The Longitudinal Structure function in SIDIS: from small to large P_T

Leonard Gamberg May 12 2026
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Preamble Motivation

Owing to the *kinematic* & *dynamical* mechanisms driving TMD SIDIS, the extraction of the independent structure functions from measured cross sections—& their interpretation in terms of the nucleon's nonperturbative partonic structure presents significant experimental and phenomenological challenges.

A quantitatively reliable description requires among other things

- consistent control of power corrections,
- QED radiative effects,
- & the matching between the small & large-$P_{h\perp}$ regimes within QCD factorization.

Regarding the latter, a unified treatment of

- low-$P_{h\perp}$ region governed by transverse momentum factorization, &
- high-$P_{h\perp}$region described by collinear factorization,

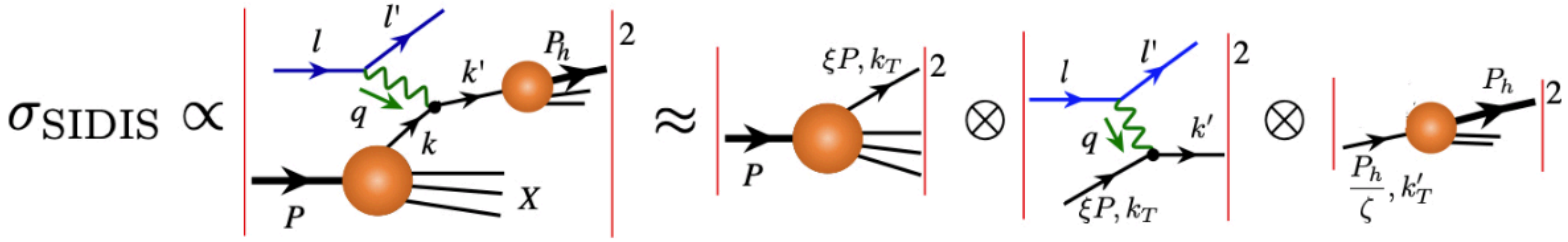
necessitates a controlled implementation of small & large--transverse-momentum matching " $W + Y$ "

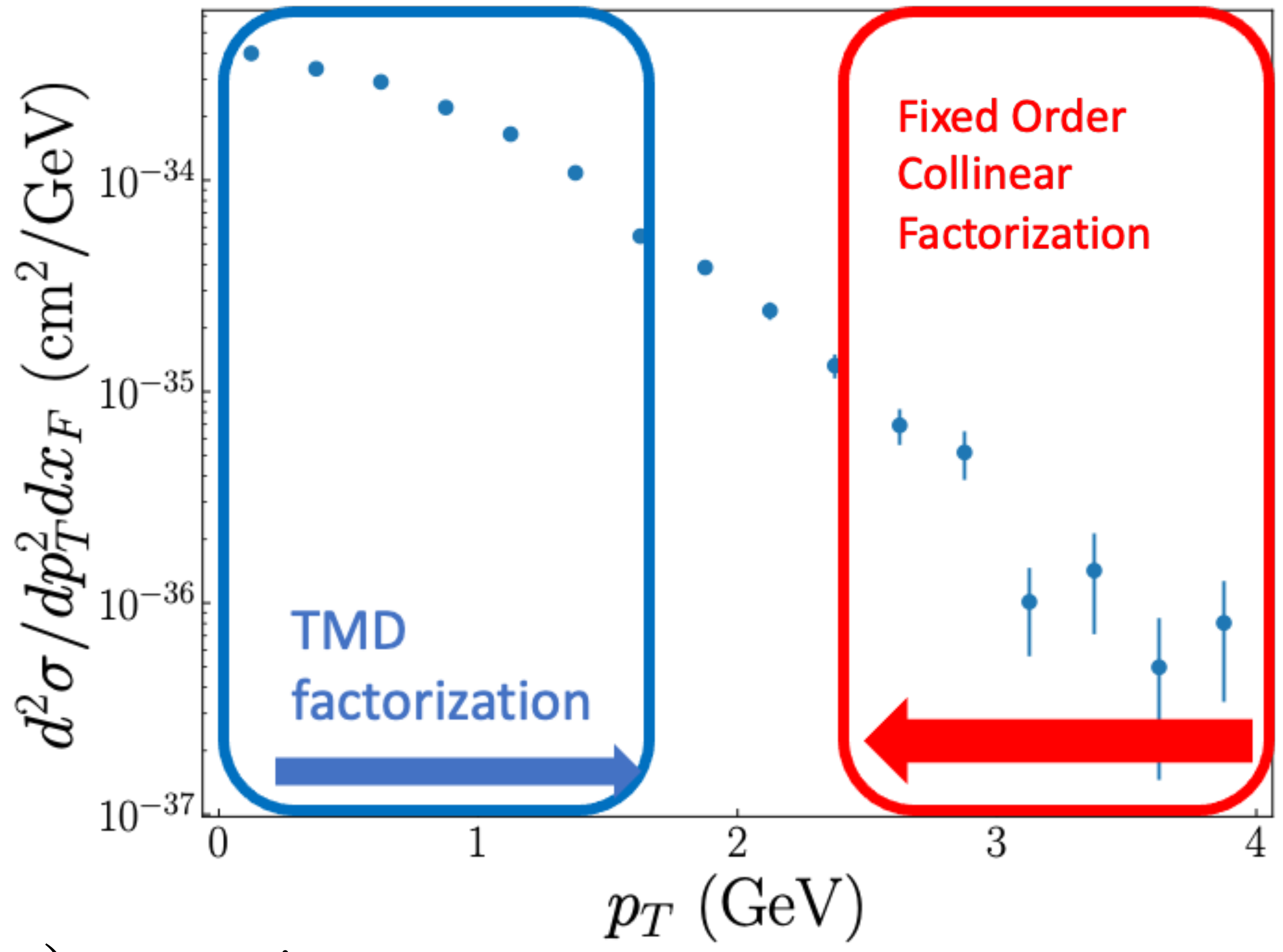
Preamble

Entails discussion factorization: TMDs & collinear factorization

- Collinear $M \ll q_T \sim Q$
- TMD $M \sim q_T \ll Q$

$$E'E_h \frac{d\sigma_{ep\to e'hX}}{d^3 l' d^3 P_h} \approx \hat{\sigma}_{eq\to e'q'} \otimes f_1 \widetilde{\otimes} D_{h/q'}.$$

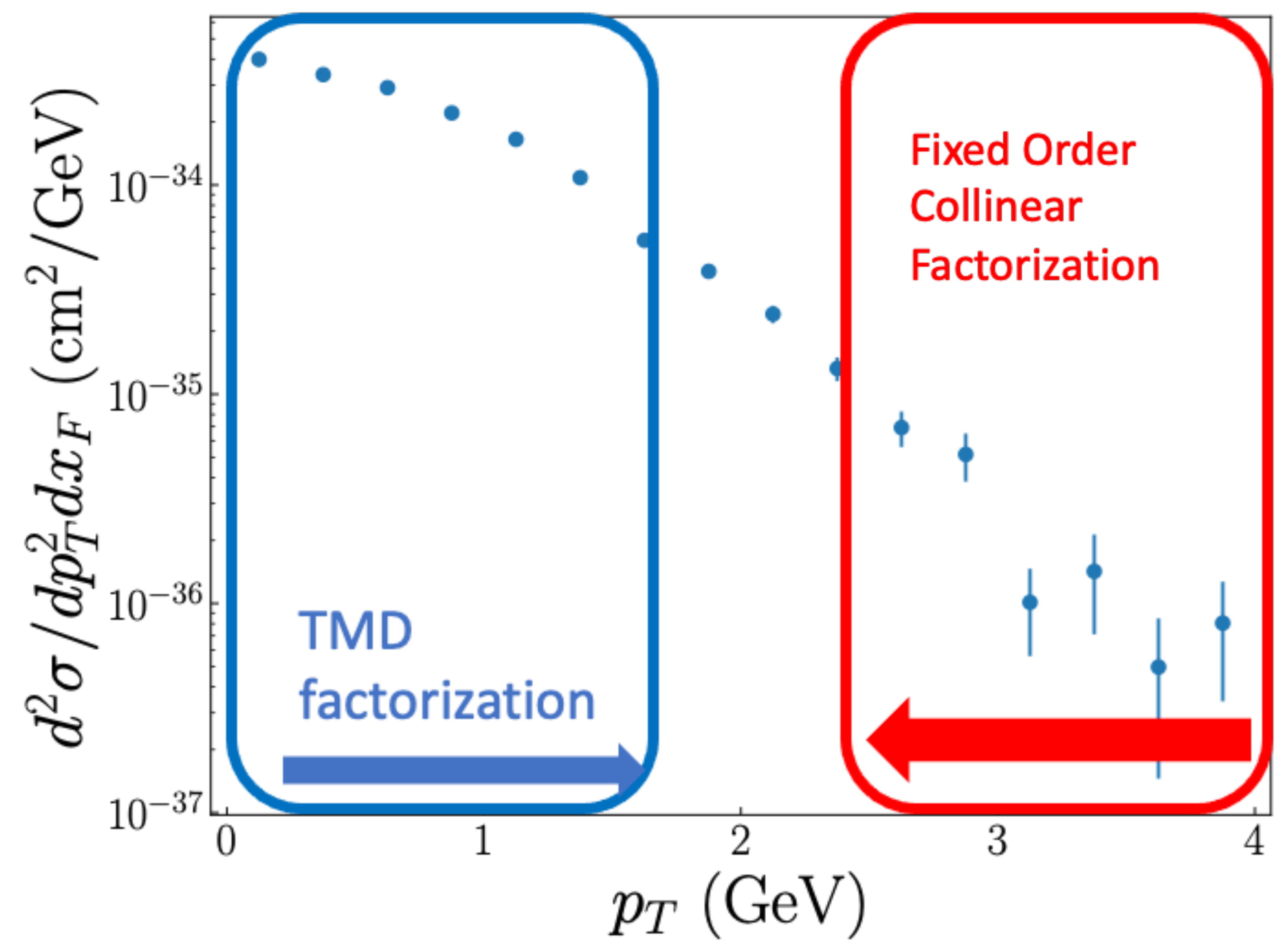




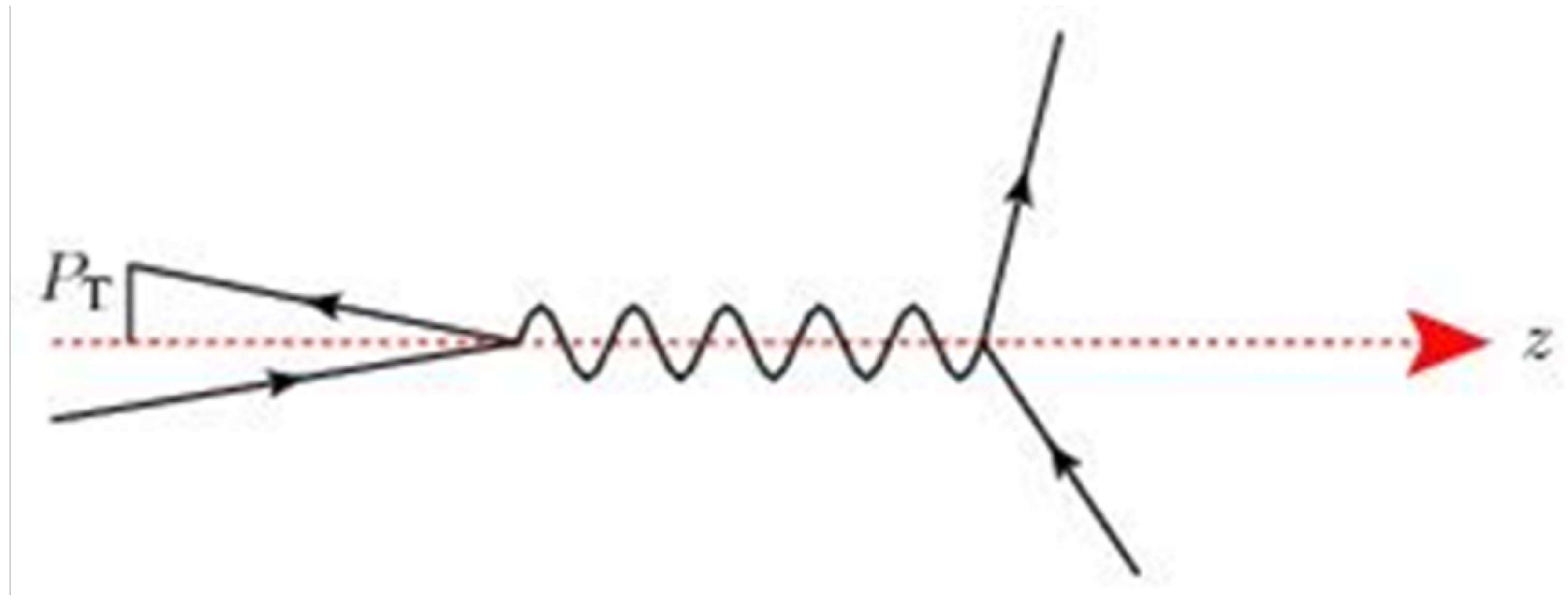
1. $\Longrightarrow$ Matching "low" (TMD) to "high" (collinear) P_T (or q_T) spectrum

CSS formalism Catani et. al match in “AY” region

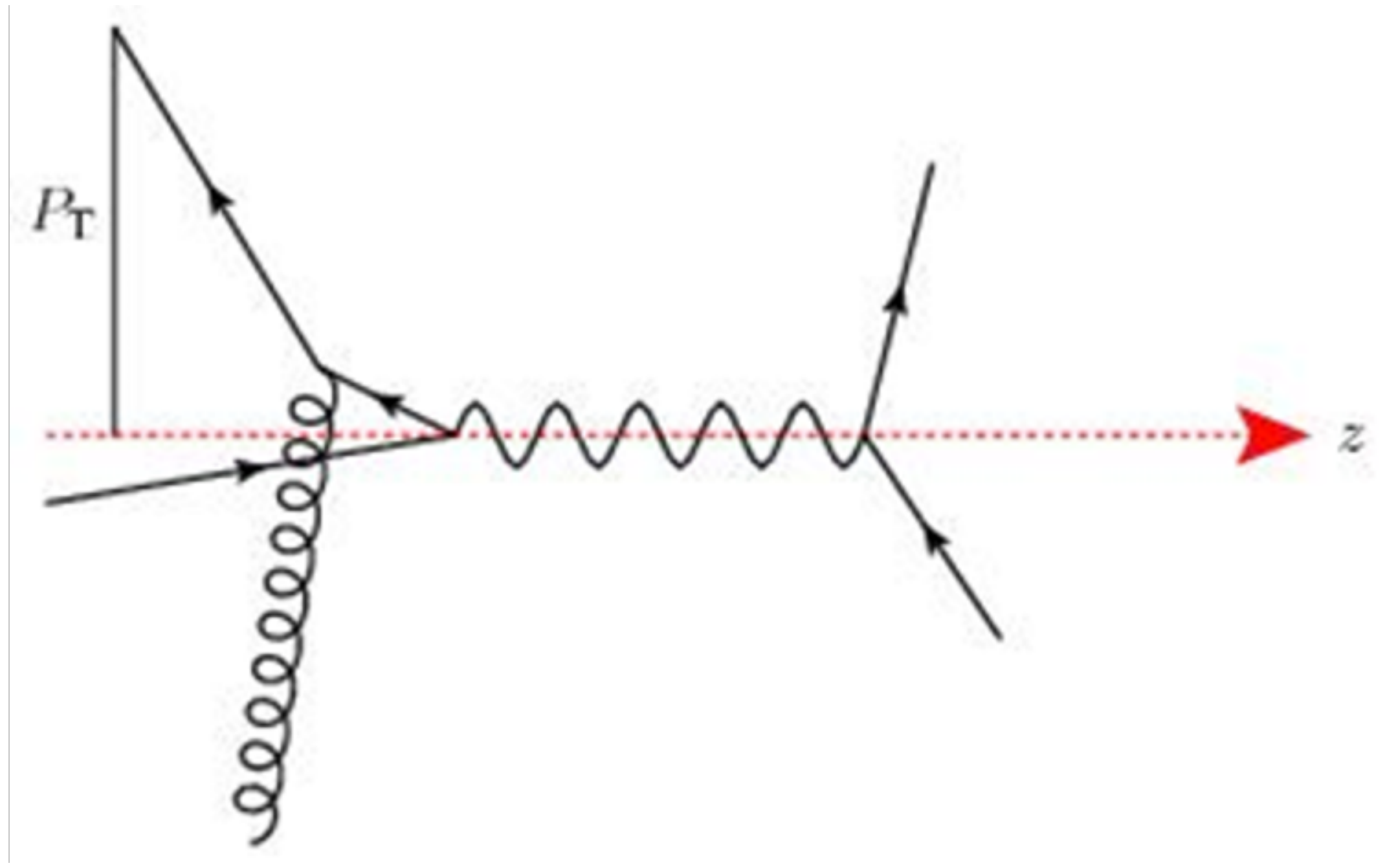
Scale separation: $F = \mathrm{W} + \mathrm{FO} - \mathrm{ASY}$



W term probes *intrinsic* transverse motion



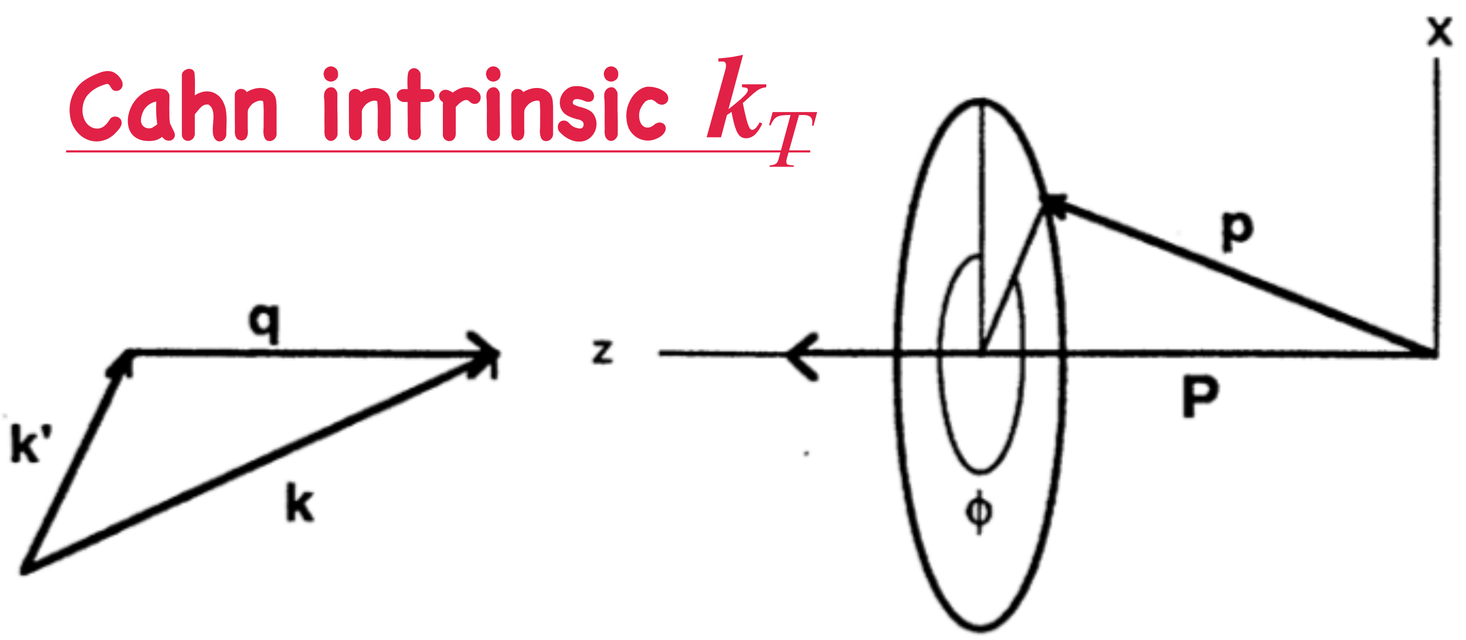
FO captures *hard* radiation



Two mechanisms ? *Matching* ...

