Heavy and heavy-light tensor and axial-tensor mesons in the Covariant Spectator Theory

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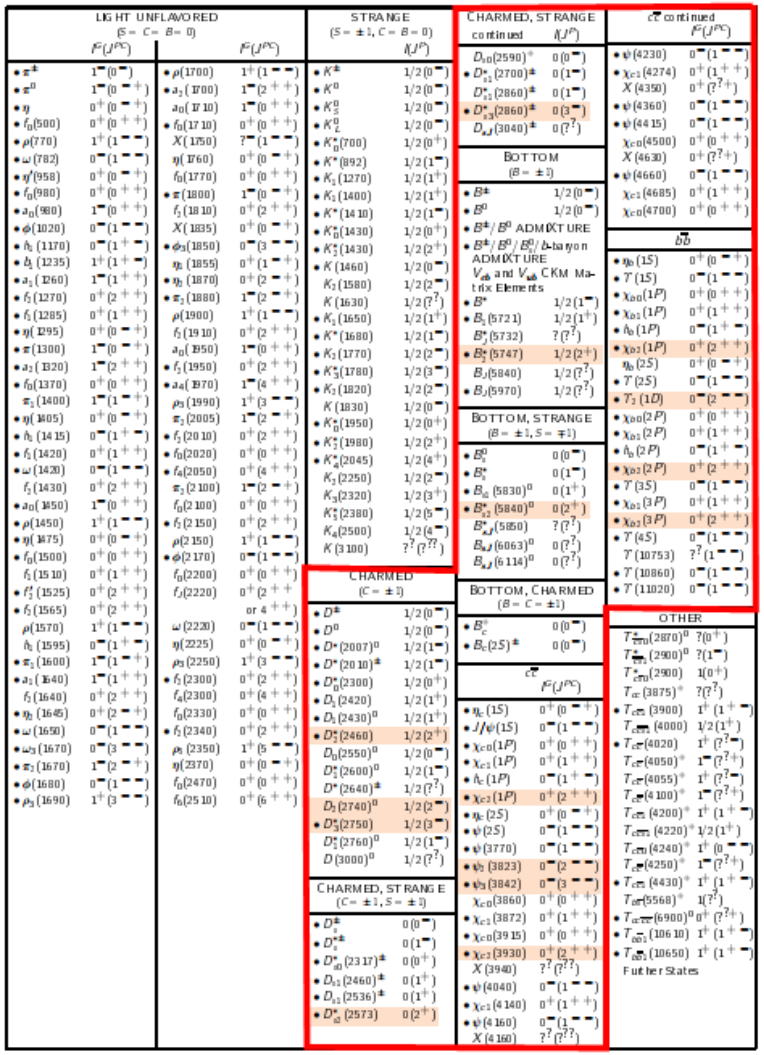
LIP

HEAVY AND HEAVY-LIGHT MESON SPECTRA

PDG: Established meson states with at least one heavy quark



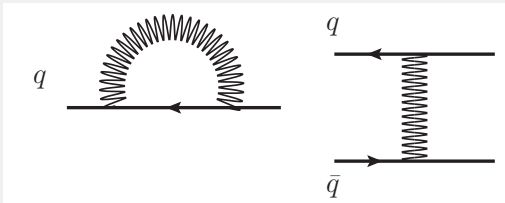
Theoretical approaches

- Lattice QCD
- Bethe-Salpeter/Dyson-Schwinger Equations
- Quasi-potential approaches, **CST**
- RSE, unitarized quark model
- Relativistic Hamiltonian Dynamics (instant, front, point form)
- Basis Light-Front Quantization
- Chiral quark model
- Effective field theories
- Two-body Dirac equation
- Relativized Schrödinger equation

Focus in this talk: Tensor states with $J \geq 2$

COVARIANT SPECTATOR THEORY (CST)

- Manifest Lorentz covariance
- Minkowski space
- Dirac and Schrödinger limits
- Confinement included in interaction kernel
- Dynamical-chiral symmetry breaking through AVWTI

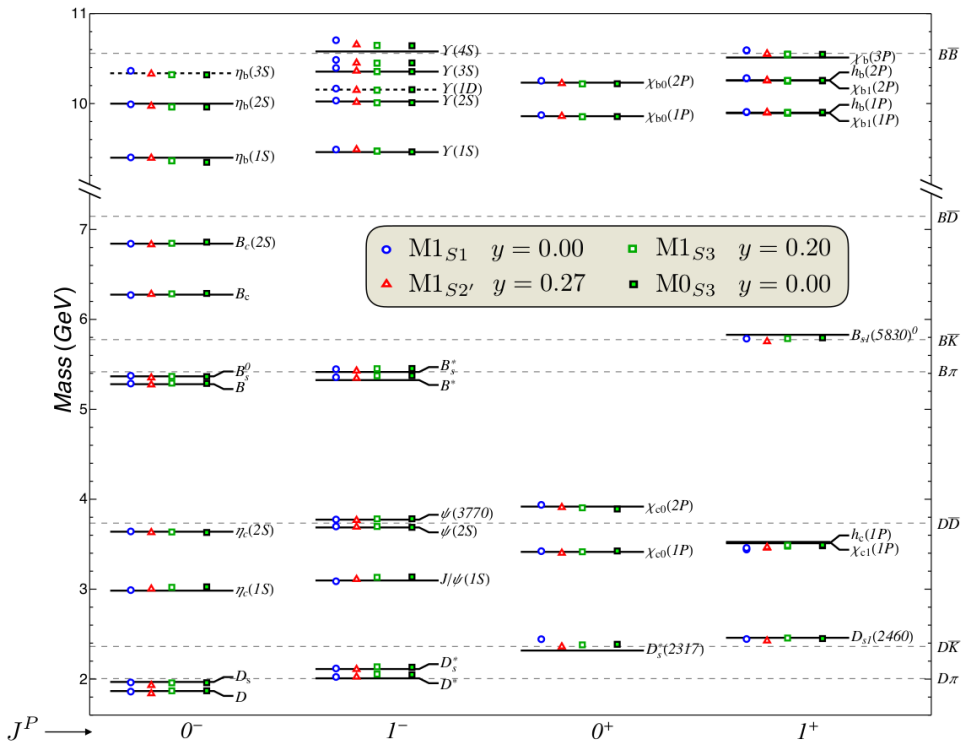


So far with CST:
Global description of 39 heavy and heavy-light states with $J^P = 0^\pm, 1^\pm$

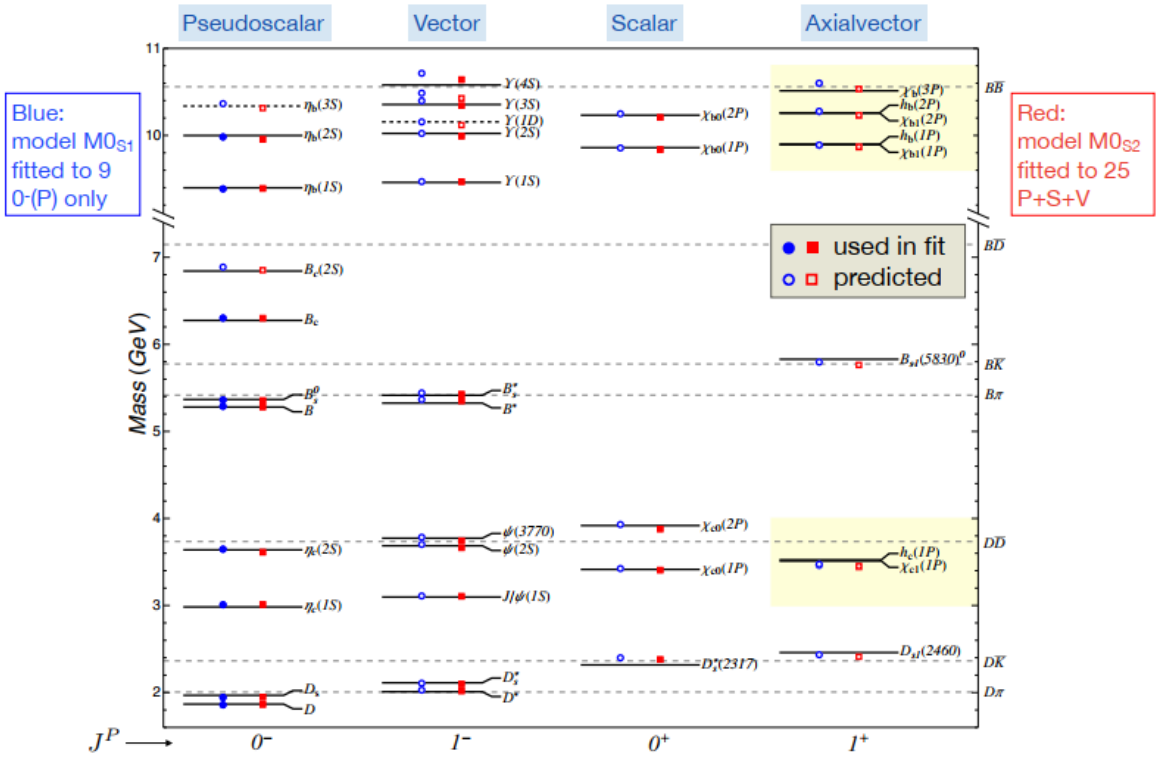
LEITÃO, STADLER, PEÑA, EB PLB 2017, PRD 2017

