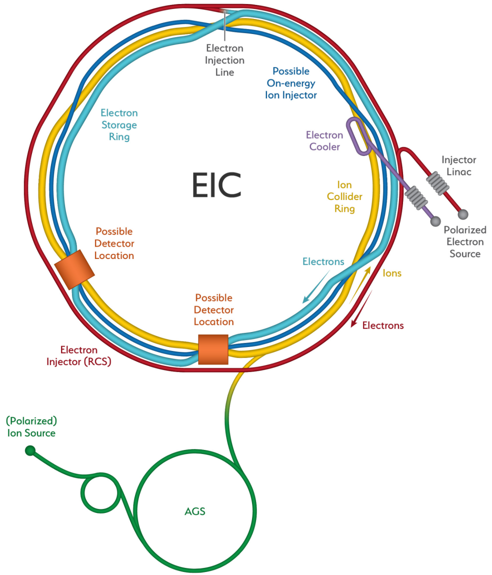
Effect of polarization on nucleon

internal structure density

polarized TMDs and anomalous magnetic moment

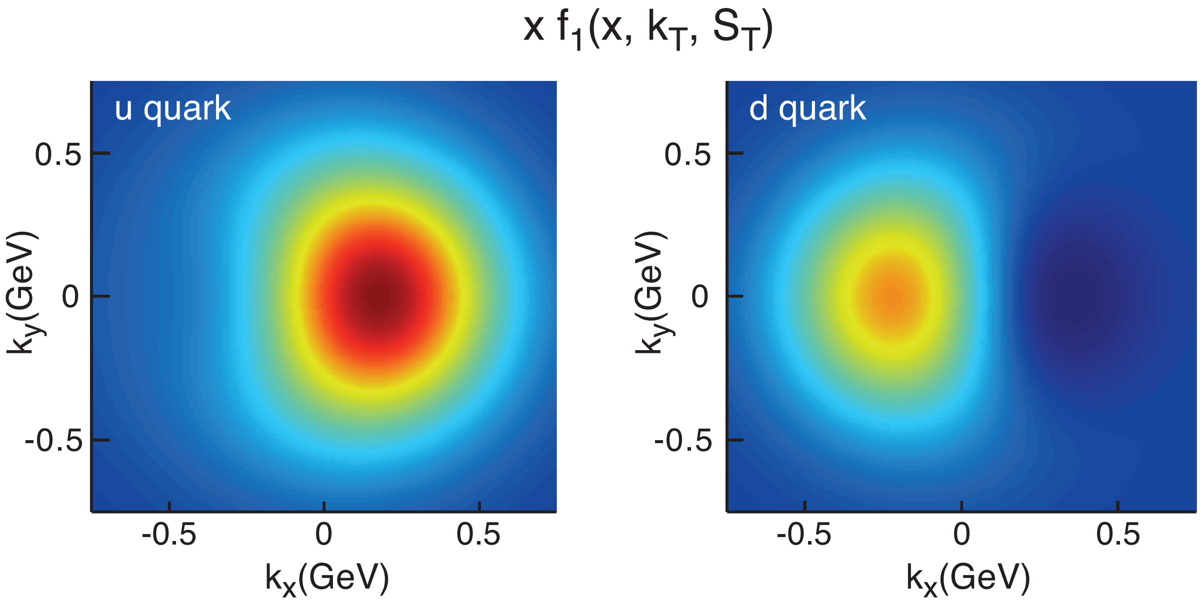
purpose-built

playground



Phenomenology of polarized TMDs

⇒ presence of a non-zero Sivers function $f_{1T}^{\perp}$ induces a dipole deformation of f_1

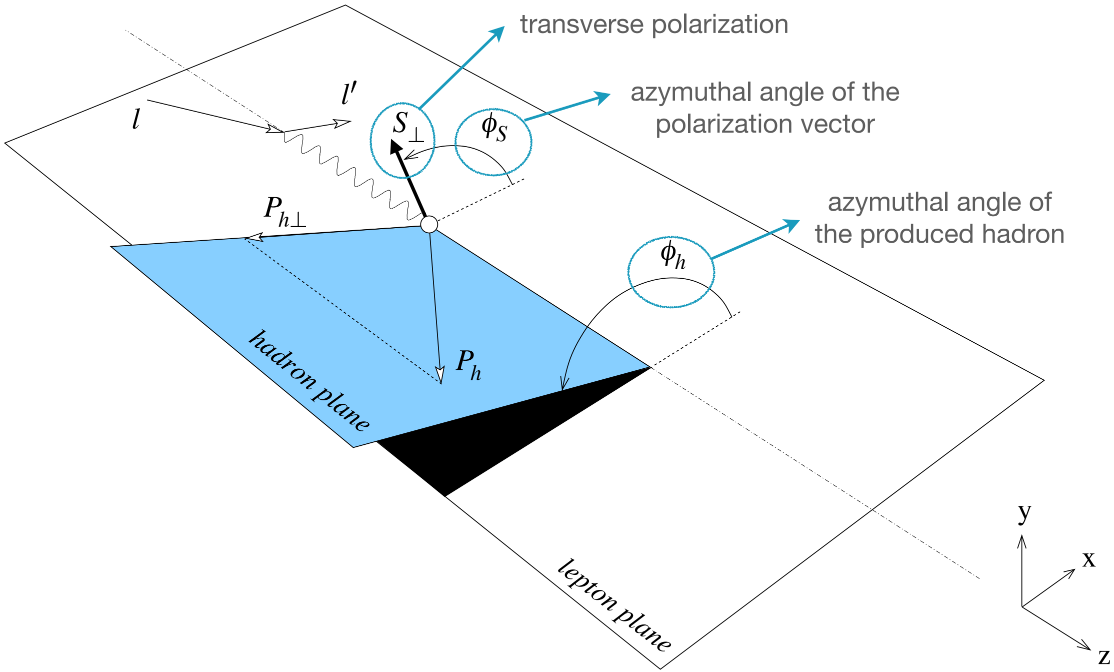


[EIC White Paper]

Extraction of Sivers Function

Determined through its contributions to

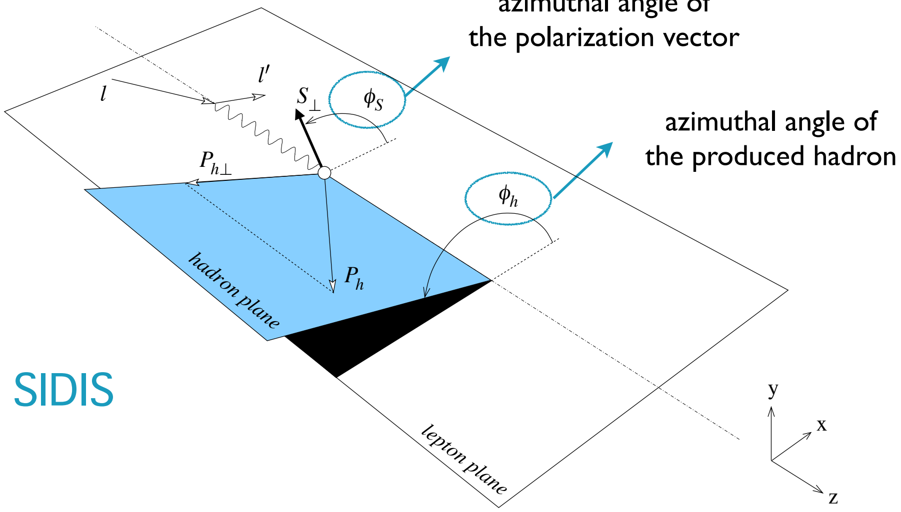
the cross section of **polarized SIDIS** **(and DY)**



Extraction of Sivers Function

Determined through its contributions to the cross section of polarized SIDIS

LO - NLL



SIDIS

$$A_{UT}^{\sin(\phi_h-\phi_S)} \equiv \langle \sin(\phi_h - \phi_S) \rangle \sim \frac{f_{1T}^{\perp,a} \otimes D_1^{a\to h}}{f_1^a \otimes D_1^{a\to h}}$$

universality

Parametrization of Sivers function

Sivers function parametrized in terms of its **first moment**

$$f_{1T}^{\perp}(x, k_{\perp}^2) = f_{1T}^{\perp(1)}(x)\, f_{1TNP}^{\perp}(x, k_{\perp}^2)$$

At NLL proportional to the **Qiu–Sterman (ETQS) quark–gluon correlator**

$$f_{1T}^{\perp(1)}(x) \equiv \int d^2\mathbf{k}_T \frac{k_T^2}{2M^2} f_{1T}^{\perp q}(x, \mathbf{k}_T^2) = -\frac{1}{2M} T_F^q(x, x)$$

Parametrization of Sivers function

Sivers function parametrized in terms of its **first moment**

$$f_{1T}^{\perp}(x,k_{\perp}^{2}) = f_{1T}^{\perp(1)}(x)\, f_{1TNP}^{\perp}(x,k_{\perp}^{2})$$

