The DIS dipole picture cross section in exact kinematics

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Based on arXiv:2601.07302 with T. Lappi, H. Mäntysaari, X.B. Tong

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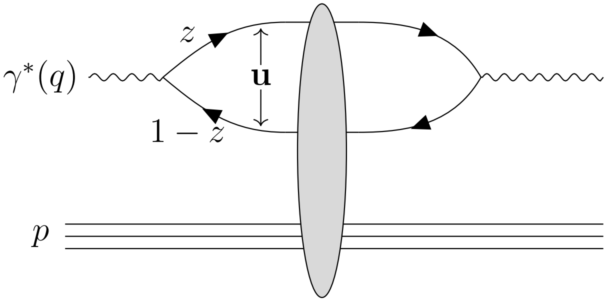
The DIPOLE PICTURE and DIS

- Requirement : High precision theory calculations at small x for EIC.

$$F_L(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{\text{em}}}\sigma_L^{\gamma^* p}, \qquad F_2(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{\text{em}}}\left(\sigma_L^{\gamma^* p} + \sigma_T^{\gamma^* p}\right).$$

- The total cross section is usually calculated with the Optical Theorem (OT).



$$\Leftrightarrow \quad \sigma_{L,T}^{\gamma^* p,\text{OT}} = \sigma_0 \int_{z,\mathbf{u}} N(\mathbf{u}) \left|\Psi_{L,T}^{\gamma^* q\bar{q}}(z,|\mathbf{u}|)\right|^2$$

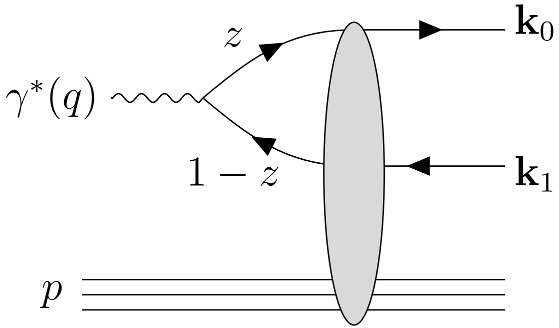
- Problem : Implicitly assumes asymptotically large $\gamma^* p$ center-of-mass energy, W.

THE FINITE ENERGY CONSTRAINT

- Colliders operate at finite energy $\Rightarrow M^2_{q\bar{q}} \leq W^2$.

$$M^2_{q\bar{q}} = (k_0 + k_1)^2 = \frac{\mathbf{P}^2 + m_q^2}{z(1-z)}$$

$$\mathbf{P} = (1-z)\mathbf{k}_0 - z\mathbf{k}_1$$



- Finite energy constrains final state phase space.
- Purpose : Impose $M^2_{q\bar{q}} \leq W^2$. Allows us to
  1. derive a finite W constrained $\sigma^{\gamma^* p}_{\text{tot}}$ in the dipole picture at leading order.
  2. numerically quantify importance of constraint for light and heavy quarks.

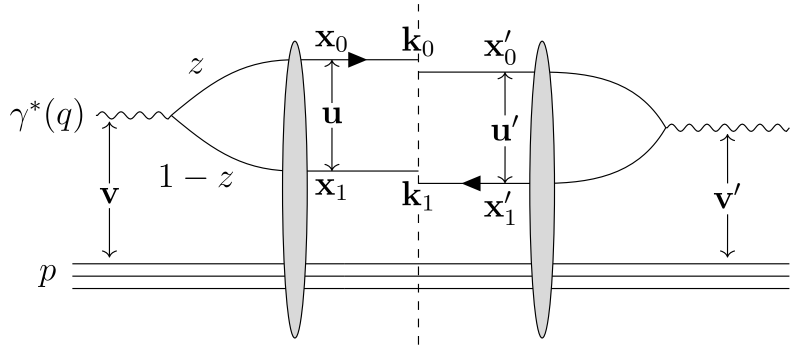
Part I

FINITE-ENERGY CONSTRAINED CROSS SECTION

DI-JET DIS CROSS SECTION IN THE DIPOLE PICTURE

Dipole size and center of mass

$$\mathbf{u} = \mathbf{x}_0 - \mathbf{x}_1, \quad \mathbf{v} = z\mathbf{x}_0 + (1-z)\mathbf{x}_1,$$
$$\mathbf{u}' = \mathbf{x}'_0 - \mathbf{x}'_1, \quad \mathbf{v}' = z\mathbf{x}'_0 + (1-z)\mathbf{x}'_1$$



