Matching relations for gluon TMDs up to one-loop accuracy

Alessio Carmelo Alvaro

In collaboration with N. Kato, B. Pasquini, C. Pisano, S. Rodini

QCD Evolution, El Escorial (Madrid) 05-12-2026

Based on 2605.07516

This talk is supported by the SHARP Cost Action CA24159

UNIVERSITÀ DI PAVIA

Istituto Nazionale di Fisica Nucleare

Outline

1) Introduction

2) LO and NLO Results

3) Mass corrections

4) Conclusions

Introduction

Matching relations

In the small-$\boldsymbol{b}$ regime:

$$f_i(x,b) = \sum_n \alpha_s^n \sum_j \left[C_{ij}^{(n)} \otimes f_j\right](x) \; + \; O(b^2)$$

TMD

Matching Coefficient

PDF

Matching relations

In the small-b regime:

$$f_i(x,b) = \sum_n \alpha_s^n \sum_j \left[C_{ij}^{(n)} \otimes f_j\right](x) \; + \; O(b^2)$$

Constraints on TMD functional form → Parameterization $f_i(x,b) \propto \sum_j \left(C_{ij} \otimes f_j\right)(x)$

Predictions for TMD observables

Actual Status of Matching Relations

Quark TMDs

Distribution	Tw2	Tw3	Accuracy
f_1	$f_1\,,f_g$	-	$\mathrm{N^3LO}$
g_1	$g_1\,,\Delta f_g$	-	$\mathrm{N^3LO}$
g_{1T}	$g_1\,,\Delta f_g$	$\mathcal{T}_g\,,\mathcal{G}$	NLO
h_1	h_1	-	$\mathrm{N^3LO}$
h_{1L}	h_1	$\mathcal{T}_h$	NLO
$h_{1T}^{\perp}$	-	$\mathcal{T}_h$	LO
$f_{1T}^{\perp}$	-	$T\,,\mathcal{G}$	NLO
$h_1^{\perp}$	-	E	NLO

Actual Status of Matching Relations

Quark TMDs

Distribution	Tw2	Tw3	Accuracy
f_1	$f_1\,, f_g$	-	$\mathrm{N^3LO}$
g_1	$g_1\,, \Delta f_g$	-	$\mathrm{N^3LO}$
g_{1T}	$g_1\,, \Delta f_g$	$\mathcal{T}_g\,, \mathcal{G}$	NLO
h_1	h_1	-	$\mathrm{N^3LO}$
h_{1L}	h_1	$\mathcal{T}_h$	NLO
$h_{1T}^{\perp}$	-	$\mathcal{T}_h$	LO
$f_{1T}^{\perp}$	-	$T\,, \mathcal{G}$	NLO
$h_1^{\perp}$	-	E	NLO

Gluon TMDs

Distribution	Tw2	Tw3	Accuracy
f_1^g	$f_g\,, f_1$	-	$\mathrm{N^3LO}$
$h_1^{\perp g}$	$f_g\,, f_1$	-	$\mathrm{N^3LO}$
g_{1L}^g	$\Delta f_g\,, g_1$		$\mathrm{N^3LO}$
g_{1T}^g			
$f_{1T}^{\perp g}$	-		
h_{1T}^g	-		
$h_{1L}^{\perp g}$			
$h_{1T}^{\perp g}$			

LO and NLO Results

Tree level computation

$$G^{\mu\nu}(x,b) = \sum_{n=0}^{\infty} \frac{1}{n!} b_{\mu_1} \dots b_{\mu_n} \left[\partial^{\mu_1} \dots \partial^{\mu_n} G^{\mu\nu}(x,b) \right]_{|b=0}$$

Each term is a sum of operators with geometrical twist $2 \leq k \leq n+2$

Issue: each operator needs to be treated separately

Tree level computation

$$G^{\mu\nu}(x,b) = \sum_{n=0}^{\infty} \frac{1}{n!} b_{\mu_1} \dots b_{\mu_n} \left[\partial^{\mu_1} \dots \partial^{\mu_n} G^{\mu\nu}(x,b)\right]_{|b=0}$$

Each term is a sum of operators with geometrical twist $2 \leq k \leq n+2$

Issue: each operator needs to be treated separately

Does a unified treatment exist?

Moos, Vladimirov, JHEP 12 (2020) 145

Rodini, Alvaro, Pasquini, PLB 845 (2023) 138163

Example of results: unpolarized

$$f_1^g(x,b) = f_g(x)$$

$$+\sum_{k=1}^{\infty} \frac{1}{k!(k-1)!} \left(\frac{x^2M^2b^2}{4}\right)^k \int_0^1 du \int dy\, \delta(x-uy) \left(\frac{1-u}{u}\right)^{k-1} f_g(y)$$

Leading Term

Mass series

Example of results: Sivers

$$f_{1T}^{\perp,g}(x,b) = \mp\frac{2\pi}{x}\left(2F_2^+ + F_4^+\right)(-x,0,x)$$

$$\mp\sum_{k=1}^{\infty}\frac{1}{(k-1)!(k+1)!}\left(\frac{x^2M^2b^2}{4}\right)^k\int_0^1 du\int dy\,\delta(x-uy)\,G_k(u)\,\frac{2\pi}{y}\left(2F_2^+ + F_4^+\right)(-y,0,y)$$

Leading Term

Mass series

T-oddness

Example of results: worm-gear T

$$g_{1T}^{g}(x,b) = -\int_0^1 du \int dy\, \delta(x-uy)\, u \left(y\Delta f_g(y) + \frac{\mathscr{F}(y)}{y} - \frac{\mathscr{T}(y)}{y} \right)$$

WW approximation

Twist 3 gluon PDF

Twist 3 quark PDF

Summary tree-level

6 TMDs match at tree-level up to twist 3 PDFs

$$\left\{f_1^g, g_{1L}^g\right\},\ \left\{g_{1T}^{\perp,g}\right\},\ \left\{f_{1T}^{\perp,g}, h_{1T}^g\right\},\ \left\{h_{1L}^{\perp,g}\right\}$$

Summary tree-level

6 TMDs match at tree-level up to twist 3 PDFs

$$\left\{f_1^g, g_{1L}^g\right\}, \left\{g_{1T}^{\perp,g}\right\}, \left\{f_{1T}^{\perp,g}, h_{1T}^g\right\}, \left\{h_{1L}^{\perp,g}\right\}$$

$h_1^{\perp,g}$: NLO matching

$h_{1T}^{\perp,g}$: twist 4 and 5 PDFs

NLO: Parton-in-parton

$$\Phi_i(x, b) \stackrel{NLO}{=} a_s \int_0^1 du \int dy\, \delta(x - uy) \sum_j C_{ij}(u)\, \Phi_j(y)$$

NLO: Parton-in-parton

$$\Phi_i(x,b) \overset{NLO}{=} a_s \int_0^1 du \int dy\, \delta(x-uy) \sum_j C_{ij}(u)\, \Phi_j(y)$$

$$C_{ij} \Longleftrightarrow \langle p_j\,, s_j \,|\, F^{\mu+}\, \Gamma^i_{\mu\nu}\, F^{\nu+} \,|\, p_j\,, s_j \rangle$$

$\Gamma_{\mu\nu}$ proper projector

$p_j\,, s_j$ gluon/quark final state

Main idea

Question: can we extend to higher-twist? and how?

$$p_j^\mu = (p^+, 0, \mathbf{0}) \Longrightarrow p_j^\mu = \left(p^+, \frac{\boldsymbol{p_T}^2}{2p^+}, \boldsymbol{p_T}\right)$$

$\left(\boldsymbol{b} \cdot \boldsymbol{p_T}\right)^n$ associated to higher-twist terms

Main idea

Question: can we extend to higher-twist? and how?

$$p_j^\mu = (p^+, 0, \mathbf{0}) \Longrightarrow p_j^\mu = \left(p^+, \frac{\boldsymbol{p_T}^2}{2p^+}, \boldsymbol{p_T}\right)$$