Its nonperturbative part is arbitrary, but constrained by the positivity bound.

$$f_{1TNP}^{\perp}(x,k_{\perp}^{2}) = \frac{1}{\pi K_f}\frac{(1+\lambda_S k_{\perp}^{2})}{M_1{}^2+\lambda_S M_1{}^4)} e^{-k_{\perp}^{2}/M_1{}^2} f_{1NP}(x,k_{\perp}^{2})$$

following the definition to the nonperturbative part of the unpolarized TMD distribution

$$f_{1,NP}(x,b_T^2) \propto \text{ F.T of } \left(e^{\frac{k_{\perp}^{2}}{g_{1,A}}} + \lambda_B\, e^{\frac{k_{\perp}^{2}}{g_{1,B}}} + \lambda_C\, e^{\frac{k_{\perp}^{2}}{g_{1,C}}} \right)$$

Parametrization of Sivers function

Sivers function parametrized in terms of its **first moment**

$$f_{1T}^{\perp}(x, k_{\perp}^2) = f_{1T}^{\perp(1)}(x)\, f_{1TNP}^{\perp}(x, k_{\perp}^2)$$

$$f_{1TNP}^{\perp}(x, k_{\perp}^2) = \frac{1}{\pi K_f} \frac{(1+\lambda_S\, k_{\perp}^2)}{(M_1{}^2+\lambda_S\, M_1{}^4)} e^{-k_{\perp}^2/M_1{}^2} f_{1NP}(x, k_{\perp}^2)$$

Free parameters $\lambda_S\,, M_1$

following the definition to the nonperturbative part of the unpolarized TMD distribution

$$f_{1,NP}(x, b_T^2) \propto \text{ F.T of } \left(e^{\frac{k_{\perp}^2}{g_{1,A}}} + \lambda_B\, e^{\frac{k_{\perp}^2}{g_{1,B}}} + \lambda_C\, e^{\frac{k_{\perp}^2}{g_{1,C}}} \right)$$

from **MAP22** TMD extraction

Parametrization of Sivers function

Sivers function parametrized in terms of its first moment

$$f_{1T}^{\perp}(x, k_\perp^2) = f_{1T}^{\perp(1)}(x)\, f_{1TNP}^{\perp}(x, k_\perp^2)$$

Its nonperturbative part is arbitrary, but constrained by the positivity bound.

$$f_{1TNP}^{\perp}(x, k_\perp^2) = \frac{1}{\pi K_f} \frac{(1 + \lambda_S k_\perp^2)}{(M_1^2 + \lambda_S M_1^4)} e^{-k_\perp^2/M_1^2} f_{1NP}(x, k_\perp^2)$$

K_f

normalization factor to guarantee that the weighted integral is unitary

$$K_f \equiv \int d^2 k_\perp \frac{k_\perp^2}{2M^2} f_{1TNP}^{\perp}$$

Parametrization of Sivers function

normalization
$(|N_{Siv}| \leq 1)$

$T_n(x)$ Chebyshev polynomials

$$f_{1T}^{\perp(1)}(x) = \frac{N_{Siv}^a}{G_{max}^a} x^{\alpha_a}(1-x)^{\beta_a}\Big(1 + A_a T_1(x) + B_a T_2(x)\Big)\, f_1(x, Q^2)$$

maximum value of the
x polynomial function

Free parameters $N_{Siv}^a\,, \alpha_a\,, \beta_a\,, A_a\,, B_a$

Positivity bound

$$\left[\frac{k_T^2}{2M^2} f_{1T}(x, k_T^2)\right]^2 \leq \frac{k_T^2}{4M^2} f_1^2(x, k_T^2)$$

Flavor dependent: distinct for **up**, **down**, **sea**

Evolution of Sivers function first moment

evolved Sivers function first moment

At NLL we approximate its evolution, through OPE, to DGLAP

$$\tilde{f}_{1T}^{\perp(1)a}(x, b_T^2; Q^2) =$$

$$= \sum_i \left(\tilde{C}_{a/i} \otimes f_1^i\right)\left(x, \overline{b}_*; \mu_b\right) e^{\tilde{S}\left(\overline{b}_*; \mu_b, \mu\right)} e^{g_K\left(b_T\right) \ln\left(\mu/\mu_0\right)} \hat{f}_{1TNP}^{\perp(1)a}\left(x, b_T\right)$$

Evolution of Sivers function first moment

evolved Sivers function first moment

$$\tilde{f}_{1T}^{\perp(1)a}(x, b_T^2; Q^2) =$$

$$= \sum_i \left(\tilde{C}_{a/i} \otimes f_1^i\right)\left(x, \overline{b}_*; \mu_b\right) e^{\tilde{S}(\overline{b}_*; \mu_b, \mu)} e^{g_K(b_T)\ln(\mu/\mu_0)} \hat{f}_{1TNP}^{\perp(1)a}\left(x, b_T\right)$$

nonperturbative part of TMD

collinear PDF

(Wilson Coefficient)

(Sudakov form factor)

pQCD

nonperturbative part of evolution

g_2 taken from **MAP22** TMD extraction

Evolution of Sivers function first moment

$$\tilde{f}_{1T}^{\perp(1)a}(x, b_T^2; Q^2) = \sum_i \left(\tilde{C}_{a/i} \otimes f_1^i\right)\left(x, \overline{b}_*; \mu_b\right) e^{\tilde{S}(\overline{b}_*; \mu_b, \mu)} e^{g_K(b_T)\ln(\mu/\mu_0)} \hat{f}_{1TNP}^{\perp(1)a}\left(x, b_T\right)$$

The renormalization group evolution of TMDs is encoded in the **Sudakov form factor** S.

Here the resummation of large logarithm contributions is performed at **NLL**

$$\mu_b = 2e^{-\gamma_E}/b_T$$

at large bT: TMD evolution runs into a nonperturbative region and becomes unreliable.

cured by b∗-prescription

$$b_*\left(b_T\right) = b_{\text{bmax}}\left(\frac{1 - e^{-b_T^4/b_{\text{max}}^4}}{1 - e^{-b_T^4/b_{\text{min}}^4}}\right)^{\frac{1}{4}}$$

$$b_{\text{bmax}} = 2e^{-\gamma_E}\ \text{GeV}^{-1}$$

$$b_{\text{min}} = 2e^{-\gamma_E}/Q$$

Evolution of Sivers function first moment

$$b_*(b_T) = b_{\text{bmax}} \left(\frac{1 - e^{-b_T^4/b_{\text{max}}^4}}{1 - e^{-b_T^4/b_{\text{min}}^4}} \right)^{\frac{1}{4}}$$

$$b_{\text{bmax}} = 2e^{-\gamma_E}\ \text{GeV}^{-1}$$

$$b_{\text{min}} = 2e^{-\gamma_E}/Q$$

$$\mu_b = 2e^{-\gamma_E}/b_T$$

at large b_T the function $b_*(b_T)$ saturates to b_{max} , μ_b frozen at 1 GeV.
the perturbative contributions to the TMD merge into the nonperturbative region

At small b_T, the TMD formalism is not valid and must match onto the fixed-order formalism.

$$\lim_{b_T \to 0} \tilde{f}_{1T}^{\perp(1)a}\left(x, b_T^2; Q^2\right) = \int d^2\boldsymbol{k}_T \frac{k_T^2}{2M^2} f_{1T}^{\perp a}\left(x, k_T^2; Q^2\right) = f_{1T}^{\perp(1)a}\left(x; Q^2\right)$$

