The analytic structure of the QCD propagators, confinement, and deconfinement

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Outline

The analytic structure of the QCD propagators

- Why we care, what we look for, what we know: recap on propagator singularities, relevance to confinement & deconfinement, high-temperature limit & more
- Recent progress: first complete calculation of the gluon propagator's IR analytic structure at $T \geq 0$ directly in complex plane, using 1-loop (deformed) perturbation theory in Landau gauge, $|\mathbf{p}| = 0$
- Interpretation and lessons learned down to $T = 0$

Propagators and their singularities

At variance with propagators and their spectral functions, poles,

$$D^{-1}(q_{\text{pole}}) = 0 \,, \qquad S^{-1}(q_{\text{pole}}) = 0$$

are gauge- and renormalization scheme-**independent**,

$$\Delta D^{-1}(q) \propto D^{-1}(q) \,, \qquad \Delta S^{-1}(q) \propto S^{-1}(q) \quad \Longrightarrow$$

$$\Longrightarrow \quad \Delta D^{-1}(q_{\text{pole}}) = 0 \,, \qquad \Delta S^{-1}(q_{\text{pole}}) = 0 \quad \Longrightarrow$$

$$\Longrightarrow \quad \Delta q_{\text{pole}} = 0$$

(ΔD^{-1}, ΔS^{-1}: gauge-dependent part of inverse propagators)

Nielsen, Nucl. Phys. B 101 (1975); Kobes, Kunstatter, Rebhan, Phys. Rev. Lett. 64 (1990)

Depending on their location/residue: physical/unphysical 1-particle/quasi-particle states. Unphysical states typical of gauge theor. + gauge-dep. ops.

QCD propagator singularities: expectations

- Pure Yang-Mills theory, full QCD: deconfinement phase transition/ smooth crossover at $T = T_c \sim \Lambda_{\text{QCD}}$.
- Physical spectrum different for $T < T_c$ (confined phase) and $T > T_c$ (deconfined phase). Can we deduce this from the singularities of the gluon and quark propagators?
  - $T > T_c$: (actually, $T \gg T_c$) quark and gluon deconfined $\Longrightarrow$ quasi-particle singularities

QCD propagator singularities: high-temperature

At very high temperatures, both the gluon and the quark propagator have poles on the second Riemann sheet, i.e. **quasi-particle** poles: e.g. on the retarded Riemann sheets, $z = \pm\omega_{P,f} - i\gamma_{P,f}$, at $\mathbf{p} = 0$,

Gluons

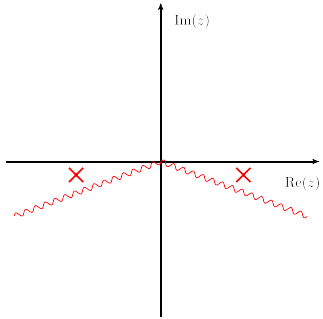
$$\omega_P = \sqrt{\frac{1+N_f/6}{3}}\, gT$$

$$\gamma_P = 6.635\, \frac{g^2 T}{8\pi}$$

Quarks (e.g. $N_f = 2$)

$$\omega_f = \frac{gT}{\sqrt{6}}$$

$$\gamma_f = 1.399\, \frac{g^2 T}{3\pi}$$



Braaten, Pisarski, Phys. Rev. D 42 (1990); Kobes, Kunstatter, Mak, Phys. Rev. D 45 (1992)

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Asymptotic freedom

$$T \to \infty \implies g(T) \to 0$$

$$g \ll 1 \implies \frac{\gamma_{P,f}}{\omega_{P,f}} \sim \frac{g}{4\pi} \ll 1$$

As $T \to \infty$, narrower and narrower peaks in $\rho(\omega)$: gluons and quarks approach free asymptotic behavior (*deconfinement*)

Braaten, Pisarski, Phys. Rev. D 42 (1990); Kobes, Kunstatter, Mak, Phys. Rev. D 45 (1992)

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Physical relevance

- Gauge-invariant (covariant, Coulomb, TA gauges)
- Same in pinch technique (gauge-invariant correlators)

  Sasaki, Nucl. Phys. B 490 (1997)
- Yield the LO contribution to the EoS of SIM (HTLs)

Braaten, Pisarski, Phys. Rev. D 42 (1990); Kobes, Kunstatter, Mak, Phys. Rev. D 45 (1992)

QCD propagator singularities: expectations and questions

- Pure Yang-Mills theory, full QCD: deconfinement phase transition/ smooth crossover at $T = T_c \sim \Lambda_{\rm QCD}$.
- Physical spectrum different for $T < T_c$ (confined phase) and $T > T_c$ (deconfined phase). Can we deduce this from the singularities of the gluon and quark propagators?
  - $T > T_c$: (actually, $T \gg T_c$) quark and gluon deconfined $\Longrightarrow$ quasi-particle singsularities ✓✓✓
  - $T < T_c$: quark and gluon confined $\Longrightarrow$ either no 1-particle singularities or unphysical complex-conjugate poles in the first Riemann sheet (allowed for correlators of gauge-dependent operators on indefinite-metric Hilbert spaces, like A^a_μ and ψ)
- How do low-T "confining" singularities evolve to high-T quasi-particle singularities? I.e. how does the physical spectrum evolve with T?

