Effective field theory of composite nucleons in high density matter

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Excited QCD 2026 Workshop

Granada – 08 - 14 January 2026

Generic Motivation

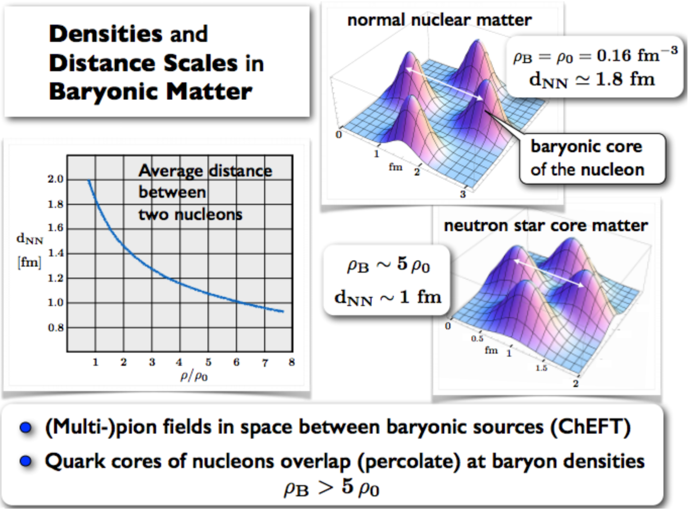


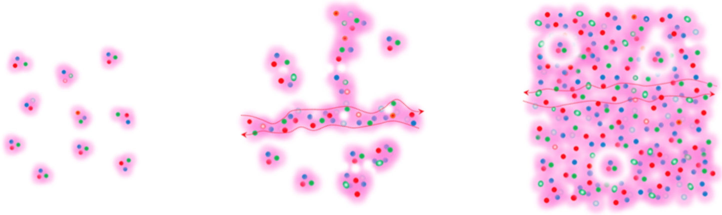
Figure from W. Weise

Long-standing problem

How to deal with a situation in which nucleons strongly overlap

but quarks and gluons do not completely deconfine,

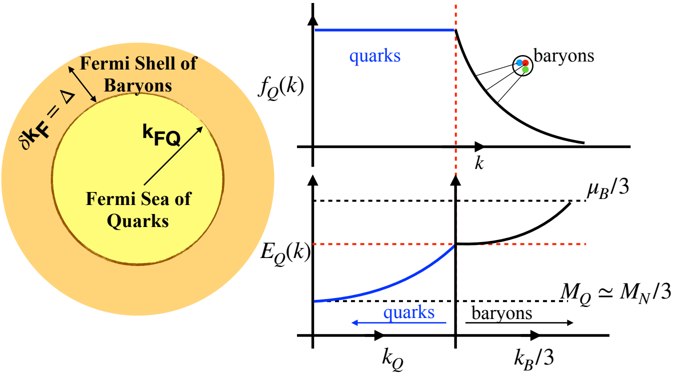
there occur percolation-type fluctuations



Percolation-type fluctuation

Figures from Fukushima et al.

Quarkyonic matter*



L. McLerran, S. Reddy, Phys. Rev. Lett. 122, 122701 (2019)

Nucleon compositeness: quark model

Baryon (nucleon) creation operator:

$$\hat{b}^\dagger_n = \frac{1}{\sqrt{3!}} \Psi_n^{\mu_1\mu_2\mu_3}\, \hat{q}^\dagger_{\mu_1} \hat{q}^\dagger_{\mu_2} \hat{q}^\dagger_{\mu_3}$$

$$\{\hat{b}_n, \hat{b}^\dagger_{n'}\} = \delta_{nn'} - \hat{\Delta}_{nn'}, \qquad \{\hat{b}_n, \hat{b}_{n'}\} = 0$$

$$\hat{\Delta}_{nn'} = 3\Psi_n^{*\mu_1\mu_2\mu_3}\, \Psi_{n'}^{\mu_1\mu_2\nu_3} \hat{q}^\dagger_{\nu_3} \hat{q}_{\mu_3} - \frac{3}{2} \Psi_n^{*\mu_1\mu_2\mu_3}\, \Psi_{n'}^{\mu_1\nu_2\nu_3} \hat{q}^\dagger_{\nu_3} \hat{q}^\dagger_{\nu_2} \hat{q}_{\mu_2} \hat{q}_{\mu_3}$$