Next

- Extend this model to **arbitrary** J^P and calculate $2^\pm, 3^\pm$ states
- Improve by including the **running** of the strong coupling



PREDICTIVE POWER OF COVARIANT INTERACTION KERNELS



Leitão, Stadler, Peña, EB PLB (2017)

- Test: Does this still apply to tensor mesons?
- Aim: Contribute to prediction of quantum numbers and identification of possibly non-$q\bar{q}$ states

Kernel parameters well determined through fit to **pseudoscalar** (0^-) states only (almost 100% $L = 0$, $S = 0$)

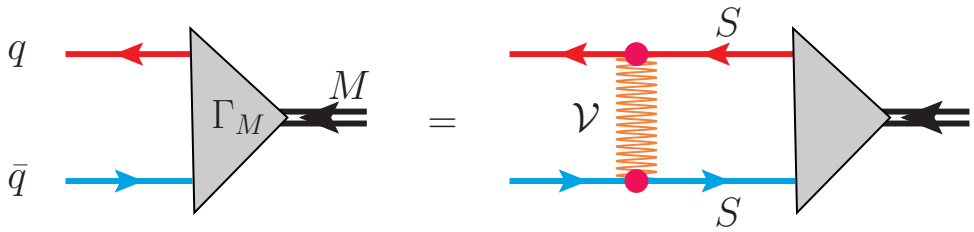
- $\langle 0^- | \boldsymbol{L} \cdot \boldsymbol{S} | 0^- \rangle = 0$
- $\langle 0^- | S_{12} | 0^- \rangle = 0$
- $\langle 0^- | \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 | 0^- \rangle = -\frac{3}{4}$

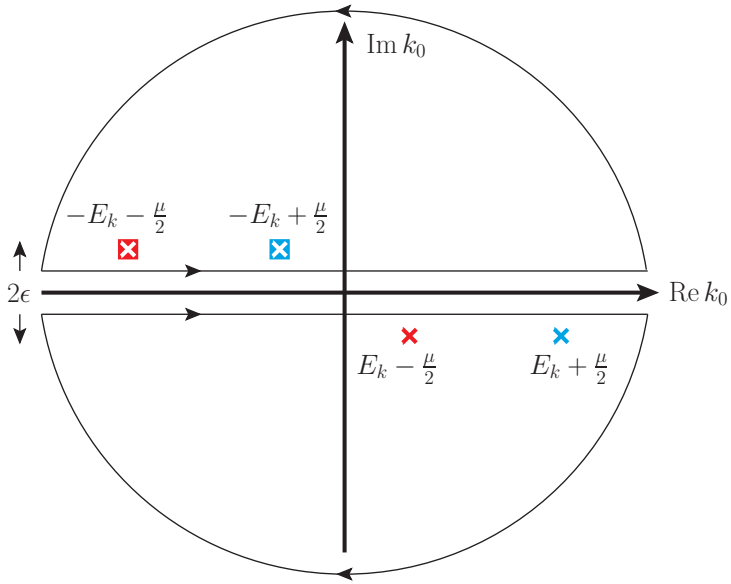
$\Longrightarrow$ do not constrain spin-orbit and tensor forces; cannot separate spin-spin from central force

But: **Covariance** of interaction kernel $(\mathbf{1} \otimes \mathbf{1}, \gamma^\mu \otimes \gamma_\mu)$ **determines** all spin-dependent forces correctly!

GROSS (CST) EQUATION

Bethe-Salpeter equation (BSE)

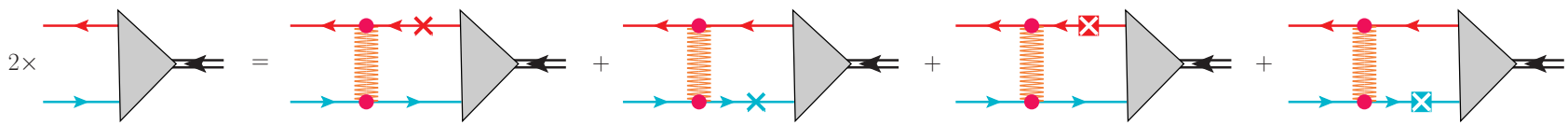




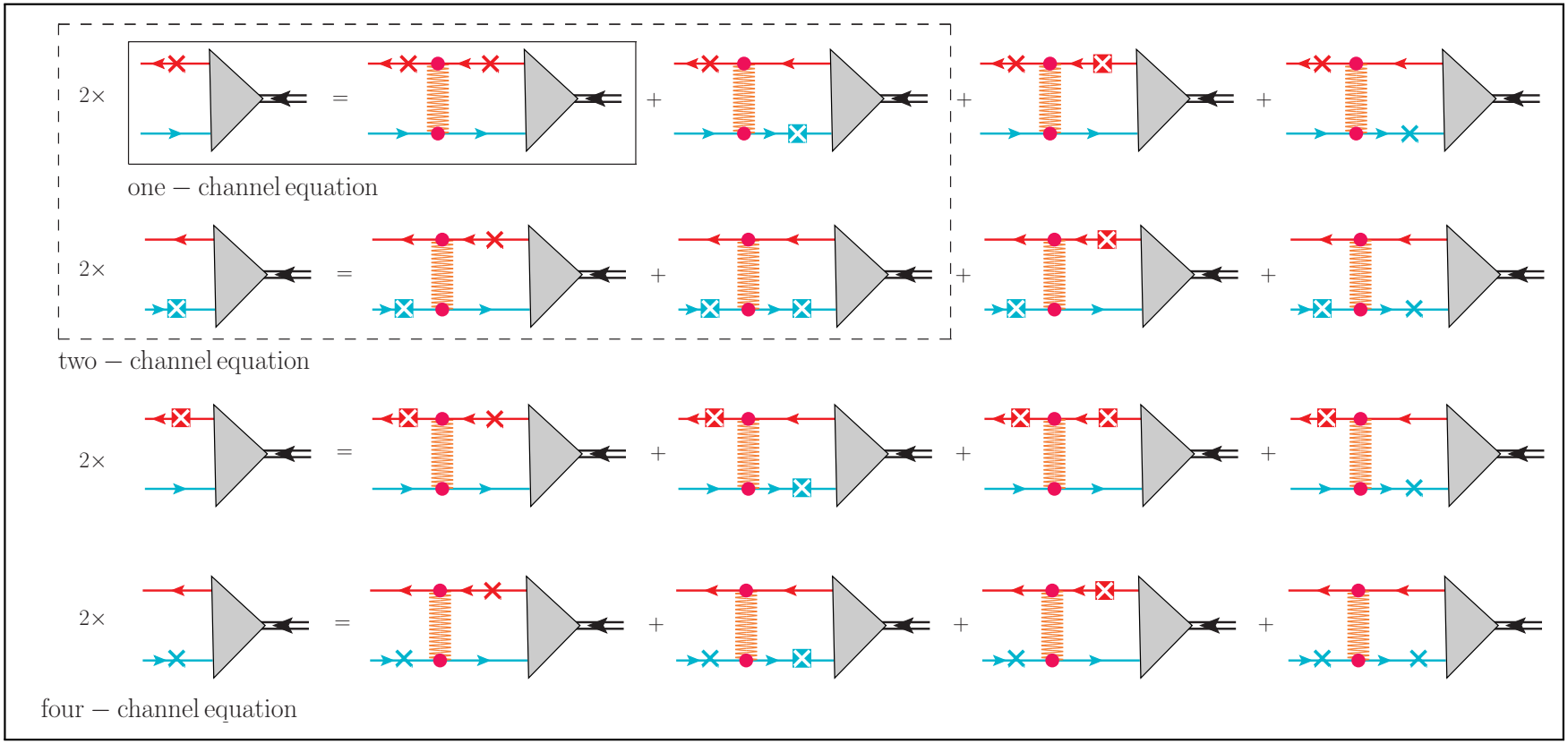
- CST: Effective treatment of full BSE
- Assume **real** quark-mass poles
- Keep only **quark pole** contributions in k_0 integration
- **3D covariant** loop integrations in Minkowski space