QCD propagator singularities: confined phase

No conclusive answer yet. Evidence of **complex-conjugate** poles from many approaches (mostly $T = 0$). E.g. for gluon propagator:

- **Lattice simulations**: Binosi, Tripolt, Phys. Lett. B 801 (2020)
- **Dyson-Schwinger equations**: Binosi, Tripolt, *ibidem*; Fischer, Huber, Phys. Rev. D 102 (2020)
- **Analytic approaches**: *Curci-Ferrari model*: Hayashi, Kondo, Phys. Rev. D 99 (2019); *screened massive expansion*: Siringo, Phys. Rev. D 94 (2016); Siringo, Comitini, Phys. Rev. D 98 (2018); *Gribov-Zwanziger scenario*: P. de Brito, Pereira, Phys. Rev. D 110 (2024)

However, it has been shown that lattice data can be reproduced accurately also **without** c.c. poles

- Dudal, Oliveira, Silva, Phys. Rev. D 89 (2014); Weishi Li, Lowdon, Oliveira, Silva, Phys. Lett. B 803 (2020)

Gluon propagator singularities: evolution with T

Of course no conclusive answer either. Not many studies, all involve fits/spectral reconstructions from lattice data or (Euclidean) DSEs:

- Silva, Oliveira, Bicudo, Cardoso, Phys. Rev. D 89 (2014) 074503
- Haas, Fister, Pawlowski, Phys. Rev. D 90 (2014) 091501(R)
- Silva, Oliveira, Dudal, Roelfs, Acta Phys. Polon. Supp. 10 (2017) 995
- Ilgenfritz, Pawlowski, Rothkopf, Trunin, Eur. Phys. J. C 78 (2018) 127

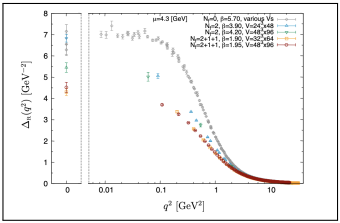
The first complete (YMT, full QCD, poles, spectral function) **direct** calculation in **complex plane** was recently carried out at $|\mathbf{p}| = 0$ using the **screened massive expansion** (massive deformation of perturbation theory) in Landau gauge to 1-loop.

G. Comitini, Phys. Rev. D 112 (2025) 094016

The screened massive expansion

F. Siringo, Nucl. Phys. B 907 (2016); F. Siringo and G. C., Phys. Rev. D 98 (2018)

In linear covariant gauges (and beyond), the transverse Euclidean gluon propagator **dynamically** acquires an **infrared mass** $\Longrightarrow$ reformulate QCD perturbation theory so this mass is accounted for



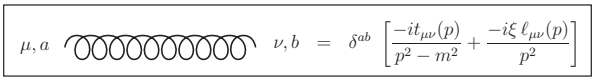
Ayala, *et al.*, PRD 86 (2012)

- Famous example: Curci-Ferrari model, $\mathcal{L}_{\text{QCD}} \to \mathcal{L}_{\text{QCD}} + \frac{m^2}{2} A^a_\mu A^{a\,\mu}$
- The gluon mass m screens the IR from divergences: no IR Landau pole $\Longrightarrow$ P.T. self-consistent

The screened massive expansion (**SME**) achieves the same without changing $\mathcal{L}_{\text{QCD}}$: only shift the expansion point of PT,

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{kin.}} + \mathcal{L}_{\text{int.}} = (\mathcal{L}_{\text{kin.}} + \delta\mathcal{L}_m) + (\mathcal{L}_{\text{int.}} - \delta\mathcal{L}_m) = \mathcal{L}'_{\text{kin.}} + \mathcal{L}'_{\text{int.}}$$

$\delta\mathcal{L}_m$ is chosen so the zero-order gluon propagator $\Delta^{ab}_{\mu\nu}(p)$ reads



Namely, in momentum space,

$$\delta S_m = \int \frac{d^d p}{(2\pi)^d} \frac{m^2}{2} A^a_\mu(-p)\, t^{\mu\nu}(p)\, A^a_\nu(p)$$

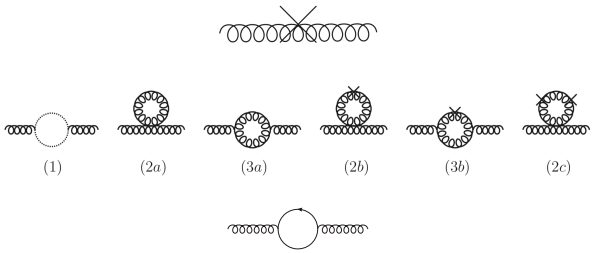
This also adds a new two-gluon vertex to the Feynman rules, the gluon "mass counterterm" $\delta\Gamma^{ab}_{\mu\nu}(p)$ ($\leftarrow$ unrelated to renormalization)

$\delta\Gamma^{ab}_{\mu\nu}(p)$ is non-perturbative: not proportional to the coupling g