Final state momenta

$$\mathbf{P} = (1-z)\mathbf{k}_0 - z\mathbf{k}_1, \quad \mathbf{K} = \mathbf{k}_0 + \mathbf{k}_1$$

The di-jet dipole picture cross section

$$\frac{\mathrm{d}\sigma^{\gamma^* p}_{L,T}}{\mathrm{d}^2\mathbf{K}\mathrm{d}^2\mathbf{P}} = \int_{z,\mathbf{u},\mathbf{v},\mathbf{u}',\mathbf{v}'} e^{i\mathbf{K}\cdot(\mathbf{v}'-\mathbf{v})} e^{i\mathbf{P}\cdot(\mathbf{u}'-\mathbf{u})} \underbrace{\mathcal{N}(z,\,\mathbf{u},\,\mathbf{v},\,\mathbf{u}',\,\mathbf{v}')}_{\text{Target interaction}} \underbrace{\Psi^{\gamma^* q\bar{q}}_{L,T}(z,\,|\mathbf{u}|)\Psi^{\gamma^* q\bar{q}}_{L,T}(z,\,|\mathbf{u}'|)^\dagger}_{\text{Lightcone wavefunction squared}}$$

Target interaction ⇔ correlators of Wilson lines in fundamental representation

$$\mathcal{N}(z,\mathbf{u},\mathbf{v},\mathbf{u}',\mathbf{v}') = \mathbb{1} - \frac{1}{N_c}\underbrace{\left\langle \mathrm{Tr}\left[U(\mathbf{x}_0)U^\dagger(\mathbf{x}_1)\right]\right\rangle}_{\text{dipole}} - \frac{1}{N_c}\left\langle \mathrm{Tr}\left[U(\mathbf{x}'_1)U^\dagger(\mathbf{x}'_0)\right]\right\rangle + \underbrace{\frac{1}{N_c}\left\langle \mathrm{Tr}\left[U(\mathbf{x}_0)U^\dagger(\mathbf{x}_1)U(\mathbf{x}'_1)U^\dagger(\mathbf{x}'_0)\right]\right\rangle}_{\text{quadrupole}},$$

$$\to 0 \quad \text{as } |\mathbf{u}| \text{ and } |\mathbf{u}'| \to 0$$
$$\to 1 \quad \text{as } |\mathbf{u}| \text{ and } |\mathbf{u}'| \to \infty$$

Recovering the optical theorem

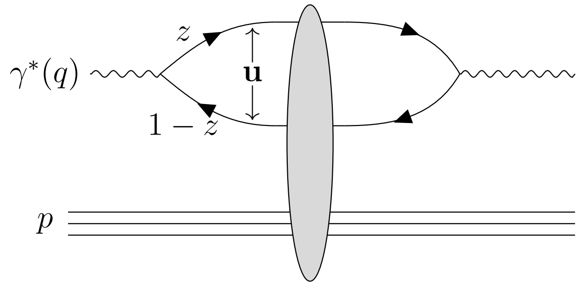
Integrating the di-jet cross section assuming translational invariance $\int_{\mathbf{v}} \to \sigma_0/2$ of the target

$$\sigma^{\gamma^* p}_{L,T} = \int_{z,\mathbf{u},\mathbf{v},\mathbf{u}',\mathbf{v}'} \frac{\mathrm{d}^2\mathbf{K}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{P}}{(2\pi)^2} e^{i\mathbf{K}\cdot(\mathbf{v}'-\mathbf{v})} e^{i\mathbf{P}\cdot(\mathbf{u}'-\mathbf{u})} \mathcal{N}(z,\mathbf{u},\mathbf{v},\mathbf{u}',\mathbf{v}') \Psi^{\gamma^* q\bar{q}}_{L,T}(z,|\mathbf{u}|) \left(\Psi^{\gamma^* q\bar{q}}_{L,T}(z,|\mathbf{u}'|)\right)^\dagger$$

Total DIS cross section of DIS

$$\sigma^{\gamma^* p,\mathrm{OT}}_{L,T} = \sigma_0 \int_{z,\mathbf{u}} N(\mathbf{u}) \left|\Psi^{\gamma^* q\bar{q}}_{L,T}(z,|\mathbf{u}|)\right|^2$$

$$N(\mathbf{u}) = \mathbb{1} - \frac{1}{N_c}\left\langle \mathrm{Tr}\left[U((1-z)\mathbf{u})U^\dagger(-z\mathbf{u})\right]\right\rangle$$



