Rapidity regulators for low x QCD: F_L at NLO in the dipole factorization

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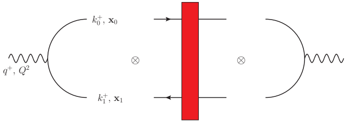
Dipole factorization of DIS at low x: from LO to NLO

Regge-Gribov limit $x_{Bj} \to 0$ at fixed $Q^2 > 1$ GeV^2

At leading power in x_{Bj} (Eikonal approximation), and using optical theorem:

Dipole factorization of inclusive DIS structure functions $F_{T,L}(x_{Bj}, Q^2)$

At LO in α_s (Nikolaev and Zakharov (1991)) :



$$F_{T,L} \propto \sigma^{\gamma}_{T,L} = 2N_c \widetilde{\sum_{q_0\bar{q}_1 \text{ F. states}}} (2q^+)2\pi\delta(k_0^+ + k_1^+ - q^+) \left|\widetilde{\psi}_{\gamma_{T,L}\to q_0\bar{q}_1}\right|^2 \mathrm{Re}\left[1 - \mathcal{S}_{01}\right] + O(\alpha_s)$$

Eikonal multiple scattering of each parton on the target resummed thanks to infinite Wilson lines $U_{F,A}(\mathbf{x}_n)$ along the x^+ direction through the hadronic target
$\to$ nonlinear effects of gluon saturation included

$q\bar{q}$ dipole operator: $$\mathcal{S}_{01} = \frac{1}{N_c} \mathrm{Tr}\left(U_F(\mathbf{x}_0)\, U_F^{\dagger}(\mathbf{x}_1)\right)$$

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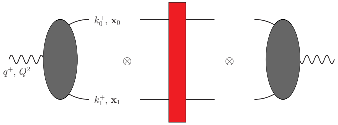
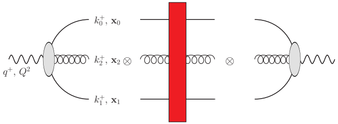
At NLO in α_s:





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$$+ 2N_c C_F \widetilde{\sum_{q_0\bar{q}_1 g_2 \text{ F. states}}} (2q^+)2\pi\delta(k_0^+ + k_1^+ + k_2^+ - q^+) \left|\widetilde{\psi}_{\gamma_{T,L}\to q_0\bar{q}_1 g_2}\right|^2 \mathrm{Re}\left[1 - \mathcal{S}^{(3)}_{012}\right] + O(\alpha_s^2)$$

$q\bar{q}g$ "tripole" operator: $$\mathcal{S}^{(3)}_{012} \equiv \frac{1}{N_c C_F} \mathrm{Tr}\left(t^b U_F(\mathbf{x}_0)\, t^a U_F^\dagger(\mathbf{x}_1)\right) U_A(\mathbf{x}_2)_{ba}$$

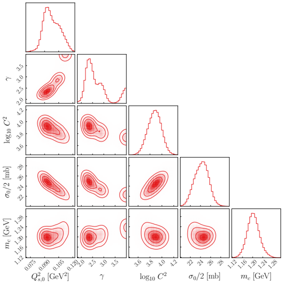
- Massless quarks case at NLO: G.B. (2016-2017) (see also Balitsky, Chirilli (2011-2013))
- Massive quarks case at NLO: G.B., Lappi, Paatelainen (2021-2022)

First dipole fit at NLO+NLL accuracy

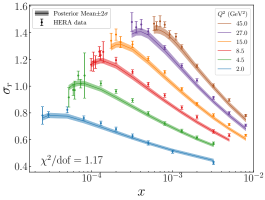
Fit to HERA data on inclusive and charm reduced DIS cross sections, using:

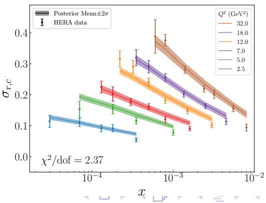
- Dipole factorization formula at NLO with massive charm quark and 3 massless flavors
- Low x evolution of dipole with full NLO BK equation

  (with Gaussian process emulator)
- Uncertainties estimated via Bayesian inference



Casuga, Mäntysaari (2026)





NLO CGC calculations : with standard cut-off

In NLO calculations at low x_{Bj} with gluon saturation, for evolution equations, or DIS or pA observables:

Most frequently used regularization technique (in particular in LFPT):

1. Perform transverse integration in dim. reg.
2. Expand in ϵ
3. And then perform integrations over k^+ momenta regulated by a cut off k^+_{min}

Issues with this regularization procedure:

- Does not distinguish clearly soft divergences from rapidity/low x divergences
- Ambiguities in the definition of non-eikonal power corrections at low x
- Difficult to compare results with other pQCD communities, like TMD, jets, etc...

NLO CGC calculations : with standard cut-off

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1. Perform transverse integration in dim. reg.
2. Expand in ϵ
3. And then perform integrations over k^+ momenta regulated by a cut off $k^+_{\min}$

Issues with this regularization procedure:

- Scheme dependence in the definition of low x evolution equations (BFKL, BK, B-JIMWLK):

  Evolution/ordering along k^+, or along k^-, or along rapidity?

  $\Rightarrow$ Identical results at LO+LL accuracy

  $\Rightarrow$ Scheme dependence at higher orders: (anti-)collinear logs in observables and in evolution kernel

  For DIS observables: only k^- evolution scheme allow smooth matching with DGLAP evolution of the target (with large P^-)

  But cut-off regularization in k^+ forces k^+ as evolution variable

Rapidity regulators from pQCD/TMD

Many new regulators for rapidity divergences have been proposed by the TMD and SCET communities in the last 15 years

Some of them should be suitable as well in the context of low x physics/CGC, for example:

- η regulator, Chiu, Jain, Neill, Rothstein, 2011-2012
- analytic regulator, Becher, Neubert, 2011; Becher, Bell, 2012
- pure rapidity regulator, Ebert, Moult, Stewart, Tackmann, Vita, Zhu, 2019

The η regulator has been used for CGC observables at NLO, but in the language of SCET, in Liu, Kang, Liu, 2020; Liu, Xie, Kang, Liu, 2022

Using rapidity regulators in NLO CGC calculations

Proposal: 3 versions of rapidity regularisation for CGC calculations at NLO:

Introduce a factor in the loop integrand (with gluon momentum k)

- regulator in k^+: $\left(\frac{k^+}{\nu_B^+}\right)^\eta$ (inspired by analytic and η regulators)
- regulator in k^-: $\left(\frac{\nu_B^-}{k^-}\right)^\eta \sim \left(\frac{2k^+\nu_B^-}{\mathbf{k}^2}\right)^\eta$
- pure rapidity regulator: $\left(\frac{k^+}{k^-}\frac{\nu_B^-}{\nu_B^+}\right)^{\frac{\eta}{2}} \sim \left(\frac{2(k^+)^2\nu_B^-}{\mathbf{k}^2\nu_B^+}\right)^{\frac{\eta}{2}}$

In the 3 cases, transforms divergent dk^+/k^+ integrals over k^+ into $dk^+(k^+)^{-1+\eta}$.