Nonetheless, n-point functions are computed using the usual P.T. rules. E.g., for the 1-loop dressed transverse gluon propagator

$$\Delta(p^2) = \frac{-i}{Z_A\, p^2 - m^2 - \Pi(p^2)}$$

one computes the diagrams



with insertions of $\delta\Gamma^{ab}_{\mu\nu}(p)$ to enforce renormalizability

The screened massive expansion at finite T

F. Siringo and G.C., Phys. Rev. D 103 (2021); G.C. and F. Siringo, Phys. Rev. D 111 (2025)

Matsubara formalism to compute the chromoelectric $\Delta_L(\omega_n, |\mathbf{p}|)$ and chromomagnetic $\Delta_T(\omega_n, |\mathbf{p}|)$ components of the gluon propagator,

$$\Delta_{\mu\nu}(\omega_n, \mathbf{p}) = \Delta_T(\omega_n, |\mathbf{p}|)\, P^T_{\mu\nu}(\omega_n, \mathbf{p}) + \Delta_L(\omega_n, |\mathbf{p}|)\, P^L_{\mu\nu}(\omega_n, \mathbf{p})$$

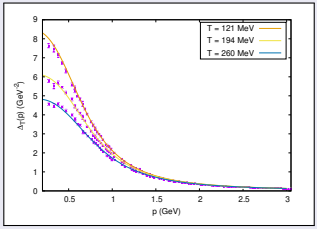
Free parameters: **gluon mass** $m(T)$ (effective mass at temperature T) and **strong coupling** $\alpha_s(T)$ – $\alpha_s(T)$ usually absorbed into other parameters.

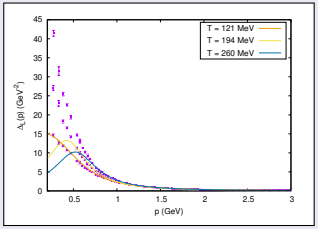
At $T = 0$ optimization based on gauge invariance of poles yields $\alpha_s(\mu_0)$ at a *fixed* scale μ_0 expressed in units of m.

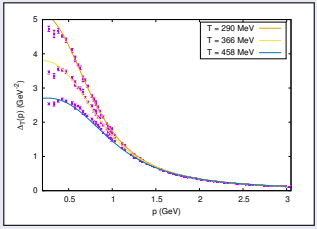
At $T > 0$, knowledge of $\alpha_s(T)$ requires the T-dependent running of the coupling in the **non-perturbative** regime: we do not have it $\implies$

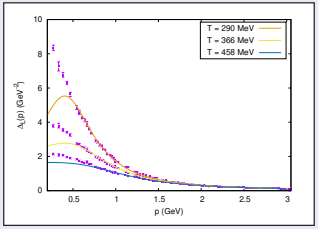
$\implies$ approximate $(m(T), \alpha_s(T)) \approx (m(0), \alpha_s(0))$, or fit both from the lattice

Pure Yang-Mills theory: Landau gauge static fits ($\omega_n = 0$)



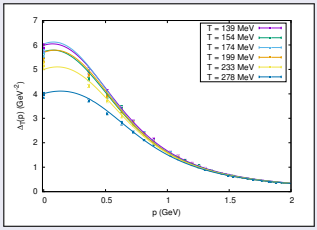


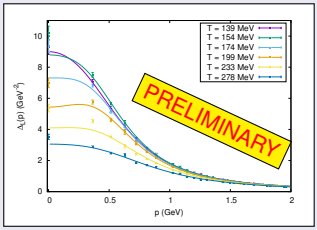




Lattice data: Silva, Oliveira, Bicudo, Cardoso, Phys. Rev. D 89 (2014) 074503

$N_f = 2$ full QCD: Landau gauge static fits ($\omega_n = 0$)





Lattice data: Salgado, Oliveira, Silva, Workshop “The complex structure of strong interactions in Euclidean and Minkowski space”, 26-30 May 2025, ECT*, Trento

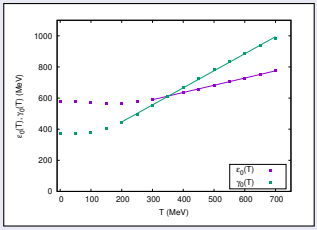
Transverse component shows better agreement $\Longrightarrow$ use params. from Δ_T

$\Delta_T(0, |\mathbf{p}|)$ not very sensitive to quark mass $\Longrightarrow$ set effective IR quark mass $M_q \approx 400$ MeV, suggested by CSB

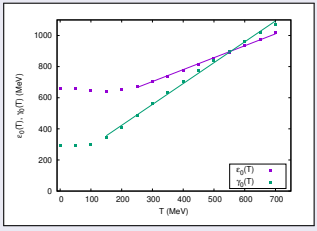
SME: $T \geq 0$ analytic struct. of the gluon propagator

G.C., Phys. Rev. D 112 (2025)

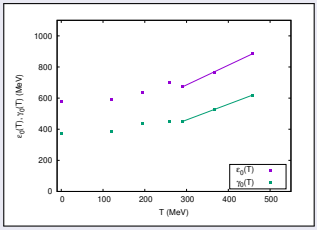
(Landau-gauge) poles @ $|\mathbf{p}| = 0$ ($\Delta_T = \Delta_L$): complex-conjugate



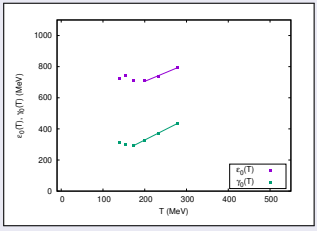
Pure Yang-Mills, opt. params. @ $T = 0$



$N_f = 2 + 1$ full QCD, opt. params. @ $T = 0$



Pure Yang-Mills, latt. fit params.



