Rapidity Dependent Beam Functions

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Based on work done in collaboration with Emmet Byrne (University of Manchester) and Jonathan Gaunt (University of Cyprus)

QCD Evolution
Madrid, 12th of May 2026

Aims of Calculation

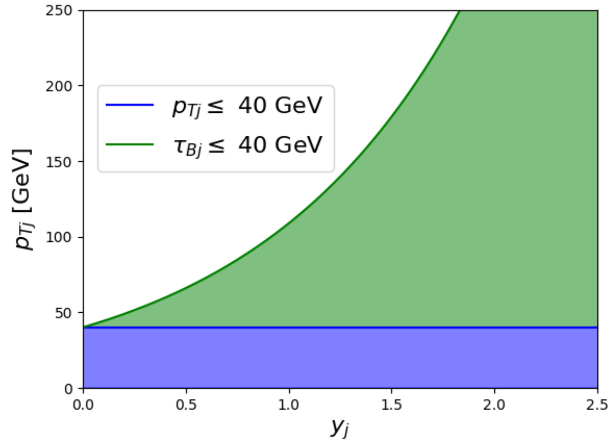
- Produce an addition piece to the leading jet p_T beam functions to incorporate an additional pseudo-rapidity cut

- Produce a correction to the resummed prediction for the 0-jet WZ production to alleviate tension between the theory and experimental result

Rapidity Dependent Jet Vetoes

Due to a lack of tracking information, high rapidity, low p_T jets are hard to resolve experimentally

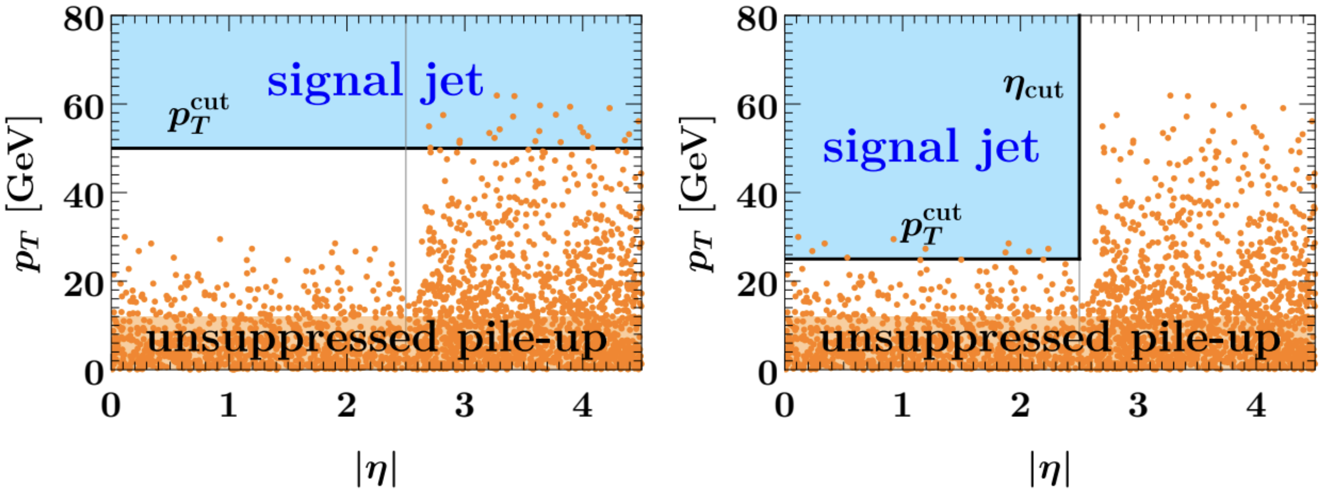
A rapidity dependent jet veto allows p_T cuts at forward rapidities to be loosened or removed, reducing sensitivity to forward low p_T jets



$$\mathcal{T}_{Bj} = m_{Tj} e^{-|y_j - Y|}$$

Tackmann, Walsh, Zuberi, arXiv:1206.4312

Clark, Gangal, Gaunt, arXiv:2504.06353



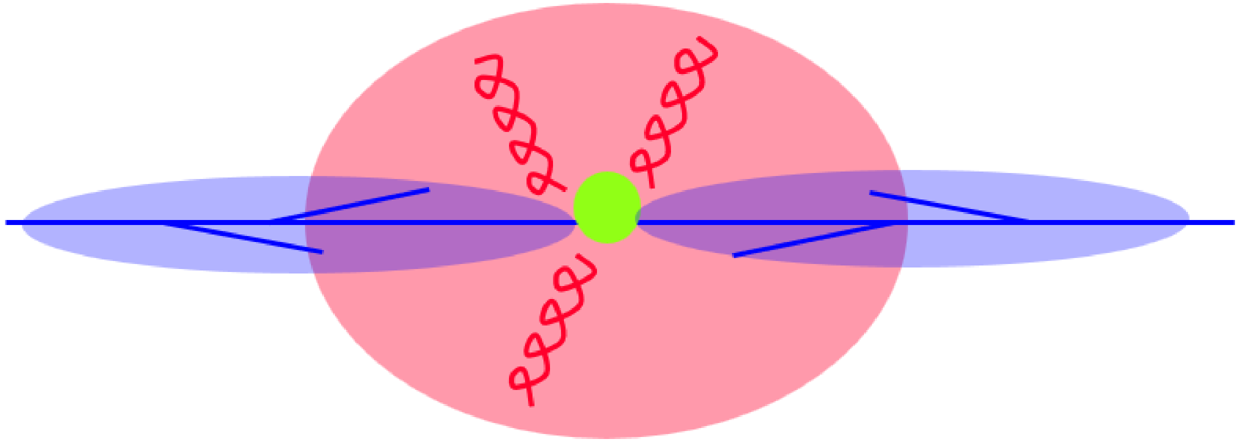
Michel, Pietrulewicz, Tackman, arXiv:1810.12911

SCET Factorisation

The below p_T jet cut cross section for quark-initiated processes can be factorised in SCET without a rapidity cut as follows:

$$\sigma\left(p_{Tj} < p_T^{\mathrm{cut}}\right) = H_{qq'}\left(Q,\mu\right) B_q\left(Q, p_T^{\mathrm{cut}}, x_q, \mu, \nu\right) B_{q'}\left(Q, p_T^{\mathrm{cut}}, x_{q'}, \mu, \nu\right) S\left(p_T^{\mathrm{cut}}, \mu, \nu\right) + \mathcal{O}\left(\frac{p_T^{\mathrm{cut}}}{Q}\right)$$