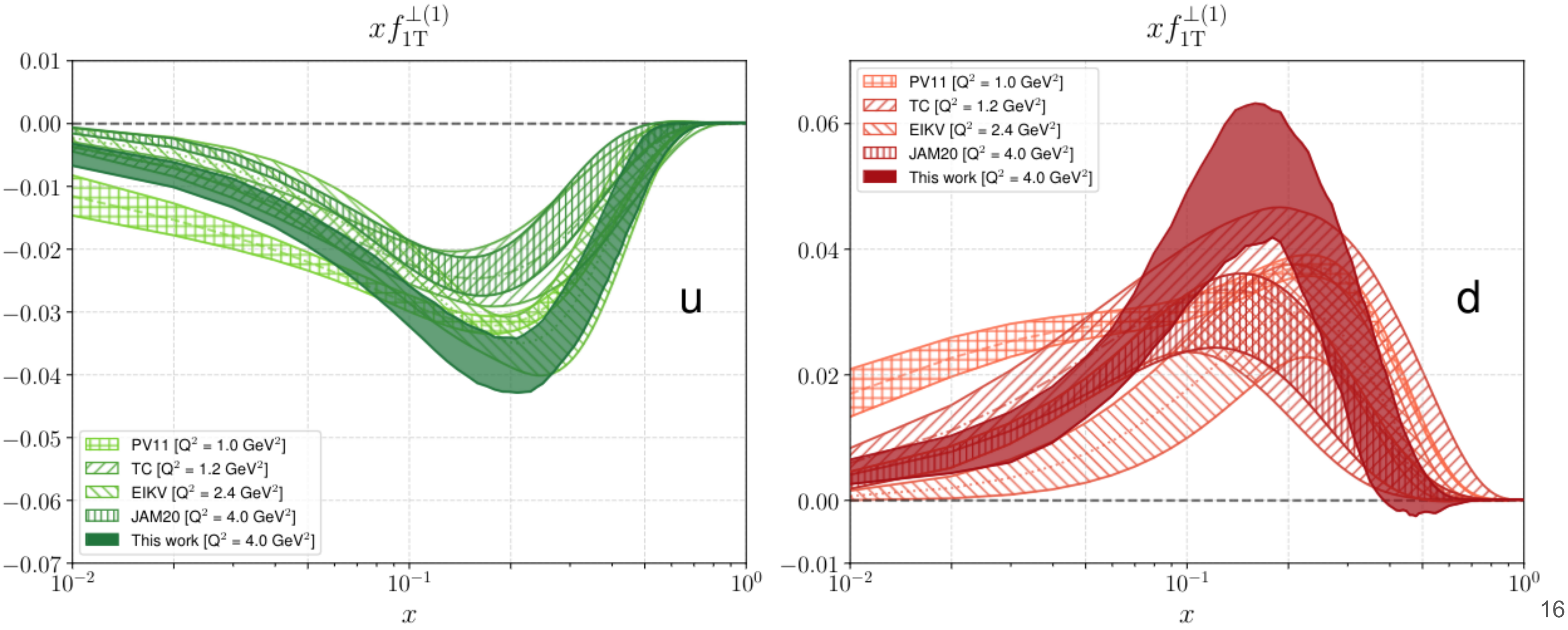
Approximation at NLL: the first moment is evolved with collinear evolution

Starting point: PV20Sivers Polarized TMDs

125 data points from SIDIS, DY

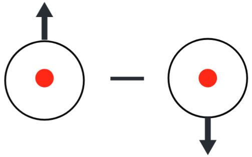
LO-NLL

$\chi^2 = 1.12$



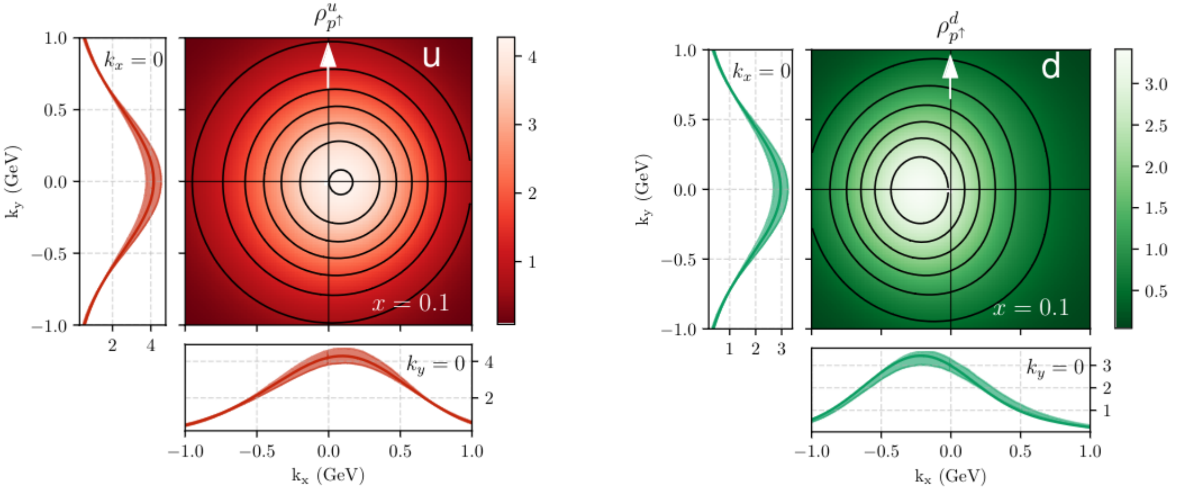
TMDs

PV20Sivers

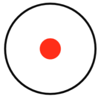


polarized

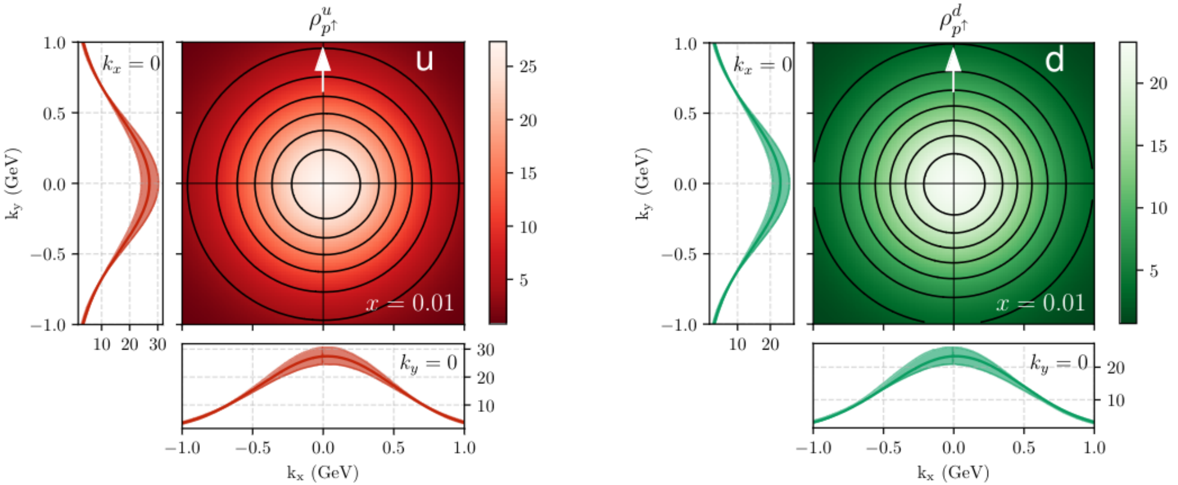
$$f_1(x, k_\perp; Q^2) - f_{1T}^\perp(x, k_\perp; Q^2)$$



PV17



unpolarized



Updated Sivers extraction

Additional data

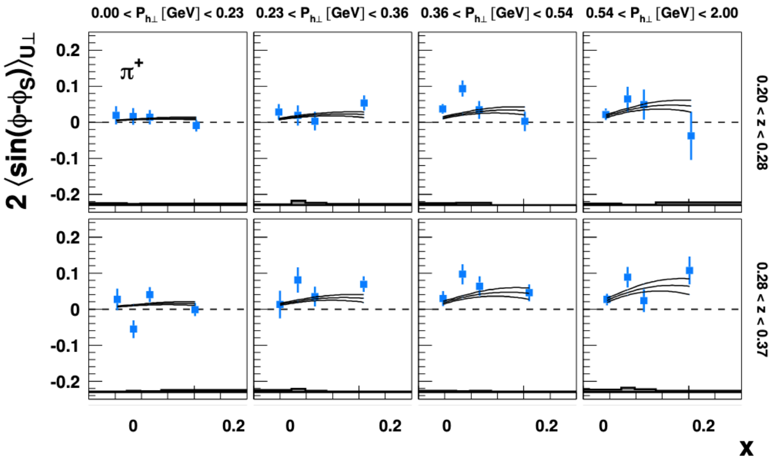
hermes

[JHEP12(2020)010]

2020

SIDIS

Multidimensional
x, P_{hT}, *z*



COMPASS

[PRL133]

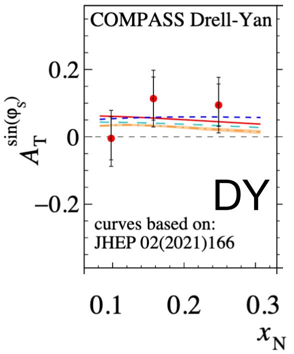
2022

SIDIS

2015-2018

pion-induced
DY

projected in *x*



Updated Sivers extraction

included datasets

SIDIS

hermes 2020

[JHEP12(2020)010]

target: proton [H]

final state hadron

π^+, π^-, K^+, K^-

250 data points

[PRL107,072003(2011)]

Jefferson Lab

t: neutron [^{3}He]

6 data points

COMPASS

[PLB673,127(2009)]

[PLB770,138(2017)]

[PRL133]

2009 t: deuteron [^{6}LiD]

2017 t: Proton [NH_3]

2022 t: deuteron [^{6}LiD]

f.s.h

h^+, h^-

98 data points

DY

W-Z production

2016

STAR

[PRL116,132301(2016)]

7 data points

2015-2018

pion-induced DY

t: Proton [NH_3]

COMPASS

3 data points

[PRL133]

Total number of data

364

Updated Sivers extraction

Additional data

hermes 2020
[JHEP12(2020)010]