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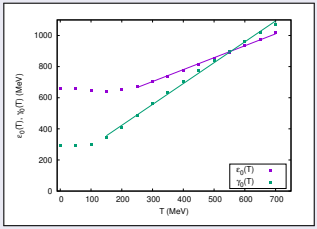

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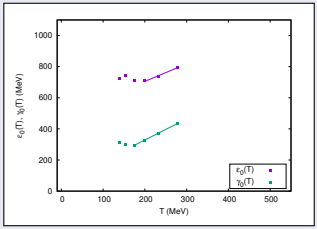


$N_f = 2$ full QCD, latt. fit params.

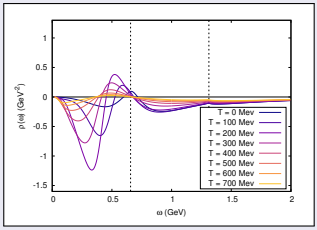
Key features

- C.c. poles up to and beyond T_c (including T's relevant to lab. QGP)
- Real and imaginary part increase with T, linearly for $T > T_c$
- Imaginary part smaller than real part when computed by lattice parameters
- Real part increases with N_f

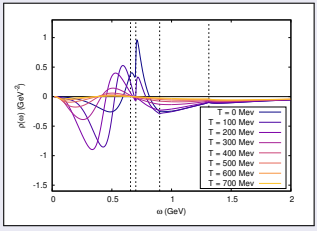
SME: $T \geq 0$ analytic struct. of the gluon propagator

G.C., Phys. Rev. D 112 (2025)

Landau-gauge spectral function @ $|\mathbf{p}| = 0$ ($\Delta_T = \Delta_L$)



Pure Yang-Mills, opt. params. @ $T = 0$



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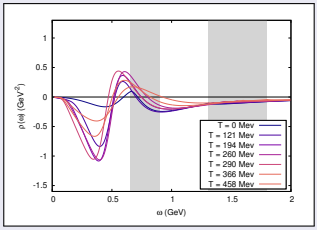
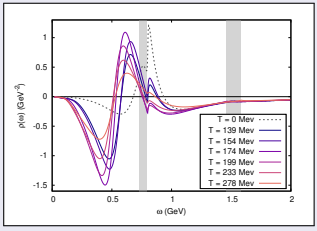
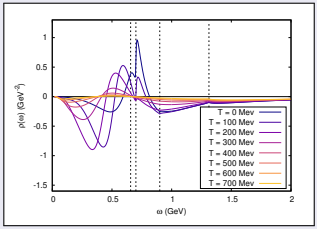


Pure Yang-Mills, latt. fit params.

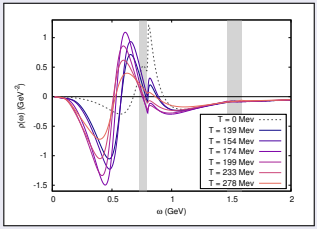


$N_f = 2$ full QCD, latt. fit params.

Landau-gauge spectral function @ $|\mathbf{p}| = 0$ ($\Delta_T = \Delta_L$)



$N_f = 2 + 1$ full QCD, opt. params. @ $T = 0$



$N_f = 2$ full QCD, latt. fit params.

Key features

- No sharp peaks up to and beyond T_c (including lab. QGP T's) $\Longrightarrow$ no evidence for quasi-particle-like behavior
- Positivity violated at low + high frequencies
- Thresholds at $2M_q$, m and $2m$
- $|\rho(\omega)|$ largest at $T \approx T_c$
- Tendency for low-ω min/max to line up for different T's when computed by lattice params
- Non-trivial dependence on M_q: possible intermediate-T enhancement, but still no peak (see paper)

Competing interpretations

- The approximation is **accurate**. Gluons still behave as **confined** degrees of freedom (no change in analytic structure) right **above** the deconfinement temperature T_c, e.g at QGP temperatures
- The approximation needs to be improved. Once improved, starting from some T, c.c. poles will **disappear** and are replaced by **no poles** – i.e. deconfined, but **strongly interacting** phase – or **sharp spectral peaks** – i.e. deconfined, **quasi-particle** picture

We **know** the approximation **does** break down at some T: the SME predicts "confining" c.c. poles up to very large temperatures $T \gg m$, but at high T we **must** recover the deconfined **quasi-particle** poles

How does this happen and what does it tell us?