Two-nucleon state:

$$|nn'\rangle = \hat{b}^\dagger_n \hat{b}^\dagger_{n'} |0\rangle$$

$$N_{nn'} = \langle nn'|nn'\rangle = 1 - \delta_{nn'} - \Delta N_{nn'}$$

$$\Delta N_{nn'} = 3\Psi_n^{*\mu_1\mu_2\mu_3}\, \Psi_{n'}^{*\nu_1\nu_2\nu_3} \left(\Psi_n^{\mu_1\mu_2\nu_3}\, \Psi_{n'}^{\nu_1\nu_2\mu_3} - \Psi_n^{\mu_1\nu_2\nu_3}\, \Psi_{n'}^{\nu_1\mu_2\mu_3}\right)$$

Explicit calculation

Using Gaussian wave functions:

$$\Delta N_{nn'} = 3 \sum_{p_1 \ldots p_6} \Phi^*_P(123)\Phi^*_{P'}(456)\Big[A_{nn'}\Phi_P(126)\Phi_{P'}(453) - B_{nn'}\Phi_P(453)\Phi_{P'}(126)\Big]$$

$$= \left(\frac{r_0^3}{V}\right) \times \frac{9\sqrt{3}}{\pi^{3/2}} \left[A_{nn'}\, e^{-\left(\mathbf{P}-\mathbf{P}'\right)^2 r_0^2/12} - B_{nn'}\, e^{-\left(\mathbf{P}-\mathbf{P}'\right)^2 r_0^2/3}\right]$$

$A_{nn'}, B_{nn'}$: numbers from color-spin-isospin

Relevance of nucleon compositeness: nucleon size $\approx$ packing volume

Field theory: "deformed" field algebra

Baryon (nucleon) field operator:

$$\psi(\boldsymbol{x},t) = \sum_{\boldsymbol{k},s} \left[u_s(\boldsymbol{k})\, e^{-ik\cdot x}\, b_s^\dagger(\boldsymbol{k}) + v_s(\boldsymbol{k})\, e^{ik\cdot x}\, d_s^\dagger(\boldsymbol{k}) \right]$$

- $b_s(\boldsymbol{k}) b^\dagger_{s'}(\boldsymbol{k}') + \lambda\, b^\dagger_{s'}(\boldsymbol{k}') b_s(\boldsymbol{k}) = \delta_{ss'}\delta_{\boldsymbol{k}\boldsymbol{k}'}, \quad \lambda\, d_s(\boldsymbol{k}) d^\dagger_{s'}(\boldsymbol{k}') + d^\dagger_{s'}(\boldsymbol{k}') d_s(\boldsymbol{k}) = \delta_{ss'}\delta_{\boldsymbol{k}\boldsymbol{k}'}$

- $\lambda \equiv 1 - x \rightarrow \{b_s(\boldsymbol{k}), b^\dagger_{s'}(\boldsymbol{k}')\} = \delta_{ss'}\delta_{\boldsymbol{k}\boldsymbol{k}'} + x\, b^\dagger_s(\boldsymbol{k}) b_s(\boldsymbol{k})$

- $\psi(\boldsymbol{x},t)\psi^\dagger(\boldsymbol{x}',t) + \lambda\, \psi^\dagger(\boldsymbol{x}',t)\psi(\boldsymbol{x},t) = \delta(\boldsymbol{x}-\boldsymbol{x}')$

Nonrelativistic: S.S. Avancini and GK, J. Phys A 28, 685 (1995)
Relativistic free gas: X-Y. Hou et al., J. Stat. Mech. (2020) 113402

Compositeness $\approx$ deformation

Quark model: $\{\hat{b}_n, \hat{b}^\dagger_{n'}\} = \delta_{nn'} - \hat{\Delta}_{nn'}$

Deformed field: $\{\hat{b}_n, \hat{b}^\dagger_{n'}\} = \delta_{nn'} + x\, \hat{b}^\dagger_{n'} \hat{b}_n$

Deformation x: compositeness in an "average way"

$$N_{nn'} = \langle nn' | nn' \rangle = \begin{cases} 1 - \delta_{n'n} - \dfrac{r_0^3}{V} f_{nn'} \\ \\ 1 - \delta_{n'n} - x\, \delta_{nn'} \end{cases} \Rightarrow \boxed{x \sim \frac{r_0^3}{V}}$$