Analogy with dim.reg. : $\eta \leftrightarrow \epsilon$ and $\nu_B^\pm \leftrightarrow \mu$

Order of limits: take $\eta \to 0$ at finite ϵ, and later expand in ϵ.

$\Rightarrow$ η regulates only rapidity/low x div., whereas ϵ regulates also soft div.

Aim: revisit the calculation of NLO DIS (F_L, massless quarks) (G.B., 2016-2017) with the $+$ and $-$ versions of the regulator validate their implementation in CGC in LFPT.

Remark: results with *pure rapidity regulator* can be obtained from the average of the $+$ and $-$ versions.

Using rapidity regulators in NLO CGC calculations

From a diagram with dim. reg. and a rapidity regulator: typical expression for *rapidity sensitive* terms of the form

$$I(\epsilon,\eta) = \int_0^1 d\xi\, \xi^{-1+\eta}\, f(\xi,\epsilon,\eta)$$

with ξ the k^+ momentum fraction of the gluon in the loop.

Expansion around $\eta = 0$ at finite ϵ:

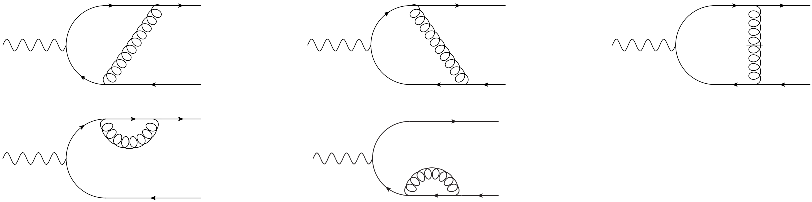
$$\begin{aligned} I(\epsilon,\eta) &= \int_0^1 d\xi\, \xi^{-1+\eta}\, f(0,\epsilon,\eta) + \int_0^1 d\xi\, \xi^{-1+\eta}\, \left[f(\xi,\epsilon,\eta) - f(0,\epsilon,\eta)\right] \\ &= \frac{1}{\eta}\, f(0,\epsilon,\eta) + \int_0^1 \frac{d\xi}{(\xi)_+}\, f(\xi,\epsilon,0) + O(\eta) \\ &= \frac{1}{\eta}\, f(0,\epsilon,0) + (\partial_\eta f)\,(0,\epsilon,\eta=0) + \int_0^1 \frac{d\xi}{(\xi)_+}\, f(\xi,\epsilon,0) + O(\eta) \end{aligned}$$

$\Rightarrow$ η pole and scheme dependent terms, and $+$ distribution

$\gamma_L^* \to q\bar{q}$ at one loop in LFPT

One loop corrections to the $\gamma_L^* \to q\bar{q}$ Light-Front wave function in momentum space found to factorize as:

$$\Psi^{NLO}_{\gamma_L^* \to q\bar{q}} = \left(1 + \frac{\alpha_s C_F}{2\pi}\, \mathcal{V}^L\right) \Psi^{LO}_{\gamma_L^* \to q\bar{q}}$$



Only vertex corrections diagrams need a rapidity regulator

But in earlier calculation with cut-off in k^+: all diagrams gave logs of the cut-off

One-loop $\gamma_L^* \to q\bar{q}$ LFWF in momentum space

With the 3 vertex corrections and 2 self-energy NLO diagrams for the $\gamma_L^* \to q\bar{q}$ LFWF:

- Result with rapidity regulator in k^+ (with $\overline{Q}^2 \equiv z(1-z)Q^2$):

$$\mathcal{V}^L\Big|^{\eta+} = \left[\frac{2}{\eta} + 2\log\left(\frac{\sqrt{z(1-z)}q^+}{\nu_B^+}\right) - \frac{3}{2}\right]\Gamma(1+\epsilon)\left[\frac{4\pi\mu^2}{\mathbf{P}^2+\overline{Q}^2}\right]^\epsilon \left[\frac{\mathbf{P}^2}{\mathbf{P}^2+\overline{Q}^2}\right]^\epsilon \mathrm{B}\left(\frac{\mathbf{P}^2}{\mathbf{P}^2+\overline{Q}^2}; -\epsilon, -\epsilon\right)$$
$$+\frac{1}{2}\left[\log\left(\frac{z}{1-z}\right)\right]^2 - \frac{\pi^2}{6} + \frac{(5+\delta_s)}{2} + O(\epsilon) + O(\eta)$$

Note: Coefficient of $1/\eta$ with full dependence in ϵ, required by proper ordering of $\eta \to 0$ and $\epsilon \to 0$ limits

- Result with rapidity regulator in k^-:

$$\mathcal{V}^L\Big|^{\eta-} = \left[\frac{2}{\eta} + 2\log\left(\frac{2\sqrt{z(1-z)}q^+\nu_B^-}{\overline{Q}^2}\right) - \frac{3}{2}\right]\Gamma(1+\epsilon)\left[\frac{4\pi\mu^2}{\mathbf{P}^2+\overline{Q}^2}\right]^\epsilon \left[\frac{\mathbf{P}^2}{\mathbf{P}^2+\overline{Q}^2}\right]^\epsilon \mathrm{B}\left(\frac{\mathbf{P}^2}{\mathbf{P}^2+\overline{Q}^2}; -\epsilon, -\epsilon\right)$$
$$+2\frac{S_\epsilon}{\epsilon^2}\left[\frac{\overline{Q}^2}{\mu^2}\right]^{-\epsilon} + 2\,\mathrm{Li}_2\left(\frac{\mathbf{P}^2}{\mathbf{P}^2+\overline{Q}^2}\right) - 3\left[\log\left(\frac{\mathbf{P}^2+\overline{Q}^2}{\overline{Q}^2}\right)\right]^2$$
$$+\frac{1}{2}\left[\log\left(\frac{z}{1-z}\right)\right]^2 - \frac{2\pi^2}{3} + \frac{(5+\delta_s)}{2} + O(\epsilon) + O(\eta)$$

$\to$ New: double pole in ϵ, and non-trivial dependence on relative momentum $\mathbf{P}$ of the dipole.