COMPASS 2022
2015-2018
[PRL133]

More accurate unpolarized TMDs
for a self-consistent global extraction

MAPTMD22 [JHEP10(2022)127]

pionMAPTMD [PRD.107.014014]

Updated Sivers extraction

Additional data

hermes 2020
[JHEP12(2020)010]

COMPASS 2022
2015-2018
[PRL133]

Accurate unpolarized TMDs

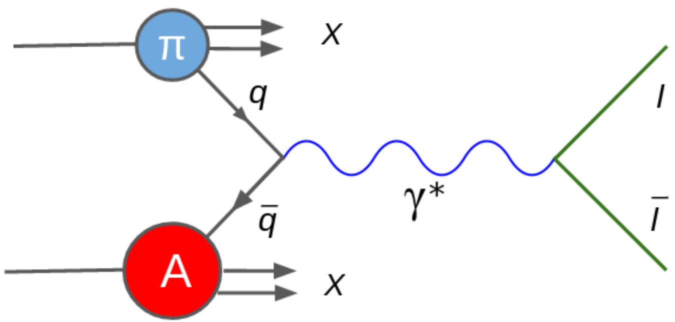
MAPTMD22
[JHEP10(2022)127]

pionMAPTMD
[PRD.107.014014]

Revised fitting framework

NangaParbat

Pion TMD extraction: MAP22Pion



accuracy

N^3LL

[PRD 107 (2023) 1, 014014]

$$\frac{d\sigma^{DY}}{d|\boldsymbol{q}_T|dydQ} \propto \int d|\boldsymbol{b}_T||\boldsymbol{b}_T|J_0(|\boldsymbol{q}_T||\boldsymbol{b}_T|)\hat{f}^{\,a}_{1\pi}(x_A, \boldsymbol{b}_T^2; \mu, \zeta_A)\hat{f}^{\,\bar{a}}_{1p}(x_B, \boldsymbol{b}_T^2; \mu, \zeta_B)$$