$\left(\boldsymbol{b} \cdot \boldsymbol{p_T}\right)^n$ associated to higher-twist terms

LO computation: $\boldsymbol{e}^{i(bp_T)}$

NLO computation: $\boldsymbol{e}^{iu(bp_T)}$

Formalization

$$p_T^\mu \to -\, i\, \partial_T^\mu(0) \Longrightarrow e^{iub\cdot p_T} \to e^{ub\cdot\partial_T(0)}$$

$$\Phi_i(x,b) \overset{NLO}{=} \int_0^1 du \int dy\, \delta(x-uy)\, \sum_j C_{ij}(u)\, \left(e^{ub\cdot\partial_T(0)}\, \Phi_j(y,b) \right)$$

Formalization

$$p_T^\mu \to -\, i\, \partial_T^\mu(0) \Longrightarrow e^{iub\cdot p_T} \to e^{ub\cdot\partial_T(0)}$$

$$\Phi_i(x,b) \overset{NLO}{=} \int_0^1 du \int dy\, \delta(x-uy)\, \sum_j C_{ij}(u)\, \left(e^{ub\cdot\partial_T(0)}\, \Phi_j(y,b)\right)$$

Inclusion of hadron-mass corrections

Formalization

$$p_T^\mu \to -i\,\partial_T^\mu(0) \Longrightarrow e^{iub\cdot p_T} \to e^{ub\cdot\partial_T(0)}$$

$$\Phi_i(x,b) \overset{NLO}{=} \int_0^1 du \int dy\, \delta(x-uy)\, \sum_j C_{ij}(u) \left(e^{ub\cdot\partial_T(0)}\,\Phi_j(y,b)\right)$$