One-loop $\gamma_L^* \to q\bar{q}$ LFWF in mixed space

Taking Fourier transform from $\mathbf{P}$ to dipole size $\mathbf{x}_{01}$:

One loop corrections to the $\gamma_L^* \to q\bar{q}$ LFWF still factorizes in mixed space:

$$\tilde{\Psi}^{NLO}_{\gamma_L^* \to q\bar{q}} = \left(1 + \frac{\alpha_s C_F}{2\pi}\, \tilde{\mathcal{V}}^L\right) \tilde{\Psi}^{LO}_{\gamma_L^* \to q\bar{q}}$$

- With rapidity regulator in k^+:

$$\tilde{\mathcal{V}}^L\Big|^{\eta+} = -\left[\frac{2}{\eta} + 2\log\left(\frac{\sqrt{z(1-z)}q^+}{\nu_B^+}\right) - \frac{3}{2}\right] \frac{\Gamma(1-\epsilon)}{\epsilon} \left[\pi\mu^2\mathbf{x}_{01}^2\right]^{\epsilon} + \frac{1}{2}\left[\log\left(\frac{z}{1-z}\right)\right]^2 - \frac{\pi^2}{6} + \frac{(5+\delta_s)}{2} + O(\epsilon) + O(\eta)$$

$\rightarrow$ Very similar as earlier results with cut-off in k^+ from G.B., 2016.

- With rapidity regulator in k^- (with $c_0 \equiv 2e^{-\gamma_E}$):

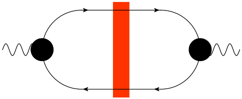
$$\tilde{\mathcal{V}}^L\Big|^{\eta-} = -\left[\frac{2}{\eta} + 2\log\left(\frac{2\sqrt{z(1-z)}q^+\nu_B^-\mathbf{x}_{01}^2}{c_0^2}\right) - \frac{3}{2}\right] \frac{\Gamma(1-\epsilon)}{\epsilon} \left[\pi\mu^2\mathbf{x}_{01}^2\right]^{\epsilon} + 2\frac{S_\epsilon}{\epsilon^2}\left[\frac{\mathbf{x}_{01}^2\mu^2}{c_0^2}\right]^{\epsilon} + \frac{1}{2}\left[\log\left(\frac{z}{1-z}\right)\right]^2 - \frac{\pi^2}{3} + \frac{(5+\delta_s)}{2} + O(\epsilon) + O(\eta)$$

Differences: double pole term in ϵ, and scale for rapidity/low x log.

In both cases: no dependence on Q^2 anymore! (like with k^+ cut-off)

One-loop $\gamma_L^* \to q\bar{q}$ LFWF in mixed space

$q\bar{q}$ contribution to F_L structure function at NLO:



$$F_L|^{q\bar{q}} = 16Q^4\, N_c \sum_f e_f^2 \int_0^1 dz\, z^2(1-z)^2 \int \frac{d^{2-2\epsilon}\mathbf{x}_0}{(2\pi)^2} \int \frac{d^{2-2\epsilon}\mathbf{x}_1}{(2\pi)^2} \mathrm{Re}\left[1-\mathcal{S}_{01}\right]$$
$$\times \left(\frac{4\pi^2\mu^2\mathbf{x}_{01}^2}{\overline{Q}^2}\right)^{\epsilon} \left[\mathrm{K}_\epsilon\left(\overline{Q}|\mathbf{x}_{01}|\right)\right]^2 \left(1 + \frac{\alpha_s C_F}{\pi}\,\tilde{\mathcal{V}}^L\right)$$

- With rapidity regulator in k^+:

$$\tilde{\mathcal{V}}^L\Big|^{\eta+} = -\left[\frac{2}{\eta} + 2\log\left(\frac{\sqrt{z(1-z)}q^+}{\nu_B^+}\right) - \frac{3}{2}\right]\frac{\Gamma(1-\epsilon)}{\epsilon}\left[\pi\mu^2\mathbf{x}_{01}^2\right]^{\epsilon}$$
$$+\frac{1}{2}\left[\log\left(\frac{z}{1-z}\right)\right]^2 - \frac{\pi^2}{6} + \frac{(5+\delta_s)}{2} + O(\epsilon) + O(\eta)$$

$\to$ Very similar as earlier results with cut-off in k^+ from G.B., 2016.

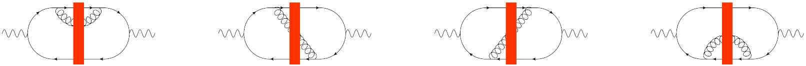
- With rapidity regulator in k^- (with $c_0 \equiv 2e^{-\gamma_E}$):

$$\tilde{\mathcal{V}}^L\Big|^{\eta-} = -\left[\frac{2}{\eta} + 2\log\left(\frac{2\sqrt{z(1-z)}q^+\nu_B^-\mathbf{x}_{01}^2}{c_0^2}\right) - \frac{3}{2}\right]\frac{\Gamma(1-\epsilon)}{\epsilon}\left[\pi\mu^2\mathbf{x}_{01}^2\right]^{\epsilon}$$
$$+2\frac{S_\epsilon}{\epsilon^2}\left[\frac{\mathbf{x}_{01}^2\mu^2}{c_0^2}\right]^{\epsilon} + \frac{1}{2}\left[\log\left(\frac{z}{1-z}\right)\right]^2 - \frac{\pi^2}{3} + \frac{(5+\delta_s)}{2} + O(\epsilon) + O(\eta)$$

Differences: double pole term in ϵ, and scale for rapidity/low x log.
In both cases: no dependence on Q^2 anymore! (like with k^+ cut-off)

$q\bar{q}g$ contribution to F_L: Rapidity safe terms

Other contributions to F_L at NLO at low x_{Bj}: $q\bar{q}g$ Fock state scattering on the target



UV divergent terms for gluon close to the quark $(\mathbf{x}_2 \to \mathbf{x}_0)$ or to the antiquark $(\mathbf{x}_2 \to \mathbf{x}_1)$ with $q\bar{q}g$ tripole operator reducing to $q\bar{q}$ dipole, thanks to color coherence

$\Rightarrow$ Build UV subtraction scheme to explicity cancel UV divergences between $q\bar{q}g$ and $q\bar{q}$ Fock states

Analog to IR subtraction schemes but virtual-virtual UV cancelation!