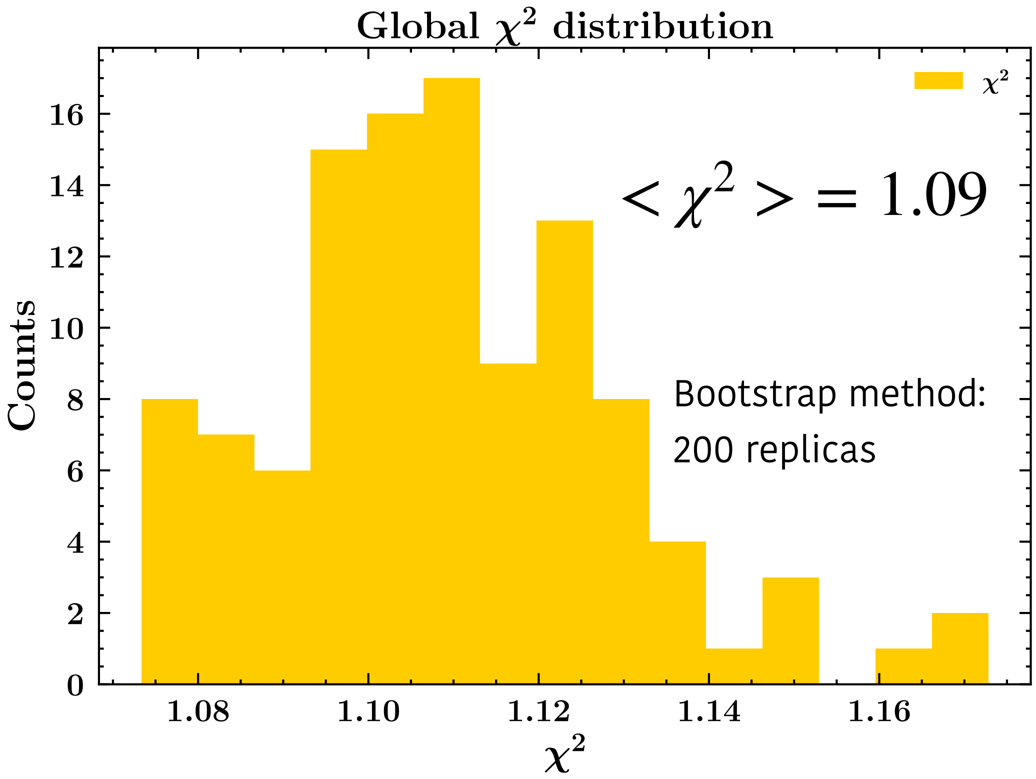
Updated MAP Sivers Fit results

Total number of data **364**

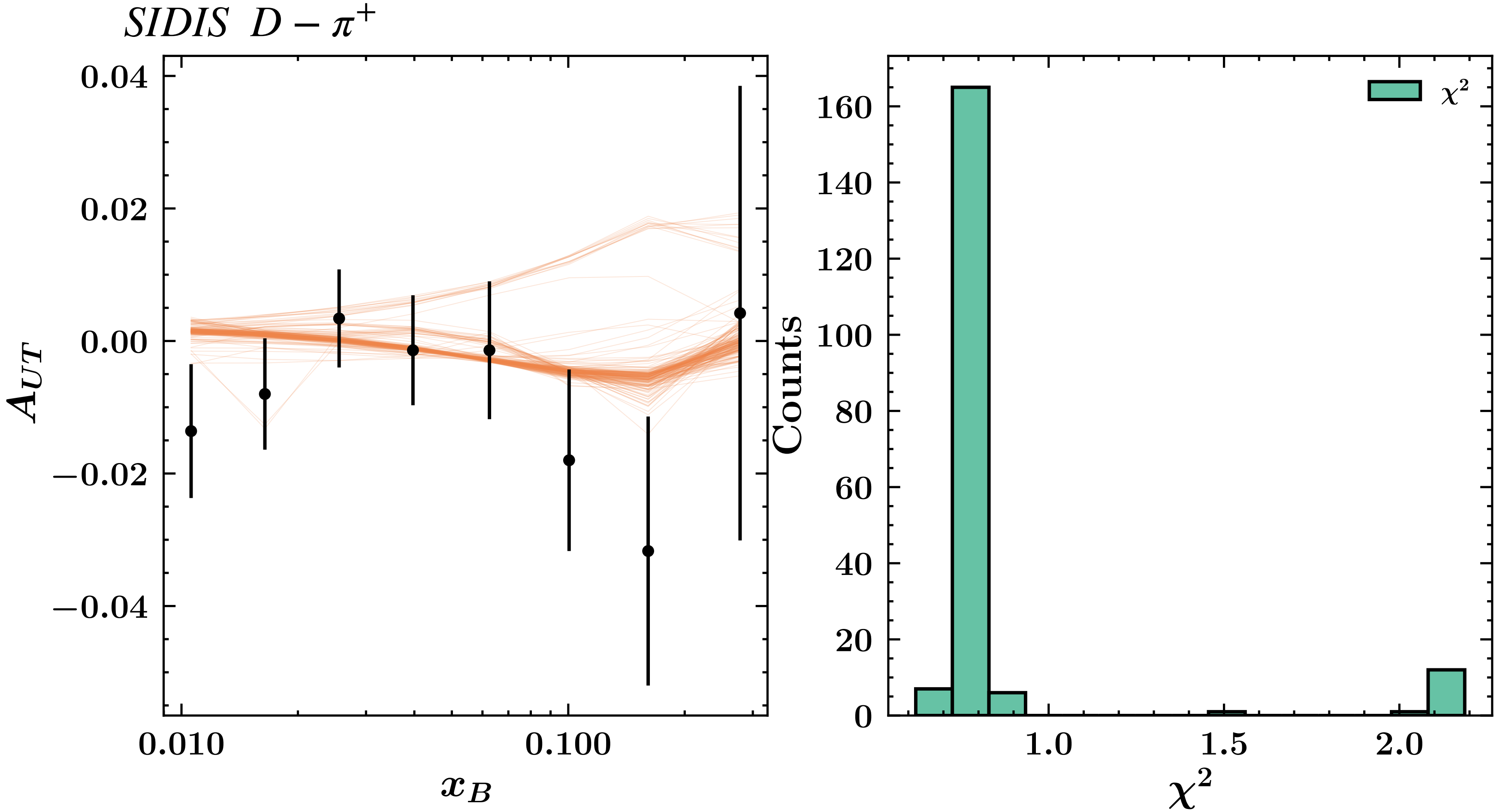
global fit from

- semi-inclusive DIS,
- pion-induced Drell-Yan
- W-Z boson production

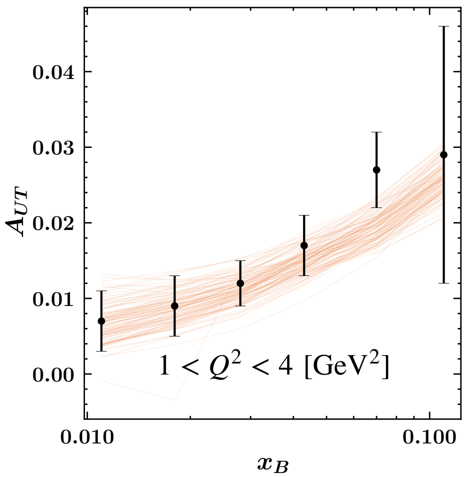
Accuracy: NLL-LO



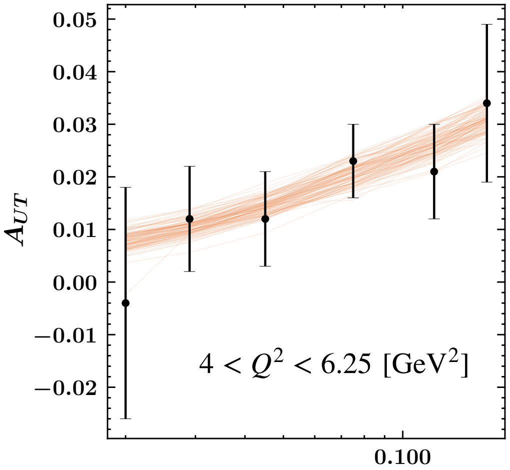
Selected results: Compass 2009