Charge-conjugation symmetric Gross equation

Savkli, Gross PRC (2001), EB, Gross, Peña, Stadler PRD (2014)



4-CHANNEL EQUATION

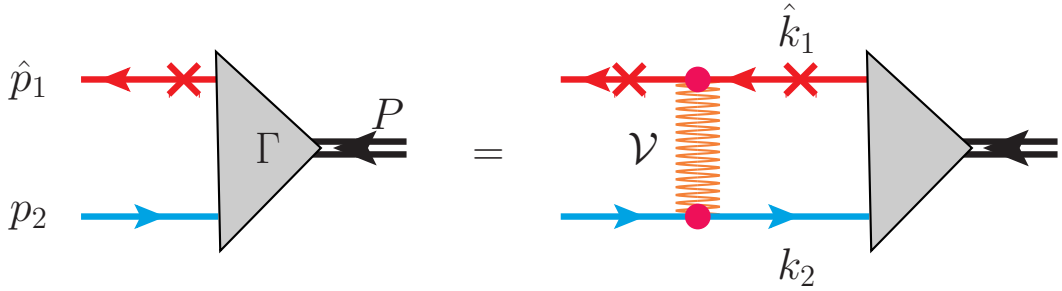


CST equation	unequal quark masses	$\mathcal{C}$-symmetry	light mesons
One-channel	✓	×	×
Two-channel	✓	✓	×
Four-channel	✓	✓	✓

Beyond BSE ladder approximation: All have **smooth one-body** (Dirac equation) and **nonrelativistic** (Schrödinger equation) limits

HEAVY AND HEAVY-LIGHT MESONS

One-channel spectator equation (1CSE)



$$\Gamma(\hat{p}_1, p_2) = -\int \frac{d^3k}{(2\pi)^3} \frac{m_1}{E_{1k}} \mathcal{V}(\hat{p}_1, \hat{k}_1) \frac{m_1 + \hat{\not{k}}_1}{2m_1} \Gamma(\hat{k}_1, k_2) \frac{m_2 + \not{k}_2}{m_2^2 - k_2^2 - i\epsilon}$$

with $\hat{k} = (E_{1k}, \boldsymbol{k})$, $m_1 \geq m_2$

- Particularly suited for **heavy-light** (and heavy) mesons
- Numerically **easier** than two- or four-channel equations
- $\mathcal{C}$-parity splittings **small** in heavy quarkonia
- Use **fixed** quark masses
- Gauge-parameter ξ independent because $\bar{u}(\boldsymbol{p}) \not{q} u(\boldsymbol{k}) = 0$

COVARIANT CONFINING KERNEL

$$\mathcal{V}(\hat{p},\hat{k}) = [(1-y)\mathbf{1}\otimes\mathbf{1} - y\gamma^\mu\otimes\gamma_\mu]V_{\rm L}(\hat{p},\hat{k}) - \gamma^\mu\otimes\gamma_\mu[V_{\rm G}(\hat{p},\hat{k}) + V_{\rm C}(\hat{p},\hat{k})]$$

mixing parameter y in $V_{\rm L}$: $y=0$ (pure S)... $y=1$ (pure V)

Nonrelativistic linear-confining potential $V_{\rm L}^{\rm nr}(r) = \sigma r$ in momentum space:

$$\langle V_{\rm L}^{\rm nr}\phi\rangle(\boldsymbol{p}) = \int \frac{d^3k}{(2\pi)^3} V_{\rm L}^{\rm nr}(\boldsymbol{p}-\boldsymbol{k})\phi(\boldsymbol{k}) = -8\pi\sigma\,{\rm P}\int \frac{d^3k}{(2\pi)^3}\frac{\phi(\boldsymbol{k})-\phi(\boldsymbol{p})}{(\boldsymbol{k}-\boldsymbol{p})^4}$$

Cauchy principal value singularity

Leitão, Peña, Stadler, EB PRD (2014)

Covariant generalization: $\boldsymbol{q}^2 \to -q^2$

$$\langle V_{\rm L}\phi\rangle(\hat{p}) = \int \frac{d^3k}{(2\pi)^3}\frac{m}{E_k} V_{\rm L}(\hat{p},\hat{k})\phi(\hat{k}) = -8\pi\sigma\,{\rm P}\int \frac{d^3k}{(2\pi)^3}\frac{m}{E_k}\frac{\phi(\hat{k})-\phi(\hat{p})}{(\hat{k}-\hat{p})^4}$$

Still confining? Yes! Meson vertex function **vanishes if both** quarks on-shell!

Savkli, Gross PRC (2001)

ONE GLUON EXCHANGE

One-gluon-exchange

$$V_{\mathrm{G}}(\hat{p},\hat{k}) = -\frac{4}{3}\frac{4\pi\alpha_s\left(Q^2\right)}{q^2}, \quad q^2 = (\hat{p}-\hat{k})^2 = -Q^2$$

running strong coupling

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \ln\left(\frac{Q^2}{\Lambda_{\mathrm{QCD}}} + \tau\right)}, \quad \beta_0 = \frac{33-2N_f}{12\pi}, \quad N_f = 2$$

parameter τ determined by value $\alpha_s(Q^2=0)$
Λ_{QCD} determined through $\alpha_s(M_Z^2) = 0.1183$

Covariant constant $V_{\mathrm{C}}(\hat{p},\hat{k}) = \frac{E_{1k}}{m_1}(2\pi)^3 C\delta^3(\boldsymbol{q})$

In nonrelativistic limit

$$\mathcal{V}(\hat{p},\hat{k}) = [(1-y)\mathbf{1}\otimes\mathbf{1} - y\gamma^\mu\otimes\gamma_\mu]V_{\mathrm{L}}(\hat{p},\hat{k}) - \gamma^\mu\otimes\gamma_\mu[V_{\mathrm{G}}(\hat{p},\hat{k}) + V_{\mathrm{C}}(\hat{p},\hat{k})]$$

reduces to Cornell-type potential $V^{\mathrm{nr}}(r) = \sigma r - \frac{4}{3}\frac{\alpha_{\mathrm{s}}(Q^2)}{r} - C$ (for any value y)

MESON VERTEX FUNCTION

Natural-parity: $J^P = 0^+, 1^-, 2^+, 3^-, \ldots$

$$\Gamma(\hat{p}_1, p_2) = \zeta^{\mu\nu\ldots\omega}[G_1 M_{\mu\nu\ldots\omega} + G_2 N_{\mu\nu\ldots\omega} + (G_3 M_{\mu\nu\ldots\omega} + G_4 N_{\mu\nu\ldots\omega})(m_2 - \not{p}_2)]$$