Optical theorem structure functions

$$F_L^{\mathrm{OT}}(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{\mathrm{em}}}\sigma_L^{\gamma^* p,\mathrm{OT}}, \quad F_2^{\mathrm{OT}}(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{\mathrm{em}}}\left(\sigma_L^{\gamma^* p,\mathrm{OT}} + \sigma_T^{\gamma^* p,\mathrm{OT}}\right)$$

Note : At LO no dependence on CM ENERGY W of $\gamma^* p$-system (No x evolution)

THE FINITE ENERGY CONSTRAINT AND TOTAL CROSS SECTION

Change variables from **P** to $M^2_{q\bar{q}}$ with delta function

$$\frac{\mathrm{d}\sigma^{\gamma^* p}_{L,T}}{\mathrm{d}M^2_{q\bar{q}}} = \int_{z,\mathbf{u},\mathbf{v},\mathbf{u}',\mathbf{v}'} \frac{\mathrm{d}^2\mathbf{K}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{P}}{(2\pi)^2} e^{i\mathbf{K}\cdot(\mathbf{v}'-\mathbf{v})} e^{i\mathbf{P}\cdot(\mathbf{u}'-\mathbf{u})} \mathcal{N}(z,\mathbf{u},\mathbf{v},\mathbf{u}',\mathbf{v}') \Psi^{\gamma^* q\bar{q}}_{L,T}(z,|\mathbf{u}|) \left(\Psi^{\gamma^* q\bar{q}}_{L,T}(z,|\mathbf{u}'|)\right)^\dagger \times \delta\left(M^2_{q\bar{q}} - \frac{\mathbf{P}^2 + m_q^2}{z(1-z)}\right)$$

Total cross section at finite energy (i.e. imposing $M^2_{q\bar{q}} \leq W^2$)

$$\sigma^{\gamma^* p}_{L,T} = \int_{M^2_{\mathrm{min}}}^{M^2_{\mathrm{max}}} \mathrm{d}M^2_{q\bar{q}} \frac{d\sigma^{\gamma^* p}_{L,T}}{\mathrm{d}M^2_{q\bar{q}}} \quad , \qquad \begin{cases} M^2_{\mathrm{max}} = W^2 \\ M^2_{\mathrm{min}} = \dfrac{m_q^2}{z(1-z)} \end{cases}$$

PURPOSE 1: THE TOTAL CROSS SECTION IN EXACT KINEMATICS

Integrating over $M^2_{q\bar{q}}$ assuming translational invariance $\int_{\mathbf{v}} \to \sigma_0/2$

$$\checkmark \quad \boxed{\sigma_{L,T}^{\gamma^* p,\mathrm{FE}} = \sigma_0 \int_{z,\mathbf{u},\mathbf{u}'} \Psi_{L,T}^{\gamma^* q\bar{q}}(z,\, |\mathbf{u}|) \left(\Psi_{L,T}^{\gamma^* q\bar{q}}(z,\, |\mathbf{u}'|)\right)^\dagger \widetilde{\mathcal{N}}(z,\, \mathbf{u},\, \mathbf{u}') \frac{\zeta(M^2_{\mathrm{max}}) J_1\left(\zeta(M^2_{\mathrm{max}})\right)}{2\pi |\mathbf{u}' - \mathbf{u}|^2}}$$

Compared to "OT": W-dependence and a multipole correlator structure in the target interaction

$$\zeta(M^2_{\mathrm{max}}) = |\mathbf{u}' - \mathbf{u}| \sqrt{M^2_{\mathrm{max}}\, z(1-z) - m_q^2}$$

$$\widetilde{\mathcal{N}}(z,\, \mathbf{u},\, \mathbf{u}') = \mathbb{1} - \frac{1}{N_c} \left\langle \mathrm{Tr}\left[U((1-z)\mathbf{u}) U^\dagger(-z\mathbf{u})\right]\right\rangle - \frac{1}{N_c} \left\langle \mathrm{Tr}\left[U(-z\mathbf{u}') U^\dagger((1-z)\mathbf{u}')\right]\right\rangle$$
$$+ \frac{1}{N_c} \left\langle \mathrm{Tr}\left[U((1-z)\mathbf{u}) U^\dagger(-z\mathbf{u}) U(-z\mathbf{u}') U^\dagger((1-z)\mathbf{u}')\right]\right\rangle$$

Finite energy constrained structure functions

$$F_L^{\mathrm{FE}}(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{\mathrm{em}}} \sigma_L^{\gamma^* p,\mathrm{FE}}, \quad F_2^{\mathrm{FE}}(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{\mathrm{em}}} \left(\sigma_L^{\gamma^* p,\mathrm{FE}} + \sigma_T^{\gamma^* p,\mathrm{FE}}\right)$$

Expect to recover OT as $M^2_{\mathrm{max}} \to \infty$.