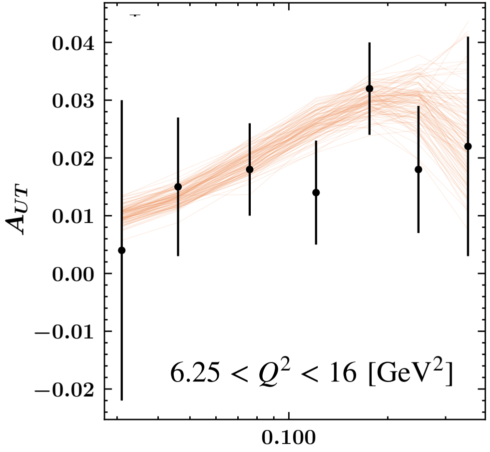


Selected results: Compass 2017 h^+

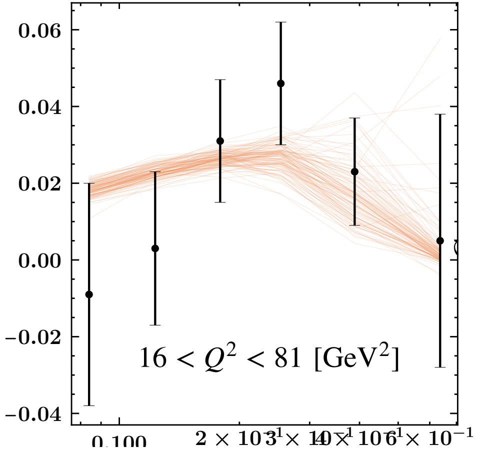


$$SIDIS \;\; p \to h^+$$





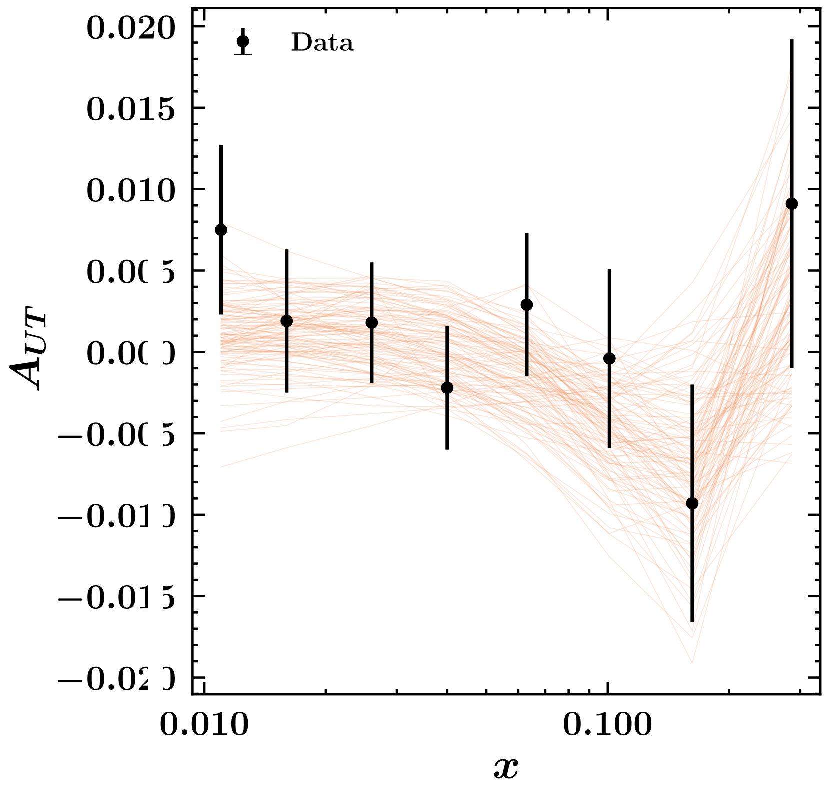
Similar results for $p \to h^-$

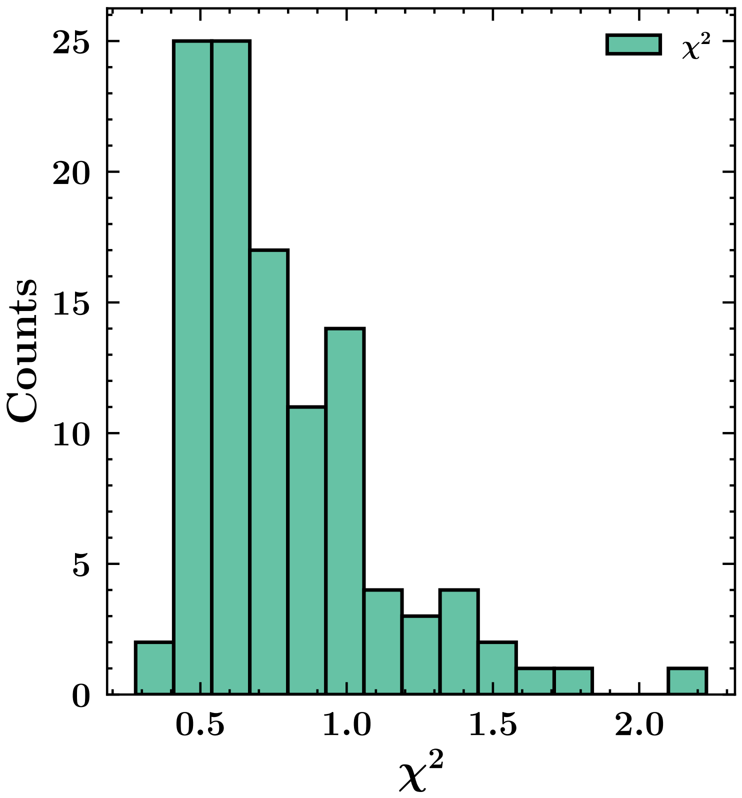


Selected results: COMPASS 2022

$SIDIS\ \ D-h^{+}$