From low to high T and back: lessons learned

Again: the SME predicts "confining" c.c. poles up to very large temperatures $T \gg m$. I expect e.g. the Curci-Ferrari model does too

$T \gg m$ is equivalent to $m \to 0$, i.e. ordinary perturbation theory at finite temperature.

Does this mean ordinary QCD perturbation theory predicts "confining" c.c. poles at very *high* temperatures? **YES**

Historically (1980s), this was known in very different terms as the **plasmon puzzle**. The connection of the plasmon puzzle to the issue of c.c. poles in QCD is little recognized, if at all

Plasmon puzzle: negative damping

- When the retarded gluon propagator is computed to 1-loop in covariant gauges by **ordinary** perturbation theory, the imaginary part γ (damping constant) of the gluon pole $z = \omega - i\gamma$ is found to be **gauge-dependent** and to have the **wrong sign** [Heinz, Kajantie, Toimela, Phys. Lett. B 183 (1987)]:

$$\gamma = -\frac{g^2 T}{8\pi}\left[5 + (1-\xi) + (1-\xi)^2/2\right] \leq -\frac{g^2 T}{8\pi} \times \frac{9}{2}$$

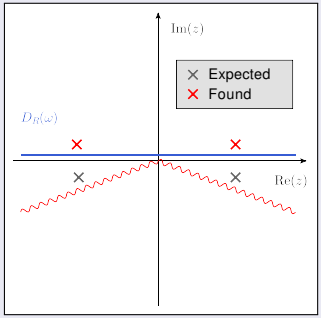
- E.g. in Landau and Feynman gauge

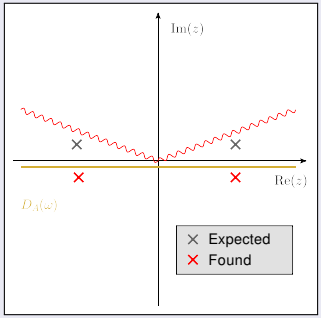
$$\gamma_{\text{Landau}} = -\frac{g^2 T}{8\pi} \times \frac{13}{2}\,, \quad \gamma_{\text{Feynman}} = -\frac{g^2 T}{8\pi} \times 5$$

From low to high T and back: lessons learned

Plasmon puzzle: negative damping $\equiv$ c.c. poles

- "Quasi-particle" poles with a "negative" damping costant are just c.c. poles! Half found by continuing $\Delta_R(\omega)$, half $\Delta_A(\omega)$

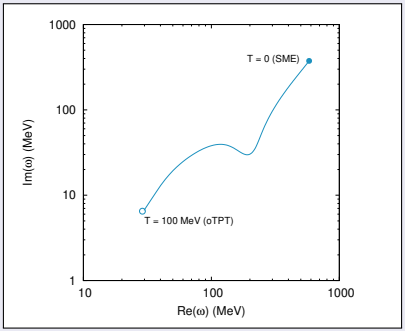




From low to high T and back: lessons learned

Plasmon puzzle: relevance to our discussion

- The c.c. poles that gave rise to the plasmon puzzle are **the same poles** found at **low** and **zero** temperatures: in analytic approaches you can draw a **continuous** line from one to the other by
  - reducing the gluon mass m and coupling constant α_s
  - increasing the temperature T



From low to high T and back: lessons learned

Plasmon puzzle: relevance to our discussion

- At high temperature, solved by resumming hard-thermal loops:

$$\Delta^{(0)}_{\mu\nu}(P) \to \Delta^{(\star)}_{\mu\nu}(P) \ , \quad \Gamma^{(0)}_{\mu\nu\sigma}(P,Q) \to \Gamma^{(\star)}_{\mu\nu\sigma}(P,Q) \ ,$$
$$\Gamma^{(0)}_{\mu\nu\sigma\tau}(P,Q,K) \to \Gamma^{(\star)}_{\mu\nu\sigma\tau}(P,Q,K)$$

- Once HTLs are resummed, gluon poles cease to be c.c. and gauge-dependent and become quasi-particle-like and gauge-independent:

$$\omega_P = \sqrt{\frac{1+N_f/6}{3}}\, gT \quad , \qquad \gamma_P = 6.635\, \frac{g^2 T}{8\pi}$$

- We **know** we **must** resum HTLs at least starting from some large temperature T. **What** temperature?

From low to high T and back: lessons learned

Plasmon puzzle: relevance to our discussion... also vacuum ??