Part II

Quantifying importance of finite-energy constraint

Expressions for evaluating correlators

Dipole correlator in MV model ($Q_{s0}^2 = 0.104\,\mathrm{GeV}^2$, $\gamma = 1$)

$$\frac{1}{N_c}\left\langle \mathrm{Tr}\left[U((1-z)\mathbf{u})U^\dagger(-z\mathbf{u})\right]\right\rangle = \exp\left\{-\frac{1}{4}\left(|\mathbf{u}|^2 Q_{s0}^2\right)^\gamma \log\left(1/(|\mathbf{u}|\Lambda_{\mathrm{QCD}}) + e\right)\right\} = \exp\left\{\frac{-C_F}{2}\Gamma(|\mathbf{u}|)\right\}$$

Gaussian approximation is used to evaluate quadrupole[1]. Exact at LO[2].

$$\frac{1}{N_c}\left\langle \mathrm{TR}\left[U((1-z)\mathbf{u})U^\dagger(-z\mathbf{u})U(-z\mathbf{u}')U^\dagger((1-z)\mathbf{u}')\right]\right\rangle = \exp\left\{\frac{-C_F}{2}\left(\Gamma(\mathbf{u}') - \Gamma(\mathbf{u})\right)\right\}$$
$$\times\left[\left(\frac{\sqrt{\Delta}+G_1}{2\sqrt{\Delta}} - \frac{G_2}{\sqrt{\Delta}}\right)\exp\left\{\frac{N_c}{4}\mu^2\sqrt{\Delta}\right\} + \left(\frac{\sqrt{\Delta}-G_1}{2\sqrt{\Delta}} + \frac{G_2}{\sqrt{\Delta}}\right)\exp\left\{\frac{-N_c}{4}\mu^2\sqrt{\Delta}\right\}\right]$$
$$\times\exp\left\{\mu^2\left(\frac{-N_c}{4}G_1 + \frac{1}{2N_c}G_2\right)\right\}$$

G_1, G_2 and Δ depend non-trivially on $\Gamma(|\mathbf{u}|)$.

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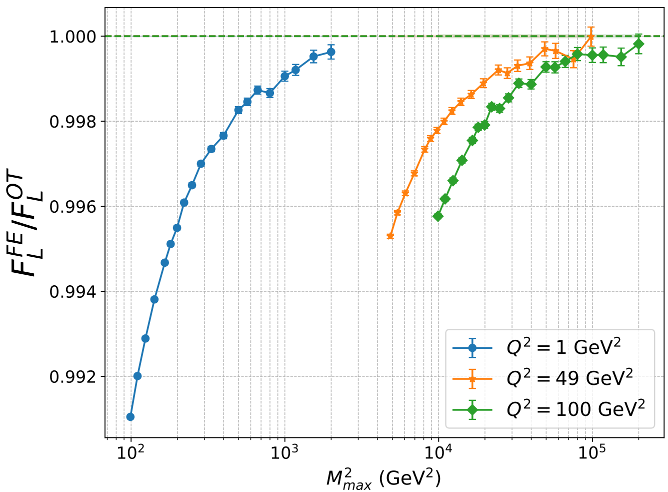
[1] See e.g. Phys. Rev. D 83, 105005, Dominguez, Marquet, Xiao and Yuan for derivation.

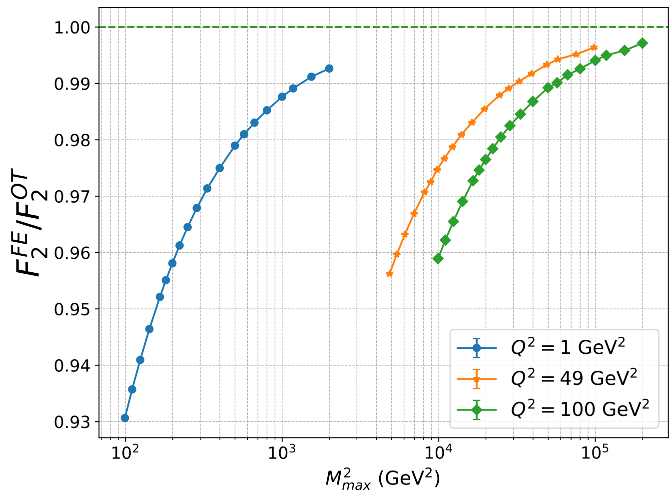
[2] Has been shown in arXiv:1108.4764, A. Dumitru, J. Jalilian-Marian, T. Lappi, B. Schenke and R. Venugopalan.

Purpose 2: Importance of finite-W constraint: Light quarks

$m_q = 0.14$ GeV

The ratio of the structure functions, $F^{\mathrm{FE}}_{\mathrm{L},2}/F^{\mathrm{OT}}_{\mathrm{L},2}$, as a function of M^2_{max}.





$$\sigma^{\gamma^* p,\mathrm{OT}}_{L,T} = \sigma_0 \int_{z,\mathbf{u}} N(\mathbf{u}) \left|\Psi^{\gamma^* q\bar{q}}_{L,T}(z,|\mathbf{u}|)\right|^2, \quad \sigma^{\gamma^* p,\mathrm{FE}}_{L,T} = \sigma_0 \int_{z,\mathbf{u},\mathbf{u}'} \Psi^{\gamma^* q\bar{q}}_{L,T}(z,|\mathbf{u}|)\left(\Psi^{\gamma^* q\bar{q}}_{L,T}(z,|\mathbf{u}'|)\right)^\dagger \widetilde{\mathcal{N}}(z,\mathbf{u},\mathbf{u}')\frac{\zeta(M^2_{\mathrm{max}})J_1(\zeta(M^2_{\mathrm{max}}))}{2\pi|\mathbf{u}'-\mathbf{u}|^2}$$

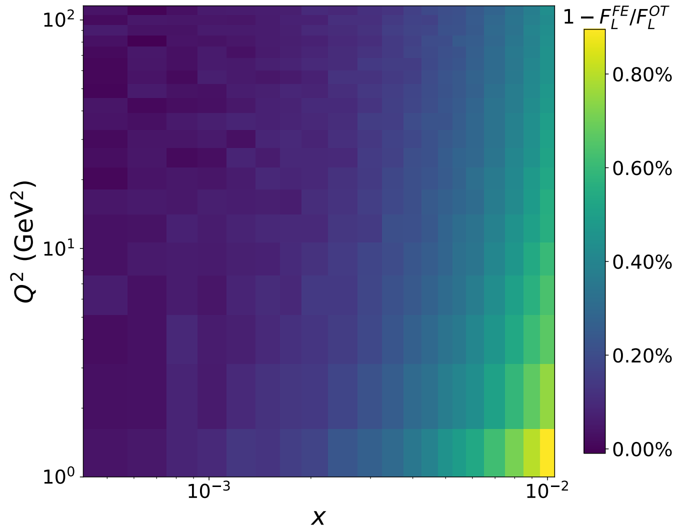
NOTE :

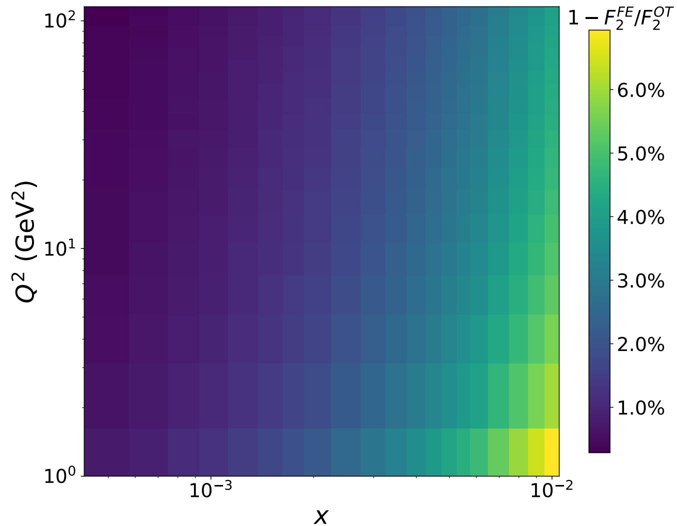
- Recovers OT as $M^2_{\mathrm{max}} \to \infty$.
- Effect larger for F_2 than F_L due to limits $z \to 0,\ 1 \Rightarrow M^2_{q\bar{q}}$ becomes large (aligned jet limit).

Purpose 2: Importance of finite-W constraint: Light quarks

$m_q = 0.14$ GeV

For physical cross sections: $M^2_{\text{max}} = W^2 \approx \frac{Q^2}{x}$. The rel. difference $1 - F^{\text{FE}}_{\text{L},2} / F^{\text{OT}}_{\text{L},2}$.



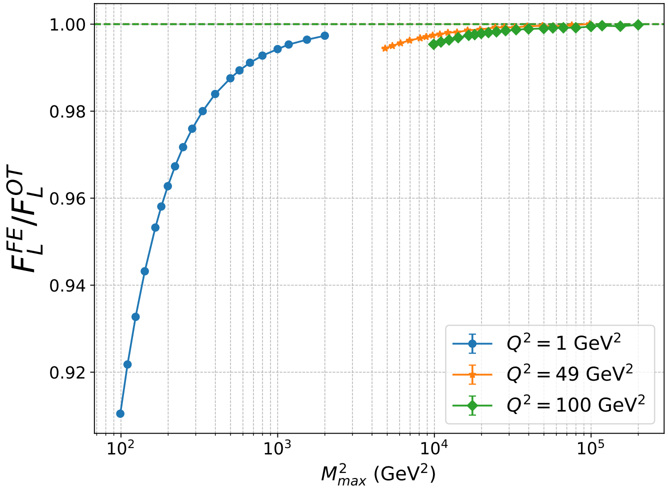


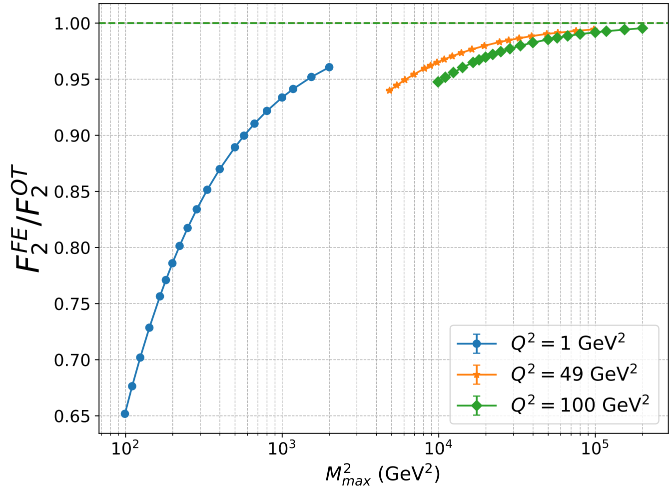
- Effect of the finite energy constraint largest at relatively small Q^2 and large x.
- Max effect: $F_L \sim 0.9\%$ and $F_2 \sim 7\%$.

Purpose 2: Importance of finite-W constraint: Heavy quarks

$m_q = 1.4$ GeV

The ratio of the structure functions, $F^{\mathrm{FE}}/F^{\mathrm{OT}}$, as a function of M^2_{max}.





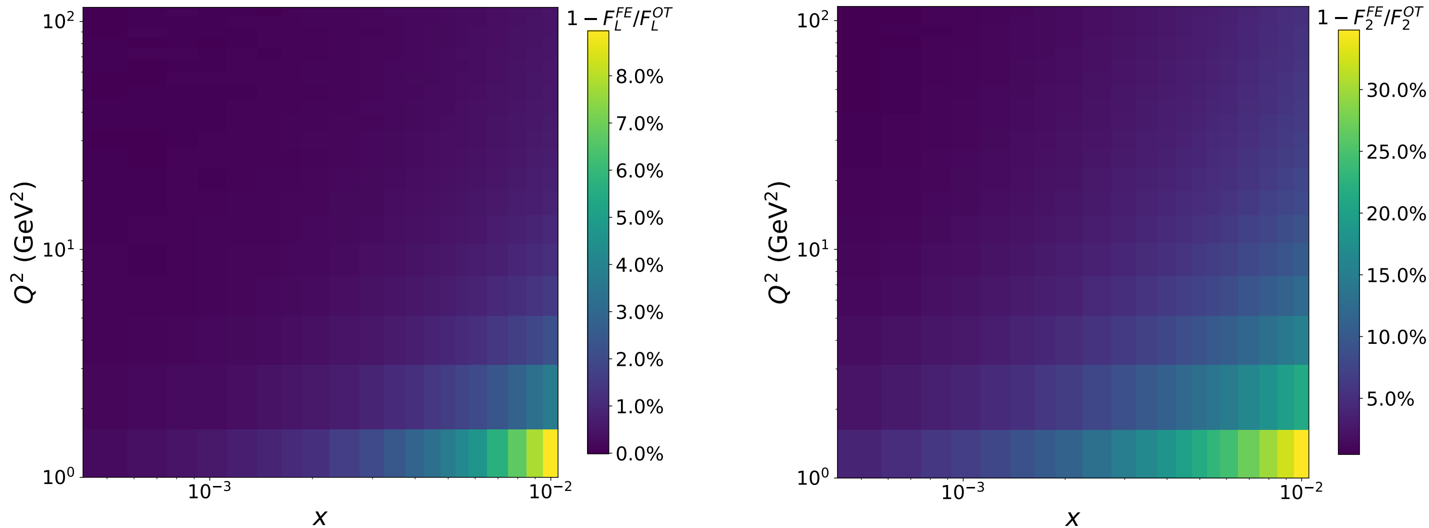
NOTE :