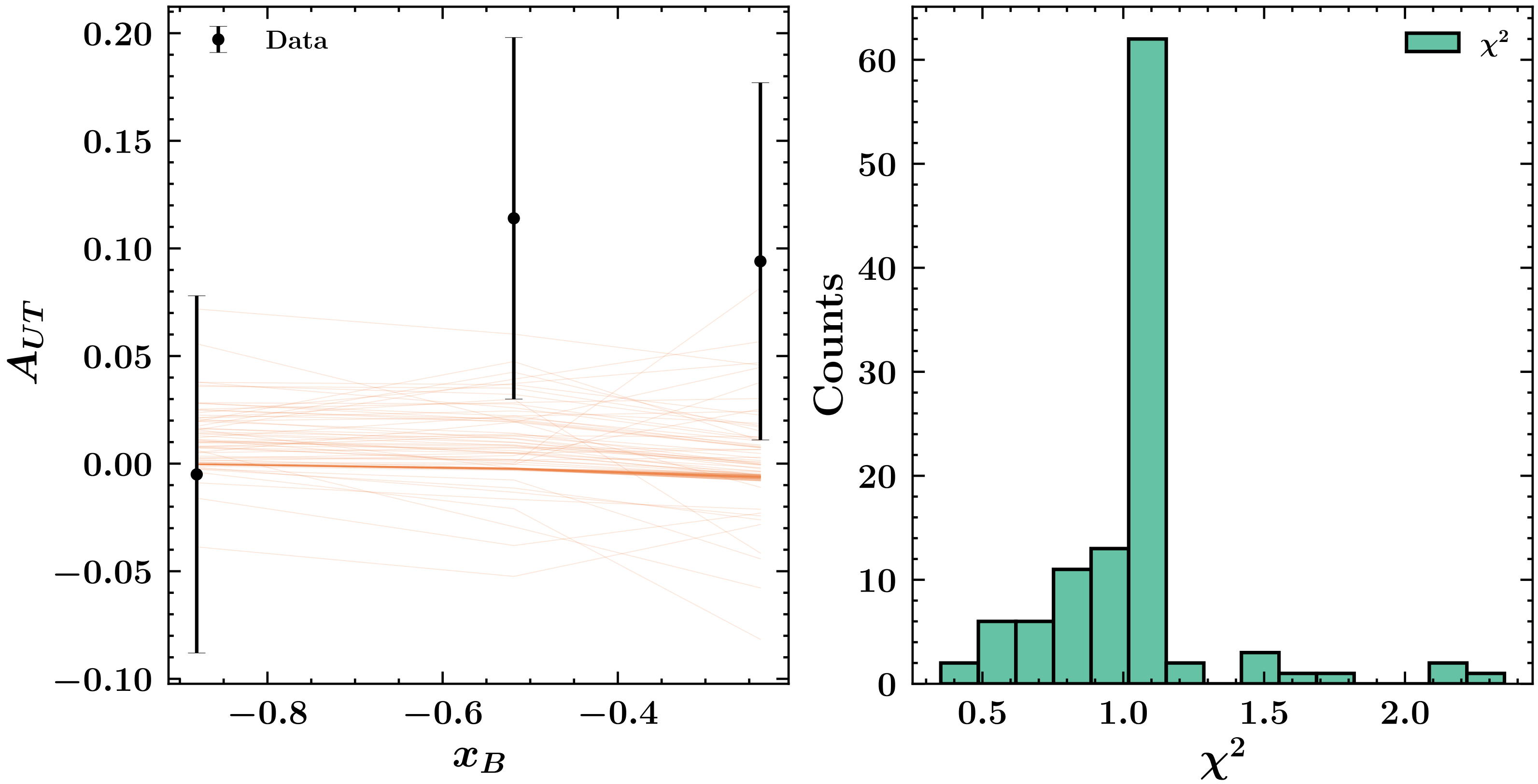
$\mu\ \ beam$



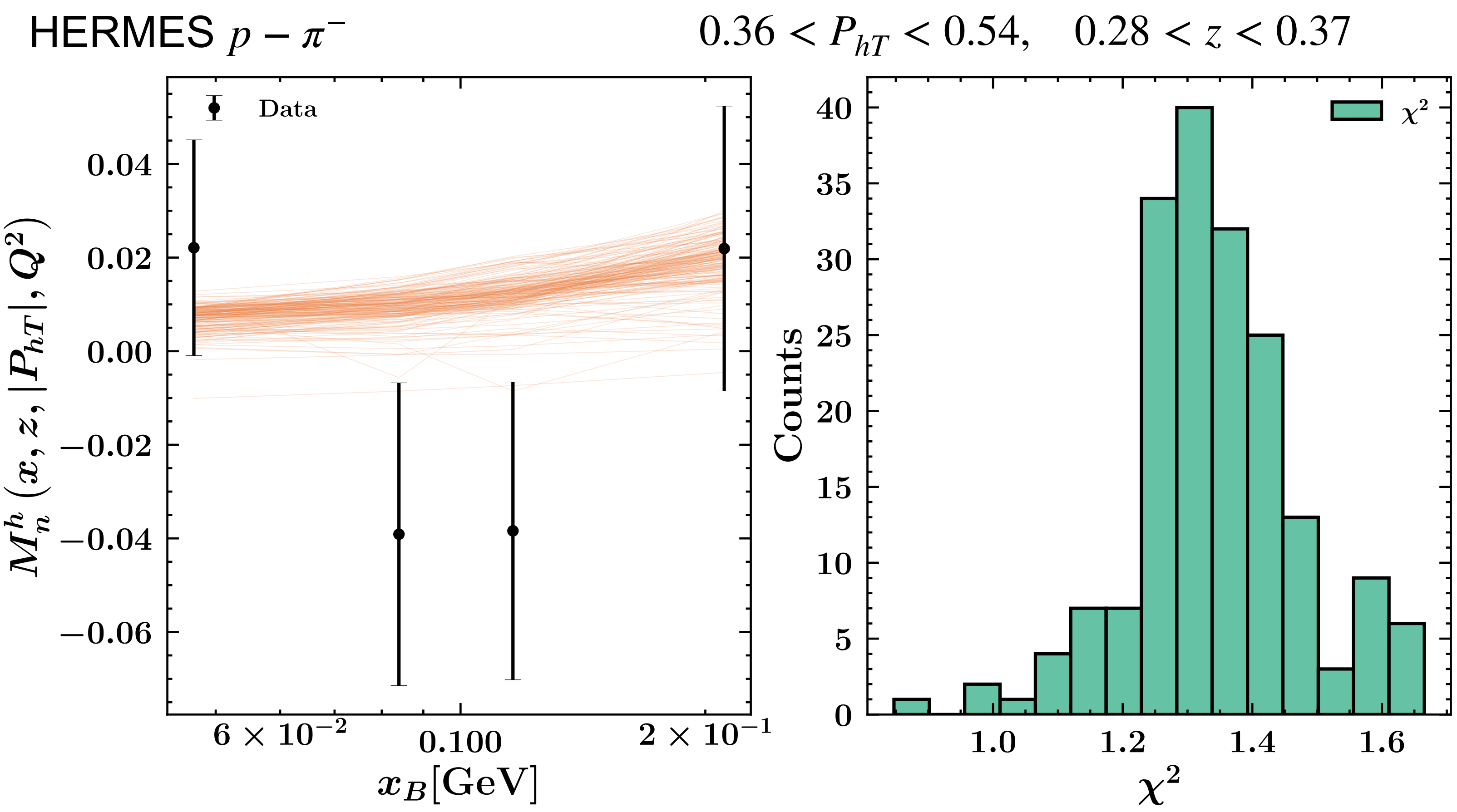


Selected results: COMPASS DY 2015

Pion-induced DY



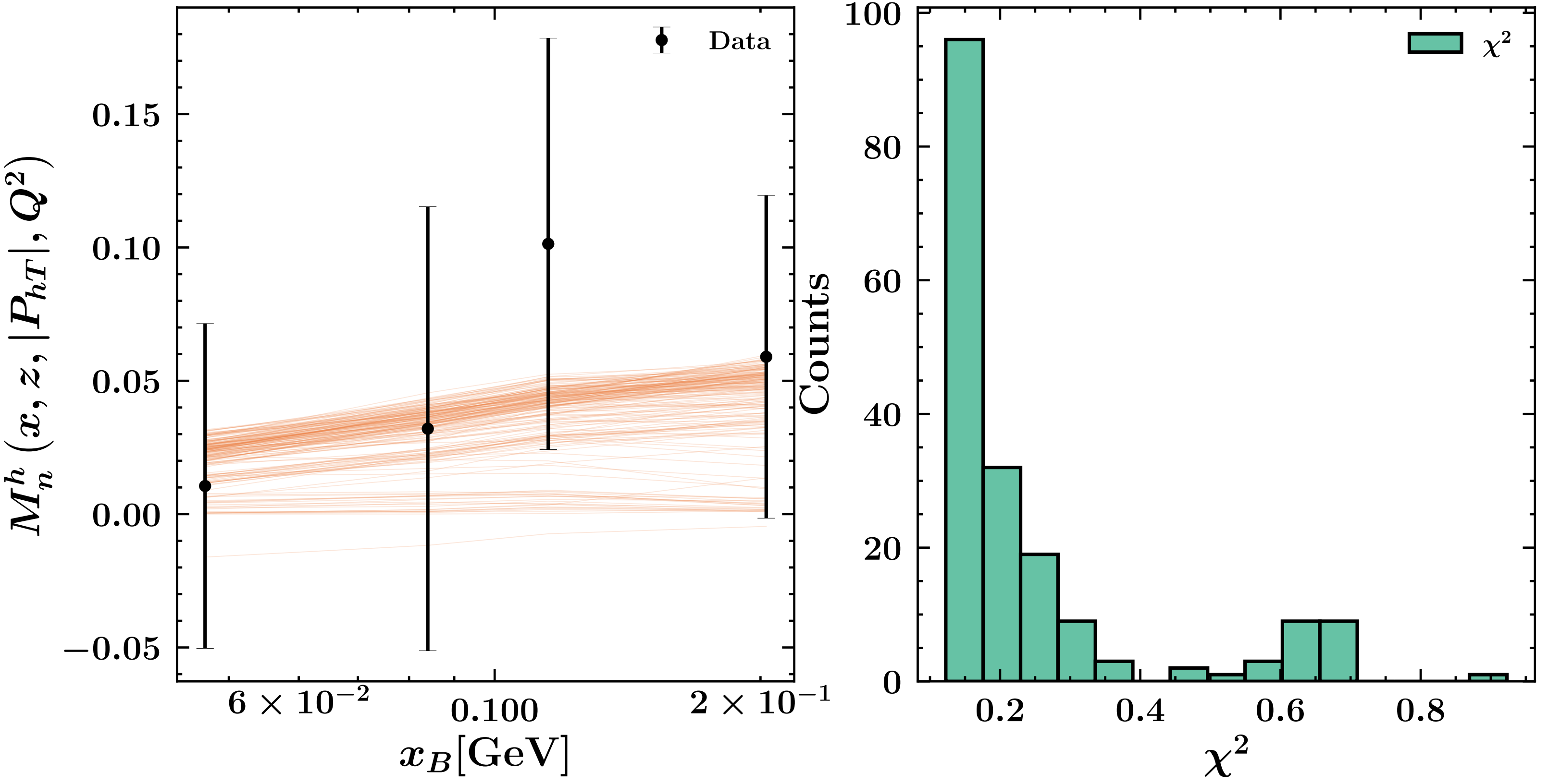
Selected results: HERMES 2020



Selected results: HERMES 2020

HERMES $p - K^+$

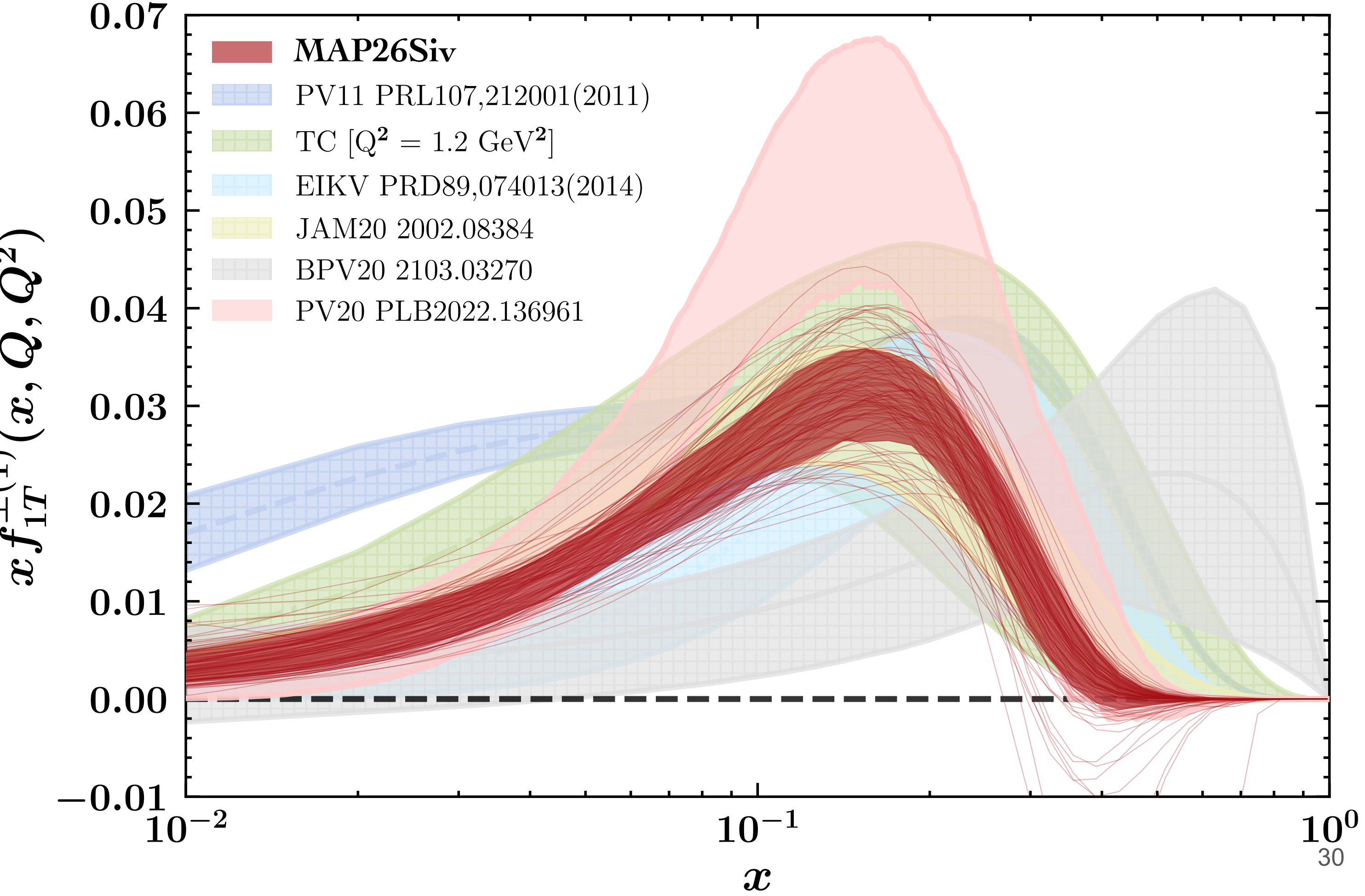
$0.36 < P_{hT} < 0.54, \quad 0.28 < z < 0.37$



Results on Sivers TMD first moment

down quark

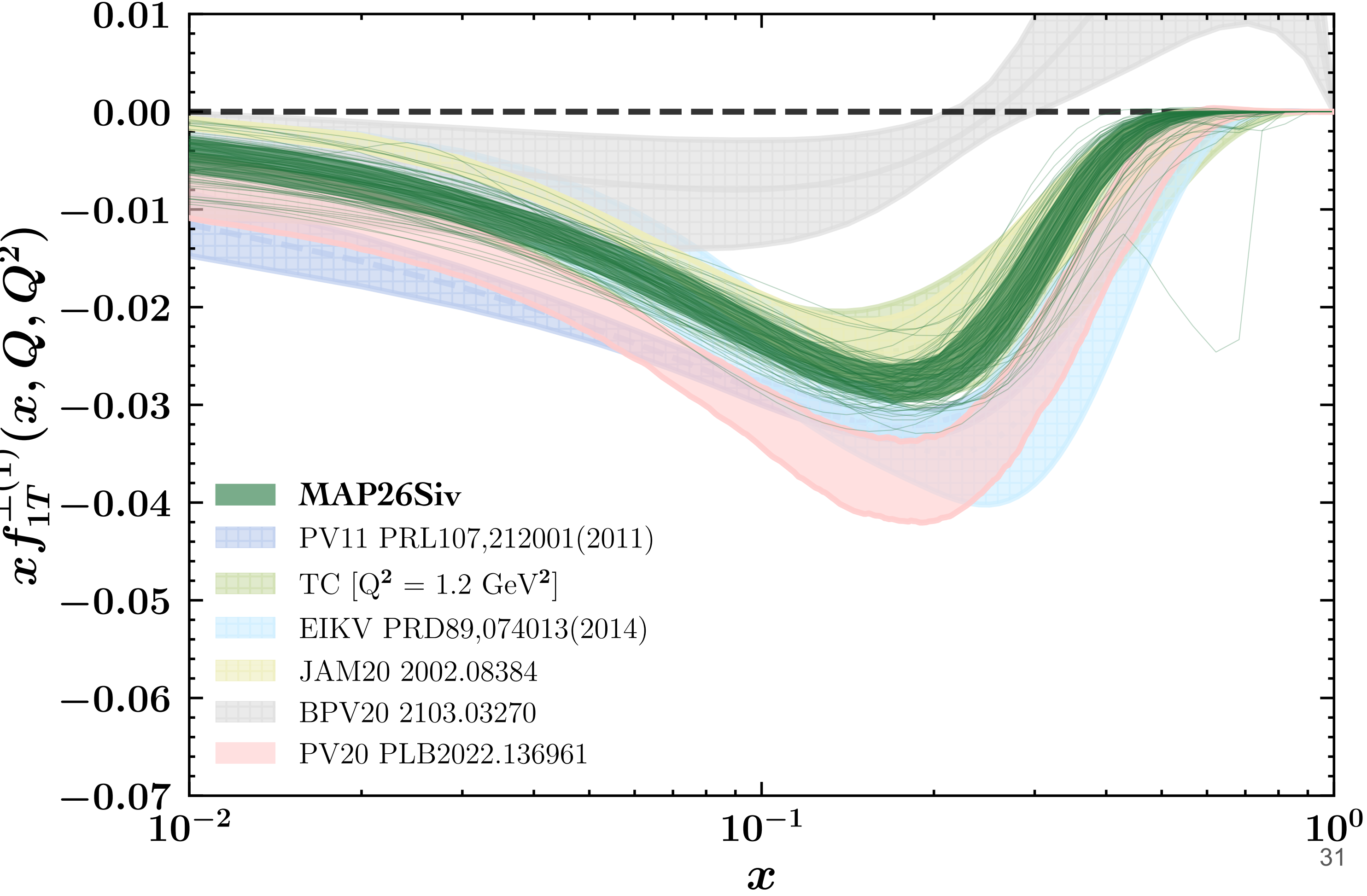
$Q^2 = 4\,GeV^2$



Results on Sivers TMD first moment

up quark

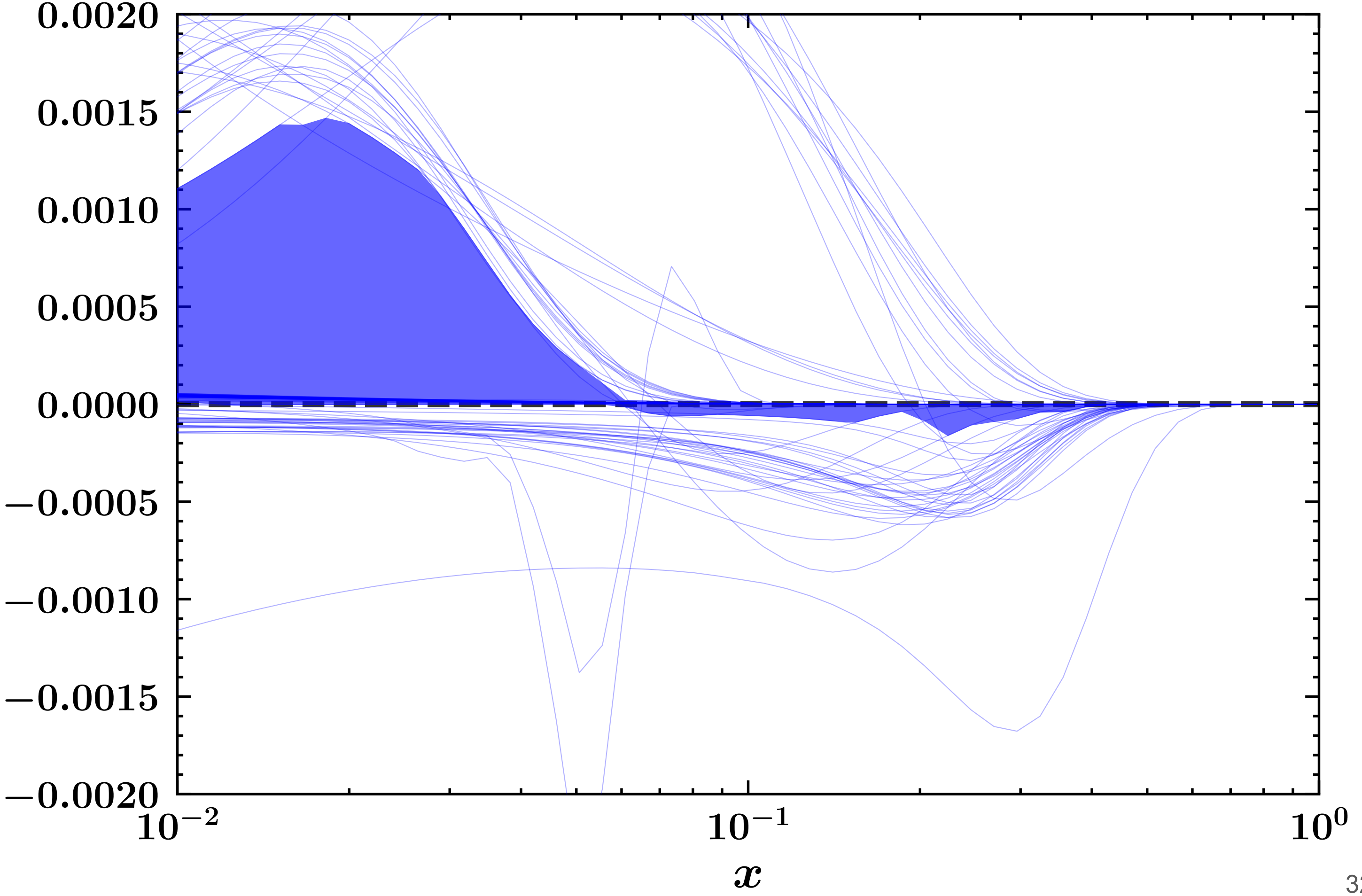
$Q^2 = 4\,GeV^2$



Results on Sivers TMD first moment

sea quark

$Q^2 = 4\,GeV^2$



Testing evolution frameworks: EKT

[2009.10710]

M.G. Echevarria, Z-B. Kang, J. Terry

At large *x*, transverse spin dynamics leads to a modification to the quark to quark splitting kernel

$$P^T_{q\leftarrow q}(x) = P_{q\leftarrow q} - N_C\,\delta(1-x)$$

standard quark to quark splitting kernel for unpolarized PDFs,

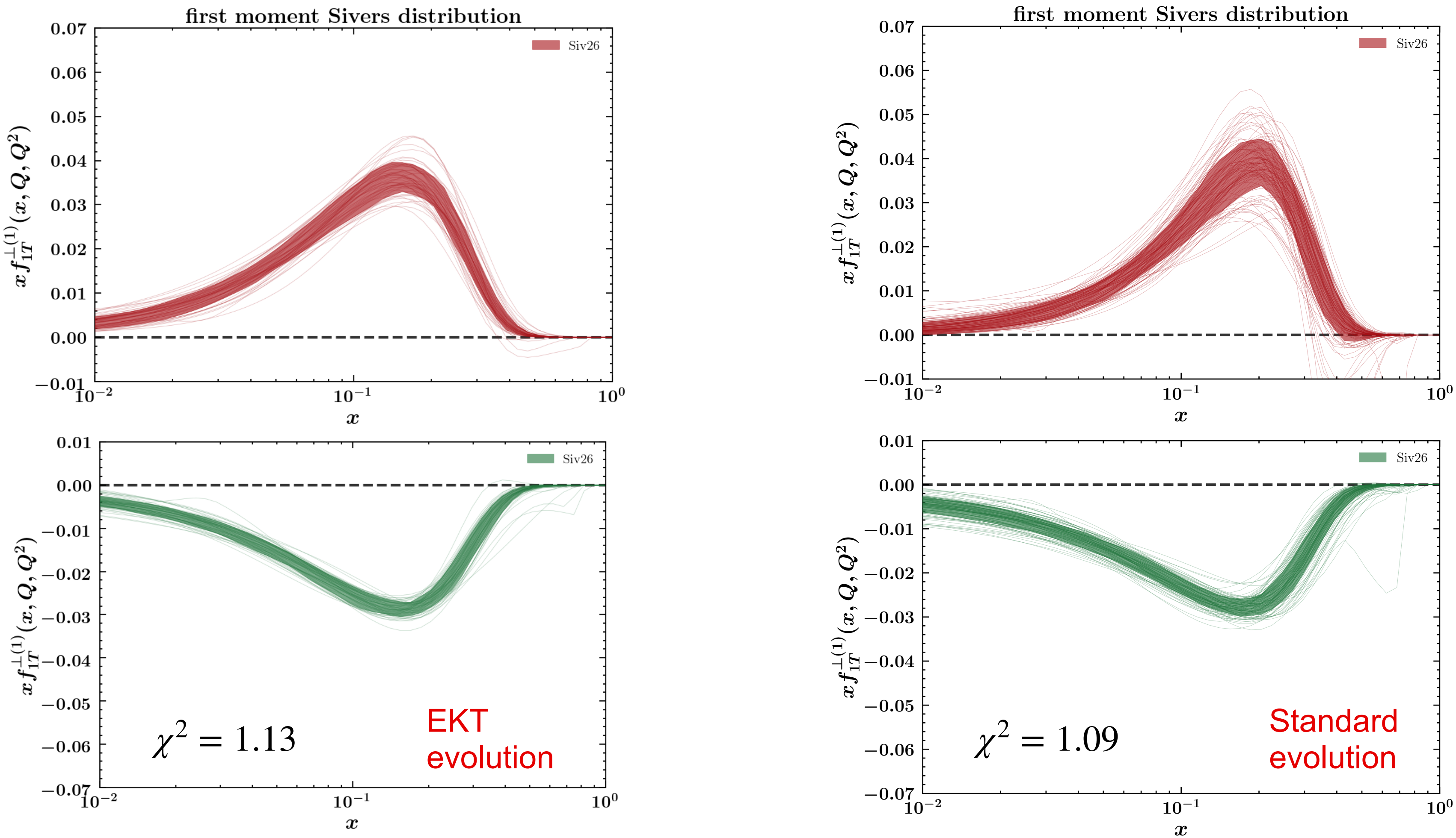
$$P_{q\leftarrow q}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]$$

> Modified evolution applied to the Sivers first moment

> Kept the standard DGLAP evolution for collinear PDFs

Changes to splitting functions in apfel++

Testing evolution frameworks: comparison



Conclusions

- Updated MAP global extraction of Sivers function with a fully consistent TMD framework, connecting different datasets, energies, processes
- New data from COMPASS, Hermes, STAR help explore new phenomenological aspects of polarized TMDs (multi dim., DY)
- New datasets confirm previous works, allowing more flexibility