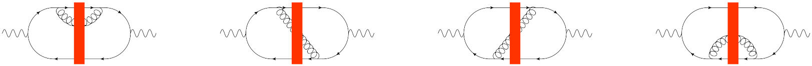
For regular terms at $\xi = 0$: no rapidity regularization needed $\Rightarrow$ same results as G.B., 2017

- Extract UV divergent dipole-like contribution (to be combined with the $q\bar{q}$ contribution)

$$\tilde{\mathcal{V}}^{L}_{q\bar{q}g;\ \xi\ \text{reg.};\ \text{UV}} = -\frac{3}{2}\frac{S_\epsilon}{\epsilon}\left[\frac{\mathbf{x}_{01}^2\mu^2}{c_0^2}\right]^{\epsilon} - \frac{\delta_s}{2} + O(\epsilon)$$

- Same UV-subtracted leftover from the terms regular terms at $\xi = 0$ as in G.B., 2017

$q\bar{q}g$ contribution to F_L: Rapidity sensitive terms



Rapidity divergent piece of the $q\bar{q}g$ contribution:

$$F_L|^{q\bar{q}g}_{1/\xi} = \frac{16Q^4}{2\pi}\, N_c \sum_f e_f^2 \int_0^1 dz\, z^2(1-z)^2 \int d^{2-2\epsilon}\mathbf{x}_0 \int d^{2-2\epsilon}\mathbf{x}_1\, \frac{\alpha_s C_F}{\pi} \int d^{2-2\epsilon}\mathbf{x}_2$$
$$\times\, \mathrm{Re}\left[1-\mathcal{S}^{(3)}_{012}\right] \int_0^1 d\xi\, \frac{2}{\xi} \left\{|\mathcal{I}^j(a)|^2 - \mathrm{Re}\left(\mathcal{I}^j(a)^* \mathcal{I}^j(b)\right)\right\} + (q \leftrightarrow \bar{q})$$

with Fourier integral (and similar for $\mathcal{I}^j(b)$)

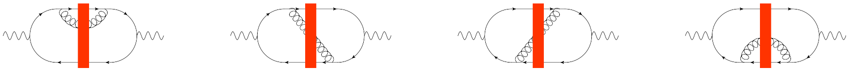
$$\mathcal{I}^j(a) \equiv \mu^{2\epsilon} \int \frac{d^{2-2\epsilon}\mathbf{P}}{(2\pi)^{2-2\epsilon}} \frac{e^{i\mathbf{P}\cdot(\mathbf{x}_{01}+\xi\mathbf{x}_{20})}}{(\mathbf{P}^2+\overline{Q}^2)} \int \frac{d^{2-2\epsilon}\mathbf{K}}{(2\pi)^{2-2\epsilon}} \frac{\mathbf{K}^j\, e^{i\mathbf{K}\cdot\mathbf{x}_{20}}}{\left[\mathbf{K}^2 + \frac{\xi(1-\xi)}{(1-z)}(\mathbf{P}^2+\overline{Q}^2)\right]}$$

Observation: taking $\xi = 0$ in $\mathcal{I}^j(a)$ is equivalent to focusing on its UV regime $\mathbf{x}_2 \to \mathbf{x}_0$ (and $\mathbf{K} \to +\infty$).

Remark on implementation of k^- rapidity reg. : different $\mathbf{K}$ gluon momentum before and after the target

$\Rightarrow$ Insert the factor $(2\xi z q^+ \nu_B^- / \mathbf{K}^2)^{\frac{\eta}{2}}$ in each integral $\mathcal{I}^j(a)$ or $\mathcal{I}^j(b)$.

$q\bar{q}g$ contribution to F_L: + distribution piece



Both rapidity regulators in k^+ and k^- lead to the same + distribution contribution:

$$F_L|^{q\bar{q}g}_{+\text{ dist.}} = \frac{16Q^4}{2\pi} N_c \sum_f e_f^2 \int_0^1 dz\, z^2(1-z)^2 \int d^{2-2\epsilon}\mathbf{x}_0 \int d^{2-2\epsilon}\mathbf{x}_1 \frac{\alpha_s C_F}{\pi} \int d^{2-2\epsilon}\mathbf{x}_2$$
$$\times \operatorname{Re}\left[1-\mathcal{S}^{(3)}_{012}\right] \int_0^1 d\xi \frac{2}{(\xi)_+} \left\{|\mathcal{I}^j(a)|^2 - \operatorname{Re}\left(\mathcal{I}^j(a)^* \mathcal{I}^j(b)\right)\right\} + (q \leftrightarrow \bar{q})$$

But subtracting the $\xi = 0$ value of the bracket simultaneously subtracts its UV behavior
$\Rightarrow$ Fully finite contribution, can take $\epsilon = 0$:

$$F_L|^{q\bar{q}g}_{+\text{ dist.}} = 16Q^4 N_c \sum_f e_f^2 \int_0^1 dz\, z^2(1-z)^2 \int \frac{d^2\mathbf{x}_0}{(2\pi)^2} \int \frac{d^2\mathbf{x}_1}{(2\pi)^2} \frac{\alpha_s C_F}{\pi} \int \frac{d^2\mathbf{x}_2}{2\pi} \left[\frac{\mathbf{x}_{20}}{\mathbf{x}_{20}^2}\cdot\left(\frac{\mathbf{x}_{20}}{\mathbf{x}_{20}^2} - \frac{\mathbf{x}_{21}}{\mathbf{x}_{21}^2}\right)\right]$$
$$\times \operatorname{Re}\left[1-\mathcal{S}^{(3)}_{012}\right] 2\int_0^1 \frac{d\xi}{\xi} \left\{\left[\mathrm{K}_0\left(\overline{Q}\sqrt{(1-\xi)\mathbf{x}_{01}^2 + \xi\mathbf{x}_{21}^2 + \frac{z\xi(1-\xi)}{(1-z)}\mathbf{x}_{20}^2}\right)\right]^2 - \left[\mathrm{K}_0\left(\overline{Q}|\mathbf{x}_{01}|\right)\right]^2\right\} + (q \leftrightarrow \bar{q})$$

However, in the regime of large daughter dipoles $\mathbf{x}_{20}^2 \sim \mathbf{x}_{21}^2 \gg \mathbf{x}_{01}^2$, the ξ integration gives a large collinear $\log(\mathbf{x}_{20}^2/\mathbf{x}_{01}^2)$.