Deformation parameter in QCD matter

- λ must be density dependent, $\lambda = \lambda(\rho)$
- Low densities, $\lambda = 0$
- High densities, λ fluctuates (percolation)
- Thermodynamic properties (timescales: “quench" or "annealed" deformation fluctuations):

$$\Omega = -\frac{1}{\beta}\,\langle \ln Z(\lambda) \rangle_\lambda \quad \text{or} \quad \Omega = -\frac{1}{\beta}\,\ln\langle\, Z(\lambda)\,\rangle_\lambda, \qquad Z(\lambda) = \mathrm{Tr}\, e^{-\beta H(\lambda)}$$

- Compute $Z(\lambda)$ with an effective field theory (of deformed fields)

Effective field theory

Write the most general Lagrangian of the deformed fields:

- Dirac kinetic term: $\overline{\psi}(i\partial\!\!\!/ - M)\psi$
- $C_s\,(\overline{\psi}\psi)^2$, $C_v\,(\overline{\psi}\gamma_\mu\psi)^2$, $C_{ps}\,(\overline{\psi}\gamma_5\psi)^2$, $\cdots$, $C_{dv}\,(\overline{\psi}\partial\!\!\!/\psi)^2$, $\ldots$
- Compute $Z(\lambda)$ for a given $\lambda = \lambda(\rho)$
- Compute the average over "realizations" of $\lambda = \lambda(\rho)$
- Start with something simple:
  - $\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ - M)\psi - C_s\,(\overline{\psi}\psi)^2 - C_v\,(\overline{\psi}\gamma_\mu\psi)^2$
  - Mean field approximation
  - Fixed $\lambda(\rho)$, no fluctuations

Quon-like quasiparticles in other contexts

— Haldane's Fractional Exclusion Statistics (FES)

- Alters Hilbert space with a statistical parameter g
- Models 1D/2D systems with constrained or emergent state spaces
- Used in spin chains, FQHE, Luttinger liquids

— Anyons in 2D Systems

- Interpolate statistics via braid group, $q = e^{i\theta}$
- Realized in fractional quantum Hall states
- Share thermodynamic nonadditivity or nonextensivity with quon gases

— Cold Atom Quantum Simulators

- Simulate anyons and fractional statistics via engineered interactions
- Enable exploration of synthetic quantum matter
- Braiding = fault-tolerant quantum gates

Thermodynamics QCD matter

$$\frac{\Omega}{V} = \frac{1}{2}C_s\rho_s^2 - \frac{1}{2}C_v\rho^2 - g\int \frac{d^3k}{(2\pi)^3}\left[\ln\left(1 + e^{-\beta(E(\boldsymbol{k})-\mu_\lambda^*)}\right) + \ln\left(1 + e^{-\beta(E(\boldsymbol{k})+\mu_\lambda^*)}\right)\right]$$

$+ \rho\,\Sigma(\rho)$ $\quad\leftarrow$ rearrangement terms, therm. consistency

- $E(\boldsymbol{k}) = \left(\boldsymbol{k}^2 + {M^*}^2\right)^{1/2}, \qquad \mu_\lambda^* = \mu^* + T\ln\lambda$
- $\rho_s = g\int d^3x\,\langle\bar{\psi}(x)\psi(x)\rangle = g\int \frac{d^3k}{(2\pi)^3}\frac{M^*}{E(\boldsymbol{k})}\left(\frac{1}{e^{\beta(E(\boldsymbol{k})-\mu_\lambda^*)}} + \frac{1}{e^{\beta(E(\boldsymbol{k})+\mu_\lambda} + 1}\right)\frac{1}{\lambda}$
- $\rho = g\int d^3x\,\langle\bar{\psi}(x)\psi(x)\rangle = g\int \frac{d^3k}{(2\pi)^3}\left(\frac{1}{e^{\beta(E(\boldsymbol{k})-\mu_\lambda^*)}} - \frac{1}{e^{\beta(E(\boldsymbol{k})+\mu_\lambda} + 1}\right)\frac{1}{\lambda}$
- $M^* = M - C_s\,\rho_s, \qquad \mu^* = \mu - C_v\,\rho + \Sigma(\rho)$