Unnatural-parity: $J^P = 0^-, 1^+, 2^-, 3^+, \ldots$

$$\Gamma^5(\hat{p}_1, p_2) = \zeta^{\mu\nu\ldots\omega}[G_1 M_{\mu\nu\ldots\omega} + G_2 N_{\mu\nu\ldots\omega} + (G_3 M_{\mu\nu\ldots\omega} + G_4 N_{\mu\nu\ldots\omega})(m_2 + \not{p}_2)]\gamma^5$$

$\zeta^{\mu\nu\ldots\omega} = \{\xi^{\mu}_{\lambda_1} \otimes \xi^{\nu}_{\lambda_2} \otimes \cdots \otimes \xi^{\omega}_{\lambda_J}\}^{(J)}_{m_J}$ rank-J spherical polarization tensor
$M_{\mu\nu\ldots\omega}(p), N_{\mu\nu\ldots\omega}(p)$ rank-J tensors, $G_i = G_i(\hat{p}_1 \cdot p_2, p_2^2)$ invariant functions

Partial-wave basis
Eigenstates of L and S useful for spectroscopic identification
Dirac-spinor decomposition of projector and propagator in 1CSE

$$m + \hat{\not{p}} \sim \sum_{\text{spins}} u(\boldsymbol{p})\bar{u}(\boldsymbol{p}), \qquad \frac{m + \not{p}}{m^2 - p^2 - i\epsilon} \sim \sum_{\text{spins}} \left[-\frac{v(-\boldsymbol{p})\bar{v}(-\boldsymbol{p})}{E_p + p^0 - i\epsilon} + \frac{u(\boldsymbol{p})\bar{u}(\boldsymbol{p})}{E_p - p^0 - i\epsilon} \right]$$

$\bar{u}\,\Gamma^{(5)}\,v$ and $\bar{u}\,\Gamma^{(5)}\,u$ can be written $\chi^\dagger \left\{ T^{(L)}_{M_L} \otimes T^{(S)}_{M_S} \right\}^{(J)}_{M_J} \mathrm{i}\sigma_2 \chi$

NATURAL-PARITY STATES

Nonrelativistic component (dominant) with $L = J \pm 1$ and $S = 1$:

$$\bar{u}_{\lambda_1}(\boldsymbol{p})\Gamma(\hat{p}_1, p_2)v_{\lambda_2}(-\boldsymbol{p}) \sim \sum_{L=J\pm1} \psi_{L,1}(p) Y_{Lm_L}(\hat{\boldsymbol{p}}) C^{1m_S}_{\frac{1}{2}\lambda_1\frac{1}{2}\lambda_2} C^{Jm_J}_{Lm_L 1m_S}$$

Relativistic component (small) with $L = J$ and $S = 0, 1$:

$$\bar{u}_{\lambda_1}(\boldsymbol{p})\Gamma(\hat{p}_1, p_2)u_{\lambda_2}(\boldsymbol{p}) \sim \sum_{S=0,1} \psi_{J,S}(p) Y_{Jm_L}(\hat{\boldsymbol{p}}) C^{Sm_S}_{\frac{1}{2}\lambda_1\frac{1}{2}\lambda_2} C^{Jm_J}_{Jm_L Sm_S}$$

Normalizations $J^P = 0^+$: $\int_0^\infty \mathrm{d}p\, p^2[\psi_P^2(p) + \psi_S^2(p)] = 1$

$J^P = 1^-$: $\int_0^\infty \mathrm{d}p\, p^2[\psi_S^2(p) + \psi_D^2(p) + \psi_{P_s}^2(p) + \psi_{P_t}^2(p)] = 1$

$J^P = 2^+$: $\int_0^\infty \mathrm{d}p\, p^2[\psi_P^2(p) + \psi_F^2(p) + \psi_{D_s}^2(p) + \psi_{D_t}^2(p)] = 1$

$J^P = 3^-$: $\int_0^\infty \mathrm{d}p\, p^2[\psi_D^2(p) + \psi_G^2(p) + \psi_{F_s}^2(p) + \psi_{F_t}^2(p)] = 1$

Relativistic components very **small** for $b\bar{b}$, up to **9%** for $c\bar{q}$

UNNATURAL-PARITY STATES

Nonrelativistic component (dominant) with $L = J$, $S = 0, 1$:

$$\bar{u}_{\lambda_1}(\boldsymbol{p})\Gamma^5(\hat{p}_1, p_2)v_{\lambda_2}(-\boldsymbol{p}) \sim \sum_{S=0,1} \psi_{J,S}(p) Y_{Jm_L}(\hat{\boldsymbol{p}}) C^{Sm_S}_{\frac{1}{2}\lambda_1\frac{1}{2}\lambda_2} C^{Jm_J}_{Jm_L Sm_S}$$

Relativistic component (small) with $L = J \pm 1$, $S = 1$:

$$\bar{u}_{\lambda_1}(\boldsymbol{p})\Gamma^5(\hat{p}_1, p_2)u_{\lambda_2}(\boldsymbol{p}) \sim \sum_{L=J\pm1} \psi_{L,1}(p) Y_{Lm_L}(\hat{\boldsymbol{p}}) C^{1m_S}_{\frac{1}{2}\lambda_1\frac{1}{2}\lambda_2} C^{Jm_J}_{Lm_L 1m_S}$$

Normalizations $J^P = 0^-$: $\int_0^\infty \mathrm{d}p\, p^2[\psi_S^2(p) + \psi_P^2(p)] = 1$

$J^P = 1^+$: $\int_0^\infty \mathrm{d}p\, p^2[\psi_{P_s}^2(p) + \psi_{P_t}^2(p) + \psi_S^2(p) + \psi_D^2(p)] = 1$

$J^P = 2^-$: $\int_0^\infty \mathrm{d}p\, p^2[\psi_{D_s}^2(p) + \psi_{D_t}^2(p) + \psi_P^2(p) + \psi_F^2(p)] = 1$

$J^P = 3^+$: $\int_0^\infty \mathrm{d}p\, p^2[\psi_{F_s}^2(p) + \psi_{F_t}^2(p) + \psi_D^2(p) + \psi_G^2(p)] = 1$

Remark: Relativistic components get multiplied with $\bar{u}$; nonrel. comp. with $\bar{v}$
opposite intrinsic parities $\Longrightarrow$ **same overall** parity!

PARAMETERS AND MODELS

For numerical solution of the 1CSE V_L and V_G need to be regularized:
Chose **Pauli-Villars subtraction** with parameters $\Lambda_L = \lambda m_1$ and $\Lambda_G = \lambda_g m_1$
Remark: OK for calculation of masses here;
in future, for decay constants, need higher power form factors

We performed **global fits** to heavy and heavy-light meson spectrum
with just **8 adjustable model parameters**
Remarks:

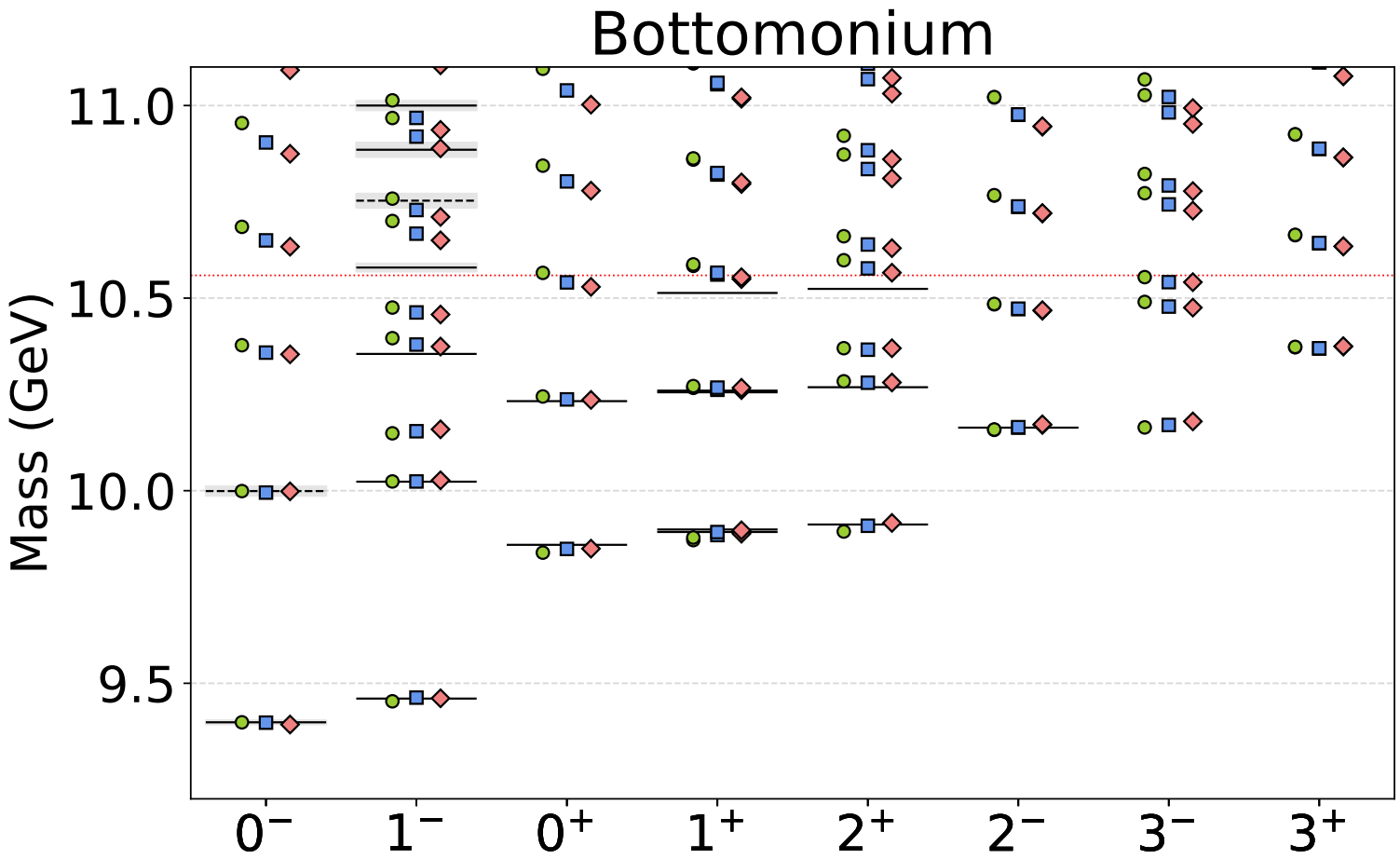
- $y = 0 \ldots 0.3$ consistent with data; set $y = 0$ (pure scalar confinement)
- Running $\alpha_s(Q^2)$ **improves** fit compared to constant α_s;
  removes constant interaction (fits give essentially $C \approx 0$)

Model	Fitted states	σ [GeV^2]	$\alpha_s(0)$	m_b [GeV]	m_c [GeV]	m_s [GeV]	m_q [GeV]	λ	λ_g
I	9: 0^-	0.2179	0.3972	4.7811	1.4289	0.2595	0.1185	1.2608	1.8780
II	25: $0^\pm, 1^-$	0.1962	0.4219	4.8032	1.4528	0.2774	0.1327	2.2588	2.3686
III	54: $0^\pm, 1^\pm, 2^\pm, 3^-$	0.1745	0.5285	4.8619	1.5190	0.3552	0.2012	2.9960	2.2285

Models give **similar** parameter values
$\Longrightarrow$ fit to pseudoscalar states **essentially predicts** the other states ✓

PRELIMINARY RESULTS: $b\bar{b}$

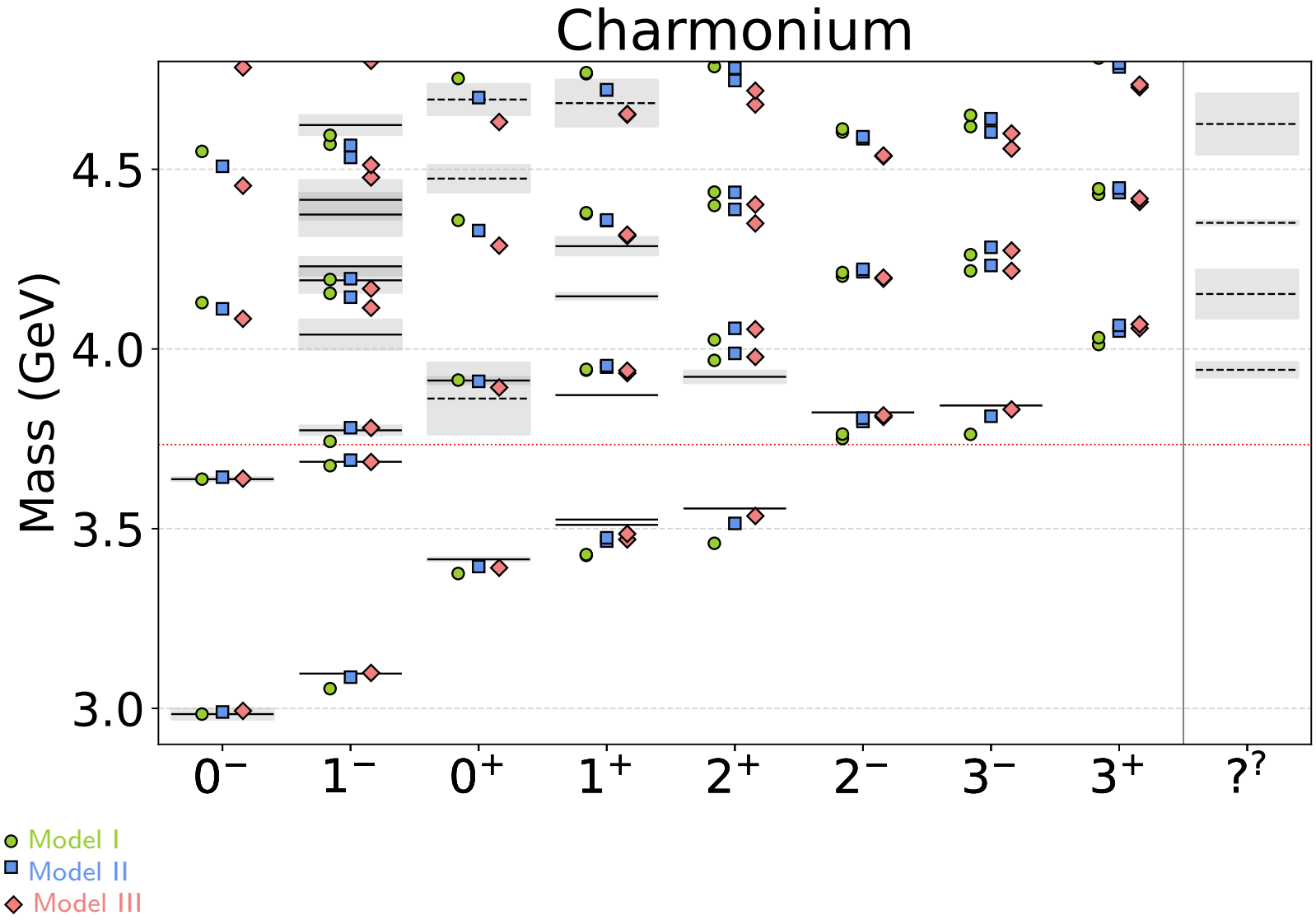
$b\bar{b}$	
• $\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$0^-(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon_2(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
• $h_b(2P)$	$0^-(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_{b1}(3P)$	$0^+(1^{++})$
• $\chi_{b2}(3P)$	$0^+(2^{++})$
• $\Upsilon(4S)$	$0^-(1^{--})$
$\Upsilon(10753)$	$?^?(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$



