Parton distribution and fragmentation functions with massive gluons

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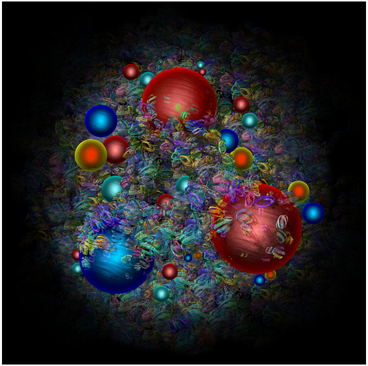
Outline

Collaboration with B. El-Bennich, G. Krein, F. Serna, R. C. Silveira, U. Reinosa and M. Peláez

Introduction

- QCD - the theory of strong interactions
- High energy regime - perturbative
  - ➢ Asymptotic freedom
- Low energy regime - nonperturbative
  - ➢ Confinement
  - ➢ Bound states



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PDFs and FFs

- Factorization
- Deep inelastic scattering (DIS), semi-inclusive DIS, drell-yan process
- Connection between PDFs and elementary FFs

$$d_q^{\pi}(z) = \frac{z}{6} f_q^{\pi}\left(x = \frac{1}{z}\right)$$

- Cascade of mesons (jet equations)

B. El-Bennich's talk

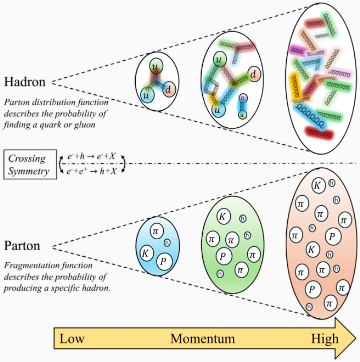
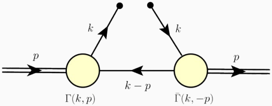


FIG. 1. Illustration of hadron and parton from low to high momentum and their relations in terms of parton distribution function and fragmentation function.

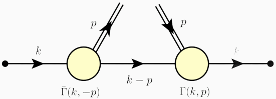
Datta et al, Phys.Rev.Lett **134**, 111902 (2025)

PDFs and FFs

Diagrammatic representation



$$f_q^{\pi}(x) = N_c C_q^{\pi} \int_k \delta\Big(k^+ - x p^+\Big) \mathrm{Tr}_{\mathrm{D}}\Big[S(k)\gamma^+ S(k) \bar{\Gamma}_{\pi}\big(-k + p/2, -p\big) S_{\mathrm{os}}(k-p) \Gamma_{\pi}\big(k - p/2, p\big)\Big]$$



$$d_q^{\pi}(z) = \frac{N_c C_q^{\pi} z}{6} \int_k \delta\Big(k^+ - \frac{p^+}{z}\Big) \mathrm{Tr}_{\mathrm{D}}\Big[S(k)\gamma^+ S(k) \bar{\Gamma}_{\pi}\big(-k + p/2, -p\big) S_{\mathrm{os}}(k-p) \Gamma_{\pi}\big(k - p/2, p\big)\Big]$$

R. Silveira, F. Serna, B. El-Bennich Phys.Rev.C **111**, 065204 (2025).
G.B., B. El-Bennich, G. Krein, F. Serna, R. Silveira, Phys.Rev.D **112**, 114023 (2025).

Chiral Symmetry

- Important symmetry in QCD lagrangian

$$\mathcal{L} = \sum_{i=1}^{N_f} \bar{\Psi}_i(\not{D} + m)\Psi_i + \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}\,, \qquad \Psi_{L,R} = \frac{1 \mp \gamma_5}{2}\Psi$$

- In the chiral limit ($m \to 0$), the QCD Lagrangian is invariant under

$$SU(N_f)_L \times SU(N_f)_R$$

- Explicit chiral symmetry breaking comes from the quark mass term

$$m\,\bar{\Psi}\Psi = m(\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L)$$

- Even for $m_q = 0$, strong interactions dynamically generate $\langle\bar{\Psi}\Psi\rangle \neq 0$
- Dynamical chiral symmetry breaking generates an effective quark mass
- Pions emerge as pseudo-Goldstone bosons

Curci-Ferrari model

- Title:

  Parton distribution and fragmentation functions with **massive gluons**

- where massive gluons do come from?

- Curci-Ferrari model   G. Curci and R. Ferrari, Nuov Cim A **32**, 151 (1976)

Curci-Ferrari model

- Motivated by lattice simulations
- Effective way deal with Gribov copies

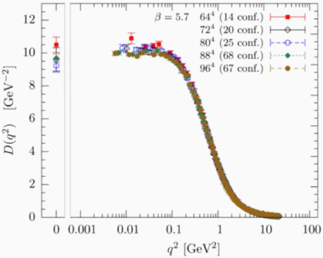


Fig. 2. The bare lattice gluon propagator $D(q^2)$ versus q^2 for $\beta = 5.70$ and various lattice sizes. We also show data on $D(0)$ (left).

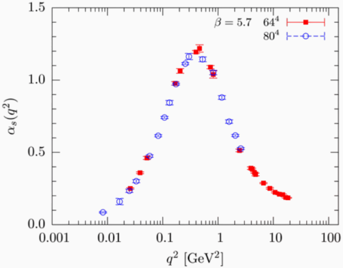


Fig. 5. Running coupling $\alpha_s(q^2)$ versus q^2 for lattice sizes 64^4 and 80^4 at $\beta = 5.70$.

- $\frac{N_c \alpha_s}{4\pi}$ small parameter - perturbative treatment!

Curci-Ferrari model

Curci-Ferrari model lagrangian (in Landau gauge)

$$\mathcal{L} = \sum_{i=1}^{N_f} \bar{\Psi}_i(\not{D} + \mathcal{M}_\Lambda)\Psi_i + \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + ih^a\partial_\mu A^a_\mu + \partial_\mu \bar{c}^a (D_\mu c)^a + \frac{1}{2}m_\Lambda^2 (A^a_\mu)^2$$

- Double expansion
  - g_g and $1/N_c$
- Renormalization group improvement
- Green's functions and beta functions
- Bound states (BSE)

$$D_{\mu\nu}(k) = \frac{1}{k^2 + m_\Lambda^2}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

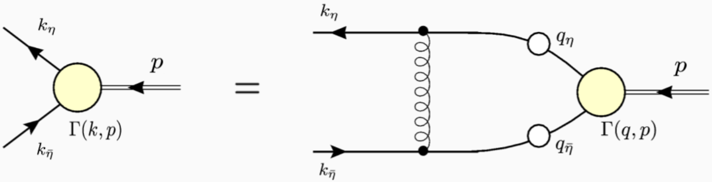
bare gluon propagator

M. Peláez, U. Reinosa, J. Serreau, M. Tissier, N. Wschebor, Phys.Rev.D **96**, 114011 (2017)
M. Peláez, U. Reinosa, J. Serreau, M. Tissier, N. Wschebor, Phys.Rev.D **103**, 094035 (2021)
M. Peláez, U. Reinosa, J. Serreau, N. Wschebor, Phys.Rev.D **107**, 054025 (2023)

Curci-Ferrari model

- Rainbow equation

- Bethe-Salpeter equation



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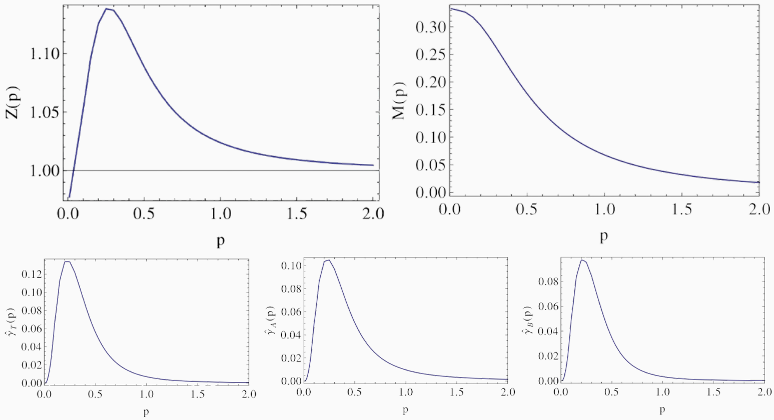
Curci-Ferrari model

Chiral limit



M. Peláez, U. Reinosa, J. Serreau, N. Wschebor, Phys.Rev.D **107**, 054025 (2023)

Curci-Ferrari model

Pion decay constant in chiral limit

$$-ip_\mu f_\pi^2 = N_c \int_q \mathrm{Tr_D}\left[\gamma_\mu\gamma_5 S(q_\eta)\Gamma_\pi(q,p)S(q_{\bar\eta})\right]$$

$$\begin{aligned} f_\pi^2 &= \frac{N_c}{4\pi^2}\int_0^\infty dx \frac{xZ^2(x)}{\left[x+M^2(x)\right]^2} \\ &\times \left\{\gamma_P(x)\left[M(x)-\frac{x}{2}M'(x)\right]\right. \\ &\quad +\frac{3}{2}M(x)\left[x\gamma_T(x)-M(x)\gamma_A(x)\right] \\ &\quad \left.\frac{x+M^2(x)}{2}\gamma_B(x)\right\} \end{aligned}$$

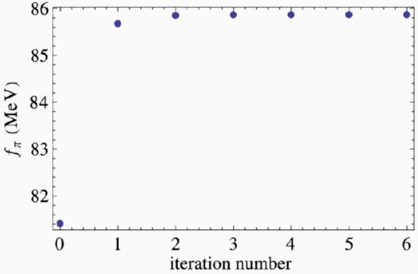


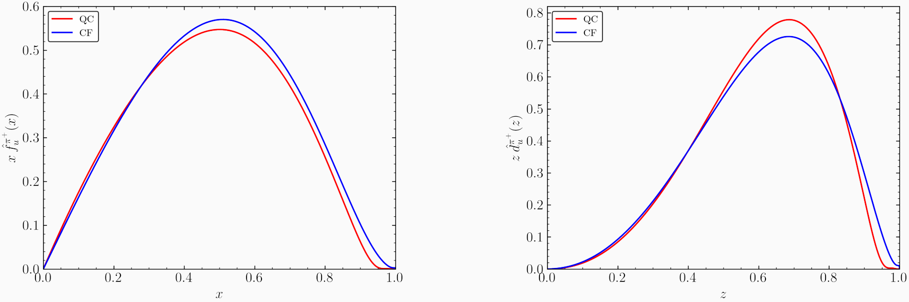
FIG. 4. Evolution of f_π with the number of iterations for $g_0 = 1.93$ and $m_0 = 0.11$ GeV.

M. Peláez, U. Reinosa, J. Serreau, N. Wschebor, Phys.Rev.D **107**, 054025 (2023)

Numerical results

$$f_q^{\pi}(x) = N_c C_q^{\pi} \int_k \delta\left(k^+ - xp^+\right) \mathrm{Tr}_{\mathrm{D}} \left[S(k)\gamma^+ S(k)\bar{\Gamma}_\pi\left(-k + p/2, -p\right) S_{\mathrm{os}}(k-p)\Gamma_\pi\left(k - p/2, p\right)\right]$$

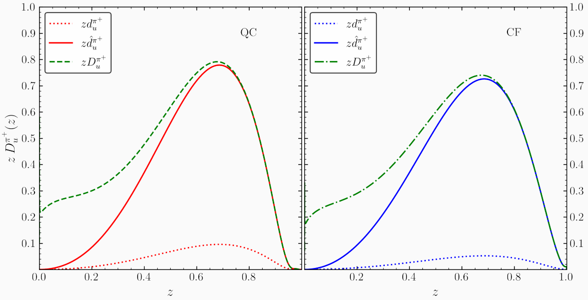
$$d_q^{\pi}(z) = \frac{z}{6} f_q^{\pi}\left(x = \frac{1}{z}\right) \qquad \text{(Drell-Levy-Yan relation)}$$



G.B., B. El-Bennich, G. Krein, F. Serna, R. Silveira, Phys.Rev.D **112**, 114023 (2025).

Numerical results

$$D_u^{\pi^+}(z) \equiv \frac{1}{3}\left[D_0(z) + D_1(z)\right] \to \begin{cases} \frac{2}{3} D_0^\pi(z) = \hat{d}_q^\pi(z) + \int_z^1 \frac{dy}{y}\, \hat{d}_u^\pi\left(1 - \frac{z}{y}\right) D_0^\pi(y) \\ \frac{2}{3} D_1^\pi(z) = \hat{d}_q^\pi(z) - \frac{1}{3}\int_z^1 \frac{dy}{y}\, \hat{d}_u^\pi\left(1 - \frac{z}{y}\right) D_1^\pi(y) \end{cases}$$



G.B., B. El-Bennich, G. Krein, F. Serna, R. Silveira, Phys.Rev.D **112**, 114023 (2025).

Conclusions and perspectives

Within Curci-Ferrari model...

- Bethe-Salpeter equation
- Pion decay constant
- Parton distribution functions
- Fragmentation functions

But only in the chiral limit!

Next step requires massive pions!

Conclusions and perspectives

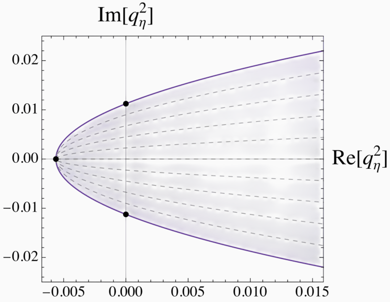
Work in progress...

- Quark propagator in the complex plane

$$S(q_\eta) = \frac{Z(q_\eta^2)}{\left[-i\not{q}_\eta + M(q_\eta^2)\right]}$$

$$q_\eta = q + \frac{p}{2}$$

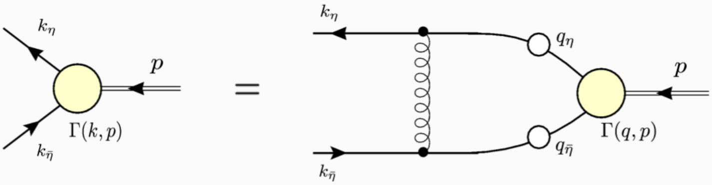
$$q_\eta^2 = q^2 - \frac{m_\pi^2}{4} + i m_\pi q z_q$$



Conclusions and perspectives

Work in progress...

- Bethe-Salpeter equation: pion mass and decay constant

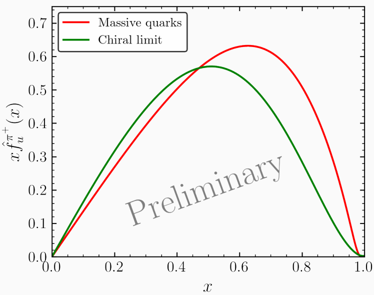


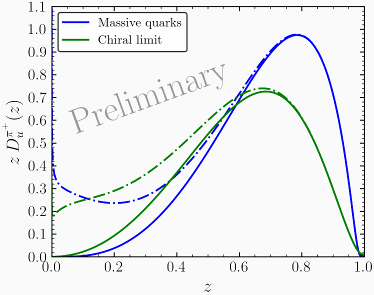
$$\lambda(p^2)\Gamma_\pi = \mathcal{K}\Gamma_\pi\,, \qquad \text{with} \quad \lambda(p^2 = -m_\pi^2) = 1$$

Conclusions and perspectives

Work in progress...

- PDFs and FFs...





Thank you!

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