- Again recovers OT as $M^2_{\mathrm{max}} \to \infty$.
- Effect larger for F_2 than F_L due to aligned jet limit.
- Much larger effect for heavy quarks!!

Purpose 2: Importance of finite-W constraint: Heavy quarks

$m_q = 1.4$ GeV

For physical cross sections: $M^2_{\rm max} = W^2 \approx \frac{Q^2}{x}$. The rel. difference $1 - F^{\rm FE}_{\rm L,2}/F^{\rm OT}_{\rm L,2}$.



NOTE :

- Effect of finite energy constraint largest at relatively small Q^2 and large x.
- Max effect: $F_L \sim 9\%$ and $F_2 \sim 35\%$.
- 10 times larger for F_L and 5 times larger for F_2. Quark mass surpresses aligned jet limit

Part III

OUTLOOK: FINITE ENERGY CONSTRAINT AND THE BK INITIAL CONDITION

Finite energy constraint for BK initial condition

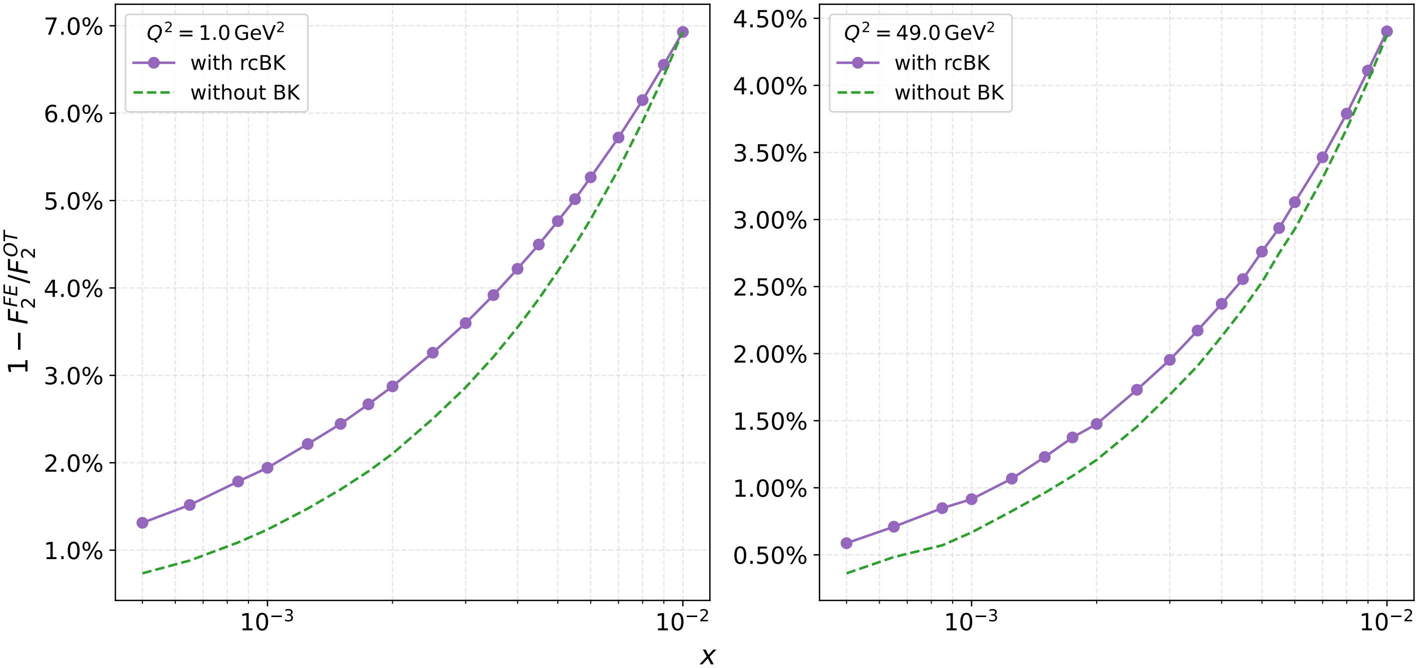
- So far, LO DIS dipole picture cross section
- Additional x-dependence from the evolution of the correlators e.g. BK or JIMWLK.
- Work in progress together with Casuga and Mäntysaari.
- Initial condition for BK given by MV model

$$\frac{1}{N_c}\left\langle \mathrm{Tr}\left[U((1-z)\mathbf{u})U^\dagger(-z\mathbf{u})\right]\right\rangle = e^{-\frac{1}{4}\left(|\mathbf{u}|^2 Q_{s0}^2\right)^\gamma \log\left(1/(|\mathbf{u}|\Lambda_{\mathrm{QCD}})+e\cdot e_c\right)}$$

- Previous fits of extracting non-perturbative parameters from data with $F_{L,2}^{\mathrm{OT}} \Rightarrow$ Replace with $F_{L,2}^{FE}$.
- For a proper uncertainty analysis $\rightarrow$ Bayesian inference

Initial results using BK evolved correlator

- $m_q = 0.14$ GeV



Using BK dipole fit[3] $\rightarrow$ effect small for light quarks

[3] Phys. Rev. D 88, 114020, Lappi and Mäntysaari

SUMMARY AND CONCLUSIONS

- Introduced W-dependence into the DIS cross section at LO. Description in exact kinematics.
- Purpose 1 : Improved cross section for DIS dipole picture calculations

$$\sigma_{L,T}^{\gamma^* p} = \sigma_0 \int_{z,\mathbf{u},\mathbf{u}'} \Psi_{L,T}^{\gamma^* q\bar{q}}(z,\,|\mathbf{u}|) \left(\Psi_{L,T}^{\gamma^* q\bar{q}}(z,\,|\mathbf{u}'|)\right)^{\dagger} \widetilde{\mathcal{N}}(z,\,\mathbf{u},\,\mathbf{u}') \frac{\zeta(M_{\mathrm{max}}^2) J_1\left(\zeta(M_{\mathrm{max}}^2)\right)}{2\pi|\mathbf{u}' - \mathbf{u}|^2}$$

- Recover Optical Theorem cross section as $M_{\mathrm{max}}^2 \to \infty$.
- Purpose 2 : Finite energy constraint has impact at LO. Larger effect for F_2 than F_L.
- At $Q^2 \sim 1$ GeV^2 and $x_B \sim 0.01$, substantial effect for F_2 ($\sim 35\%$) for heavy quarks.
- A finite energy constrained fit of the BK IC and the impact on F_L and F_2 is in the works

Part IV

BACKUP SLIDES

Lightcone wavefunctions at LO

Longitudinal wavefunciton squared

$$\Psi_L^{\gamma^* q\bar{q}}(z,\, |\mathbf{u}|) \left(\Psi_L^{\gamma^* q\bar{q}}(z,\, |\mathbf{u}'|)\right)^\dagger = N_c \sum_f \frac{\alpha_{em} Q_f^2}{4\pi^2} 8 Q^2 z^2 (1-z)^2 K_0(\varepsilon_f |\mathbf{u}|) K_0^*(\varepsilon_f |\mathbf{u}'|)$$

Transverse wavefunction squared

$$\Psi_T^{\gamma^* q\bar{q}}(z,\, |\mathbf{u}|) \left(\Psi_T^{\gamma^* q\bar{q}}(z,\, |\mathbf{u}'|)\right)^\dagger = 2N_c \sum_f \frac{\alpha_{em} Q_f^2}{4\pi^2} \big[(z^2 + (1-z)^2)\varepsilon_f^2 \cos(\alpha) K_1(\varepsilon_f |\mathbf{u}|) K_1^*(\varepsilon_f |\mathbf{u}'|) + m_f^2 K_0(\varepsilon_f |\mathbf{u}|) K_0^*(\varepsilon_f |\mathbf{u}'|)\big].$$

$$\varepsilon_f = Q^2 z(1-z) + m_f^2$$