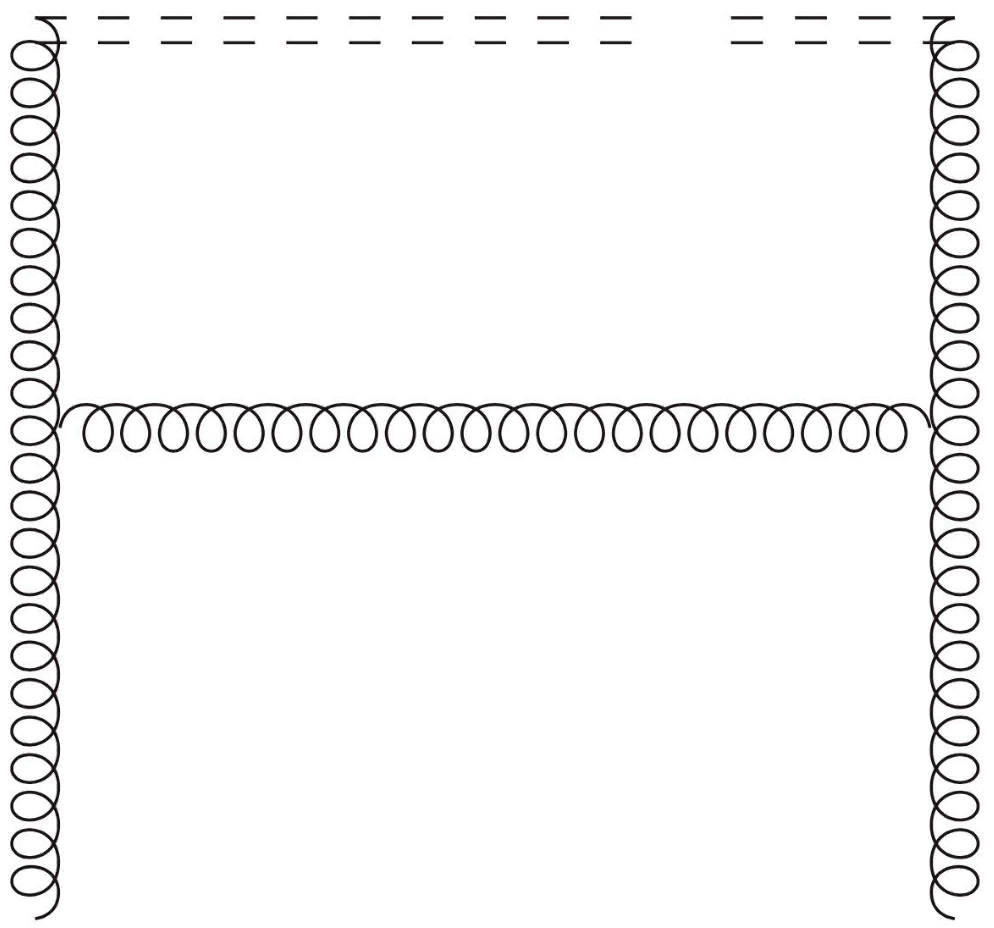
Inclusion of hadron-mass corrections

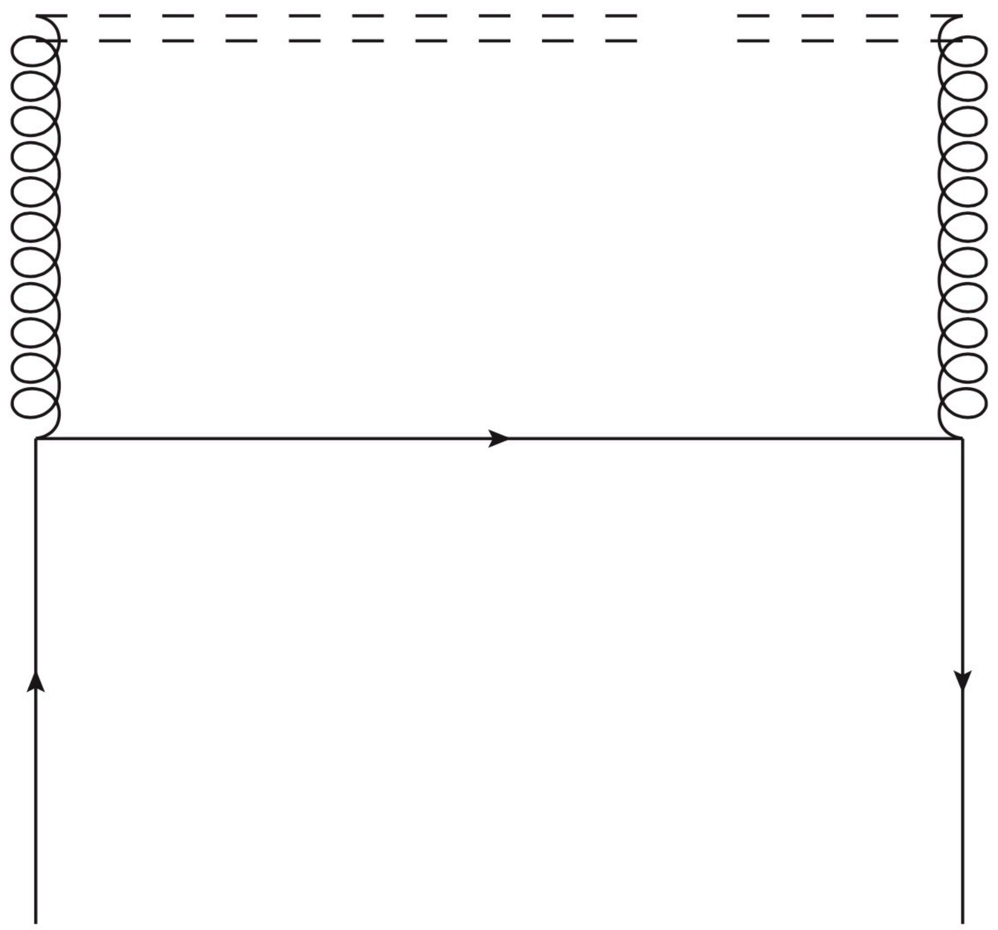
At the moment only twist 2 PDFs

Gluon results

Light-cone gauge

Recover known results





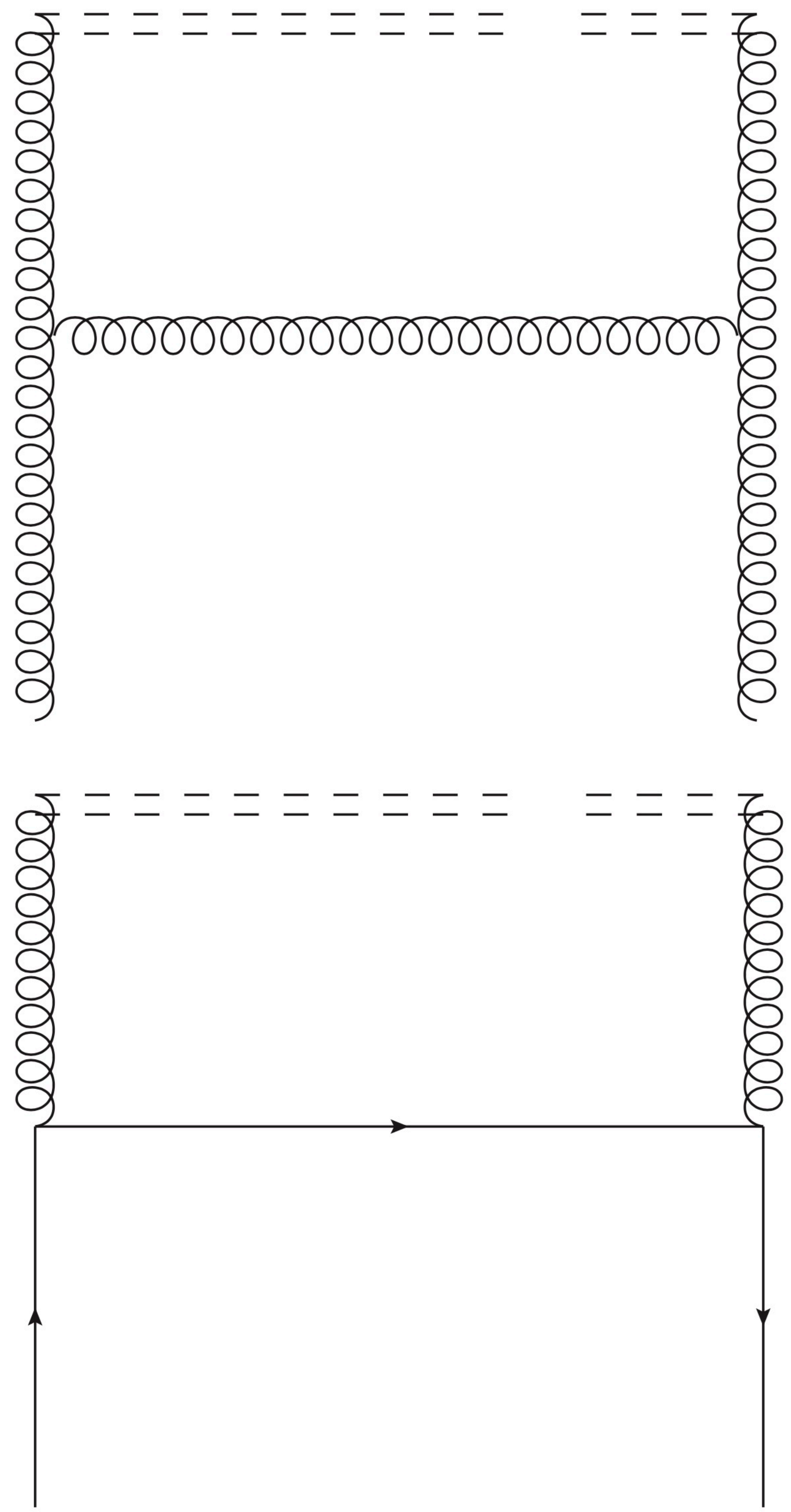
Gluon results

Light-cone gauge

Recover known results

Wandzura-Wilczek for the $g_{1T}^{\perp,g}$:

$$g_{1T}^{g}(x,b) = \int_0^1 du \int dy\, \delta(x-uy)$$
$$\times \left\{ 4\, C_{gg}(\mu^2, b^2, \epsilon)\, u \left(\Delta p_{gg} + 2\epsilon(1-u) \right) \left(-y g_T^g(y) \right) + 2\, C_{gq}(\mu^2, b^2, \epsilon)\, u \left(\Delta p_{gq} + 2\epsilon(1-u) \right) \left(y g_T^q(y) \right) \right\}$$



Mass Corrections

‘Universality’ of mass corrections

Exact correspondence of the mass series between gluon and quark TMDs

f_1 ,$f_{1T}^{\perp}$, g_{1L}, g_{1T}: operators have similar mathematical structure

‘Universality’ of mass corrections

Exact correspondence of the mass series between gluon and quark TMDs

f_1 ,$f_{1T}^{\perp}$, g_{1L} , g_{1T}: operators have similar mathematical structure

Quark h_{1T} , $h_{1L}^{\perp}$: twist 2 PDF; gluon h_{1T} , $h_{1L}^{\perp}$: twist 3 PDF

$\Longrightarrow$ operators with different mathematical structures

‘Universality’ of mass corrections

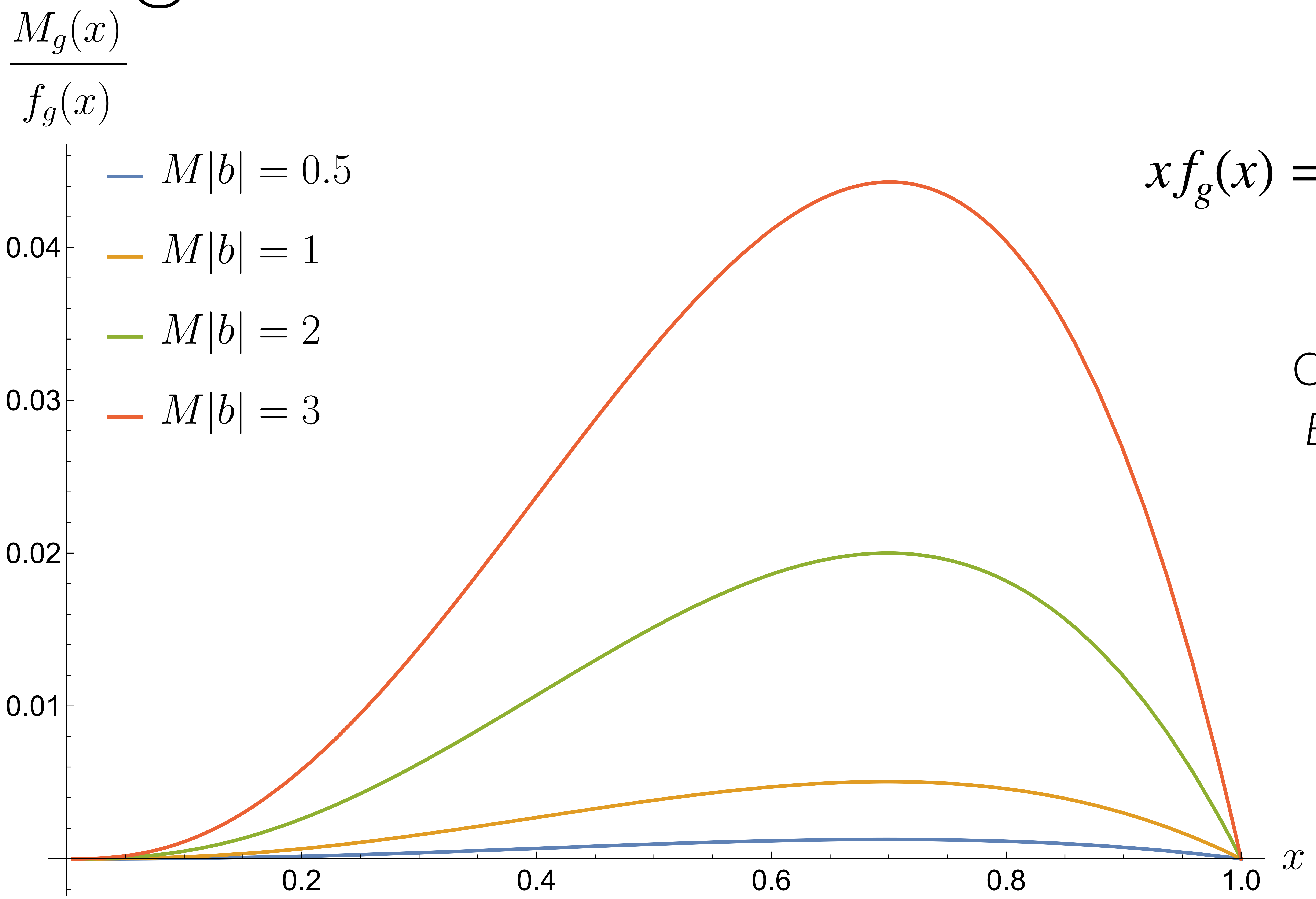
Exact correspondence of the mass series between gluon and quark TMDs