$q\bar{q}g$ contribution to F_L: UV term from the η pole

From the rapidity sensitive $q\bar{q}g$ term, apart from the $+$ distribution piece, one gets the η pole piece:
Contains UV divergences that can be isolated into a dipole-like combination by writing

$$\left(1-\mathcal{S}_{012}^{(3)}\right) = (1-\mathcal{S}_{01}) + \left(\mathcal{S}_{01}-\mathcal{S}_{012}^{(3)}\right)$$

- With rapidity regulator in k^+:

$$\tilde{\mathcal{V}}^L_{q\bar{q}g;\ \eta\text{ pole.; UV}}\Big|^{\eta+} = \left[\frac{2}{\eta} + 2\log\left(\frac{\sqrt{z(1-z)}q^+}{\nu_B^+}\right)\right]\frac{\Gamma(1-\epsilon)}{\epsilon}\left[\pi\mu^2\mathbf{x}_{01}^2\right]^{\epsilon} + O(\epsilon) + O(\eta)$$

- With rapidity regulator in k^-:

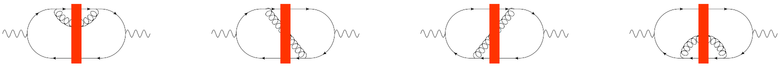
$$\tilde{\mathcal{V}}^L_{q\bar{q}g;\ \eta\text{ pole.; UV}}\Big|^{\eta-} = \left[\frac{2}{\eta} + 2\log\left(\frac{2\sqrt{z(1-z)}q^+\nu_B^-\mathbf{x}_{01}^2}{c_0^2}\right)\right]\frac{\Gamma(1-\epsilon)}{\epsilon}\left[\pi\mu^2\mathbf{x}_{01}^2\right]^{\epsilon} - 2\frac{S_\epsilon}{\epsilon^2}\left[\frac{\mathbf{x}_{01}^2\mu^2}{c_0^2}\right]^{\epsilon} + \frac{\pi^2}{6} + O(\epsilon) + O(\eta)$$

In both cases: total dipole-like contribution to NLO F_L ($q\bar{q}$ terms $+$ all dipole-like UV terms from $q\bar{q}g$):

$$\tilde{\mathcal{V}}^L_{\text{total}} = \frac{1}{2}\left[\log\left(\frac{z}{1-z}\right)\right]^2 - \frac{\pi^2}{6} + \frac{5}{2} + O(\epsilon) + O(\eta)$$

Same result, finite, as with cut-off in k^+, G.B., 2017.

UV subtracted η pole piece with $\eta+$ regulator



Expanding in η and then taking $\epsilon = 0$ in the leftover contribution, in the case of rapidity regulator in k^+:

$$F_L\big|^{q\bar{q}g\ ;\ \eta+}_{\eta\ \text{pole, UV sub.}} = 16Q^4\, N_c \sum_f e_f^2 \int_0^1 dz\, z^2(1-z)^2 \int \frac{d^2\mathbf{x}_0}{(2\pi)^2} \int \frac{d^2\mathbf{x}_1}{(2\pi)^2} \Big[\mathrm{K}_0\left(\overline{Q}|\mathbf{x}_{01}|\right)\Big]^2$$
$$\times \frac{2\alpha_s C_F}{\pi} \int \frac{d^2\mathbf{x}_2}{2\pi} \frac{\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2\mathbf{x}_{21}^2} \mathrm{Re}\left[\mathcal{S}_{01} - \mathcal{S}_{012}^{(3)}\right] \left[\frac{1}{\eta} + \log\left(\frac{q^+\sqrt{z(1-z)}}{\nu_B^+}\right)\right] + O(\epsilon) + O(\eta)$$

Need to define a *rapidity subtracted* (or renormalized) dipole operator to absorb the $1/\eta$ into the LO, as

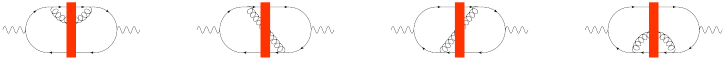
$$\mathcal{S}_{01}\big|_{\text{rap. sub.}}(\nu^+) \equiv \mathcal{S}_{01}\big|_{\text{unsub.}} + \frac{1}{\eta}\left[\frac{\nu^+}{\nu_B^+}\right]^{\eta} \frac{2\alpha_s C_F}{\pi} \left\{\int \frac{d^2\mathbf{x}_2}{2\pi} \frac{\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2\mathbf{x}_{21}^2} \mathrm{Re}\left[\mathcal{S}_{01} - \mathcal{S}_{012}^{(3)}\right] + O(\epsilon)\right\}$$

The rapidity subtracted dipole operator should then depend on ν^+, according to the standard B-JIMWLK evolution.

Natural scale choice: $\nu^+ = q^+\sqrt{z(1-z)}$, to resum low x leading logs.

However: large collinear logs mentioned earlier for large daughter dipoles $\mathbf{x}_{20}^2 \sim \mathbf{x}_{21}^2 \gg \mathbf{x}_{01}^2$ still there.

UV subtracted η pole piece with $\eta-$ regulator



Expanding in η and then taking $\epsilon = 0$ in the leftover contribution, in the case of rapidity regulator in k^-:

$$F_L|^{q\bar{q}g\ ;\ \eta-}_{\eta\text{ pole, UV sub.}} = 16Q^4\, N_c \sum_f e_f^2 \int_0^1 dz\, z^2(1-z)^2 \int \frac{d^2\mathbf{x}_0}{(2\pi)^2} \int \frac{d^2\mathbf{x}_1}{(2\pi)^2} \Big[\mathrm{K}_0\left(\overline{Q}|\mathbf{x}_{01}|\right)\Big]^2$$
$$\times \frac{2\alpha_s C_F}{\pi} \int \frac{d^2\mathbf{x}_2}{2\pi}\, \mathrm{Re}\left[\mathcal{S}_{01} - \mathcal{S}^{(3)}_{012}\right] \left\{ \left[\frac{1}{\eta} + \log\left(\frac{2zq^+\nu_B^-\,\mathbf{x}_{20}^2}{c_0^2}\right)\right] \left[\frac{\mathbf{x}_{20}}{\mathbf{x}_{20}^2}\cdot\left(\frac{\mathbf{x}_{20}}{\mathbf{x}_{20}^2} - \frac{\mathbf{x}_{21}}{\mathbf{x}_{21}^2}\right)\right]\right.$$
$$\left. + \left[\frac{1}{\eta} + \log\left(\frac{2(1-z)q^+\nu_B^-\,\mathbf{x}_{21}^2}{c_0^2}\right)\right] \left[\frac{\mathbf{x}_{21}}{\mathbf{x}_{21}^2}\cdot\left(\frac{\mathbf{x}_{21}}{\mathbf{x}_{21}^2} - \frac{\mathbf{x}_{20}}{\mathbf{x}_{20}^2}\right)\right] \right\} + O(\epsilon) + O(\eta)$$

After similar *rapidity subtraction* of dipole operator, it should depend on ν^-, according to the standard B-JIMWLK evolution.
Results reminiscent of Liu, Xie, Kang, Liu, 2022 for NLO single jet in pA from SCET.

Natural scale choice: $\nu^- = c_0^2/\left(2q^+\sqrt{z(1-z)}\mathbf{x}_{01}^2\right)$, to resum low x leading logs.

Leftover after this choice: terms in $\log(\mathbf{x}_{20}^2/\mathbf{x}_{01}^2)$ and in $\log(\mathbf{x}_{21}^2/\mathbf{x}_{01}^2)$:

- Cancel the large collinear logs mentioned earlier for large daughter dipoles $\mathbf{x}_{20}^2 \sim \mathbf{x}_{21}^2 \gg \mathbf{x}_{01}^2$
- Become new large anticollinear logs in the small daughter dipole regimes $\mathbf{x}_{20}^2 \ll \mathbf{x}_{21}^2 \sim \mathbf{x}_{01}^2$ or $\mathbf{x}_{21}^2 \ll \mathbf{x}_{20}^2 \sim \mathbf{x}_{01}^2$

Summary and comments

- Modern rapidity regulators used to rederive:
  - $\gamma_L^* \to q\bar{q}$ LFWF at one loop
  - DIS structure function F_L at NLO

  $\to$ Shows the feasibility of NLO CGC calculations with these rapidity regulators from the SCET/TMD communities

In this calculation:

- LL B-JIMWLK evolution recovered, with either scale ν^+ or ν^- as evolution variable (or rapidity), depending on the type of rapidity regulator used
- Expected scheme-dependent pattern of large (anti)collinear logs recovered

Using these rapidity regulators: new insights on (anti)collinear logs in BK/B-JIMWLK and their resummation?

To be done:

- Case of DIS structure function F_T at NLO : calculations being finalized
- Low x evolution from operator definition with these rapidity regulators
- Less inclusive observables: interplay of low x evolution with soft and collinear physics

$q\bar{q}g$ contribution to F_L: Rapidity safe terms

- Same UV-subtracted leftover from the terms regular terms at $\xi = 0$ as in G.B., 2017:

$$F_L\Big|^{q\bar{q}g\text{ reg.}} = 16Q^2\, N_c \left(\frac{\alpha_s C_F}{\pi}\right) \sum_f e_f^2 \int_0^1 dz\, z(1-z) \int \frac{d^2\mathbf{x}_0}{(2\pi)^2} \int \frac{d^2\mathbf{x}_1}{(2\pi)^2} \int \frac{d^2\mathbf{x}_2}{2\pi} \int_0^1 d\xi$$
$$\times \Bigg\{ \left(-2+\xi\right) \left[\frac{\mathbf{x}_{20}}{\mathbf{x}_{20}^2} \cdot \left(\frac{\mathbf{x}_{20}}{\mathbf{x}_{20}^2} - \frac{\mathbf{x}_{21}}{\mathbf{x}_{21}^2}\right)\right] \left\{ \Big[\mathrm{K}_0\left(QX_{012}\right)\Big]^2 \mathrm{Re}\left[1-\mathcal{S}_{012}^{(3)}\right] - \left(\mathbf{x}_2 \to \mathbf{x}_0\right) \right\}$$
$$+ \xi\, \frac{\mathbf{x}_{20}\cdot\mathbf{x}_{21}}{\mathbf{x}_{20}^2\, \mathbf{x}_{21}^2} \Big[\mathrm{K}_0\left(QX_{012}\right)\Big]^2 \mathrm{Re}\left[1-\mathcal{S}_{012}^{(3)}\right] \Bigg\} + O(\epsilon) + (q \leftrightarrow \bar{q})$$

$$X_{012}^2 \equiv \frac{1}{(q^+)^2}\left[k_0^+ k_1^+ \mathbf{x}_{01}^2 + k_2^+ k_0^+ \mathbf{x}_{20}^2 + k_2^+ k_1^+ \mathbf{x}_{21}^2\right] = z(1-z)(1-\xi)\mathbf{x}_{01}^2 + z^2\xi(1-\xi)\mathbf{x}_{20}^2 + z(1-z)\xi\mathbf{x}_{21}^2$$
$$\mathcal{S}_{012}^{(3)} \equiv \frac{1}{N_c C_F}\, \mathrm{Tr}\left(t^b U_F(\mathbf{x}_0) t^a U_F(\mathbf{x}_1)^\dagger\right)\, U_A(\mathbf{x}_2)_{ba}$$

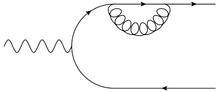
Term $(q \leftrightarrow \bar{q})$: similar integrand, up to the exchanges $\mathbf{x}_0 \leftrightarrow \mathbf{x}_1$ and $z \leftrightarrow 1-z$.
$\Rightarrow$ same contribution to F_L, after the integrations.

Quark off-shell self-energy diagram

One loop corrections to the $\gamma_L^* \to q\bar{q}$ Light-Front wave function found to factorize as:

$$\Psi^{NLO}_{\gamma_L^*\to q\bar{q}} = \left(1 + \frac{\alpha_s C_F}{2\pi}\,\mathcal{V}^L\right)\,\Psi^{LO}_{\gamma_L^*\to q\bar{q}}$$

Contribution of quark self-energy diagram ($\overline{Q}^2 \equiv z(1-z)Q^2$), with dim. reg. only:



$$\mathcal{V}^L_{q\,\text{S. E.}} = \int_0^1 \frac{d\xi}{\xi}\Big[-2+O(\xi)\Big]\,4\pi\,\mu^{2\epsilon}\int\frac{d^{2-2\epsilon}\mathbf{K}}{(2\pi)^{2-2\epsilon}}\,\frac{1}{\left[\mathbf{K}^2 + \frac{\xi(1-\xi)}{(1-z)}(\mathbf{P}^2+\overline{Q}^2)\right]}$$

$$= \Gamma(\epsilon)\left[\frac{\mathbf{P}^2+\overline{Q}^2}{4\pi\mu^2(1-z)}\right]^{-\epsilon}\int_0^1 d\xi\;\xi^{-1-\epsilon}\,(1-\xi)^{-\epsilon}\Big[-2+O(\xi)\Big]$$

Scale $\propto \xi$ in the denominator of $\mathbf{K}$ integral $\Rightarrow$ $\xi^{-\epsilon}$ factor regulating the $\xi = 0$ IR div.