Tackmann, Walsh, Zuberi, arXiv:1206.4312

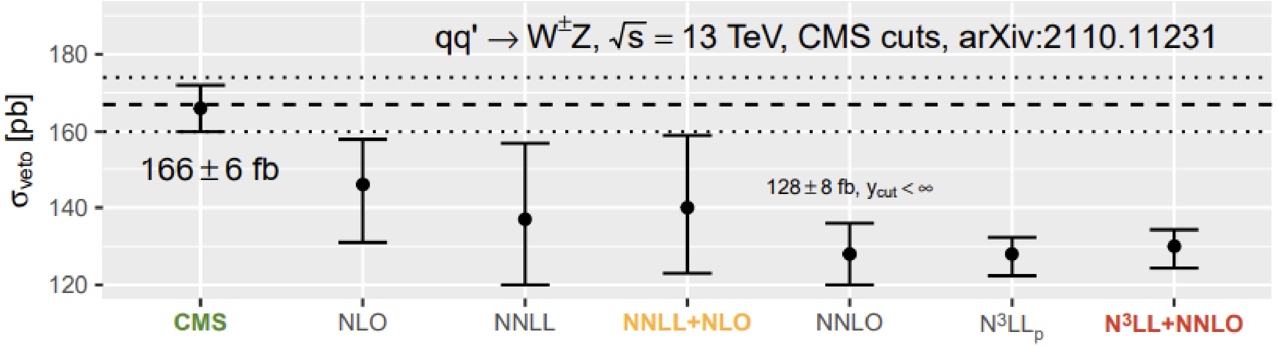


$$B_i\left(x\right) = \sum_j \int_x^1 \frac{dz}{z} \mathcal{I}_{ij}\left(z\right) f_j\left(\frac{x}{z}\right)$$

Motivation: WZ Production at the LHC

The previous factorisation formula does not include any rapidity cut in the jet definition

This can lead to mismatch between theory and experimental predictions where such a cut is often used



in this case, and that the use of y_{cut}-dependent beam functions is necessary to provide a reliable theoretical prediction. Overall, these results highlight the importance of using appropriate

Campbell, Ellis, Neumann, Seth arXiv:2301.11768

$$\sigma_0\left(p_{Tj} < p_T^{\mathrm{cut}}, \eta_{\mathrm{cut}}, R\right) = H_{q\bar{q}}\left(Q^2,\mu\right) B_q\left(p_T^{\mathrm{cut}}, \eta_{\mathrm{cut}}, R, \omega_q, \mu, \nu\right) B_{\bar{q}}\left(p_T^{\mathrm{cut}}, \eta_{\mathrm{cut}}, R, \omega_{\bar{q}}, \mu, \nu\right) S\left(p_T^{\mathrm{cut}}, R, \mu, \nu\right) + \mathcal{O}\left(\frac{p_T^{\mathrm{cut}}}{Q}, e^{-\eta_{\mathrm{cut}}}\right)$$

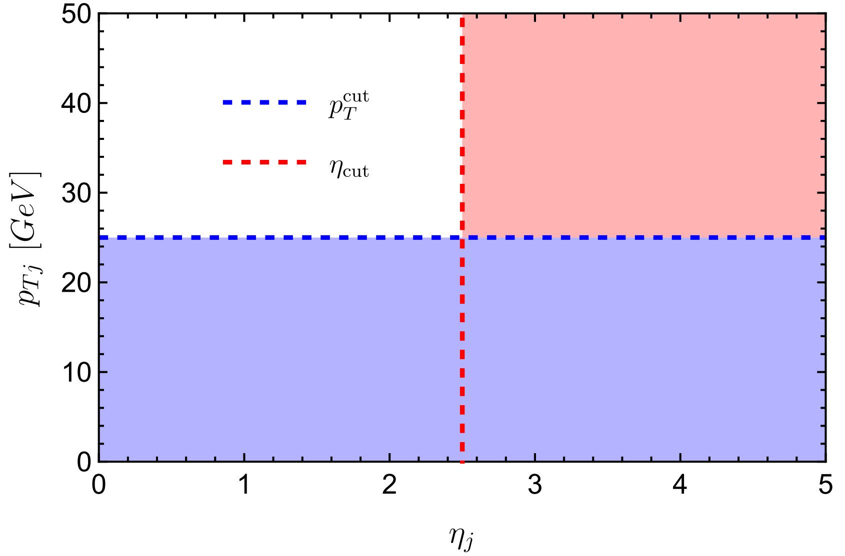
Michel, Pietrulewicz, Tackman, arXiv:1810.12911

Calculation: Overview

Most of the phase space is shared between the standard leading jet transverse momentum beam functions and the rapidity dependent beam functions

The calculation has been done at NLO (see arXiv:1810.12911), in this work the NNLO calculation is completed

$$B_q\left(p_T^{\rm cut},\eta_{\rm cut}\right) = B_q\left(p_T^{\rm cut}\right) + \Delta B_q\left(p_T^{\rm cut},\eta_{\rm cut}\right)$$



$$\Delta\mathcal{I}_{ij}^{(1)}\left(z,\zeta_{\rm cut}\right) = \theta\left(\frac{\zeta_{\rm cut}}{1+\zeta_{\rm cut}} - z\right) 4P_{ij}\left(z\right)\ln\left(\frac{1-z}{z}\zeta_{\rm cut}\right)$$

$$\begin{aligned}\mathcal{M}_{\Delta B}^{RR} =& \theta(\Delta R < R)\theta(\eta_t > \eta_{\rm cut})\theta(k_{t\perp} > p_T^{\rm cut}) \\ &+\theta(\Delta R > R)\left[\theta(k_{2T} < p_T^{\rm cut})\theta_{k_1} + \theta(k_{1T} < p_T^{\rm cut})\theta_{k_2} + \theta_{k_1}\theta_{k_2}\right]\end{aligned}$$

$$\theta_{k_i} = \theta(y_i > \eta_{\rm cut})\theta(k_{iT} > p_T^{\rm cut})$$

Calculation: Simple Addition

The calculation is further split by introducing a simplified measurement where all emissions are clustered regardless of the separation:

$$\mathcal{M}^{\text{Simple}} = \theta\left(\eta_t > \eta_{\text{cut}}\right)\theta\left(k_{t\perp} > p_T^{\text{cut}}\right) \qquad \mathcal{M}^{\mathcal{A}} = \mathcal{M}^{\text{Total}} - \mathcal{M}^{\text{Simple}}$$

See arXiv:1102.4344, 1206.4312, 1412.2126, 1608.01999, 2204.02987, 2512.23398, 2604.13176 etc

This simple measurement contribution will contain all the real-virtual contributions and acts as a 'counter term' for divergences from collinear splittings