- Model I: 9 states fitted (0^-)
- Model II: 25 states fitted ($0^\pm, 1^-$)
- Model III: 54 states fitted ($0^\pm, 1^\pm, 2^\pm, 3^-$)

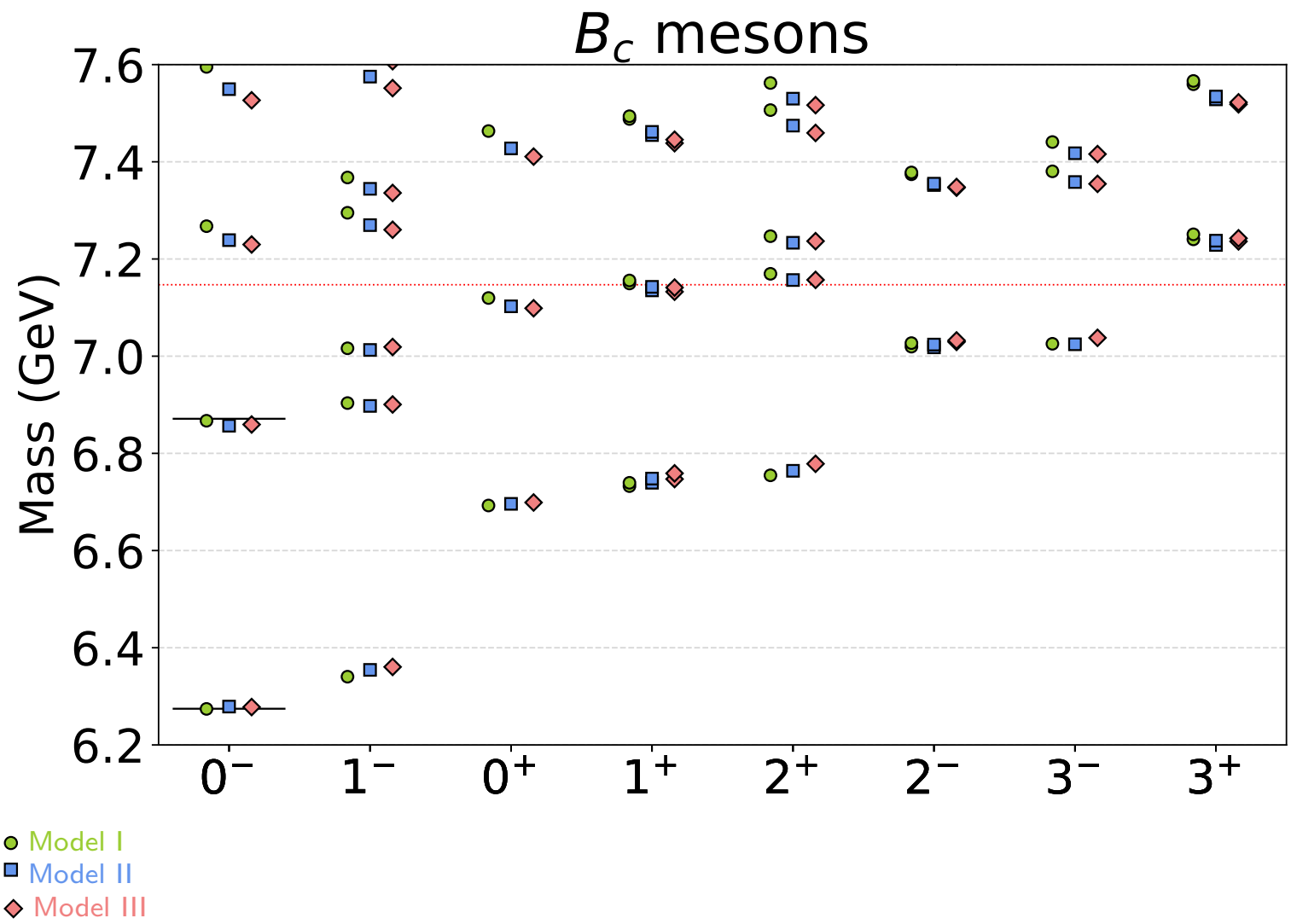
PRELIMINARY RESULTS: $c\bar{c}$

$c\bar{c}$	$I^G(J^{PC})$
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$0^-(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
• $\psi_2(3823)$	$0^-(2^{--})$
• $\psi_3(3842)$	$0^-(3^{--})$
$\chi_{c0}(3860)$	$0^+(0^{++})$
• $\chi_{c1}(3872)$	$0^+(1^{++})$
• $\chi_{c0}(3915)$	$0^+(0^{++})$
• $\chi_{c2}(3930)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
• $\psi(4040)$	$0^-(1^{--})$
• $\chi_{c1}(4140)$	$0^+(1^{++})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
• $\psi(4230)$	$0^-(1^{--})$
• $\chi_{c1}(4274)$	$0^+(1^{++})$
$X(4350)$	$0^+(?^{?+})$
• $\psi(4360)$	$0^-(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
$\chi_{c0}(4500)$	$0^+(0^{++})$
$X(4630)$	$0^+(?^{?+})$
• $\psi(4660)$	$0^-(1^{--})$
$\chi_{c1}(4685)$	$0^+(1^{++})$
$\chi_{c0}(4700)$	$0^+(0^{++})$



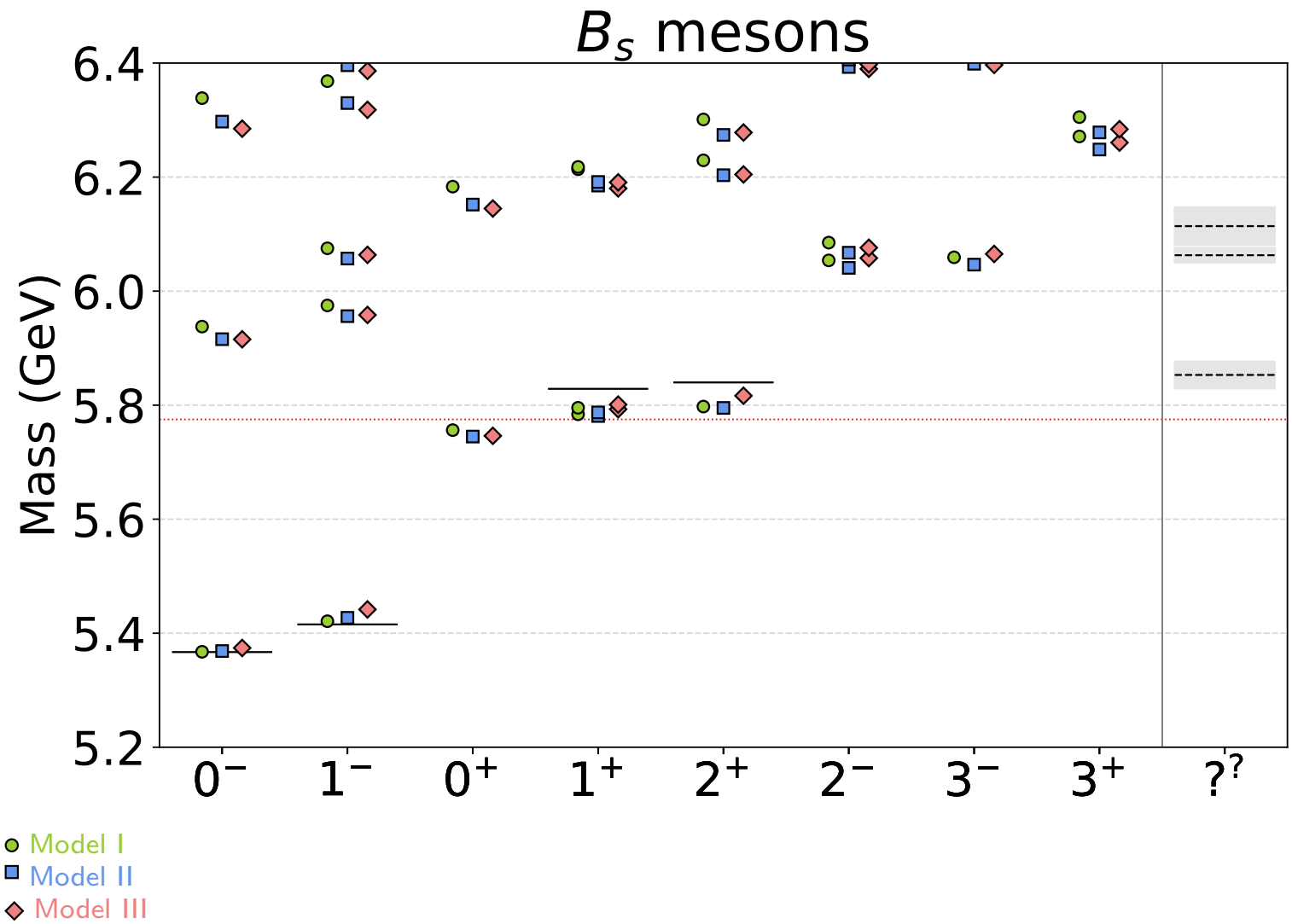
PRELIMINARY RESULTS: $b\bar{c}$

BOTTOM, CHARMED ($B = C = \pm 1$)	
• B_c^+	$0(0^-)$
• $B_c(2S)^\pm$	$0(0^-)$



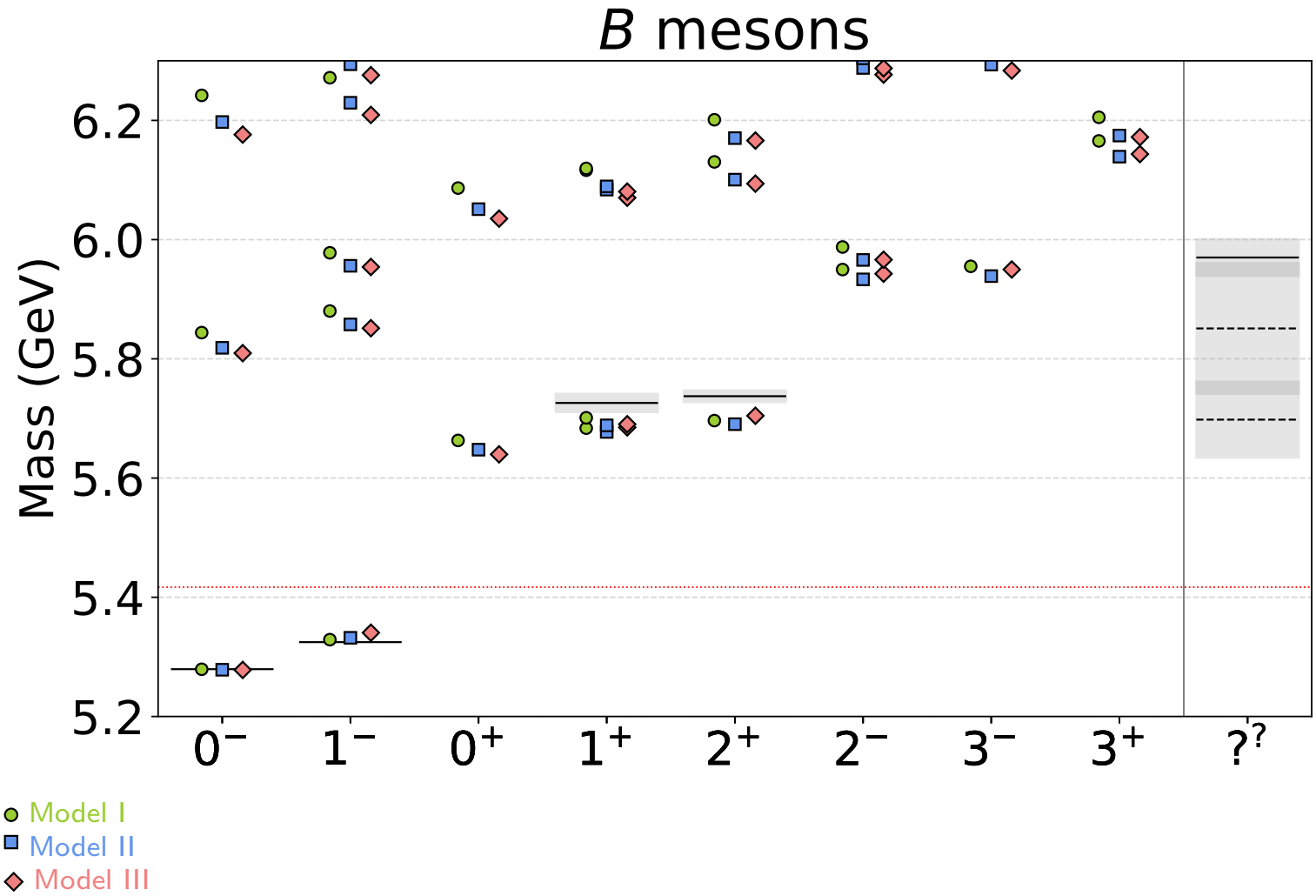
PRELIMINARY RESULTS: $b\bar{s}$

BOTTOM, STRANGE ($B = \pm 1, S = \mp 1$)	
• B_s^0	$0(0^-)$
• B_s^*	$0(1^-)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B_{s2}^*(5840)^0$	$0(2^+)$
$B_{sJ}^*(5850)$	$?(?^?)$
$B_{sJ}(6063)^0$	$0(?^?)$
$B_{sJ}(6114)^0$	$0(?^?)$



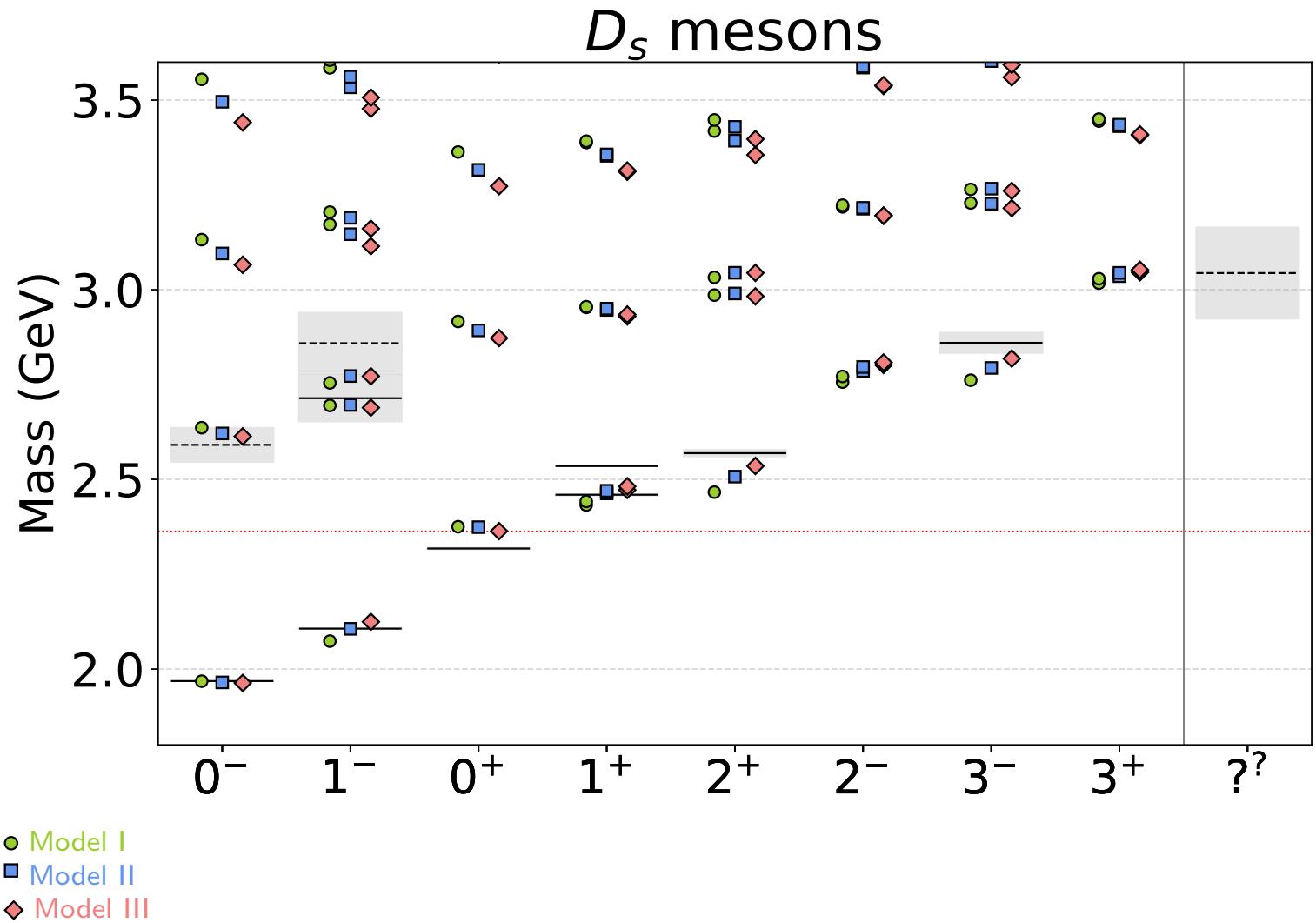
PRELIMINARY RESULTS: $b\bar{q}$

BOTTOM ($B = \pm 1$)	
• $B^{\pm}$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^{\pm}/B^0$ ADMIXTURE	
• $B^{\pm}/B^0/B_s^0$/b-baryon ADMIXTURE V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)$	$1/2(1^+)$
$B_J^*(5732)$	$?(?^?)$
• $B_2^*(5747)$	$1/2(2^+)$
$B_J(5840)$	$1/2(?^?)$
• $B_J(5970)$	$1/2(?^?)$



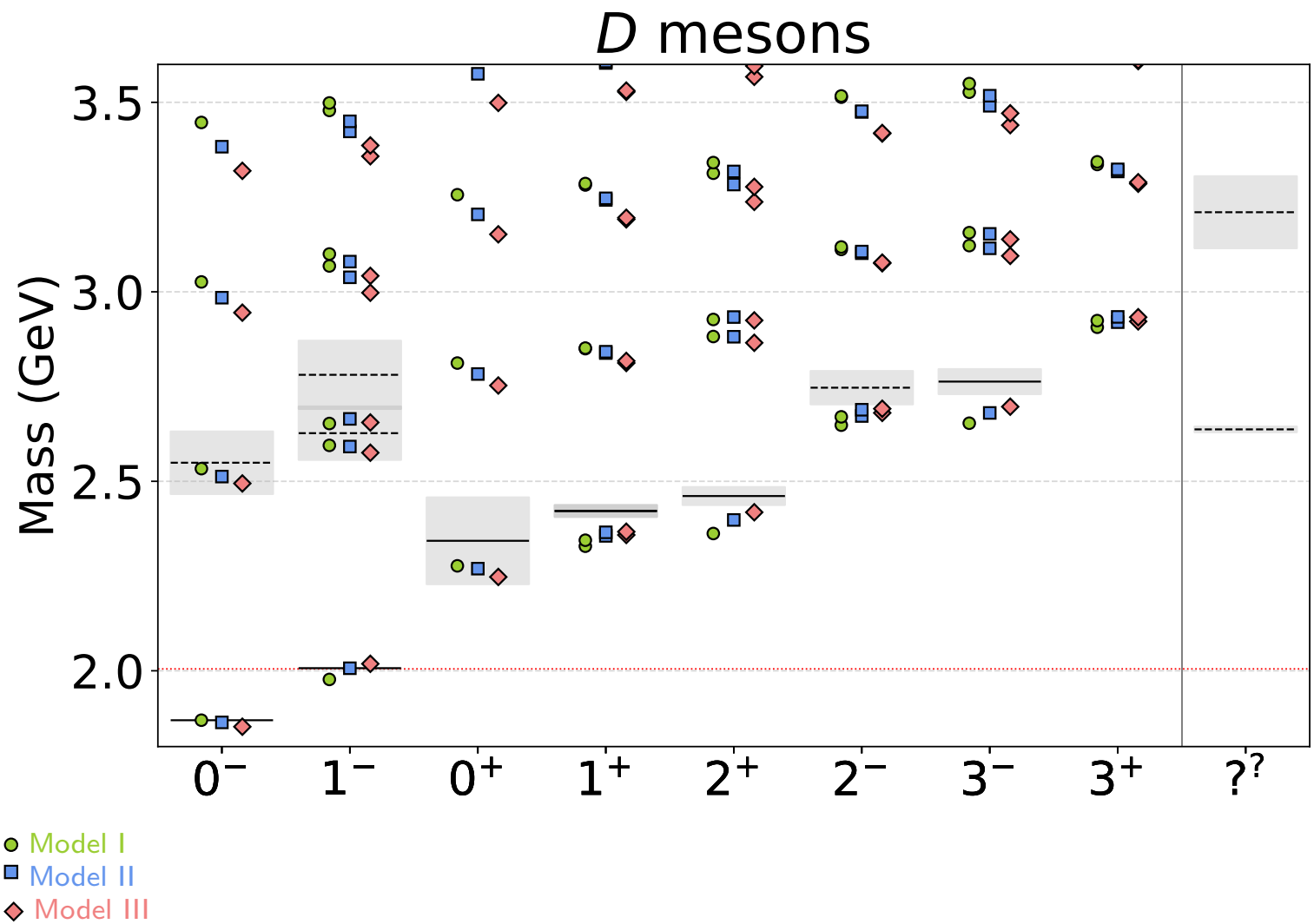
PRELIMINARY RESULTS: $c\bar{s}$

CHARMED, STRANGE ($C = \pm 1$, $S = \pm 1$)	
• $D_s^{\pm}$	$0(0^-)$
• $D_s^{*\pm}$	$0(1^-)$
• $D_{s0}^*(2317)^{\pm}$	$0(0^+)$
• $D_{s1}(2460)^{\pm}$	$0(1^+)$
• $D_{s1}(2536)^{\pm}$	$0(1^+)$
• $D_{s2}^*(2573)$	$0(2^+)$
$D_{s0}(2590)^+$	$0(0^-)$
• $D_{s1}^*(2700)^{\pm}$	$0(1^-)$
$D_{s1}^*(2860)^{\pm}$	$0(1^-)$
• $D_{s3}^*(2860)^{\pm}$	$0(3^-)$
$D_{sJ}(3040)^{\pm}$	$0(?^?)$



PRELIMINARY RESULTS: $c\bar{q}$

CHARMED ($C = \pm 1$)	
• $D^{\pm}$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^{\pm}$	$1/2(1^-)$
• $D_0^*(2300)$	$1/2(0^+)$
• $D_1(2420)$	$1/2(1^+)$
• $D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)$	$1/2(2^+)$
$D_0(2550)^0$	$1/2(0^-)$
$D_1^*(2600)^0$	$1/2(1^-)$
$D^*(2640)^{\pm}$	$1/2(?^?)$
$D_2(2740)^0$	$1/2(2^-)$
• $D_3^*(2750)$	$1/2(3^-)$
$D_1^*(2760)^0$	$1/2(1^-)$
$D(3000)^0$	$1/2(?^?)$



CONCLUSIONS

SUMMARY

- We have extended the one-channel CST formalism to **arbitrary** J^P
- We generalized the OGE for a **running coupling** $\alpha_s(Q^2)$: improves the model; removes constant interaction
- With only few global parameters we obtain **good description** of the heavy and heavy-light meson spectrum for $J^P = 0^{\pm}, 1^{\pm}, 2^{\pm}, 3^{-}$

OUTLOOK

- Include **dynamical** quark mass (mass function) from quark self-interactions
- Calculate **decay constants** (requires different regularization of OGE)
- Solve charge-conjugation invariant **four-channel** CST equation for **light mesons**