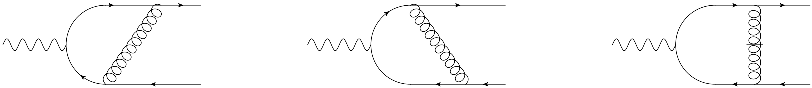
Dim. reg. enough in that case: no rapidity divergence!

Full result, with UV times IR double ϵ pole (with $S_\epsilon \equiv [4\pi\,e^{-\gamma_E}]^\epsilon$):

$$\mathcal{V}^L_{q\,\text{S. E.}} = 2\,\frac{S_\epsilon}{\epsilon^2}\left[\frac{\mathbf{P}^2+\overline{Q}^2}{\mu^2(1-z)}\right]^{-\epsilon} + \frac{3}{2}\,\frac{S_\epsilon}{\epsilon}\left[\frac{\mathbf{P}^2+\overline{Q}^2}{\mu^2(1-z)}\right]^{-\epsilon} - \frac{\pi^2}{6} + \frac{\delta_s}{2} + 3 + O(\epsilon)$$

Vertex correction

3 LFPT diagrams with vertex correction topology:



Individual diagrams have power divergences at $\xi = 0$ on top of log divergences
But power divergences (and some log) cancel between vertex correction LFPT diagrams

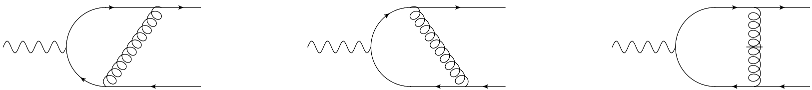
In the total the vertex correction:

- Terms with no potential div at $\xi = 0 \Rightarrow$ dim. reg. enough (single ϵ UV pole)

$$\mathcal{V}^{L}_{\text{v. corr.}}\Big|_{\text{rap. safe, 1}} = \frac{(z-2)}{2}\,\frac{S_\epsilon}{\epsilon}\left[\frac{\overline{Q}^2}{\mu^2}\right]^{-\epsilon} - \frac{3}{2}\log(1-z) - \frac{\delta_s\, z}{2} + \frac{z}{2} - 2 + O(\epsilon) + (z \leftrightarrow 1-z)$$

Vertex correction

3 LFPT diagrams with vertex correction topology:



Individual diagrams have power divergences at $\xi = 0$ on top of log divergences
But power divergences (and some log) cancel between vertex correction LFPT diagrams

In the total the vertex correction:

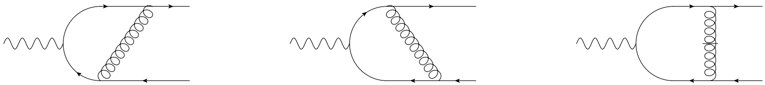
- Terms with no potential div at $\xi = 0 \Rightarrow$ dim. reg. enough (single ϵ UV pole)
- Terms of the same type as quark self-energy $\Rightarrow$ dim. reg. enough (double ϵ pole)

$$\mathcal{V}^L_{\text{v. corr.}}\Big|_{\text{rap. safe, 2}} = \int_0^1 \frac{d\xi}{\xi}\, 4\pi\, \mu^{2\epsilon} \int \frac{d^{2-2\epsilon}\mathbf{K}}{(2\pi)^{2-2\epsilon}} \frac{1}{\left[\mathbf{K}^2 + \frac{\xi(1-\xi)}{(1-z)}(\mathbf{P}^2 + \overline{Q}^2)\right]} + (z \leftrightarrow 1-z)$$

$$= -\frac{S_\epsilon}{\epsilon^2}\left[\frac{\mathbf{P}^2 + \overline{Q}^2}{\mu^2(1-z)}\right]^{-\epsilon} + \frac{\pi^2}{12} + O(\epsilon) + (z \leftrightarrow 1-z)$$

Vertex correction

3 LFPT diagrams with vertex correction topology:



Individual diagrams have power divergences at $\xi = 0$ on top of log divergences
But power divergences (and some log) cancel between vertex correction LFPT diagrams

In the total the vertex correction:

- Terms with no potential div at $\xi = 0 \Rightarrow$ dim. reg. enough (single ϵ UV pole)
- Terms of the same type as quark self-energy $\Rightarrow$ dim. reg. enough (double ϵ pole)
- Terms with potential div at $\xi = 0$ but finite $\mathbf{K}$ integral:

$$\mathcal{V}^{L}_{\text{v. corr.}}\Big|_{\mathcal{B}_0/\xi} = \int_0^1 \frac{d\xi}{\xi}\,(1-\xi)\left[\left(1+\frac{z\xi}{(1-z)}\right)\mathbf{P}^2 + (1-\xi)\,\overline{Q}^2\right]\mathcal{B}_0 + (z \leftrightarrow 1-z)$$

$$\mathcal{B}_0 \equiv 4\pi\,(\mu^2)^{\epsilon}\int \frac{d^{2-2\epsilon}\mathbf{K}}{(2\pi)^{2-2\epsilon}}\,\frac{1}{[\mathbf{K}^2+\Delta_1]\,[(\mathbf{K}+\mathbf{L})^2+\Delta_2]}$$

Dim. reg. insufficient in such term: Rapidity regulator needed!

Remark need to calculate the finite integral $\mathcal{B}_0$ with full ϵ dependence because of the ordering of limits.

Rapidity singular contribution with $\eta+$ regulator

Introducing the factor $(\xi z q^+/\nu_B^+)^\eta$, performing the $\mathbf{K}$ integral thanks to Feynman parametrization, and changing variables:

$$\mathcal{V}^L_{\text{v. corr.}}\Big|^{\eta+}_{\mathcal{B}_0/\xi} = \left[\frac{zq^+}{\nu_B^+}\right]^\eta \Gamma(1+\epsilon)\,\left[4\pi\,\mu^2\right]^\epsilon \int_0^1 dy\, y^{-1-\epsilon+\eta} \int_0^1 d\zeta\, \zeta^{\eta-1} \left[1+\frac{z\zeta}{(1-z)}\right]^{-1-\epsilon}$$
$$\times \left[(1-y)\,\mathbf{P}^2 + (1-y\zeta)\,\overline{Q}^2\right]^{-1-\epsilon} \left[\left((1-y)\,\mathbf{P}^2 + (1-y\zeta)\,\overline{Q}^2\right) + y\mathbf{P}^2\left(1+\frac{z\zeta}{(1-z)}\right)\right] + (z\leftrightarrow 1-z)$$

Dim. reg. can regulate the $y=0$ div, but rapidity regulator needed for the $\zeta=0$ div.

Separating the η pole piece and the $+$ prescription piece:

$$\mathcal{V}^L_{\text{v. corr.}}\Big|^{\eta+}_{\mathcal{B}_0/\xi;\ \eta\text{ pole}} = \frac{1}{\eta}\left[\frac{zq^+}{\nu_B^+}\right]^\eta \Gamma(1+\epsilon)\,\left[4\pi\,\mu^2\right]^\epsilon \left[\mathbf{P}^2+\overline{Q}^2\right] \int_0^1 dy\, y^{-1-\epsilon+\eta}\,\left[(1-y)\,\mathbf{P}^2+\overline{Q}^2\right]^{-1-\epsilon}$$
$$+ (z\leftrightarrow 1-z)$$
$$= \left[\frac{1}{\eta} + \log\left(\frac{zq^+}{\nu_B^+}\right)\right]\left[-\frac{S_\epsilon}{\epsilon}\left[\frac{\overline{Q}^2}{\mu^2}\right]^{-\epsilon} + 2\,\log\left(\frac{\mathbf{P}^2+\overline{Q}^2}{\overline{Q}^2}\right) + O(\epsilon)\right]$$
$$-\frac{S_\epsilon}{\epsilon^2}\left[\frac{\mathbf{P}^2+\overline{Q}^2}{\mu^2}\right]^{-\epsilon} - \mathrm{Li}_2\left(\frac{\mathbf{P}^2}{\mathbf{P}^2+\overline{Q}^2}\right) - \frac{\pi^2}{12} + O(\epsilon) + O(\eta) + (z\leftrightarrow 1-z)$$

Note: double pole in ϵ is a consequence of expanding in η first, at finite ϵ.

Rapidity singular contribution with $\eta+$ regulator

Introducing the factor $(\xi z q^+/\nu_B^+)^\eta$, performing the $\mathbf{K}$ integral thanks to Feynman parametrization, and changing variables:

$$\mathcal{V}^L_{\text{v. corr.}}\Big|^{\eta+}_{\mathcal{B}_0/\xi} = \left[\frac{zq^+}{\nu_B^+}\right]^\eta \Gamma(1+\epsilon)\,\left[4\pi\,\mu^2\right]^\epsilon \int_0^1 dy\, y^{-1-\epsilon+\eta} \int_0^1 d\zeta\, \zeta^{\eta-1}\left[1+\frac{z\zeta}{(1-z)}\right]^{-1-\epsilon}$$
$$\times\left[(1-y)\,\mathbf{P}^2+(1-y\zeta)\,\overline{Q}^2\right]^{-1-\epsilon}\left[\left((1-y)\,\mathbf{P}^2+(1-y\zeta)\,\overline{Q}^2\right)+y\mathbf{P}^2\left(1+\frac{z\zeta}{(1-z)}\right)\right]+(z\leftrightarrow 1-z)$$

Dim. reg. can regulate the $y=0$ div, but rapidity regulator needed for the $\zeta=0$ div.

Separating the η pole piece and the $+$ prescription piece:

$$\mathcal{V}^L_{\text{v. corr.}}\Big|^{\eta+}_{\mathcal{B}_0/\xi;\,+\,\text{prescr.}} = \Gamma(1+\epsilon)\,\left[4\pi\,\mu^2\right]^\epsilon \int_0^1 \frac{d\zeta}{(\zeta)_+} \int_0^1 dy\, y^{-1-\epsilon}\left[1+\frac{z\zeta}{(1-z)}\right]^{-1-\epsilon}$$
$$\times\left[\left((1-y)\,\mathbf{P}^2+(1-y\zeta)\,\overline{Q}^2\right)\right]^{-1-\epsilon}\left\{\left[(1-y)\,\mathbf{P}^2+(1-y\zeta)\,\overline{Q}^2\right]+y\mathbf{P}^2\left(1+\frac{z\zeta}{(1-z)}\right)\right\}+O(\eta)+(z\leftrightarrow 1-z)$$
$$=-\log(1-z)\frac{S_\epsilon}{\epsilon}\left[\frac{\mathbf{P}^2+\overline{Q}^2}{\mu^2}\right]^{-\epsilon}-\frac{1}{2}\Big[\log(1-z)\Big]^2-\text{Li}_2\left(-\frac{z}{(1-z)}\right)+\text{Li}_2\left(\frac{\mathbf{P}^2}{\mathbf{P}^2+\overline{Q}^2}\right)$$
$$+O(\epsilon)+O(\eta)+(z\leftrightarrow 1-z)$$

Rapidity singular contribution with $\eta-$ regulator

Introducing instead the factor $(2\xi z q^+ \nu_B^- / \mathbf{K}^2)^\eta$, and following similar steps:

- The η pole piece is now obtained as

$$\mathcal{V}^{L}_{\text{v. corr.}}\Big|^{\eta-}_{\mathcal{B}_0/\xi;\ \eta\ \text{pole}} = \left[\frac{1}{\eta} + \log\left(\frac{2zq^+\nu_B^-}{\mathbf{P}^2+\overline{Q}^2}\right)\right]\left[-\frac{S_\epsilon}{\epsilon}\left[\frac{\overline{Q}^2}{\mu^2}\right]^{-\epsilon} + 2\,\log\left(\frac{\mathbf{P}^2+\overline{Q}^2}{\overline{Q}^2}\right) + O(\epsilon)\right]$$

$$-\frac{\pi^2}{3} + O(\epsilon) + O(\eta) + (z \leftrightarrow 1-z)$$

- Same $+$ prescription piece is obtained as with the rapidity regulator in k^+