- HTLs are characterized by:
  - Poles proportional to the gluon mass, $m_g^2/P^2 \propto g^2T^2/P^2$ in Green functions: when $P \propto m_g \sim gT$, HTLs become of the same order as tree level
  - Being determined by ghost-free Ward identities [Braaten, Pisarski, Nucl. Phys. B 339 (1990)]
- But... when a gluon mass is generated by e.g. the Schwinger mechanism, radiative corrections to the gluon propagator and vertices are expected to have similar terms also at $T = 0$!
- **Conclusion**: If at $T = 0$ dynamical gluon mass generation is realized e.g. via the Schwinger mechanism, the gluon polarization and vertices contain terms whose structure is known to make gluon c.c. poles disappear at large T's $\implies$ investigate

Thank you

Backup slides

Propagators and their singularities: a quick recap

QCD + gauge fixing:

$$\mathcal{L}_{\text{FP}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{g.f.}} + \mathcal{L}_{\text{ghost}}$$

Can define gauge- and ren. scheme-dependent propagators:

$$D_F(x)^{ab}_{\mu\nu} = i\langle T\left\{A^a_\mu(x)A^b_\nu(0)\right\}\rangle\ , \quad S_F(x) = i\langle T\left\{\psi(x)\overline{\psi}(0)\right\}\rangle$$

$$D_R(x)^{ab}_{\mu\nu} = i\langle[A^a_\mu(x), A^b_\nu(0)]\rangle\,\Theta(x^0)\ , \quad S_R(x) = i\langle\{\psi(x), \overline{\psi}(0)\}\rangle\,\Theta(x^0)$$

$$D_A(x)^{ab}_{\mu\nu} = -i\langle[A^a_\mu(x), A^b_\nu(0)]\rangle\,\Theta(-x^0)\ , \quad S_A(x) = -i\langle\{\psi(x), \overline{\psi}(0)\}\rangle\,\Theta(-x^0)$$

$$D_E(x_E)^{ab}_{\mu\nu} = \langle T_E\left\{A^a_\mu(x_E)A^b_\nu(0)\right\}\rangle\ , \quad S_E(x) = \langle T_E\left\{\psi(x_E)\overline{\psi}(0)\right\}\rangle$$

Propagators and their singularities: a quick recap

Analytic continuation*: $D_{F,R,A}(p^0), D_E(p^4) \to D(z)$

$$D(z) = \int_{-\infty}^{+\infty} d\mu\, \mu\, \frac{\rho(\mu)}{-z^2 + \mu^2}$$

$$\rho(\mu) = \frac{1}{\pi} \mathrm{Im}\{D(\mu + i\epsilon)\}$$

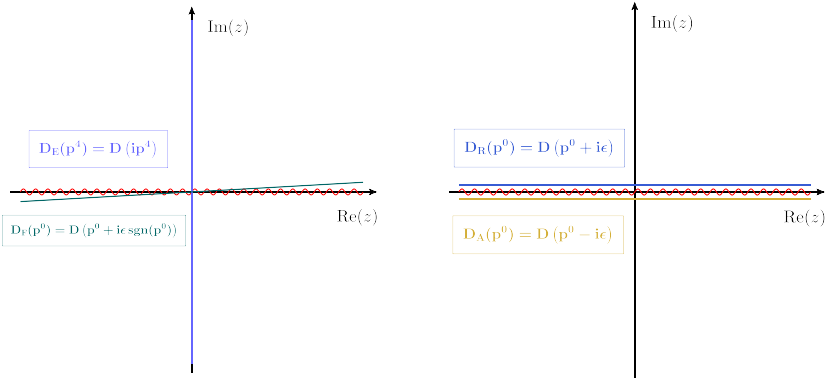
$$\rho(\mu) = \mathrm{sgn}(\mu) \sum_n \frac{\sigma_n}{2E_n} |\langle \Omega | \mathcal{F}(0) | n \rangle|^2\, \delta(|\mu| - E_n)$$

$z \in \mathbb{C}$: complexified frequency, $\sigma_n = \langle n|n\rangle = \pm 1$

* For the moment we ignore c.c. (and generalized) eigenvalues

Propagators and their singularities: a quick recap

Analytic continuation: $D_{F,R,A}(p^0), D_E(p^4) \to D(z)$

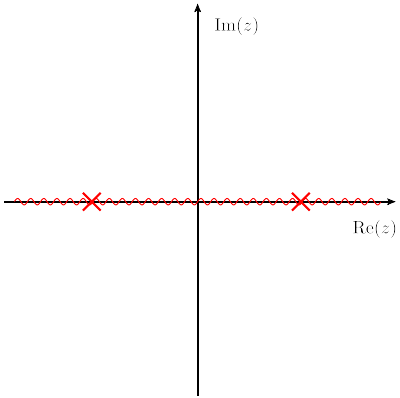


$z \in \mathbb{C}$: complexified frequency, all props. share singularities

Propagators and their singularities: a quick recap

Three types of poles:

- 1. Real poles, $z \in \mathbb{R}$: asymptotic (stable) states

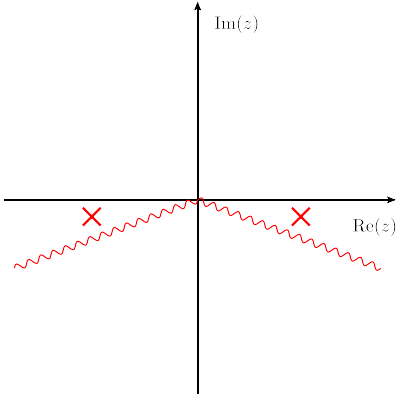


Note: we ignore generalized (a.k.a. multipole) eigenstates

Propagators and their singularities: a quick recap

Three types of poles:

- 2. “Complex” poles on secondary Riemann sheets: resonances ($T = 0$), quasi-particles ($T > 0$)

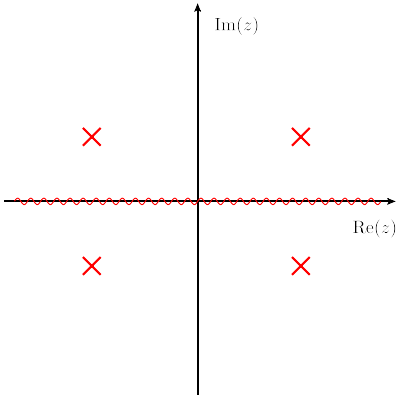


Found by analytically continuing beyond the branch cut, physically meaningful only if close to the real axis, manifest as peaks in the spectral function

Propagators and their singularities: a quick recap

Three types of poles:

- 3. C.c. poles on the principal Riemann sheet, $z \in \mathbb{C}$, $\mathrm{Im}(z) \neq 0$: "confining" poles?



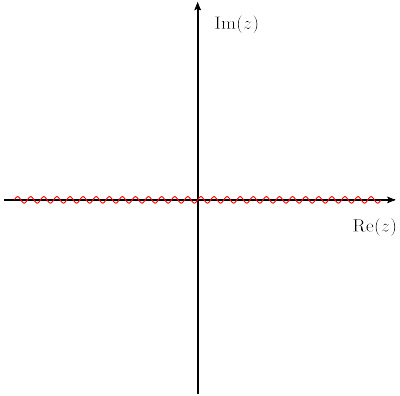
Unclear interpretation, zero-norm thus no physical states/resonances/quasi-particles, break Wick rotation (Euclidean $\leftrightarrow$ Minkowski via $ip^4 \leftrightarrow p^0$) and ret. + adv.

Propagators and their singularities: a quick recap

Kinda four

~~Three~~ types of poles:

- 4. No poles; if on secondary sheets, away from $\mathbb{R}$ (no spectral peak): no asymptotic states, no resonances, no quasi-particles



Not always possible, e.g. if $\rho(\omega) < 0$, $\Delta(ip^4) > 0$ (but same goes at least for secondary poles)

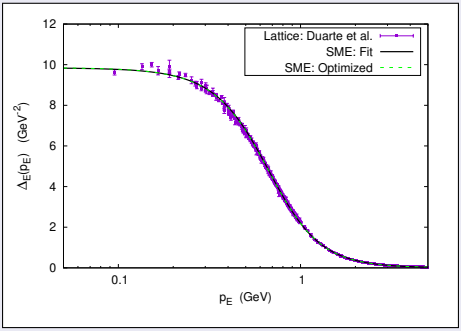
QCD propagator singularities: confined phase

The confined phase is non-perturbative $\Longrightarrow$ need to resort to **non-perturbative methods**

- **Lattice simulations**: Euclidean only, need to solve ill-defined problem of reconstruction of Minkowski/complex plane from finite Euclidean data
- **Dyson-Schwinger equations**: not easy to solve in complex plane (or reconstruction from Euclidean data), highly truncation-dependent
- **Analytic approaches**: based on modified perturbation theories, excellent agreement with Euclidean data, direct access to complex plane, whether they capture full non-perturbative information to finite order is currently unknown

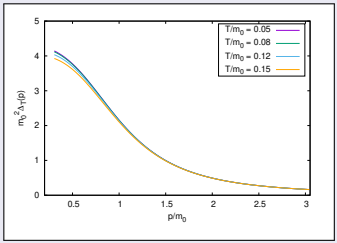
Screened massive expansion: optimization via gauge invariance

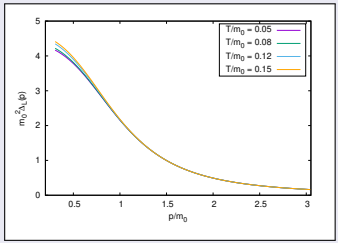
- At $T = 0$ one can optimize π_0 based on principles of gauge invariance: $\pi_0 = -0.876$. m^2 is then the only free parameter. Excellent agreement is found with the IR lattice data[a]

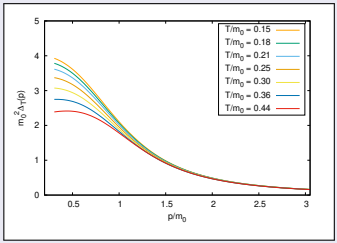


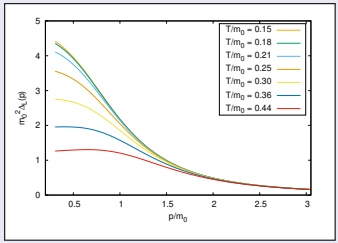
[a] $m = 0.657$ GeV for Duarte, Oliveira and Silva, Phys. Rev. D 94 (2016)

Screened massive expansion: parameters fixed from $T = 0$ ($\omega_n = 0$)