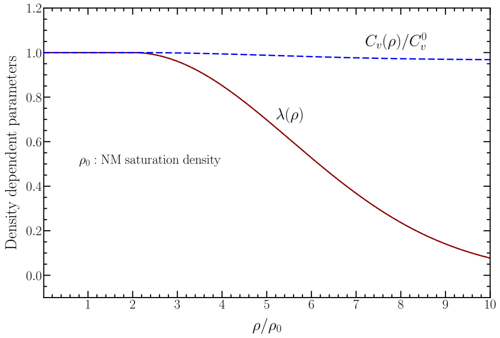
Work with Randall H. Vargas

Results for $T = 0$

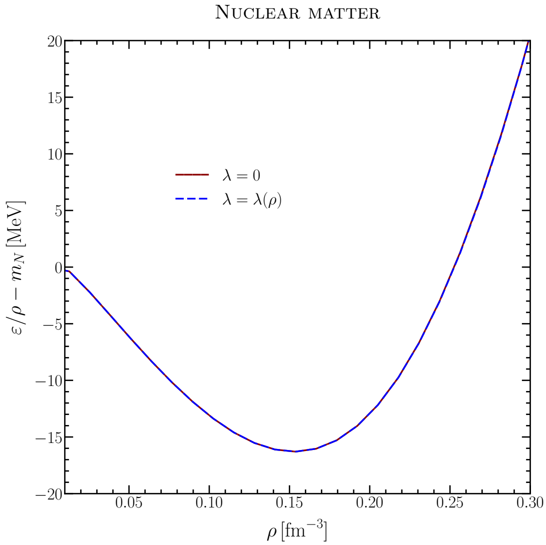
Fix parameters to saturate symmetric nuclear matter:

—This model (at the MFA level) = Walecka model

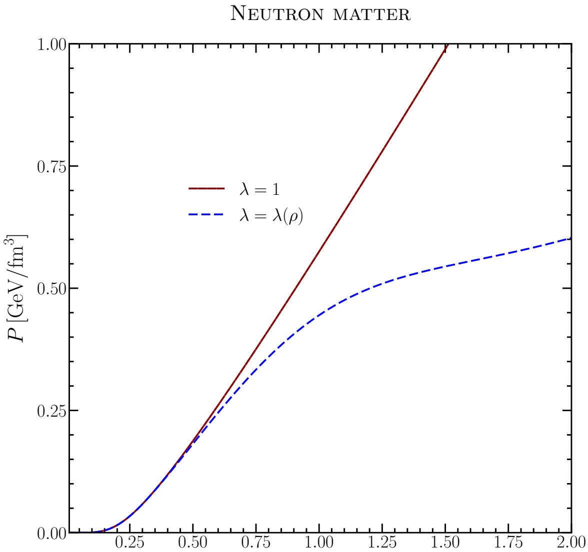
— Model for $\lambda = \lambda(\rho)$ & $C_v(\rho)$