Factorization & Matching collinear to TMD unpolarized/angle independent Collins Soper Sterman NPB 1985

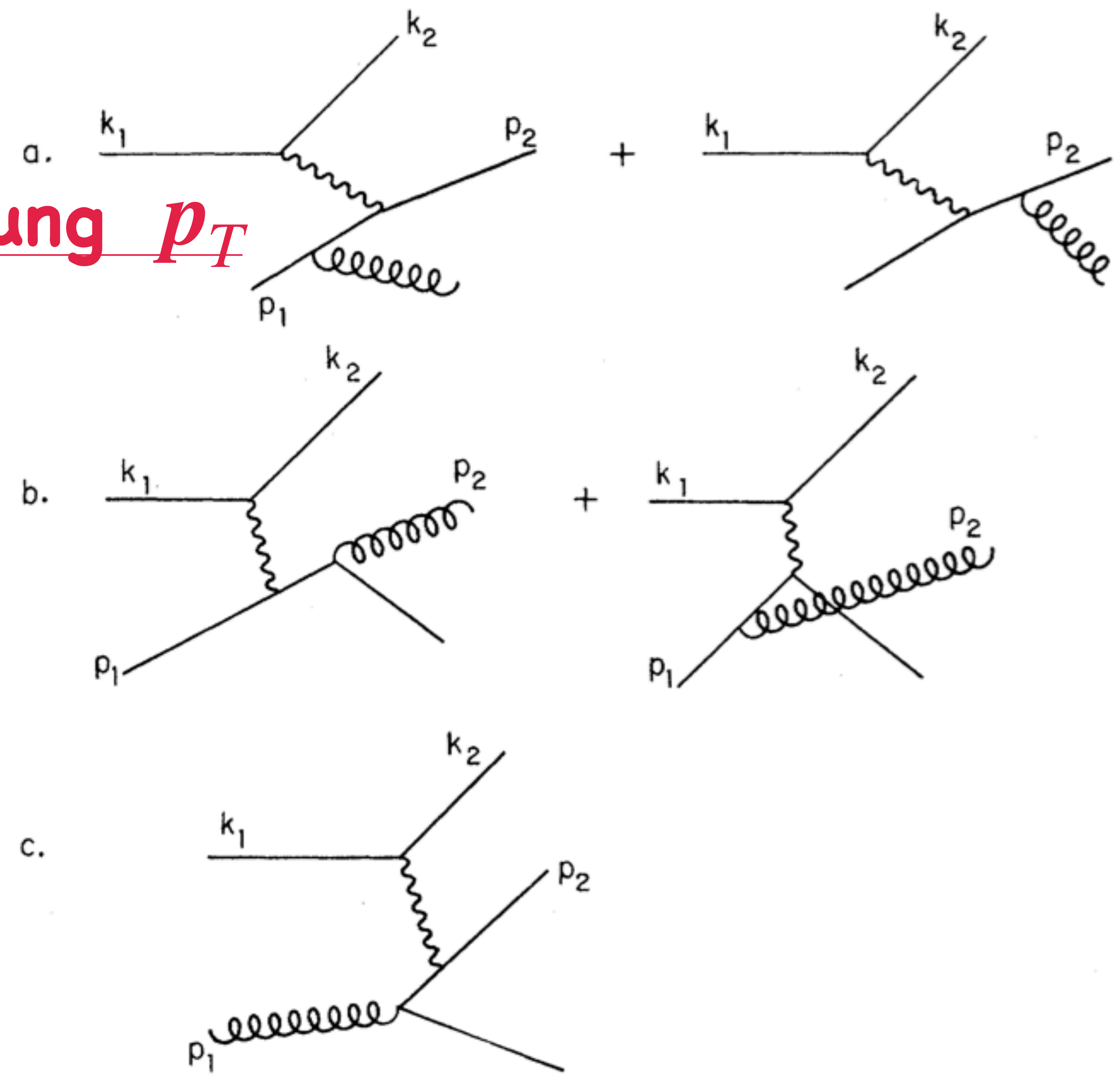
Cahn intrinsic k_T

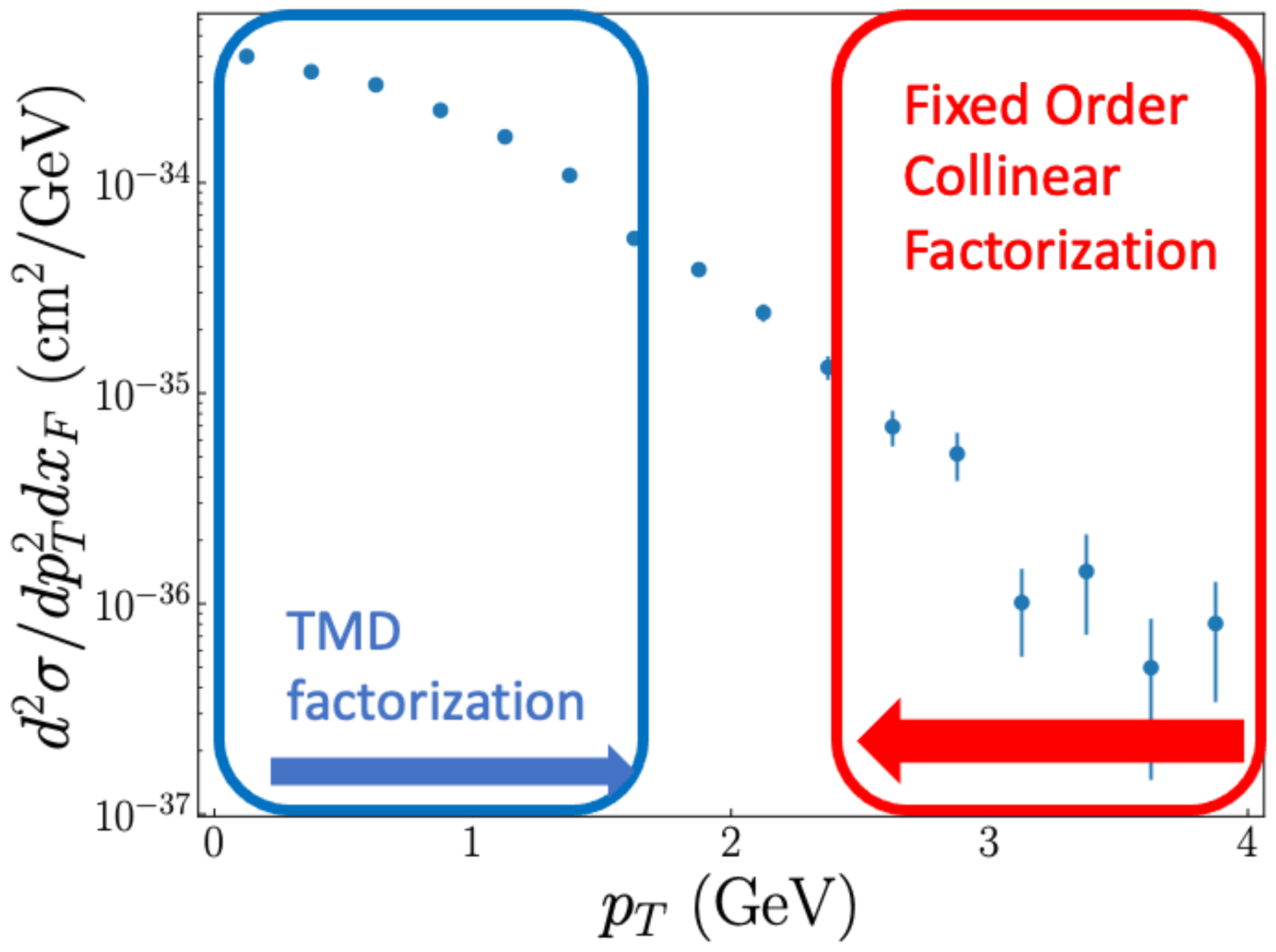


- **"TMD" region**

$(p_T \sim k_T) \sim q_T \ll Q$

Georgi & Politzer
hard gluon bremsstrahlung p_T





- **"Collinear " region**

$\Lambda_{qcd} \ll q_T \sim Q$

Comprehensive study of matching the hi & low Q_T in the AY (overlap) region in SIDIS was carried out by Bacchetta, Boer, Diehl, Mulders JHEP (2008) attention was given to azimuthal and polarization dependence

Preamble

- Within this context an important and comparatively underexplored ingredient is the contribution of longitudinally polarized virtual photons to the unpolarized SIDIS cross section from $F_{UU,L}$

- Esp. given the substantial difficulty of describing the unpolarized SIDIS cross section across the full transverse-momentum range, our limited quantitative understanding of longitudinal photon contributions introduces additional systematic uncertainties.

- These could directly impact precision extractions of unpolarized TMDs which to reliably constrain without dedicated measurements and an improved phenomenological treatment.

- Therefore imperative to extend global SIDIS analyses to incorporate longitudinal photon effects in a consistent factorization framework spanning the entire transverse-momentum range.

$$M^h(Q^2, x, z, P_{h\perp}) = \frac{F_{UU,T} + \epsilon F_{UU,L}}{F_T + \epsilon F_L}$$

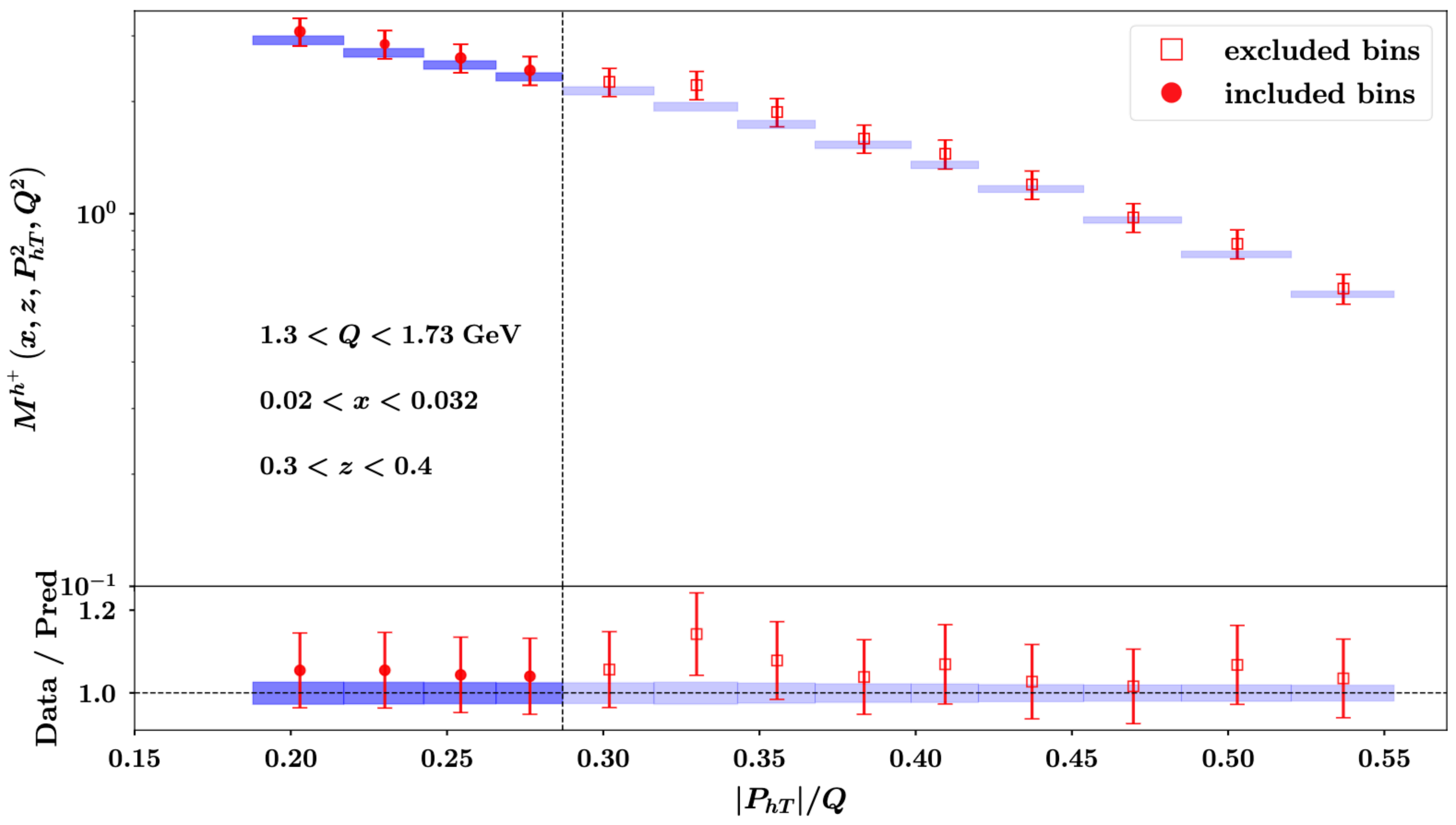
Preamble

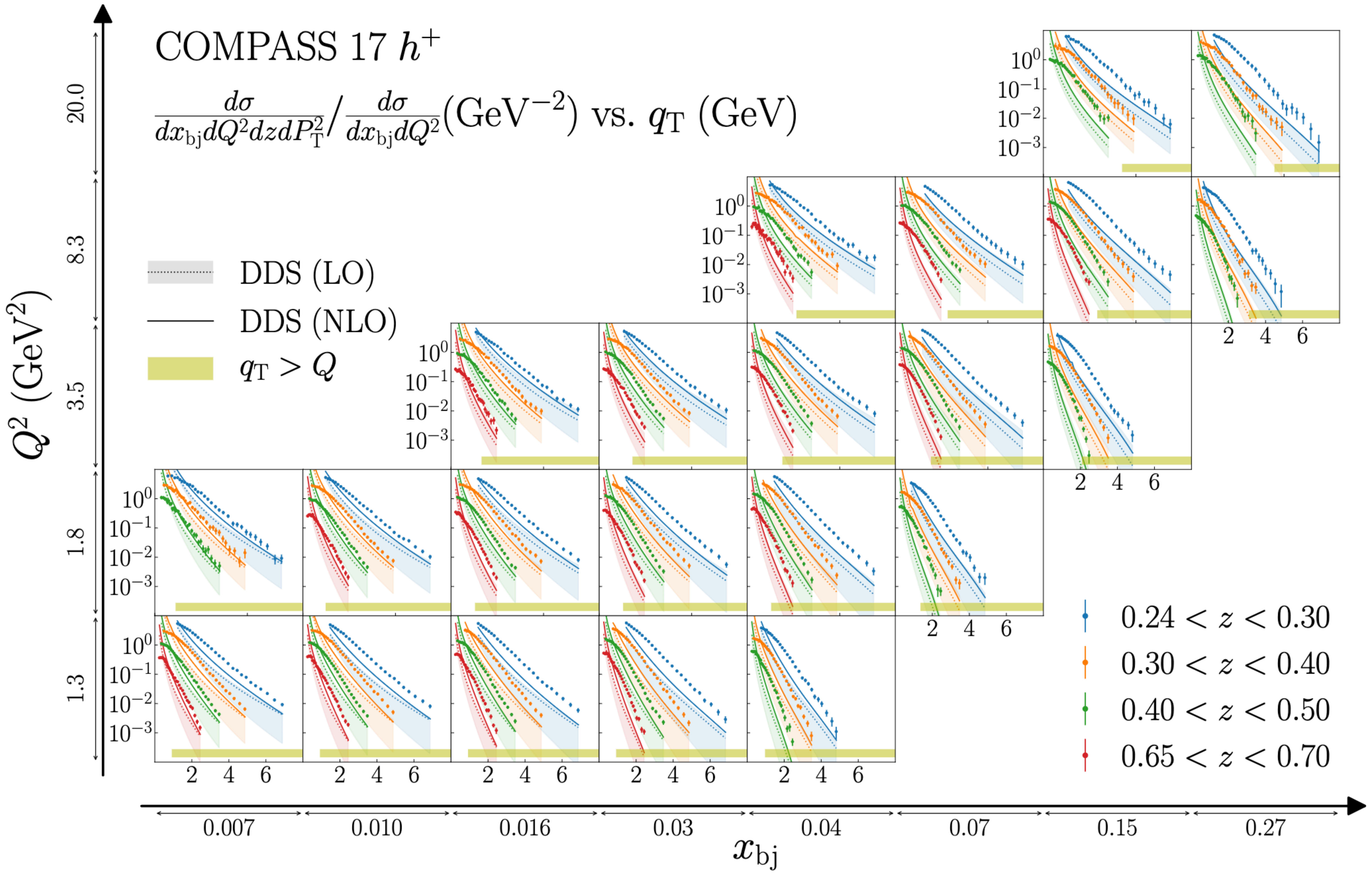
Multiplicity: SIDIS cross section normalized to DIS

$$M^h(Q^2, x, z, P_{h\perp}) \equiv \frac{dxdQ^2}{d^2\sigma^{\mathrm{DIS}}(Q^2, x)} \frac{d^4\sigma^h(Q^2, x, z, P_{h\perp})}{dxdQ^2dzdP_{h\perp}}$$

Gonzalez-Hernandez et. al., arXiv:1808.04396v2 [hep-ph] (2018)

MAP Bacchetta et. al., arXiv:2206.07598v2 [hep-ph] (2022)





W term describes “large” P_{hT} data surprisingly well ?

FO theory does not describe “large” P_{hT} data so well

Preamble

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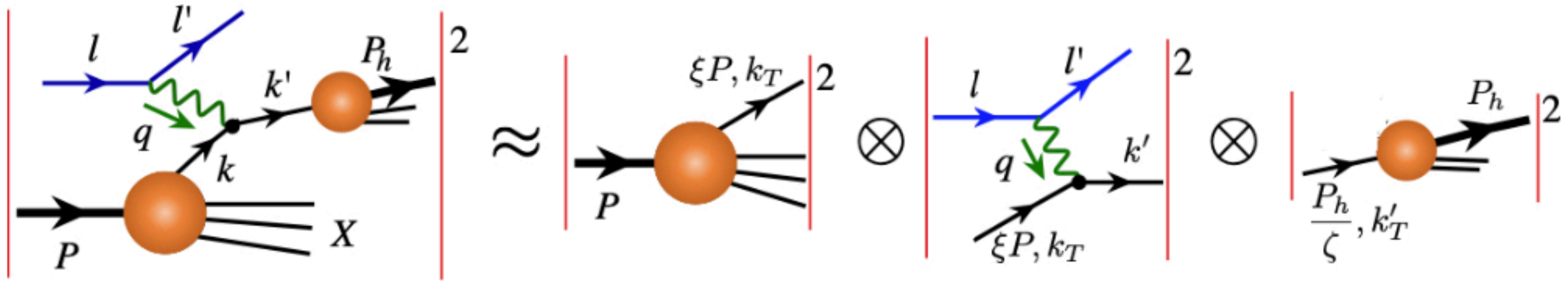
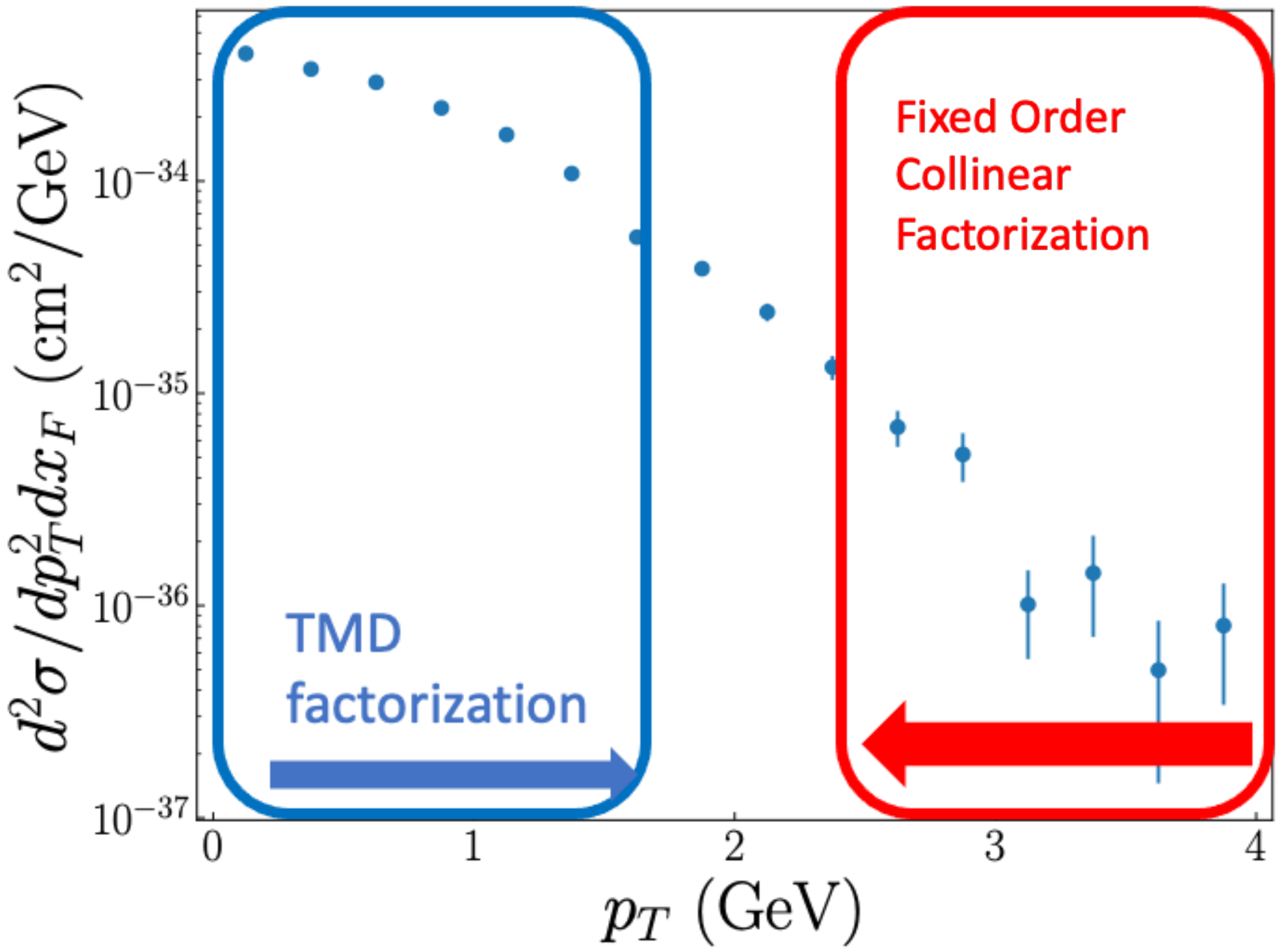
$\sigma_{\rm SIDIS} \propto$ 



1. $\Longrightarrow$Matching "low" (TMD) to "high" (collinear) P_T (or q_T) spectrum
2. Discussion: power counting & $\exists$ a resummation formalism generaized CSS?
3. Challenge of Factorization at LP NLP NNLP in the hard scale Q

*First prelim SIDIS studies beyond tree level:
"Matches & Mis-matches" Bacchetta et al. JHEP 2008, Chen & Ma PLB 2017

Literature
Bacchetta et al. PLB 2019
MIT group, Gao, Ebert, Stewart JHEP 2022
Gamberg, Kang, Shao, Terry, Zhao arXiv:221.13209 … & …Page Gamberg Kang Stasto 2026…
Madrid group Vladimirov, Rodini, Scimemi, Piloñeta et al JHEP 2021, 2022, 2023, 2024 2025
Balitsky 2023 rapidity TMD evolution Balitsky & Prokudin 2026

TMD Collaboration

Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

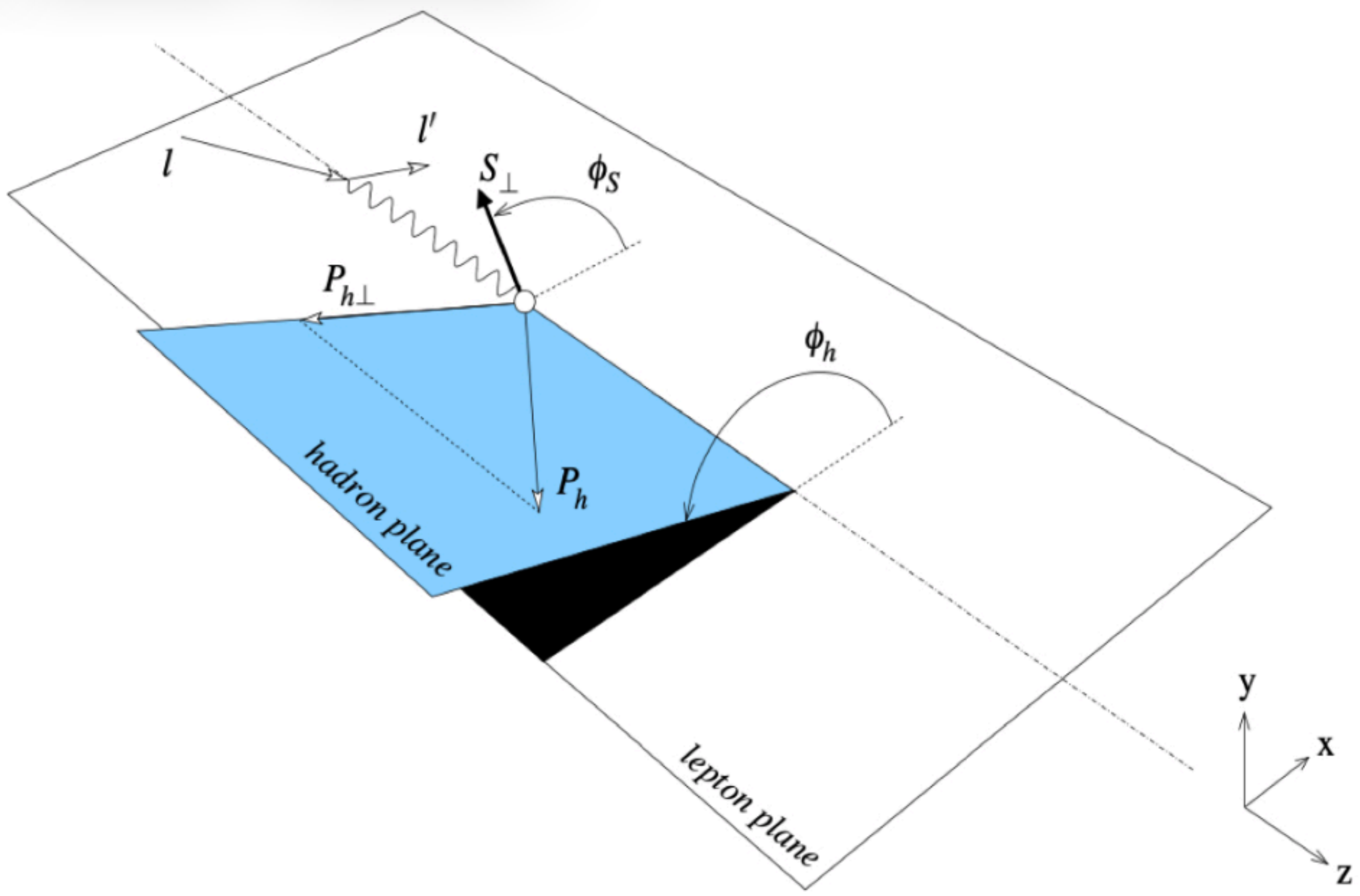
TMD Handbook

Advertisement

L. Gamberg, A. Metz, I. Stewart
w/ Tom Mehen

Focus on two related NNLP unpol. observables

- $F_{UU,L}$ & $R_{SIDIS} = \frac{\sigma_L}{\sigma_T} \sim \frac{F_{UU,L}}{F_{UU,T}}$



- $F_{UU}^{\cos\phi_h} \to \langle \cos\phi_h \rangle$ n.b. L/T inteference

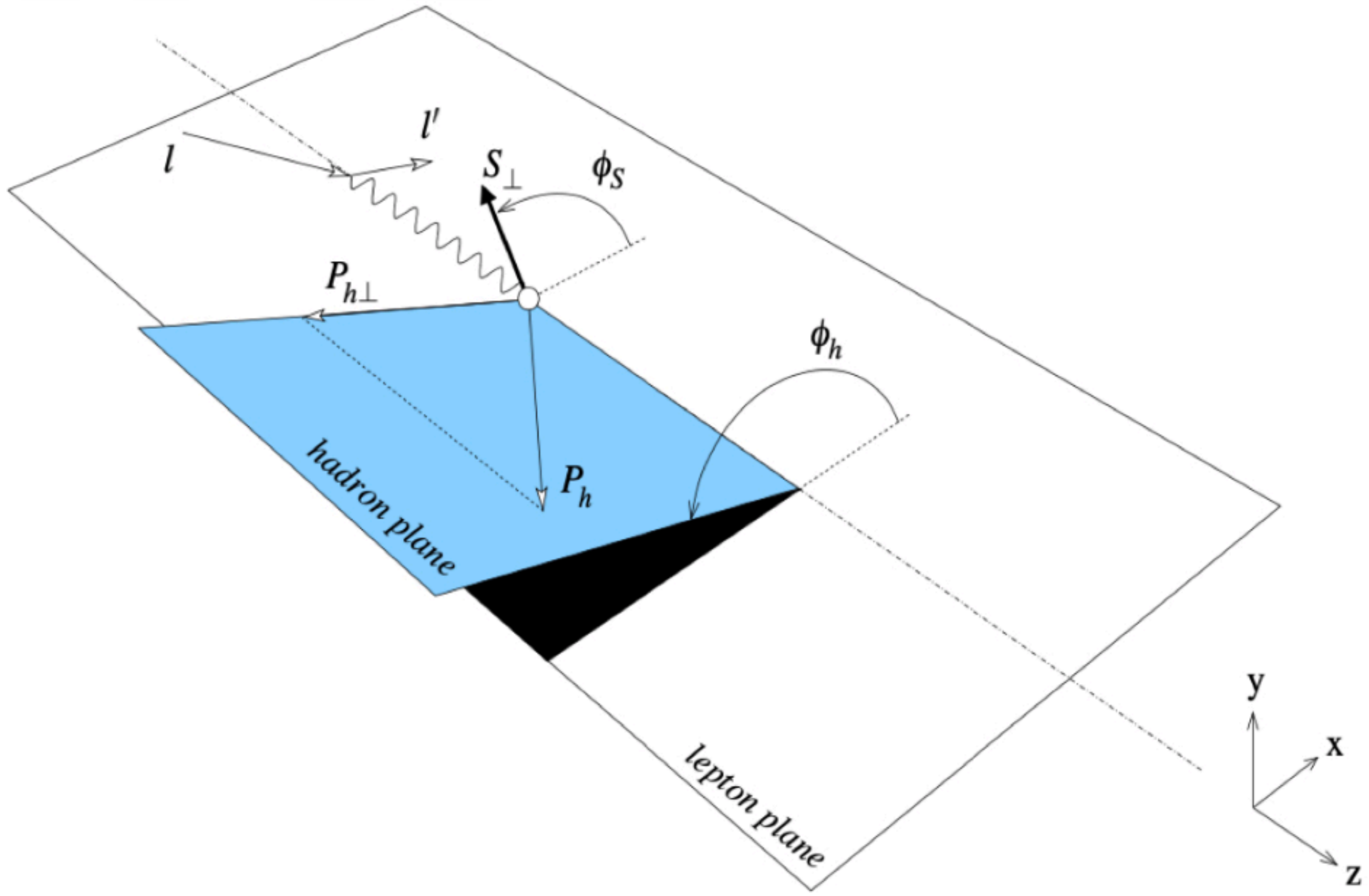
Georgi & Cahn, PRL 1978, PLB 1978

- Critique of perturbative QCD calculation of azimuthal dependence in SIDIS
- Emphasize importance intrinsic k_T the early days/birth of "TMD physics"

Focus on two related NNLP unpol. observables

- $F_{UU,L}$ & $R_{SIDIS} = \frac{\sigma_L}{\sigma_T} \sim \frac{F_{UU,L}}{F_{UU,T}}$

Feynman "Photon-Hadron Phys." 1972, Ravndal, PLB 1973



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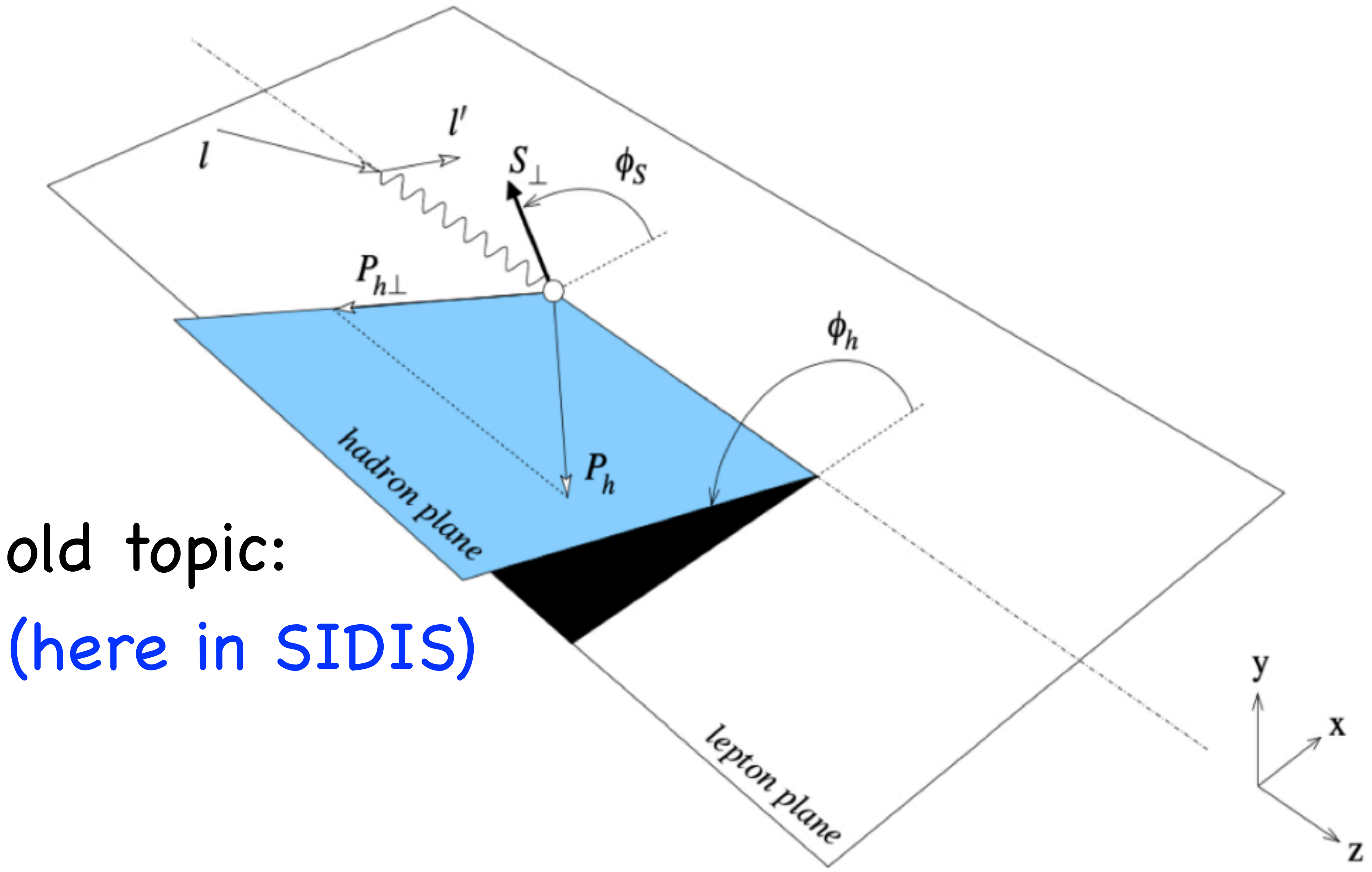

Feynman "Photon-Hadron Phys." 1972, Ravndal, PLB 1973

- Interesting opportunity to study hadron structure, age old topic:
  Ratio of longitudinal and transverse $\gamma^*_{T/L}$ cross section (here in SIDIS)
  classic power supressed $\sim (1/Q)^2$ in DIS …

Focus on two related NNLP unpol. observables

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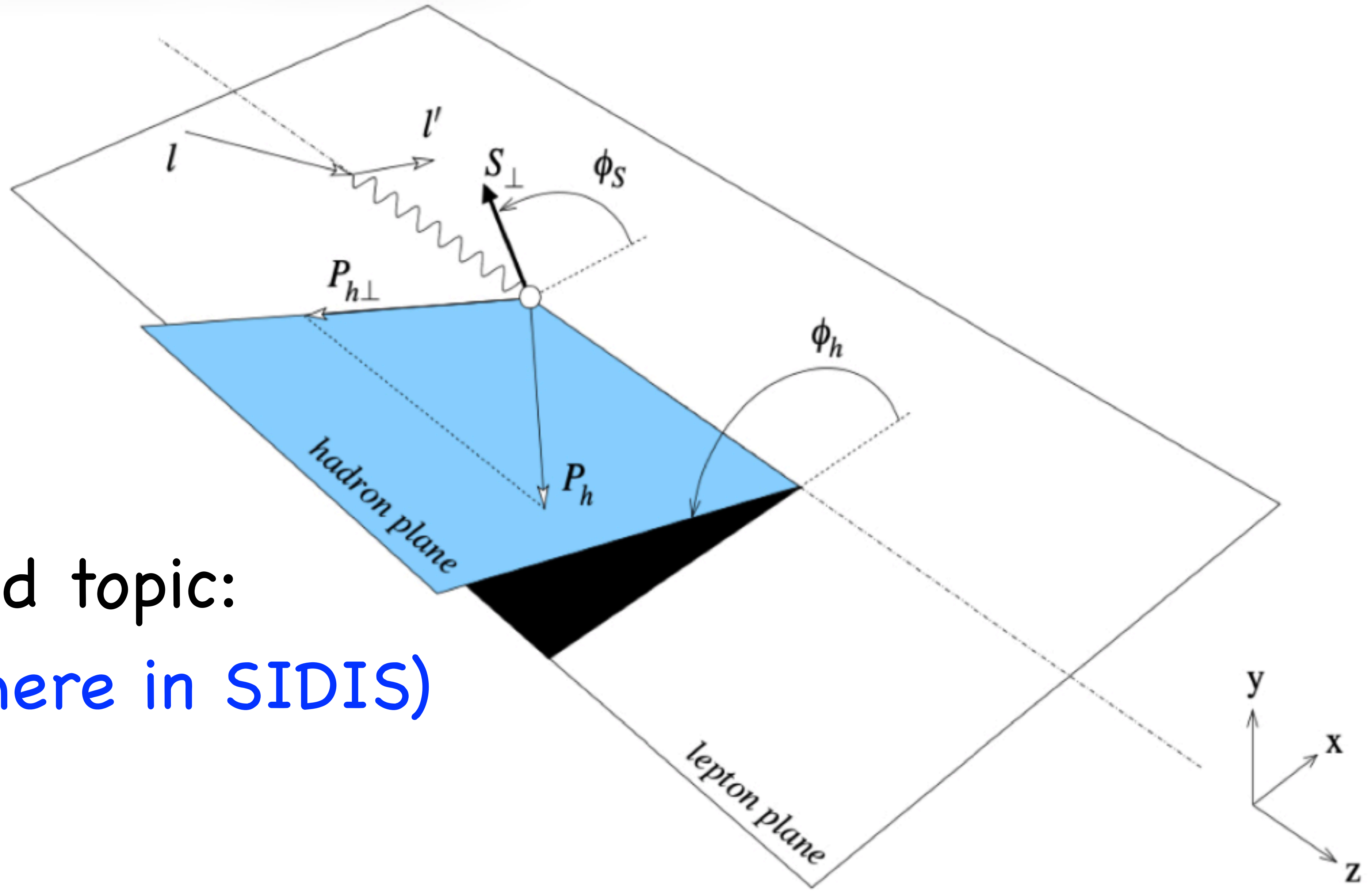


- Interesting opportunity to study hadron structure, age old topic: Ratio of longitudinal and transverse $\gamma^*_{T/L}$ cross section (here in SIDIS) classic power supressed $\sim (1/Q)^2$ in DIS ...

- Suggested extension beyond DIS: Feynman 1972 "Photon Hadron Interactions" & Ravndal PLB 1973, Cahn 1989 PRD 1989:

$$R = \frac{\sigma_L}{\sigma_T} = \frac{4\left(m^2 + \langle p_\perp^2 \rangle\right)}{Q^2} \overset{?}{\Longrightarrow} \frac{F_L}{F_T} \overset{?}{\sim} \frac{F_{UU,L}}{F_{UU,T}}$$

$\langle p_\perp^2 \rangle$ intrinsic parton transverse momentum

R_{SIDIS} What do we know

- $F_{UU,L}$ & $R_{SIDIS} = \frac{\sigma_L}{\sigma_T} \sim \frac{F_{UU,L}}{F_{UU,T}}$

- Often assumed $R_{DIS} \approx R_{SIDIS}$ however is independent of z , P_T, and φ

- Few measurements of $F_{UU,L}(x, z, q_T)$ JLab Hall C

- In TMD pheno @ low transverse momentum often assumed negligible

20^{th} century interpretation collinear DIS physics

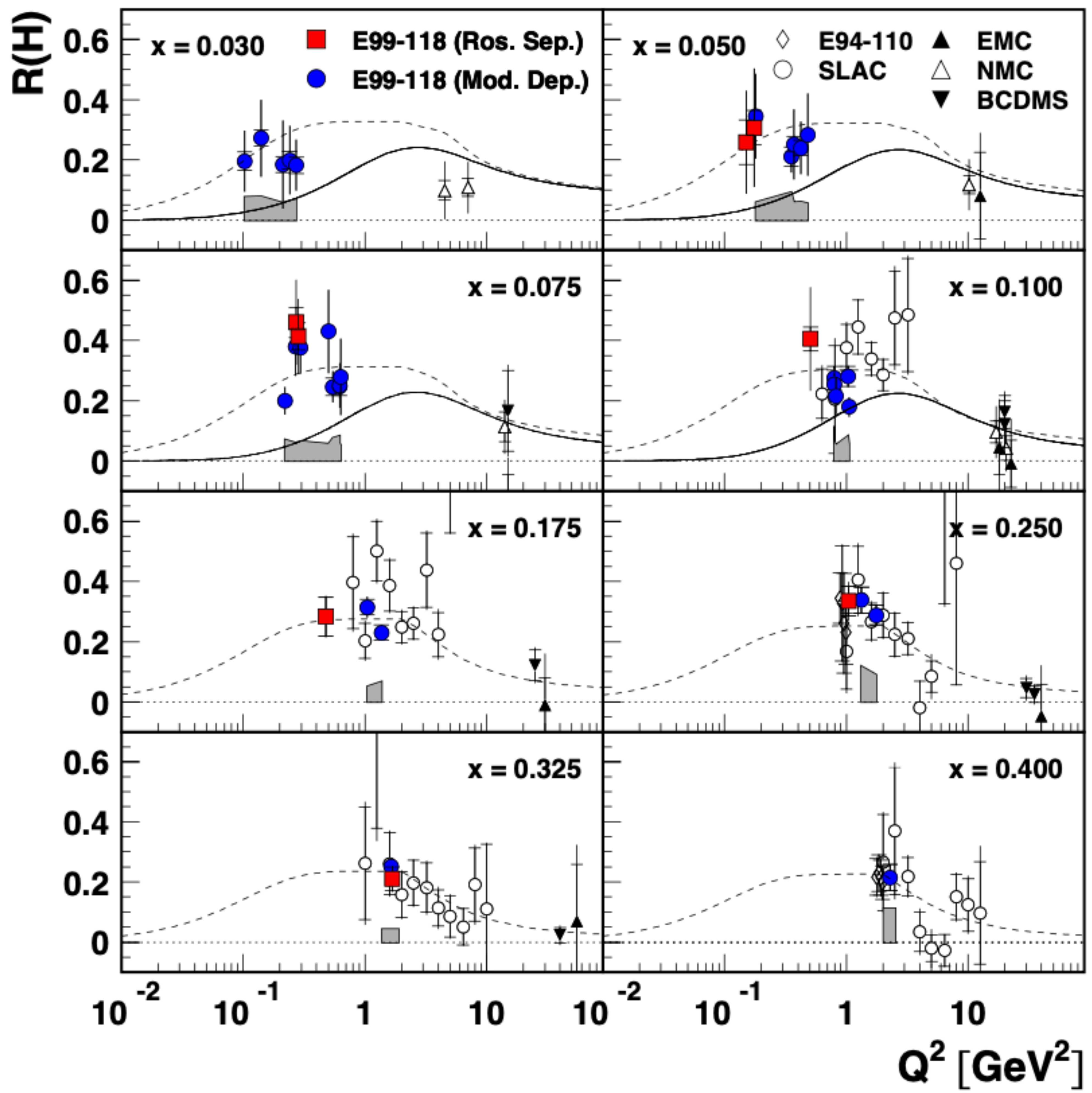
Recall inclusive cross section: σ_T and σ_L
or structure functions $F_L(F_2 \,\&\, F_1)$ & $F_T(F_1)$,
via absorption of longitudinal vs. transverse photons

$$R = \frac{\sigma_L}{\sigma_T} = \frac{F_L}{F_T} = \frac{1}{2xF_1}\left\{F_2\left(1+\frac{4x^2M^2}{Q^2}\right) - 2xF_1\right\}$$

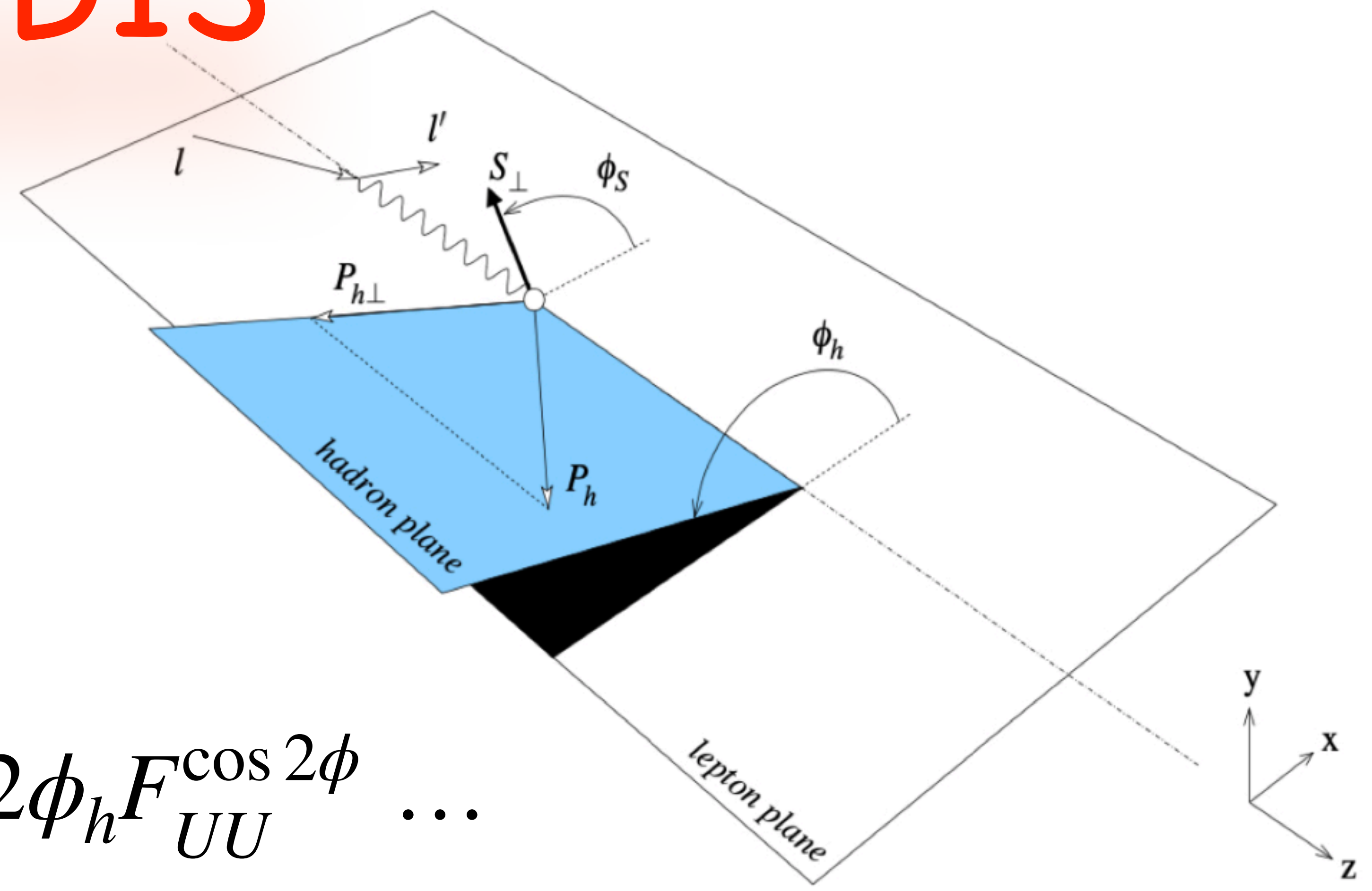
- Zero in parton model scaling limit $\lim_{Q^2\to\infty} R \approx \frac{4x^2M^2}{Q^2} \to 0\,,$
- non-trivial pQCD process $\{F_2 - 2xF_1\} \sim \frac{\alpha_s(Q)}{2\pi} C_2(F)\, x$

Comparison of values of $R(x,Q^2)$ for hydrogen from JLab exp. (E99-118) to results of other exps.

JLab exp. (E99-118) Prl 2007



21^{st} century interpretation: from DIS to SIDIS



$$R = \frac{\sigma_L}{\sigma_T} \longrightarrow \frac{F_{UU,L}}{F_{UU,T}}$$

$$\frac{d\sigma}{dxdydzd\phi_h dP_{h\perp}} = \frac{\alpha^2}{xQ^2}\frac{y}{2(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi} + \varepsilon\cos 2\phi_h F_{UU}^{\cos 2\phi} \ \ldots\right.$$

Does ∃ TMD observable for $F_{UU,L}$?

Tree level TMD factorization Mulders et al. NPB 1996 Bacchetta et al. JHEP 2007

$$F_{UU,T} = C[f_1\, D_1]\ , \qquad F_{UU,L} = C[\,.\,?\,.\,]$$

$$\mathcal{C}[wfD] = \sum_a x\, e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{q}_T)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

↕ decomposition

$$W^{\mu\nu} \propto \sum_a e_z^2 \int d^2p_T\, d^2k_T \delta^2\left(\mathbf{p}_T - \mathbf{k}_T + \mathbf{q}_T\right) \mathrm{Tr}\left\{\Phi^a(x, \mathbf{p}_T)\gamma^\mu \Delta^a(z, \mathbf{k}_T)\gamma^\nu\right\}$$

Reminder: recall Quark correlator LP, NLP

n.b. "tree level" Mulders Tangerman 1996 Bacchetta 2007

$$\Phi(x, p_T) = \frac{1}{2}\left\{ f_1 \not{n} + f_{1T}^{\perp} \frac{S_{T\,i}\varepsilon^{ij} p_{T\,j}}{M} \not{n} + \dots \right\} + \frac{M}{2P^+}\left\{ e + \dots \quad + f^{\perp} \frac{\not{p}_T}{M} \dots \right\}$$

Leading Quark TMDPDFs (Nucleon Spin, Quark Spin)

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^{\perp}$ = Boer-Mulders
L		g_1 = Helicity	$h_{1L}^{\perp}$ = Worm-gear
T	$f_{1T}^{\perp}$ = Sivers	$g_{1T}^{\perp}$ = Worm-gear	h_1 = Transversity; $h_{1T}^{\perp}$ = Pretzelosity

Subleading Quark TMDPDFs

Nucleon Polarization	Quark Chirality: Chiral Even	Chiral Odd
U	$f^{\perp}, g^{\perp}$	$e\,,\ h$
L	$f_L^{\perp}, g_L^{\perp}$	e_L, h_L
T	$f_T, f_T^{\perp}, g_T, g_T^{\perp}$	$e_T, e_T^{\perp}, h_T, h_T^{\perp}$

Reminder: recall Quark correlator LP, NLP, NNLP

n.b. "tree level" Mulders Tangerman 1996 Bacchetta 2007

$$\Phi(x, p_T) = \frac{1}{2}\left\{ f_1 \not{n} + f_{1T}^{\perp} \frac{S_{Ti}\varepsilon^{ij} p_{Tj}}{M} \not{n} + \ldots \right\} + \frac{M}{2P^+}\left\{ e + \ldots \qquad + f^{\perp} \frac{\not{p}_T}{M} \ldots \right\} + \frac{M^2}{(2P^+)^2}\left\{ \ldots \qquad ?? \qquad \ldots \right\}$$

Leading Quark TMDPDFs

Nucleon Spin — Quark Spin

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U	$f^{\perp}$, $g^{\perp}$	e , h
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T	f_T, $f_T^{\perp}$, g_T, $g_T^{\perp}$	e_T, $e_T^{\perp}$, h_T, $h_T^{\perp}$

Context TMD Correlator at tree level NNLP "twist 4"

Goeke, Metz, Schlegel PLB 2005

NNLP:

$$\Phi^{[\gamma^-]} = \frac{M^2}{(P^+)^2}\left[f_3(x,\vec{k}_T^2) - \frac{\varepsilon_T^{ij} k_{Ti} S_{Tj}}{M} f_{3T}^{\perp}(x,\vec{k}_T^2) \right]$$

Correlator at tree level @ "twist" 4

previously of academic interest

factorization relatively unexplored

Recall *LP* & *NLP* :

$$\Phi^{[\gamma^+]} = f_1(x,\vec{k}_T^2) - \frac{\varepsilon_T^{ij} k_{Ti} S_{Tj}}{M} f_{1T}^{\perp}(x,\vec{k}_T^2),$$

$$\Phi^{[\gamma^i]} = \frac{M}{P^+}\left[\frac{k_T^i}{M}\left(f^{\perp}(x,\vec{k}_T^2) - \frac{\varepsilon_T^{jk} k_{Tj} S_{Tk}}{M} f_T^{\perp\prime}(x,\vec{k}_T^2) \right) + \quad \ldots \right.$$

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Context TMD Correlator at tree level NNLP "twist 4"

NNLP:

NNLP: some discussion in Bacchetta et al.
Matches and mismatches JHEP 2008
& recent discussion w/ M. Cerruti

$$F_{UU,L} = \frac{4M^2}{Q^2}\, \mathcal{C}\left[\frac{\boldsymbol{p}_T^2}{M^2} f_1 D_1\right]$$

Correlator at tree level @ "twist" 4
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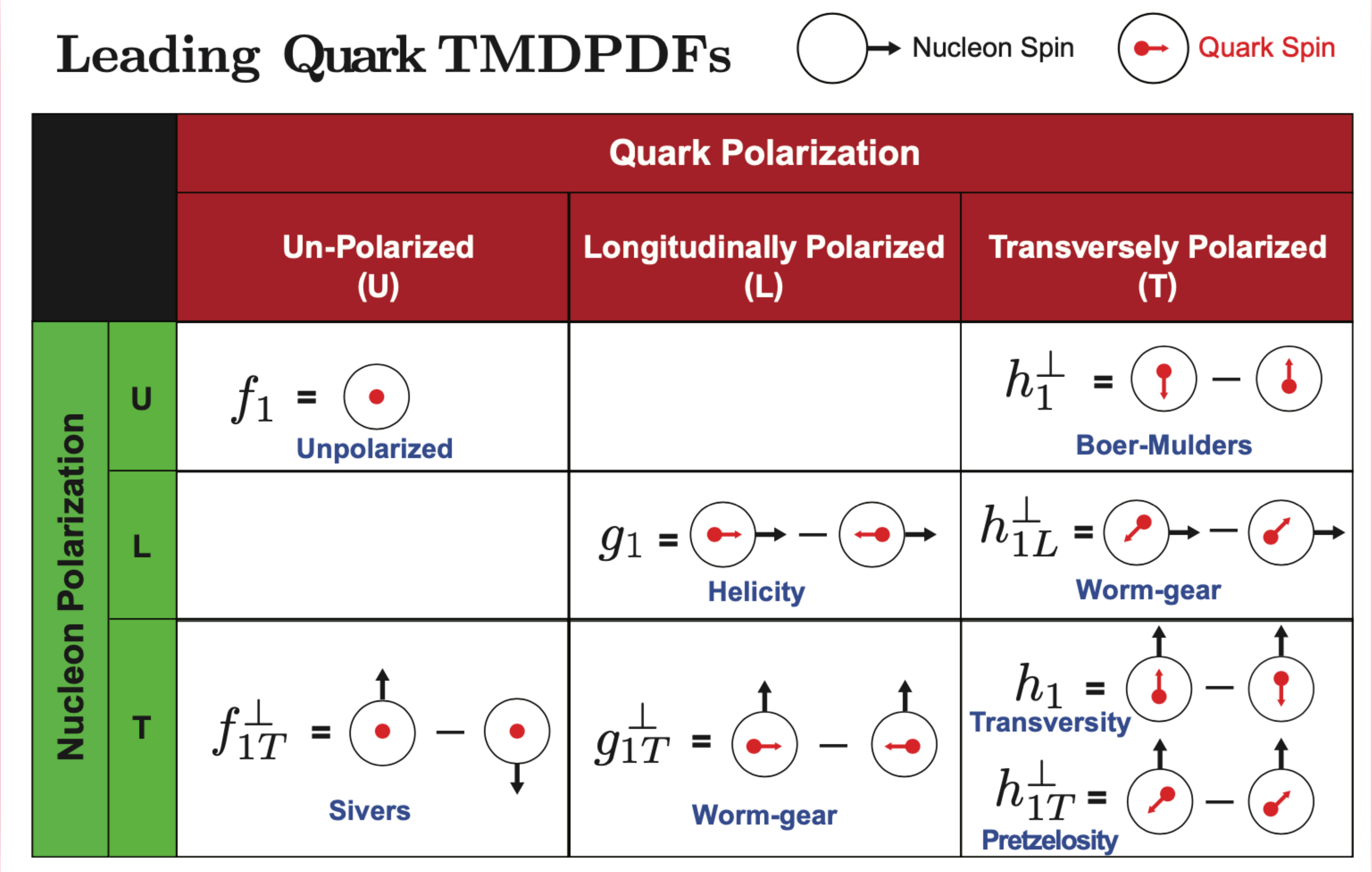

Subleading Quark TMDPDFs

Nucleon Polarization		Quark Chirality: Chiral Even	Quark Chirality: Chiral Odd
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	L	$f_L^\perp,\ g_L^\perp$	$e_L,\ h_L$
	T	$f_T,\ f_T^\perp,\ g_T,\ g_T^\perp$	$e_T,\ e_T^\perp,\ h_T,\ h_T^\perp$

R_{SIDIS} NLP TMDs ?

R_{sidis} esitmate MAP sizable contribution up to 20%

$$\frac{d\sigma}{dx\, dy\, dz\, d\phi_S\, d\phi_h\, dP_{h\perp}} = \frac{\alpha^2}{xQ^2}\frac{y}{2(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon F_{UU,L} \dots\right.$$

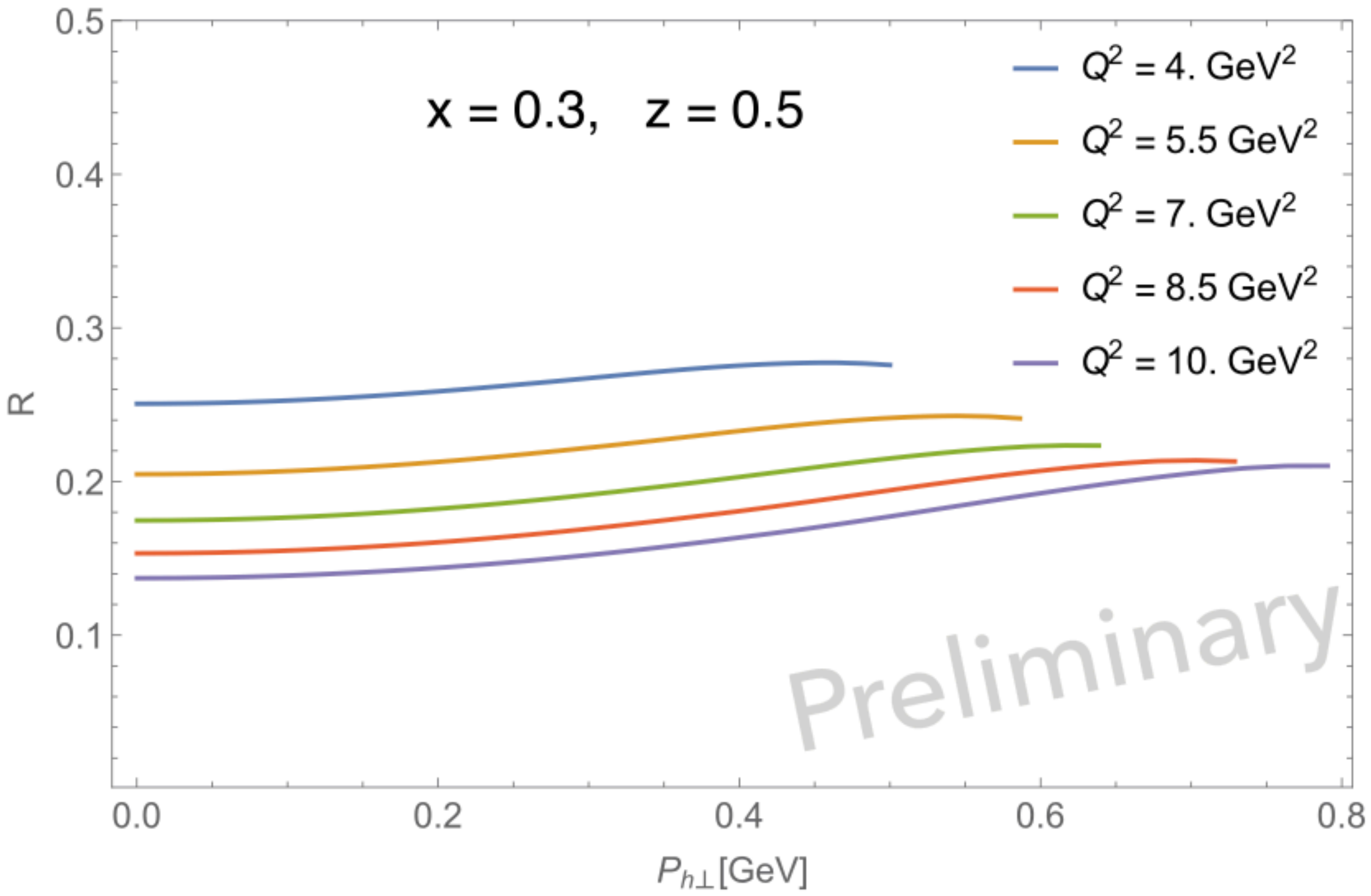
$$F_{UU,L} = \frac{4M^2}{Q^2} C\left[\frac{p_T^2}{M^2} f_1 D_1\right]$$

ε ratio of longitudinal and transverse photon flux

$$\varepsilon = \frac{1-y-\frac{1}{4}\gamma^2 y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2 y^2}, \quad \gamma^2 \equiv \frac{4M^2x^2}{Q^2}$$

- **Findings demonstrate $F_{UU,L}$ can't be ignored**
- substantial & essential for an accurate interpretation of $F_{UU,T}$
- which is associated with LP TMDs
- SIDIS normalization in TMD factorization ?

Bacchetta & Cerruti MAP Eur. Phys. J. A (2024) 60:173

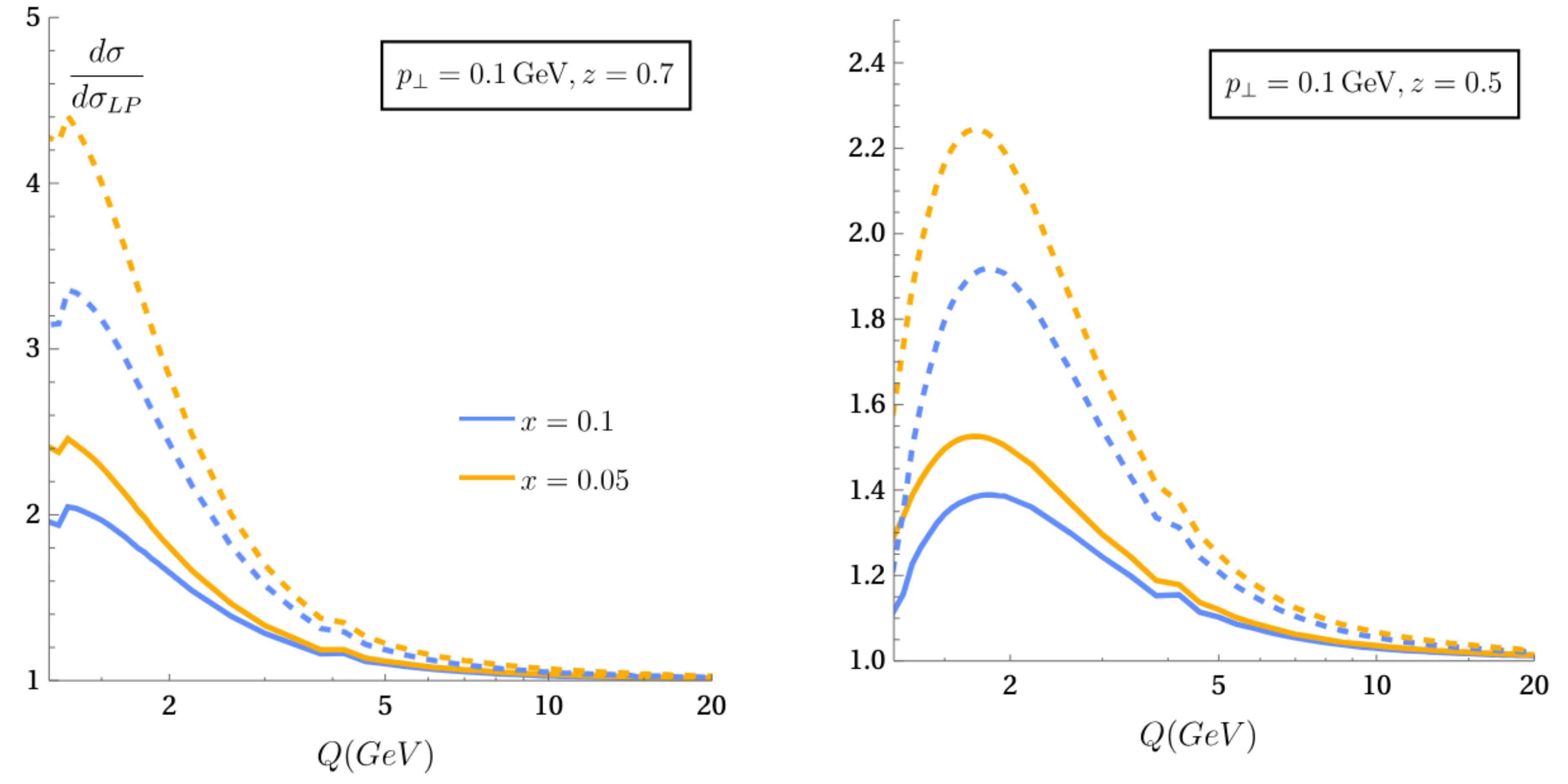


Estimate of $R_{SIDIS} = F_{UU,L}/F_{UU,T}$ versus hadron transverse momentum P_{hT} @ fixed x and z and for values Q2, compatible with JLab22 kinematics, using MAP22 TMD analysis

Recent wk. on NNLP contributions in W term

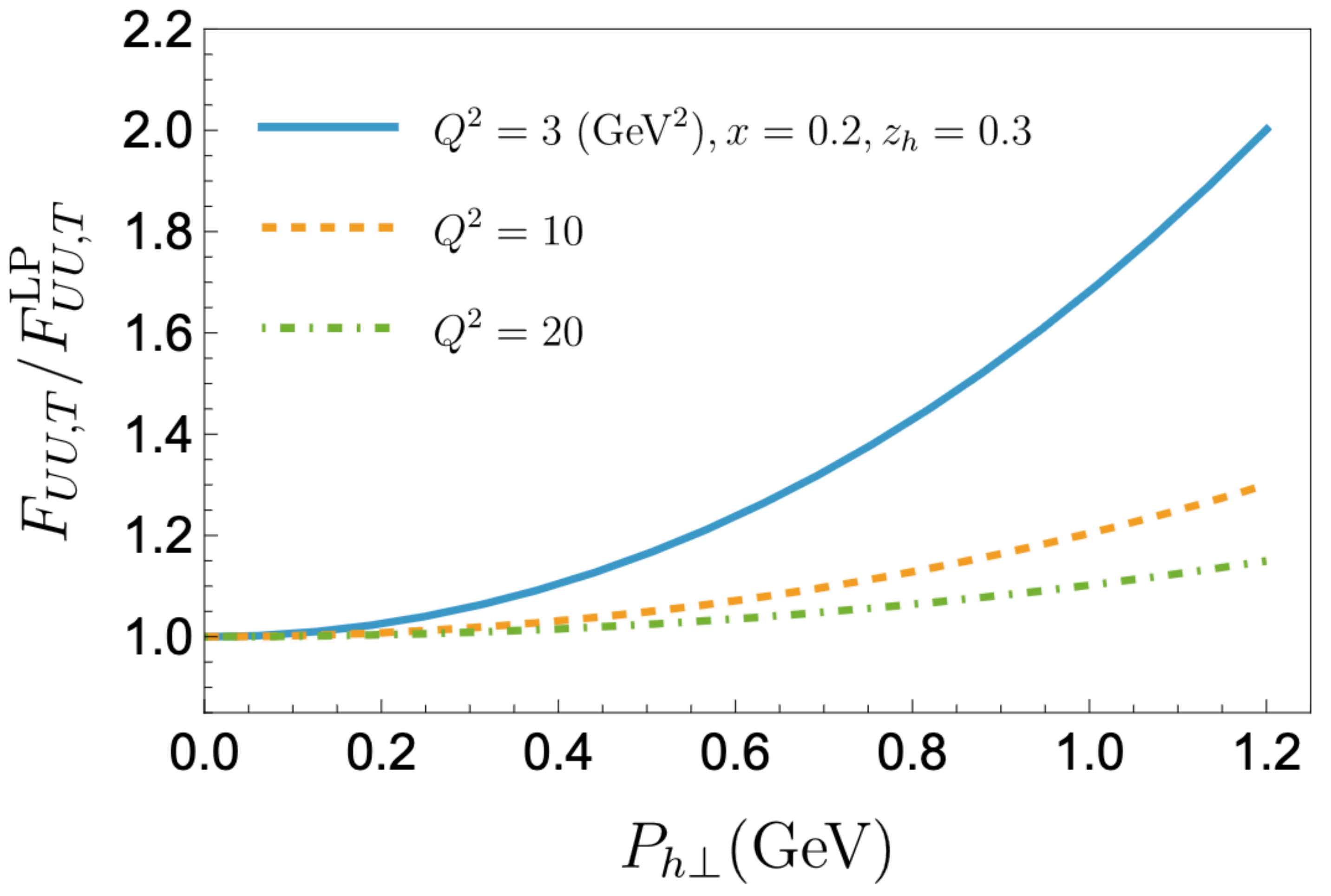
Kinematic power corrections for TMD factorization theorem of semi-inclusive deep-inelastic scattering

Sara Piloñeta [a] **and Alexey Vladimirov** [a]



Next-to-next-to-leading power corrections to unpolarized Semi-Inclusive Deep Inelastic Scattering

Ian Balitsky [a,c] **Alexei Prokudin** [b,c]



To learn more we consider lg $q_T \; F_{UU,L}(x,z,q_T,Q)$

Take "granular" look: consider both @ small & large $q_T \approx \frac{P_{hT}}{z}$ structure functions

- $F_{UU,T}(x,z,q_T,Q)$ &
- $F_{UU,L}(x,z,q_T,Q)$

Key to understanding predictive power in SIDIS & global analysis is whether the low & high q_T physics due to common mechanism ?

- **Explore thru factorization low & high q_T "matching"**

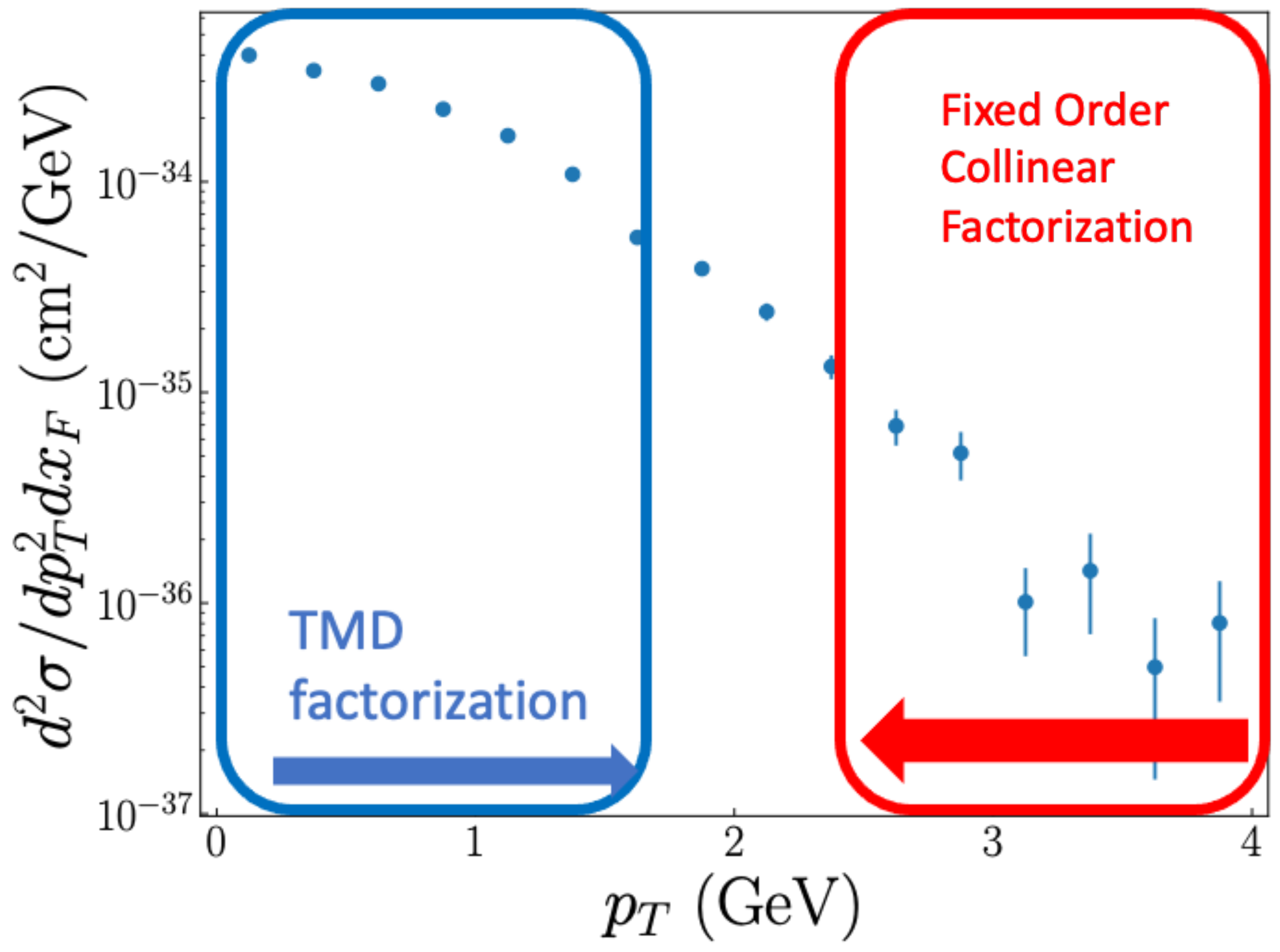
i) consider power counting: necessary conditions for match
ii) direct calculation: sufficient conditions for match

e.g. Allows extension of simulataneous fits of colliner pdfs & TMDs
W+ FO

$$\tilde{f}(x,b_T;\mu,\zeta) = \left[C \otimes f\right](x,b_T;\mu_0,\zeta_0) \times e^{S(b_T;\mu,\mu_0,\zeta,\zeta_0)} f_{NP}(x,b_T)$$

Matching of TMD & FO-large $q_T \approx P_{hT}/z$ Rigorous proof @ Leading power

- Factorization & Matching unpolarized Collins Soper Sterman NPB (1985), Bozzi Catani Floraian Grazzini NPB (2006), Bacchetta, Boer, Diehl, Mulders JHEP (2008) Collins, Gamberg, Prokudin, Rogers, Sato, Wang PRD (2016) ….
- N.B. Transverse polarization Ji, Qiu, Vogelsang Yuan PRL (2006); PRD (2006)



- **Cross section in terms of different "regions"**
- W valid for $q_T \sim k_T \ll Q$ TMD factorization
- FO valid for $k_T \ll p_T \sim Q$ Collinear factorization
- AY subtracts d.c. & in principle,

  $AY \to W,\ p_T \to \infty$ and $AY \to FO, p_T \to 0$
- $Y \equiv \to FO - AY$

$$\frac{d\sigma(m \lesssim p_T \lesssim Q, Q)}{dy\, dq^2 d^2 p_T} = W(p_T, Q) - AY(p_T, Q) + FO(p_T, Q) + \mathcal{O}\left(\frac{M}{Q}\right)^c$$

Explore thru factorization low & high q_T "matching"

i) consider power counting: necessary conditions for match

Power counting in regions

Bacchetta et al. JHEP 2008

Power counting $F_{XY,Z}$ → $F_{UU,L}$ & $F_{UU,T}$ ...

- Low TMD factorization

$M \sim q_T \ll Q$

$$M^2 F_{UU,T} \sim \mathcal{F}\left[f_1 D_1\right]$$

Leading power

$$M^2 F_{UU,L} \sim \frac{M^2}{Q^2}\mathcal{F}\left[\,..?..\,\right]$$

Sub sub leading

- High collinear factorization

$M \ll q_T \sim Q$

$$Q^2 F_{UU,T} = \alpha_s \mathcal{F}\left[f_1 D_1\right]$$

Leading power

$$Q^2 F_{UU,L} = \alpha_s \mathcal{F}\left[f_1 D_1\right]$$

Leading power

Power counting in "intermediate region

Power counting $F_{UU,L}$

- High collinear factorization $\longrightarrow M \ll q_T \ll Q \longleftarrow$ • Low TMD factorization

$$M^2 F_{UU,L}^{M\ll q_T\ll Q} \sim \alpha_s \frac{q_T^2}{Q^2}\frac{M^2}{q_T^2}\mathcal{F}[f_1 D_1]$$

$$M^2 F_{UU,L}^{M\ll q_T\ll Q} \sim \alpha_s \frac{M^2}{q_T^2}\frac{q_T^2}{Q^2}\mathcal{F}[f_1 D_1]$$

∃ Match

$$F_{UU,L} \sim \frac{1}{Q^2}\alpha_s \mathcal{F}[f_1 D_1]$$

An aside Power counting in "intermediate region

Power counting $F_{UU}^{\cos\phi}$

- High collinear factorization $\longrightarrow M \ll q_T \ll Q \longleftarrow$ • Low TMD factorization

$$M^2 F_{UU}^{\cos\varphi} \;{}^{M \ll q_T \ll Q} \sim \alpha_s \frac{q_T}{Q} \frac{M^2}{q_T^2} \mathcal{F}[f_1 D_1]$$

$$M^2 F_{UU}^{\cos\varphi} \;{}^{M \ll q_T \ll Q} \sim \alpha_s \frac{M^2}{q_T^2} \frac{q_T}{Q} \mathcal{F}[f_1 D_1]$$

$\exists$ Match

$$F_{UU}^{\cos\phi_h} \sim \frac{1}{Q\,q_T} \alpha_s \mathcal{F}[f_1 D_1]$$

Explore thru factorization low & high q_T "matching"

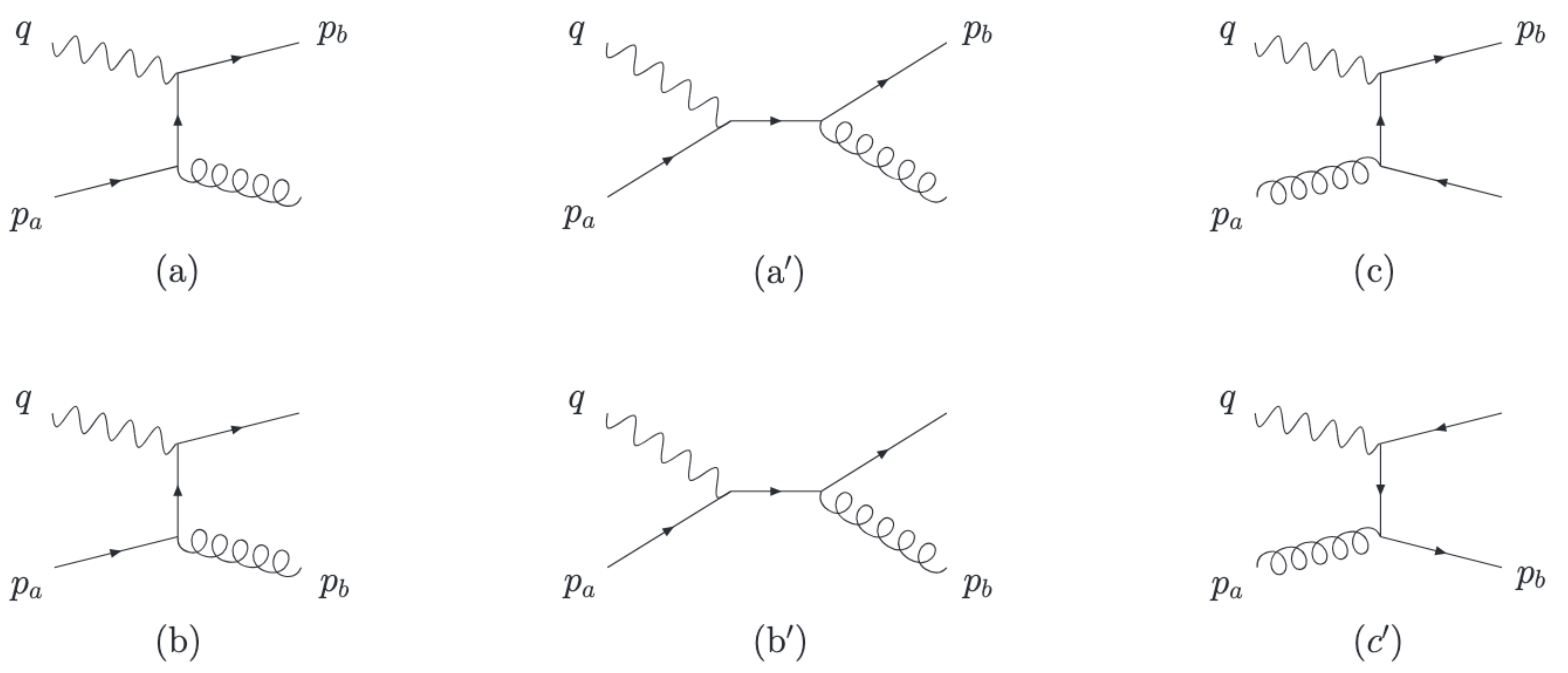
ii) direct calculation: sufficient conditions for match

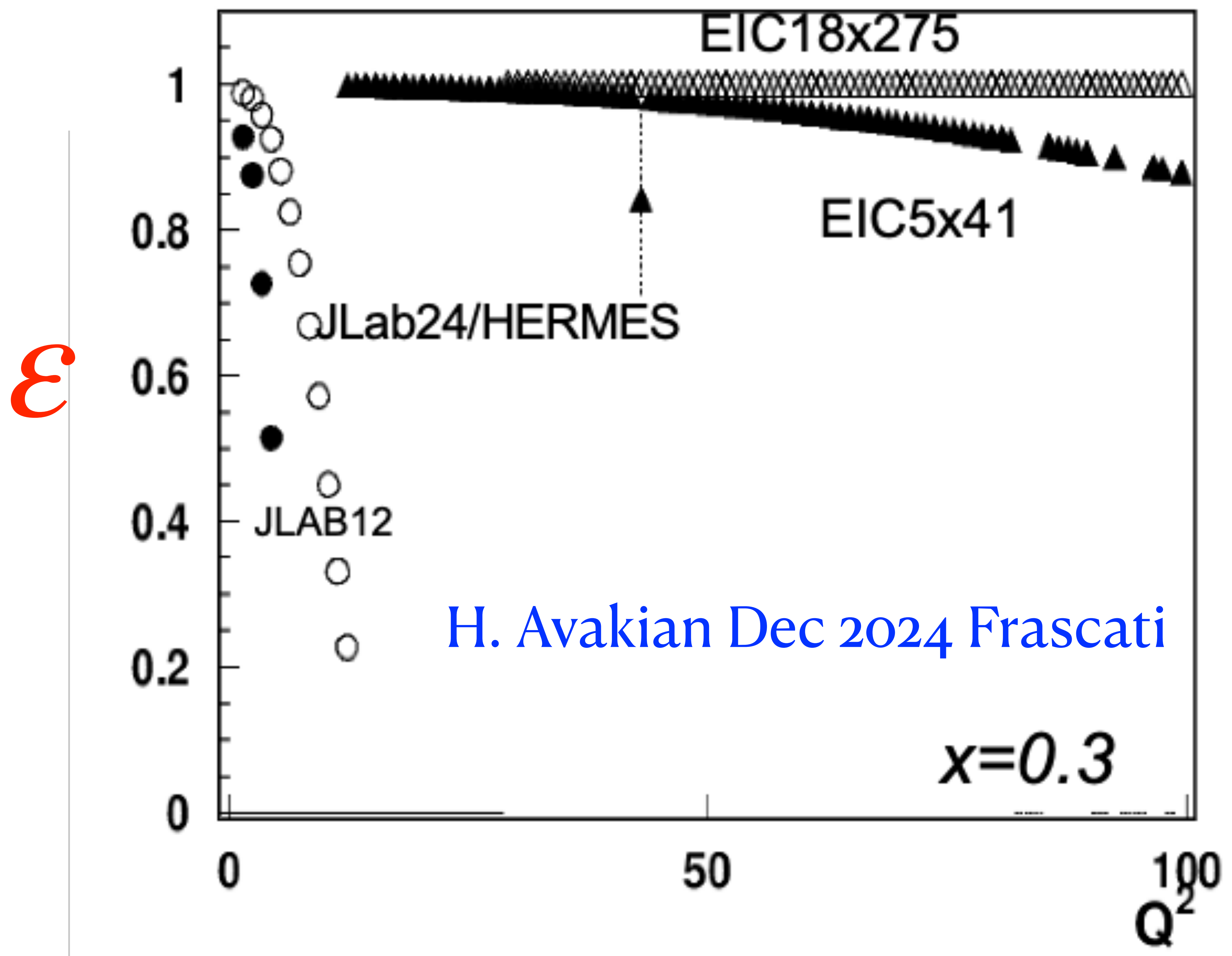
Consider $F_{UU,L}$ & R_{SIDIS} @ large P_T

- @ large P_T , $F_{UU,L} \sim F_{UU}^{\cos 2\phi_h}$ — see Bacchetta et al. JHEP 2008 "Matches & Mis-matches":

in principle hard gluon raditation — "collinear P_T factorization applies CSS 1985
Catani et al. 1997-2015, Nadolsky, Vogelsang Koike NPB 2005 … many others

$$\frac{d\sigma}{dxdydzd\phi_h dP_{h\perp}} = \frac{\alpha^2}{xQ^2}\frac{y}{2(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon\, F_{UU,L} + \ldots\right.$$





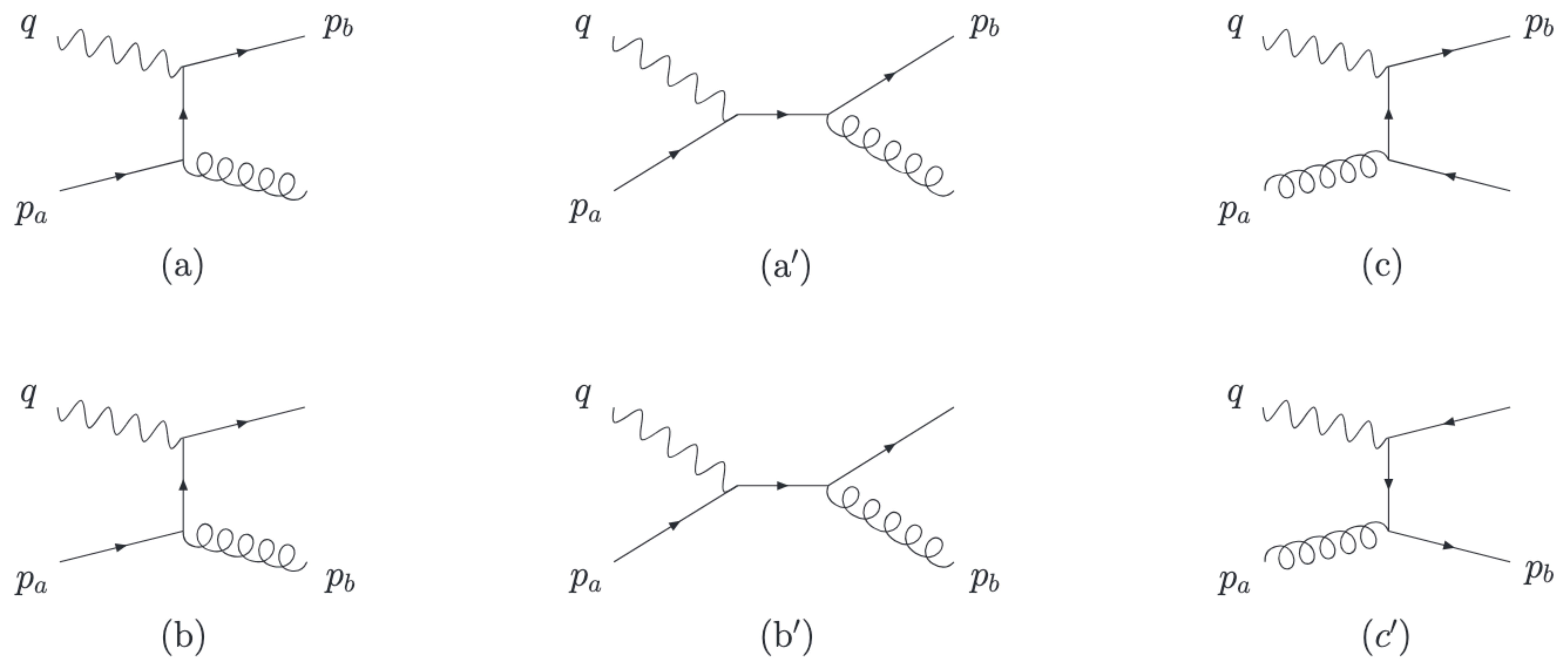
Large $q_T \sim P_{hT}/z$ well established factorization Leading POWER

$M \ll q_T \sim Q$

e.g.

$$F_{UU,T} = \frac{1}{Q^2}\frac{\alpha_s}{(2\pi z)^2}\sum_a x\, e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}}\, \delta\!\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\,\hat{z}}\right)$$

$$\times \left[f_1^a\!\left(\frac{x}{\hat{x}}\right) D_1^a\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\!\left(\frac{x}{\hat{x}}\right) D_1^g\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\!\left(\frac{x}{\hat{x}}\right) D_1^a\!\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* g\to q\bar{q})} \right]$$



- $\gamma^* q \to qg$ $\quad C_{UU}^{\cos 2\phi_h} = 4C_F\,\hat{x}\hat{z},$
- $\gamma^* q \to gq$ $\quad C_{UU}^{\cos 2\phi_h} = 4C_F\,\hat{x}\,(1-\hat{z}),$
- $\gamma^* g \to q\bar{q}$ $\quad C_{UU}^{\cos 2\phi_h} = 8T_R\,\hat{x}\,(1-\hat{x}),$

Power behavior $F_{UU,L}$ & $F_{UU,T}$ in AY region, $M \ll q_T \ll Q$

$M \ll q_T \sim Q$

$$F_{UU,T} = \frac{1}{Q^2}\frac{\alpha_s}{(2\pi z)^2}\sum_a x\,e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}}\int_z^1 \frac{d\hat{z}}{\hat{z}}\,\delta\!\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\,\hat{z}}\right)$$
$$\times\left[f_1^a\!\left(\frac{x}{\hat{x}}\right)D_1^a\!\left(\frac{z}{\hat{z}}\right)C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\!\left(\frac{x}{\hat{x}}\right)D_1^g\!\left(\frac{z}{\hat{z}}\right)C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\!\left(\frac{x}{\hat{x}}\right)D_1^a\!\left(\frac{z}{\hat{z}}\right)C_{UU,T}^{(\gamma^* g\to q\bar{q})}\right]$$

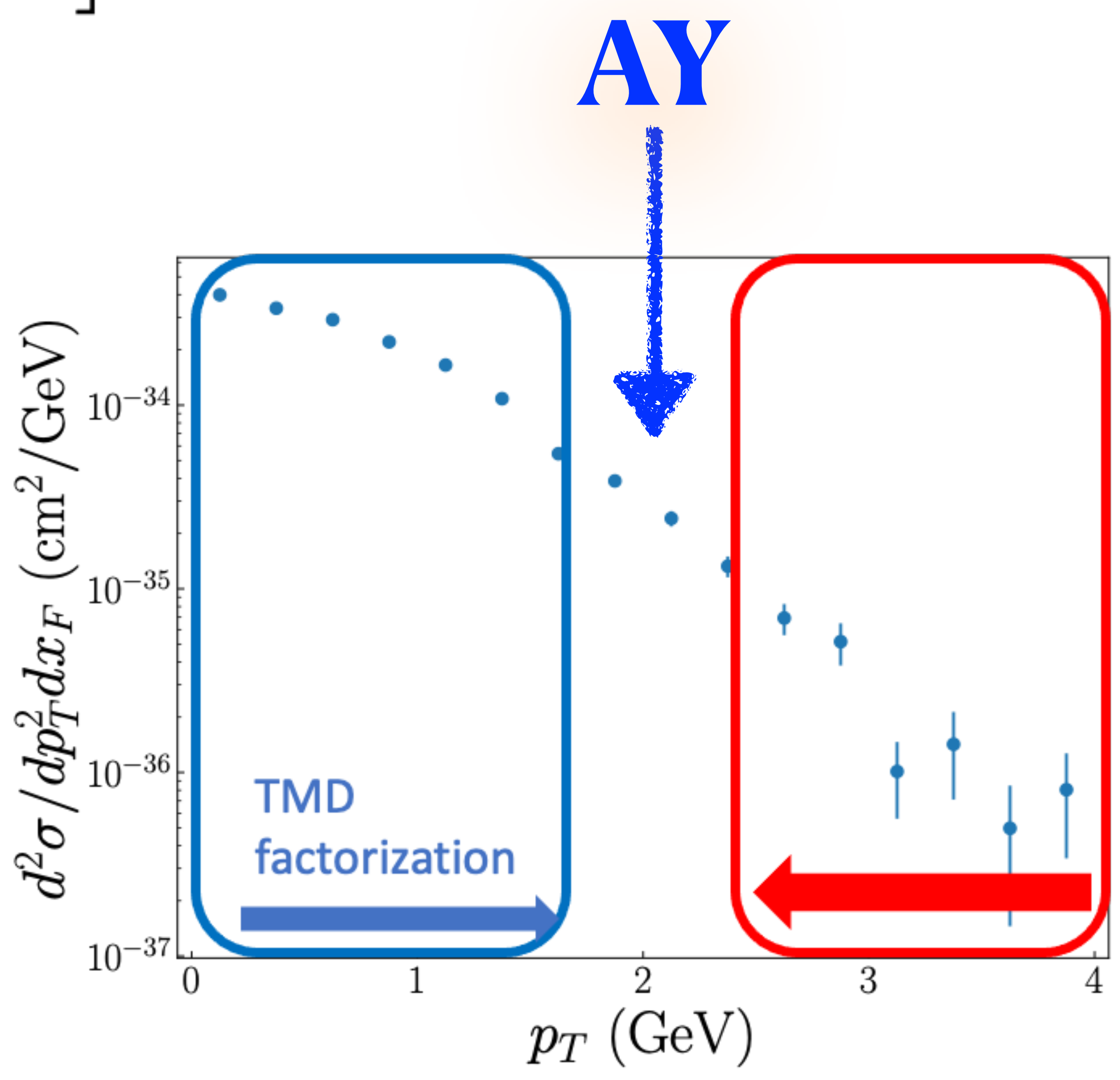
Asymptotic region

$M \ll q_T \ll Q$

$$F_{UU,T} = \frac{1}{q_T^2}\frac{\alpha_s}{2\pi^2 z^2}\sum_a x\,e_a^2\Big[f_1^a(x)\,D_1^a(z)\,L\!\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\,(D_1^a\otimes P_{qq} + D_1^g\otimes P_{gq})(z)$$
$$+ (P_{qq}\otimes f_1^a + P_{qg}\otimes f_1^g)(x)\,D_1^a(z)\Big]$$

$$F_{UU,L} = \frac{2}{Q^2}\frac{\alpha_s}{2\pi^2 z^2}\sum_a x\,e_a^2\Big[f_1^a(x)\,D_1^a(z)\,L\!\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x)\,(D_1^a\otimes P''_{qq} + D_1^g\otimes P''_{gq})(z)$$
$$+ (P''_{qq}\otimes f_1^a + P''_{qg}\otimes f_1^g)(x)\,D_1^a(z)\Big]$$

$$L\!\left(\frac{Q^2}{q_T^2}\right) \equiv C_F\left[2\ln\!\left(\frac{Q^2}{q_T^2}\right) - 3\right]$$



Can $F_{UU,L}$ be resummed ?

See e.g. Berger Qiu Zhang, PRD D65 (2002)

Probe for sub-leading physics

Large → intermediate → low q_T regions

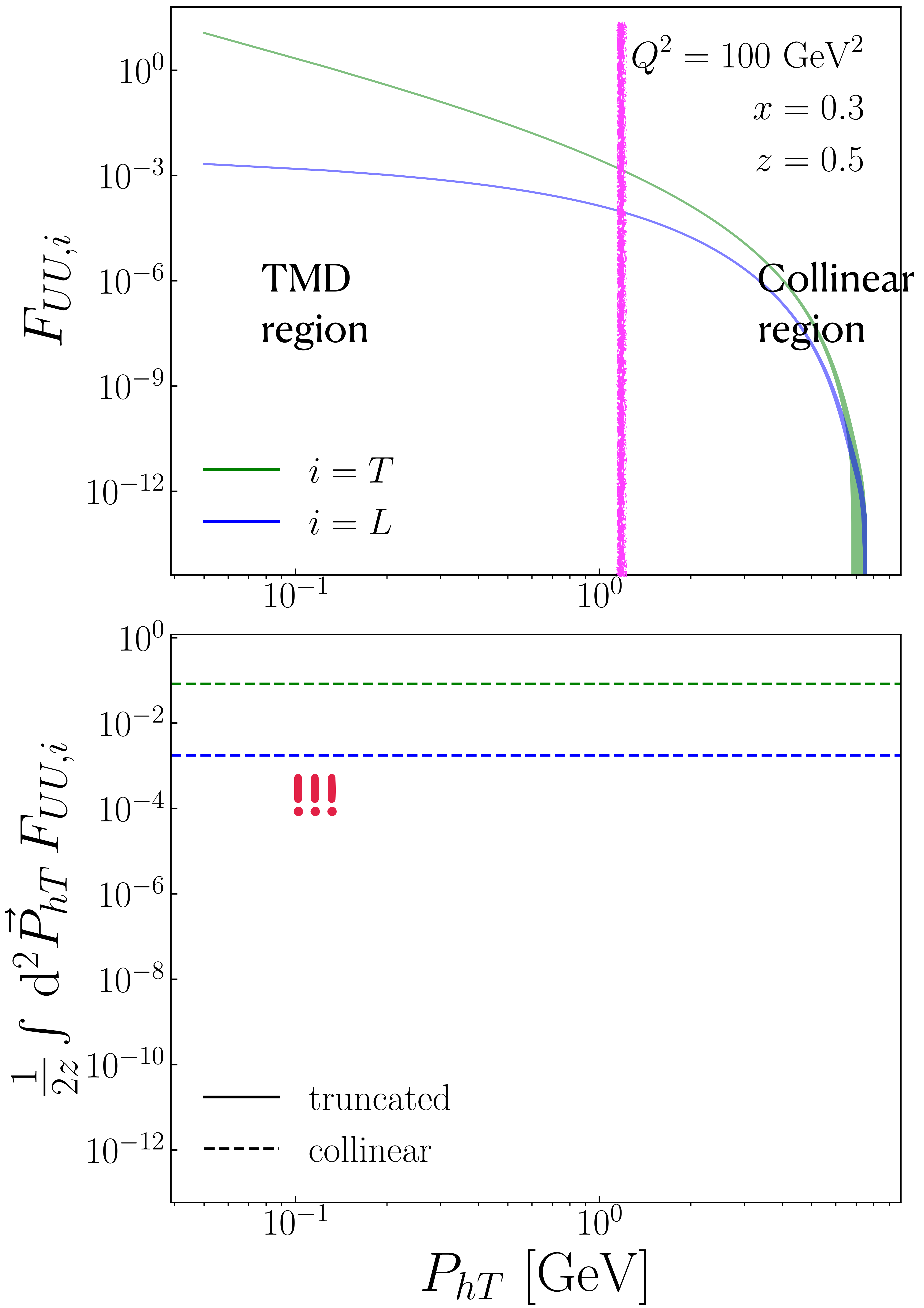
- $F_{UU,T}(x, z, q_T, Q)$ & $F_{UU,L}(x, z, q_T, Q)$

- **Collinear SIDIS vs. truncated moments**
  the dashed line for $F_{UU,L}$ is a leading power statement

$$\int_{P^2_{hT\,\min}/z^2}^{q^2_{T\,\max}} F(x, z, Q^2, P_{hT})$$

- $F_{UU,L}$ truncated moment converges "P_T **integrable**"
- $F_{UU,T}$ truncated moment as expected **diverges**

- Small TMD contribution ? Small power corrections ?



Probe for sub-leading physics

Truncated moments of R_{SIDIS} from large p_T

e.g.

$$F_{UU,T} = \frac{1}{Q^2}\frac{\alpha_s}{(2\pi z)^2}\sum_a x\, e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}}\int_z^1 \frac{d\hat{z}}{\hat{z}}\,\delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\,\hat{z}}\right) \times \left[f_1^a\left(\frac{x}{\hat{x}}\right) D_1^a\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q \to qg)} + f_1^a\left(\frac{x}{\hat{x}}\right) D_1^g\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q \to gq)} + f_1^g\left(\frac{x}{\hat{x}}\right) D_1^a\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* g \to q\bar{q})} \right]$$

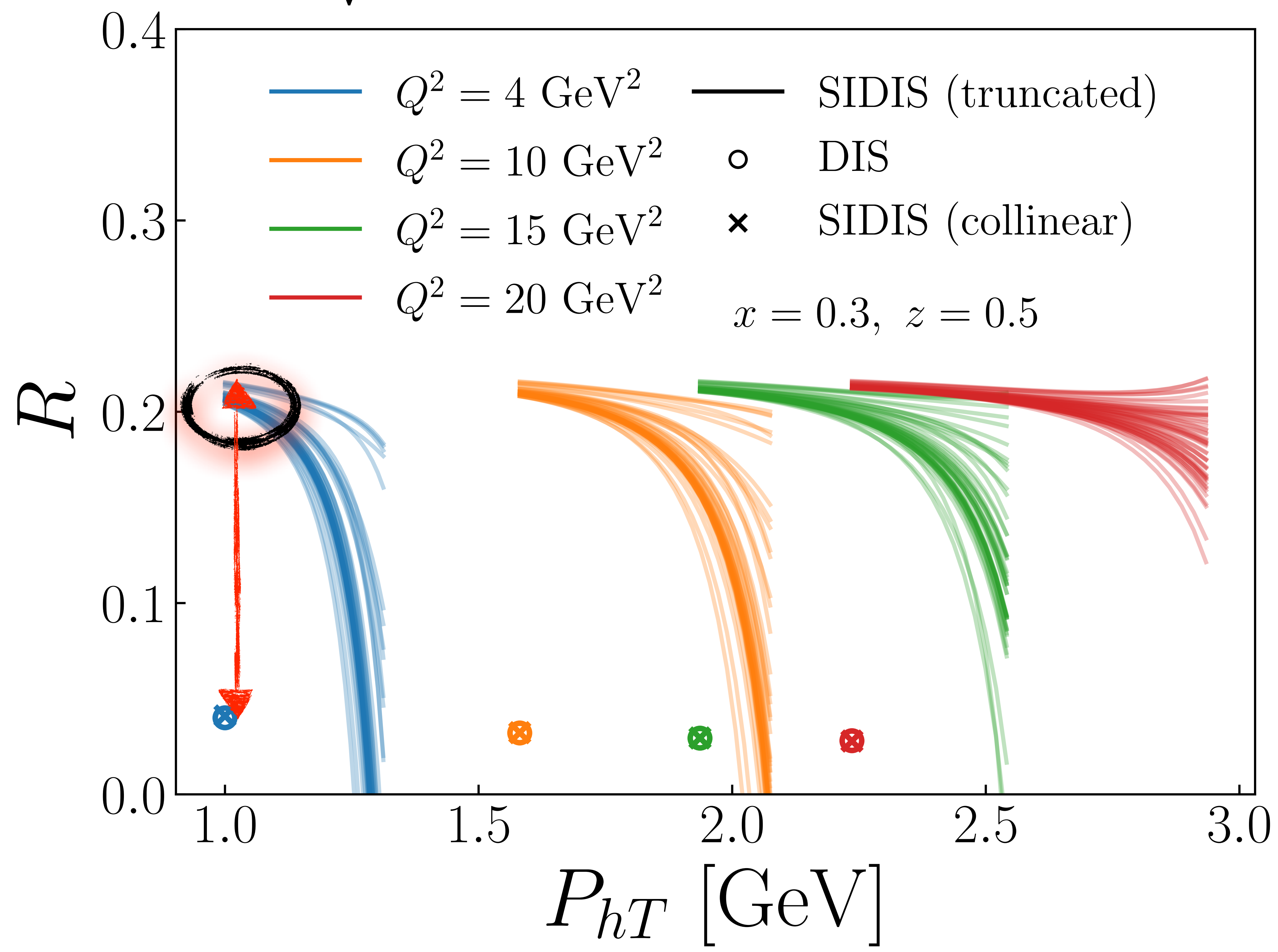
$$R_{SIDIS} = \int_{P_{hT}} \mathrm{d}^2 \vec{P}'_{hT}\, F_{UU,L}^{(\mathrm{FO})} \Big/ \int_{P_{hT}} \mathrm{d}^2 \vec{P}'_{hT}\, F_{UU,T}^{(\mathrm{FO})}$$

SIDIS truncated moments

$$\int_{P^2_{hT\,\mathrm{min}}/z^2}^{q^2_{T\,\mathrm{max}}} F(x, z, Q^2, P_{hT})$$

Nb: Bands are generated by computing the observable on subset of JAM replicas (from recent W+charm analysis) & taking the mean ± standard deviation

$\sqrt{s} = 140\,\mathrm{GeV},\ x = 0.3,\ z = 0.5$

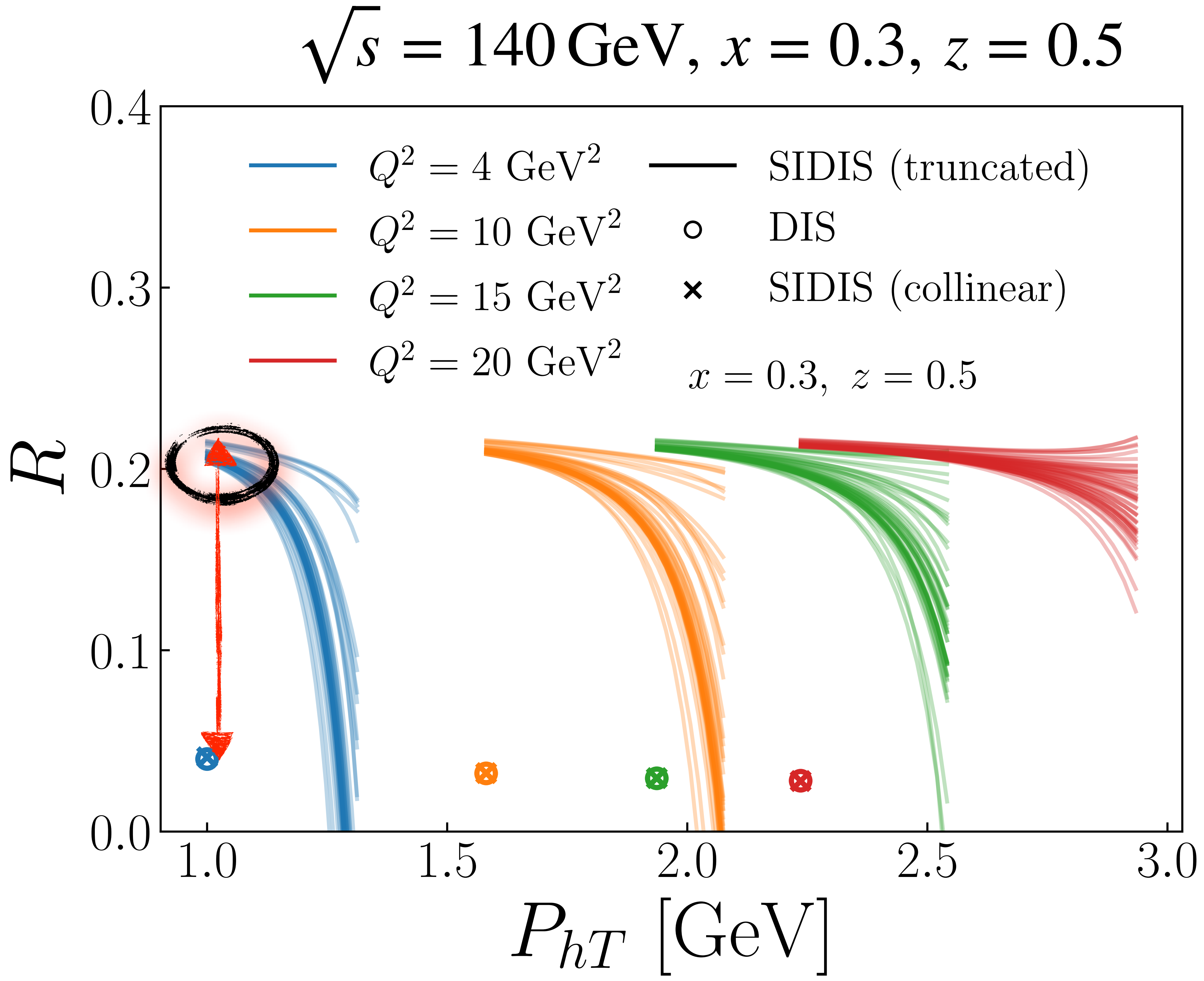


Probe for sub-leading physics

Truncated moments of R_{SIDIS} from large p_T

$$F_{UU,T} = \frac{1}{Q^2}\frac{\alpha_s}{(2\pi z)^2}\sum_a x\, e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}}\, \delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\,\hat{z}}\right)$$
$$\times \left[f_1^a\left(\frac{x}{\hat{x}}\right) D_1^a\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\left(\frac{x}{\hat{x}}\right) D_1^g\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\left(\frac{x}{\hat{x}}\right) D_1^a\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* g\to q\bar{q})} \right]$$

$$R_{SIDIS} = \frac{F_{UU,L}^{FO}}{F_{UU,T}^{FO}}$$



Comments:

- Truncated moment is sig. larger than P_T integrated SIDIS—indication of
  - power corrections ?
  - TMD contribution ?

W-term is important for R_{SIDIS}!

What role does sub-leading $F_{UU,L}^{(W)}$ play?

Stitching together TMD + FO with goal of estimating subleading power "resummed W" term

Since $\mathrm{W}+\mathrm{Y}$ not feasible phenomenologically yet …

→ approximate ASY by interpolating W & FO

→ use intersection points between W & FO to separate scales

→ ensure analyticity point-by-point

$$F_{UU,L} = \left(1 - w_L\right) F_{UU,L}^{[M\sim q_T \ll Q]} + w_L\, F_{UU,L}^{[M \ll q_T \sim Q]} \overset{?}{\Longrightarrow} W + Y + \mathcal{O}\left(\frac{M}{Q}\right)^c$$

Where w_L is-transition function

$$F_{UU,L}^{[M\sim q_T \ll Q]} = \frac{M^2}{Q^2} C\Big[\frac{p_T^2}{M^2} f_1 D_1\Big] \qquad F_{UU,L}^{[M \ll q_T \sim Q]} = Fixed\, order$$

Building our model

Since W + Y not feasible phenomenologically yet …

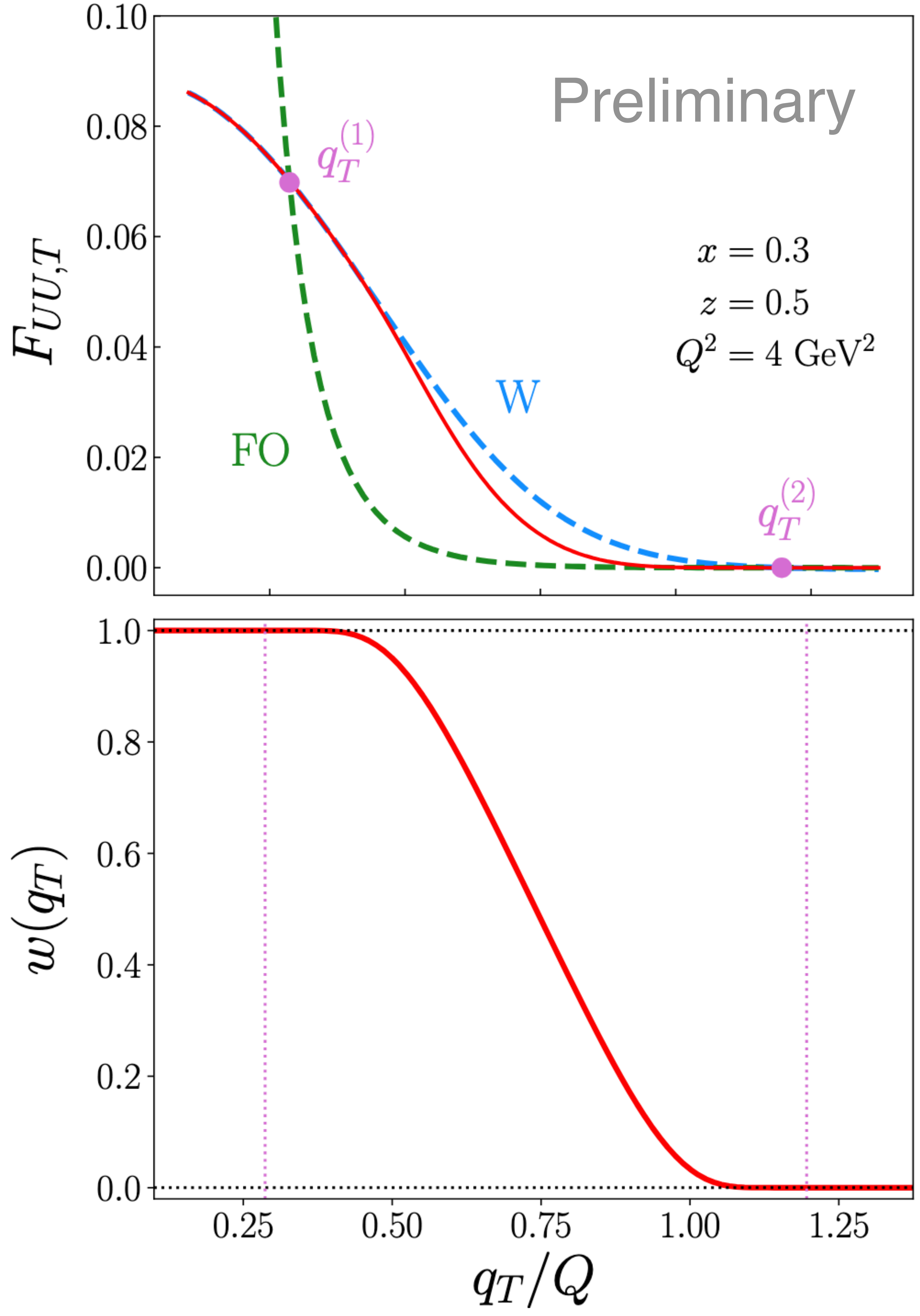
→ approximate ASY by interpolating W & FO

→ use intersection points between W & FO to separate scales

→ ensure analyticity point-by-point

$$F = w(q_T)\,\mathrm{W} + [1 - w(q_T)]\,\mathrm{FO}$$

$$w(q_T) = \begin{cases} 1 & q_T < q_T^{(1)} \\ f_{\mathrm{interp}}(q_T^{(c)}, \sigma) & q_T^{(1)} < q_T < q_T^{(2)} \\ 0 & q_T > q_T^{(1)} \end{cases}$$



Uncertainty quantification

Use same interpolation for $F_{UU,T/L}$

$\rightarrow$

$$\omega_L = \frac{k_T^2}{k_T^2 + Q^2/4} \rightarrow \frac{k_T^2}{4Q^2} \text{ when } k_T \sim 0$$

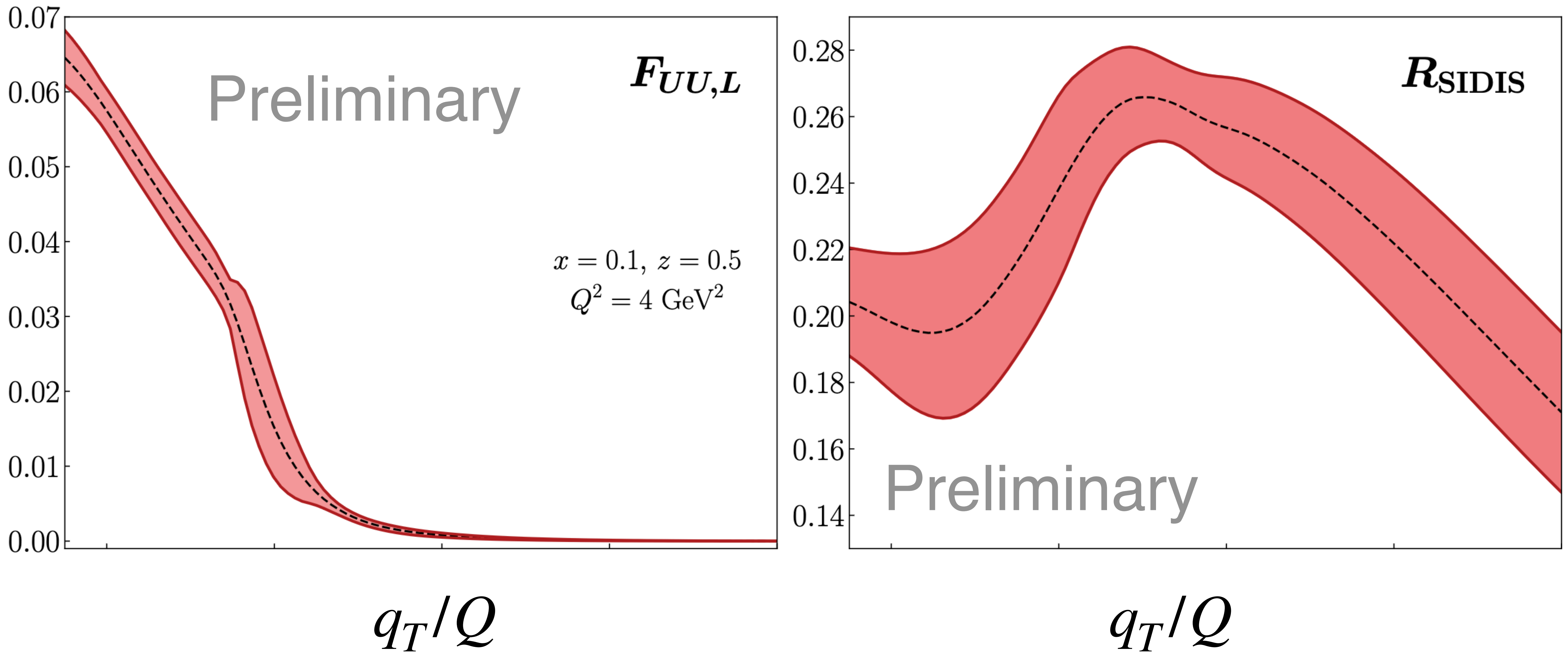
$\rightarrow$ 4 types of uncertainty:

(1) interpolation between W & FO

(2) Scale variation

(3) TMD replicas

(4) Collinear hessian sets



Pheno impacts/opportunities

R_{SIDIS} predictions at Compass kinematics

→ full error budget

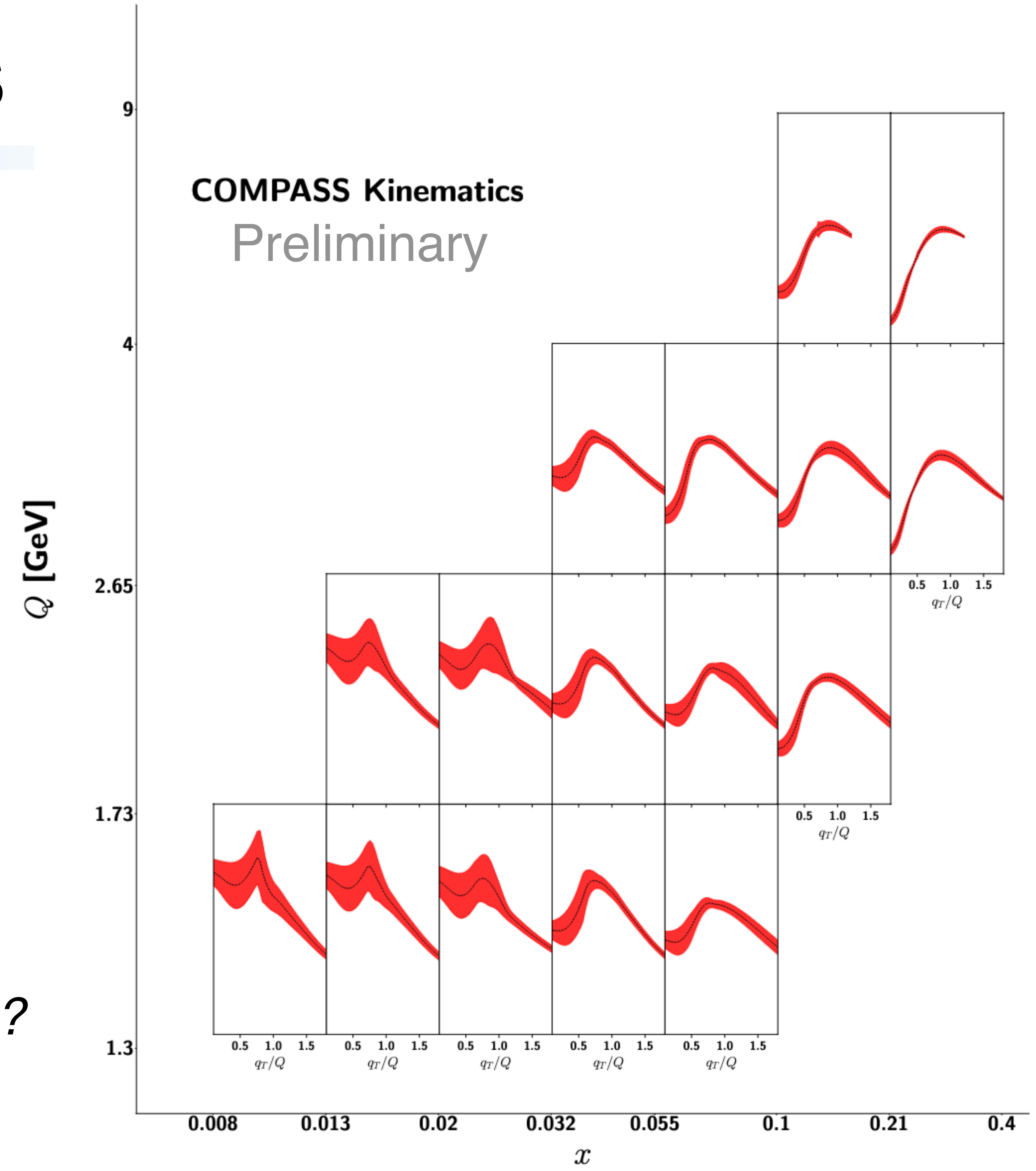
→ trends with increasing Q

(1) Magnitude of $R_{\text{SIDIS}}(q_T \to 0)$ decreases

(2) $R_{\text{SIDIS}}(q_T \to 0) \sim (q_T/Q)^2$

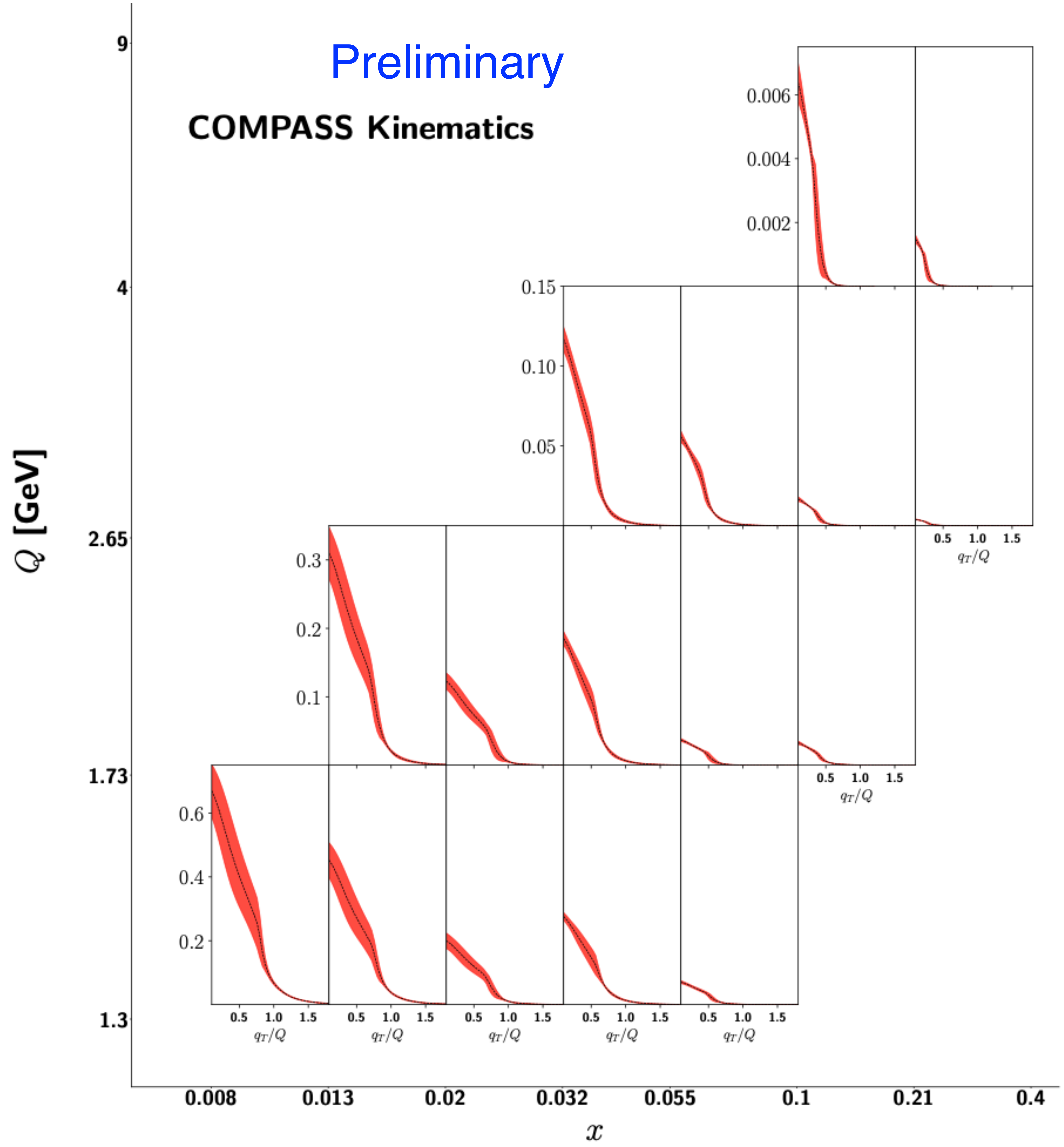
(3) Uncertainty decreases

How do theory & experimental uncertainties compare?



Pheno cont.

$F_{UU,L}$ predictions @ Compass kinematics



Summary

Longitudinal Structure function $F_{UU,L}$

→ indispensable to probe transverse proton structure

→ lots of challenges in formal/ phenomenological description of q_T spectrum

Progress in sub-leading TMD physics:

→ JLab experiments: E12-06-104, E12-09-017, E12-09-002

→ theory: Balitsky & Prokudin, Piloñeta & Vladimirov

This work:

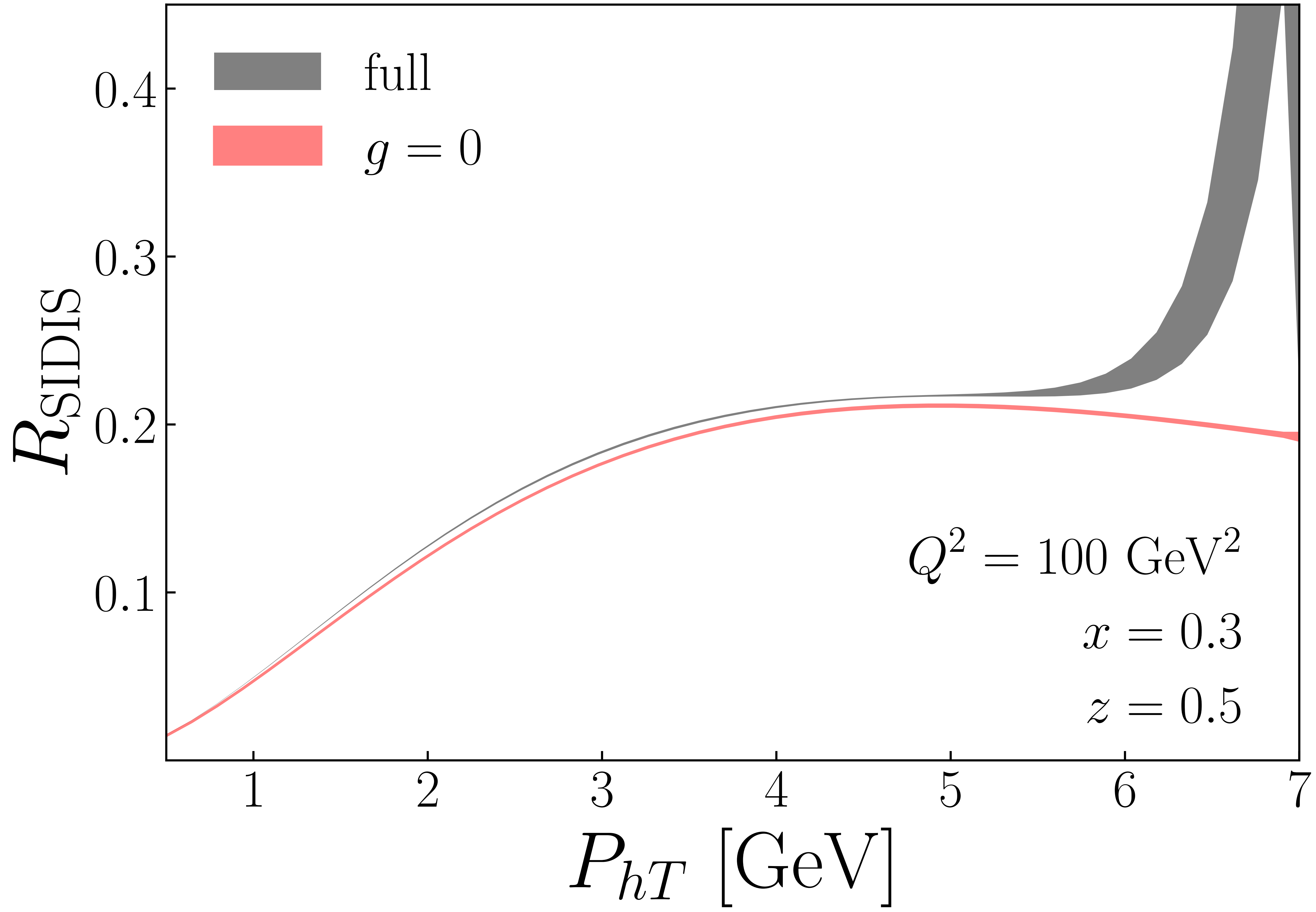
→ pheno estimates with uncertainties of $F_{UU,L}$ and R_{SIDIS} considering interplay of W & FO

→ Future — impacts on multiplicities/collinear SIDIS

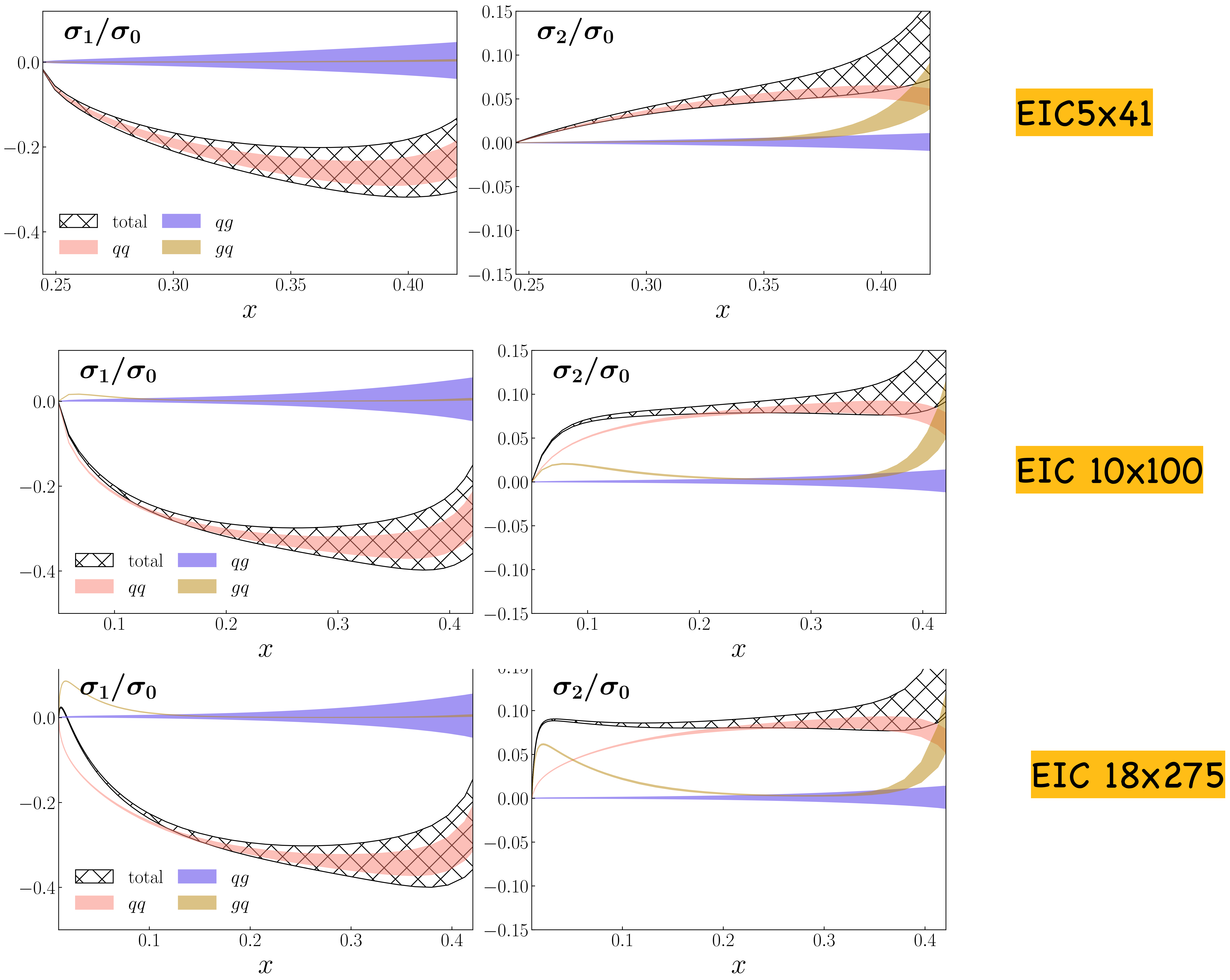
R_{SIDIS} & $\sigma_L \sim F_{UU,L}$ at large p_T

- gluon contribute large uncertainty @ hi-x (see delta function)
- $g \to 0$ gluon PDF set to zero
- R_{SIDIS} could be useful to pin down the g @ $lg\ x$

$$\text{FO} = \sum_q e_q^2 \int_{\frac{q_T^2}{Q^2}\frac{xz}{1-z}+x}^{1} \frac{d\xi}{\xi - x} H(\xi)\ \boldsymbol{f}_q(\xi,\mu)\ \boldsymbol{d}_q(\zeta(\xi),\mu) + O(\alpha_S^2) + O(m^2/q^2)$$



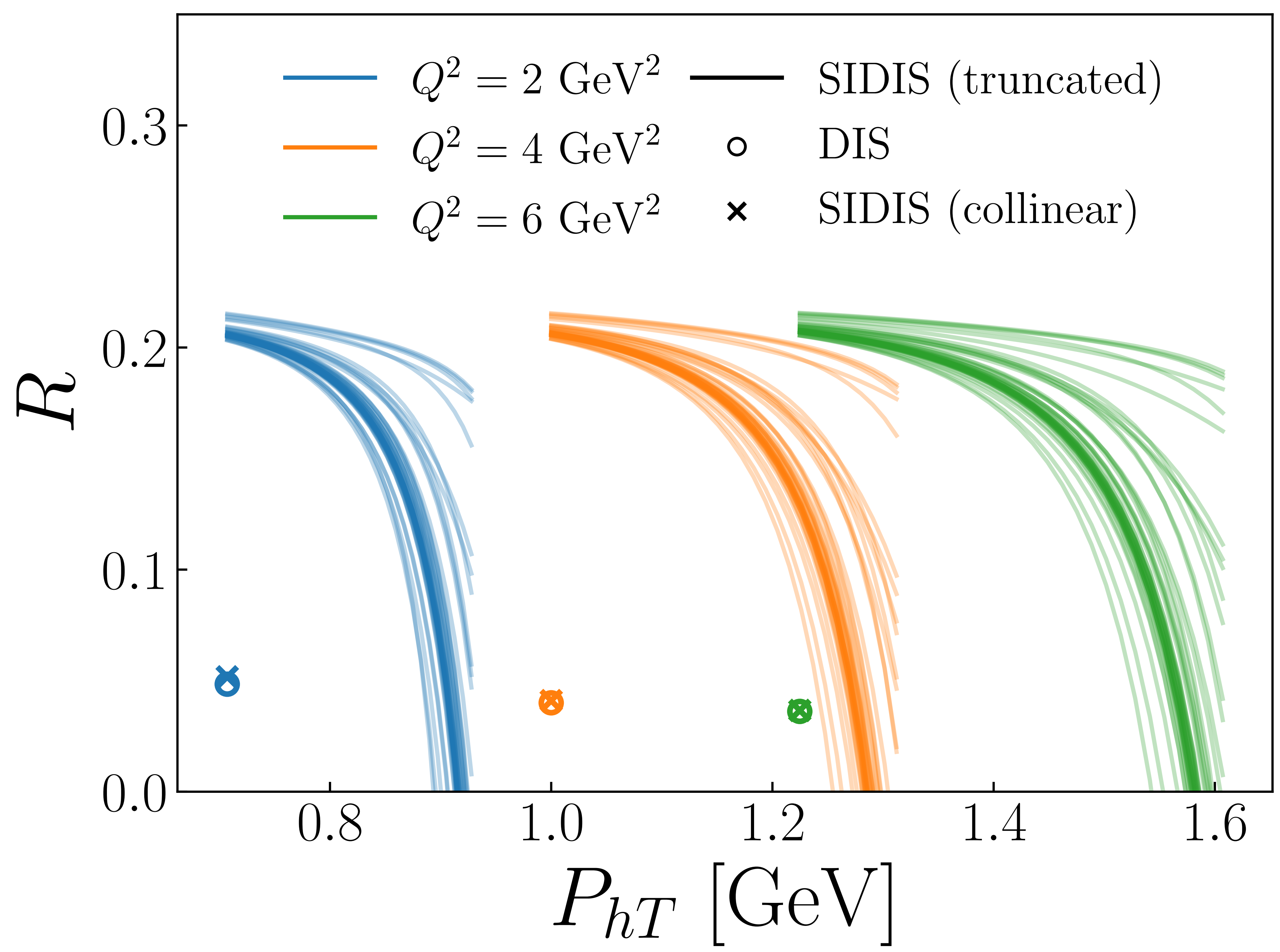
+ **Attention**: $\left(\frac{q_T^2}{Q^2}\frac{xz}{1-z} + x\right) < \xi < 1$
+ **large q_T probes large ξ in PDFs**
+ Can be useful in collinear global fits



EIC5x41
EIC 10x100
EIC 18x275
σ_1/σ_0
σ_2/σ_0
total
qq
qg
gq
x

R_{SIDIS} & $\sigma_L \sim F_{UU,L}$ at large p_T

Jlab 11 GeV $x = 0.3$ & $z = 0.5$



Jlab 22 GeV $x = 0.3$ & $z = 0.5$