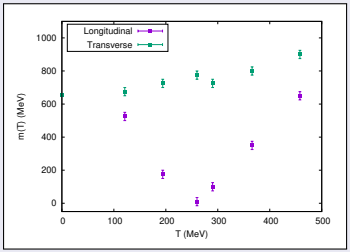


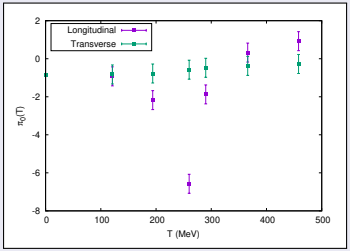






Screened massive expansion: parameters from the lattice ($\omega_n = 0$)

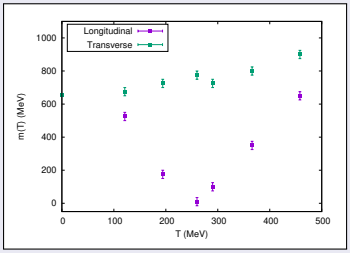


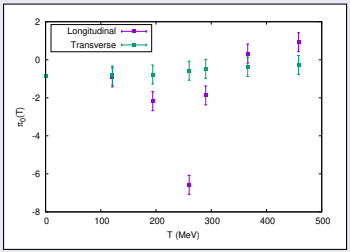


Silva, Oliveira, Bicudo, PRD 89 (2014)

- The transverse fit is quite good up to $T = 458$ MeV, the longitudinal one isn't below $|\mathbf{p}| \approx 0.7$ GeV around $T \approx T_c$
- Need to fit the parameters separately for the two components
- Possible explanations: needs separate mass parameters $m^2_{T,L}(T)$ for the two components, Polyakov loop unaccounted for, etc.

Screened massive expansion: parameters from the lattice ($\omega_n = 0$)





Silva, Oliveira, Bicudo, PRD 89 (2014)

- The transverse mass parameter grows from 650 to 900 MeV from $T = 0$ to $T = 458$ MeV
- Consistent with a transverse gluon mass still influenced by the behavior of vacuum, enhanced by thermal effects
- We'll come back to this (hopefully) at the end

Screened massive expansion: modeling IR quarks

- Obv., quark chiral symmetry breaking cannot be described by keeping the quark sector decoupled from the gluons'
- In the spirit of the screened massive expansion, what we would do in the quark sector is shift the expansion point of the quarks' perturbative series too:

$$\overline{\psi}(i\not{D} - m_q)\psi = \left[\overline{\psi}(i\not{\partial} - M_q)\psi\right] + \left[\overline{\psi}(g\not{A} + m_q - M_q)\psi\right]$$

- This strategy proved successfull in vacuum[a], but it is overshoot if the aim is to obtain qualitative results *in the gluon sector*

[a] See e.g. G.C., D. Rizzo, M. Battello and F. Siringo, PRD 104 (2021)

Screened massive expansion: modeling IR quarks

- Instead, we employ the simplest possible model for IR quarks: we use an enhanced mass $M_q \approx 350 - 450$ MeV, as suggested by lattice, SDE, etc., and keep it constant at all momenta, temperatures and densities:

$$(\mathcal{L}_{\text{QCD}})_0 = (\mathcal{L}_{\text{YM}})_{0,\text{SME}} + \sum_q \overline{\psi}_q(\partial\!\!\!/ + M_q)\psi_q$$

  without further subtracting the quark mass

- We work with $N_f = 2 + 1$, $M_1 = 350$ MeV, $M_2 = 450$ MeV

Analytic structure at finite T and μ

Beyond Euclidean space

- The screened massive expansion is a modified perturbation theory $\Longrightarrow$ it provides analytic expressions up to a 1D integral at finite (T, μ) (Bose/Fermi distributions)
- Very easy to extend to the complex plane: replace $\omega \in 2\pi\mathbb{Z}$ (Matsubara freq.) with $\omega = p^4 \in \mathbb{C}$ in functions/integrands and if complex singularities are where expected that's the (unique) analytic continuation
- On indefinite-metric Hilbert spaces, propagators can only have two kinds of singularities:
  - a branch cut or poles for $\omega \in i\mathbb{R}$ (Minkowski axis)
  - complex-conjugate singularities for $\omega \notin \mathbb{R}, i\mathbb{R}$ (e.g. c.c. poles)

Beyond Euclidean space

- Branch cut and c.c. singularities fully determine the propagator over the whole $\mathbb{C}$ via spectral representations. E.g., in the presence of simple c.c. poles $\{z_i(|\mathbf{p}|)\}$,

$$\Delta_{T,L}(z,|\mathbf{p}|) = \sum_i \frac{Z_i(|\mathbf{p}|)}{z - z_i(|\mathbf{p}|)} + \int_{-\infty}^{+\infty} \mathrm{d}\omega \, \frac{\rho_{T,L}(\omega,|\mathbf{p}|)}{\omega + iz}$$

where the spectral functions $\rho_{T,L}(\omega,|\mathbf{p}|)$ live on the branch cut,

$$\rho_{T,L}(\omega,|\mathbf{p}|) = -\frac{1}{\pi} \, \mathrm{Im} \left\{ \Delta_{T,L}(i\omega + \epsilon, |\mathbf{p}|) \right\}$$