$f_1, f_{1T}^{\perp}, g_{1L}, g_{1T}$: operators have similar mathematical structure

Quark $h_{1T}, h_{1L}^{\perp}$: twist 2 PDF; gluon $h_{1T}, h_{1L}^{\perp}$: twist 3 PDF
$\Longrightarrow$ operators with different mathematical structures

Underlying symmetry?

Magnitude of mass corrections: unpolarized

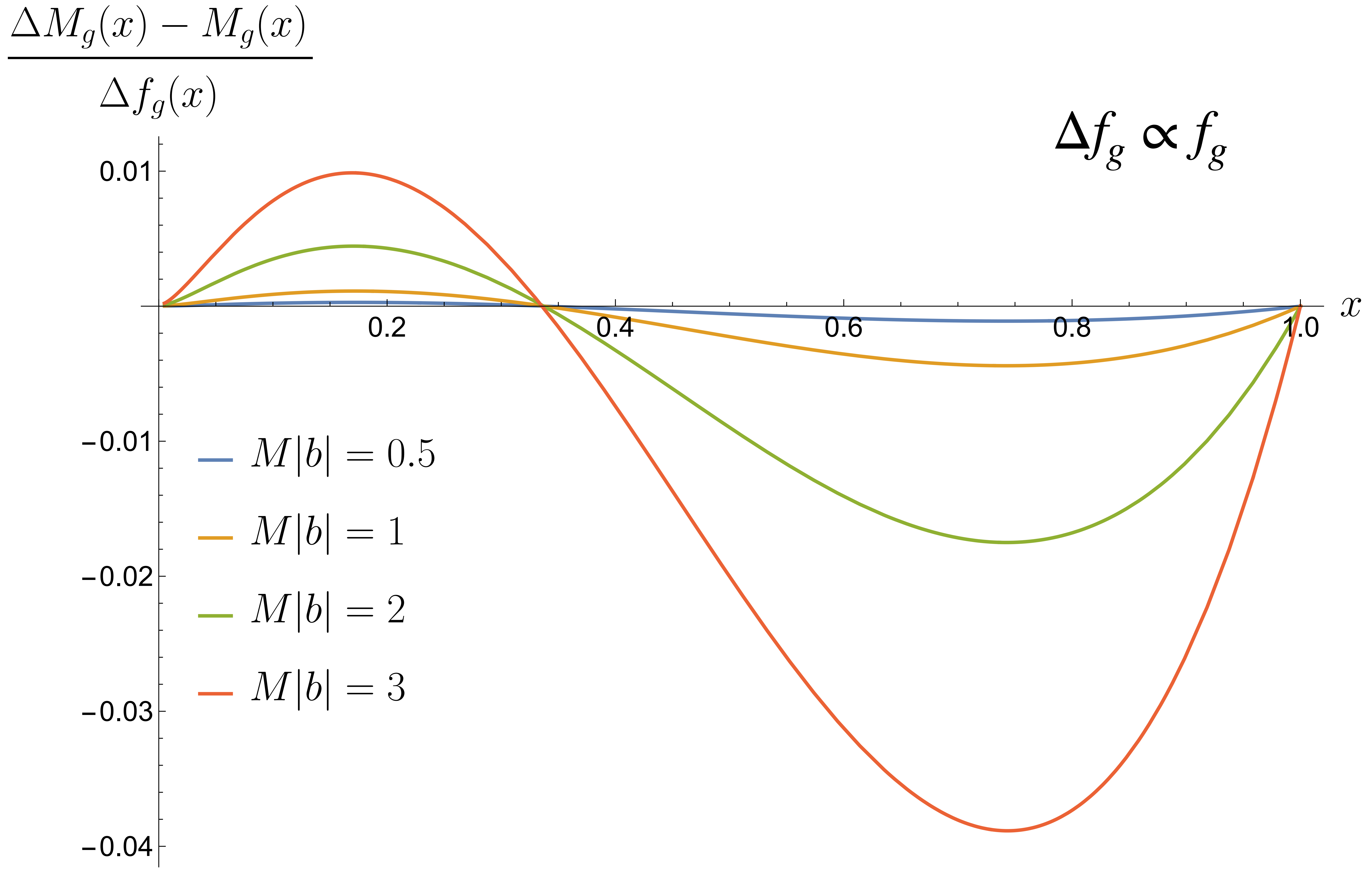


$$x f_g(x) = A_g\, x^{\alpha_g} (1-x)^{\beta_g} - A'_g\, x^{\alpha'_g} (1-x)^{\beta'_g}$$

$$\mu^2 = 1.9\,\mathrm{GeV}^2$$

Central values of HeraPDF2.0
Eur.Phys.J.C 75 (2015) 12, 580

Magnitude of mass corrections: helicity



Mass corrections at one-loop

$$\Phi_i(x,b) \overset{NLO}{=} a_s \int_0^1 du \int dy\, \delta(x-uy) \sum_j C_{ij}(u) \left(e^{ub\cdot\partial_T(0)} \Phi_j(y,b) \right)$$

Mass corrections at one-loop

$$f_1^g(x,b) \stackrel{NLO}{=} \text{leading term} +$$

$$\int_0^1 du \int_0^1 dv \int dy \sum_{k=1}^{\infty} \frac{\delta(x-uvy)}{k!(k-1)!} \left(\frac{x^2M^2b^2}{4}\right)^k \left(\frac{\bar{v}}{v}\right)^{k-1} \left(C_{gg}^{U/U}(u,b) f_g(y) + C_{gq}^{U/U}(u,b) f_1(y)\right)$$

$\int du$ from the loop-computation

$\int dv$ from the mass series

Conclusions

Summary

Distribution	Tw2	Tw3	Accuracy
f_1^g	$f_g\,,f_1$	-	N^3LO
$h_1^{\perp g}$	$f_g\,,f_1$	-	N^3LO
g_{1L}^g	$\Delta f_g\,,g_1$	-	N^3LO
g_{1T}^g	$\Delta f_g\,,g_1$	$\mathcal{F}\,,\mathcal{T}$	NLO/LO
$f_{1T}^{\perp g}$	-	$2F_2^+ + F_4^+$	LO
h_{1T}^g	-	$2F_2^+ - 2F_4^+$	LO
$h_{1L}^{\perp g}$	-	$2F_2^+ - 2F_4^+$	LO
$h_{1T}^{\perp g}$	-	-	LO

Summary

Tree-Level: matching up to twist-3 PDFs (T-Odd effects)

One-loop: WW for worm-gear T

Summary

Tree-Level: matching up to twist-3 PDFs (T-Odd effects)

One-loop: WW for worm-gear T

'Extended' parton-in-parton method

Hadron-mass corrections to NLO

Outlook

Inclusion of higher-twist PDFs

Inclusion of T-odd effects

Outlook

Inclusion of higher-twist PDFs

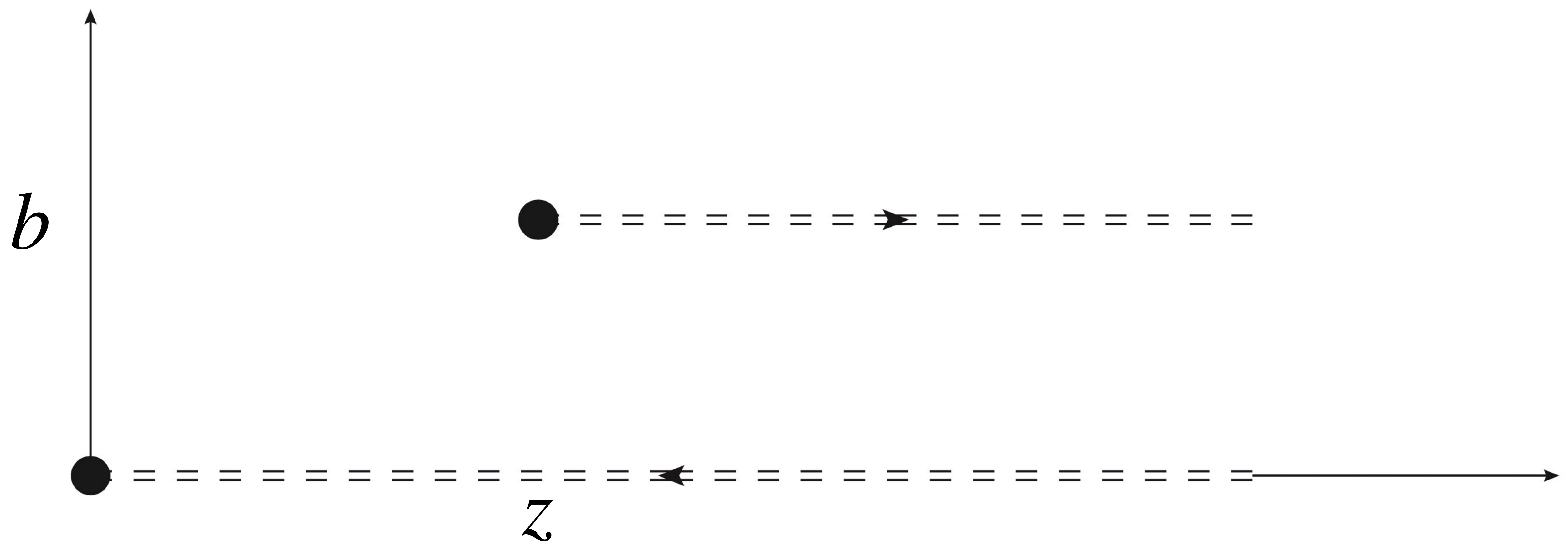
Inclusion of T-odd effects

Extension to NLP TMDs

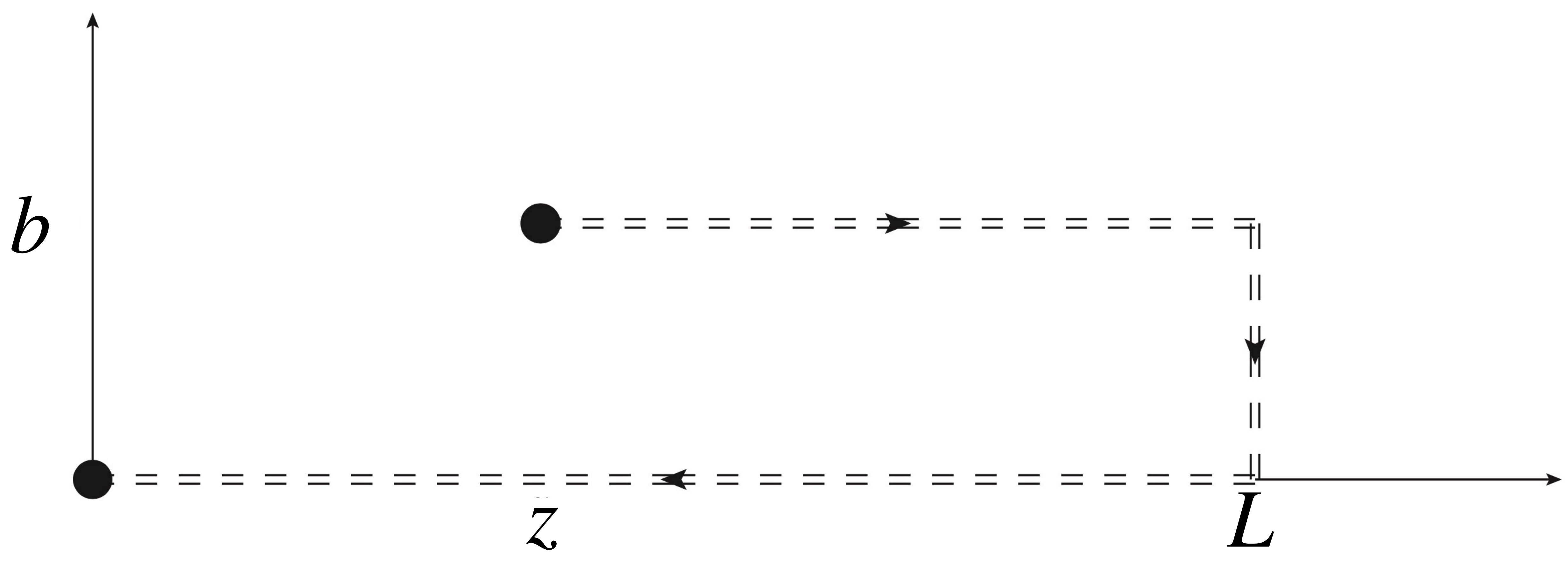
Extension to GTMDs

Backup

Tree level computation

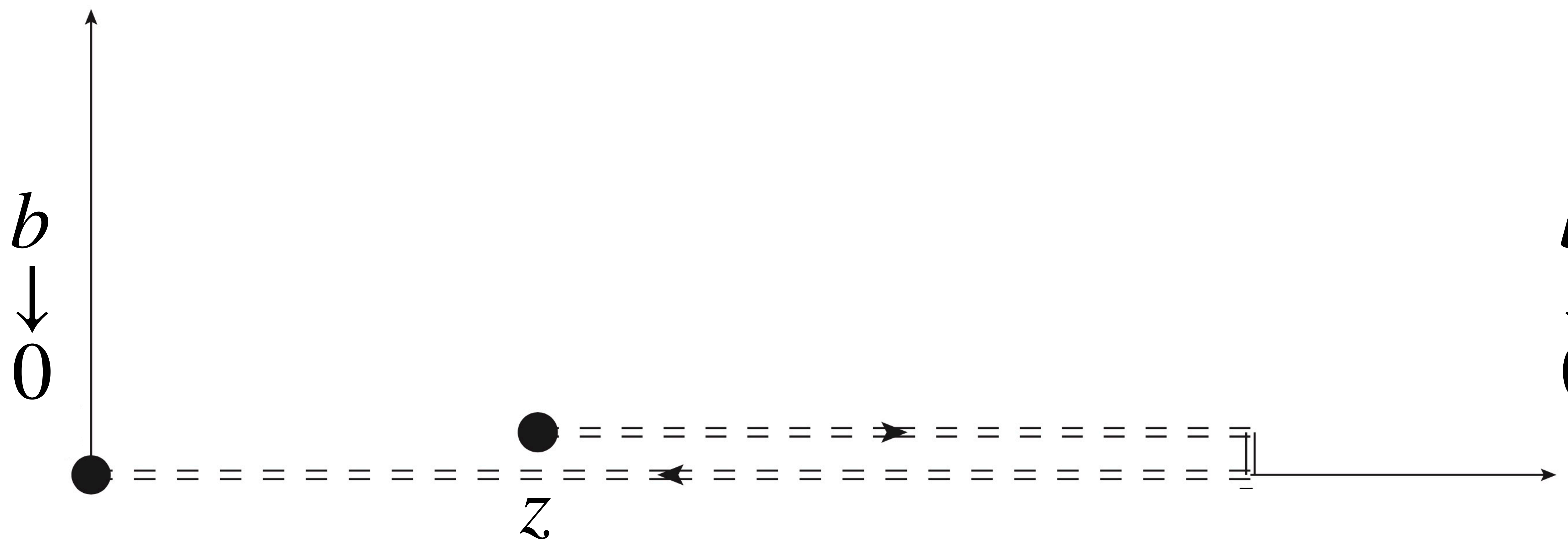


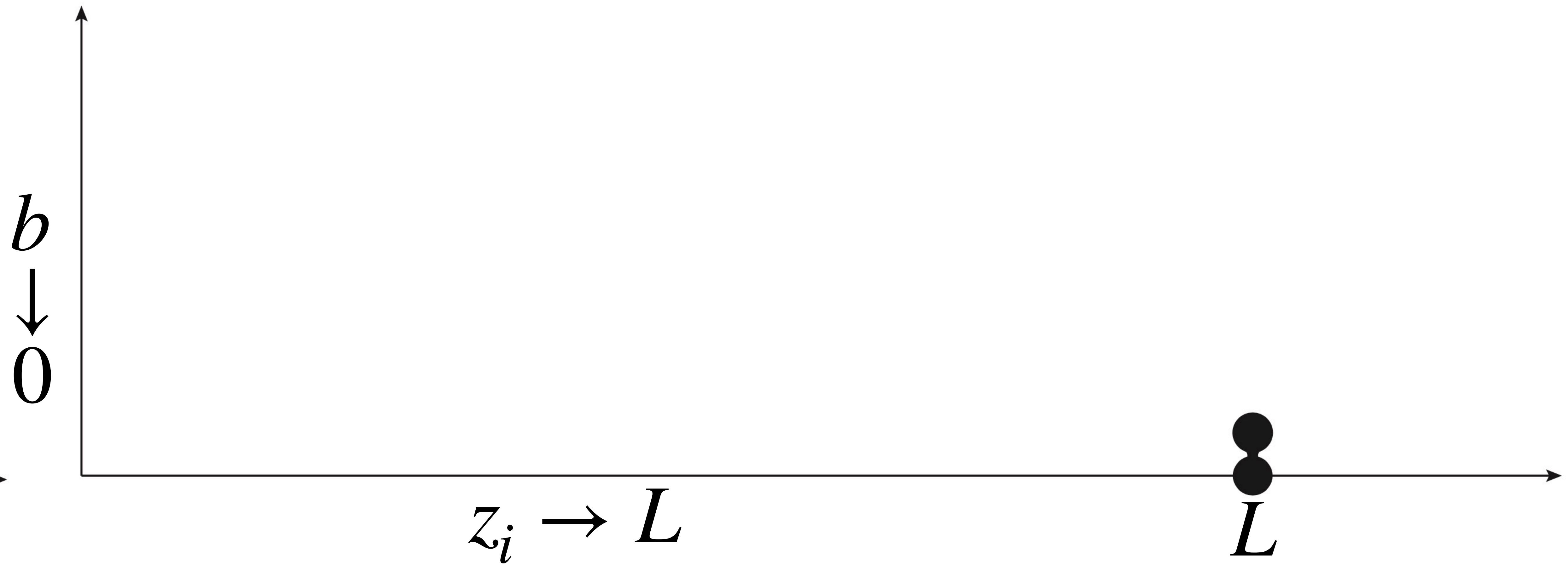
$$F^{\mu+}(zn+b)\,[zn+b\,,\infty n+b]$$
$$[\infty n+b\,,\infty n]\,[\infty n\,,0]\,F^{\nu+}(0)$$



$$F^{\mu+}(zn+b)\,[zn+b\,,Ln+b]$$
$$[Ln+b\,,Ln]\,[Ln\,,0]\,F^{\nu+}(0)$$

Tree level computation





$$\sum_{n=0}^{\infty} \frac{1}{n!} \left(F^{\mu+}(zn)\,[zn\,, Ln]\right) \left(b \cdot \overleftarrow{D}\right)^n [Ln\,,0]\, F^{\nu+}(0) \quad \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{s,t=0}^{\infty} \frac{(z-L)^s(-L)^t}{s!t!} F^{\mu+} \overleftarrow{D}_+^s \left(b \cdot \overleftarrow{D}\right)^n D_+^t F^{\nu+}$$

Tree level computation

Spinor Formalism: $x_\mu \to x_{\alpha\dot{\alpha}} = x_\mu \sigma^\mu_{\alpha\dot{\alpha}}$

Twist decomposition → (anti)symmetrization of spinor indeces

Hadronic matrix element

$$L \to \mp \infty$$

Fourier transform to x-space

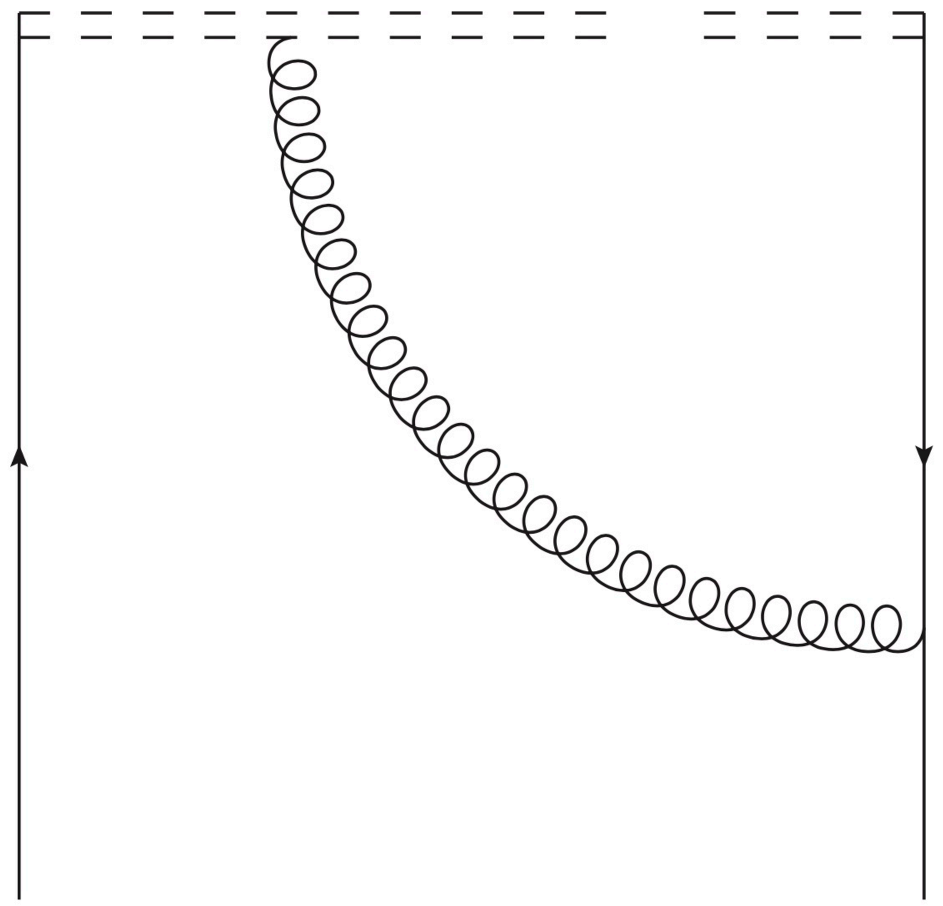
Check: quark worm-gears (one-loop)

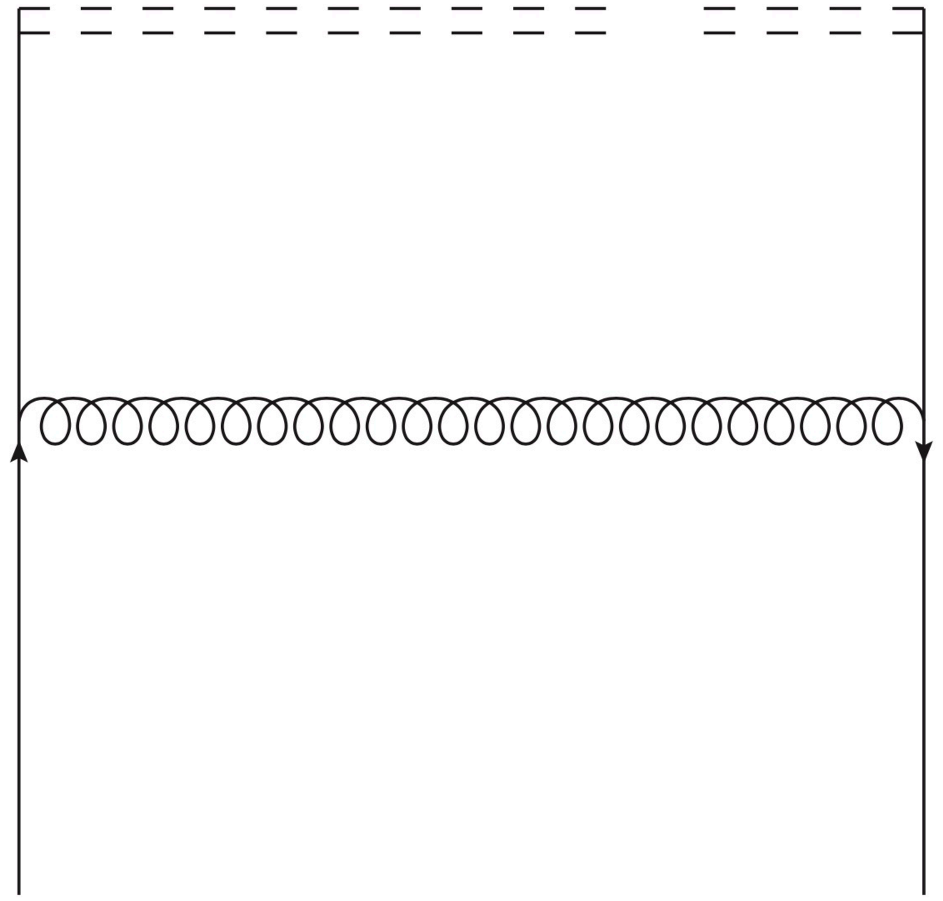
Comparison with position-space computation
(Feynman gauge)

For each diagram:

LT → LP + NLP

NLT → LP





Check: quark worm-gears (one-loop)

Position space:

LT→ $\gamma^+\gamma^5 + (b\partial)\gamma^+\gamma^5 + \ldots \sim O(b^0) + O(b^1) + \ldots$

NLT→ $\gamma_T^\mu\gamma^5 \sim O(b^1) + \ldots$

Parton-in-parton:

Global phase $e^{iub\cdot p_T} = 1 + iub \cdot p_T + \ldots$

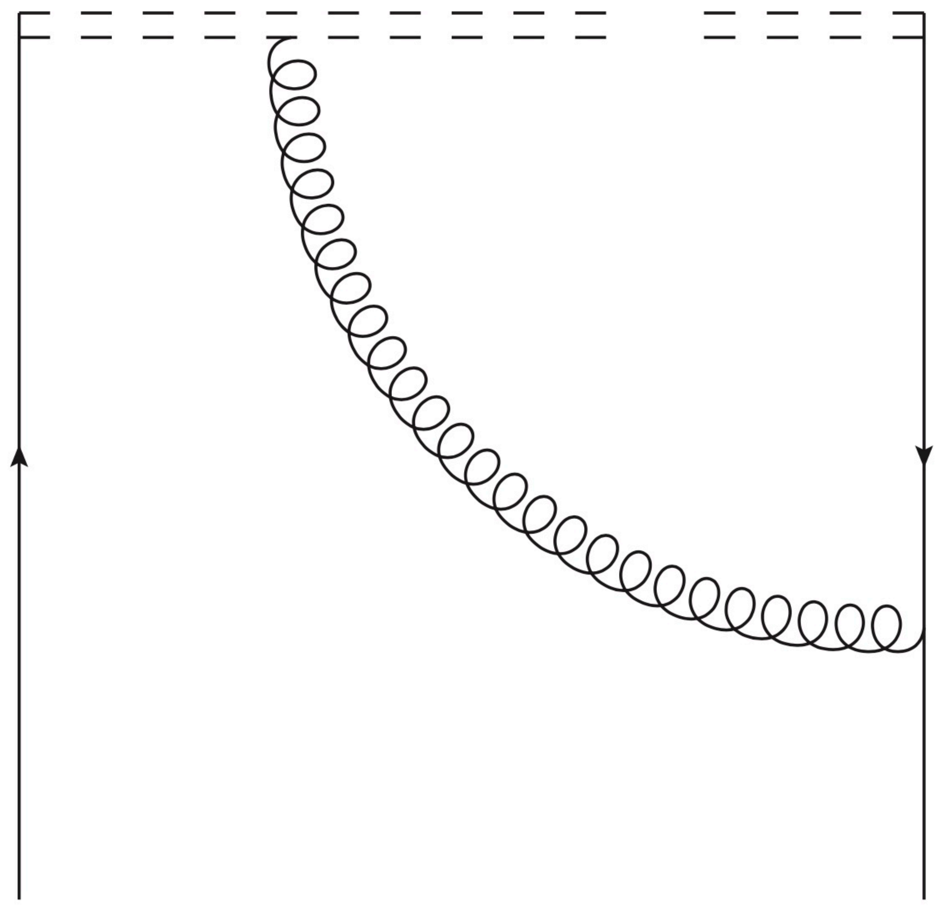
Projector $\gamma^5\gamma^\alpha P_\alpha = \gamma^5\gamma^- p^+ + \gamma^5\gamma_T^\alpha p_{T,\alpha} + \gamma^5\gamma^+ p^-$

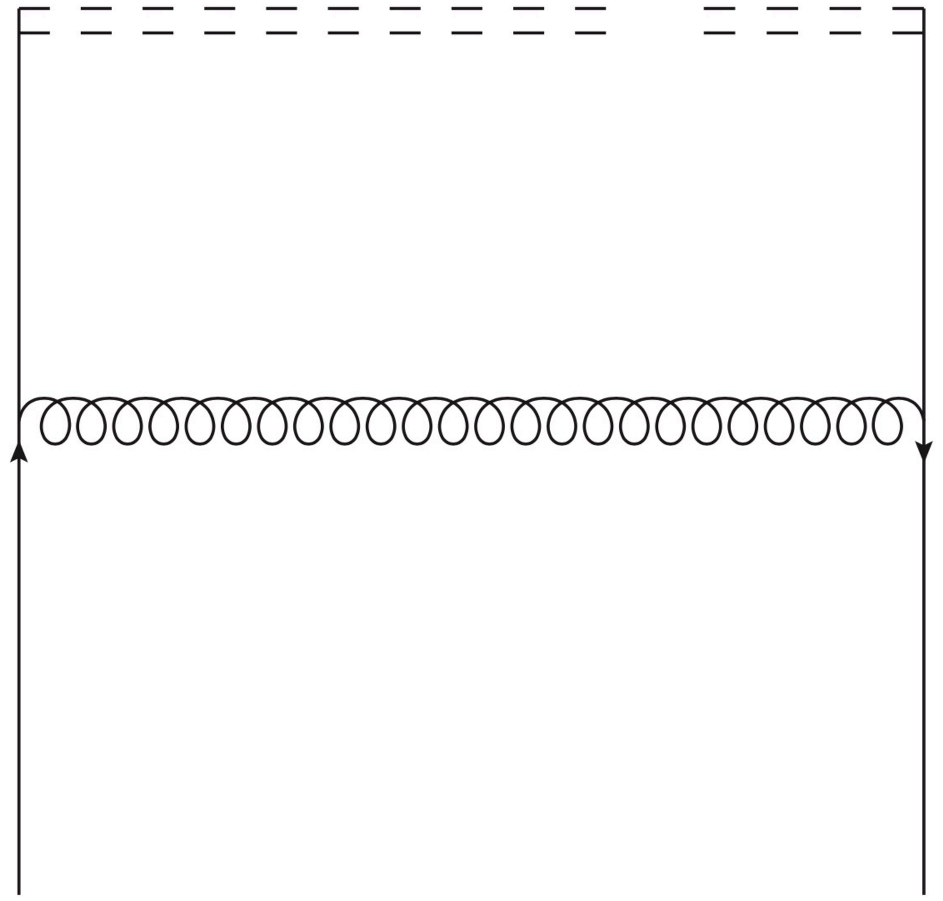
LT→$O(b^0) + O(b^1) + \ldots$

NLT→$O(b^1) + \ldots$

$O(b^0) \sim tw2 \to g_1$

$O(b^1) \sim tw2 + tw3 \to g_{1T}$





Check: quark worm-gears (one-loop)

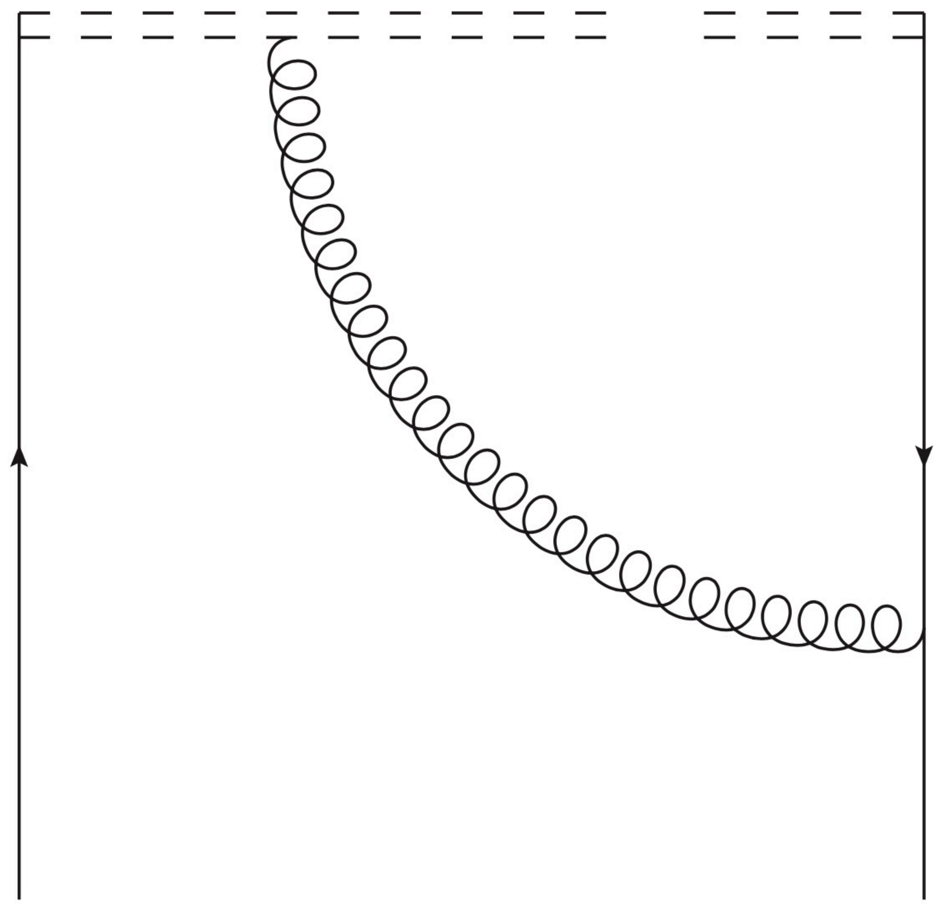
Comparison with position-space computation
(Feynman gauge)

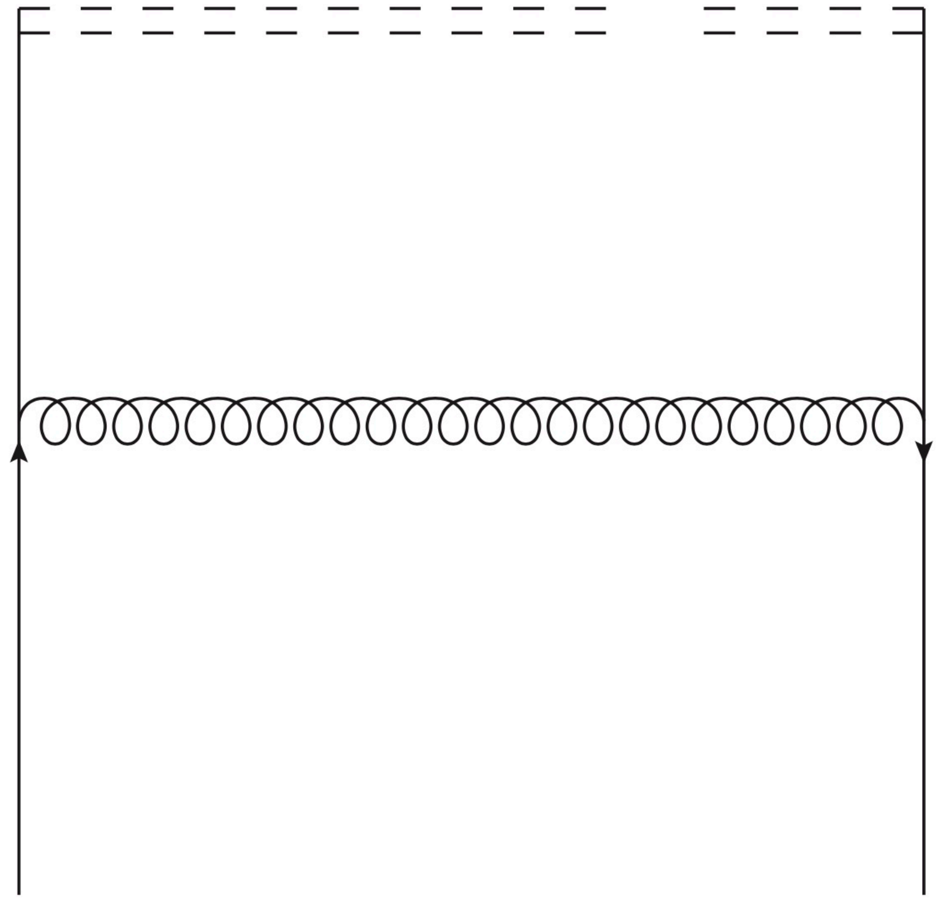
For each diagram:

LT → LP + NLP

NLT → LP

Exact correspondence!





Mass corrections beyond one-loop

Under the hypothesis that $e^{ub\cdot\partial_T(0)} \to e^{b\cdot\partial_T(0)\prod_{l=1}^n u_l}$

$$\Phi_i^{(n)}(x,b) = a_s^n \left(\prod_{l=1}^{n}\int_0^1 du_l\right)\int dy\,\delta\left(x - y\prod_{l=1}^{n}u_l\right)\sum_j C_{ij}^{(n)}(\boldsymbol{u})\,\left(e^{b\cdot\partial_T(0)\prod_{l=1}^n u_l}\Phi_j(y,b)\right)$$

$$F_i^{(n)}(x,b) = \sum_{k=1}^{\infty}\frac{a_s^n}{k!k!}\left(\frac{x^2M^2b^2}{4}\right)^k\left(\prod_{l=1}^{n}\int_0^1 du_l\right)\int_0^1 dv\int dy\,\delta\left(x - vy\prod_{l=1}^{n}u_l\right)\sum_j G_j(k,v)C_{ij}^{(n)}(\boldsymbol{u})f_j(y)$$