Previously existing fully differential beam functions in the virtuality and transverse momentum of the incoming quark can be used to calculate the simple addition contribution:

$$I_G[f(t,\vec{k}_\perp^2)] = \theta\left(\frac{\zeta_{\text{cut}}}{1+\zeta_{\text{cut}}} > x\right)\int_{\frac{x}{1-x}(p_T^{\text{cut}})^2}^{\frac{1-x}{x}(\zeta_{\text{cut}}p_T^{\text{cut}})^2} \mathrm{d}t \int_{(p_T^{\text{cut}})^2}^{\frac{1-x}{x}t} \mathrm{d}(\vec{k}_\perp^2) f(t,\vec{k}_\perp^2)$$

$$\Delta B_q^{\text{Simple}}\left(p_T^{\text{cut}},\zeta_{\text{cut}},x_q,\mu\right) = I_G\left[B_q\left(t,\vec{k}_\perp^2,x_q,\mu\right)\right] \qquad \zeta_{\text{cut}} = \frac{x_q E_{\text{CM}}}{p_T^{\text{cut}}}e^{-\eta_{\text{cut}}}$$

Gaunt, Stahlhofen, arXiv:1409.8281

Calculation: Renormalization

By considering the renormalization of all beam functions involved, a total expression for the addition can be found:

$$\Delta\mathcal{I}_{ij}^{(2)}(\eta_{\text{cut}}, p_T^{\text{cut}}, z) = I_G\left[\mathcal{I}_{ij}^{(2)}(\vec{k}_\perp^2, t, z)\right] + I_{\mathcal{A}}\left[\mathcal{M}^{\mathcal{A}}\mathcal{A}_{ij}^{(2)}\right] + I_{\mathcal{A}}\left[\mathcal{M}^{y\to\eta}\mathcal{A}_{ij}^{(2)}\right]$$
$$+2\sum_k I_G\left[\mathcal{I}_{ik}^{(1)}(\vec{k}_\perp^2, t, z)\otimes_z f_{k/j}^{(1)}(z)\right] + I_G\left[Z_i^{B(1)}(t,z)\otimes_t \mathcal{I}_{ij}^{(1)}(\vec{k}_\perp^2, t, z)\right]$$
$$-2\sum_k \Delta\mathcal{I}_{ik}^{(1)}(\eta_{\text{cut}}, p_T^{\text{cut}}, z)\otimes_z f_{k/j}^{(1)}(z) - Z_i^{B(1)}(p_T^{\text{cut}}, z)\times\Delta\mathcal{I}_{ij}^{(1)}(\eta_{\text{cut}}, p_T^{\text{cut}}, z)$$

Simple addition terms | Amplitude terms | Renormalization terms

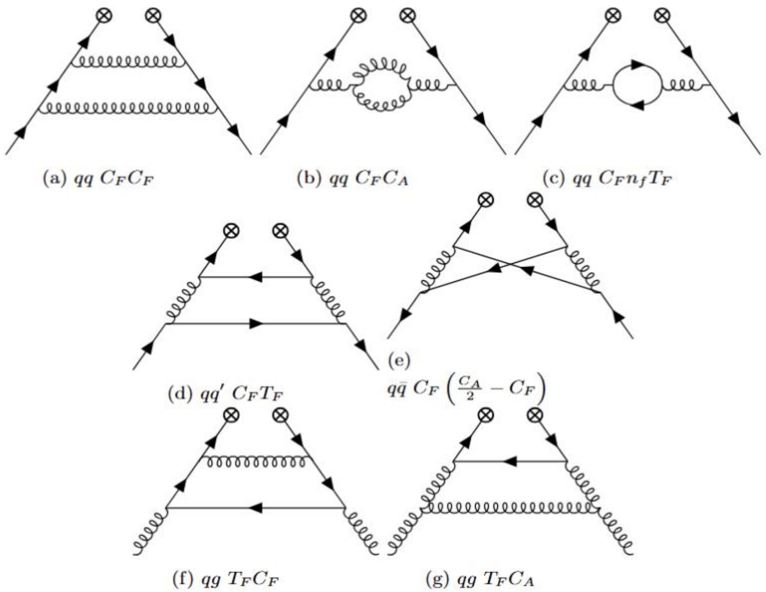
Most terms can be calculated in a similar manner to the simple addition; the amplitude terms are the most interesting

Calculation: Amplitude Terms

The remaining terms can be calculating using known double-real emission amplitudes (see arXiv:1401.5478, arXiv:1405.1044)

Subtraction terms are calculated by analytically integrating the measurement and amplitude simplified in various divergent limits

Dimensional regularization and a rapidity regulator are used to extract the poles



$$I_{\mathcal{A}}[\mathcal{M}^{\mathcal{A}}\mathcal{A}_{ij}^{(2)} - \sum_n \hat{L}_n[\mathcal{M}^{\mathcal{A}}\mathcal{A}_{ij}^{(2)}]] + I_{\mathcal{A}}[\sum_n \hat{L}_n[\mathcal{M}^{\mathcal{A}}\mathcal{A}_{ij}^{(2)}]]$$

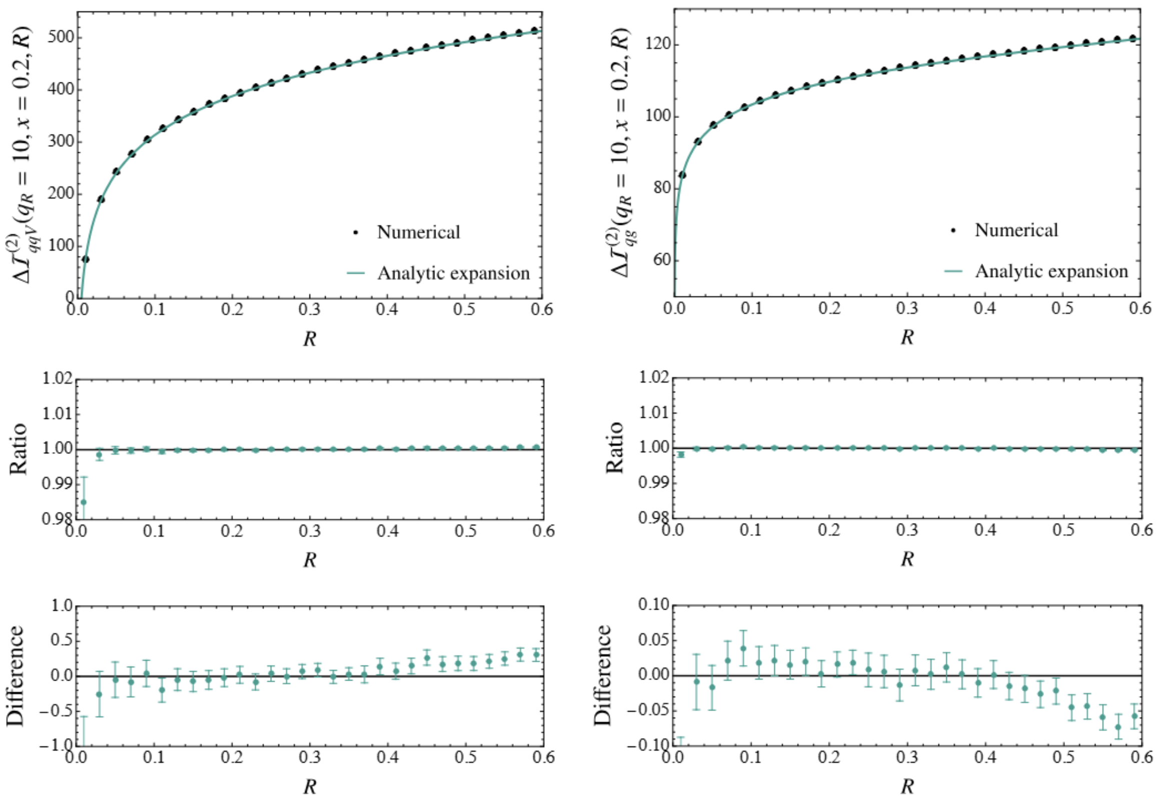
$$d = 4 - 2\epsilon, \left(\frac{\nu}{k_2^-}\right)^{\eta} \to \frac{1}{\epsilon^n}, \frac{1}{\eta^m}, \frac{1}{\epsilon^n \eta^m}$$

Calculation: Jet Radius Dependence

Calculating the full jet radius dependence analytically is beyond the scope of this calculation

By taking small R limits of the measurement and amplitudes, the log R term along with all terms up to R^2 have been calculated analytically (and up to R^4 in some channels)

These can be used to extrapolate from R = 0.2, at which the numerical results are fitted



Global Slicing

Below-cut cross sections can be calculated using both the jet p_T and rapidity-dependent beam functions and combined with an above-cut cross section to obtain an inclusive cross section for a test process (e.g. on-shell Z production)

This above-cut term can be easily calculated using programs such as MadGraph as it only contains NLO type divergences

$$\sigma_{\mathrm{FO}} = \sigma_{\mathrm{Below}}^{\mathrm{SCET}}\left(p_T^{\mathrm{cut}}\right) + \sigma_{\mathrm{Above}}\left(p_T^{\mathrm{cut}}\right) + \mathcal{O}\left(\frac{p_T^{\mathrm{cut}}}{Q}\right)$$

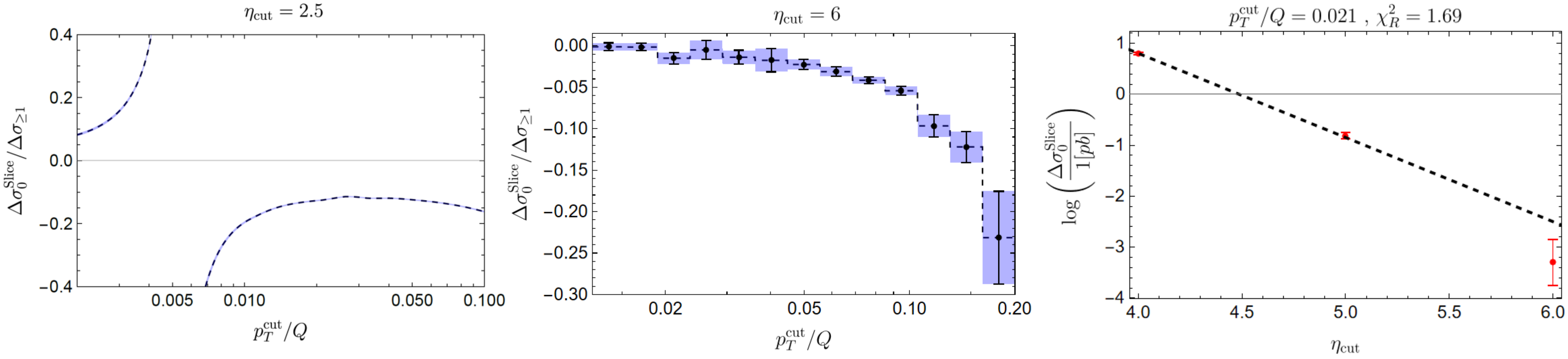
$$\sigma_{\mathrm{FO}} = \sigma_{\mathrm{Below}}^{\mathrm{SCET}}\left(p_T^{\mathrm{cut}}, \eta_{\mathrm{cut}}\right) + \sigma_{\mathrm{Above}}\left(p_T^{\mathrm{cut}}, \eta_{\mathrm{cut}}\right) + \mathcal{O}\left(\frac{p_T^{\mathrm{cut}}}{Q}, e^{-\eta_{\mathrm{cut}}}\right)$$

$$\Delta\sigma_{\mathrm{Below}}^{\mathrm{SCET}}\left(p_T^{\mathrm{cut}}, \eta_{\mathrm{cut}}\right) = -\Delta\sigma_{\mathrm{Above}}\left(p_T^{\mathrm{cut}}, \eta_{\mathrm{cut}}\right) + \mathcal{O}\left(\frac{p_T^{\mathrm{cut}}}{Q}, e^{-\eta_{\mathrm{cut}}}\right)$$

Global Slicing

By choosing specific cut values, the power corrections can be reduced to validate the below cut prediction and therefore the calculation of the beam functions

The size of deviation can also be compared to the expected size of the power corrections and behave as expected



WZ Addition Calculation

We follow the theoretical setup seen in arXiv:2301.11768 (complex mass scheme, fiducial cuts, no use of profile scales etc)

The 0-jet WZ cross section including the rapidity cut moves closer to the experimental prediction; an ~ 3σ discrepancy is still present (compared to the initial ~ 4σ discrepancy)

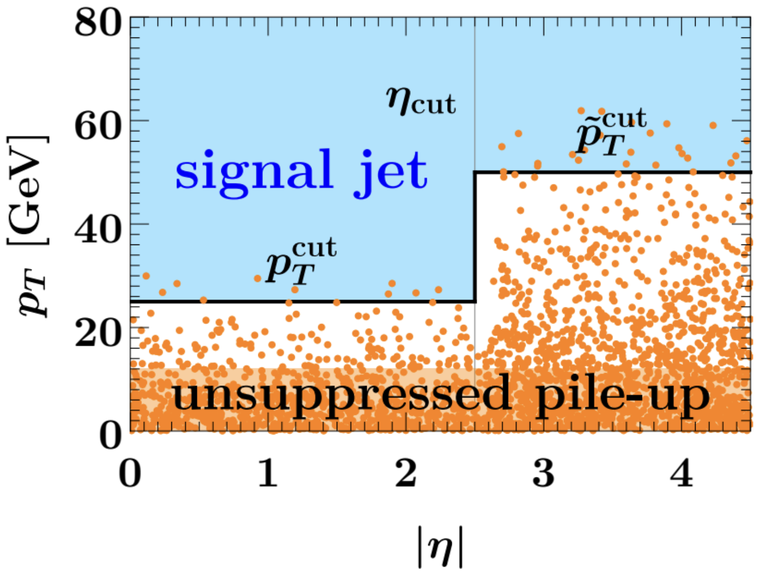
	$\sigma_0\left(p_T^{\rm cut}\right)$ [fb]	$\sigma_0\left(p_T^{\rm cut}, \eta_{\rm cut}\right)$ [fb]
NNLO		
σ_0	128 ± 8	137 ± 8
$\lvert\sigma_0^{\rm CMS} - \sigma_0\rvert$	$38 \pm 10\,(3.8\sigma)$	$29 \pm 10\,(2.9\sigma)$
NNLL′		
σ_0	128^{+4}_{-5}	139^{+4}_{-5}
$\lvert\sigma_0^{\rm CMS} - \sigma_0\rvert$	$38^{+7}_{-8}\,(4.9\sigma)$	$27^{+7}_{-8}\,(3.5\sigma)$
NNLO+NNLL′		
σ_0	130^{+4}_{-6}	139^{+4}_{-6}
$\lvert\sigma_0^{\rm CMS} - \sigma_0\rvert$	$36^{+7}_{-8}\,(4.2\sigma)$	$27^{+7}_{-8}\,(3.5\sigma)$

The CMS Collaboration arXiv:2110.11231
Campbell, Ellis, Neumann, Seth arXiv:2301.11768

What’s Next?

Calculation of gluon rapidity dependent beam functions

Calculation of finite step rapidity dependent beam function



Michel, Pietrulewicz, Tackman, arXiv:1810.12911

Summary

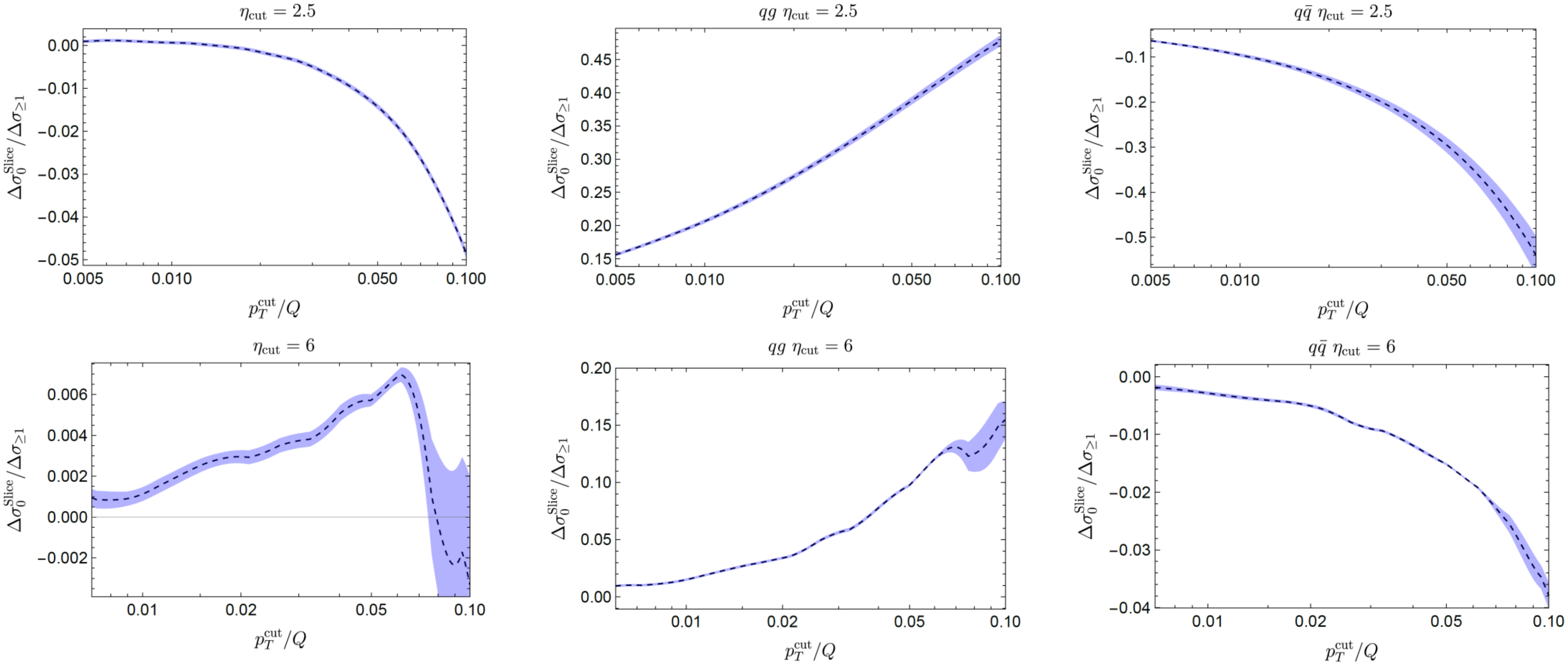
Successfully calculate the quark rapidity dependent beam function at NNLO

Working semi-analytic + numerical implementations

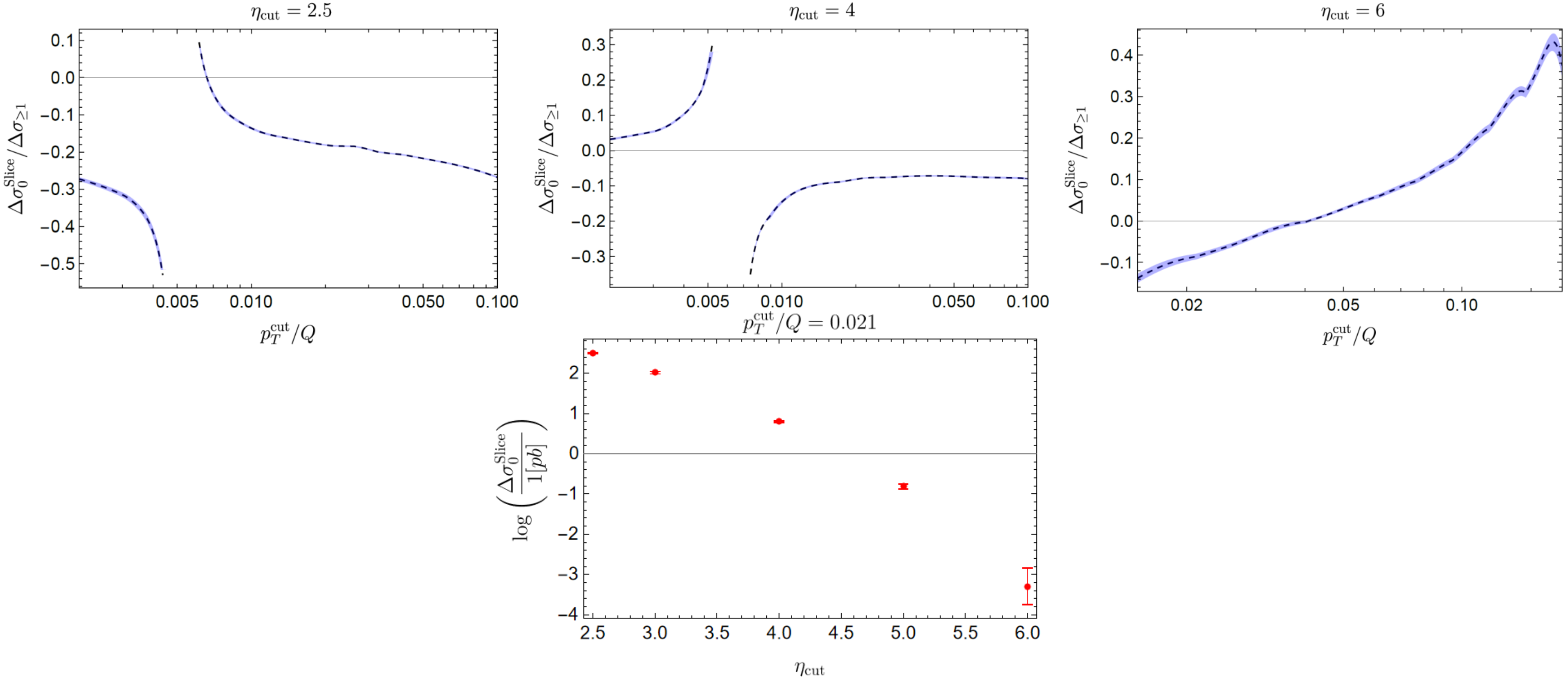
Calculated the resummed addition to the WZ 0-jet cross section; showed this was not sufficient to explain the discrepancy

Predictions for gluon rapidity dependent beam functions and finite step veto to come in the future

Backup: NLO Slicing

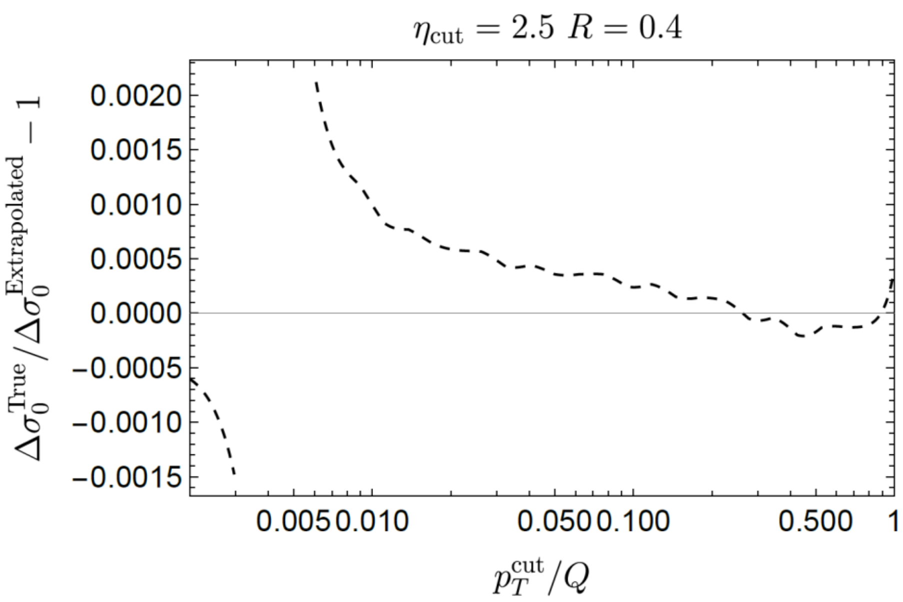


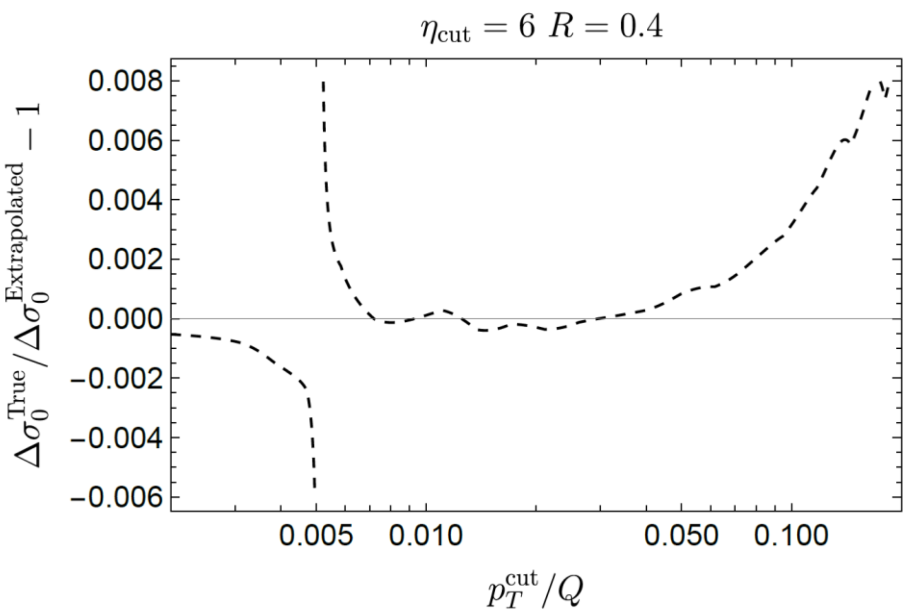
Backup: NNLO Slicing



Backup: R Extrapolation

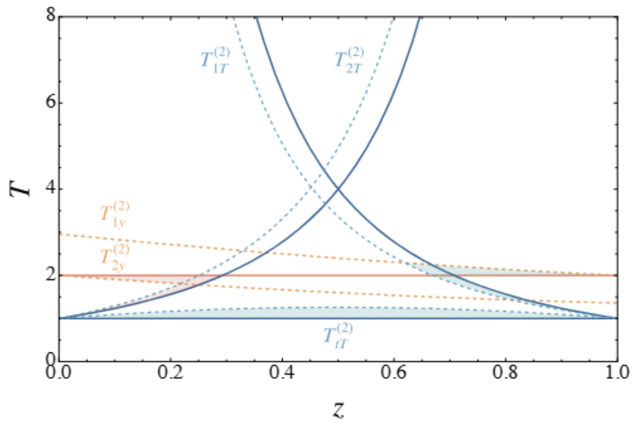
$$\Delta\mathcal{I}_{ij}^{(2)}\left(\zeta_{\text{cut}}, x, R\right) = \Delta\mathcal{I}_{ij}^{(2)}\left(\zeta_{\text{cut}}, x, R = 0.2\right) + \log\left(R/0.2\right)\Delta I_{ij}^{(2,\log R)}\left(\zeta_{\text{cut}}, x\right) + \sum_{i=1}^{n}\left(R^i - 0.2^i\right)\Delta I_{ij}^{(2,R^i)}\left(\zeta_{\text{cut}}, x\right)$$



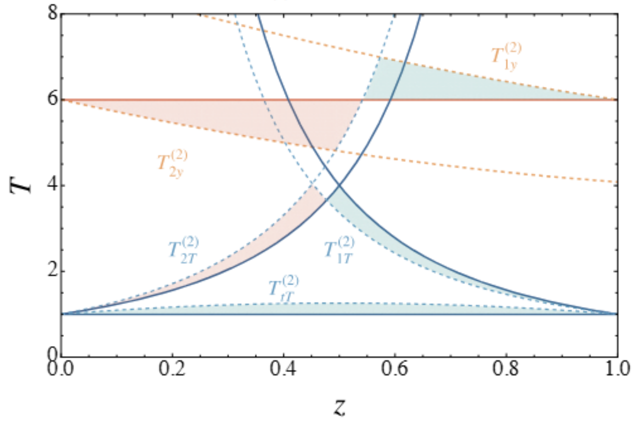


Backup: R Calculation

	$\mathcal{A}^{(2,-2)}$	$\mathcal{A}^{(2,-1)}$	$\mathcal{A}^{(2,0)}$
$\mathcal{M}^{\mathcal{A}}_{\Delta R^0}$	$\log R$	$0 \times R$	R^2
$\mathcal{M}^{\mathcal{A}}_{\Delta R^1} - \mathcal{M}^{\mathcal{A}}_{\Delta R^0}$	R	R^2	R^3
$\mathcal{M}^{\mathcal{A}}_{\Delta R^2} - \mathcal{M}^{\mathcal{A}}_{\Delta R^1}$	R^2	R^3	R^4



(a) $Q_1 < 4Q_0$



(b) $Q_1 > 4Q_0$

Figures made by
Emmet Byrne

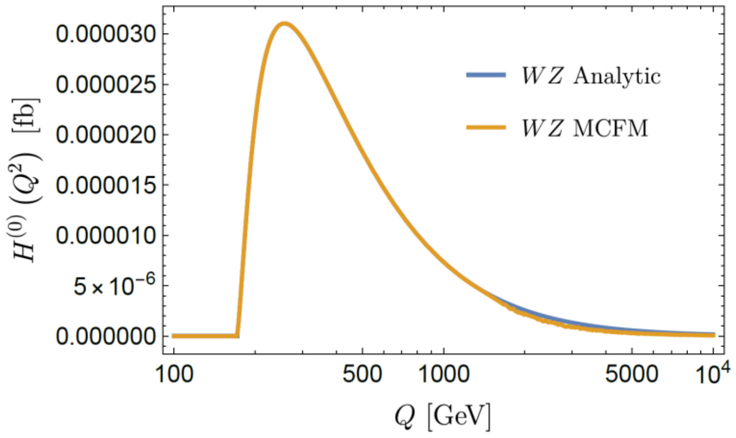
Backup: Hard Function Extraction

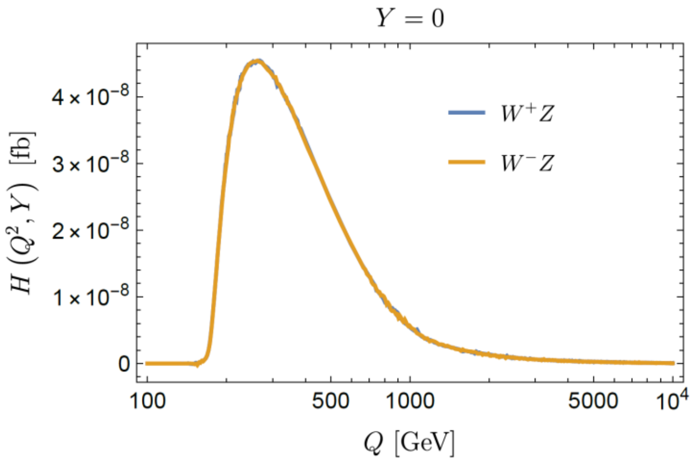
Extract the FO differential cross section numerically from implementation in MCFM

$$\left.\frac{d\sigma_0^{\mathrm{Bin}}}{dQ^2 dY}\right|_{Q=(Q_L^{\mathrm{Bin}}+Q_U^{\mathrm{Bin}})/2 \& Y=(Y_L^{\mathrm{Bin}}+Y_U^{\mathrm{Bin}})/2} = \frac{\sigma_0^{\mathrm{Bin}}}{\left((Q_U^{\mathrm{Bin}})^2-(Q_L^{\mathrm{Bin}})^2\right)\left(Y_U^{\mathrm{Bin}}-Y_L^{\mathrm{Bin}}\right)}$$

Match order by order to SCET factorised form

$$\frac{d\sigma_0}{dQ^2 dY} = H^{W^\pm Z} \sum_{q_i, q_j} S_{qq}\left(p_T^{\mathrm{cut}}\right) B_{q_i}\left(p_T^{\mathrm{cut}}\right) B_{q_j}\left(p_T^{\mathrm{cut}}\right) + \mathcal{O}\left(\frac{p_T^{\mathrm{cut}}}{Q}\right)$$





Backup: WZ Addition Setup

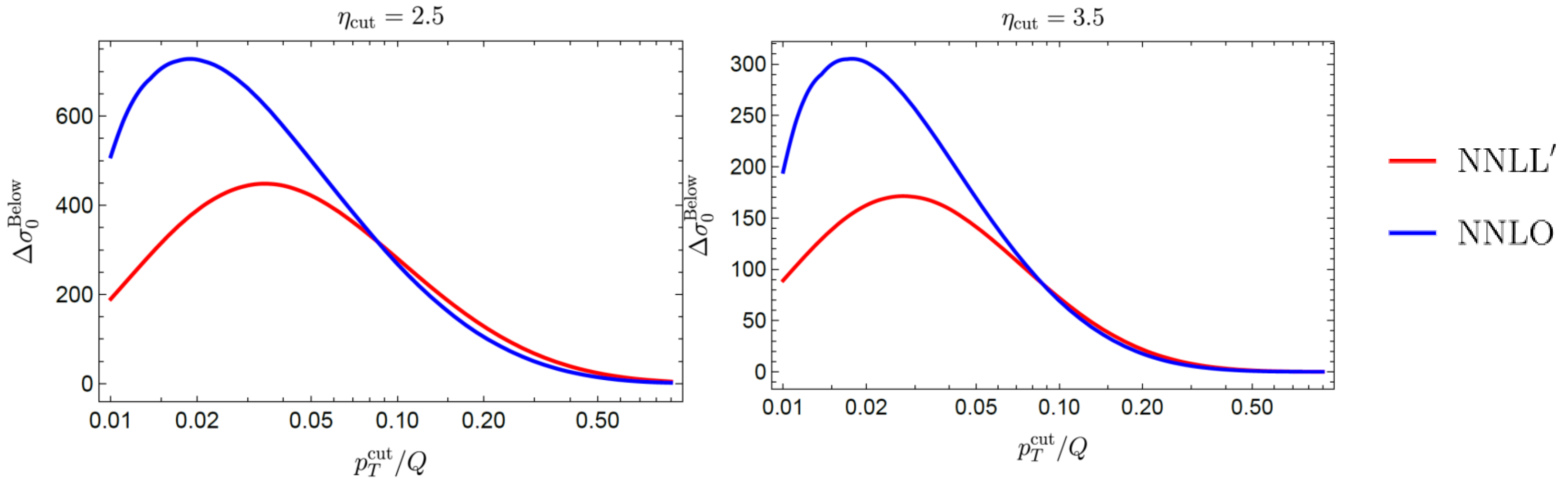
Resummed term is calculated using a SCET factorisation formula, where all scales are evolved to resumm the large logs using the factorisation functions corresponding RGEs, e.g.:

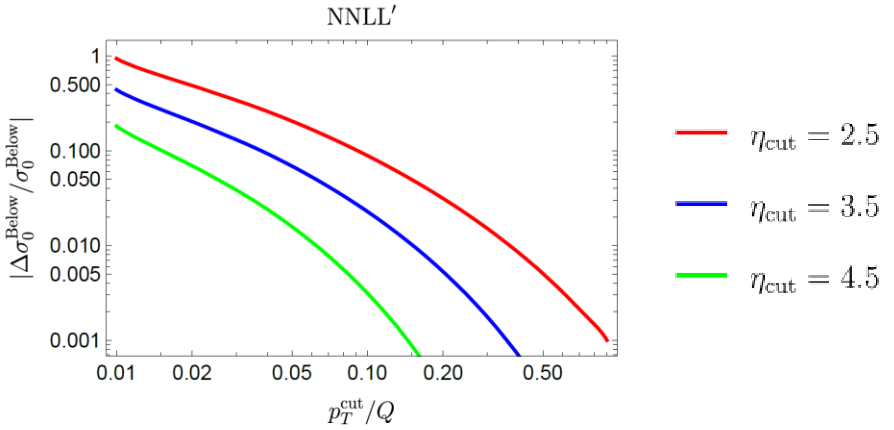
$$H\left(Q^2, Y, \mu_H, \mu\right) = U_H\left(Q^2, \mu_H, \mu\right) H\left(Q^2, Y, \mu_H\right) \qquad U_H\left(Q^2, \mu_H, \mu\right) = \left| e^{-2K_\Gamma(\mu_H,\mu)+K_\gamma(\mu_H,\mu)} \left(\frac{-Q^2 - i0}{\mu_H^2}\right)^{\eta_\Gamma(\mu_H,\mu)} \right|^2$$

Non-singular term calculated by matching to the FO result

$$\Delta\sigma_0^{\rm NS}\left(p_T^{\rm cut}, \eta_{\rm cut}\right) = \Delta\sigma_0^{\rm N^kLO}\left(p_T^{\rm cut}, \eta_{\rm cut}\right) - \Delta\sigma_0^{\rm N^kLL'}\left(p_T^{\rm cut}, \eta_{\rm cut}\right)\Big|_{\rm exp.to N^kLO}$$

Backup: PS Importance





Backup: Fitting

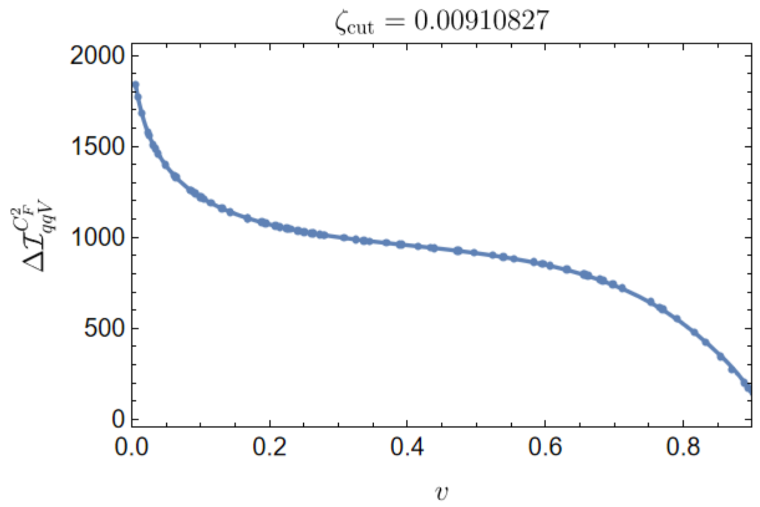
The remaining numerical parts of the calculation depend on 2 variables:

$$\{v, \zeta_{\text{cut}}\},\ v = \frac{1+\zeta_{\text{cut}}}{\zeta_{\text{cut}}} z$$

To fit we use a 2-D Bernstein basis along with carrier functions to describe endpoint behaviour:

$$F_{\text{Base}} = F_{\text{car}}(v, \zeta_{\text{cut}}) \{1, \binom{n_v}{i} \binom{n_\zeta}{j} (1-v)^{n_v - i} v^i (1-\zeta_{\text{cut}})^{n_\zeta - j} \zeta_{\text{cut}}{}^j\}$$

$$i \in \{0, n_v\}, j \in \{0, n_\zeta\}$$



Fitting procedure based on Fantomas, arXiv:2511.15657