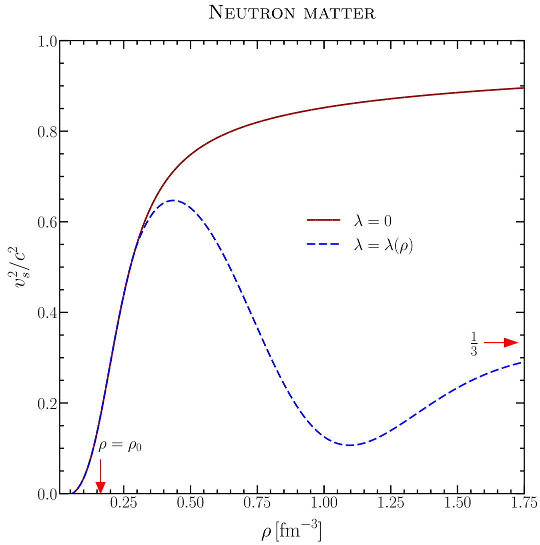
Nuclear matter binding energy



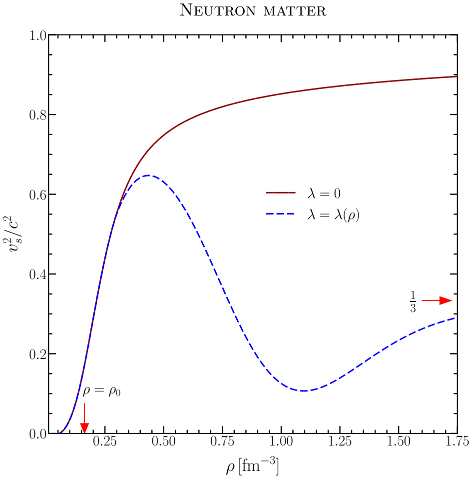
Equation of State - EOS

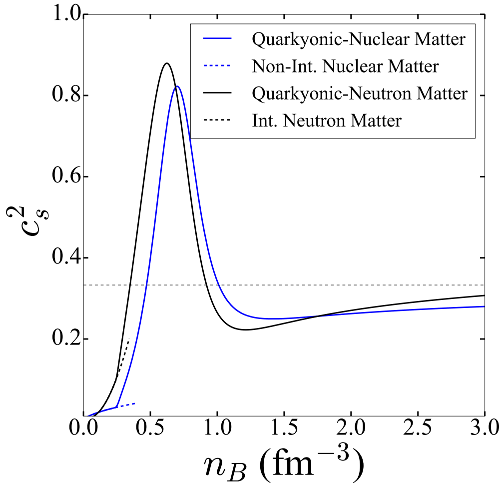


Speed of sound

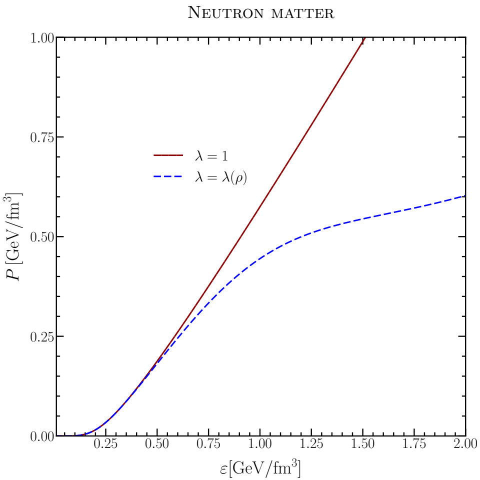


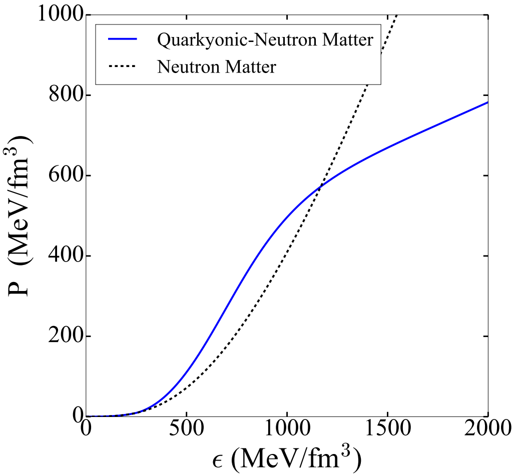
Speed of sound - quonic x quarkyonic



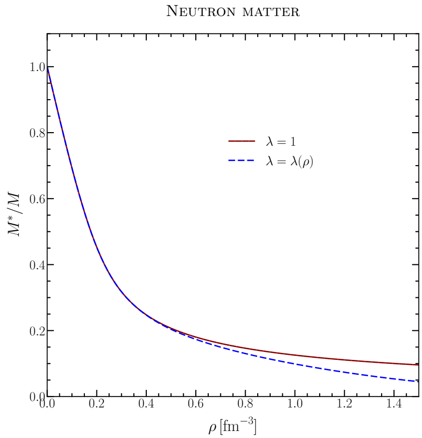


EOS - quonic x quarkyonic





Effective mass



At high density, reaching conformal limit

Conclusions & Perspectives

- Presented a framework to tackle "percolating" high density matter
- Deformed fields to model internal degrees of freedom
- Effective field theory perspective
- Made a first application: fixed, nonfluctuating deformation,
  used MFA with a lowest-order Lagrangian
- Plenty of optimism that it might bring insight into a very difficult problem

Thank you

Funding

FAPESP
SÃO PAULO RESEARCH FOUNDATION

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico